Research Article

Towards an Advanced New Emerging Method of Determination of Mohr–Coulomb Parameters of Soils from at the Oedometric Test: Case Study-Lateritic Soils of Cameroon

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In this paper, we present a newly emerging approach for the use of the oedometer test for the determination of some important mechanical properties of an elastic-plastic soil using the Mohr–Coulomb yield criterion and an associated flow rule. Analysis of the oedometric test is performed in low strains. The identification of constitutive parameters is carried out by means of the Newton optimization procedure used to solve inverse problems. The consolidation test problem is solved using new equations proposed in this paper. The solution of the problem according to the equations developed in this paper was validated by the numerical modeling of an oedometric test by following the requirements and the experimental protocol from the Plaxis Finite Element code. The method has been validated through Plaxis’ software, making it reliable. The sensitivity study of the model with respect to its parameters was carried out. Laboratory results analysis over nine samples of the southern Cameroon soils are compared to those computed analytically. The major parameters are Young’s modulus, Poisson’s ratio, the internal friction angle, the cohesion, and the dilatancy angle. The mechanical properties of those soils obtained from the proposed model are very close to, or almost identical in some cases with values obtained in the laboratory. The practical validation and the application of our method have been successfully applied on the lateritic soils of Cameroon. The compressibility curves reconstituted from the proposed theoretical solution are superimposed on those resulting from laboratory tests with low dispersions.

1. Introduction

The soils behavior law are built from laboratory tests. The basis tests for these laws are the oedometric and the triaxial test [1]. Given the complexity of the behavior of soils, the difficulties encountered to determine their properties and the technical and professional requirements (deadlines, appropriate equipment, costs, etc.), engineers often use a perfectly plastic elastic model based on the Mohr–Coulomb criterion for the analysis of geotechnical structures [2].

The consolidation test makes it possible to simulate the stress that the ground has undergone in its history and that it will undergo under the loading of the work to be carried. The principle of this test is to measure the settlement \( W \) of a cylindrical soil test and subjected to a uniaxial compression \( \sigma_z \) increasing by preventing any lateral deformation \( \varepsilon_x = \varepsilon_y = 0 \). The two lower and upper faces of the sample are drained. Generally, the test is performed on saturated samples. During the test, we measured for each bearing the stress \( \sigma_z \) and the settlement as a function of time. For the whole of the test, we plot the oedometric curve, volume variation, or more specifically variation of the voids ratio index \( e \) as a function of the decimal logarithm of the stress. By simplifying, one obtains a first straight line of low slope and a second right of much higher slope. The intersection of the two lines is the maximum preconsolidation stress \( \sigma^p \) that the soil has known in its history. This test allows to know the initial state of the soil. The slopes of the two straight lines \( C_g \)
and \( C_\epsilon \) account for the compressibility of the soil, respectively, in the overconsolidated domain and in the normally consolidated domain [3–5]. The entire test protocol is described in French Standard Institution XP 94-090-1 of December 1997.

The Mohr–Coulomb model is characterized by five parameters: \( E, \nu, \Phi, \Psi \), and \( c \). These parameters are usually obtained on a triaxial test in Consolidation Isotropic Drained (CID) or undrained (CIU) on a soil sample by exploiting the experiment curves obtained as a result of this test. Young’s modulus \( E \) is extracted from the graph \( \varepsilon / \varepsilon_c \); (i.e: \( q = \sigma_{oo} - \sigma_{oo}\); deviatoric stress of triaxial test). It adjusts the slope of the elastic part of the curve. For Finite Element Software applications, \( E_{50} \) is generally considered. Poisson’s ratio \( \nu \) is determined from the CID or CIU tests, by calculating the initial slope in the graph \( (\varepsilon, \varepsilon_c) \). Indeed, \( \varepsilon / \varepsilon_c = 1 - 2\nu \). The value of dilatancy is determined from these tests, by calculating the slope of the ascending curve in the graph \( (\varepsilon, \varepsilon_c) \). Indeed, \( \varepsilon / \varepsilon_c = 2 \sin \Psi / 1 - \sin \Psi \) [2]. The internal friction angle \( \Phi \) and shear strength \( c \) are deduced from the graph \( (p, q) \).

In this paper, we develop a method for determining these same parameters from an oedometric test on a various soil (mixture of sand and clay or lateritic soil) sample meeting the Mohr–Coulomb criterion, with associated flow rule \( (\Phi = \Psi) \). Such a method has not been developed anywhere in the world. The motivations for this approach come from the fact that in most developing countries, Geotechnicals laboratories and Engineering’s offices often do not have triaxial equipment for laboratory testing under real stress condition; they are generally limited to performing direct shear tests. Knowing the value of a triaxial test in a drained or undrained condition and information on a test specimen of soil subject to this test, we have initiated research in this area. That of exploiting the oedometric test (all geotechnical laboratories have an oedometric cell) to quickly determine by the inverse method the parameters of the Mohr–Coulomb model of a soil sample under drained conditions. The approach leading to this method is developed as follows.

2. Empirical Relations between Current Geotechnical Parameter

Different analytical and empirical relations have been proposed in the literature in order to link the geotechnical characteristics of soils. Authors have shown that these relations depend both on nature and stress state of the soil, although this last point is less considered in the proposed relations. In the following, different approaches proposed in the literature are presented in order to obtain Young’s Modulus of the soil as a function of the dynamic cone resistance. One drawback of such relations is that it is not always defined at which deformation level Young’s Modulus is obtained. Chua and Fernandes [6, 7] proposed an analytical solution to calculate the elastic modulus of a medium from a 1-D model for penetration analysis of a rigid projectile into an ideally locking material. A relation between the penetration index and the elastic modulus is therefore obtained for different soil natures. The results from Chua and Fernandes [6, 7] are transposed to the Panda dynamic cone penetrometer by Haddani et al. [7, 8], which are presented in Table 1. Bellotti et al. [6, 9] presented relations for the secant Young Modulus obtained from a calibration chamber for an average axial strain of 0.1.

\[
E = 4q_d \forall q_d \leq 10 \text{[MPa]},
\]

\[
E = 2q_d + 20\nu 10 \leq q_d \leq 50 \text{[MPa]},
\]

Correlations between the dynamic cone resistance \( q_d \) and the California Bearing Ratio (CBR) have been extensively proposed in the literature. Some of these correlations are presented in Table 2. According to Heukelom [7, 18], the CBR and Young’s Modulus can be related by the following simple linear relation:

\[
E = A^* \text{CBR},
\]

where \( A \) is obtained experimentally and depends on both soil nature and the mean effective stress. Mohammed et al. [7, 19] evaluated both in situ static cone resistance of different cohesive soils and Young’s Modulus by laboratory tests. The following relation has been proposed:

\[
E = aq_c^n + bf_c + ca_n + d\rho + e,
\]

where \( E \) is Young’s Modulus, \( q_c \) is the static cone resistance, \( f_c \) is the frictional resistance, \( a_n \) is the natural water content, \( \rho \) is the dry volumetric mass density, \( n \) is an integer (1, 2, 3), and \( a, b, c, d, \text{and } e \) are regression constants. Simpler direct relations have also been proposed by [7, 20, 21], as reported by [7, 22, 23], according to the following expression:

\[
E = B^* q_c.
\]

Cassan [7, 23] proposed to link the static cone resistance \( q_c \) and the dynamic cone resistance \( q_d \) for different soils by a linear relation (equation (5)). Values of \( C \) for different materials are summarized in Table 3.

\[
q_c = C^* q_d.
\]

Empirical relations have also been proposed between the oedometric deformation modulus \( E_{\text{eod}} \) and the static cone resistance \( q_c \). Buisman [7, 24] was the first to propose a linear relation (equation (6)) between \( E_{\text{eod}} \) and \( q_c \). The value of \( a \) (also called Buisman coefficient) for different soil natures was largely studied by authors. The recommended values are summarized in Table 4.

\[
E_{\text{eod}} = a^* q_c.
\]

The oedometric deformation modulus \( E_{\text{eod}} \) and Young’s Modulus \( E \) are well defined on a linear elastic material by the following relation:

\[
E = E_{\text{eod}} \frac{(1 + \nu)(1 - 2\nu)}{(1 - \nu)}.
\]

In order different direct relations that vary according to the sites proposed in the literature (Table 5) between compressibility index \( C_c \) and liquid limit of the soil [25]. The unit weight of soil \( \gamma \) is determined by the following formula
from the correlation between cone resistance $q_c$ and the unit weight of soil $\gamma_w$ [27–30]:

$$\frac{v}{\gamma_w} = 0.27(\log R_f) + 0.36 \left( \log \frac{q_c}{\rho_a} \right) + 1.236, \quad (8)$$

where $\gamma_w$ unit weight of water (10 kN/m$^3$), $\rho_a$ atmospheric pressure (100 kPa); $R_f = f_s / q_c$, 100%, where $R_f$ friction ratio between skin friction and cone resistance.

For granular soils, the angle of friction, $\varphi$ drops with the cone resistance, $q_c$ value [29], and the Standard Penetration
Test $N_{60}$ value [31]. Several correlations can be identified, among corresponding to sand [29, 31]:

$$
\phi' = \tan^{-1} \left[ 0.1 + 0.38 \log \left( \frac{\sigma'_{v0}}{\sigma_{v0}} \right) \right],
$$

$$
\phi' = \tan^{-1} \left[ \frac{N_{60}}{12.2 + 20.3 \left( \sigma'_{v0}/p_n \right)^{0.34}} \right],
$$

$$
\phi' = 54 - 27.6034 e^{-0.014 (N_1)_{60}} \quad ,
$$

$$
\phi' = 27.1 + 0.3 (N_1)_{60} - 0.00054 (N_1)_{60}^2 \quad .
$$

For a given clay, the angle of friction, $\phi$ drops with the plasticity index, $I_p$ value. Several correlations can be identified, among which let us cite Fahri’s [31] corresponding to French clays:

$$
\tan \phi' = 0.21 + \frac{8}{I_p + 6}. \quad (10)
$$

Based on the plasticity Index, for normally consolidated clays: on any given sites following correlation can be identified [31]:

$$
c_u = (0.11 + 0.0037 I_p) \sigma'_{v0}. \quad (11)
$$

Based on Menard’s Pressurometer test, the main correlations between the Standard net limit pressure corrected from the total horizontal stress acting in the soil at test elevation ($p^*_{t}$) and the undrained shear strength $c_u$ are given as follows [32]:

$$
c_u = \frac{p^*_{t}}{5.5} \quad \text{for soft soils } (p^*_{t} < 50 \text{kpa}),
$$

$$
c_u = \frac{p^*_{t}}{10} + 25f \quad \text{for hardening soils } (p^*_{t} > 50 \text{kPa}),
$$

$$
c_u = 0.67 \left( p^*_{t} \right)^{0.75} \quad \text{for all soils}. \quad (12)
$$

The main correlations between Young’s modulus $E$ and Normalized Menard pressurometer modulus $E_M$ are given (Figure 1) as follows [33]:

$$
k = \frac{E}{E_M}. \quad (13)
$$

The complete methodology adopted in this article is presented in the following section.

3. Methodology

The method consists in determining the analytical solution of a soil sample subjected to an oedometric test (with a soil satisfying the Mohr–Coulomb criterion, with associated flow rule). For a given soil sample, the game settings from Mohr–Coulomb ($E, \nu, \Phi, \Psi, c$) is injected into the solution to obtain a compressibility curve (volume variation curve or more specifically, variation of the void ratio index $e$ as a function of the logarithm of the stress) theoretic equivalent to the compressibility curve obtained following an oedometric test. The two curves are superimposed by minimizing at maximum the gap between the points of curve experimental and theoretical proposed. Such an approach is commonly used in Civil Engineering. This is the case of bituminous materials [34–36], the case of determination of the shear wave velocity in saturated soft clay using measurements of the liquid limit, plastic limit, and natural water content [37], the case of the solution of the cone penetration test using triaxial parameters [38–40], the case of the solution of the expansion of a cylindrical cavity such as on a pressure meter test [41–50], and the case of analytical solution for estimation of undrained shear strength of soft soil obtained by cylinder vertical penetration [51–54].

3.1. Resolution of the Inverse Problem. The determination of the parameter of the soil behavior model from the oedometric test consists in the resolution of the inverse problem following: find a parameter set (s) that minimizes the difference between the measurements of a real test and the results obtained by integrating the behavior model expressed by the following formula (14) [42, 55–57]:

$$
s = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(\delta_{ci} - \delta_{mi})^2}, \quad (14)
$$

$\delta_{mi}$ et $\delta_{ci}$, respectively, denote the laboratory measurements and the integration result of the model for the number $n$ stress level $\Delta \sigma_i$. The search for the minimum requires the use of an optimization technique. Newton’s method is used in this paper. The implementation of the said method requires solving the problem of a soil sample governed by the Mohr–Coulomb criterion, under the loading of an oedometer test. In this paper, the associated flow rule is used. The complete resolution of this problem is developed in the following section.

3.2. Problem Description. A cylindrical specimen of a material which follows the Mohr–Coulomb model is subjected to one-dimensional compression. This cylindrical specimen is pressed on one surface while all the other surfaces are confined such that they have free-slip boundary conditions (Figure 2). This oedometer test provides a direct test in this regard because the postyielding stress state resides on one of the edges of the Mohr–Coulomb yield surface.

All of the processes involved in loading and unloading a natural soil deposit can be simulated in the laboratory. An oedometer test subjects a cylindrical sample of soil to conditions similar to the deposit of silt. Basically, the oedometer consists of a very stiff steel ring enclosing the soil sample as depicted in Figure 2. The sample is loaded vertically through the loading cap by the applied load. The applied load is increased in increments and then decreased again to simulate the processes of overburden accumulation and removal that occurred in the natural soil deposit. In most oedometer tests we would not attempt to measure the horizontal stress applied to the sample by the rigid ring, but it would be possible to do so if we wished and
we could determine $K_0$ (coefficient of earth pressure at rest) directly. Results from oedometer tests are usually presented in terms of the vertical stress and the void ratio index of the soil. In most geomechanics texts the void ratio index is denoted by $e$, but we have used $e$ earlier to represent volumetric strain. Typical data from an oedometer test are illustrated in Figure 3. The graph shows how the void ratio changes as the applied load increases. The load is represented by the logarithm of the vertical effective stress. It is usually assumed that uniform vertical stress is applied to the sample, although this may not be exactly true. A more correct assessment of the problem would suggest that we should use a mixed boundary condition on all the sample surfaces.

The uniform stress approximation will be sufficiently accurate in an average sense. There are three segments of more or less linear response in Figure 3. In the early stages of loading, the graph is linear and this portion may be interpreted as primarily an elastic response (region A). In this region, the soil particle structure is basically unchanged from its original state and little if any fragmentation and rearrangement have occurred [58]. The void ratio is decreasing,
but this is primarily because of the elastic deformation of the particles themselves. The first important particle fracture corresponds to the point at which the graph becomes nonlinear (region B). The graph begins to steepen smoothly at this point as more fractures and particle rearrangement occurs. The void ratio decreases more quickly as these irreversible processes take place. A second region of linear response then appears (region C). In this region, the soil structure continues to evolve in the sense that fracture and fragmentation lead to further particle rearrangement. The distribution of particle sizes is changing as more and more particles are broken.

Finally, the applied load is reduced and unloading commences. Once again we find an approximately linear response, which we now identify as the elastic rebound of the compressed soil skeleton (region D). If the applied load is reduced to zero the void ratio index is permanently reduced and some degree of volumetric strain is permanently locked into the particle structure.

We can clearly see the region of inelastic response in Figure 3. It begins when the first particle fractures occur and the loading curve begins to bend downward and it ends when unloading commences. One of the main challenges geomechanics poses to the theory of plasticity is how to model this type of inelastic behaviour. An interesting point about the oedometer test response or the response of natural soil deposits described earlier, is that while there are significant shear stresses developed in the soil during loading, the inelastic behaviour we observe is not directly related to shearing. In fact, we could produce a similar inelastic response in a test where the soil is subjected to a purely isotropic stress. The classical theories of plasticity relate solely to inelastic response caused by shear [59]. Some important changes are required to develop a theory that encompasses behavior similar to that shown in Figure 3.

3.3. Analytics Solutions-Loading and Unloading Phase

3.3.1. Elastic Solution. The vertical elastic deformation of a soil column of height $H$, a function of the time and the loading speed $V$ in an oedometer mold is developed as follows:

$$\Delta \varepsilon_x = \frac{V \Delta t}{H - V t},$$
$$\Delta \varepsilon_y = \Delta \varepsilon_z = 0,$$
$$\Delta \sigma_z = (K + 2G) \Delta \varepsilon_z,$$
$$\Delta \sigma_y = K \Delta \varepsilon_z,$$
$$\Delta \sigma_z = \sigma_y.$$

(15)

3.3.2. Plastic Solution. Yield criteria Mohr-Coulomb plasticity are defined as follows:

$$F^1 = \sigma_z - \sigma_y N_\psi + 2c \sqrt{N_\psi},$$
$$F^2 = \sigma_z - \sigma_x N_\psi + 2c \sqrt{N_\psi}. $$

(16)

During plastic flow, the strain increments are composed of elastic and plastic parts and we have the following equation:

$$\Delta \varepsilon_x = \Delta \varepsilon_x^e + \Delta \varepsilon_x^p,$$
$$\Delta \varepsilon_y = \Delta \varepsilon_y^e + \Delta \varepsilon_y^p,$$
$$\Delta \varepsilon_z = \Delta \varepsilon_z^e + \Delta \varepsilon_z^p.$$ 

(17)

Using the boundary conditions, we may write:

$$\Delta \varepsilon_x = \frac{V \Delta t}{H - V t} - \Delta \varepsilon_x^p,$$
$$\Delta \varepsilon_y = -\Delta \varepsilon_y^p,$$
$$\Delta \varepsilon_z = -\Delta \varepsilon_z^p.$$ 

(18)

The flow rule for plastic flow along the edge of the Mohr–Coulomb criterion corresponding to $\sigma_y = \sigma_z$ has the form:

$$\Delta \varepsilon_x^p = \lambda_1 \frac{\partial G^1}{\partial \sigma_x} + \lambda_2 \frac{\partial G^2}{\partial \sigma_x},$$
$$\Delta \varepsilon_y^p = \lambda_1 \frac{\partial G^1}{\partial \sigma_y} + \lambda_2 \frac{\partial G^2}{\partial \sigma_y},$$
$$\Delta \varepsilon_z^p = \lambda_1 \frac{\partial G^1}{\partial \sigma_z} + \lambda_2 \frac{\partial G^2}{\partial \sigma_z},$$

(19)

where $G^1$ and $G^2$ are the potential functions corresponding to $F^1$ and $F^2$:

$$G^1 = \sigma_z - \sigma_y N_\psi,$$
$$G^2 = \sigma_z - \sigma_x N_\psi.$$ 

(20)

After substitution,

$$\Delta \varepsilon_x^p = \lambda_1 + \lambda_2,$$
$$\Delta \varepsilon_y^p = -\lambda_1 N_\psi,$$
$$\Delta \varepsilon_z^p = -\lambda_2 N_\psi,$$

(21)

where $N_\psi = 1 + \sin \phi/1 - \sin \phi$ and $N_\psi = 1 + \sin \psi/1 - \sin \psi$, (choice of drained or undrained parameters; using $\Phi' = \phi'$, for drained condition or $\Phi = \psi$, for undrained condition).

By symmetry, we know $\lambda_1 = \lambda_2$:

$$\Delta \varepsilon_x^p = 2\lambda_1,$$
$$\Delta \varepsilon_y^p = -\lambda_1 N_\psi,$$
$$\Delta \varepsilon_z^p = -\lambda_1 N_\psi.$$ 

(22)

The stress increments are given as follows:
Deformation in the elastic domain is described as follows:

\[ \Delta \sigma_z = (K + 2G)\Delta \varepsilon_z + 2K\Delta \varepsilon_y, \]
\[ \Delta \sigma_y = (K + 2G)\Delta \varepsilon_y + K(\Delta \varepsilon_z + \Delta \varepsilon_y), \]
\[ \Delta \varepsilon_z = \Delta \sigma_y, \]
\[ \Delta \sigma_z = (K + 2G)\left( \frac{V \Delta t}{H - Vt} - 2\lambda_1 \right) + 2K\lambda_1 N_\varphi, \]
\[ \Delta \sigma_y = (K + 2G)\lambda_1 N_\varphi + K\left( \frac{V \Delta t}{H - Vt} - 2\lambda_1 + \lambda_1 N_\varphi \right). \]

During plastic flow, the consistency condition that \( \Delta F^1 = 0 \) should be satisfied, which takes the following form:

\[ \Delta \sigma_z - \Delta \sigma_y, N_\varphi = 0. \] (24)

Solving for \( \lambda_1 \), we get the following equation:

\[ \lambda_1 = \frac{k(2K + 2G) - 2K(N_\varphi + N_\psi) + 2(K + G)N_\varphi N_\psi}{2(k + 2G) - 2K(N_\varphi + N_\psi) + 2(K + G)N_\varphi N_\psi}. \] (25)

3.4. Direct Oedometer Test Solution from the Applied Vertical Stress. In most soil laboratories, the oedometer test is carried out by vertical load bearing (loading-unloading-reloading). In this case, the vertical deformation of the soil in the elastic zone is equivalent to the following expression [60]:

\[ \Delta \varepsilon_z = \frac{(1 + \nu)(1 - 2\nu)}{E(1 - \nu)}\Delta \sigma_z. \] (26)

In the plastic zone, the vertical deformation of the specimen soil test which follows Mohr–Coulomb model has expressed:

\[ \Delta \varepsilon_z = \left[ 1 - 4\nu \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) + 2(1 - \nu) \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 \right] \Delta \sigma_z \frac{E}{2(1 - \nu)\sigma_c \cos \phi - 2\nu - \sin \phi}. \] (27)

The limit of elasticity of the soil body in compression corresponds to the following expression:

\[ P_c = \frac{2(1 - \nu)\sigma_c \cos \phi}{1 - 2\nu - \sin \phi}. \] (28)

According to the formula (26) and (28); the equation (29), makes finding the goind their maximum elastic deformation in compression in the ground. This maximum deformation in the elastic domain is described as follows:

\[ \Delta \varepsilon_{z_{\text{max}}} = \frac{(1 + \nu)(1 - 2\nu)}{E(1 - \nu)}P_c. \] (29)

The slopes of the straight lines of the vertical stress curve (\( \Delta \sigma_z \)) versus vertical deformation (\( \Delta \varepsilon_z \)) of the soil in the elastic and plastic areas are expressed by the respective following expressions: \( E(1 - \nu)/(1 + \nu)(1 - 2\nu) \) and \( E/(1 - 4\nu(1 - \sin \phi/1 + \sin \phi) + 2(1 - \nu). (1 - \sin \phi/1 + \sin \phi)^2) \)

Using the formula (15), (18), (22), (26), (27), and (29), the settlement \( W \) (elastic and plastic domain) of soil body at \( z = H \), is obtained by applying the following expression:

\[ W = \Delta \varepsilon_z H. \] (30)

For the unloading phase, the residual settlement \( W_r \) of soil body is expressed by the relationships (31).

\[ W_r = W - W_u. \] (31)

4. Verification and Validation of Elastic-Plastic Solution of Oedometer Test

We compare the approximate solutions from the PLAXIS Finite Element code [61] with analytic solutions to a simple problem of the oedometer test at loading, including plasticity. The following parameters are used for the oedometer test for this verification: Young’s modulus of soil sample: 500 MPa; Poisson’s ratio: 0.25; friction angle: \( \phi \); dilatancy angle: \( \phi \); shear strength (cohesion): 0.1 MPa; \( \Delta t \): incremental time (s); \( t \): Time of loading/unloading/reloading (s); \( V \): Loading speed (cm/s); \( H \): Initial height of specimen or thickness of soil (cm); \( W_r \): settlement of the soil body for the unloading stress \( i \); \( \sigma_{i - u} \).

4.1. Analysis of the Sensitivity. The analysis of the sensitivity of the method with respect to its parameters makes it possible to determine the parameters to be favored in the optimization process and the influence domain of each of them. Figures 6(a)–6(d), illustrate the sensitivity of the simulation of the oedometer test compared to Mohr–Coulomb parameters.

It is found through Figure 6(a), that disruption of Young’s modulus strongly affects the behavior of the soil (curve \( \sigma_{z_c}, \varepsilon_z \)). Vertical deformations increase as Young’s modulus decreases; on the other hand, the soil enters plasticity rapidly when Young’s modulus increases while keeping the other parameters. There is also a high degree of sensitivity in the simulations with respect to Poisson’s ratio, friction angle, and soil dilatancy angle (Figures 6(b) and
6(c)). Whatever the behavior domain (elastic or plastic), the deformations increase rapidly with the increase of the values of these parameters ($\phi$, $\Psi$, $\gamma$). Figure 6(d) shows that the curve is insensitive to the disruption of cohesion. In this Figure 6(d), the three curves representing the cohesion effect of the model for respective values of 50, 100, and 150 kPa are superimposed. It is also observed that the compressive yield strength is reached quickly for low values of the cohesion and the slope of the curves in the elastic range is almost the same.

5. Application of the Method on the Lateritic Soil of Cameroon

The methodology outlined is used to determine the mechanical properties of Cameroonian lateritic clays and sandy clay (Figures 7 and 8). The theoretical solution developed in the previous section is used, the curves of compressibility of soil are thus superimposed by comparing theoretical Mohr–Coulomb parameters and those from triaxial and oedometer tests on several nine samples of soil. The load
applied theoretical on soil samples is the same as that which was applied in the laboratory when performing these oedometric tests. The theoretical curve of the behavior of each soil specimen (curve $\sigma_z$, $\varepsilon_z$) is then transformed into a compressibility curve by calculating the void ratio in the specimen under each stress. The voids ratio $\varepsilon_i$ under each stress $\sigma_{z,i}$ is obtained by applying the following formula:

$$\varepsilon_i = \frac{H(1 - \varepsilon_{z,i}) - (W_d/g_i, s)}{(W_d/g_i, s)} ,$$  \hspace{1cm} (32)

where $H$ is the initial height of specimen, $s$ is the inner area of the cylindrical mold (specimen area), $g_i$ is the gravity specific, and $W_d$ is the weight of the dry soil slice constituting the test specimen.

In a conventional triaxial test, the following expressions are used to determine the parameters of Mohr–Coulomb: $\varepsilon_{v,i} = (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$: volumetric strain of triaxial test; $\varepsilon_{zz,i}$: axial strain of triaxial test; $q = \sigma_{zz} - \sigma_{yy}$: deviatoric stress of triaxial test (MPa) and $p' = (\sigma'_{zz} + \sigma'_{xx} + \sigma'_{yy})/3$: effective mean stress of triaxial test (MPa). Figure 9 clarifies the verification process for our method step by step.

Table 6 illustrates the laboratory results on nine soil samples (engineering properties of lateritic soil tested). The geotechnical properties of these nine application samples are derived from the South of Cameroon. The geological substratum of the plateau of South Cameroon consists essentially of metamorphic and plutonic rocks of the Precambrian age, dominated by calc-alkaline granites (Figure 7). The values of some geotechnical properties (determined according to the French Standard Institution) of this soil are gathered in Table 6. It is a reddish lateritic soil, classified as a fine grained soil (A-7-5 and A-2-7) in the AASHTO (American Association of State Highway and Transportation Officials) soil classification system [63], and sandy clay or silt (A2 to B6) in the GTR (French "Guide des Terrassements Routiers") soil classification [64]. Table 7 and Figures 10 and 11, illustrate the laboratory results on soil samples (triaxial in Consolidation Isotropic Drained (CID) and oedometer) compared to those theoretically obtained from the method developed in this paper. Nine soils body (lateritic clay and sandy clay) are tested in a conventional laboratory tests (oedometer and triaxial) and the integrating the soil behavior model, the difference between the parameter of the laboratory test and the results obtained by integrating the behavior model expressed by formula (14) using optimization technique is small ($s < 0.2$ for all nine case).

This study shows that the optimization methodology operates correctly to determine the four remaining parameters of the Mohr–Coulomb model (because here
Figure 8: Soil type—reddish lateritic clays.

Figure 7: Geological map of Cameroon and site location.
Figure 9: Iterative procedure used and verification process for our method step by step.

Table 6: Engineering properties $p_l^*$ of nine soils application.

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<td>A1</td>
<td>&lt;2 mm (%) 85</td>
<td>&lt;80 μm (%) 58</td>
<td>&lt;2 μm (%) 40</td>
<td>26.9</td>
<td>0.57</td>
<td>23.1</td>
<td>68 35 33 38 26.7 0.625 125 0.104 0.0053</td>
<td>A-2-7 (15) A3</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>91.2 49 39.7 26.5</td>
<td>0.6</td>
<td>26.7 64 34 30 45 29.2 0.65 100 0.064 0.0048</td>
<td>A-7-5 (17) A3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>95 56 56 26.1</td>
<td>0.51</td>
<td>20.9 61 36 25 142 15.1 0.55 110 0.144 0.005</td>
<td>A-7-5 (18) A2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>47.6 34 21 26.5</td>
<td>0.5</td>
<td>25 66 35 31 29 31.2 0.625 125 0.19 0.0073 A-7-5 (17) A3</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>A5</td>
<td>77.1 64 39.2 27</td>
<td>0.7</td>
<td>29 53 28 25 44 27.2 0.625 91 0.053 0.0053 A-7-5 (14) A2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>90.7 63 38.5 26.9</td>
<td>0.68</td>
<td>27 62 32 30 77 23.7 0.722 87.5 0.056 0.0096 A-7-5 (17) A3</td>
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<td></td>
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<tr>
<td>A7</td>
<td>80.2 63 38.7 28</td>
<td>0.71</td>
<td>30.1 65 35 30 36 30.9 0.695 32.5 0.079 0.0102 A-7-5 (17) A3</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>A8</td>
<td>37.2 33 22.2 28.5</td>
<td>0.37</td>
<td>17.7 65 30 35 28 32 0.328 67.5 0.185 0.0063 A-2-7 (4) B6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A9</td>
<td>54.5 32 19.8 28.3</td>
<td>0.41</td>
<td>23.3 58 27 31 72 28.8 0.365 99 0.086 0.0059 A-2-7 (4) B6</td>
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</table>


Table 7: Comparison of the parameters of soil obtained by conventional laboratory triaxial and oedometric tests and those obtained by our theoretical method.

<table>
<thead>
<tr>
<th>Soil body</th>
<th>Parameter</th>
<th>Test value</th>
<th>M-C solution</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>$\sigma_p'$ (bar)</td>
<td>1.25</td>
<td>1.3</td>
<td>-3.85</td>
</tr>
<tr>
<td></td>
<td>$C_c$</td>
<td>0.104</td>
<td>0.128</td>
<td>-18.5</td>
</tr>
<tr>
<td></td>
<td>$C_g$</td>
<td>0.0053</td>
<td>0.007</td>
<td>-18.5</td>
</tr>
<tr>
<td></td>
<td>$E$ (MPa)</td>
<td>6.59</td>
<td>7</td>
<td>-5.82</td>
</tr>
<tr>
<td></td>
<td>$\nu$</td>
<td>—</td>
<td>0.42</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$\epsilon'$ (kPa)</td>
<td>38</td>
<td>38</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\Phi' = \Psi'$ (°)</td>
<td>26.7</td>
<td>28</td>
<td>-4.64</td>
</tr>
<tr>
<td>A2</td>
<td>$\sigma_p'$ (bar)</td>
<td>1.25</td>
<td>1.3</td>
<td>-3.85</td>
</tr>
<tr>
<td></td>
<td>$C_c$</td>
<td>0.064</td>
<td>0.0075</td>
<td>-14.7</td>
</tr>
<tr>
<td></td>
<td>$C_g$</td>
<td>0.0048</td>
<td>0.005</td>
<td>-6.77</td>
</tr>
<tr>
<td></td>
<td>$E$ (MPa)</td>
<td>10.92</td>
<td>11</td>
<td>-0.71</td>
</tr>
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</table>
\[ \Phi' = \Psi' \text{ for the associated flow rule). Figures 10 and 11 show the laboratory and theoretical compressibility curves of the different samples tested. The theoretical solution is very close to the experimental solution, and the differences between these different curves are relatively small. Analysis of the results of Table 7 and Figure 11 shows the essential parameters of the Mohr–Coulomb model derived from the theoretical solution of the oedometer test are almost identical to those obtained triaxial. The values of internal friction angle given by the laboratory test for the samples A5, A6, and A7, are small different from the values obtained for the other samples,}

<table>
<thead>
<tr>
<th>Soil body</th>
<th>Parameter</th>
<th>Test value</th>
<th>M-C solution</th>
<th>Error (%)</th>
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<td>( \nu )</td>
<td>—</td>
<td>0.42</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( c' ) (kPa)</td>
<td>45</td>
<td>45</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( \Phi' = \Psi' ) (°)</td>
<td>29.2</td>
<td>29.2</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( \sigma_p' ) (bar)</td>
<td>1.1</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( C_c )</td>
<td>0.144</td>
<td>0.155</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( C_g )</td>
<td>0.005</td>
<td>0.006</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( A_3 )</td>
<td>( E ) (MPa)</td>
<td>8.5</td>
<td>8</td>
<td>6.25</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
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<tr>
<td>( c' ) (kPa)</td>
<td>142</td>
<td>132</td>
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<td>7.58</td>
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<td>( \Phi' = \Psi' ) (°)</td>
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<tr>
<td>( \sigma_p' ) (bar)</td>
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<td>1.25</td>
<td>0</td>
<td>0</td>
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<td>0.19</td>
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<td>0</td>
</tr>
<tr>
<td>( C_g )</td>
<td>0.0073</td>
<td>0.007</td>
<td>1</td>
<td>1.39</td>
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<td>( A_4 )</td>
<td>( E ) (MPa)</td>
<td>2.31</td>
<td>2.5</td>
<td>7.68</td>
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<td>( \nu )</td>
<td>—</td>
<td>0.44</td>
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<td>—</td>
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<td>( c' ) (kPa)</td>
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<td>29</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>31.1</td>
<td>32</td>
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<tr>
<td>( \sigma_p' ) (bar)</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( C_c )</td>
<td>0.053</td>
<td>0.058</td>
<td>1</td>
<td>1.39</td>
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<tr>
<td>( C_g )</td>
<td>0.005</td>
<td>0.005</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( A_5 )</td>
<td>( E ) (MPa)</td>
<td>6.76</td>
<td>7</td>
<td>3.48</td>
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<td>—</td>
<td>0.47</td>
<td>—</td>
<td>—</td>
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<tr>
<td>( c' ) (kPa)</td>
<td>44</td>
<td>44</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( \Phi' = \Psi' ) (°)</td>
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<td>30</td>
<td>—9.33</td>
<td>9.33</td>
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<tr>
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<td>0.9</td>
<td>1</td>
<td>—9</td>
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<tr>
<td>( C_c )</td>
<td>0.056</td>
<td>0.05</td>
<td>12</td>
<td>12</td>
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<tr>
<td>( C_g )</td>
<td>0.0065</td>
<td>0.006</td>
<td>18.18</td>
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<td>( A_6 )</td>
<td>( E ) (MPa)</td>
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<td>6</td>
<td>0</td>
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<td>—</td>
<td>0.45</td>
<td>—</td>
<td>—</td>
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<td>( c' ) (kPa)</td>
<td>77</td>
<td>77</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( \Phi' = \Psi' ) (°)</td>
<td>23.2</td>
<td>28</td>
<td>—17.14</td>
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<td>( \sigma_p' ) (bar)</td>
<td>0.325</td>
<td>0.36</td>
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<td>( C_c )</td>
<td>0.099</td>
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<tr>
<td>( A_7 )</td>
<td>( E ) (MPa)</td>
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<td>3.9</td>
<td>—7.49</td>
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<td>( \nu )</td>
<td>—</td>
<td>0.47</td>
<td>—</td>
<td>—</td>
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<tr>
<td>( c' ) (kPa)</td>
<td>36</td>
<td>36</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( \Phi' = \Psi' ) (°)</td>
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<td>34</td>
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<td>( \sigma_p' ) (bar)</td>
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<td>0.195</td>
<td>—5.13</td>
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<td>—9.73</td>
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<td>( \nu )</td>
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<td>0.45</td>
<td>—</td>
<td>—</td>
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<tr>
<td>( c' ) (kPa)</td>
<td>28</td>
<td>28</td>
<td>0</td>
<td>0</td>
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<tr>
<td>( \Phi' = \Psi' ) (°)</td>
<td>32</td>
<td>31</td>
<td>3.23</td>
<td>3.23</td>
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<tr>
<td>( \sigma_p' ) (bar)</td>
<td>0.99</td>
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<td>—13.57</td>
<td>13.57</td>
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<td>0.105</td>
<td>—8.57</td>
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<td>0.007</td>
<td>—10.94</td>
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<td>—</td>
<td>0.41</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( c' ) (kPa)</td>
<td>72</td>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \Phi' = \Psi' ) (°)</td>
<td>28.8</td>
<td>29</td>
<td>—0.69</td>
<td>0.69</td>
</tr>
</tbody>
</table>
giving the impression that these laboratory results (A5, A6, and A7) are erroneous.

The analysis of the parameters summarized in Table 7 also shows that the shear strength values obtained by our method are identical to those resulting from the triaxial test. The friction angle obtained by our method is slightly higher than the friction angle resulting from the triaxial for the majority of the samples. The error related to the method developed in this paper, supposed to describe all the phenomena observed in the laboratory, depends on the mathematical equations of the model and the uncertainties of experimental measurements. This error is obtained from the expression (33) and its values for the cases studied are summarized in Table 7.

\[
e_{r\text{-method}} (%) = \frac{p_{\text{exp}} - p_{\text{theo}}}{p_{\text{theo}}} \times 100, \quad (33)
\]

with \(p_{\text{th}}\) or \(p_{\text{exp}}\): theoretical or experimental parameter.

In the samples tested, the difference between the laboratory values and the values obtained by our method in terms of a single parameter is less than 19%.

6. Conclusion

The method developed in this paper does not replace conventional shear tests of soil. She is complementary insofar as in certain projects, the number of shear tests carried out is insufficient to really characterize the sequence according to its spatial variability. In addition to determining settlement and consolidation parameters, it will also be possible to determine shear parameters on the same samples, reducing coats and waiting times for obtaining laboratory test results. It will be possible to obtain direct correlations between the shear and deformability parameters.
Figure 11: Continued.
on each soil sample tested. We have proposed in this paper, the theoretical solution for obtaining the oedometric curve compressibility of soil. Triaxial tests and oedometric tests were conducted to compare the experimental compressibility responses of soil with the theoretical solutions, which is considered to be the verification of the theory. The study for the determination of the mechanical properties of soils whose behavior can be described by an elastic model, perfectly plastic, using the Mohr–Coulomb model with an associated flow rule was carried out. The method developed showed its reliability by comparing the solution obtained on a soil sample to the solution of Plaxis Finite Element software [60]. The inverse problem for this determination was solved by the optimization method works correctly for determining the necessary parameters:

(i) The study of the sensitivity of the method to its parameters has shown that the parameters of major influences are respectively Young’s modulus, Poisson’s ratio, the friction angle, and the dilatance angle of the soils;

(ii) The influence of cohesion is relatively weak in the conditioning of the method. In the last phase, the method was applied to nine soils samples and its results were compared to those obtained experimentally in the laboratory;

(iii) The difference between the laboratory values and the values obtained by our method is less than 19%;

(iv) The results obtained in this paper are in accordance with the recent previous studies notably those of Zoa et al., [56] and Zoa and Amba [55, 57], concerning the influence of all Mohr–Coulomb parameters;

(v) Our method is a good approximation of conventional laboratory tests and will probably be used by practitioners, students, and civil and Geotechnical Engineering researchers to calculate (numerical and analytical) geotechnical structures [65].

**Notation**

\( c' \): Shear strength in terms of effective stress

\( c \): Cohesion

\( c_u \): Undrained shear strength

CBR: California bearing ratio

**Figure 11**: Comparison of theoretical and laboratory compressibility curves on soil samples tested.
Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

Several people were involved in obtaining the results from this paper; there are very many of them and we cannot name them. Find our warm thanks. The authors acknowledge particularly the GéoMécanique team of the Laboratoire 3SR at Grenoble, France, and Geotechnical’s Laboratories and Engineering’s Offices of Cameroon for the facilities and support to perform the experiments.

References


