Research Article

Seismic Signal Denoising Based on Adaptive Wavelet Modulus Maximum Method

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Seismic noise suppression plays an important role in seismic data processing and interpretation. Aiming at remedying the problem of low quality of seismic data acquired by a seismometer, a novel denoising method based on wavelet maximum modulus and an adaptive threshold is designed. This adaptive wavelet maximum modulus (ATWMM) seismic signal noise reduction method is using the opposite polarity of the Lipschitz index with seismic signal and noise to extract the seismic signal of original data. Setting adaptive thresholding function related to wavelet decomposition scale to solve the problem of effective signal losing on a small decomposition scale. The experimental results show that the ATWMM method can extract more seismic signal from the noisy seismic data. Using RMSE and SNR evaluation, the synthetic Ricker seismic dataset experimental results show that the indexes are 0.0502 and 20.1617 dB. Compared with the wavelet modulus maximum (WMM) method, it has a 25.7% reduction and 14.6% increase. The real-field seismic data came from the JZW51 seismic monitoring station in China experimental results indicate that the proposed ATWMM method is effective in seismic signal denoising with SNR above 30.1855 dB of approximately 15.8% enhancement compared with the WMM method, that has improvement for quality of seismic data.

1. Introduction

An earthquake is a natural disaster that endangers human life and property security. Seismic data processing and interpretation plays an important role in the seismic monitoring before and after earthquakes. However, seismic data are usually subject to varying degrees of noise in field acquisition at seismic monitoring stations, resulting in the reduction of data quality. Seismic noise suppression can bring more accuracy of subsequent seismic data analysis, so data denoising is of significance in increasing the quality of seismic data [1–4].

How to remove noise from seismic data is an essential research field in seismic signal processing. At present, there are many methods for denoising seismic signals widely used, including the t-x domain and transformed domain. Denoising methods in the t-x domain consist of the median filtering [5], polynomial fitting [6], and nonlocal means filtering [7]. Those methods use the differences between seismic signal and noise in the t-x domain of travel time and apparent velocity. Denoising methods in transformed domain consist of f-x predictive filtering [8], Fourier transform [9], wavelet transform [10–12], curvelet transform [13], seislet transform [14], shearlet transform [15, 16], mode decomposition [17], and so on. These methods apply a transformation to the seismic data from the t-x domain to another domain such as the f, s domain, where signal and noise can be separated, then remove noise from seismic data via setting a threshold in the transformed domain, and finally inverse transform the data back to t-x domain [18].

Among them, wavelet transform has received much more attention in seismic noise suppression research because of its powerful nonstationary signal time-frequency analysis capability, computation-efficient and maintenance, low cost, and acceptable detection signal quality. Meanwhile, the denoising effect of the wavelet-based method is highly
dependent on a suitable thresholding function. Choice of an appropriate thresholding function is challenging [19–21].

Donoho and Johnstone introduced a wavelet threshold denoising method, with a threshold processing of wavelet coefficients and reconstruct the original signal, which can effectively suppress random interference to realize the separation of signal and noise. The hard-thresholding and soft-thresholding functions have been widely used in the analysis and denoising of seismic data. However, hard-thresholding induces additional resonance because of discontinuous thresholding values. Soft-thresholding brings information loss due to excessive shrinkage of signal-related coefficients [22–24]. Yu et al. developed a combination method of complex wavelet transform and normal moveout transform to noise attenuation to achieve signal-to-noise ratio improvements without compromising data bandwidth in seismic data [25]. Langston et al. presented a procedure for removing noise from seismic time series using the continuous wavelet transform. Its noise sample used to estimate the thresholding parameter is stationary throughout the seismic time series. This shows that CWT techniques offer a technique for analyzing noise and signal useful for the detection of the seismic signal [26]. Long et al. proposed an improved empirical mode decomposition-wavelet threshold denoising method. The test result shows that the improved denoising method can effectively remove noise in seismic signals and preserve the effective signals of the target [27]. Mallat introduced a wavelet transform modulus maxima denoising algorithm using the local maximum point of wavelet transform coefficients to reconstruct the signal, which is suitable for time varying filtering of nonstationary signals, such as seismic signal [28]. Xiong et al. present an analysis by using wavelet maxima and methods of Eddy field calculation mean to detect seismic anomalies. Using wavelet transformations as data mining tools by computing the maxima that can be used to identify obvious anomalies in seismic data [29]. Lu et al. invited method based on wavelet transform and fast Fourier transform and carries out threshold denoising on seismic signal to reduce noise. [30]. The rest of this paper is structured as follows: Section 2 explains the ATWMM method principles. Experiments and discussions are presented in Section 3, including synthetic seismic datasets and real-field seismic dataset noise reduction experiments. Section 4 represents conclusions.

2. ATWMM Noise Reduction Method

A recorded seismic signal $x(t)$ can be expressed as follows:

$$x(t) = s(t) + n(t),$$

where $s(t)$ is the superposition of seismic signal and $n(t)$ is the noise, i.e., some additional natural/environmental/instrumental noise or nonseismic signal. The goal of noise reduction is to estimate the denoised seismic signal $\hat{s}(t)$ with a minimized root mean square error (RMSE) between $\hat{s}(t)$ and $s(t)$.

$$\text{RMSE} = \sqrt{\frac{1}{m} \sum_{n=1}^{m} [\hat{s}(t) - s(t)]^2},$$

Decomposing recorded seismic signal based on wavelet transform, we obtain wavelet coefficients $W_x(2^j, \tau)$, where $j$ and $\tau$ are scaling parameter and shifting parameter, respectively. Since seismic signal and noise elements produce wavelet coefficients with different propagation characteristics, given a suitable threshold $TH_x$ to suppress noise by eliminating the magnitude of noise-related coefficients. The denoised seismic signal $\hat{s}(t)$ can be obtained by inverse transform the reserved coefficients after thresholding into the time domain. Wavelet modulus maxima $\Omega$ correspond to the singularities of the recorded seismic signal, which can characterize by Lipschitz Index $\alpha$. Seismic signals have positive Lipschitz Index values, but noise elements have negative values [28]. The relationship theorem between wavelet modulus maxima and Lipschitz Index is expressed as follows:

$$\log_2|W_x(2^j, \tau)| \leq \log_2 K + ja,$$

where $K$ is constant. According to propagation characteristics of wavelet modulus maxima, seismic signals have larger wavelet modulus maxima values with scaling parameter increase [28]. However, the noise element’s wavelet modulus maxima values are smaller with scaling parameter increase.

In the traditional wavelet modulus maximum (WMM) method, the threshold function is only related to the maximum decomposition scale of the wavelet transform, not related to the other decomposition scales. The threshold function of the WMM algorithm is

$$TH = C \cdot \max_{j} \frac{|W_x(2^j, \tau)|}{J},$$

where $J$ is the maximum scale of wavelet decomposition, and $C$ is the threshold parameter. Because small decomposition scales are not considered, it is reserved noise and lost effective signal on a small decomposition scale. The proposed ATWMM method is to remove more noise and retain more
useful signals on small decomposition scales. Suppose $n(t)$ is a generalized stationary white noise with variance $\sigma^2$, its autocorrelation function is as follows:

$$E[n(u)n(v)] = \sigma^2 \delta(u-v).$$  \hfill (5)

Wavelet transform expressed as follows:

$$W_n(2^j, \tau) = \int_{-\infty}^{\infty} n(u) \psi(\tau-u)du,$$

$$|W_n(2^j, \tau)|^2 = \int n(u)n(v) \psi(\tau-u) \psi(\tau-v)dv,$$

$$E\left(|W_n(2^j, \tau)|^2\right) = \frac{\sigma^2 \|\psi\|^2}{2^j}.$$  \hfill (6)

It shows that $E(|W_n(2^j, \tau)|^2)$ is inversely proportional to scale $j$. According to equation (6), setting a new threshold function when $j<j$ is as follows:

$$TH_n = C \cdot \left( \frac{\max|W_n(2^j, \tau)|}{j} \right)^2.$$  \hfill (7)

With the increase of scale, some wavelet transforms modulus maxima at noise with negative Lipschitz Index decreases sharply, but some wavelet transforms modulus maxima at a seismic signal with positive Lipschitz Index increases rapidly [28]. The thresholding has adaptive variation based on a scale with inversely proportional, with superior noise attenuation capacity because it has better noise elimination at a small scale. In this paper, we combine the threshold function of the WMM algorithm with the new function is as follows:

$$TH_n = \begin{cases} 
C \cdot \left( \frac{\max|W_n(2^j, \tau)|}{j} \right)^2, & j \leq J, \\
C \cdot \left( \frac{\max|W_n(2^j, \tau)|}{j} \right), & j = J, 
\end{cases}$$  \hfill (8)

where $J$ is the maximum scale of wavelet decomposition, $j$ is the scale of wavelet decomposition, and $C$ is the threshold parameter. The thresholding has adaptive variation based on a scale with inversely proportional, with superior noise attenuation capacity and reservation seismic signal because it has better noise elimination at small scale and seismic signal retention at large scale, which can solve the problem of effective signal losing on small decomposition scale.

Therefore, the procedures of ATWMM are summarized as follows.

Step 1: Calculation of wavelet modulus maxima set of recorded seismic signal

Step 2: Setting threshold $TH_n(j)$ at the maximum scale $J$ with (8), reservation wavelet modulus maxima larger than $TH_n(j)$, and elimination wavelet modulus maxima smaller than $TH_n(j)$

Step 3: Setting threshold $TH_n(j)$ at the scale $j$ with (8), reservation wavelet modulus maxima larger than $TH_n(j)$, and elimination wavelet modulus maxima smaller than $TH_n(j)$

Step 4: Setting $j = j-1$, repeat step 3 until $j = 1$, collect retention wavelet modulus maxima in steps 2–4

Step 5: Reconstruct the denoised seismic signal $\tilde{s}(t)$ by inverse transform the reserved coefficients in step 4 into the time domain.

3.3. Experiments and Discussions

The synthetic seismic dataset and the real-field seismic dataset are used to compare the denoising performance of the proposed ATWMM method and the wavelet modulus maxima (WMM) method. The synthetic seismic dataset is generated by Ricker wave with a single-channel and profile of 30 seismic sections. The real-field seismic dataset is from the IZWS1 seismic monitoring station in Sichuan Jiuzhaigou, China. Considering the character of the signal dataset, the mother wavelet used here is sym6, and the decomposition level is fixed to 4.

To evaluate the denoising quality, we define root mean square error (RMSE) as (2) and signal-to-noise ratio (SNR). The smaller RMSE indicates a better denoising ability, and the larger SNR means a better noise reduction result. SNR is defined as follows:

$$\text{SNR} = 20 \log \frac{\sum_{i=1}^{m} (s(i))^2}{\sum_{i=1}^{m} (\tilde{s}(i) - s(i))^2},$$

where $s(i)$ is noise-free seismic signal and $\tilde{s}(i)$ is the denoised seismic signal.

3.2. Synthetic Ricker Profile Tests. Figure 1(a) shows the single-channel noise-free Ricker signal dataset, including 700 sampling points with 1 ms as a time sampling interval. The dominant frequency of the Ricker wave is set to 35 Hz. Figure 1(d) illustrates the noisy Ricker data that the SNR is 3 dB, where the seismic signal is almost submerged in noise. Figures 1(g)–1(i) and Figures 1(j)–1(l) demonstrate the denoising results used WMM and ATWMM methods, respectively. Compared to Figure 1(a), both the two methods can remove noise and improve data quality, but the proposed ATWMM method has significant performance improvement in noise reduction with the WMM method especially below 35 Hz and above 50 Hz.

Table 1 shows the statistical comparison of denoised results of the two methods in terms of SNR and RMSE. The proposed ATWMM method improves SNR better than the WMM method from 17.5922 dB to 20.1617 dB with an increase of 14.6% and reduces RMSE effectively from 0.0676 to 0.0502 with a decrease of 25.7%. It shows that the ATWMM method can filter out random noise and reserve effective signal outperforms the WMM method.
Figure 1: Noise reduction performance comparison on single-channel Ricker data: (a) the noise-free seismic data, (b) frequency-domain information of (a), (c) time-frequency signal of (a), (d) the noisy seismic data, (e) frequency-domain information of (d), (f) time-frequency signal of (d), (g) the denoised seismic signal based-WMM, (h) frequency-domain information of (g), (i) time-frequency signal of (g), (j) the denoised seismic signal based-ATWMM, (k) frequency-domain information of (j), and (l) time-frequency signal of (j).

Table 1: Comparison of denoising method in terms of SNR and RMSE with single-channel Ricker data.

<table>
<thead>
<tr>
<th>Noise reduction method</th>
<th>SNR (dB)</th>
<th>RMSE</th>
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<tbody>
<tr>
<td>WMM method</td>
<td>17.5922</td>
<td>0.0676</td>
</tr>
<tr>
<td>ATWMM method</td>
<td>20.1617</td>
<td>0.0502</td>
</tr>
</tbody>
</table>

traces with 700 sampling points per trace with 1 ms as the time sampling interval. It has a dominant frequency of 35 Hz with a peak of 1 and a superimposed domain frequency of 20 Hz with a peak of 1.5. Its trace interval distance is set to 25 m, and the spindle speeds are 1800 m/s and 2100 m/s. Figure 2(b) illustrates the noisy synthetic seismic profile that the SNR is 3 dB.

Figures 2(c) and 2(d) demonstrate the denoised seismic profile results based-WMM and ATWMM, respectively. The ATWMM method has the better denoising results, where much more noise is eliminated and useful seismic signal is reserved in the residual profile as shown in Figure 2(d). To better simulate real environment noise, the noise with SNR of −10 dB, −5 dB, −3 dB, 3 dB, and 5 dB are employed in the noise-free synthetic Ricker seismic profile. The denoised results are shown in Table 2.

From Table 2 we can see that the proposed method can improve the SNR of the noisy synthetic Ricker seismic profile at least from 21.9921 to 23.8136 dB, which is over 108.3% higher than that obtained by the WMM method. Moreover, the ATWMM method can achieve an RMSE from 0.0742 to 0.0638, which reduces above 14.0%.

3.3. Real-Field Seismic Data Tests. To prove the advantages of the ATWMM method, real-field seismic signals are recorded at the JZW51 seismic monitoring station in Sichuan Jiuzhaigou to conduct tests. Figures 3(a)–3(c) shows the real-field seismic data at 51JZW station in NS, EW, and UD direction, respectively. Figures 3(d)–3(f) demonstrates the denoised seismic in NS, EW, and UD direction based-WMM, respectively, and Figures 3(g)–3(i) show the denoised seismic in NS, EW, and UD direction based-ATWMM, respectively. As shown in Figure 3, the results illustrate that the ATWMM method can obtain a better noise suppression result than that of the WMM method. The WMM method can usefully remove the high-frequency noise, but the noise in lower frequency cannot be well.
suppressed. However, the ATWMM method can eliminate
the noise in almost frequency and preserve the most effective
seismic signal. Therefore, it indicates that the ATWMM
method is superior to the WMM method in the real-field
seismic signal noise reduction.

Table 3 shows the statistical comparison of the two
denoising methods of real-field seismic data. The SNRs of
the NS, EW, and UD direction based-WMM are 27.6050 dB,
25.8299 dB, and 25.4971 dB, respectively. After applying the
ATWMM method, the SNRs are increased to 31.9519 dB,
31.9083 dB, and 30.1855 dB, with an improvement of 15.8%,
23.5%, and 18.4%, respectively. So the ATWMM method
adopted to denoise real-field seismic signal has the advan-
tages of high SNR.
In this paper, we propose a novel ATWMM noise reduction method via setting an adaptive thresholding function that aims to improve seismic noise suppression. The adaptive thresholding function related to the wavelet decomposition scale can remove noise and reserve the effective signal losing on a small decomposition scale. Experimental results from both the synthetic Ricker seismic datasets and the real-field seismic signal demonstrate that its validity in improving SNRs and RMSEs, which can eliminate noise and preserve effective seismic signal more effectively. In conclusion, the ATWMM method shows significant noise reduction improvements over the WMM method.

### Data Availability

All experimental data and calculated data that support the findings of this paper are available from the corresponding author upon reasonable request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest.

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