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Research Article

Solitary Wave and Singular Wave Solutions for Ivancevic Option Pricing Model

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The nonlinear option pricing model presented by Ivancevic is investigated. By using travelling wave transforming method, the nonlinear option pricing equation is transformed into a differential equation with constant coefficients. By solving the differential equation with F-expansion method, a series of exact solutions have been obtained for the Ivancevic option pricing model. By choosing appropriate parameter values, the dark-soliton and dark-soliton-like solutions, periodic wave solutions, and rogue wave solutions are obtained. These solutions will enrich the types of exact waves in the existing literature of the Ivancevic option pricing model. Furthermore, they may have potential uses in describing the possible physical mechanisms for wave phenomenon in financial markets.

1. Introduction

The mathematical study of economy and finance problems has been one of the most interesting topics in financial risk field. Sharp proposed the stochastic differential equations (SDEs) in financial market [1]. Irving and Dewson investigated the mixed linear-nonlinear coupled differential equations based on multivariate discrete time series sequences [2]. Later, a continuous-time finance model was constructed according to the optimal consumption and portfolio rules [3]. Priya and Muthukumar studied controllability in terms of fractional-order impulsive stochastic differential equation [4]. Decardi-Nelson and Liu proposed an algorithm to determine the economic zone to be tracked based on the robust economic model [5]. As the economy and financial markets are nonlinear systems, nonlinear science is widely applied in constructing mathematical models to find deep properties of economy and financial problems. Most of nonlinear phenomena are generally presented in the form of nonlinear partial differential equations (NPDEs) [6–8]. To conduct the mathematical models of finance, it is imperative to determine that the models are either complex-valued or real values with wave function. Many researchers developed the models to extract wave distributions. In recent years, experts have investigated different types of waves based on NPDEs, such as mixed lump wave [9], three-wave [10], breather [11], rogue waves [12], multiple complex soliton [13], bright and dark-soliton [14], complex wave [15], soliton solution [16], travelling wave solutions [17], dark waves [18], and double-wave solutions [19]. On the other hand, soliton theory is one of the widely used theories due to the exact information, such as periodic, singular, dark, bright, complex, and travelling, which can be obtained. These kinds of dynamical information help to understand, predict, and control the complex behaviors of financial market. Nowadays, various powerful technical tools have been proposed to explore these NPDEs, such as inverse scattering method [20], Hirota's bilinear method [21], F-expansion method [22–30], and auto-Bäcklund transformation method [31].

As for the option pricing model, Black and Scholes proposed the classic Black–Scholes model [32], which determines a fair market value of an option. Over the past several decades, the Black–Scholes model has been increasingly popular because it provided an effective method to model the option value and extract implied volatilities.

Later, a quantum-probability-based option-pricing model was presented by Ivancevic to describe the controlled Brownian behavior of financial markets. This adaptive-wave model is formally defined by the adaptive nonlinear Schrödinger equation, defining the option-pricing wave function in terms of the stock price and time [33]. Later, this model was renamed as the Ivancevic option pricing model (IOPM). As an alternative model of Black–Scholes equation, the Ivancevic option pricing model has been increasingly attracting over the last few years since it provides the values of options effectively.

The Ivancevic option pricing model can be written by [33]

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\sigma\frac{\partial^2\psi}{\partial s^2}+\lambda|\psi|^2\psi=0, \tag{1}$$

where ψ (s, t) represents the complex-valued option price wave function and $|\psi|^2$ denotes the probability density function for the option price in terms of the stock price s and time t. The volatility σ can be either constant or stochastic process, which is dispersion frequency coefficient. In the Black–Scholes model, the underlying volatility is assumed to be a constant over the life of the derivative and unaffected by the changes in the price level. λ signifies the Landau coefficient, representing the adaptive market potential. In simplest nonadaptive scenario, λ is equal to r, which represents interest rate.

The nonlinear evolution (1) has wide applications in optics, fluid dynamics, and finance. Note that some kink wave solutions and travelling wave solutions have also been generated for the Zoomeron equation, which has a similar form as (1) [34]. The trial function method, tanh expansion method, and direct perturbation method have been explored to get the exact solutions of the Ivancevic option pricing model. Financial rogue wave solutions were studied on the Ivancevic option pricing model [35]. The dynamics of financial rogue wave solutions may be used to explain real financial crisis/storms (e.g., 1997 Asian financial crisis/storm and the current global financial crisis/storm). Later, Yan proposed a coupled nonlinear volatility and option pricing model to propose the vector financial rogue waves [36]. Vector financial one and two-rogon solutions were found for the coupled nonlinear volatility and option pricing model without embedded w-learning. In 2017, a model augmented by external atomic potentials was proposed to price European call options on a stock index [37]. In [38], a nonzero adaptive market potential was studied. Later, novel types of rogue wave and dark wave solutions were constructed for the Ivancevic option pricing model based on the trial function method [39]. The Ivancevic option pricing model also can be solved by rational sine-Gordon expansion method and modified exponential method. Through these two methods, complex, periodic, mixed dark-bright, singular, travelling, and hyperbolic functions have been extracted [40]. Very recently, a novel semi-analytical technique was carried out to solve the time-fractional Ivancevic option pricing model [41]. Since linearization is not required in this method, complex numerical computations were significantly reduced compared to the existing perturbation technique.

In this paper, the F-expansion method based on hyperbolic secant functions and tangent functions is used to reach the exact solutions for the Ivancevic option pricing model. To enrich the wave solution types, a series of novel wave solutions are found for the probability density function. The organization of this paper is as follows. In Section 2, the nonlinear option pricing equation is transformed into a differential equation with constant coefficients by using the travelling wave transforming method. In Section 3, dark-soliton solutions, periodic wave solutions, and singular wave solutions are obtained by solving the differential equation with the F-expansion method. Section 4 will conclude this paper.

2. Methodology

To get various kinds of financial wave solutions, (1) will be solved exactly using the F-expansion method [42]. We look for the travelling wave solution to (1) in the form

$$\psi(s,t)=\varphi(\xi)\exp[i(ks-\omega t)], \tag{2}$$

where $\phi(\xi)$ is a function depending on $\xi=s-\sigma kt$. The substitution of (2) into (1) leads to the nonlinear oscillator ordinary differential equation:

$$\frac{1}{2}\sigma\varphi''(\xi)+\left[\omega-\frac{1}{2}\sigma k^2\right]\varphi(\xi)+\beta\varphi^3(\xi)=0. \tag{3}$$

Let $\gamma=(1/2)\sigma k^2-\omega$, $\alpha=(\sigma/2)$, and then (3) reads as

$$\alpha\varphi''(\xi)+\gamma\varphi(\xi)+\beta\varphi^3(\xi)=0. \tag{4}$$

We assume that (3) has the following solution [42]:

$$\varphi(\xi)=\sum_{i=-N}^{N}a_iF^i. \tag{5}$$

In (5), a_i is a real constant to be determined, and the function $F(\xi)$ should obey the following ordinary differential equation:

$$F'=AF^3+\frac{1}{2}F^{-1}, \tag{6}$$

where $F'=(\mathrm{d}F/\mathrm{d}\xi)$ and A is an unknown constant to be determined. By solving (6), one can get a series of general solutions. When $A>0$, the general solution can be read as

$$F_1=\sqrt{\frac{\tan\left(\sqrt{2A}\,\xi+C_1\right)}{\sqrt{2A}}}. \tag{7}$$

When $A<0$, (6) has two kinds of general solutions in the following form:

$$F_2=\sqrt{\frac{\tanh\left(\sqrt{-2A}\,\xi+C_2\right)}{\sqrt{-2A}}}, \tag{8}$$

or

$$F_3 = \sqrt{\frac{\coth\left(\sqrt{-2A}\,\xi + C_3\right)}{\sqrt{-2A}}}. \tag{9}$$

In equations (6)–(8), C_1, C_2, and C_3 are free integral constants. Changing the values of these parameters will not result in the change of the overall shape of the wave function but only affect the position of the wave.

Due to the requirements of the homogeneous equilibrium, each derivative will increase the power of the equation by 2. If the power of the function F is n, i.e., $d(F) = n$, we can easily get $d(F') = n + 2$. To be sure that the highest derivative term is in equilibrium with the nonlinear term, $N = 2$ must be ensured in (5). So, the solution of (3) is

$$\varphi(\xi) = \frac{a_{-2}}{F^2} + \frac{a_{-1}}{F} + a_0 + a_1 F + a_2 F^2. \tag{10}$$

Insert (5) and (6) into (3), and let the coefficient before each power of F be zero, and we can get the following set of super-algebra equations:

$$-\beta a_{-2}^3 - 2\alpha a_{-2} = 0, \tag{11}$$

$$-\beta a_1^3 - \alpha a_{-1} A^2 - 3\beta a_{-1} a_2^2 - 6\beta a_0 a_1 a_2 = 0, \tag{12}$$

$$-3\beta a_{-2}^3 a_0 - 3\beta a_{-2} a_{-1}^2 = 0, \tag{13}$$

$$-8\alpha a_2 A^2 - \beta a_2^3 = 0, \tag{14}$$

$$\begin{aligned} &-\gamma a_0 - 6\beta a_{-2} a_0 a_2 - 3\beta a_{-1}^3 a_2 - \beta a_0^3 \\ &-6\beta a_{-1} a_0 a_1 - 3\beta a_{-2} a_1^2 = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} &-3\beta a_0 a_1^2 - 3\beta a_0^3 a_2 + 4\alpha a_2 A \\ &-6\beta a_{-1} a_1 a_2 + \gamma a_2 - 3\beta a_{-2} a_2^2 = 0, \end{aligned} \tag{16}$$

$$-\beta a_{-1}^3 + \frac{1}{4}\alpha a_1 - 6\beta a_{-2} a_{-1} a_0 - 3\beta a_{-2}^3 a_1 = 0, \tag{17}$$

$$\begin{aligned} &-6\beta a_{-1} a_0 a_2 + \gamma a_1 + \alpha a_1 A \\ &-3\beta a_0^2 a_1 - 6\beta a_{-2} a_1 a_2 - 3\beta a_{-1} a_1^2 = 0, \end{aligned} \tag{18}$$

$$\begin{aligned} &-6\beta a_{-2} a_0 a_1 - 3\beta a_{-1}^2 a_1 - 6\beta a_{-2} a_{-1} a_2 \\ &-3\beta a_{-1} a_0^2 - \gamma a_{-1} + \alpha a_{-1} A = 0, \end{aligned} \tag{19}$$

$$-3\beta a_0 a_2^2 - 3\beta a_1^2 a_2 = 0, \tag{20}$$

$$-3\beta a_1 a_2^2 - 3\alpha a_1 A^2 = 0, \tag{21}$$

$$\begin{aligned} &4\alpha a_{-2} A - 3\beta a_{-2} a_0^2 - 3\beta a_{-1}^2 a_0 + \gamma a_{-2} \\ &-3\beta a_{-2}^2 a_2 - 6\beta a_{-2} a_{-1} a_1 = 0. \end{aligned} \tag{22}$$

From (11), we have $a_{-2} = \sqrt{(-2\alpha/\beta)}$ or $a_{-2} = 0$. The values of the remaining parameters can be reached by solving the rest of the equations, i.e., equations (11)–(21).

3. Results and Discussion

Case 1. To obtain nonzero solutions of (3), we suppose that all the coefficients a_1 $(i \neq 2)$ are 0, and thus we have

$$\begin{aligned} a_2 &= \frac{\gamma}{\sqrt{-2\alpha\beta}}, \\ A &= -\frac{\gamma}{4\alpha}. \end{aligned} \tag{23}$$

Therefore, (5) reads as

$$\varphi(\xi) = \begin{cases} \dfrac{\gamma}{\sqrt{-2\alpha\beta}} \dfrac{\tan\left(\sqrt{2A}\,\xi + C_1\right)}{\sqrt{2A}} = \sqrt{\dfrac{\gamma}{\beta}} \tan\left(\sqrt{\dfrac{-\gamma}{2\alpha}}\,\xi + C_1\right), \text{when}\, \alpha\beta > 0, \alpha\gamma < 0; \\ \dfrac{\gamma}{\sqrt{-2\alpha\beta}} \sqrt{\dfrac{\tanh\left(\sqrt{-2A}\,\xi + C_2\right)}{\sqrt{-2A}}} = \sqrt{-\dfrac{\gamma}{\beta}} \tanh\left(\sqrt{\dfrac{\gamma}{2\alpha}}\,\xi + C_2\right), \text{when}\, \alpha\beta < 0, \alpha\gamma > 0; \\ \dfrac{\gamma}{\sqrt{-2\alpha\beta}} \sqrt{\dfrac{\coth\left(\sqrt{-2A}\,\xi + C_3\right)}{\sqrt{-2A}}} = \sqrt{-\dfrac{\gamma}{\beta}} \coth\left(\sqrt{\dfrac{\gamma}{2\alpha}}\,\xi + C_3\right), \text{when}\, \alpha\beta < 0, \alpha\gamma > 0. \end{cases} \tag{24}$$

In the following, we discuss the case of $a_{-2} = \sqrt{(-2\alpha/\beta)}$.

Obviously, the intensity distribution $|\psi(s,t)|^2$ depends on the values of the parameters according to equations (24) and (25). Figure 1 shows the propagation of the financial wave given by (24) for the chosen adaptive market potential $\alpha = -1$, $\beta = 1$, $\gamma = 1$, $k = 0.1$, and $C_1 = 0$. In this figure, a periodic solution is obtained. $\varphi(\xi)$ will be infinity when $\sqrt{(-\gamma/2\alpha)}\xi + C_1 = (\pi/2) + p\pi$, where p is an arbitrary integer. If the volatility is fixed, the probability for the option price will become particularly large when the relationship between asset price and time maintains that $\sqrt{(-\gamma/2\alpha)}(s - \sigma k t) + C_1 = (\pi/2) + p\pi$. Once the values of s and t deviate from this relation, the probability intensity becomes zero quickly.

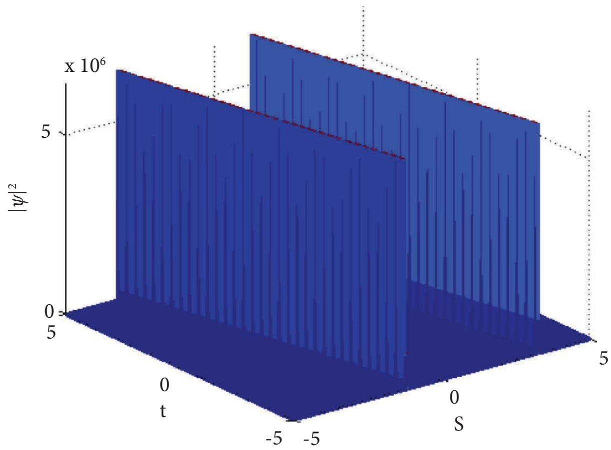

FIGURE 1: Propagation of a periodic soliton solution (24) for $\alpha = -1$, $\beta = 1$, $\gamma = 1$, $k = 0.1$, and $C_1 = 0$.

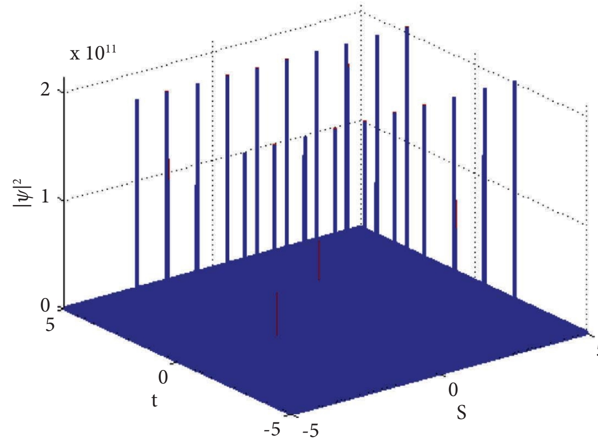

FIGURE 2: Propagation of a rogue soliton solution (24) for $\alpha = -1$, $\beta = 1$, $\gamma = 1$, $k = 50$, and $C_1 = 0$.

(24) also admits a type of financial rogue waves. Figure 2 shows a typical example of rogue waves when $\alpha = -1$, $\beta = 1$, $\gamma = 1$, $k = 50$, and $C_1 = 0$. This solution uniformly approaches the constant background zero at most values of s and t. But in some intermediate time, it blows up to infinity. The existence of exploding rogue waves in the Ivancevic option pricing equation is a distinctive phenomenon, and their occurrence would be catastrophic in real financial markets.

(24) gives a dark-soliton solution for the Ivancevic option pricing model. A typical intensity propagation is shown in Figure 3 when $\alpha = 1$, $\beta = -2$, $\gamma = 3$, $k = 0.1$, and $C_2 = 0$. It is found that the financial soliton keeps its shape unchanged during the propagation process. During the long-time evolution, the financial soliton keeps the energy and momentum unchanged.

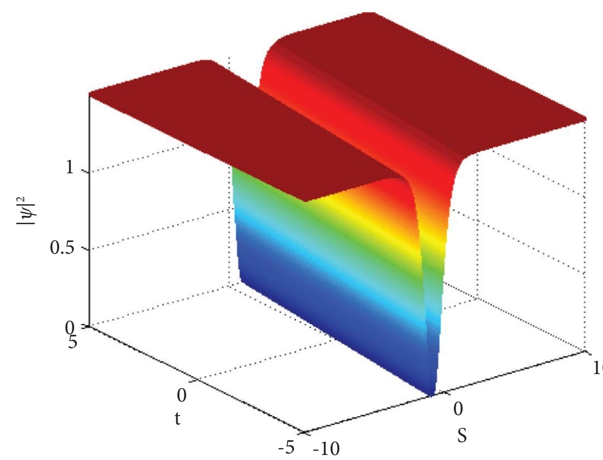

FIGURE 3: Propagation of a dark-soliton solution (24) for $\alpha = 1$, $\beta = -2$, $\gamma = 3$, $k = 0.1$, and $C_2 = 0$.

Case 2. If $a_i\,(i \neq -2) = 0$, we can obtain from equations (11) to (21) that

$$a_{-2} = \sqrt{\frac{-2\alpha}{\beta}},\qquad A = -\frac{\gamma}{4\alpha}. \tag{25}$$

When $\alpha\beta > 0, \alpha\gamma < 0$, we can obtain a new solution of (5):

$$\varphi(\xi) = \sqrt{\frac{-2\alpha}{\beta}}\left[\frac{\tan(\sqrt{2A}\,\xi + C_1)}{\sqrt{2A}}\right]^{-1} = \sqrt{\frac{\gamma}{\beta}}\cot\left(\sqrt{-\frac{\gamma}{2\alpha}}\,\xi + C_4\right). \tag{26}$$

Figure 4 gives a typical example of the propagation of a singular wave solution when $\alpha = 1$, $\beta = 2$, $\gamma = 1$, $k = 0.1$, and $C_2 = -2$.

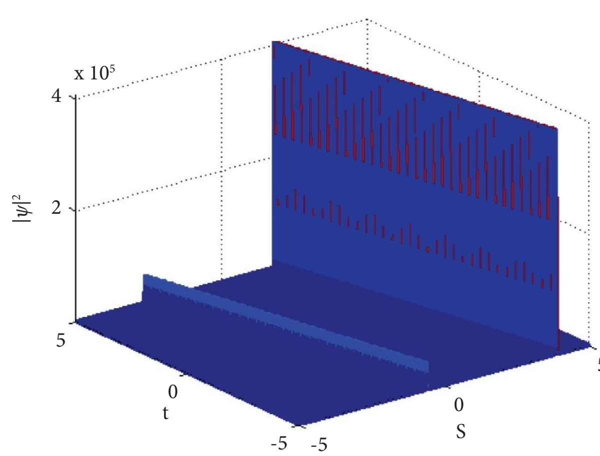

FIGURE 4: Wave propagation of solution (27) for $\alpha = 1$, $\beta = 2$, $\gamma = 1$, $k = 0.1$, and $C_2 = -2$.

Case 3. If $a_i\,(i = \pm 2) \neq 0$, we can obtain from (11) and (14) that

$$a_{-2} = \sqrt{\frac{-2\alpha}{\beta}},$$
$$a_2 = -\frac{\gamma}{\sqrt{-8\alpha\beta}}, \quad (27)$$
$$A = \frac{\gamma}{8\alpha},$$

or

$$a_{-2} = \sqrt{\frac{-2\alpha}{\beta}},$$
$$a_2 = -\frac{\gamma}{\sqrt{-32\alpha\beta}}, \quad (28)$$
$$A = -\frac{\gamma}{16\alpha}.$$

When the coefficients get the value as those in equation (28), a series of exact solutions can be obtained.

When $\alpha\beta < 0$ and $\alpha\gamma > 0$, we have

$$\varphi(\xi) = \sqrt{-\frac{\gamma}{2\beta}}\left[\cot\left(\sqrt{\frac{\gamma}{4\alpha}}\xi + C_{5a}\right) - \tan\left(\sqrt{\frac{\gamma}{4\alpha}}\xi + C_{5b}\right)\right]. \quad (29)$$

When $\alpha\beta > 0$ and $\alpha\gamma > 0$, we have

$$\varphi(\xi) = \pm\sqrt{\frac{\gamma}{4\beta}}\left[\coth\left(\sqrt{\frac{\gamma}{8\alpha}}\xi + C_{6a}\right) - \tanh\left(\sqrt{\frac{\gamma}{8\alpha}}\xi + C_{6b}\right)\right]. \quad (30)$$

When $\alpha\beta > 0$ and $\alpha\gamma < 0$, we have

$$\varphi(\xi) = \sqrt{\frac{\gamma}{4\beta}}\left[\cot\left(\sqrt{-\frac{\gamma}{8\alpha}}\xi + C_{7a}\right) - \tan\left(\sqrt{-\frac{\gamma}{8\alpha}}\xi + C_{7b}\right)\right], \quad (31)$$

$$\varphi(\xi) = \pm\sqrt{\frac{\gamma}{2\beta}}\left[\coth\left(\sqrt{-\frac{\gamma}{4\alpha}}\xi + C_{8a}\right) - \tanh\left(\sqrt{-\frac{\gamma}{4\alpha}}\xi + C_{8b}\right)\right]. \quad (32)$$

Obviously, equations (30)–(32) give a series of exact solutions of (2), including bright soliton solutions, rogue wave solutions, and dark-soliton-like wave solutions. Figure 5 gives a typical example of a financial bright wave solution of (30) when $\alpha = 1$, $\beta = 100$, $\gamma = 1$, $k = 1, C_{6a} = 0.5$, and $C_{6b} = -0.5$. In this case, the probability density function for the option price remains unchanged with the evolution of time. Therefore, the financial market will be stable and the chance of large-scale financial risks is low.

When the free parameters α, β, andγ get some certain values, (31) can admit dark-soliton-like wave solutions. Figure 6 gives a typical example of a dark-soliton-like wave solution of (31) when $\alpha = 10, \beta = 2, \gamma = 7, k = 0.02, C_{7a} = 2$, and $C_{7b} = 2$. In most time, the probability density function for the option price remains are a dark-soliton embedded in a nonzero background. Different from that described in

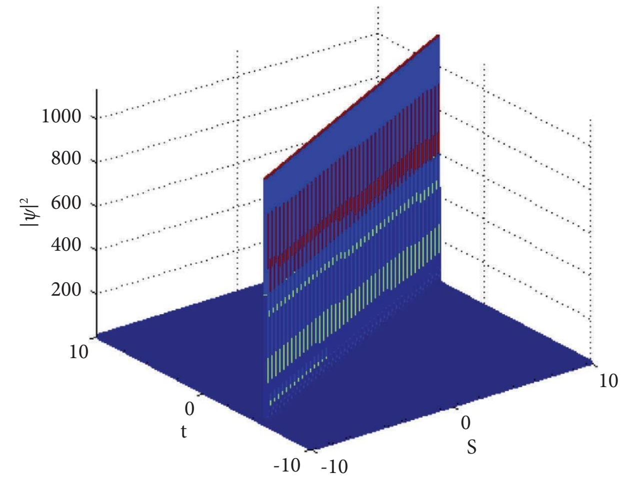

FIGURE 5: A financial bright wave solution of equation (31) when $\alpha = 1$, $\beta = 100$, $\gamma = 1$, $k = 1, C_{6a} = 0.5$, and $C_{6b} = -0.5$.

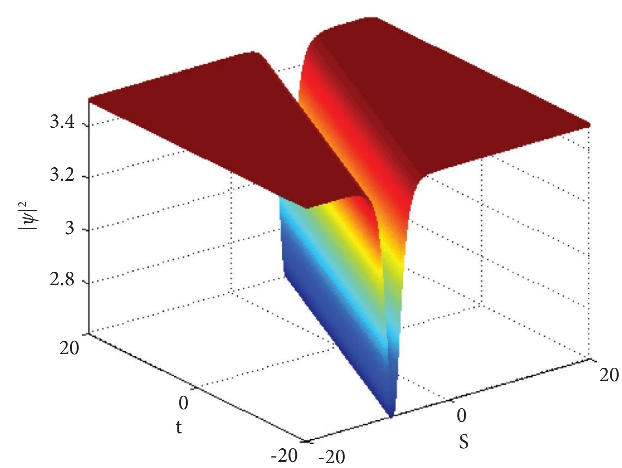

FIGURE 6: A dark-soliton-like wave solution of equation (31) when $\alpha = 10$, $\beta = 2$, $\gamma = 7$, $k = 0.02, C_{7a} = 2$, and $C_{7b} = 2$.

Figure 4, the probability density is a nonzero finite minimum value at the center in Figure 6.

4. Conclusions

In conclusion, we have investigated the exact solutions of the option pricing model. Using travelling wave transformation and the F-expansion method, abundant exact solutions of the nonlinear option pricing equation are obtained in terms of hyperbolic secant functions and tangent functions. If the parameters take special values, we get the existing solitary wave solutions, singular soliton solution, periodic solutions, including dark-soliton solutions, periodic wave solutions, rogue wave solutions, and singular wave solutions. These results may be used to explain some financial crisis.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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