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Research Article

Design of Asynchronous Motor Controller Based on Controlled Lagrangians Method

Yunbo Gao,[1] Jianqiang Liu,[2] Maoqing Li ,[1] Zhiyu Tian,[1] and Cuiran Li[1]

[1] *School of Automation and Electrical Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China*
[2] *Lanzhou Wanli Aviation Electromechanical Limited Liability Company, Lanzhou 730070, China*

Correspondence should be addressed to Maoqing Li; limq@mail.lzjtu.cn

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Asynchronous motor system has the characteristics of high order, strong coupling, and nonlinearity. From the dynamical model, it is the underactuated mechanical system, which means that the dimension of its input space is fewer than the degree of freedom. Following this perspective, the energy based nonlinear control technology-CL (controlled Lagrangians) method is used to solve the control problem in this paper. Based on the expected controlled energy and its derivative with respect to time, controlled Lagrangians and generalized force are constructed, and they produce the controlled equations. In order to ensure the complete matching between the controlled equation and the original equation, the gyroscopic forces containing the first-order term of velocity are innovatively introduced into the generalized force, and the matching conditions are obtained. By solving the matching conditions composed of some partial differential equations, the nonlinear smooth feedback control law can realize the global asymptotic stabilization of not only velocity but also position. Finally, the controlled energy is selected as the Lyapunov function, and the stability is proved according to the LaSalle invariant theorem. The effectiveness of the designed control law is demonstrated in the results of the simulation.

1. Introduction

With the advantages of low price, simple structure, convenient maintenance, and reliable operation, the asynchronous motor has always been in a leading position in today's social industrial production. Under the concept of advocating production environmental protection and low-carbon economy, the research on the control performance of the asynchronous motor has important theoretical significance and practical value [1].

The asynchronous motor is a nonlinear system. However, the traditional linear control method cannot reveal its nonlinear nature. Therefore, the research on the control method of nonlinear theory is of great significance to improve the dynamic and static performance of the AC asynchronous motor. At present, the nonlinear control methods applied to the asynchronous motor mainly include feedback linearization control [2], backstepping control [3], sliding mode control [4], active disturbance rejection control [5, 6], and passive control theory [7, 8]. The control performance of the system has been significantly improved for application of the above method.

Sun developed chopping control and energy-saving controller of a three-phase AC asynchronous motor [2]. Yu et al. designed the nonlinear adaptive controller of the asynchronous motor system by using the subsystem separation method and backstepping technology to ensure the stability of the system [3]. Lekhchine et al. designed a renewable energy storage electrical system for asynchronous motors. In this system, the motor is driven by sliding mode control, which can overcome the chattering phenomenon through the sliding surface based on fuzzy logic [4]. Li et al. proposed a second-order ADRC and AC excitation control system based on stator flux oriented control to control the active and reactive power of the variable-speed pumped storage unit [5, 6]. Wu et al. discussed the problem of asynchronous passive control of Markov jump systems and obtained three equivalent sufficient conditions to ensure the random passivity of hidden Markov jump systems by using matrix inequality technology. Based on the established conditions, an asynchronous controller is designed [7–9]. Yu et al. studied the tracking control of the underactuated dynamic system and proposed a six-step motion strategy of the pendulum driven vehicle rod system [10, 11]. By implementing feedback and other measures, the asymptotic stability of the control Hamiltonian system can be realized. The literature [12–15] reports some new results about the control of underactuated dynamic systems.

In terms of mathematical equivalence, some studies analyze the controlled Lagrange (CL) method [16–19]. Compared with PBC, the CL method has a simpler mathematical form and clearer physical meaning, which is easy to understand. Usoro et al. described a Lagrangian method for solving nonlinear constrained optimization problems in set theory control problems. By introducing the matrix Lagrange multiplier, the problem is simplified to solve a set of nonlinear simultaneous matrix equations [16]. Műller et al. proved the possibility of maximizing the torque without exceeding the limit value of magnetic flux and stator current, which is independent of the number of revolutions of an asynchronous motor [17]. Lindgren et al. gave the exact slope distribution and other characteristic distributions of symmetric and asymmetric Lagrangian spatiotemporal waves at level crossings [18, 19]. In addition, some studies are extended to the general PBC method from the perspective of robust control and optimal control of general PCH systems [20–26]. These results have been proved to be expressed in a Lagrangian form.

This paper applys CL method to analyze the asynchronous motor system from the perspective of an underdrive mechanical system. We will use the electromagnetic energy generated by the stator and rotor windings and the mechanical energy generated by the rotor to construct a controlled energy controller [12]. The controlled system maintains the Lagrangian mechanical structure in form, obtains the smooth nonlinear feedback control law, has a large convergence range, and helps to realize the CL method robust control and optimal control [14]. Compared with the port controlled dissipative Hamiltonian system, the nonlinear smooth feedback control law obtained in this paper can realize the global asymptotic stabilization of position and velocity at the same time.

2. Mathematical Model

For the convenience of writing, we will indicate the independent variables of the functions and matrices that appear, which will be omitted when they appear below. $l, m, n \in N$, and $N_i = \{1, \cdots, i\}$, N_n means a collection consisting of the first n quantity of natural numbers. Let $z(\mathbf{e})$ represents the function of the vector $\mathbf{e} = [e_1, \cdots, e_5]^\mathrm{T}$, Y_{kj} corresponds to the element at the kth row and the jth column of the function matrix vector $\mathbf{Y}(\mathbf{e})$, x_i denotes the ith element of function vector $\mathbf{X}(\mathbf{e})$: $R^5 \longrightarrow R^5$, where $i \in N_5$, $\mathbf{I}$ means the five-order identity matrix. Besides, some notations as given below:

$$z_{,i} = \frac{\partial z(\mathbf{e})}{\partial e_i}, Y_{,i} = \begin{bmatrix} Y_{11,i} & \cdots & Y_{1m,i} \\ \vdots & & \vdots \\ Y_{l1,i} & \cdots & Y_{lm,i} \end{bmatrix}$$
$$\partial_\mathbf{e} z = \begin{bmatrix} z_{,1} \\ \vdots \\ z_{,n} \end{bmatrix}, \partial_{\mathbf{e}^\mathrm{T}} X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & & \vdots \\ x_{m1} & \cdots & x_{m,n} \end{bmatrix}. \quad (1)$$

The generalized coordinate variables of the AC asynchronous motor is $\mathbf{q} = [q_1, \ldots, q_5]^\mathrm{T}$, where q_1, q_2, q_3, q_4 are the components of stator inductance charge and rotor inductance charge on $d - q$ axis, and q_5 is the angular position of motor rotor. $\mathbf{u} = [u_1, \ldots, u_4, 0]^\mathrm{T}$ is the original control input, where u_1, u_2, u_3, u_4 are the elements of the stator voltage and rotor voltage on the $d - q$ axis. In addition, $\mathbf{u} = \mathbf{O}(\mathbf{q})\mathbf{v}$, where the input coupling matrix $\mathbf{O} = [I_{01}, I_{02}, I_{03}, I_{04}]$, and $\mathbf{v} \in R^4$.

In view of the mathematic model of a three-phase asynchronous motor, the following assumptions are expressed as follows [9]:

(1) The spatial harmonic and the spatial difference between three-phase windings are ignored. Meanwhile, it is assumed that the generated magnetomotive force is distributed sinusoidally along the circumference of the air gap.

(2) Magnetic circuit saturation and core loss are ignored. At the same time, it is assumed that the inductance parameters of every phase winding, not only self-inductance but also mutual one, are constant.

(3) It is not considered of the influence of frequency and temperature changes on the variation of winding resistance.

According to the above assumptions for the AC asynchronous motor, its mathematical model in the $d - q$ coordinate system can be obtained by

$$\begin{bmatrix} L_{11} & 0 & L_{13} & 0 & 0 \\ 0 & L_{11} & 0 & L_{13} & 0 \\ L_{13} & 0 & L_{33} & 0 & 0 \\ 0 & L_{13} & 0 & L_{33} & 0 \\ 0 & 0 & 0 & 0 & J/n_\mathrm{p} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} + \begin{bmatrix} 0 & L_{11}\dot{q}_5 & 0 & L_{13}\dot{q}_5 & -n_\mathrm{p}\psi_\mathrm{sq} \\ -L_{11}\dot{q}_5 & 0 & -L_{13}\dot{q}_5 & 0 & n_\mathrm{p}\psi_\mathrm{sd} \\ 0 & L_{13}\dot{q}_5 & 0 & L_{33}\dot{q}_5 & 0 \\ -L_{13}\dot{q}_5 & 0 & -L_{33}\dot{q}_5 & 0 & 0 \\ n_\mathrm{p}\psi_\mathrm{sq} & -n_\mathrm{p}\psi_\mathrm{sd} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} + \begin{bmatrix} R_1 & -\omega_1 L_{11} & 0 & -\omega_1 L_{13} & 0 \\ \omega_1 L_{11} & R_1 & \omega_1 L_{13} & 0 & 0 \\ 0 & -\omega_1 L_{13} & R_2 & -\omega_1 L_{33} & 0 \\ \omega_1 L_{13} & 0 & \omega_1 L_{33} & R_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ T_\mathrm{L} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ 0 \end{bmatrix}. \quad (2)$$

In (2), $\psi_{sd} = L_{11}\dot{q}_1 + L_{13}\dot{q}_3$, $\psi_{sq} = L_{11}\dot{q}_2 + L_{13}\dot{q}_4$, L_{11}, and L_{33} denote the equivalent self-inductance of the stator and rotor phase windings, respectively. And L_{13} is the equivalent mutual inductance of the stator and rotor phase windings. The load torque is denoted by T_L, and $T_L = T_1 + T_L'$, where T_1 includes no-load torque and the external one, and $T_L' = Hq_5$ denotes the torsional torque generated when the motor and the mechanical load are connected with a relatively long shaft, where H is the deformation coefficient. The equation (2) is abbreviated as

$$M\ddot{q} + C(q,\dot{q})\dot{q} + C_0\dot{q} + \partial_q E_p = u, \quad (3)$$

where $(\mathbf{q}^T, \dot{q}^T) = (\mathbf{q}_d^t, 0^T)$ is the equilibrium point, and its input must satisfy $\overline{O}\mathbf{u} = 0$, where $\overline{O} \in (\mathbf{O}^{\perp})^T$ and $(\mathbf{O}^{\perp})^T\mathbf{O} = 0$, so $\overline{O} = [0,0,0,0,1]$.

3. Design of Asynchronous Motor Controller Based on CL Method

3.1. Construction of Controlled Energy and Generalized Force. According to (3), the controlled kinetic energy of the controlled system is $\overline{E}_k(\mathbf{q},\dot{q}) = (1/2)\dot{q}^T\overline{M}\dot{q}$. The controlled kinetic energy matrix satisfies $\overline{M}: R^5 \longrightarrow R^{5\times5}, \overline{M} = \overline{M}^T$ and $|\overline{M}| \neq 0$.

Take the controlled potential energy as $\overline{E}_p: R^5 \longrightarrow R$, and generalized force $\overline{u} \in R^5$, then controlled Lagrangian function $\overline{L}(\mathbf{q},\dot{q})$ and controlled energy $\overline{E}(\mathbf{q},\dot{q})$ of the system are as follows:

$$\begin{cases} \overline{L} = \overline{E}_k - \overline{E}_p, \\ \overline{E} = \overline{E}_k + \overline{E}_p. \end{cases} \quad (4)$$

Sometimes, the controlled energy has physical meaning, such as controlled kinetic energy or controlled potential energy, and maybe it has only a mathematical meaning which is sufficient and necessary. According to $\overline{L}$ and $\overline{u}$, we obtain the controlled equations of the system as follows:

$$\overline{u} = \overline{M}\ddot{q} + \partial_{\mathbf{q}^T}(\overline{M}\dot{q})\dot{q} - \frac{1}{2}\partial_{\mathbf{q}}\left(\dot{q}^T\overline{M}\dot{q}\right) + \partial_{\mathbf{q}}\overline{E}_p. \quad (5)$$

Using (4) and (5), we get

$$\dot{\overline{E}} = \dot{q}^T\left[\frac{d}{dt}\partial_{\dot{q}}\overline{L} - \partial_{\mathbf{q}}\overline{L}\right] = \dot{q}^T\overline{u}. \quad (6)$$

The generalized force of the system consists of two parts, namely gyroscopic forces $\mathbf{G}(\mathbf{q},\dot{q})\dot{q}$ and dissipation force $-\mathbf{D}(\mathbf{q})\dot{q}$, where gyroscopic forces matrix is $\mathbf{G} = -\mathbf{G}^T = \sum_{i\in N_5}\mathbf{I}_{oi}\mathbf{G}_{io} = \sum_{i,j\in N_5}\mathbf{I}_{oi}\dot{q}^T\mathbf{g}_{io}(\mathbf{q}) + \mathbf{G}^0$.

Furthermore, $g_{ij}^{(k)}(\mathbf{q})$ represents the element at the kth row and jth column of function matrix $\mathbf{g}_{io}$, and for $i,j,k \in N_5$, $g_{ij}^{(k)}$ is the kth component function of the element G_{ij} of the gyroscopic forces matrix.

Since the gyroscopic forces matrix is an anti-symmetric one, there is $g_{ij}^{(k)} = -g_{ji}^{(k)}$. Similarly, the constant elements of the gyroscopic forces matrix $\mathbf{G}$ also satisfy $\mathbf{G}_{ij}^0 = -\mathbf{G}_{ji}^0$.

Remark 1. Since there is $\mathbf{C}_0\dot{q}$ term in the original system, constant term $\mathbf{G}^0$ is introduced into the matrix $\mathbf{G}$ to construct the gyroscopic forces consisting of the first term of the velocity, which matches $\mathbf{C}_0\dot{q}$ of the original system. Maybe these forces do not exist in real world, and only mathematical meaning is necessary for them. As we know, this introduction is for the first time.

When generalized force $\overline{u} = (\mathbf{G} - \mathbf{D})\dot{q}$, we obtain from (5) that

$$\dot{\overline{E}} = \dot{q}^T(\mathbf{G} - \mathbf{D})\dot{q} \leq 0, \quad (7)$$

which indicates that the energy of the closed system is decreasing. Multiplying $\mathbf{M}\overline{M} = \mathrm{N}(\mathbf{q})$ −N(q) at the two sides of (4) at the same time, we obtain

$$\mathbf{N}\overline{u} = \mathbf{M}\ddot{q} + \mathbf{N}\left[\partial_{\mathbf{q}^T}(\overline{M}\dot{q})\dot{q} - \frac{1}{2}\partial_{\mathbf{q}}\left(\dot{q}^T\overline{M}\dot{q}\right) + \partial_{\mathbf{q}}\overline{E}_p\right]. \quad (8)$$

According to (2) and (7), the original control input u and the control input $\overline{u}$ of the controlled system are obtained. The relationship between them can be given as

$$\begin{aligned} u = {} & (C + C_0)\dot{q} - N\partial_{\mathbf{q}}\overline{E}_p + \partial_{\mathbf{q}}E_p + N\overline{u} \\ & - N\left[\partial_{\mathbf{q}^T}(\overline{M}\dot{q})\dot{q} - \frac{1}{2}\partial_{\mathbf{q}}\left(\dot{q}^T\overline{M}\dot{q}\right)\right] \end{aligned}. \quad (9)$$

According to (9)–(14) in [14] and (8) in this article, we obtain

$$\begin{aligned} u = {} & (C + C_0)\dot{q} + \partial_{\mathbf{q}}E_p - N\partial_{\mathbf{q}}\overline{E}_p \\ & - \sum_{i\in N_5}\frac{1}{2}NI_{oi}\dot{q}^T T^{(i)}\dot{q} + N(\overline{u} - \overline{G}\dot{q}). \end{aligned} \quad (10)$$

The deduction of (9) is tedious, so we borrow the similar process in literature [12] for abbreviation.

3.2. Determination of the Matching Conditions. If the controlled equation (5) matches the original (3), the control input determined by (9) is true; that is, $u_5 = 0$ is true for any point $(\mathbf{q},\dot{q})$.

In the same form as gyroscopic forces matrix $\mathbf{G}$, matrix $\widehat{G}(\mathbf{q},\dot{q})$ is given as follows:

$$\widehat{G}(\mathbf{q},\dot{q}) = \mathbf{G}(\mathbf{q},\dot{q}) - \overline{G}(\mathbf{q},\dot{q}), \quad (11)$$

where $\widehat{G}$ gives out functions $\widehat{g}_{ji}^{(k)}(\mathbf{q})$ and $\widehat{g}_{ij}$ similar to $g_{ji}^{(k)}(\mathbf{q})$ and g_{ij}. Substituting $\overline{u} = (\mathbf{G} - \mathbf{D})\dot{q}$ and (11) into (10), we get

$$\begin{aligned} u = {} & -\sum_{i\in N_5}\frac{1}{2}NI_{oi}\dot{q}^T T^{(i)}\dot{q} + (C + C_0)\dot{q} \\ & + N(\widehat{G} - D)\dot{q} - N\partial_{\mathbf{q}}\overline{E}_p + \partial_{\mathbf{q}}E_p. \end{aligned} \quad (12)$$

Let $\overline{N} = \overline{O}\mathbf{N}$, multiplying the line vector $\overline{O}$ from the left to (12) and taking the obtained left side zero constantly, we acquire the matching conditions as follows (13)–(16):

$$0=\sum_{i\in N_5}\overline{N}_iT_{jj}^{(i)}+\sum_{i\neq j}2\overline{N}_i g_{ji}^{(i)}, \quad (13)$$

where $j\in N_5$ and each j represents an equation, and

$$0=\sum_{i\in N_5}\overline{N}_i\left(T_{fj}^{(i)}+g_{ji}^{(f)}+g_{fi}^{(j)}\right)+hn_{\mathrm{p}}L_{13}. \quad (14)$$

In equation (14), each pair of (j, f) corresponds to an equation. Otherwise, $j, f\in N_5$ and $j>f$, When $f+j\neq 5$, $h=0$; then $h=(-1)f$.

$$0=\sum_{i\in N_5}\overline{N}_iD_{ij}+\sum_{i\neq j}\overline{N}_i\widehat{g}_{ji}, \quad (15)$$

where $j\in N_5$, and each j corresponds to an equation.

$$0=\sum_{i\in N_5}\overline{N}_i\overline{E}_{\mathrm{p},i}-E_{\mathrm{p},5}. \quad (16)$$

Multiplying (13) by $\overline{N}_1^2,\cdots,\overline{N}_5^2$, and multiplying (14) by $2\overline{N}_1\overline{N}_2$, $2\overline{N}_1\overline{N}_3$, $2\overline{N}_1\overline{N}_4$, $2\overline{N}_1\overline{N}_5$, $2\overline{N}_2\overline{N}_3$, $2\overline{N}_2\overline{N}_4$, $2\overline{N}_2\overline{N}_5$, $2\overline{N}_3\overline{N}_4$, $2\overline{N}_3\overline{N}_5$, and $2\overline{N}_4\overline{N}_5$, then the sum to obtain a equation has nothing to do with gyroscopic forces:

$$2n_{\mathrm{p}}L_{13}\left(\overline{N}_2\overline{N}_3-\overline{N}_1\overline{N}_4\right)=\sum_{i\in N_5}\overline{O}\overline{N}_i\mathbf{W}_{,i}^{-1}\overline{O}^{\mathrm{T}}. \quad (17)$$

Remark 2. Equation (17) is obtained without gyroscopic forces terms due to anti-symmetric property of gyroscopic forces matrix, which indicates that the quadratic form of the anti-symmetric matrix is equal to zero.

Let $\mathbf{W}^{-1}=\mathbf{K}(\mathbf{q})$, then (17) can be concisely expressed as

$$2n_pL_{13}\left(\overline{N}_2\overline{N}_3-\overline{N}_1\overline{N}_4\right)=\sum_{i\in N_5}\overline{O}\overline{N}_i\mathbf{K}_{,i}\overline{O}^{T}. \quad (18)$$

For the controlled kinetic energy matrix, its regular condition is $|K|\neq 0$. According to literature [12], $\mathbf{W}(\mathbf{q})=\mathbf{M}^{-1}\overline{M}\mathbf{M}^{-1}$ is known, so $\mathbf{N}=\mathbf{K}\mathbf{M}^{-1}$ can be obtained from the definition of matrixes $\mathbf{N},\mathbf{W}$, and K. If $\overline{N}_1,\cdots,\overline{N}_5$ are zero, then $|K|=0$ is available. Therefore, $\overline{N}_1,\cdots,\overline{N}_5$ cannot be zero at the same time. In order to facilitate the subsequent calculations, suppose $\overline{N}_5\neq 0$. Multiplying (15) by $\overline{N}_1,\cdots,\overline{N}_5$ and summing them, we get

$$\overline{N}\mathbf{D}\overline{N}^{T}=0. \quad (19)$$

In summary, the combination of (16), (18), and (19) and $|K|\neq 0$ is the condition under which the controlled system matches the original one.

Remark 3. Equations (16) and (18) are partial differential equations, and equations (13)–(15) related to gyroscopic forces terms are linear algebraic equations. They are cascaded. PDEs are resolved at first to decrease difficulty, and then there are only linear algebraic equations which could be solved explicitly with introducing the previous solution.

Except algebraic equations, there are only two PDEs contained in the matching condition for the underactuation degree one system. They could be solved with involving enough independent variables according to some examples applied in CL methods.

3.3. Determination of the Matching Controller. For convenience, some notations can be expressed as follows:

$$\widehat{O}=\left(O^{\mathrm{T}}O\right)^{-1}O^{\mathrm{T}},\widehat{N}=\widehat{O}N. \quad (20)$$

Multiplying $\widehat{O}$ from the left to (12), the matching control of the system is

$$\begin{aligned}\mathbf{v}=&-\frac{1}{2}\widehat{N}\sum_{i\in N_5}I_{oi}\dot{q}^{\mathrm{T}}T^{(i)}\dot{q}+\widehat{O}(C+C_0)\dot{q}\\&+\widehat{N}(\widehat{G}-D)\dot{q}-\widehat{N}\partial_{\mathbf{q}}\overline{E}_{\mathrm{p}}+\widehat{O}\partial_{\mathbf{q}}E_{\mathrm{p}}.\end{aligned} \quad (21)$$

4. Matching Conditions Solution

For the asynchronous motors model, there is no control input ($u_5=0$) at the fifth degree of freedom except the ath degree of freedom, where $a\in N_4$, at the same time, the following notations are defined as

$$\widehat{O}=\mathbf{O}^{T},\overline{N}=\mathbf{N}_{50},\mathbf{O}_{ao}\widehat{N}=\mathbf{N}_{ao}. \quad (22)$$

Introduce the function vector $\Gamma^{\mathrm{T}}=-N_{5a}/N_{55}$ from the definition of matrixes $\mathbf{N}$, $\mathbf{W}$, and $\mathbf{K}$, we get $K_{50}=-N_{55}\Phi(q)^{\mathrm{T}}$, where $\Phi^{\mathrm{T}}=[\Phi_1,\cdots,\Phi_5]=\Gamma^{\mathrm{T}}\mathbf{M}$, and thus the element in the fifth row of matrix $\mathbf{K}$ is $K_{5a}=K_{55}\Phi_a/\Phi_5$. To ensure $\mathbf{K}>0$, we choose $K_{aa}=K_{55}[k_a\Phi_a^2/\Phi_5^2+k_{(a+n)}]$, where k_a and $k_{(a+n)}$ are constants. The system matching conditions expressed by Γ, K_{55}, and $\overline{E}_{\mathrm{p}}$ are

$$\sum_{i\in N_5}\Gamma_iK_{55,i}=\frac{2n_pL_{13}\left(\Gamma_2\Gamma_3-\Gamma_1\Gamma_4\right)K_{55}}{\left(\Gamma^{T}\mathbf{M}_{05}\right)}, \quad (23)$$

$$\sum_{i\in N_5}\Gamma_i\overline{E}_{p,i}=\frac{\Gamma^{T}\mathbf{M}_{05}}{K_{55}E_{\mathrm{p},5}}, \quad (24)$$

$$\Gamma^{\mathrm{T}}\mathbf{D}\Gamma=0. \quad (25)$$

Assume $\Gamma_1=0,\cdots,\Gamma_4=0,\Gamma_5=-1$, and find a special solution to (23) as follows:

$$K_{55}=k_5. \quad (26)$$

In the above (23), k_5 is a constant.

The $\mathbf{K}$ matrix of the system is

$$\mathbf{K}=k_5\begin{bmatrix}k_6&0&0&0&0\\0&k_7&0&0&0\\0&0&k_8&0&0\\0&0&0&k_9&0\\0&0&0&0&1\end{bmatrix}. \quad (27)$$

It can be seen from the $\mathbf{K}$ matrix that its determinant is $|K| = k_5^5 k_6 k_7 k_8 k_9$, so a sufficient condition for $\mathbf{K} > 0$ is

$$k_5, k_6, k_7, k_8, k_9 > 0. \tag{28}$$

From (24), the positive definite solution to the controlled potential energy $\overline{E}_p$ and the satisfied conditions are

$$\begin{cases} \overline{E}_p = \sum_{i \in N_5} (q_i - a_i)^2; \\ k_5 = \dfrac{JH}{2n_p}, \\ a_5 = -\dfrac{JT_1}{2n_p k_5}, \end{cases} \tag{29}$$

where $a_1, \cdots, a_5$ are constants. After some calculations, the Hessian matrix of $\overline{E}_p$ is

$$\frac{\partial^2 \overline{E}_p}{\partial \mathbf{q} \partial \mathbf{q}^T} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}. \tag{30}$$

It is clear that the Hessian matrix is positive definite. And from $\overline{E}_{p,1}(a_1), \cdots, \overline{E}_{p,5}(a_5) = 0$, we know that $(a_1, \cdots, a_5)$ is the minimum point of the controlled potential energy.

Remark 4. The controlled energy consists of controlled kinetic energy and potential energy. On the one hand, the controlled kinetic energy is in quadratic form of velocity and achieves positive definiteness with supplied condition. On the other hand, the controlled potential energy is constructed in the form to be positive definite conveniently.

According to (30), the dissipation matrix is chosen in diagonal form as follows:

$$\mathbf{D} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ 0 \end{bmatrix}, \tag{31}$$

where $d_1, \cdots, d_4$ can be any value greater than zero.

5. Matching Control Law and Stability Analysis

Based on the above analysis, $\mathbf{N} = \mathbf{KM}^{-1}$ is known, so the matrix $\mathbf{N}$ of the system is

$$\mathbf{N} = \beta_1 \begin{bmatrix} k_6 L_{33} & 0 & -k_6 L_{13} & 0 & 0 \\ 0 & k_7 L_{33} & 0 & -k_7 L_{13} & 0 \\ -k_8 L_{13} & 0 & k_8 L_{11} & 0 & 0 \\ 0 & -k_9 L_{13} & 0 & k_9 L_{11} & 0 \\ 0 & 0 & 0 & 0 & \beta_2 \end{bmatrix}. \tag{32}$$

In equation (32), $\beta_1 = k_5/(L_{11}L_{33} - L_{13}^2)$ and $\beta_2 = n_p(L_{11}L_{33} - L_{13}^2)/J$, because $\mathbf{T}^{(\mathbf{i})} = \mathbf{MK}_{,i}^{-1}\mathbf{M}$, where $i \in N_5$, so we get

$$\mathbf{T}^{(i)} = 0, i \in N_5. \tag{33}$$

From (13)–(15) and from (30)–(32), we have the gyroscopic forces component function $\widehat{g}_{ij}^{(k)}$ as follows:

$$\widehat{g}_{ij}^{(k)} = \gamma_{ij}^{(k)}, \widehat{g}_{ij} = \gamma_{ij}, i, j, k \in N_5. \tag{34}$$

In equation (33), one part of the value of gyroscopic forces component function $\gamma_{ij}^{(k)}$ needs to satisfy $\gamma_{15}^{(2)} = -\gamma_{25}^{(1)}, \gamma_{15}^{(3)} = -\gamma_{35}^{(1)}, \gamma_{25}^{(4)} = -\gamma_{45}^{(2)}, \gamma_{35}^{(4)} = -\gamma_{45}^{(3)}$ and be arbitrary values, the other part needs to be taken according to the following (35), and the rest can be arbitrary values.

$$\begin{aligned} &\gamma_{15}^{(1)} = 0, \gamma_{15}^{(4)} = 2JL_{13}/k_5, \\ &\gamma_{15}^{(5)} = 0, \gamma_{25}^{(2)} = 0, \\ &\gamma_{25}^{(3)} = -2JL_{13}/k_5, \gamma_{25}^{(5)} = 0, \\ &\gamma_{35}^{(2)} = JL_{13}/k_5, \gamma_{35}^{(3)} = 0, \\ &\gamma_{35}^{(5)} = 0, \gamma_{45}^{(1)} = -JL_{13}/k_5 \\ &\gamma_{45}^{(4)} = 0, \gamma_{45}^{(5)} = 0, \\ &\gamma_{15} = 0, \gamma_{25} = 0, \\ &\gamma_{35} = 0, \gamma_{45} = 0. \end{aligned} \tag{35}$$

To keep the control law simple, take the value of $\gamma_{12}^{(5)}, \gamma_{15}^{(3)}, \gamma_{25}^{(4)}, \gamma_{35}^{(4)}, \gamma_{12}, \gamma_{13}, \gamma_{14}, \gamma_{23}, \gamma_{24}, \gamma_{34}$ to be one and the rest to be zero.

Remark 5. A large part of variables, $\widehat{g}_{ij}^{(k)}$ and $\widehat{g}_{ij}$ in equation (34), take the value zero for convenience to get the simpler control law. The different evaluation for them could still assure stability of the system and maybe affect the convergence rate of the system. Further, there is a room to search control law for better performance in some respects.

Substituting (20), (27)–(35) into (21), the obtained matching control law of the motor system is described as

$$u_1 = R_1\dot{q}_1 + (\dot{q}_5 - \omega_1)(L_{11}\dot{q}_2 + L_{13}\dot{q}_4) - n_p\psi_{sq}\dot{q}_5 + \frac{k_5k_6}{(L_{11}L_{33} - L_{13}^2)}\{[L_{33}(\dot{q}_2 + \dot{q}_3 + \dot{q}_4 + \dot{q}_3\dot{q}_5 + \dot{q}_2\dot{q}_5) + L_{13}(\dot{q}_1 + \dot{q}_2 - \dot{q}_4 - \dot{q}_4\dot{q}_5)] + L_{13}J\dot{q}_5 * (2L_{33}\dot{q}_4 - L_{13}\dot{q}_2)/k_5 + (L_{13}d_3\dot{q}_3 - L_{33}d_1\dot{q}_1) - 2[L_{33}(q_1 - a_1) - L_{13}(q_3 - a_3)]\}, \quad (36)$$

$$u_2 = R_1\dot{q}_2 + (\omega_1 - \dot{q}_5)(L_{11}\dot{q}_1 + L_{13}\dot{q}_3) + n_p\psi_{sd}\dot{q}_5 + \frac{k_5k_7}{(L_{11}L_{33} - L_{13}^2)}\Big\{[L_{33}(\dot{q}_3 + \dot{q}_4 - \dot{q}_1 - \dot{q}_1\dot{q}_5 + \dot{q}_4\dot{q}_5) + L_{13}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3)] + \frac{L_{13}J\dot{q}_5}{k_5} * (L_{13}\dot{q}_1 - 2L_{33}\dot{q}_3) + (L_{13}d_4\dot{q}_4 - L_{33}d_2\dot{q}_2) - 2[L_{33}(q_2 - a_2) - L_{13}(q_4 - a_4)]\Big\}, \quad (37)$$

$$u_3 = (\dot{q}_5 - \omega_1)(L_{13}\dot{q}_2 + L_{33}\dot{q}_4) + R_2\dot{q}_3 + \frac{k_5k_8}{(L_{11}L_{33} - L_{13}^2)}\{[L_{11}(\dot{q}_4 - \dot{q}_1 - \dot{q}_2 + \dot{q}_4\dot{q}_5) - L_{13}(\dot{q}_2 + \dot{q}_3 + \dot{q}_2\dot{q}_5 + \dot{q}_3\dot{q}_5)] + L_{13}J\dot{q}_5(L_{11}\dot{q}_2 - 2L_{13}\dot{q}_4)/k_5 + (L_{13}d_1\dot{q}_1 - L_{11}d_3\dot{q}_3) - 2[L_{11}(q_3 - a_3) - L_{13}(q_1 - a_1)]\} \quad (38)$$

$$u_4 = (\omega_1 - \dot{q}_5)(L_{13}\dot{q}_1 + L_{33}\dot{q}_3) + R_2\dot{q}_4 + \frac{k_5k_9}{(L_{11}L_{33} - L_{13}^2)}\{[L_{13}(\dot{q}_1 - \dot{q}_3 - \dot{q}_4 + \dot{q}_1\dot{q}_5 - \dot{q}_4\dot{q}_5) - L_{11}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3)] + L_{13}J\dot{q}_5(2L_{13}\dot{q}_3 - L_{11}\dot{q}_1)/k_5 + (L_{13}d_2\dot{q}_2 - L_{11}d_4\dot{q}_4) - 2[L_{11}(q_4 - a_4) - L_{13}(q_2 - a_2)]\}. \quad (39)$$

In summary, the conclusion is listed as follows:.

Proposition 1. *For AC asynchronous motor systems, if the parameters of controller meet the following conditions:*

$$\begin{cases} k_5, \cdots, k_9 > 0, \\ d_1, \cdots, d_4 > 0, \end{cases} \quad (40)$$

then the smooth feedback control law expressed by equations (35)–(38) can stabilize the motor globally asymptotically at $(_{\mathbf{q}}^{dT}, 0^T)$. *It is the desired equilibrium point at which the controlled potential energy achieves the minimum.*

Proof. Let Lyapunov candidate function $V = \overline{E}$. If the system controller parameters are selected according to (39), then the function is positive definite. From equation (6), there is $\dot{V} \le 0$. Therefore, the control law given by equations (35)–(38) enables the induction motor to achieve global asymptotic stabilization at $(_{\mathbf{q}}^{\mathrm{dT}}, 0^{\mathrm{T}})$.

For the asymptotic stability, it can be proved that there is no trajectory of isolated points other than equilibrium points in the set of $\dot{V} = 0$.

Assuming that there is such a trajectory in the set, it can be obtained as

$$\dot{q}_i \equiv 0, i \in N_4. \quad (41)$$

There is a certain point q_0 on this trajectory, and at the same time, (39) also holds in a certain area δ_0 of this point q_0. Differentiating and integrating (40) along the trajectory, we get

$$\begin{cases} \ddot{q}_i \equiv 0, \\ q_i = \alpha_i, i \in N_4, \end{cases} \quad (42)$$

where $\alpha_1, \cdots, \alpha_4$ are constant.

Substituting (35)–(38), (40), and (41) into the first four equations of (1), we get

$$\begin{cases} \frac{2k_5k_6}{L_{11}L_{33} - L_{13}^2}[L_{33}(\alpha_1 - a_1) - L_{13}(\alpha_3 - a_3)] = 0 \\ \frac{2k_5k_7}{L_{11}L_{33} - L_{13}^2}[L_{33}(\alpha_2 - a_2) - L_{13}(\alpha_4 - a_4)] = 0 \end{cases}$$
$$\begin{cases} \frac{2k_5k_8}{L_{11}L_{33} - L_{13}^2}[L_{11}(\alpha_3 - a_3) - L_{13}(\alpha_1 - a_1)] = 0 \\ \frac{2k_5k_9}{L_{11}L_{33} - L_{13}^2}[L_{11}(\alpha_4 - a_4) - L_{13}(\alpha_2 - a_2)] = 0 \end{cases} . \quad (43)$$

It can be seen from observation that (43) is true if and only if $\alpha_1 = a_1, \cdots, \alpha_4 = a_4$, and vice versa. Therefore, the assumption that there are isolated points belonging to the point group of $\dot{V} = 0$ does not hold.

From (4) and (41), we obtain $\partial_q\overline{E}_{\mathrm{p}} = 0$. This shows that the trajectory in the set $\dot{V} = 0$ can only be the equilibrium point, and the Hessian matrix of $\overline{E}_{\mathrm{p}}$ is positive definite, so point $\mathbf{q}_{\mathrm{d}}$ is the sole extreme point of $\overline{E}_{\mathrm{p}}(\mathbf{q})$. According to the

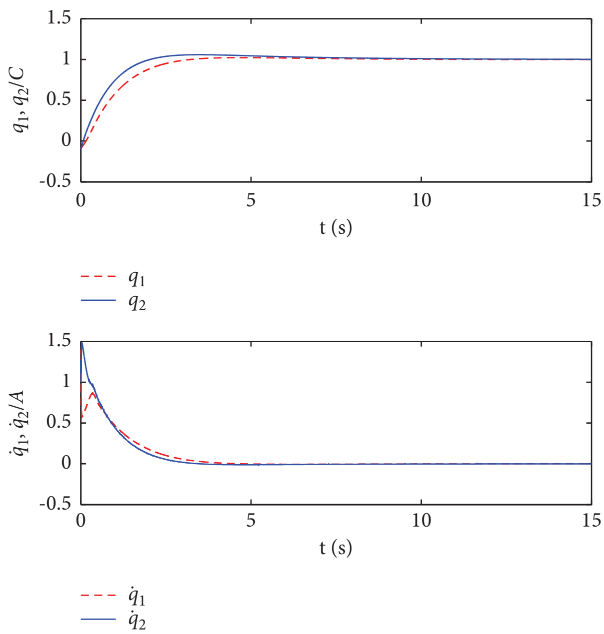

FIGURE 1: Stator inductive charge and current.

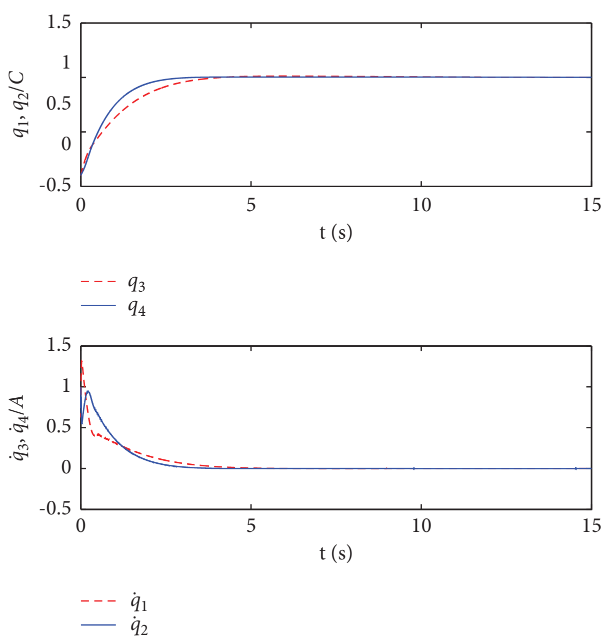

FIGURE 2: Charge and current of rotor.

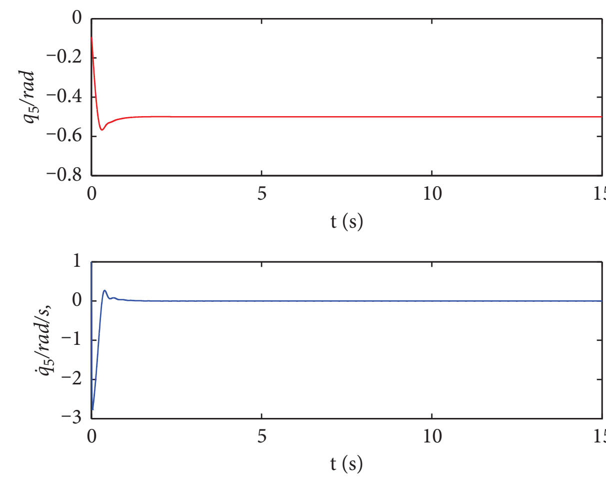

FIGURE 3: Rotor angular displacement and angular velocity.

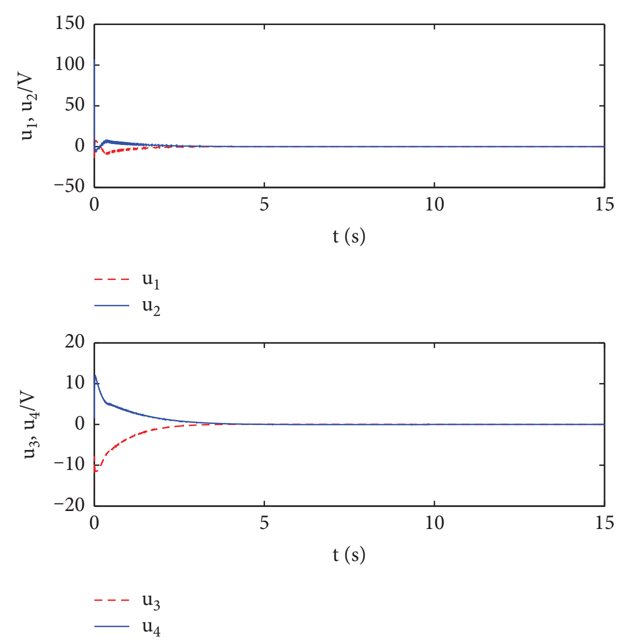

FIGURE 4: Control input.

LaSalle principle, the system can achieve global asymptotic stabilization by the proposed control law. □

6. Simulation Result Analysis

The simulation parameters of the system are selected as $\mathbf{L}_{11} = 0.45$ H, $\mathbf{L}_{13} = 0.42$ H, $\mathbf{L}_{33} = 0.45$ H, $\mathbf{J} = 0.2$ kg m^{2}H, $\omega_1 =$ 10 rad/s, $\mathbf{T}_1 = 8$Nm, $\mathbf{H} = 8$Nm, $n_p = 8$, and $\mathbf{R}_1 = \mathbf{R}_2 = 0.97\ \Omega$. The parameters of the controller that satisfy the positive definite of the controlled energy are $k_5 = 0.1$, $k_6 = 4$, $k_7 = 6$, $k_8 = 0.5$, and $k_9 = 0.4$.

When $d_1 = 7$, $d_2 = 7$, $d_3 = 5$, and $d_4 = 5$, the simulation results are as follows:

The desired equilibrium point of the system is $(\mathbf{q}_d^t, 0^T)$, where $_{\mathbf{q}}^{dT} = [1, 1, 1, 1, -0.5]$. It can be seen from Figures 1–3 that when the AC asynchronous motor is disturbed by some uncertain factors, the control target of the system deviates from the expected balance point. Under the control input of Figure 4, the control target of the system can return to the desired equilibrium point as soon as possible. In this process, the change of electromagnetic torque, original, and controlled energy are showed in Figures 5 and 6.

Remark 6. Parameters k_5, k_6, k_7, k_8, and k_9 in controller are connected with the controlled kinetic energy, and their evaluation should ensure controlled kinetic energy positive

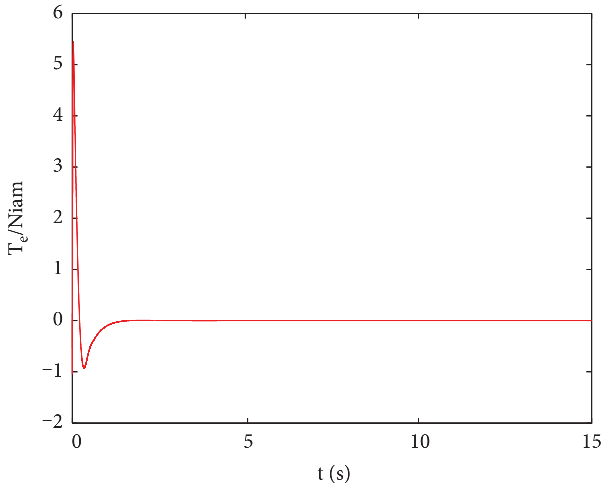

FIGURE 5: Electromagnetic torque.

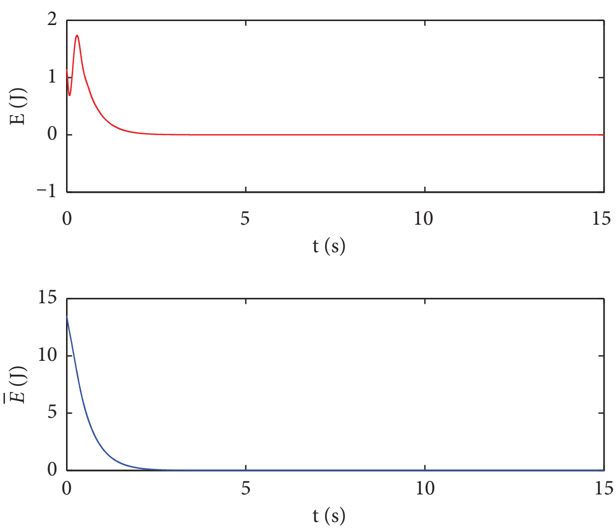

FIGURE 6: Original energy and controlled energy.

definite. Furthermore, they will affect primarily the amplitude of inputs.

Remark 7. Parameters d_1, d_2, d_3, and d_4 are related only to the dissipated forces, and thus the changing of them could be able to improve the convergence rate.

Compared with other control methods [8, 9], the nonlinear smooth feedback control law obtained in this article has a larger convergence range for its global asymptotic stabilization.

7. Conclusion

The control of an asynchronous motor is studied in this paper. By applying the controlled Lagrange function method to high-order, strongly coupled, and time-varying nonlinear systems, the controlled equation matching the original equation of the system is derived by using the expected controlled energy and its time derivative. Because the primary term of velocity exists in the original equation, the gyroscopic forces of the generalized primary term of velocity is innovatively introduced into the controlled equation, and the condition of complete matching between the original equation and the controlled equation can be obtained. By solving the matching condition composed of some partial differential equations, the specific matching control law of the system is obtained, and the global asymptotic stabilization of not only velocity but also position can be realized at the desired equilibrium point at the same time. Finally, the controlled energy of the system is chosen as the Lyapunov function for its property, which facilitates the proof of stability.

In the research process, we can see that the CL method analyzes the asynchronous motor system from the perspective of underactuated mechanical system, so that the controlled system maintains the Lagrangian mechanical structure in form, and the nonlinear smooth feedback control law can be obtained, which has a large convergence range and is helpful to realize robust control and optimal control. On the basis of the work in this paper, the nonlinear CL method will be improved by the introducing observer and be intended to solve tracking problem in following work.

Data Availability

The data supporting the findings of this study are available within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding this research.

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