Research Article

New Classifications of All Single Traveling Wave Solutions of the Fractional Approximate Long Water Wave Equations by Using the Complete Discrimination System

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First, our work is to transform the space-time fractional approximate long water wave equations into nonlinear ordinary differential equations via the traveling wave transformation in the sense of conformable fractional derivative. Second, we simplify the nonlinear ordinary differential equations into an ordinary differential equation with only one variable by integration and some transformations. Finally, we can further get all single traveling wave solutions of the space-time fractional approximate long water wave equations by the complete discrimination system for the four-order polynomial method; these solutions include the hyperbolic function solutions, rational function solutions, and implicit solutions.

1. Introduction

The exact traveling wave solutions of fractional partial differential equations have attracted much attention from mathematicians and engineering experts [1–10]. In recent years, some methods for constructing fractional partial differential equations have been proposed, such as the fractional double function method [11], improved fractional subequation method [12], $(G'/G1/G)$ -expansion method [13], plane dynamic system analysis method [14], and complete discrimination method of polynomial [15]. The complete discrimination method of polynomials was first proposed by Liu [16]. Using this method, the classification of all single traveling wave solutions of fractional partial differential equations can be constructed.

In this study, we consider the space-time fractional approximate long water wave equations [17]:

$$\begin{cases} D_t^\alpha u - u D_x^\beta u - D_x^\beta v + \gamma D_x^\beta D_x^\beta u = 0, \\ D_t^\alpha v - D_x^\beta(uv) - \gamma D_x^\beta D_x^\beta v = 0, t > 0, 0 < \alpha, \beta \leq 1, \end{cases} \quad (1)$$

where $u = u(x, t)$, $v = v(x, t)$. D_t^α and D_x^β represent the conformable fractional derivative. (1) is a very important shallow water waves model, which is usually used to describe the propagation of shallow water waves.

The work of this study is as follows. In Section 2, we give the construction steps of traveling wave solutions for fractional partial differential equations. In Section 3, we apply the method in Section 2 to the space-time fractional approximate long water wave equations. In Section 4, we give a conclusion.

2. Preliminaries

First, we consider the following integrated fractional partial differential equations:

$$\begin{cases} P(u, u_t, u_x, D_t^\alpha u, D_x^\beta u, D_x^\beta(uv), D_{tt}^{2\alpha} u, D_{xx}^{2\beta} u, \dots) = 0, \\ Q(v, v_t, v_x, D_t^\alpha v, D_x^\beta v, D_x^\beta(uv), D_{tt}^{2\alpha} v, D_{xx}^{2\beta} v, \dots) = 0, \end{cases} \quad (2)$$

where $u = u(t, x)$, $v = v(t, x)$, P and Q are the two polynomial in u , v , and its various fractional derivatives.

Step 1. Consider the traveling wave transformation as follows:

$$u(t, x) = U(\xi), v(t, x) = V(\xi), \xi = a \frac{t^\alpha}{\alpha} + b \frac{x^\beta}{\beta}, \quad (3)$$

where a and b are the constants. Using traveling wave transformation, (2) can be simplified to

$$P(U, U', U'', \dots) = 0, \quad (4)$$

and satisfy the relation

$$Q(U, U', U'', V, V', V'', \dots) = 0, \quad (5)$$

where P is a polynomial of U and its derivatives. Q is a polynomial of U , the derivative of U and the derivative of V .

Step 2. The above nonlinear ordinary differential equation (4) can be reduced to the following ordinary differential form:

$$(U')^2 = U^4 + pU^2 + qU + r. \quad (6)$$

Here, the integral form of (6) is

$$\int \frac{dU}{\sqrt{U^4 + pU^2 + qU + r}} = \pm (\xi - \xi_0), \quad (7)$$

where ξ_0 is an arbitrary constant. Here, we assume that the fourth-order polynomial is

$$F(U) = U^4 + pU^2 + qU + r, \quad (8)$$

and its complete discrimination system is

$$D_1 = 4, D_2 = -p, D_3 = -2p^3 + 8pr - 9q^2, E_2 = 9p^2 - 32pr,$$

$$D_4 = -p^3q^2 + 4p^4r + 36pq^2r - 32p^2r^2 - \frac{27}{4}q^4 + 64r^3. \quad (9)$$

Next, we can obtain the classification of solutions of (6) according to polynomial complete discrimination system (9) ([18–23]).

3. Single Traveling Wave Solutions of System (1)

Consider the traveling wave transformation as follows:

$$u(t, x) = U(\xi), v(t, x) = V(\xi), \xi = a \frac{t^\alpha}{\alpha} + b \frac{x^\beta}{\beta}, \quad (10)$$

where a and b are the constants.

Substituting (10) into (1), we obtain the ordinary differential equations as follows:

$$\begin{cases} aU' - bUU' - bV' + \gamma b^2U'' = 0, \\ aV' - b(UV' + VU') - \gamma b^2V'' = 0. \end{cases} \quad (11)$$

Integrating the second equation of (11) with respect to ξ , we have

$$V = \frac{a}{b}U - \frac{c_1}{b} - \frac{U^2}{2} + \gamma bU', \quad (12)$$

where c_1 is the integration constant. Substituting (12) into the first equation of (11), we obtain

$$-\gamma^2 b^3 U'' + \frac{b}{2} U^3 - \frac{3a}{2} U^2 + \left(c_1 + \frac{a^2}{b}\right) U - c_2 = 0, \quad (13)$$

where c_2 is the integration constant.

Multiplying both ends of (13) by U' and integrating once, we obtain

$$(U')^2 = \frac{1}{4\gamma^2 b^2} U^4 - \frac{a}{\gamma^2 b^3} U^3 + \frac{c_1 b + a^2}{\gamma^2 b^4} U^2 - 2c_2 U + 2c_3, \quad (14)$$

where c_3 is the integration constant.

In order to give the classification of traveling wave solutions of (1), we need to assume that the coefficient of U^3 is zero. So, we make the following transformations:

$$\begin{cases} \psi = \left(\frac{1}{4\gamma^2 b^2}\right)^{(1/4)} \left(U - \frac{a}{b}\right), \\ \xi_1 = \left(\frac{1}{4\gamma^2 b^2}\right)^{(1/4)} \xi. \end{cases} \quad (15)$$

Substituting (15) into (14), it becomes

$$(\psi_{\xi_1})^2 = \psi^4 + p\psi^2 + q\psi + r, \quad (16)$$

where $p = (2|\gamma b|(c_1 b + a^2)/\gamma^2 b^4) - (3/|\gamma||b|^3)$, $q = (4\gamma^2 b^2)^{1/4}[-(2a^3/\gamma^2 b^5) + (2a(a^2 + bc_1)/\gamma^2 b^5) - 2c_2]$, $r = -(3a^4/4\gamma^2 b^6) + (a^2(a^2 + bc_1)/\gamma^2 b^6) - (2ac_2/b) + 2c_3$.

The solution of (16) can be written in the following integral form:

$$\int \frac{d\psi}{\sqrt{\psi^4 + p\psi^2 + q\psi + r}} = \xi_1 - \xi_0, \quad (17)$$

where ξ_0 is an arbitrary constant.

Case 1. If $D_2 < 0$, $D_3 = 0$, $D_4 = 0$,

$$F(\psi) = [(\psi - l)^2 + s^2]^2, \quad (18)$$

where $s > 0$.

Substituting (18) into (17) and combining (14), we can get the solution of (1) as

$$\begin{aligned} u_1(t, x) = & \sqrt{[4]4\gamma^2 b^2} s \tan \left[s \sqrt{[4]4\gamma^2 b^2} \left(a \frac{t^\alpha}{\alpha} + b \frac{x^\beta}{\beta} \right) - s \xi_0 \right] \\ & + \left(l \sqrt{[4]4\gamma^2 b^2} + \frac{a}{b} \right). \end{aligned} \quad (19)$$

For example, when $a = 1, b = 1, c_1 = -1, c_3 = -(1/8), \gamma = 1$, and $\xi_0 = 0$, we obtain $u_1(t, x) = 1 + \tan(t^\alpha/\alpha + x^\beta/\beta)$. Therefore, we draw the three-dimensional diagram of solution $u_1(t, x)$ through Maple software, as shown in Figure 1.

Case 2. If $D_2 = 0, D_3 = 0, D_4 = 0$,

$$F(\psi) = \psi^4. \quad (20)$$

Substituting (20) into (17) and combining (14), we can get the solutions of (1) as

$$u_2(t, x) = -\frac{\sqrt{[4]}4\gamma^2b^2}{\sqrt{[4]}4\gamma^2b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0} + \frac{a}{b}. \quad (21)$$

Case 3. If $D_2 > 0, D_3 = 0, D_4 = 0, E_2 > 0$,

$$F(\psi) = (\psi - \theta)^2(\psi - \vartheta)^2, \quad (22)$$

where $\theta > \vartheta$.

Next, substituting (22) into (17) and combining (14), when $\psi > \theta$ or $\psi < \vartheta$, we can get the solution of equation (10) as

$$u_3(t, x) = \frac{(\theta + \vartheta)\sqrt{[4]}4\gamma^2b^2}{2} + \frac{\sqrt{[4]}4\gamma^2b^2(\vartheta - \theta)}{2} \coth \frac{(\theta - \vartheta)(\sqrt{[4]}4\gamma^2b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0)}{2}. \quad (23)$$

When $\vartheta < \psi < \theta$, we can attain

$$u_4(t, x) = \frac{(\theta + \vartheta)\sqrt{[4]}4\gamma^2b^2}{2} + \frac{\sqrt{[4]}4\gamma^2b^2(\vartheta - \theta)}{2} \tanh \frac{(\theta - \vartheta)(\sqrt{[4]}4\gamma^2b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0)}{2}. \quad (24)$$

For example, when $a = 1, b = 1, c_1 = -(1/2), c_2 = -(1/2), c_3 = (1/8), \gamma = 1$, and $\xi_0 = 0$, we obtain $u_4(t, x) = -\tanh(t^\alpha/\alpha + x^\beta/\beta)$. Therefore, we draw the kink solitary wave solution $u_4(t, x)$ through Maple software, as shown in Figure 2.

Case 4. If $D_2 > 0, D_3 > 0, D_4 = 0$,

$$F(\psi) = (\psi - \theta_1)^2(\psi - \theta_2)(\psi - \theta_3), \quad (25)$$

where θ_1, θ_2 , and θ_3 are the real numbers, and $\theta_2 > \theta_3$.

If $\theta_1 > \theta_2$ and $\psi < \theta_2$ or $\theta_1 < \theta_3$ and $\psi < \theta_3$, the implicit analytical solution of (13) is obtained:

$$\begin{aligned} & \pm (\xi_1 - \xi_0), \\ & = \int \frac{d\psi}{\sqrt{(\psi - \theta_1)^2(\psi - \theta_2)(\psi - \theta_3)}}, \\ & = \frac{1}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)} \ln \frac{\left[\sqrt{(\psi - \theta_2)(\theta_1 - \theta_3)} - \sqrt{(\theta_1 - \theta_2)(\psi - \theta_3)} \right]^2}{|\psi - \theta_1|}. \end{aligned} \quad (26)$$

If $\theta_1 > \theta_2$ and $\psi < \theta_3$ or when $\theta_1 < \theta_3$ and $\psi < \theta_2$, the implicit analytical solution of (13) is obtained:

$$\begin{aligned} & \pm (\xi_1 - \xi_0), \\ & = \int \frac{d\psi}{\sqrt{(\psi - \theta_1)^2(\psi - \theta_2)(\psi - \theta_3)}}, \\ & = \frac{1}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)} \ln \frac{\left[\sqrt{(\psi - \theta_2)(\theta_3 - \theta_1)} - \sqrt{(\theta_2 - \theta_1)(\psi - \theta_3)} \right]^2}{|\psi - \theta_1|}. \end{aligned} \quad (27)$$

If $\theta_2 > \theta_1 > \theta_3$, the implicit analytical solution of (13) is obtained:

$$\begin{aligned} & \pm (\xi_1 - \xi_0), \\ & = \int \frac{d\psi}{\sqrt{(\psi - \theta_1)^2(\psi - \theta_2)(\psi - \theta_3)}}, \\ & = \frac{1}{(\theta_2 - \theta_1)(\theta_1 - \theta_3)} \arcsin \frac{(\psi - \theta_2)(\theta_1 - \theta_3) + (\theta_1 - \theta_2)(\psi - \theta_3)}{[(\psi - \theta_1)(\theta_2 - \theta_3)]}. \end{aligned} \quad (28)$$

Case 5. If $D_2 > 0, D_3 = 0, D_4 = 0, E_2 = 0$,

$$F(\psi) = (\psi - \nu)^3(\psi - \omega), \quad (29)$$

where ν and ω are the real numbers.

Substituting (29) into (17) and combining (14), when $\psi > \nu$ and $\psi > \omega$ or when $\psi < \nu$ and $\psi < \omega$, we can yield

FIGURE 1: The solution $u_1(t, x)$ of equation (19) with $a = 1$, $b = 1$, $c_1 = -1$, $c_2 = -1$, $c_3 = -(1/8)$, $\gamma = 1$, and $\xi_0 = 0$.

FIGURE 2: The solution $u_4(t, x)$ of equation (24) with $a = 1$, $b = 1$, $c_1 = -(1/2)$, $c_2 = -(1/2)$, $c_3 = (1/8)$, $\gamma = 1$, and $\xi_0 = 0$.

$$u_5(t, x) = \frac{4(\nu - \omega)\sqrt{[4]}\gamma^2 b^2}{(\omega - \nu)^2 \left[\sqrt{[4]}\gamma^2 b^2 (at^\alpha/\alpha + bx^\beta/\beta) - \xi_0 \right]^2 - 4} + \left(\nu\sqrt{[4]}\gamma^2 b^2 + \frac{a}{b} \right). \quad (30)$$

Case 6. If $D_2 D_3 < 0$, $D_4 = 0$,

$$F(\psi) = (\psi - \vartheta_1)^2 \left[(\psi - \vartheta_2)^2 + \vartheta_3^2 \right], \quad (31)$$

where ϑ_1 , ϑ_2 , and ϑ_3 are the real numbers.

Substituting (31) into (14), we obtain the solution of equation (1):

$$u_6(t, x) = \sqrt[4]{4\gamma^2 b^2} \frac{\left[e^{\pm \sqrt{(\vartheta_1 - \vartheta_2)^2 + \vartheta_3^2}} \left(\sqrt[4]{4\gamma^2 b^2} (at^\alpha/\alpha + bx^\beta/\beta) - \xi_0 \right) - \varepsilon \right] + \sqrt{(\vartheta_1 - \vartheta_2)^2 + \vartheta_3^2} (2 - \varepsilon)}{\left[e^{\pm \sqrt{(\vartheta_1 - \vartheta_2)^2 + \vartheta_3^2}} \left(\sqrt[4]{4\gamma^2 b^2} (at^\alpha/\alpha + bx^\beta/\beta) - \xi_0 \right) - \varepsilon \right]^2 - 1} + \frac{a}{b}, \quad (32)$$

where $\varepsilon = (\vartheta_1 - 2\vartheta_2)/\sqrt{(\vartheta_1 - \vartheta_2)^2 + \vartheta_3^2}$.

Case 7. If $D_2 > 0$, $D_3 > 0$, $D_4 > 0$,

$$F(\psi) = (\psi - \rho_1)(\psi - \rho_2)(\psi - \rho_3)(\psi - \rho_4), \quad (33)$$

where ρ_1 , ρ_2 , ρ_3 , and ρ_4 are the real numbers and $\rho_1 > \rho_2 > \rho_3 > \rho_4$.

Then, we can obtain the solution of (1):

$$u_7(t, x) = \sqrt{[4]} 4\gamma^2 b^2$$

$$\frac{\rho_2(\rho_1 - \rho_4)sn^2\left(\sqrt{(\rho_1 - \rho_3)(\rho_2 - \rho_4)}/2\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m\right) - \rho_1(\rho_2 - \rho_4)}{(\rho_1 - \rho_4)sn^2\left(\sqrt{(\rho_1 - \rho_3)(\rho_2 - \rho_4)}/2\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m\right) - (\rho_2 - \rho_4)} + \frac{a}{b}, \quad (34)$$

$$u_8(t, x) = \sqrt{[4]} 4\gamma^2 b^2$$

$$\frac{\rho_4(\rho_2 - \rho_{34})sn^2\left(\sqrt{(\rho_1 - \rho_3)(\rho_2 - \rho_4)}/2\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m\right) - \rho_2(\rho_2 - \rho_4)}{(\rho_2 - \rho_3)sn^2\left(\sqrt{(\rho_1 - \rho_3)(\rho_2 - \rho_4)}/2\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m\right) - (\rho_2 - \rho_4)} + \frac{a}{b}, \quad (35)$$

where $m^2 = (\rho_1 - \rho_4)(\rho_2 - \rho_3)/(\rho_1 - \rho_3)(\rho_2 - \rho_4)$.

Case 8. If $D_2 D_3 \geq 0$, $D_4 < 0$,

$$F(\psi) = (\psi - s_1)(\psi - s_2)[(\psi - s_3)^2 + s_4^2], \quad (36)$$

where s_1 , s_2 , s_3 , and s_4 are the real constants, $s_1 > s_2$ and $s_4 > 0$.

Then, we can give the solution of (1):

$$u_9(t, x) = \sqrt{[4]} 4\gamma^2 b^2 \frac{\varepsilon_1 cn^2\left(\sqrt{\mp 2s_4 m_1(s_1 - s_2)}/2mm_1\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m\right) + \varepsilon_2}{\varepsilon_3 cn^2\left(\sqrt{\mp 2s_4 m_1(s_1 - s_2)}/2mm_1\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m\right) + \varepsilon_4} + \frac{a}{b}, \quad (37)$$

where $\varepsilon_1 = (1/2)(s_1 + s_2)\varepsilon_3 - (1/2)(s_1 + s_2)\varepsilon_4$, $\varepsilon_2 = (1/2)(s_1 + s_2)c_4 - (1/2)(s_1 + s_2)\varepsilon_3$, $\varepsilon_3 = s_1 - s_3 - (s_4/m_1)$, $\varepsilon_4 = s_1 - s_3 - s_4 m_1$, $E = (s_4^2 + (s_1 - s_3)(s_2 - s_3)/s_4(s_1 - s_2))$, $m_1 = E \pm \sqrt{E^2 + 1}$.

Case 9. If $D_2 D_3 \leq 0$, $D_4 > 0$,

$$u_{10}(t, x) = \sqrt{[4]} 4\gamma^2 b^2$$

$$\left[\frac{\varepsilon_1 sn\left(\mu\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m_2\right) + \varepsilon_2 cn\left(\mu\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m_2\right)}{\varepsilon_3 sn\left(\mu\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m_2\right) + \varepsilon_4 cn\left(\mu\left(\sqrt{[4]} 4\gamma^2 b^2(at^\alpha/\alpha + bx^\beta/\beta) - \xi_0\right), m_2\right)} \right] + \frac{a}{b}, \quad (39)$$

where $\varepsilon_1 = \chi_1 \varepsilon_3 + l_1 \varepsilon_4$, $\varepsilon_2 = \chi_1 \varepsilon_4 - l_1 \varepsilon_3$, $\varepsilon_3 = -l_1 - (l_2/m_1)$, $\varepsilon_4 = \chi_1 - \chi_2$, $m^2 = (m_1^2 - 1/m_1^2)$, $E = ((\chi_1 - \chi_2)^2 + l_1^2 + l_2^2/2l_1 l_2)$, $m_1 = E + \sqrt{E^2 - 1}$, and $\mu = (l_2 \sqrt{(\varepsilon_3^2 + \varepsilon_4^2)(m_1^2 \varepsilon_3^2 + \varepsilon_4^2)}/\varepsilon_3^2 + \varepsilon_4^2)$.

Remark 1. In this section, we obtain all single traveling wave solutions $u(t, x)$ of (1) by using the complete discriminant of fourth-order polynomials. Accordingly, we can obtain the solutions $v(t, x)$ of equation (1) through the relationship (12) and the solutions $u_1(t, x) \sim u_{10}(t, x)$.

$$F(\psi) = [(\psi - \chi_1)^2 + l_2^2][(\psi - \chi_2)^2 + l_2^2], \quad (38)$$

where χ_1 , χ_2 , l_1 , and l_2 are the real constants, and $l_1 \geq l_2 > 0$.

So, we can give the solution of (1):

4. Conclusion

The construction of exact solutions of fractional partial differential equations has always been a very important research field. In this study, the fractional partial differential equations are simplified into an ordinary differential equation with only one variable by integration and some transformations. In this way, through the complete discriminant system of polynomials, we can easily obtain the classification of the exact solutions of the ordinary differential equation. We apply the method to the classification of

the single traveling wave solutions of the fractional approximate long water wave equations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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