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Research Article

Long-Term Bidding Scheduling of a Price-Maker Cascade Hydropower Station Based on Supply Function Equilibrium

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How cascade hydropower stations (CHSs) play the electricity market game is regarded as an important issue. The majority of the work to date has focused on short-term horizons and several simplifications in the hydropower system. If future prices are expected to be higher than the current price, CHSs with large reservoirs allow the bidder to postpone energy production for a longer time scale, such as several months or more, which generally matches with the regulation period of the reservoir. Rejecting the assumption of simplifying the hydropower system (ASHS), the long-term bidding strategy of CHSs is discussed for a price-maker based on the supply function equilibrium (SFE). This study considers multiple price-makers, time-coupling, and the characteristics of the hydropower system, with significant challenges. The difference from the conventional model is that the long-term optimal scheduling model of CHSs is added. Moreover, a new methodology is proposed, the equilibrium curve of the uniform clearing price (UCP) is introduced, and the Nash equilibrium solutions are solved based on the nonlinear complementarity approach. In a simulated electricity market, the result can validate the feasibility of the model and adopted methodology and the rationality of the results by taking certain CHSs with multireservoir as an example. The negative influence of ASHS is analyzed, which shows that the characteristics of the hydropower system should be fully considered in the long-term bidding research. Future study aspects are also considered, which are presented as the key issues such as market assumptions and randomness.

1. Introduction

As the new round of power system reform in China is developed, market participants are faced with a series of large-scale, highly complex, and nonlinear multiperson decision-making problems. In an electricity market, independent power producers (IPPs) exercise market power by adopting certain strategic behaviors to earn greater profit. Researchers focus on the investigation of the bidding strategy in the short-term or day-ahead electricity market to explore the bidding strategy of IPPs in the electricity market. The currently available methods include (1) a method to forecast the market clearing price [1–4]; (2) a method based on risk management [5, 6]; (3) a method to forecast and estimate the strategies of competitors [7]; and (4) a method based on game theory. Much research has been conducted on the first method, which is applicable to IPPs as the price-taker. In the electricity market, which is a typical oligopoly, the method based on game theory confers more theoretical advantages compared with other methods, so it is applicable to IPPs as the price-maker. The equilibrium models of oligopolistic competition based on game theory include the Cournot equilibrium model [8–10], Bertrand equilibrium model [11], Stackelberg equilibrium model [12, 13], and SFE model [14–18]. Among them, the SFE model is akin to the bid-based pool (BBP) mode as is mainly used in the electricity market, so it is extensively studied. The strategic bidding problem for price-maker agents based on the equilibrium model is usually formulated as a bilevel optimization problem, which is in turn transformed into a mixed nonlinear complementarity problem (NCP) by applying Karush–Kuhn–Tucker (KKT) complementarity conditions, known as mathematical programming with equilibrium constraints (MPECs). There have been many solution methods, such as interior point method with penalty term [14, 19], heuristics and iterative procedures [15, 20, 21], and the nonlinear complementarity approach [8, 22–27]. At present, there has been much research into this topic [8, 14–24, 28]. However, most of them are applied in the electricity market with only thermoelectric systems and are based on the market-wide equilibrium at a certain point in time. Such scheduling decisions are generally decoupled in time.

A pattern of cascade hydropower systems with the basin development company as the main body has been formed in China, in which CHSs have become important participants in the competition in the electricity market. Since the reservoir has the ability to store water (energy), it can be decided whether to generate in the present or store water for future use, which makes the operation of CHSs coupled in time. CHSs allow the bidder to postpone energy production through reservoir regulation if future prices are expected to be higher than the current price. Furthermore, the interconnection between the CHSs makes the operation coupled in space. This means that the bidding strategy should consider the characteristics of the hydropower system, making strategic bidding among cascade hydropower companies a large-scale multiperiod bilevel optimization. The strategic bidding problem solution is therefore more subtle.

For the issue of bidding for hydropower and thermal power based on the game theory, the dynamics of the hydropower station bidding game were considered [29]. A bilevel optimization problem to electricity generation scheduling in the wholesale market environment is proposed [30]. The numerical example of a 15 bus energy power system with thermal and hydropower plants is used to test the applicability of the approaches. The coalitional strategy, forms, and coalitional conditions of CHSs were studied based on coalitional game theory [31]. Moreover, it is supposed that the output of hydropower stations is a quadratic function of inflows and storage capacity. The equilibrium of a Cournot game between multiple firms that each possesses a mixture of hydropower and thermal generation resources was studied [8]. A price-taking fringe was introduced as a new element, and a mixed linear complementarity model was established. A test case with 21 scenarios (i.e., three months and seven load levels) is proposed to analyze the equilibrium characteristics under different market structures, which is still the equilibrium at a single point in time. Moreover, the model simplified the hydroelectric system with the maximum and minimum output parameters.

The above-mentioned research mainly studied the bidding equilibrium at a single point in time and has made simplified assumptions, but all ignored the time coupling of hydropower systems, which is necessary for practical applications. A multiyear model of strategic hydroscheduling was examined through the use of a stochastic dynamic programming recursion technique [32]. A dual dynamic programming approach to the numerical solution for a medium-term optimal hydroschedule was used [33], and the submodels at each stage are Cournot duopolies. But the model only considers a single price-maker hydropower company. Based on a review of the methodology adopted for solving the problems of single-period and multiperiod stochastic strategic bidding scenarios, a new methodology for long-term strategic bidding of a price-maker hydropower-based company was proposed [34]. The proposed approach considers the uncertainty of inflow and the interconnection between the CHSs. A deterministic residual demand curve (RDC) and a piecewise linear approximation of the expected future benefit function were proposed. The model is solved based on stochastic dual dynamic programming; however, the model discussed in the present work considers a single price-maker company and follows a quantity-only bid approach, which is a particular case of a more general market equilibrium model. Several reservoirs are not aggregated into one "equivalent reservoir", but the production coefficients of hydropower stations were assumed to be constant. The midterm stochastic mixed-integer linear programming model for a price-maker hydroproducer with pumped storage in the day-ahead market was proposed [7]. A yearly stochastic self-scheduling model is presented because there is only one price-maker. The proposed approach considers the modification of the RDC based on pumped storage characteristics and a three-stage scenario tree with 90 scenarios to simulate uncertainty in inflow and demand load. But similar to most models, the output is considered a linear function of inflow.

Although these studies have considered time-coupling of CHSs, they are not a market-wide equilibrium model, but a single-firm profit-maximization model that focuses on the self-scheduling of price-maker. The short-term bidding strategy for hydropower stations based on a static game was studied [35], but by assigning weights to each generator, the multiobjective programming problem was transformed into a single-objective optimization problem to be solved approximately. An equilibrium model to determine the price and quantity of strategic bids was studied in a day-ahead electricity market, with predominantly hydropower stations [20]. A new solution method is proposed, which replaces the MPEC complementary condition with the strong dual condition and solves the competition between several leaders through a step-by-step iterative process, which we expect to be a Nash equilibrium. Two price-makers are considered, and each company has multiple thermal power and CHSs, but the output is still calculated with a constant production coefficient. There have been many reports based on market-wide equilibrium, the vast majority of which focused on short-term horizons.

All the aforementioned research simplifies the hydropower system: first, the production coefficient and the available head of water at each hydropower plant are assumed to be constant, which will make the total amount of electricity for the same amount of water constant. However, in actual operation, because of the same amount of water, different operation modes have different total electricity, so it is often necessary to ensure maximum power generation by optimizing the operation of the reservoir. This is mainly because the production coefficient is a nonlinear factor, which is related to the available head of water, discharge flow, and other factors; especially in the long-term operation, the range of variation in the available head is larger, as is the variation in production coefficient. This assumption will lead to significant discrepancies in model output data. At the same time, the difference between electricity quantity and actual power generation will bring various risks. In the period of low water level, because the actual water head is far less than the fixed water head, the actual hydropower generation is lower than expected, and there are performance risks. In the period of high water level, because the actual water head is much larger than the fixed water head and the actual power generation is higher than the expectation, abandoned water will occur. Second, the model aggregates several hydropower stations into one "equivalent hydropower station", which is a numerical accumulation. Each hydropower station is isolated from the others. The model does not consider various characteristics such as upstream and downstream hydraulic connectivity and interval inflow between CHSs, which is quite different from the actual situation.

Hence, based on the aforementioned comments, the main aims of the paper are to (1) build a long-term strategic bidding model of CHSs based on the linear supply function equilibrium (LSFE). This model considers time-coupling, so it is multiperiod. It also considers the market-wide equilibrium resulting from the competition between multiple price-makers on the quantity and price of electricity in an electricity market with multiple types of generation sources; (2) reject ASHS, be full considered the characteristics of the hydropower system, such as changes in water head, the water balance, overflow or surplus water flow, upstream and downstream hydraulic connection and interval inflow, and make a comparative analysis; (3) propose a solution methodology and introduce the equilibrium curve of the UCP.

2. Model Description and Formulation

Strategic bidding for price-maker agents is usually formulated as a bilevel optimization problem for the electricity market, in which the internal layer corresponds to the minimization of the power purchase cost of the independent system operator (ISO) under the mechanism of a UCP; the external layer represents the maximizing the profits to all bidders. The inner layer problem can be modeled by a competitive equilibrium model. The BBP mode is still the most widely used transaction mode in the actual electricity market, and the competition of power producers is more akin to the competition of supply function. Each power producers submit an incremental supply function to ISO to quote. This paper adopts the LSFE model to simulate an electricity market. In terms of long-term bidding games among hydropower generators, a multiperiod dimension is added due to hydraulic and electric connections between periods. Moreover, due to changes in inflows and outputs in different periods, the market equilibrium of the internal layer also varies synchronously, so the model is more complex.

2.1. Market Clearing Model. In the electricity market with UCP, ISO determines the power purchase plan and the UCP according to the objective of minimizing power purchase cost based on the predicted future load curve. It is supposed that there are n strategic IPPs in an electricity market, and the game between these IPPs is a static noncooperative game with complete information. It is supposed that, without considering transmission constraints and network losses, only constraints on the outputs of generator units and electricity price in the electricity market are considered; thus, the model for market clearing in the period t is given by

$$\text{Min} \sum_{i=1}^{n} P_{it} \cdot q_{it}, \tag{1}$$

$$\begin{aligned} \text{s.t.} \sum_{i=1}^{n} q_{it} &= D(t, P_t) \\ &= L_t - \delta_t P_t, \end{aligned} \tag{2}$$

$$q_{i,\min} \le q_{it} \le q_{i,\max}, \tag{3}$$

$$p_{\min} \le P_t \le p_{\max}, \tag{4}$$

$$q_{\text{it}} \ge 0, \tag{5}$$

where q_{it} and P_{it} denote the output and bidding price of the IPP i in period t; $D(t, P_t)$ denotes the demand function for power load in period t; L_t represents the predicted value of load demand in the period t as obtained by ISO prediction; δ_t ($\delta_t \ge 0$) represents the price elasticity coefficient of the load demand; P_t refers to the market electricity price in period t; $q_{i,\min}$ and $q_{i,\max}$ denote the min and max output of the IPP i in period t; $p_{\min}$ and $p_{\max}$ denote the price floor and price cap of the electricity market.

It is supposed that IPP i makes a tender offer to ISO based on LSFE so $P_{\text{it}} = \beta_{\text{it}} q_{\text{it}} + \gamma_{\text{it}}$, in which $\beta_{\text{it}} \ge 0$. β_{it} and γ_{it} are parameters pertaining to linear supply function of the IPP i in the period t. Generally, the parameter β_{it} is defined as the decision variable of the strategy used by IPPs.

2.2. Profit Maximization Model. According to the principles of microeconomics, all parties in the game aim to maximize their own benefit; that is, they are expected to maximize their own profit by calculating their own bidding functions; therefore, the long-term decision model for any IPP based on profit maximization is expressed as follows:

$$\max \pi_i = \max \sum_{t=1}^{T} \left(P_t \cdot q_{\text{it}} - c_i q_{\text{it}} \right), \tag{6}$$

where π_i denotes the amount (in CNY) of annual profitability through IPP i (other parameters are as defined above). It is supposed that IPP i features a linear function of variable operation cost: $c_i = a_i q_{\text{it}} + b_i$, in which a_i and b_i are parameters pertaining to the variable cost.

Based on equations (1) to (5) and equation (6) for n IPPs, the long-term bidding game of IPPs is formed.

2.3. Profit Maximization Model for CHSs. In terms of multiple periods and stations, CHSs present a close hydraulic connection; the same water use for power generation corresponds to multiple different combinations of outputs, and the market equilibrium also varies. It is therefore necessary to ensure the overall equilibrium of the market in multiple periods within the operating period during the long-term bidding game. Equation (6) is transformed into equations (7) to (15).

The long-term optimal operation model for CHSs based on profit maximization is given by

$$\operatorname{Max} \pi_i = \operatorname{Max} \sum_{t=1}^{T} \left(\left(P_t - b_i\right) \cdot A_{ij} \cdot Q_{ij,t} \cdot H_{ij,t} \cdot M_t \right). \quad (7)$$

The constraint on the water balance is

$$V_{ij,t+1} = V_{ij,t} + \left(R_{ij,t} - Q_{ij,t} - S_{ij,t}\right) \cdot \Delta t, \quad \forall t \in T. \quad (8)$$

The constraint on water storage in reservoirs is

$$V_{ij,t,\min} \leq V_{ij,t} \leq V_{ij,t,\max}, \quad \forall t \in T. \quad (9)$$

The constraint on discharge flow from reservoirs is

$$Q_{ij,t,\min} \leq Q_{ij,t} \leq Q_{ij,t,\max}, \quad \forall t \in T, \quad (10)$$

$$S_{ij,t} \geq 0, \quad \forall t \in T. \quad (11)$$

The constraint on the output of a station is

$$N_{ij,t} = A_{ij} \cdot Q_{ij,t} \cdot H_{ij,t}, \quad (12)$$

$$N_{ij,\min} \leq N_{ij,t} \leq N_{ij,\max}, \quad \forall t \in T. \quad (13)$$

The constraint on the total output of CHSs is

$$\sum_{i}^{k} \sum_{j}^{m} q_{ij,t} \leq L_{t\max}. \quad (14)$$

The constraint on the electricity price is

$$P_{\min} \leq P_t \leq P_{\max}. \quad (15)$$

Nonnegative constraint: all the above variables are nonnegative (≥0).where A_{ij} denotes the output factor of the j^{th} hydropower station of the IPP i; Q_{ijt} and $H_{ij,t}$ separately refer to the discharge flow (m^3/s) and the mean average net head of water (m) during power generation at the hydropower station in period t; M_t is the duration (in hours) of period t; T denotes the total number of periods within the operating period (if that lasts for one year and the calculation is undertaken on a monthly basis, then $T = 12$); $V_{ij,t}$ refers to the initial water storage (m^3) of the reservoirs; $R_{ij,t}$ and $S_{ij,t}$ represent reservoir inflow (m^3/s) and surplus water flow (m^3/s) to the hydropower station in period t, respectively; Δt is the duration (s) of the calculation period; $V_{ij,t,\min}$, $V_{ij,t,\max}$, $Q_{ij,t,\min}$, and $Q_{ij,t,\max}$ represent the minimum and maximum water storages (m^3) of reservoirs as well as the minimum discharge flow (m^3/s) and the allowable maximum discharge flow (m^3/s) of the hydropower station in period t, respectively; $N_{ij,\min}$ and $N_{ij,\max}$ separately denote the minimum operating output and the rated installed capacity (MW) of the hydropower station; $N_{ij,t}$ and $L_{t\max}$ denote the output (MW) of the hydropower station in period t and the maximum adjustable load of the electricity market in period t, respectively. If the total output of these CHSs exceeds the adjustable load demand of the market, surplus water is generated; $P_{\min}$ and $P_{\max}$ separately refer to the lowest and the highest electricity prices in the electricity market, with $i = 1, 2, \ldots, k$; $j = 1, 2, \ldots, m_i$.

Moreover, three conversion formulae for the water level of reservoirs with storage capacity, tail water level at a hydropower station with a certain discharge flow, and the water level for a given available head of water are as follows:

$$L_{ij,t}^{\text{up}} = f_{ij}\left(V_{ij,t}\right), \quad (16)$$

$$L_{ij,t}^{\text{down}} = f_{ij}'\left(Q_{ij,t} + S_{ij,t}\right), \quad (17)$$

$$H_{ij,t} = L_{ij,t}^{\text{up}} - L_{ij,t}^{\text{down}} - \Delta H_{ij}, \quad (18)$$

where $L_{ij,t}^{\text{up}}$, $f_{ij}(\cdot)$, $L_{ij,t}^{\text{down}}$, and $f_{ij}'(.)$ refer to the water level (m) of reservoirs of the j^{th} hydropower station of IPP i in period t, the water level–storage capacity curve of the station, the tailwater level (m) of the station in period t, and the nonlinear relationship between the tailwater level and the discharge flow in the lower reaches of the station, respectively; ΔH_{ij} represents the head loss (m) during power generation at the hydropower station, with $\Delta H_{ij} > 0$.

It can be seen from equation (8) and equations (16) to (18) that the storage capacity and head of water are both an implicit function of flows for power generation. Assuming that $H_{ij,t} = x_{ij}(Q_{ij,t}$ and $V_{ij,t} = y_{ij}(Q_{ij,t})$, the decision variable of the bidding strategy for CHSs is shifted from outputs to flows for power generation.

2.4. Multiperiod Bilevel Optimization Problem. Equations (1) to (5), equation (6) for n-k thermal power plants, and equations (7) to (18) for k CHSs constitute a long-term bidding strategy model of price-maker CHSs based on the LSFE equilibrium, which is a multiperiod bilayer optimization problem. The bidding strategy of each IPP is related to the strategy of its competitors. In order to achieve Nash equilibrium, it is not allowed to solve the bidding strategy of each IPP separately but must solve the bidding strategy problems of all IPPs simultaneously.

3. Proposed Methodology

The model involves many different types of IPPs, with many equality or inequality constraints, taking the form of implicit functions for multiple variables. Therefore, the equilibrium problem of large-scale electricity market is sophisticated mathematical programs. Together with the requirement for overall equilibrium for multiple hydropower stations in multiple periods among CHSs, the model is difficult to solve. It is necessary to optimize and decompose the model as described below:

3.1. Time Decoupling and Time Coupling. For thermal power, wind power, and solar photovoltaic power stations as well as run-of-the-river hydropower stations (or hydropower stations with a regulation capacity less than the duration of the chosen time period), the outputs in different time periods are independent. This means that the bids made at one given period had no effect on the following periods; that is, the problem was decoupled in time. The maximization of the long-term profits means profit maximization in each period.

As previously mentioned, a time-coupling characteristic is inherent to the problem of CHSs because of the existence of water reservoirs, which enable them to transfer available energy from one period to subsequent ones. So the profit maximization at each period is time coupled.

3.2. Price-Takers and Price-Makers. For generator units (such as run-of-the-river hydropower stations, wind power, and solar photovoltaic power stations) with a constant and low marginal cost, it is common to apply a low bidding price to realize the possible maximum output to guarantee consumption and reduce the risk of surplus water, surplus wind, and surplus photovoltaic power in the long-term schedule. These units are price-takers: it can be thought that competition occurs on the basis of the adjustable load on the system excluding outputs from the aforementioned generator units [8, 36]; that is, a bidding game occurs between hydropower stations with regulation capacity and thermal power plants as price-makers.

3.3. The Effect of Output Regulation of CHSs on the UCP. The short-term bidding strategy and long-term bidding strategy of CHSs are different. Due to the existence of the reservoir, CHSs can be scheduled according to the bidding result in the day-ahead market, without abandoning water or performing the contract risk. It can become the marginal unit and participate in the bidding game like a thermal power unit to reach the market equilibrium. However, the long-term bidding strategy is more complicated because of the seasonality of the inflow and the limited regulation ability of the reservoir, as well as the full and empty periods of the reservoir. In general, the marginal cost of hydropower station is constant and low, and the loss of profit from abandoning water often exceeds the gain from higher electricity prices. Therefore, it is assumed that CHSs are generally not taken as marginal units to determine directly the price, but as units below the marginal unit to indirectly affect the price by regulating the output of unit during the long-term bidding game.

The mechanism of influence is described as follows: it is supposed that the equilibrium curve of the UCP is given by $P = B(L)$, and the corresponding UCP of the load L_a of the system is expressed as $P_a = B(L_a)$. If the output of CHSs (located below the marginal units) is increased by ΔN ($\Delta N = L_a - L_b$), which is equivalent to the adjustable load on the system being reduced from L_a to L_b, the UCP is given by $P_b = B(L_b)$, and thus, the electricity price decreases. On the contrary, if the output of the CHSs decreases by ΔL, implying that the adjustable load on the system increases by ΔL and the electricity price increases, as shown in Figure 1. On this basis, the functional relationship between the electricity price and the output (load) when CHSs participate in the long-term game of the electricity market is established thus.

$$P_t = B(L_t' + \Delta N), \tag{19}$$

where L_t' denote the adjustable load of the system after excluding output from stations of price-takers in period t.

Hence, when the equilibrium curve of UCP is known, the different electricity prices corresponding to different outputs of CHSs can be solved, and the correlations of the inflow, output, load, and electricity price can be established, and it is possible to transform the problem of the long-term bidding game of CHSs based on LFSE into a long-term optimal operation problem of CHSs in the case that the electricity price changes with the output.

Based on the characteristics whereby CHSs are permitted to change the UCP by regulating outputs, the electricity market bidding is divided into two parts. The first part is the bidding between thermal power plants, as price-maker. The second part is the bidding between CHSs and all thermal power plants. In this way, a bilevel optimization problem is divided into two independent problems to be solved step-by-step, thus decreasing the complexity of the problem and difficulty of calculation. The first step is to calculate the equilibrium curve of the UCP excluding CHSs; the second step is to solve the long-term optimal operation problem of CHSs based on the equilibrium solution curve.

3.4. The Equilibrium Curve of the UCP Excluding CHSs. The bidding game of the electricity market without CHSs is usually the strategic bidding problem of thermal power as the price-makers. This is a one-period, bilevel optimization problem, and the scheduling decision is decoupled in time. Based on the static game of complete information, the equilibrium solution of the quantity of the bidding parties and the UCP can be obtained by specifying the load demand. According to different load demands, the equilibrium curve of the UCP at each period can be obtained. When the number and installed capacity of thermal power units, as the price-makers, change with factors such as the new units put into operation, maintenance, shutdown, and decommissioning, the equilibrium curve of UCP will change. The influence of these factors should be considered in each period of long-term bidding scheduling.

The market equilibrium solution excluding CHSs, including the equilibrium solution of the UCP, can be derived by solving equations (1) to (5) and equation (6) for $n-k$ IPPs. Therefore, the equilibrium curve of UCP describing the relationship between load demand and UCP equilibrium

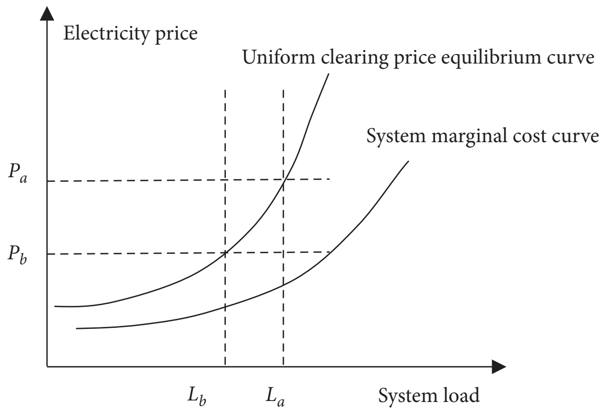

Figure 1: The effect of output regulation of CHSs on the UCP.

solution can be derived. The calculation is conducted by using a nonlinear complementarity approach and the improved Levenberg–Marquardt algorithm [32, 37–39].

3.4.1. Nonlinear Complementarity Approach. By applying the KTT complementarity condition of each participant, this bilayer optimization problem is transformed into a mixed NCP. There are many ways to solve the mixed NCP. In theory, the heuristic iterative algorithm cannot guarantee that the solution obtained by convergence is the Nash equilibrium solution, and the convergence of the algorithm cannot be guaranteed either. Direct solution methods are mainly aimed at the linear complementarity problem (LCP), and corresponding commercial LCP software packages are used to solve it [8, 25]. When the model becomes more complex and NCP occurs, the interior point method and the nonlinear complementarity approach are usually used [22]. The interior point method requires the initial value and iteration value to be in the positive octant, and additional conditions are required. Compared with other methods, the nonlinear complementarity approach is chosen because of its advantages in the following aspects: reduction of the difficulty of solution, more convenient calculation, and wider applicability and is more suitable for solving large-scale and complex equilibrium problems [26].

By introducing the generalized Lagrange multiplier, the equivalent KKT complementarity conditions for profit maximization of IPPs are obtained, and then, the KTT complementarity condition of each IPPs is transformed into a group of nonlinear algebraic equations by using the nonlinear complementary function $\psi(a,b) = a + b - \sqrt{a^2 + b^2}$. Thus, multiple MPECs are transformed into multiple ordinary nonlinear programming problems with standard constraint qualification [27].

3.4.2. The Improved Levenberg–Marquardt Algorithm. The nonlinear complementary function is nondifferentiable and nonsmooth at the origin (0, 0), which leads to the nonexistence of its gradient. When solving the above equations, semismooth algorithms should be used. Compared with other algorithms, the improved Levenberg–Marquardt algorithm considers the search direction of the Newton method and gradient method at the same time, and the algorithm has better stability performance and faster convergence speed, which is more suitable for solving large-scale NCP problems.

Definition 1. Let g: $R^n \longrightarrow R$ is a function under the local Lipschitzian condition and D_g is the set of all differentiable points of g, then the subdifferential $\partial_B g(x)$ of g is defined as follows:

$$\partial_B g(x) = \left\{ h \in R^{n\times 1} | h = \lim_{x^k \in D_g, x^k \longrightarrow x} \nabla g\left(x^k\right) \right\}, \tag{20}$$

where $\nabla g(x^k)$ is the gradient of g at a differentiable point x^k. If g is differentiable at point x, then $\partial_B g(x)$ is equal to $\nabla g(x)$. The convex closure of $\partial_B g(x)$ is the generalized gradient.

The main calculation steps are as follows.

Step 1. Initialize parameters, set the computational precision ε, and the initial point x^0.

Step 2. Calculate the search direction Δx^k.

Select any $H^k \in \partial_B E(x^k)$ and solve for Δx^k according to the following equation:

$$\left(\left(H^k\right)^T H^k + \sigma^k I\right)\Delta x^k = -\left(H^k\right)^T E\left(x^k\right) + r^k, \tag{21}$$

where σ^k is the algorithm parameter, and $\sigma^k \geq 0$; r^k is a variable vector whose norm is small enough.

Step 3. Calculate the iteration step size h^k.

In this paper, the iterative step size is determined by minimizing the merit function using the line search method. $h^k = 2^{-i}$, $i \in \{0, 1, 2, \ldots, n\}$, and i take the minimum value satisfying the following equation:

$$\Phi\left(x^k + 2^{-i}\Delta x^k\right) \leq \Phi\left(x^k\right) + \beta 2^{-i} \nabla \Phi^T\left(x^k\right)\Delta x^k, \tag{22}$$

where the merit function $\Phi(x) = 1/2E(x)^T E(x)$, and $\beta \in (0, 0.5)$.

Step 4. Variable iteration.

Iterate according to the following formula:

$$x^{k+1} = x^k + h^k \Delta x^k, \tag{23}$$

Step 5. Iteration termination criterion.

When $\|\nabla\Phi(x^k)\| \leq \varepsilon$, meeting the accuracy requirements, the iteration ends.

3.5. Long-Term Bidding Problem of CHSs: The Improved POA. Compared with other optimization algorithms, the POA is chosen because of its advantages in the following aspects: reduction of the calculation of dimensions, faster convergence speed, less parameter adjustment, and excellent practicality [40]. The POA has been improved several times and successfully applied to reservoir scheduling [41–43].

The essence of the POA is to decompose a multistate decision problem into a series of two-stage subproblems. Firstly, the variables of other stages are fixed, and the decision variables of the selected two-stage are searched for optimization and then optimized stage by stage. The optimization result of this round is taken as the initial condition of the next round of optimization, and a new round of optimization is started until the convergence condition is satisfied.

The improved POA proposed in this paper mainly has the following two improvements:

(1) Variable step size search: Considering the accuracy and time of searching for the optimal solution, a larger step length is adopted at the beginning of searching for the optimal solution, and the step size is gradually reduced according to the level of iteration accuracy until the global optimal solution is found.

(2) Consider market constraints, namely cascade total output constraints and price constraints: In each two-stage optimization process, it is necessary to judge whether the total output of the steps exceeds the adjustable load of the system, and water must be abandoned if the total output exceeds the adjustable load of the system. To judge whether the electricity price derived from the equilibrium curve of the UCP exceeds the price cap or the price floor of the electricity market, the maximum and minimum allowable values will be taken if the price exceeds the price cap or the price floor.

4. Case Study

4.1. Description of the Case Study. We simulate the competitive behavior of a regional electricity market with multiple types of generation resources. The total installed capacity of the electricity market is 7,900 MW, of which 25.6% are hydropower stations with regulating capacity, namely A1 hydropower station and A2 hydropower station, both belonging to the same IPP, and is the price-maker. Six thermal power stations, including five coal units and one gas-fired one, belong to different IPPs and are all the price-makers. Run-of-the-river hydropower station group, wind farms group, and photovoltaic power station group are composed of many different IPPs, and they are all price-takers. The installed capacities of the regional electricity market and their main parameters are listed in Table 1.

IPP A manages two hydropower stations with reservoirs, which lie on the upper and lower reaches of the same river basin. Hydropower station A1 has an incomplete annual regulating capacity, with the dead water level of 540 m and the normal water level of 600 m; hydropower station A2 also has an incomplete seasonal regulation capacity, with the dead water level of 370 m and the normal water level of 380 m. This is a multireservoir joint optimal operation problem. Monthly inflow was selected from the data of typical normal flow year in the river basin; the operating period started at the beginning of June and ran to the end of the following May, and the calculation was conducted on a monthly basis. The initial and final water levels of the operating period were both set as the respective dead water levels.

The system loads and the adjustable loads of the system are different from month to month, as shown in Figure 2. The adjustable load of the system is the system load minus the output of power stations of price-takers such as run-of-the-river hydropower station group, wind farms group, and photovoltaic power station group. Assuming that during the operating period, all IPPs in the electricity market participate in the market competition every month, and the installed capacity, number of competitors, and operation cost parameters are the same, and the same equilibrium curve of the UCP excluding CHSs is adopted every month.

4.2. Case Study Results. According to the model for SFE consisting of equations (1) to (5) and (6) for $n-\mathrm{k}$ thermal power plants, the equilibrium curve of the UCP excluding CHSs is deduced by using the nonlinear complementarity approach and the improved Levenberg–Marquardt algorithm; that is,

$$P_t = B\left(L_t'\right) = 0.000014155 L_t'^{2} + 0.01839 L_t' + 260.35, \quad L_t' \geq 0. \tag{24}$$

According to equations (7) to (18) and (24), the result of the long-term bidding game of CHSs based on SFE was calculated by using the improved POA, as shown in Table 2, Figures 3, and 4. For the convenience of comparison, the bidding game was conducted based on the optimal operation result obtained by using the objective functions of maximizing annual power generation (Figure 5).

As shown in Figure 5, when the objective function is to maximize the annual power generation, the CHSs scheduling process is as follows: for the leading reservoir A1, the highest operating water level, that is, the normal water level, will be maintained from the beginning of October to the end of April of the following year. In May, the water level of the reservoir A1 will be decreased and fell to the dead level at the end of May. From the beginning of June to the end of September, the water level of the reservoir A1 will be raised from the dead water level to the normal water level. The maximum annual power generation of the CHSs can be achieved through the above long-term operation schedule that keeps the water level as high as possible.

If ASHS is accepted and a fixed production coefficient of the CHSs is adopted, the power generation cannot be optimized. When the inflow is fixed, the power generation is fixed. In this case, the annual power generation of the CHSs that accept ASHS is 11.41% less than those that reject ASHS, and the deviation ratio of monthly output ranges from -18.3% to 1.1%. Moreover, since it is maintained at a high water level most of the time, the monthly output after accepting ASHS is generally lower than that reject ASHS, and the higher the average water level, the greater the deviation ratio. Only the monthly output in June was 1.1%

Table 1: Installed capacities of the regional electricity market and their main parameters.

Power station	Installed capacity (MW)	Maximum output (MW)	Minimum output (MW)	a_i	b_i
Hydropower station A1	1,386	1,386	0	0	60
Hydropower station A2	640	640	0	0	60
Coal-fired unit 1	1,000	1,000	100	0.102	179
Coal-fired unit 2	1,000	1,000	100	0.104	185.3
Coal-fired unit 3	600	600	50	0.208	191.6
Coal-fired unit 4	600	600	50	0.212	196.2
Coal-fired unit 5	300	300	50	0.432	206.7
Gas-fired unit 6	600	600	50	0.448	268.6
Run-of-the-river hydropower station group	1,200	1,200	0	0	50
Wind farm group	214	214	0	0	30
Photovoltaic power station group	360	360	0	0	20
Sum	7,900				

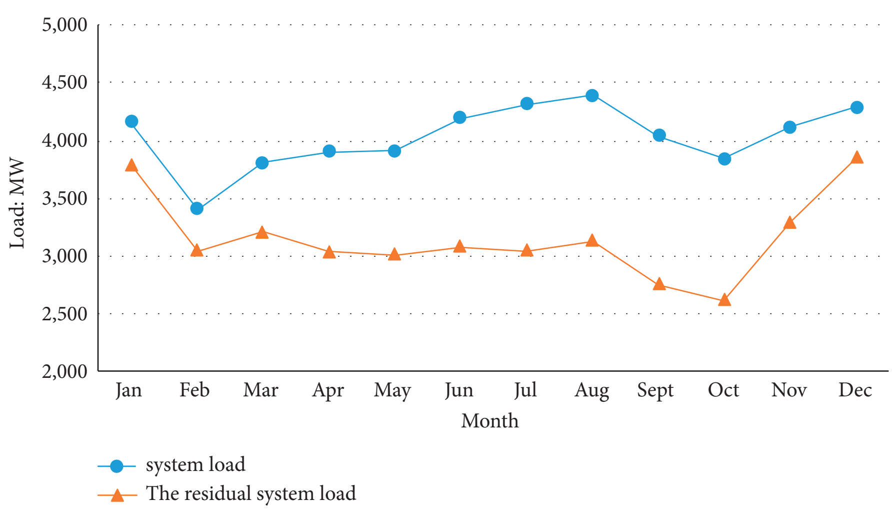

Figure 2: The curve of the system loads and the adjustable system loads.

Table 2: Result of the long-term bidding game of CHSs based on SFE.

Time period	Hydropower station A1		Hydropower station A2		Average output of CHSs (MW)	UCP (CNY/MW·h)	Hydro generation of CHSs (MW·h)	Profit of CHSs ($\times 10^6$ CNY)
	Inflow (m^3/s)	Water level at the end of the period (m)	Interval inflow (m^3/s)	Water level at the end of the period (m)				
May	331.5	540	9.6	370	1,138.1	344.74	846,765	291.91
June	383.1	540.01	11.1	380	847.3	371.85	610,060	226.85
July	910.9	599.82	26.2	380	1,696.7	310.82	1,262,362	392.37
August	726.4	597.88	20.9	380	1,975.6	300.44	1,469,837	441.6
September	776.6	599.99	22.4	379.99	2,026.0	281.38	1,458,704	410.45
October	641.8	600	18.5	380	1,785.9	285.41	1,328,728	379.24
November	288.4	600	8.3	380	828.4	392.05	596,426	233.83
December	221.2	600	6.4	379.99	640.5	465.21	476,556	221.7
January	198.9	600	5.7	380	577.4	465.84	429,610	200.13
February	186.7	597.44	5.4	380	581.6	391.76	390,868	153.13
March	193.1	600	5.6	379.99	522.1	411.86	388,433	159.98
April	184.7	575.82	5.3	380	831.4	370.13	598,589	221.55
Sum							9,856,938	3,332.74

higher than that of reject ASHS, which was caused by the lowest average water level in June.

However, when the CHSs participate in the electricity market game as price-maker, the long-term optimal operation scheme of the CHSs will be fundamentally changed, and if the assumption of the simplified hydropower system is accepted, it will also bring large losses and risks in the long-term bidding game of CHSs.

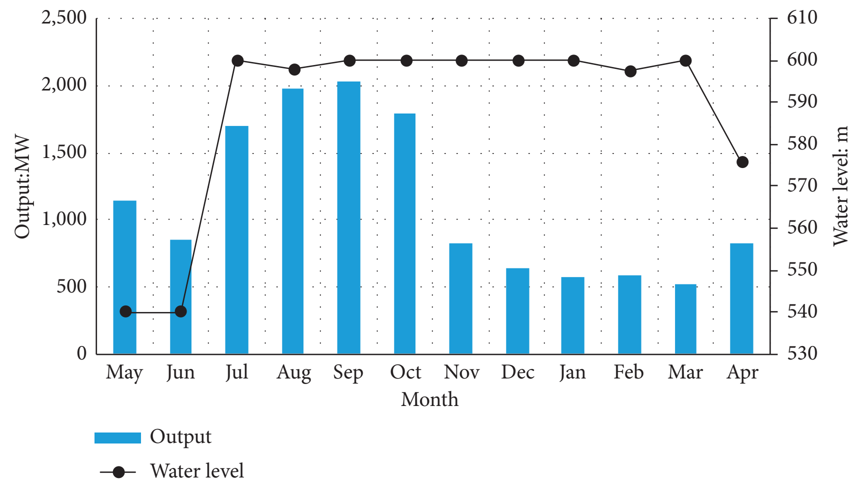

Figure 3: Output and water level based on SFE.

As shown in Table 2 and Figure 3, the CHSs scheduling process is as follows: the amount of power generation of the CHSs is determined by the inflow, and the dead water level of the reservoir A1 will be maintained in June, so as to reserve water storage capacity for subsequent flood mitigation and ensure that the monthly UCP in the flood season is relatively average, because if the discharge flow in the subsequent months of the flood season is too large, the UCP will be too low, and the profit of CHSs will decrease instead. At the end of September, the water level of the reservoir A1 will be raised to the normal water level. From the beginning of October to the end of January of the following year, the water level of the reservoir A1 will be kept at the normal water level. From the beginning of April to the end of May, the water level of the reservoir A1 gradually decreased from the normal water level to the dead water level. From February to March and from August to September, there is a process in which the water level of the reservoir A1 is first decreased and then increased, in order to obtain an optimal value in the multiperiod equilibrium.

In summer, the system load is the highest in the year, especially in July and August, but this is also the high-flow season for hydropower, which means more hydropower available output and the lowest UCP of the year. Wind power and photovoltaic outputs are greatly affected especially on rainy days. At this time, the energy structure of the grid is dominated by hydropower.

In winter, the system load is higher in the year, only lower than the load in summer, except in February. At this time, the inflow of hydropower is reduced month by month due to the decline of rainfall and the decrease in temperature, which means that the hydropower available output is also gradually reduced. Although the output of wind power and photovoltaic power is more than that in summer, the complementary influence is limited, and these factors ultimately result in the highest UCP of the year. Affected by the Chinese New Year, the system load in February was the lowest in the year. Due to the decline in demand, the UCP also decreased.

The negative correlation between the UCP and the output of CHSs shows that the UCP changes dynamically with variations in the output of CHSs and the system load in the market game, as shown in Figures 2 and 4. It can be seen that this bidding model based on SFE can better respond to the demand imposed by the electricity market and more effectively participate in electricity market competition, thus realizing an overall equilibrium across multiple periods.

If ASHS is accepted, compared with rejecting ASHS, the annual power generation and profit will be reduced by 10.8% and 13.8%, and the deviation ratio of monthly output will range from -46.8% to 22.5%, which is a large deviation. Therefore, if IPPs use the model results that accept ASHS to bid, it will bring significant economic losses and performance risks.

As shown in Figure 6, during the period from July to April of the following year, the reservoir is operating at a high water level, and the deviation of monthly output is negative; that is, the actual available output is higher than the model result. This means that in order to reduce profit losses and abandon water, it is necessary to sell unscheduled power generation that exceeds the model results in the short-term electricity market. This will further lower the UCP in the short-term market. The superposition of the two will bring about a greater loss of profit. At the same time, due to the higher expected electricity price, there is a risk of bid failure.

In May and June, the reservoir is operating at a low water level, and the deviation of the monthly output is positive; that is, the actual available output is lower than the model result. This means not only to sell the power generation in the long-term market at a lower price but also to purchase the power generation in the short-term market at a higher price to fulfill the contract.

Therefore, in the long-term bidding, the impact of various main factors of the hydropower system should be fully considered, such as changes in water head, the water balance, overflow or surplus water flow, upstream and downstream hydraulic connection, and interval inflow. This will make the model more realistic and the model results more reasonable and more accurate.

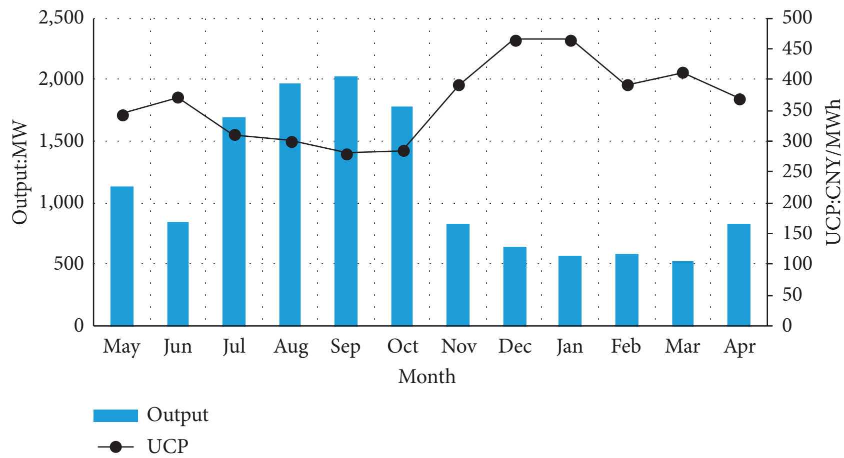

Figure 4: UCP and output based on SFE.

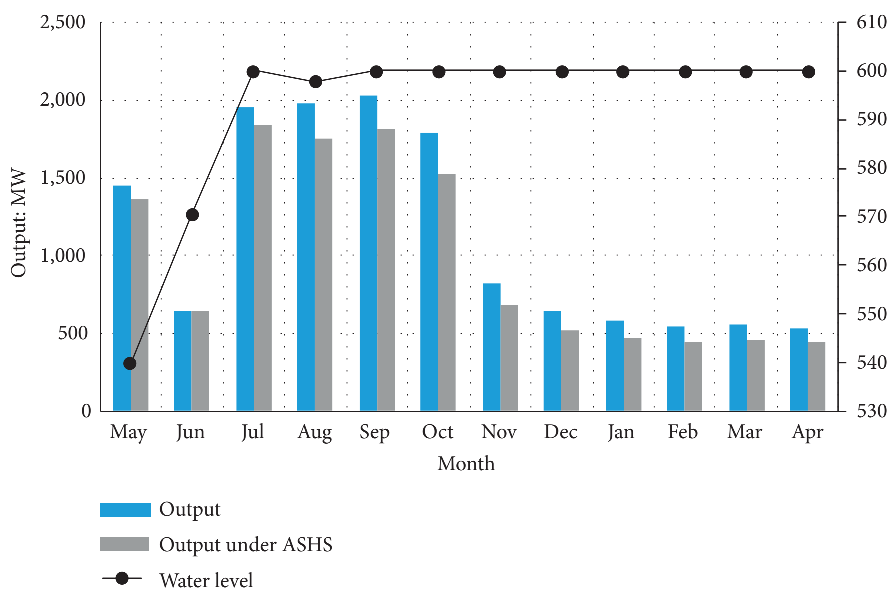

Figure 5: Influence of output under ASHS based on maximizing power generation.

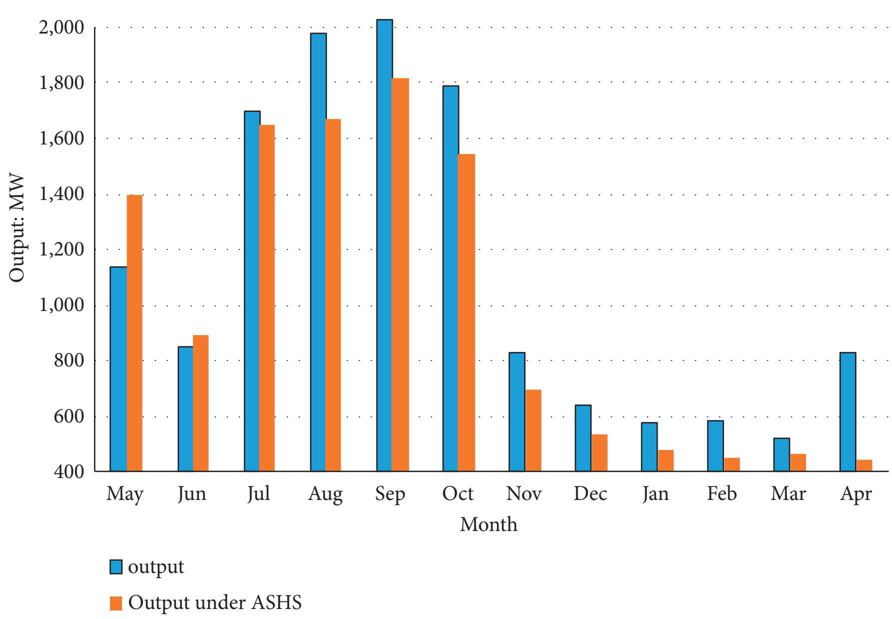

Figure 6: Influence of output under ASHS based on SFE.

5. Conclusions

The long-term bidding of CHSs as price-maker was explored based on game theory and the model for SFE, which made the market-wide equilibrium from a certain point in time to a certain longer time range. Through analysis of the major deviations and risks brought about by ASHS, it was shown that the characteristics of the hydropower system should be fully considered in the long-term bidding. The model was more complicated than the conventional model because it rejected ASHS and considered multiple price-makers, time coupling, and the characteristics of the hydropower system. The correlation among inflows, outputs, loads, and UCPs was established for CHSs through the model. In this paper, based on the characteristics of hydropower, thermal power, and other conventional types of power stations, a new methodology was proposed, the equilibrium curve of the UCP was introduced, and the market-wide equilibrium during the dispatching period was solved based on the nonlinear complementarity approach. Finally, the feasibility of the model and methodology and the rationality of the results are confirmed by way of a practical case study.

In terms of future work, we highlight three possible areas of investigation: the first is to consider more hydropower companies as price-makers. The second extension is to consider stochastic inflows, and the stochasticity of inflows is an important consideration in the operation of hydro-resources. Finally, the third relates to market-related assumptions. The present study was conducted on the assumption of noncooperative behavior with complete information: in reality, market information is generally incompletely asymmetric, and participants also tend to collude and form alliances.

Data Availability

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to reasons related to commercial confidentiality.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Gang Liu conducted the main research tasks and wrote the manuscript; Guangwen Ma contributed with comments and advice throughout the development of the manuscript. Shijun Chen contributed to the research material; Weibin Huang supervised and checked the research. All authors have read and agreed to the submitted version of the manuscript.

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