

Research Article

Group Decision Analysis Based on Complex m -Polar Fuzzy N -Soft Environment

Muhammad Akram ¹, Maheen Sultan,¹ and Ahmad N. Al-Kenani ²

¹Department of Mathematics, New Campus, University of the Punjab, Lahore, Pakistan

²Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80219, Jeddah, 21589, Saudi Arabia

Correspondence should be addressed to Muhammad Akram; m.akram@pucit.edu.pk

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This research article presents a novel multicriteria group decision-making (MCGDM) technique, namely, complex m -polar fuzzy N -soft TOPSIS (CmPFNS-TOPSIS), that incorporates the remarkable features of manipulating the complex two-dimensional data and the multipolarity of the modern era with the help of CmPFNS and the multicriteria group decision making potential of the TOPSIS technique. This newly proposed technique shares a very general parametric structure of the CmPFMS and enables us to make the suitable decisions in this multipolar complex two-dimensional domain. The incredible CmPFNS-TOPSIS technique works on the principle of finding the optimal solution nearest to the positive ideal solution (PSS) and farthest from the negative ideal solution (NIS) by evaluating the Euclidean distance between the alternatives and the optimal solutions. The support of complex m -polar fuzzy N -soft weighted averaging operator (CmPFNSWA), Euclidean distance, score function, and the revised closeness index is utilized for uncovering our optimal solutions. The alternatives, with respect to the revised closeness index, are arranged in the descending order and the alternative with the least closeness index is preferred. The methodology of the CmPFNS-TOPSIS technique is illustrated with the help of the flow chart. The proficiency of this technique is proved by considering a case study of selection of the suitable surgical equipment in the oncology department of Shaukat Khanum Hospital, Lahore (Pakistan). To prove its validity and credibility, a comparative analysis between CmPFNS-TOPSIS and m -polar fuzzy N -soft TOPSIS (mFNS-TOPSIS) is pictured with the help of a bar chart displaying the same end results of the CmPFNS-TOPSIS and mFNS-TOPSIS.

1. Introduction

A man makes numerous decisions in his daily life courses where he has multiple choices having various features, no doubt such circumstances hinder the certainty of his accurate decisions. To make the decisions worthy and acceptable, there was a need for some decision-making techniques that enable a man to pass through all the hurdles in the path of making the accurate decisions. There are many types of decision-making techniques, like multicriteria group decision making (MCGDM) technique where we have multiple attributes of the alternatives and a panel of experts is being selected to analyze the alternatives. Several scholars, including [1–4] have developed numerous MCGDM techniques and methods to account for the multiplicity of

sources of information, but there are some deficiencies in these methods as some are useful in dealing with multipolarity and some with two-dimensional data. The widespread utilization of MCGDM techniques in medical, engineering, social sciences, robotics and many other areas of science and technology have made these techniques notable and admirable [5–8]. To manipulate the unambiguous consequences of the human commitments, complexity and the multipolarity of the modern era, we put forward a novel MCGDM technique, namely, CmPFNS-TOPSIS that embraces the complex two-dimensional data and the multipolar information, shares the burden of making the apt decisions in the daily routine tasks.

The classical MCGDM techniques integrate the evaluations and the data in a crisp form, including TOPSIS [9],

VIKOR [7], ELECTRE [10], analytic hierarchy process (AHP) [11] and PROMETHEE [12]. All these techniques deal with the membership values 0 and 1. Since there are no such words like exact or certain in the daily routine tasks, the scientists looked forward towards the vagueness and uncertainty of human nature. In this scenario, Zadeh's fuzzy set [13] proved as a hallmark in characterizing the ambiguous nature of human obligations. Chen [4] extended the classical TOPSIS technique to the fuzzy TOPSIS to operate in the uncertain and ambiguous environment. It revolutionized the decision-making abilities by considering the fuzziness of the present time. Torlak et al. [8] employed the fuzzy TOPSIS method to rank the domestic airlines in Turkey. Lately, Chu and Kysely [5] implemented the fuzzy TOPSIS in ranking the objectives in the advertisements in Facebook. Furthermore, Li et al. [6] implemented the fuzzy TOPSIS method to investigate the service quality of Beijing metro system by using trapezoidal fuzzy numbers. Chen et al. [1] developed the concept of the m-polar fuzzy set to highlight the prominence of the multipolarity of the modern world. The world is getting complex day-by-day, Ramot et al. [14] put forward the notion of complex fuzzy set in order to tackle the complex two-dimensional data.

In a different vein, soft set theory [15] is concerned with a setup where the subsets of a universal set are described by their fulfilment of various attributes. It is said that soft sets produce a parameterized family of subsets, indexed by the required parameters. Because the parameters that describe the alternatives often have an intrinsically fuzzy nature (like expensive, beautiful, famous, et cetera), natural extensions of soft set theory for this case have been formalized by Maji et al. [16] or Alcantud et al. [17], who also produced an application to the valuation of assets. However, the widespread utilization of rating systems regarding games, hotels, rental rooms, movies, et cetera, called for an updated version of soft sets, independent of a fuzzy description of the attributes. Fatimah et al. [4] thus initiated the concept of N-soft set as a generalization of the primitive soft set model. It soon became a trendy topic due to its applicability, and also because it was successfully merged with the idea of fuzziness and vagueness from multiple perspectives. Let us give a short sample of recent extensions. Akram et al. [8] combined fuzzy set and N-soft set theory and put forward the concept of fuzzy N-soft set. Fuzzy N-soft sets were further extended by Akram et al. [5] who put forward the theory of m-polar fuzzy N-soft sets (mFNSS). They proved their versatility by quoting applications in routine tasks like the selection of a hotel, resort, restaurant and laptop. Fatimah and Alcantud [18] insisted on the multifuzzy abilities of N-soft sets and their applications. Many other extensions exist that incorporate for example, the traits of hesitancy [19]. Eraslan and Karaaslan [3] extended the TOPSIS method under fuzzy soft information and illustrated its application for house selection. By adding up the N ordered grades in the fuzzy soft set theory, Akram et al. [1,20] put forward the concept of fuzzy N soft set and m-polar fuzzy N soft set (mFNSS), proved their credibility by quoting the examples of daily routine tasks, like selection of a hotel, resort, restaurant and laptop. Fatimah and Alcantud

[18] insisted on the multifuzzy abilities of N-soft sets and their applications. Many other extensions exist that incorporate for example, the traits of hesitancy [21] or roughness [22].

As the world is evolving day-by-day, becoming more complex by embracing the multipolarity, so in order to face the multipolarity and the complexity of the two-dimensional data at the same time, Akram [23] and Sultan [14,24,25] come up with a novel hybrid model, namely, complex m-polar fuzzy N soft set (*CmPFNSS*) that indexed the m-polar information as well as the complex two-dimensional data. The very general structure of (*CmPFNSS*) is benefited with both the multipolarity of *mFNS* and the ability of dealing with complex two-dimensional data of the complex fuzzy set simultaneously. Therefore, there is a dire need of some MCGDM technique that enhances our decision-making abilities while facing the multipolar information and the complex two-dimensional data all together. To meet this imperative demand, a very general MCGDM technique, namely, *CmPFNS-TOPSIS* is introduced in this article along with the flow-chart of its algorithm, a case study of selecting the suitable surgical equipment for the oncology department of the Shaukhat Khanum Hospital, Lahore and a comparative analysis of *CmPFNS-TOPSIS* with the m-polar fuzzy N-soft TOPSIS representing by a bar-chart to prove its credibility and validity. The main factors that encourage us to come up with the novel *CmPFNS-TOPSIS* are as follows:

- (i) The *mFNS-TOPSIS* excels in dealing with multipolarity of the modern era, but as the world is getting more and more complex so this technique impedes in tackling with the complex two-dimensional data and proved to be insufficient in making the right decisions while facing with the complexity of the present time.
- (ii) The complex fuzzy set is more skilled to address the complex two-dimensional data, but it is incompetency in the multipolar environment and the absence of the *CmFNS-TOPSIS* technique handicap us to make the apt decisions in daily life courses.
- (iii) The pitfall of the *mFNS-TOPSIS* and the nonexistent *CFNS-TOPSIS* have grasped our attention towards the formation of the *CmPFNS-TOPSIS* which stands with the multipolarity of the present-day and its complex two-dimensional nature.

Encouraged by all these facts, we manifest a novel hybrid decision-making strategy, namely, *CmPFNS-TOPSIS* that assimilates the astonishing features of *CmPFNS* and spectacular aspects of the fuzzy TOPSIS. This impressive technique is manifested with the principle of finding the optimal solution nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS) by calculating the Euclidean distance between the alternatives and the ideal solutions. This technique is quite helpful in the ordered grading of the alternatives and considering the benefit-type as well as cost-type criteria. We illustrated its methodology by considering a case study of the selection of the surgical equipment in the oncology department of the Shaukat

Khanum Hospital, Lahore. A very comprehensive comparative analysis between CmPFNS-TOPSIS and mFNS-TOPSIS is being pictured with the help of a bar-chart to depict its credibility.

The most notable contributions made in this research article are as follows:

- (i) This research article depicts the hybrid extension of the TOPSIS under the CmPFNS environment that can easily take care of the multipolar information and the complex two-dimensional data, enabling us to make the most accurate decisions in the daily routine tasks
- (ii) The working mechanism of CmPFNS-TOPSIS is established by illustrating each and every step of its working principle
- (iii) The complex m-polar fuzzy N soft weighted averaging operator (CmPFNS-TOPSIS) and the normalized Euclidean distance for the CmPFNS is introduced
- (iv) A case study of the selection of the suitable surgical equipment for the Shaukat Khanum Hospital, Lahore is being quoted to display its practicality and potential
- (v) The algorithm of the working rule of the mFNS-TOPSIS is also summarized
- (vi) The similar case study is considered for the implication of the mFNS-TOPSIS
- (vii) A comparative analysis is being picture with the help of the bar-chart to prove the credibility of CmPFNS-TOPSIS

This research article is organized as follows: Section 2 comprises of the preliminaries to remind our previous

knowledge. Section 3 includes the algorithm of the working mechanism of the novel hybrid decision-making strategy, CmPFNS-TOPSIS. Section 4 is composed of the case study of the selection of the best surgical equipment for the oncology department in Shaukat Khanum Hospital, Lahore. Section 5 covers a very comprehensive algorithm of the working principle of the mFNS-TOPSIS. Section 6 provides a detailed comparison between CmPFNS-TOPSIS and Mfns-TOPSIS to prove the credibility of the CmPFNS-TOPSIS. Section 7 concludes the research article.

2. Complex M-polar Fuzzy N-Soft Sets

In this section, some basic concepts of complex m-polar fuzzy N-soft sets CmPFNS are presented for the better understanding of the concepts.

Definition 2.1. (see [1]). An m-polar fuzzy N-soft set (m, N) is a triple (f, D, m) , where $D = (F, C, N)$ is an N-soft set on universal set (U) and f maps any attribute in C with an m-polar fuzzy set $(mF)A$ on $F(c_k)$, which is a convenient subset of $V \times M$ and $c_k \in C$. Therefore, the domain of f is of course C , and it's codomain is $M^l (V \times M)$, the family of all sub- mF sets over $V \times M$.

Definition 2.2. Let X be a non-empty set. A complex m-polar fuzzy set on X is a set C characterized by a complex m-polar membership function $\rho_i^{\circ}C$, which maps every element of X to a closed unit circle, with m -poles.

$$C = \{ (x, \rho_1^{\circ}C(x), \rho_2^{\circ}C(x), \dots, \rho_m^{\circ}C(x)) : x \in X \}. \quad (1)$$

Equation (1) can be written as

$$C = \{ (x, r_1(x)e^{i2\pi w_1(x)}, r_2(x)e^{i2\pi w_2(x)}, \dots, r_m(x)e^{i2\pi w_m(x)}) : x \in X, 0 \leq r_i(x), w_i(x) \leq 1 \}. \quad (2)$$

The concept of CmPFNSS was first introduced by Akram and Sultan [25] in 2022. The notion of CmPFNSS merges the uniqueness of multipolar information and the complexity of two-dimensional data.

Definition 2.3. Let U be a universe of objects, S be a set of attributes, $P \subseteq S$ and $G = \{0, 1, 2, \dots, N - 1\}$ with $N \in \{2, 3, 4, \dots\}$. A triplet (μ, K, m) is called a complex multipolar fuzzy N-soft set (CmPFNSS) when $K = (F, P, N)$ is an N-soft set on U and μ is a mapping as $\mu: P \rightarrow \eta(U \times G)$, where $\eta(U \times G)$ is a family of complex m-polar fuzzy sets on $U \times G$. Consequently, for every $p \in P$ and $u \in U$, there exists a unique $(u, g_p) \in U \times G$ such that $g_p \in G$ and $\langle (u, g_p), J_1^{\circ}\mu(p), J_2^{\circ}\mu(p), \dots, J_n^{\circ}\mu(p) : i = 1, 2, \dots, m \rangle$, where $J_i^{\circ}\mu(p)$ assigns any attribute $p \in P$ to a complex m-polar membership degree.

Definition 2.4. Let $\mu_1 = \langle (d_1, \rho_1^{\circ}\mu_1 e^{2\pi i\alpha_1}, \rho_2^{\circ}\mu_1 e^{2\pi i\alpha_2}, \dots, \rho_m^{\circ}\mu_1 e^{2\pi i\alpha_m}) \rangle$ and $\mu_2 = \langle (d_2, \rho_1^{\circ}\mu_2 e^{2\pi i\alpha_1}, \rho_2^{\circ}\mu_2 e^{2\pi i\alpha_2}, \dots, \rho_m^{\circ}\mu_2 e^{2\pi i\alpha_m}) \rangle$ and $\mu = \langle (d, (\rho_1^{\circ}\mu e^{2\pi i\alpha_1}, \rho_2^{\circ}\mu e^{2\pi i\alpha_2}, \dots, \rho_m^{\circ}\mu e^{2\pi i\alpha_m})) \rangle$ be the three complex m-polar fuzzy N-soft numbers (CmPFNSS) and δ be a real constant. The basic operations on these numbers are defined as follows:

- (1) $\mu_1 \oplus \mu_2 = \langle \max(d_1, d_2), (\rho_1^{\circ}\mu_1 + \rho_1^{\circ}\mu_2 - (\rho_1^{\circ}\mu_1) (\rho_1^{\circ}\mu_2)) e^{2\pi i(\alpha_1 + \beta_1 - \alpha_1\beta_1)}, \dots, (\rho_m^{\circ}\mu_1 + \rho_m^{\circ}\mu_2 - (\rho_m^{\circ}\mu_1) (\rho_m^{\circ}\mu_2)) e^{2\pi i(\alpha_m + \beta_m - \alpha_m\beta_m)} \rangle$
- (2) $\mu_1 \otimes \mu_2 = \langle \min(d_1, d_2), (\rho_1^{\circ}\mu_1) (\rho_1^{\circ}\mu_2) e^{2\pi i(\alpha_1\beta_1)}, \dots, (\rho_m^{\circ}\mu_1) (\rho_m^{\circ}\mu_2) e^{2\pi i(\alpha_m\beta_m)} \rangle$
- (3) $\delta\mu = \langle d, (1 - (1 - \rho_1^{\circ}\mu)^{\delta}) e^{2\pi i(1 - (1 - \alpha_1)^{\delta})}, \dots, (1 - (1 - \rho_m^{\circ}\mu)^{\delta}) e^{2\pi i(1 - (1 - \alpha_m)^{\delta})} \rangle$
- (4) $\mu^{\delta} = \langle d, (\rho_1^{\circ}\mu)^{\delta} e^{2\pi i(\alpha_1)^{\delta}}, \dots, (\rho_m^{\circ}\mu)^{\delta} e^{2\pi i(\alpha_m)^{\delta}} \rangle$
- (5) $S(\mu) = (d/N - 1)^2 + (\rho_1^{\circ}\mu + \rho_2^{\circ}\mu + \dots + \rho_m^{\circ}\mu/m) + (\alpha_1 + \alpha_2, \dots, \alpha_m/m)$

Proposition 2.1. Let $\mu_1 = \langle d_1, (\rho_1 \circ \mu_1 e^{2\pi i \alpha_1}, \rho_2 \circ \mu_1 e^{2\pi i \alpha_2}, \dots, \rho_m \circ \mu_1 e^{2\pi i \alpha_m}) \rangle$ and $\mu_2 = \langle d_2, (\rho_1 \circ \mu_2 e^{2\pi i \alpha_1}, \rho_2 \circ \mu_2 e^{2\pi i \alpha_2}, \dots, \rho_m \circ \mu_2 e^{2\pi i \alpha_m}) \rangle$ and $\mu = \langle d, (\rho_1 \circ \mu e^{2\pi i \alpha_1}, \rho_2 \circ \mu e^{2\pi i \alpha_2}, \dots, \rho_m \circ \mu e^{2\pi i \alpha_m}) \rangle$ be the three complex m -polar fuzzy N -soft numbers $CmPFNSS$ and δ be a real constant [26–28]. The following equalities hold for the basic operations on $CmPFNSS$ s:

- (i) $\bullet \mu_1 \oplus \mu_2 = \mu_2 \oplus \mu_1$
- (ii) $\bullet \mu_1 \otimes \mu_2 = \mu_2 \otimes \mu_1$
- (iii) $\bullet \delta(\mu_1 \oplus \mu_2) = \delta(\mu_1) \oplus \delta(\mu_2)$
- (iv) $\bullet \delta(\mu_1 \otimes \mu_2) = \delta(\mu_1) \otimes \delta(\mu_2)$
- (v) $\bullet ((\mu)^{\delta_1})^{\delta_2} = (\mu)^{\delta_1 \delta_2}$

3. Why Do We Need a Novel Hybrid Model $CmPFNSS$?

This section facilitates us with the answer to the question that why do we need a novel hybrid model, $CmPFNSS$? A detailed discussion is entailed in this section, by considering the natural phenomena occur on earth called volcanic eruption, that how our proposed set can be beneficial in studying and predicting these eruptions so that a lot of damage can be prevented timely as a result of the volcanic explosions.

3.1. Factors on Which Volcanic Eruption Depends. The dynamical nature of the earth is a result of the natural phenomenon occurring inside the earth's crust called "volcanic eruption." When the magmastic pressure crosses a certain level inside the earth's crust the lava erupts from the volcanoes, causes a huge loss of lives and becomes a major source of destruction. Therefore, it is imperative to predict the occurrence of this phenomenon timely so that millions of lives could be saved.

The volcanic eruption depends on various factors or attributes like level of atmospheric pressure, geological factors, change in climatic conditions, pressure inside the earth's crust that further depends on the m -poles giving multipolar information including density of ocean, margins between tectonic plates, density of magma, lithostatic pressure, land sliding on mountains, melting of glaciers, presence of acidic gases and dissolved components like sulphur and water inside the earth's crust. The major factors cause the volcanic eruption are pictured in Figure 1, (<https://kidspressmagazine.com/science-for-kids/misc/misc/volcanic-eruptions-2.html>).

3.2. Role of $CmPFNSS$ in Volcanic Eruption. The question arises that how our proposed hybrid model $CmPFNSS$ is beneficial in studying and predicting this natural phenomenon? The very dual nature of $CmPFNSS$ of dealing with multipolar information, complex two-dimensional data and the N -grading enables us to be benefitted in this scenario. All the aforementioned multipolar information exhibits not only the amplitude term but also depends on the phase (time) term. For example, melting of glaciers is one of the m -poles responsible for the volcanic eruption and this

melting definitely depends on time. Similarly, pressure density, landsliding, ocean density of ocean etc all varies with respect to time. To deal with all these multipolar complex two-dimensional data, $CmPFNSS$ is proved to be a helping hand where N -grading shows the relative importance of the m -poles. Consider a $C3PFNSS$ as $B = \{(j, 5, 0.9 e^{2\pi i(0.3)}, 0.72e^{2\pi i(0.45)}, 0.19e^{2\pi i(0.82)}\}$, where j represents the atmospheric pressure, grade 5 shows its relative importance, and the complex membership degrees $(0.9e^{2\pi i(0.3)}, 0.72e^{2\pi i(0.45)}, 0.19e^{2\pi i(0.82)})$ show the amplitude and phase term of the lithostatic pressure, magmastic pressure and density of magma. Hence, this natural phenomenon can be thoroughly examined by utilizing the proposed hybrid model $CmPFNSS$.

3.3. Superiority of $CmPFNSS$ over the Existing Ones. No doubt, the fuzzy sets system is equipped with numerous worthy sets or models that are proved to be of great assistance with their remarkable features but our proposed hybrid model excelled from them in all aspects. As m -polar fuzzy set [29] deals only with multipolar information, but not with complex two-dimensional data and N -grading. Akram et al. [20] introduced the fuzzy N -soft set that handles the N -grading but fails to tackle multipolar complex two-dimensional data. The notion of m -polar fuzzy N -soft set was put forward by Akram et al. [1] that addresses the multipolar information with N -grading but collapses when we have complex two-dimensional data. To overcome this hurdle, Ramot et al. [14] gave the concept of a complex fuzzy set that can easily handle the complex two dimensional data but fails to deal with multipolarity with N -grading. Hence, in order to occupy the complexity of multipolar information with N -grading, Akram and Sultan introduced the concept of $CmPFNSS$ that covers all the aspects of multipolar complex two-dimensional data with N -grading. In the phenomenon of volcanic eruption, $CmPFNSS$ provides us with the most useful, realistic and practical information.

4. $CmPFNS$ -TOPSIS Technique for MCGDM

In this section, each and every step of $CmPFNS$ -TOPSIS technique has been explained in detail, so that we have a better on-look upon the strategy being used in performing this technique. The motive of this technique is to calculate the Euclidean distance of each alternative from the positive ideal solution (PIS) and the negative ideal solution (NIS) with respect to each alternative, where PIS deals with benefit-type criteria and NIS deals with cost-type criteria. At the end, by computing the relative closeness index, the best alternative along with the rankings of all alternatives has been selected. The algorithm of $CmPFNS$ -TOPSIS technique can be su m -up in the following steps:

4.1. Construction of Independent Decision Matrices of Each Expert. Since the technique under consideration is $CmPFNS$ -TOPSIS technique, where each decision-making expert $\Psi = \{\psi_h\}, h = 1, 2, \dots, l$ allots the N -grades and membership degrees to each alternative $\Omega = \{\omega_p\}$,

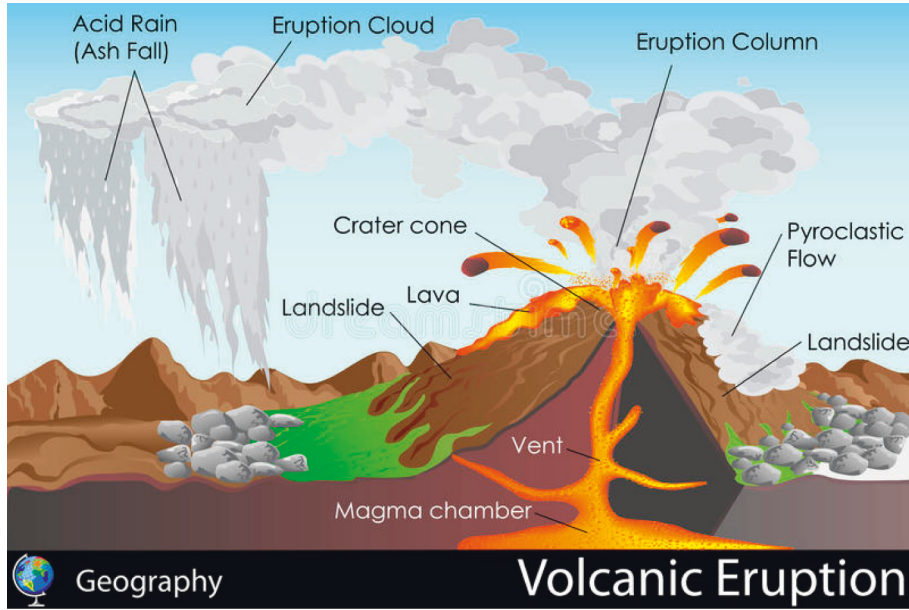


FIGURE 1: Volcanic eruption.

$p = 1, 2, \dots, r$ with respect to the decision attributes $\Phi = \{\phi_t\}, t = 1, 2, \dots, q$ that depend on the complex m -polar decision criteria $\rho_i \circ \mu e^{2\pi i \alpha_i}, i = 1, 2, \dots, m$. The result is evaluated in the form of a complex m -polar fuzzy N-soft decision matrix $\mathfrak{F}^{(h)^i} = (\xi_{pt}^{(h)})_{r \times q}$, where $\xi_{pt}^{(h)} = \langle d_{pt}^h, (\xi_{pt}^{(h)^1},$

$\xi_{pt}^{(h)^2}, \dots, \xi_{pt}^{(h)^m}) \rangle$, and $\xi_{pt}^{(h)^i} = \rho_{pt}^{(h)^i} \circ \mu e^{2\pi i \alpha_{pt}^{(h)^i}}, \rho_{pt}^{(h)^2} \circ \mu e^{2\pi i \alpha_{pt}^{(h)^2}}, \dots, \rho_{pt}^{(h)^m} \circ \mu e^{2\pi i \alpha_{pt}^{(h)^m}}, i = 1, 2, \dots, m$.

$$\mathfrak{F}^{(h)^i} = \begin{pmatrix} \langle d_{11}^h, (\xi_{11}^{(h)^1}, \xi_{11}^{(h)^2}, \dots, \xi_{11}^{(h)^m}) \rangle & \langle d_{12}^h, (\xi_{12}^{(h)^1}, \xi_{12}^{(h)^2}, \dots, \xi_{12}^{(h)^m}) \rangle & \dots & \langle d_{1q}^h, (\xi_{1q}^{(h)^1}, \xi_{1q}^{(h)^2}, \dots, \xi_{1q}^{(h)^m}) \rangle \\ \langle d_{21}^h, (\xi_{21}^{(h)^1}, \xi_{21}^{(h)^2}, \dots, \xi_{21}^{(h)^m}) \rangle & \langle d_{22}^h, (\xi_{22}^{(h)^1}, \xi_{22}^{(h)^2}, \dots, \xi_{22}^{(h)^m}) \rangle & \dots & \langle d_{2q}^h, (\xi_{2q}^{(h)^1}, \xi_{2q}^{(h)^2}, \dots, \xi_{2q}^{(h)^m}) \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle d_{r1}^h, (\xi_{r1}^{(h)^1}, \xi_{r1}^{(h)^2}, \dots, \xi_{r1}^{(h)^m}) \rangle & \langle d_{r2}^h, (\xi_{r2}^{(h)^1}, \xi_{r2}^{(h)^2}, \dots, \xi_{r2}^{(h)^m}) \rangle & \dots & \langle d_{rq}^h, (\xi_{rq}^{(h)^1}, \xi_{rq}^{(h)^2}, \dots, \xi_{rq}^{(h)^m}) \rangle \end{pmatrix}. \quad (3)$$

4.2. Construction of Aggregated Complex m -Polar Fuzzy N-Soft Decision Matrix. The gradings and membership degrees of the individual expert is of no use because there is a panel of experts who are analyzing each and every attribute of the alternatives so their assessments must be accumulated into a collective grade and membership degree in a matrix called Aggregated Complex m -Polar Fuzzy N-Soft Decision Matrix (ACmPFNSDM) [30, 31]. The entries of this matrix ACmPFNSDM $\mathfrak{F} = (\xi_{pt})_{r \times q}$ are evaluated by

using the following complex m -polar fuzzy N-soft weightage averaging (CmPFNSWA) operator:

$$\begin{aligned} \mathfrak{F}_{pt} &= CmPFNSWA_{\epsilon}(\xi_{pt}^{(1)}, \xi_{pt}^{(2)}, \dots, \xi_{pt}^{(l)}) \\ &= \epsilon_1 \xi_{pt}^{(1)} \oplus \epsilon_2 \xi_{pt}^{(2)} \oplus \dots \oplus \epsilon_l \xi_{pt}^{(l)} \\ &= (d_{pt}^*), \end{aligned} \quad (4)$$

where $\Delta = (\Delta^1, \Delta^2, \dots, \Delta^m)$,

$$\Delta^i = 1 - \prod_{h=1}^l (1 - \rho_{pt}^{(h)^i} \circ \mu)^{\epsilon_h} e^{2\pi i (1 - \prod_{h=1}^l (1 - \alpha_{pt}^{(h)^i})^{\epsilon_h})}, i=1,2,\dots,m, d_{pt}^* = \max(d_{pt}^1, d_{pt}^2, \dots, d_{pt}^l). \quad (5)$$

The constructed ACmPFNSDM can be framed as

$$\mathfrak{S} = \begin{pmatrix} \langle d_{11}^*, (\xi_{11}^1, \xi_{11}^2, \dots, \xi_{11}^m) \rangle & \langle d_{12}^*, (\xi_{12}^1, \xi_{12}^2, \dots, \xi_{12}^m) \rangle & \dots & \langle d_{1q}^*, (\xi_{1q}^1, \xi_{1q}^2, \dots, \xi_{1q}^m) \rangle \\ \langle d_{21}^*, (\xi_{21}^1, \xi_{21}^2, \dots, \xi_{21}^m) \rangle & \langle d_{22}^*, (\xi_{22}^1, \xi_{22}^2, \dots, \xi_{22}^m) \rangle & \dots & \langle d_{2q}^*, (\xi_{2q}^1, \xi_{2q}^2, \dots, \xi_{2q}^m) \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle d_{r1}^*, (\xi_{r1}^1, \xi_{r1}^2, \dots, \xi_{r1}^m) \rangle & \langle d_{r2}^*, (\xi_{r2}^1, \xi_{r2}^2, \dots, \xi_{r2}^m) \rangle & \dots & \langle d_{rq}^*, (\xi_{rq}^1, \xi_{rq}^2, \dots, \xi_{rq}^m) \rangle \end{pmatrix}, \quad (6)$$

where $\xi_{pt}^i = \rho_{pt}^i \circ \mu e^{2\pi i \alpha_{pt}^i}$, $i = 1, 2, \dots, m$.

4.3. Allocation of Weightage to Decision Criteria by Each Expert. In any project or the real-world scenario, the criteria or alternatives do not have the same relative importance. Each criterion has its own benefit, importance and credibility. And to consider one of them or to make a decision, it is mandatory for the decision makers to consider the fact of relative importance of each and every criterion. On the same lines, in a MCGDM techniques, each expert allots the weights to each criterion according to their reliability. In a CmPFNS-TOPSIS technique, not only the membership grades, as weights, but also the ordered grades, as weights, are assigned to each attribute by each expert. The individual weights assigned to each criteria is in the form of $\chi_t^{(h)} = \langle d_t^{(h)}, \bar{\sigma}_t^{(h)1}, \bar{\sigma}_t^{(h)2}, \dots, \bar{\sigma}_t^{(h)m} \rangle$ where $d_t^{(h)}$ is the weightage grade given to the criterion ϕ_t by the expert ψ_h , $\bar{\sigma}_t^{(h)i} = \rho_t^{(h)i} \circ \mu e^{2\pi i \alpha_{pt}^{(h)i}}$, $i = 1, 2, \dots, m$ is the weightage membership degree given to each criterion ϕ_t by the expert ψ_h with respect to the i -th pole. These individual assessments of the decision makers to the criteria are assembled to construct a weightage vector $\chi = (\chi_1, \chi_2, \dots, \chi_q)^T$ of the decision criteria, by utilizing the following CmPFNSWA operator:

$$\begin{aligned} \chi_t &= \text{CmPFNSWA}_\varepsilon(\bar{\sigma}_t^{(1)}, \bar{\sigma}_t^{(2)}, \dots, \bar{\sigma}_t^{(l)}) \\ &= \varepsilon_1 \bar{\sigma}_t^{(1)} \oplus \varepsilon_2 \bar{\sigma}_t^{(2)} \oplus \dots \oplus \varepsilon_l \bar{\sigma}_t^{(l)} \\ &= (d_t^*, \Delta), \end{aligned} \quad (7)$$

where $\Delta = (\Delta^1, \Delta^2, \dots, \Delta^m)$.

$$\mathfrak{S}' = \begin{pmatrix} \langle d'_{11}, (B_{11}^1, B_{11}^2, \dots, B_{11}^m) \rangle & \langle d'_{12}, (B_{12}^1, B_{12}^2, \dots, B_{12}^m) \rangle & \dots & \langle d'_{1q}, (B_{1q}^1, B_{1q}^2, \dots, B_{1q}^m) \rangle \\ \langle d'_{21}, (B_{21}^1, B_{21}^2, \dots, B_{21}^m) \rangle & \langle d'_{22}, (B_{22}^1, B_{22}^2, \dots, B_{22}^m) \rangle & \dots & \langle d'_{2q}, (B_{2q}^1, B_{2q}^2, \dots, B_{2q}^m) \rangle \\ \vdots & \vdots & \dots & \vdots \\ \langle d'_{r1}, (B_{r1}^1, B_{r1}^2, \dots, B_{r1}^m) \rangle & \langle d'_{r2}, (B_{r2}^1, B_{r2}^2, \dots, B_{r2}^m) \rangle & \dots & \langle d'_{rq}, (B_{rq}^1, B_{rq}^2, \dots, B_{rq}^m) \rangle \end{pmatrix}, \quad (13)$$

where $B_{pt}^i = \tilde{\rho}_{pt}^i \circ \mu e^{2\pi i \tilde{\alpha}_{pt}^i}$, $i = 1, 2, \dots, m$.

4.5. Construction of Score Matrix for Designation of Ideal Solutions. Since the considered environment is a complex m -polar system having complex m -polar values which are

$$\begin{aligned} \Delta^i &= 1 - \prod_{h=1}^l \left(1 - \rho_t^{(h)i} \circ \mu\right)^{\varepsilon_h} \\ &e^{2\pi i \left(1 - \prod_{h=1}^l (1 - \alpha_t^{(h)i})^{\varepsilon_h}\right)}, \quad i=1, 2, \dots, m, \end{aligned} \quad (8)$$

$$d_t^* = \max(d_t^{(1)}, d_t^{(2)}, \dots, d_t^{(l)}). \quad (9)$$

After performing these calculations, the entries of χ_t are formulated in the form of $\chi_t = \langle d_t^*, (\nabla_t^1, \nabla_t^2, \dots, \nabla_t^m) \rangle$ where $\nabla_t^i = \rho_t^{i0} \circ \mu e^{2\pi i \alpha_t^i}$.

4.4. Construction of Aggregated Weighted Complex m -Polar Fuzzy N -Soft Decision Matrix. After the formation of ACmPFNSDM and the weight vector of decision-criteria, the mutual decision of decision-making experts and the weights of the decision-criteria are merged to construct the aggregated weighted complex m -polar fuzzy N -soft decision matrix (AWCmPFNS DM) $\mathfrak{S}' = (\xi_{pt}')_{r \times q}$ can be determined as follows:

$$\begin{aligned} \xi_{pt}' &= \xi_{pt} \otimes \chi_t \\ &= (d_{pt}', B), \end{aligned} \quad (10)$$

where $B = (B^1, B^2, \dots, B^m)$,

$$B^i = (\rho_{pt}^i \circ \mu)(\rho_t^i \circ \mu) e^{2\pi i \left(\alpha_{pt}^i\right) \left(\alpha_t^i\right)}, \quad i = 1, 2, \dots, m, \quad (11)$$

$$d_{pt}' = \min(d_{pt}, d_t). \quad (12)$$

The constructed AWCmPFNSDM can be presented as follows:

very complicated, having no specific order and incomparable. That is why it is very tricky to calculate positive and negative ideal solutions from the AWCmPFNSDM. To overcome this hurdle, score function helps us to convert all the complex m -polar values into the crisp ones, which are easy to handle. The score value of each entry of

AWCmPFNSDM can be calculated by using the following formula:

$$R_{pt} = \left[\frac{d'_{pt}}{N-1} \right] + \left[\frac{\rho_{pt}^1 \circ \mu + \rho_{pt}^2 \circ \mu + \dots + \rho_{pt}^m \circ \mu}{m} \right] + \left[\frac{\alpha_{pt}^1 + \alpha_{pt}^2 + \dots + \alpha_{pt}^m}{m} \right], \quad (14)$$

where $p = 1, 2, \dots, r$ and $t = 1, 2, \dots, q$. The assembled score matrix can be represented as

$$E = \begin{pmatrix} R_{11} & R_{12} & \dots & R_{1q} \\ R_{21} & R_{22} & \dots & R_{2q} \\ \vdots & \vdots & \vdots & \vdots \\ R_{r1} & R_{r2} & \dots & R_{rq} \end{pmatrix}. \quad (15)$$

4.6. Formulation of Complex m -Polar Fuzzy N -Soft Positive Ideal Solution CmPFNS-PIS and Negative Ideal Solution CmPFNS-NIS. Let Υ_b and Υ_c represents the benefit-type criteria and cost-type criteria respectively. Now, calculate the CmPFNS-PIS (P) and CmPFNS-NIS (N) for the maximization of all benefit-type criteria and the maximization of all cost-type criteria respectively. The estimated CmPFNS-PIS (P) and CmPFNS-NIS (N) with respect to each criterion are given as follows:

$$\begin{aligned} P &= \{P_t^+ = \langle d_t^+, D_t^+ \rangle, t = 1, 2, \dots, q\}, \\ N &= \{N_t^- = \langle d_t^-, D_t^- \rangle, t = 1, 2, \dots, q\}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} D_t^+ &= \langle D_t^{+1}, D_t^{+2}, \dots, D_t^{+m} \rangle, \\ D_t^- &= \langle D_t^{-1}, D_t^{-2}, \dots, D_t^{-m} \rangle, \end{aligned} \quad (17)$$

and

$$\begin{aligned} D_t^{+i} &= \left\{ \rho_t^{i+} \circ \mu e^{2\pi i \alpha_t^+}, i = 1, 2, \dots, m \right\}, \\ D_t^{-i} &= \left\{ \rho_t^{i-} \circ \mu e^{2\pi i \alpha_t^-}, i = 1, 2, \dots, m \right\}. \end{aligned} \quad (18)$$

Now, P_t^+ and N_t^- can be formulated as

$$P_t^+ = \begin{cases} \mathfrak{S}'_{jk}: R(\mathfrak{S}'_{jk}) = \max_{1 \leq p \leq r} R((\mathfrak{S}'_{pt})), & \text{if } \phi_t \in \Upsilon_b, \\ \mathfrak{S}'_{jk}: R(\mathfrak{S}'_{jk}) = \min_{1 \leq p \leq r} R((\mathfrak{S}'_{pt})), & \text{if } \phi_t \in \Upsilon_c. \end{cases} \quad (19)$$

$$N_t^- = \begin{cases} \mathfrak{S}'_{jk}: R(\mathfrak{S}'_{jk}) = \min_{1 \leq p \leq r} R((\mathfrak{S}'_{pt})), & \text{if } \phi_t \in \Upsilon_b, \\ \mathfrak{S}'_{jk}: R(\mathfrak{S}'_{jk}) = \max_{1 \leq p \leq r} R((\mathfrak{S}'_{pt})), & \text{if } \phi_t \in \Upsilon_c. \end{cases} \quad (20)$$

4.7. Calculation of Distance Measures between Alternatives and Ideal Solutions. It is quite tricky to determine the actual CmPFNS-PIS and CmPFNS-NIS in the real-world problems related to complex two-dimensional data. To overcome this hurdle, we calculate the normalized Euclidean distance of each alternative from the CmPFNS-PIS and CmPFNS-NIS to determine the best alternative which is nearest to the CmPFNS-PIS and farthest from the CmPFNS-NIS. The normalized Euclidean distance of the alternative (ω_p) from the ideal solutions can be evaluated as follows:

$$d(\omega_p, P) = \sqrt{\frac{\sum_{t=1}^q \left[(1/(N-1))^2 (d'_{pt} - d_t^+)^2 + \sum_{i=1}^m (\tilde{\rho}_{pt}^i \circ \mu - \rho_t^{i+} \circ \mu)^2 + \sum_{i=1}^m (\tilde{\alpha}_{pt}^i - \alpha_t^{i+})^2 \right]}{rm}}, \quad (21)$$

$$d(\omega_p, N) = \sqrt{\frac{\sum_{t=1}^q \left[(1/(N-1))^2 (d'_{pt} - d_t^-)^2 + \sum_{i=1}^m (\tilde{\rho}_{pt}^i \circ \mu - \rho_t^{i-} \circ \mu)^2 + \sum_{i=1}^m (\tilde{\alpha}_{pt}^i - \alpha_t^{i-})^2 \right]}{rm}}, \quad (22)$$

where r is the number of criteria and m is the number of poles.

4.8. Evaluation of Revised Closeness Index. The revised closeness index is used to determine the extent of closeness of the alternative from the positive ideal solution and the extent of the fairness of the alternative from the negative ideal solution. The alternative with least revised closeness

index will be considered as the most suitable one. To calculate the revised closeness index, we use the formula introduced by Gundogdu and Kahraman (2019) as follows:

$$I(\omega_p) = \frac{d(\omega_p, P)}{d_{\min}(\omega_p, P)} - \frac{d(\omega_p, N)}{d_{\max}(\omega_p, N)}, \quad p = 1, 2, \dots, r, \quad (23)$$

where

$$\begin{aligned} d_{\min}(\omega_p, P) &= \min_p d(\omega_p, P), \\ d_{\max}(\omega_p, P) &= \max_p d(\omega_p, N). \end{aligned} \quad (24)$$

4.9. *Determination of the Ranking of the Alternatives.* After calculating the revised closeness index, the last step is to organize the alternatives in the descending order, that is, the alternative with the least index will come at the first place, most preferable, and the alternative having largest index will come at the last, least preferable, that is,

$$\omega' = \left\{ \omega_n: I(\omega_n) = \min_p I(\omega_p) \right\}. \quad (25)$$

5. Selection of Surgical Equipment in the Shaukhat Khanum Hospital, Lahore: A Case Study

In this section, the reliability and credibility of the proposed CmpFNS-TOPSIS is illustrated with the help of a real-life example, named as selection of the suitable equipment for the surgical oncology department of the Shaukhat Khanum Hospital, so that we have a better on-look on the practicability of our veracious technique [32].

One of the world's most well-known cancer hospitals is Shaukhat Khanum Memorial Cancer Hospital, where the patients suffering from cancer and tumor like chronic diseases are being treated with the help of the modern machinery. Since the hospital is run by the donations of the public and the government support, this is its foremost duty to maintain its standards of machines, staff and treatment. Because only this way it will remain trustworthy for its patients as ever before. To upgrade the system and keep up the level of quality of the machines, used in diagnosis and medication, the higher authority of the hospital always in endeavor to choose the finest quality of machines, trained staff for the prime treatment of their respected patients. In this scenario, the selection of such type of machines and devices is quite a hectic task to be performed because even a single machine has multiple attributes of its own and not every attribute is of equal importance. This complication can be handled with our sensational proposed CmpFNS-TOPSIS technique that can elegantly share this burden of the hospital. The data has been gathered from the website (<https://shaukatkhanum.org.pk>), where a surgical oncology department in Shaukat Khanum Hospital wants to import some medical equipment for the department. The higher authorities put forward the universe five medical equipments $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, after analyzing the department's requirements. The detailed information of the chosen alternatives is enlightened in Table 1:

To select the most suitable medical equipment for the surgery oncology department, the department has hired a panel of four decision making experts $\Psi = \{\psi_1, \psi_2, \psi_3, \psi_4\}$ who shall thoroughly examine all the attributes of the given devices, where $\psi_1 =$ Project Supervisor, $\psi_2 =$ Finance

Supervisor, $\psi_3 =$ Health Supervisor, $\psi_4 =$ Technical Supervisor.

The attributes of the alternatives $\Phi = \{\phi_1, \phi_2, \phi_3, \phi_4\}$, being evaluated by the decision making experts for their comprehensive mutual decision, further depend upon three parameters, discussed in detail:

5.1. ϕ_1 : *Cost.* In any type of project, the criterion "cost" always plays a vital role in accomplishing the given task of the project. Maintenance cost, the expenses of upgrading the machines, electricity cost and the cost of import are those parameters upon which the cost attribute depend upon.

5.2. ϕ_2 : *Accessibility.* Whenever there is a need for a machine or a device, one question always arises in our mind whether that device is in our access or not. By the accessibility of an equipment, we mean the availability of a machine, access of spare parts, and the availability of the required area or space being occupied by the machine.

5.3. ϕ_3 : *Quality.* One of the salient features of the medical devices is their quality because the sub-standard medical equipment not only disturbs the diagnosis but also has the pernicious effects on the health of the patients. This feature occupies the durability of the equipment, long lasting-not easily replaceable, viscosity of the devices and ease of sterilization.

5.4. ϕ_4 : *Health Risks.* Since the equipment under consideration are used for the medical treatment purposes, there must be very low health risks, not only for the patients but also for the doctors and surgeons using them. This attribute incorporates effects on user's health, emission of radiations and potential to deal with health sensitivity. The main intention behind this procedure is to allocate the best medical equipment, in the surgery oncology department, that will serve the patients in its best way, that is, the accuracy of the diagnosis of the ailment, minimum health risks and having the finest quality in a reasonable price of cost. The description of the alternatives, attributes, m -polar criteria and decision-making experts is synchronized in Figure 2. The graphical representation of the attributes with their parameters is in Figure 2.

The step-by-step solution for the selection of the most valuable medical equipment using recommended CmpFNS-TOPSIS method is elucidated as follows:

- (1) The administration of the oncology department assigns weights to each decision-making expert with respect to their capability and competency. The weights granted to each expert is arranged in Table 2.
- (2) The appointed decision-makers, after detailed examination of the devices under consideration according to each decision criteria, put forward their assessed data in the form of individual C3PF6SDMs, exhibited in Tables 3–6, respectively.

TABLE 1: Information related to the selected alternatives.

Alternatives	Representation	Details
ω_1	Cysto urethroscope	Its mechanism includes mainly the detection of several conditions, including bladder tumors, stones, cancer and enlarged prostate gland etc. It is a thin tube with a camera and light on the end. A doctor inserts the tube through your urethra and into your bladder so he can visualize the inside of your bladder. Magnified images from the camera are seen on the screen viewed by doctors.
ω_2	Laparoscopic 3-D unit	This is a very efficient, safe and sustainable innovation used by the surgeons to have a 3-D vision. Having a more realistic standard and closer to “open surgery” vision is one of the remarkable uses of this unit. This system equipped with 3-D technology is grasping the attention of all surgeons around the world. It is widely used to remove a damaged or diseased organ, to diagnose a wide range of conditions that develop inside the abdomen or pelvis.
ω_3	Digital fluoroscopy	This form of machine is an X-ray machine that allows the doctors to have a deep view of body structures in real time. It projects the detailed images of functions and structure of the body parts like intestine, bladder, cardiac muscle and stomach. The patient is positioned on a large moveable, flat table, a moveable X-ray camera extends over a portion of a table, captures images at different angles and send the images to nearby television monitor which is viewed by the doctor or radiologist.
ω_4	Laparoscopic gas insufflator	Laparoscopic surgery involves insufflation of a gas, usually CO_2 , into the peritoneal cavity producing a pneumoperitoneum. CO_2 is insufflated into a peritoneal cavity at a rate of $4 - 6 \text{ litermin}^{-1}$ to a pressure of 10–20 mmHg
ω_5	Abdominal perineal resection instruments set	This set has the most significant use in the treatment of rectal cancers located very low in the rectum. The instruments must have a very fine quality so that they do not form rashes on the body parts.

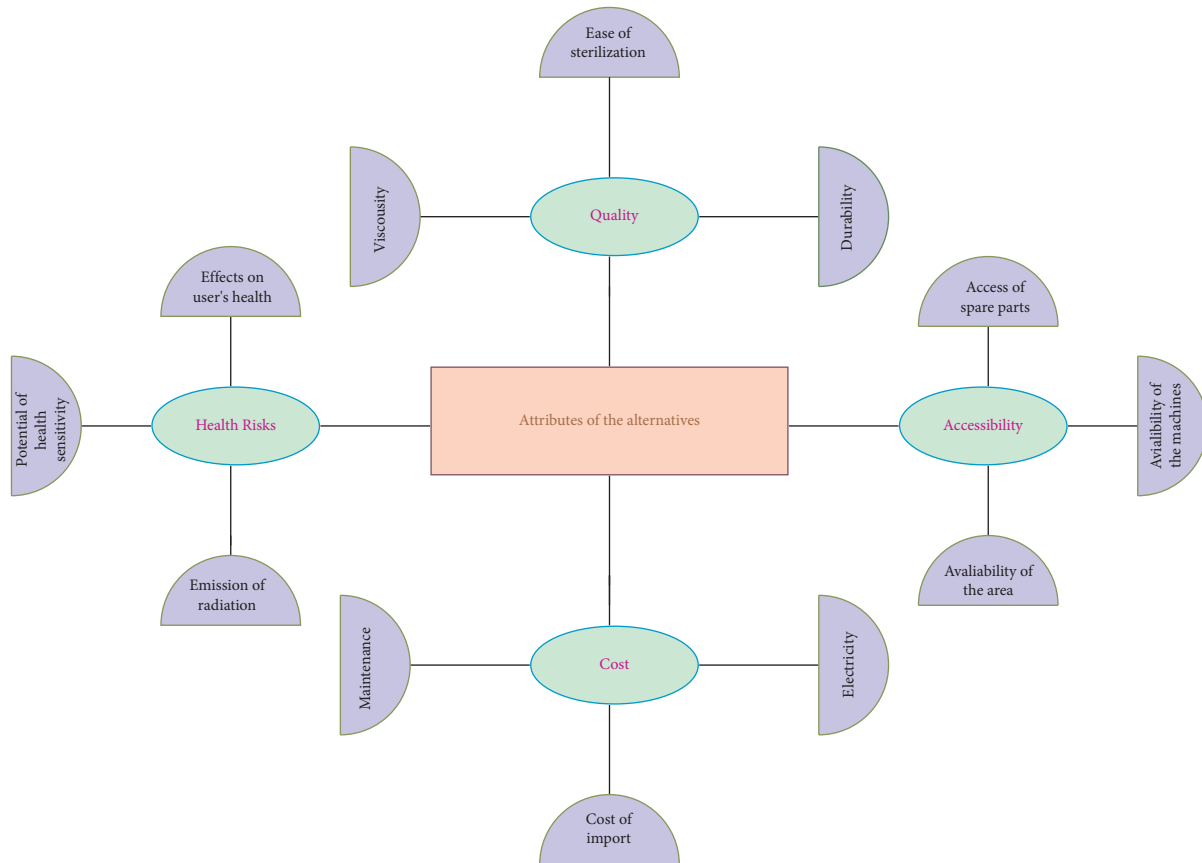


FIGURE 2: Graphical representation of the alternatives with their attributes.

TABLE 2: Weightage of experts.

Δ/Ψ	Ψ_1	Ψ_2	Ψ_3	Ψ_4
Δ_n	0.21	0.24	0.37	0.18

TABLE 3: C3PF6S DM of the project supervisor (ψ_1).

Γ^{-1}	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	$(4, 0.62e^{i2\pi(0.71)}, 0.68e^{i2\pi(0.75)}, 0.79e^{i2\pi(0.60)})$	$(2, 0.29e^{i2\pi(0.30)}, 0.22e^{i2\pi(0.35)}, 0.33e^{i2\pi(0.27)})$	$(3, 0.42e^{i2\pi(0.45)}, 0.54e^{i2\pi(0.49)}, 0.59e^{i2\pi(0.42)})$	$(4, 0.75e^{i2\pi(0.79)}, 0.63e^{i2\pi(0.78)}, 0.69e^{i2\pi(0.79)})$
ω_2	$(5, 0.89e^{i2\pi(0.90)}, 0.92e^{i2\pi(0.95)}, 0.99e^{i2\pi(0.88)})$	$(2, 0.21e^{i2\pi(0.35)}, 0.24e^{i2\pi(0.38)}, 0.34e^{i2\pi(0.27)})$	$(3, 0.52e^{i2\pi(0.50)}, 0.49e^{i2\pi(0.55)}, 0.57e^{i2\pi(0.41)})$	$(5, 0.82e^{i2\pi(0.90)}, 0.93e^{i2\pi(0.92)}, 0.95e^{i2\pi(0.97)})$
ω_3	$(5, 0.92e^{i2\pi(0.89)}, 0.89e^{i2\pi(0.95)}, 0.87e^{i2\pi(0.99)})$	$(2, 0.22e^{i2\pi(0.35)}, 0.28e^{i2\pi(0.32)}, 0.34e^{i2\pi(0.29)})$	$(3, 0.45e^{i2\pi(0.55)}, 0.56e^{i2\pi(0.41)}, 0.49e^{i2\pi(0.50)})$	$(5, 0.97e^{i2\pi(0.95)}, 0.93e^{i2\pi(0.89)}, 0.95e^{i2\pi(0.85)})$
ω_4	$(2, 0.21e^{i2\pi(0.30)}, 0.35e^{i2\pi(0.25)}, 0.29e^{i2\pi(0.31)})$	$(3, 0.42e^{i2\pi(0.52)}, 0.48e^{i2\pi(0.50)}, 0.47e^{i2\pi(0.55)})$	$(1, 0.17e^{i2\pi(0.12)}, 0.15e^{i2\pi(0.14)}, 0.18e^{i2\pi(0.19)})$	$(3, 0.43e^{i2\pi(0.49)}, 0.57e^{i2\pi(0.51)}, 0.48e^{i2\pi(0.54)})$
ω_5	$(0, 0.02e^{i2\pi(0.07)}, 0.05e^{i2\pi(0.01)}, 0.03e^{i2\pi(0.05)})$	$(4, 0.62e^{i2\pi(0.75)}, 0.69e^{i2\pi(0.71)}, 0.65e^{i2\pi(0.68)})$	$(5, 0.88e^{i2\pi(0.95)}, 0.97e^{i2\pi(0.82)}, 0.87e^{i2\pi(0.93)})$	$(1, 0.11e^{i2\pi(0.16)}, 0.15e^{i2\pi(0.12)}, 0.13e^{i2\pi(0.17)})$

TABLE 4: C3PF6S DM of the finance supervisor (ψ_2).

Γ^{-2}	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	$(3, 0.42e^{2\pi(0.52)}, 0.47e^{2\pi(0.51)}, 0.55e^{2\pi(0.44)})$	$(3, 0.43e^{2\pi(0.45)}, 0.47e^{2\pi(0.52)}, 0.53e^{2\pi(0.41)})$	$(4, 0.62e^{2\pi(0.72)}, 0.68e^{2\pi(0.70)}, 0.64e^{2\pi(0.67)})$	$(4, 0.71e^{2\pi(0.65)}, 0.66e^{2\pi(0.72)}, 0.73e^{2\pi(0.60)})$
ω_2	$(4, 0.62e^{2\pi(0.65)}, 0.67e^{2\pi(0.71)}, 0.72e^{2\pi(0.70)})$	$(3, 0.55e^{2\pi(0.42)}, 0.45e^{2\pi(0.59)}, 0.58e^{2\pi(0.48)})$	$(3, 0.44e^{2\pi(0.52)}, 0.45e^{2\pi(0.53)}, 0.49e^{2\pi(0.55)})$	$(5, 0.82e^{2\pi(0.83)}, 0.85e^{2\pi(0.90)}, 0.87e^{2\pi(0.86)})$
ω_3	$(4, 0.77e^{2\pi(0.68)}, 0.79e^{2\pi(0.65)}, 0.69e^{2\pi(0.78)})$	$(2, 0.22e^{2\pi(0.39)}, 0.35e^{2\pi(0.21)}, 0.29e^{2\pi(0.36)})$	$(4, 0.69e^{2\pi(0.75)}, 0.76e^{2\pi(0.68)}, 0.79e^{2\pi(0.66)})$	$(5, 0.85e^{2\pi(0.92)}, 0.89e^{2\pi(0.88)}, 0.99e^{2\pi(0.82)})$
ω_4	$(3, 0.43e^{2\pi(0.50)}, 0.55e^{2\pi(0.45)}, 0.44e^{2\pi(0.56)})$	$(1, 0.15e^{2\pi(0.18)}, 0.16e^{2\pi(0.12)}, 0.11e^{2\pi(0.14)})$	$(2, 0.25e^{2\pi(0.31)}, 0.27e^{2\pi(0.32)}, 0.35e^{2\pi(0.29)})$	$(3, 0.47e^{2\pi(0.54)}, 0.55e^{2\pi(0.49)}, 0.56e^{2\pi(0.53)})$
ω_5	$(1, 0.17e^{2\pi(0.12)}, 0.15e^{2\pi(0.18)}, 0.16e^{2\pi(0.11)})$	$(5, 0.88e^{2\pi(0.92)}, 0.96e^{2\pi(0.83)}, 0.84e^{2\pi(0.91)})$	$(4, 0.62e^{2\pi(0.79)}, 0.69e^{2\pi(0.72)}, 0.77e^{2\pi(0.73)})$	$(2, 0.21e^{2\pi(0.24)}, 0.25e^{2\pi(0.30)}, 0.27e^{2\pi(0.22)})$

TABLE 5: C3F6SDM of the health supervisor (ψ_3).

Γ^3	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	$(4, 0.75e^{2\pi(0.69)}, 0.67e^{2\pi(0.78)}, 0.63e^{2\pi(0.65)})$	$(3, 0.42e^{2\pi(0.56)}, 0.46e^{2\pi(0.54)}, 0.58e^{2\pi(0.42)})$	$(2, 0.22e^{2\pi(0.39)}, 0.34e^{2\pi(0.32)}, 0.26e^{2\pi(0.28)})$	$(4, 0.69e^{2\pi(0.71)}, 0.75e^{2\pi(0.67)}, 0.72e^{2\pi(0.65)})$
ω_2	$(4, 0.65e^{2\pi(0.71)}, 0.76e^{2\pi(0.61)}, 0.77e^{2\pi(0.60)})$	$(3, 0.40e^{2\pi(0.52)}, 0.55e^{2\pi(0.41)}, 0.52e^{2\pi(0.45)})$	$(3, 0.52e^{2\pi(0.47)}, 0.54e^{2\pi(0.56)}, 0.48e^{2\pi(0.45)})$	$(4, 0.65e^{2\pi(0.64)}, 0.75e^{2\pi(0.74)}, 0.63e^{2\pi(0.72)})$
ω_3	$(5, 0.84e^{2\pi(0.92)}, 0.86e^{2\pi(0.89)}, 0.89e^{2\pi(0.95)})$	$(2, 0.22e^{2\pi(0.31)}, 0.33e^{2\pi(0.28)}, 0.36e^{2\pi(0.32)})$	$(3, 0.45e^{2\pi(0.57)}, 0.52e^{2\pi(0.43)}, 0.54e^{2\pi(0.46)})$	$(4, 0.65e^{2\pi(0.72)}, 0.71e^{2\pi(0.67)}, 0.65e^{2\pi(0.75)})$
ω_4	$(3, 0.51e^{2\pi(0.42)}, 0.43e^{2\pi(0.59)}, 0.55e^{2\pi(0.47)})$	$(1, 0.14e^{2\pi(0.12)}, 0.15e^{2\pi(0.17)}, 0.18e^{2\pi(0.11)})$	$(2, 0.24e^{2\pi(0.30)}, 0.25e^{2\pi(0.34)}, 0.36e^{2\pi(0.22)})$	$(2, 0.29e^{2\pi(0.32)}, 0.35e^{2\pi(0.27)}, 0.33e^{2\pi(0.28)})$
ω_5	$(1, 0.16e^{2\pi(0.11)}, 0.11e^{2\pi(0.18)}, 0.15e^{2\pi(0.13)})$	$(5, 0.84e^{2\pi(0.97)}, 0.92e^{2\pi(0.86)}, 0.96e^{2\pi(0.84)})$	$(4, 0.75e^{2\pi(0.67)}, 0.62e^{2\pi(0.78)}, 0.68e^{2\pi(0.73)})$	$(1, 0.12e^{2\pi(0.13)}, 0.14e^{2\pi(0.15)}, 0.13e^{2\pi(0.18)})$

TABLE 6: C3F6SDM of the technical supervisor (ψ_4).

Γ^{-4}	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(5, 0.98e ^{2π(0.85)} , 0.82e ^{2π(0.99)} , 0.85e ^{2π(0.89)})	(1, 0.18e ^{2π(0.17)} , 0.12e ^{2π(0.13)} , 0.17e ^{2π(0.15)})	(3, 0.52e ^{2π(0.44)} , 0.55e ^{2π(0.46)} , 0.47e ^{2π(0.53)})	(4, 0.71e ^{2π(0.62)} , 0.73e ^{2π(0.64)} , 0.65e ^{2π(0.78)})
ω_2	(4, 0.68e ^{2π(0.71)} , 0.67e ^{2π(0.74)} , 0.75e ^{2π(0.62)})	(3, 0.42e ^{2π(0.55)} , 0.44e ^{2π(0.56)} , 0.52e ^{2π(0.43)})	(2, 0.22e ^{2π(0.32)} , 0.38e ^{2π(0.29)} , 0.31e ^{2π(0.37)})	(4, 0.66e ^{2π(0.77)} , 0.77e ^{2π(0.68)} , 0.68e ^{2π(0.62)})
ω_3	(5, 0.82e ^{2π(0.91)} , 0.88e ^{2π(0.92)} , 0.87e ^{2π(0.93)})	(0, 0.02e ^{2π(0.05)} , 0.05e ^{2π(0.03)} , 0.07e ^{2π(0.01)})	(1, 0.15e ^{2π(0.11)} , 0.15e ^{2π(0.19)} , 0.18e ^{2π(0.12)})	(5, 0.82e ^{2π(0.91)} , 0.95e ^{2π(0.85)} , 0.87e ^{2π(0.94)})
ω_4	(3, 0.48e ^{2π(0.52)} , 0.53e ^{2π(0.45)} , 0.54e ^{2π(0.54)})	(1, 0.12e ^{2π(0.14)} , 0.15e ^{2π(0.12)} , 0.18e ^{2π(0.13)})	(2, 0.24e ^{2π(0.25)} , 0.28e ^{2π(0.29)} , 0.25e ^{2π(0.35)})	(4, 0.66e ^{2π(0.70)} , 0.74e ^{2π(0.61)} , 0.63e ^{2π(0.79)})
ω_5	(1, 0.12e ^{2π(0.17)} , 0.15e ^{2π(0.11)} , 0.18e ^{2π(0.13)})	(4, 0.72e ^{2π(0.62)} , 0.71e ^{2π(0.83)} , 0.65e ^{2π(0.64)})	(5, 0.82e ^{2π(0.96)} , 0.94e ^{2π(0.83)} , 0.95e ^{2π(0.97)})	(2, 0.22e ^{2π(0.34)} , 0.27e ^{2π(0.37)} , 0.29e ^{2π(0.31)})

- (3) The individual evaluation of all the decision-making experts are assembled by using C3PF6SWA operator, given in Equations 3 and 4, to formulate the AC3PF6SDM presented in Table 7.
- (4) Each decision-making expert allots the C3PF6S weights to all decision criteria, tabulated in Table 8, depending upon their relative importance.
- (5) The given C3PF6S weights are now aggregated by using C3PF6SWA operator, as defined in Equations 5 and 6, to formulate a C3PF6S weightage vector of decision-criteria, shown in Table 9.
- (6) The entries of AC3PF6SDM are calculated by using the AC3PF6SDM and weightage vector of decision-criteria, as specified in Equations 7 and 8. The formulated AC3PF6SDM is specified in Table 10.
- (7) The score matrix is constructed by calculating the score degree of each entry of AC3PF6SDM, given in Equation 9. The constructed score matrix is compiled in Table 11.
- (8) The decision-criteria cost and health risks are the cost-type criteria while accessibility and quality are the benefit-type criteria. The ideal solutions C3PF6S-PIS and C3PF6S-NIS are evaluated by using Equations 10 and 11 respectively. The C3PF6S-PIS and C3PF6S-NIS corresponding to each criterion are represented in Table 12.
- (9) The normalized Euclidean distance measures of each medical equipment from C3PF6S-PIS and C3PF6S-NIS are calculated by using Equations 12 and 13. The calculated normalized Euclidean distance measures are given in Table 13.

- (10) The revised closeness index of each medical equipment, evaluated by using Equation 14, is displayed in Table 14.
- (11) The medical equipments are categorized in descending order according to their revised closeness index. The ranking of medical equipments is displayed in Table 15.

The pictorial representation of the step-by-step algorithm of the MCGDM problem is given in Figure 3.

Hence, after all the calculations performed, we come to know that alternative ω_5 that is Abdominal Perineal Resection Instruments Set will be the most suitable equipment for the oncology surgery department in Shaukat Khanum cancer hospital.

6. Comparative Analysis

The concept of m -polar fuzzy N -soft set is introduced by Akram et al. [1]. In this section, the technique of m -polar fuzzy N -soft TOPSIS mFNS-TOPSIS is described precisely. This strategy covers the data consisting of the m poles, N ordered grades and the soft sets. The succinct steps of this technique are as follows:

- (1) The weights are allotted to each decision-making expert (Δ).
- (2) Each decision making expert (ψ_h) assigns the m -polar fuzzy N -soft numbers ($d_{pt}^h, \rho_{pt}^{(h)} \circ \mu, \dots, \rho_{pt}^{(h)m} \circ \mu$) to the alternative (ω_p) with respect to the criterion (ϕ_t).
- (3) All the m -polar fuzzy n -soft fuzzy numbers are aggregated by using m -polar fuzzy N -soft weightage averaging operator (mFNSWA) as follows:

$$Z = \langle \max(d_{pt}^1, \dots, d_{pt}^l), 1 - \left\{ \left(1 - \rho_{pt}^{(h)1} \circ \mu\right)^{\epsilon_h} \left(1 - \rho_{pt}^{(h)2} \circ \mu\right)^{\epsilon_h}, \dots, \left(1 - \rho_{pt}^{(h)m} \circ \mu\right)^{\epsilon_h} \right\} \rangle. \tag{26}$$

- (4) The weights given by the expert (ψ_t) to the criterion (ϕ_t) is $D_t^{(h)} = \langle d_t^h, \rho_{pt}^{(h)1} \circ \mu, \dots, \rho_{pt}^{(h)m} \circ \mu \rangle$ and these individual assessments are assembled to construct a

weightage vector by m -polar fuzzy soft weighted averaging operator (mFNSWA $_{\epsilon_h}$) as follows:

$$D_t = \langle \max(d_t^1, \dots, d_t^l), 1 - \left(\left(1 - \rho_{pt}^{(h)1} \circ \mu\right)^{\epsilon_h} \left(1 - \rho_{pt}^{(h)2} \circ \mu\right)^{\epsilon_h}, \dots, \left(1 - \rho_{pt}^{(h)m} \circ \mu\right)^{\epsilon_h} \right) \rangle. \tag{27}$$

- (5) Now, the mutual decision of the experts and weights of the decision criteria are merged in aggregated m -polar fuzzy N -soft decision matrix (AmFNSDM), $J^i = (L_{pt}^i)_{r \times q}$ in which the entries are determined by $F^i = \langle \min(d_{pt}, d_t), (\rho_{pt}^1 \circ \mu)(\rho_t^1 \circ \mu), \dots, (\rho_{pt}^m \circ \mu)(\rho_t^m \circ \mu) \rangle$ where $d_{pt}^i = \min(d_{pt}, d_t)$ and $\bar{\rho}_{pt}^i \circ \mu = (\rho_{pt}^i \circ \mu)(\rho_t^i \circ \mu)$.

- (6) After forming the AmFNSDM, the score matrix is evaluated by using $S_{pt} = (d_{pt}^i / (N - 1)) + ((\rho_{pt}^1 \circ \mu + \dots + \rho_{pt}^m \circ \mu) / m)$, $p = 1, \dots, r$, $t = 1, \dots, q$.
- (7) The very next step after the score matrix is the formulation of m -polar fuzzy N -soft positive ideal solution (mFNS - PIS) and negative ideal solution (mFNS - NIS) as follows:

TABLE 7: Aggregated complex 3-Polar fuzzy 6-soft decision matrix (AC3P6FSDM).

Γ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	$(5, 0.73e^{2\pi(0.70)}, 0.67e^{2\pi(0.84)}, 0.70e^{2\pi(0.67)})$	$(3, 0.36e^{2\pi(0.47)}, 0.37e^{2\pi(0.44)}, 0.47e^{2\pi(0.35)})$	$(4, 0.44e^{2\pi(0.51)}, 0.52e^{2\pi(0.52)}, 0.48e^{2\pi(0.50)})$	$(4, 0.71e^{2\pi(0.70)}, 0.70e^{2\pi(0.70)}, 0.71e^{2\pi(0.68)})$
ω_2	$(5, 0.73e^{2\pi(0.76)}, 0.78e^{2\pi(0.78)}, 0.87e^{2\pi(0.71)})$	$(3, 0.48e^{2\pi(0.41)}, 0.45e^{2\pi(0.48)}, 0.50e^{2\pi(0.42)})$	$(3, 0.46e^{2\pi(0.47)}, 0.48e^{2\pi(0.51)}, 0.48e^{2\pi(0.56)})$	$(5, 0.75e^{2\pi(0.79)}, 0.82e^{2\pi(0.83)}, 0.81e^{2\pi(0.86)})$
ω_3	$(5, 0.84e^{2\pi(0.88)}, 0.86e^{2\pi(0.88)}, 0.85e^{2\pi(0.95)})$	$(2, 0.19e^{2\pi(0.30)}, 0.28e^{2\pi(0.23)}, 0.30e^{2\pi(0.27)})$	$(4, 0.48e^{2\pi(0.57)}, 0.56e^{2\pi(0.47)}, 0.57e^{2\pi(0.88)})$	$(5, 0.77e^{2\pi(0.82)}, 0.83e^{2\pi(0.78)}, 0.88e^{2\pi(0.81)})$
ω_4	$(3, 0.39e^{2\pi(0.39)}, 0.40e^{2\pi(0.45)}, 0.44e^{2\pi(0.43)})$	$(3, 0.20e^{2\pi(0.24)}, 0.24e^{2\pi(0.24)}, 0.24e^{2\pi(0.24)})$	$(2, 0.23e^{2\pi(0.26)}, 0.24e^{2\pi(0.29)}, 0.30e^{2\pi(0.26)})$	$(4, 0.44e^{2\pi(0.50)}, 0.54e^{2\pi(0.45)}, 0.48e^{2\pi(0.53)})$
ω_5	$(1, 0.13e^{2\pi(0.12)}, 0.12e^{2\pi(0.13)}, 0.13e^{2\pi(0.11)})$	$(5, 0.80e^{2\pi(0.91)}, 0.89e^{2\pi(0.82)}, 0.87e^{2\pi(0.81)})$	$(5, 0.78e^{2\pi(0.86)}, 0.85e^{2\pi(0.79)}, 0.83e^{2\pi(0.86)})$	$(2, 0.16e^{2\pi(0.21)}, 0.20e^{2\pi(0.23)}, 0.21e^{2\pi(0.20)})$

TABLE 8: Weightage of criteria assign by the experts.

$\Upsilon_k^{(m)}$	Ψ_1	Ψ_2	Ψ_3	Ψ_4
ϕ_1	$(3, 0.41e^{2\pi(0.53)}, 0.57e^{2\pi(0.47)}, 0.44e^{2\pi(0.52)})$	$(5, 0.82e^{2\pi(0.84)}, 0.93e^{2\pi(0.81)}, 0.84e^{2\pi(0.90)})$	$(2, 0.23e^{2\pi(0.35)}, 0.33e^{2\pi(0.27)}, 0.37e^{2\pi(0.28)})$	$(4, 0.64e^{2\pi(0.68)}, 0.73e^{2\pi(0.64)}, 0.75e^{2\pi(0.72)})$
ϕ_2	$(4, 0.48e^{2\pi(0.49)}, 0.52e^{2\pi(0.46)}, 0.58e^{2\pi(0.41)})$	$(4, 0.64e^{2\pi(0.71)}, 0.74e^{2\pi(0.63)}, 0.50e^{2\pi(0.68)})$	$(3, 0.54e^{2\pi(0.43)}, 0.58e^{2\pi(0.44)}, 0.49e^{2\pi(0.51)})$	$(4, 0.64e^{2\pi(0.7)}, 0.75e^{2\pi(0.65)}, 0.73e^{2\pi(0.68)})$
ϕ_3	$(5, 0.84e^{2\pi(0.87)}, 0.92e^{2\pi(0.94)}, 0.98e^{2\pi(0.82)})$	$(5, 0.92e^{2\pi(0.93)}, 0.84e^{2\pi(0.91)}, 0.87e^{2\pi(0.89)})$	$(4, 0.64e^{2\pi(0.68)}, 0.72e^{2\pi(0.78)}, 0.72e^{2\pi(0.84)})$	$(4, 0.75e^{2\pi(0.71)}, 0.65e^{2\pi(0.66)}, 0.72e^{2\pi(0.77)})$
ϕ_4	$(4, 0.63e^{2\pi(0.71)}, 0.67e^{2\pi(0.73)}, 0.62e^{2\pi(0.78)})$	$(3, 0.47e^{2\pi(0.51)}, 0.49e^{2\pi(0.55)}, 0.47e^{2\pi(0.59)})$	$(5, 0.87e^{2\pi(0.92)}, 0.84e^{2\pi(0.94)}, 0.82e^{2\pi(0.97)})$	$(4, 0.64e^{2\pi(0.72)}, 0.66e^{2\pi(0.78)}, 0.69e^{2\pi(0.71)})$

TABLE 9: Aggregated weightage of criteria.

Φ/Υ	C3PF6S weights
ϕ_1	$(5, 0.55e^{i2\pi(0.62)}, 0.70e^{i2\pi(0.57)}, 0.70e^{i2\pi(0.65)})$
ϕ_2	$(4, 0.57e^{i2\pi(0.58)}, 0.65e^{i2\pi(0.54)}, 0.62e^{i2\pi(0.57)})$
ϕ_3	$(5, 0.80e^{i2\pi(0.82)}, 0.80e^{i2\pi(0.85)}, 0.87e^{i2\pi(0.81)})$
ϕ_4	$(5, 0.73e^{i2\pi(0.80)}, 0.72e^{i2\pi(0.83)}, 0.70e^{i2\pi(0.87)})$

$$U = \{U_t^+ = \langle d_t^+, W_t^+ \rangle, t = 1, 2, \dots, q\}, \quad (28)$$

$$V = \{V_t^- = \langle d_t^-, W_t^- \rangle, t = 1, 2, \dots, q\},$$

where

$$W_t^+ = \{ \langle W_t^{+1}, W_t^{+2}, \dots, W_t^{+m} \rangle \}, \quad (29)$$

$$W_t^- = \{ \langle W_t^{-1}, W_t^{-2}, \dots, W_t^{-m} \rangle \}.$$

$$W_t^{+i} = \{ \rho_t^{i+} \circ \mu e^{2\pi i \alpha_t^{i+}}, i = 1, 2, \dots, m \}, \quad (30)$$

$$W_t^{-i} = \{ \rho_t^{i-} \circ \mu e^{2\pi i \alpha_t^{i-}}, i = 1, 2, \dots, m \}.$$

Now, U_t^+ and V_t^- can be formulated as

$$U_t^+ = \begin{cases} J'_{jk}: S(J'_{jk}) = \max_{1 \leq p \leq r} S((J'_{pt})), & \text{if } \phi_t \in B_e, \\ J'_{jk}: S(J'_{jk}) = \min_{1 \leq p \leq r} S((J'_{pt})), & \text{if } \phi_t \in C_e. \end{cases} \quad (31)$$

$$V_t^- = \begin{cases} J'_{jk}: S(J'_{jk}) = \min_{1 \leq p \leq r} S((J'_{pt})), & \text{if } \phi_t \in B_e, \\ J'_{jk}: S(J'_{jk}) = \max_{1 \leq p \leq r} S((J'_{pt})), & \text{if } \phi_t \in C_e. \end{cases} \quad (32)$$

(8) After evaluation of the mFNS-PIS and mFNS-NIS, we calculate the distance between the alternatives and the ideal solutions by using the distance formula:

$$d(\omega_p, U) = \sqrt{\frac{\sum_{t=1}^q \left[(1/(N-1))^2 (d_{pt}' - d_t^+)^2 + \sum_{i=1}^m (\tilde{\rho}_{pt}^i \circ \mu - \rho_t^{i+} \circ \mu)^2 \right]}{rm}}, \quad (33)$$

$$d(\omega_p, V) = \sqrt{\frac{\sum_{t=1}^q \left[(1/(N-1))^2 (d_{pt}' - d_t^-)^2 + \sum_{i=1}^m (\tilde{\rho}_{pt}^i \circ \mu - \rho_t^{i-} \circ \mu)^2 \right]}{rm}}, \quad (34)$$

where r is the number of criteria and m is the number of poles.

(9) For the ranking of the alternatives, the revised closeness index is calculated and the alternative with the minimum revised closeness index is the most preferable one. For the calculation of the revised closeness index, we use $I(\omega_p) = d(\omega_p, U) / \min(d(\omega_p, U)) - d(\omega_p, U) / \max(d(\omega_p, U))$, $p = 1, 2, \dots, r$.

The theory of m -polar fuzzy N -soft set is presented by Akram et al. [1]. The MCGDM technique, that is, TOPSIS is applied on mFNSS by MAHEEN. Now, a comparative analysis between TOPSIS on CmPFNSS and mFNSS is described in this section. We have considered the case study of the need of surgical equipments in Shaukhat Khanum Hospital by using the MCGDM technique TOPSIS for the credibility and authenticity of our presented decision-making strategy. The main steps involved in this technique are briefly mentioned as:

(1) The attributes and their criteria are thoroughly examined by the experts and the weight is assigned to each expert shown in Table 16.

- (2) Each expert gives the individual assessment in the form of ordered grades and membership degrees to each attribute with respect to its criterion given in Tables 17–20.
- (3) The individual assessments are assembled by using the mFNSSWA operator shown in Table 21.
- (4) Each expert assigns the weight to each criterion according to its credibility given in Table 22 and these weights are assembled in a weightage vector by using mFNSSWA $_{\epsilon_h}$ given in Table 23.
- (5) The mutual decision of the experts and the weights of the criteria are merged in AmFNSS DM by using AmFNSSWA given in Table 24.
- (7) The mFNSS – PIS and mFNSS – NIS are given in Table 26.
- (8) The distance measures between alternatives and the ideal solutions are shown in Table 27.
- (9) The revised closeness index of the alternatives is given in Table 28.
- (10) Rank the alternatives in the descending order and the alternative with the least closeness index is the

TABLE 10: Aggregated weighted complex 3-Polar fuzzy 6-soft decision matrix (AWC3PF6SDM).

Γ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	$(5, 0.40e^{2\pi(0.43)}, 0.47e^{2\pi(0.48)}, 0.50e^{2\pi(0.44)})$	$(3, 0.21e^{2\pi(0.25)}, 0.24e^{2\pi(0.24)}, 0.29e^{2\pi(0.20)})$	$(4, 0.35e^{2\pi(0.42)}, 0.42e^{2\pi(0.44)}, 0.42e^{2\pi(0.40)})$	$(4, 0.52e^{2\pi(0.56)}, 0.51e^{2\pi(0.59)}, 0.49e^{2\pi(0.59)})$
ω_2	$(5, 0.40e^{2\pi(0.47)}, 0.55e^{2\pi(0.44)}, 0.61e^{2\pi(0.47)})$	$(3, 0.24e^{2\pi(0.28)}, 0.29e^{2\pi(0.26)}, 0.31e^{2\pi(0.24)})$	$(3, 0.37e^{2\pi(0.38)}, 0.39e^{2\pi(0.44)}, 0.41e^{2\pi(0.37)})$	$(5, 0.55e^{2\pi(0.63)}, 0.59e^{2\pi(0.69)}, 0.57e^{2\pi(0.75)})$
ω_3	$(5, 0.47e^{2\pi(0.54)}, 0.60e^{2\pi(0.50)}, 0.60e^{2\pi(0.62)})$	$(2, 0.11e^{2\pi(0.17)}, 0.18e^{2\pi(0.12)}, 0.18e^{2\pi(0.16)})$	$(4, 0.39e^{2\pi(0.46)}, 0.45e^{2\pi(0.40)}, 0.49e^{2\pi(0.39)})$	$(5, 0.56e^{2\pi(0.66)}, 0.59e^{2\pi(0.65)}, 0.62e^{2\pi(0.70)})$
ω_4	$(3, 0.22e^{2\pi(0.24)}, 0.28e^{2\pi(0.26)}, 0.31e^{2\pi(0.28)})$	$(3, 0.12e^{2\pi(0.14)}, 0.15e^{2\pi(0.13)}, 0.15e^{2\pi(0.14)})$	$(2, 0.18e^{2\pi(0.21)}, 0.19e^{2\pi(0.25)}, 0.26e^{2\pi(0.21)})$	$(4, 0.32e^{2\pi(0.40)}, 0.39e^{2\pi(0.37)}, 0.34e^{2\pi(0.46)})$
ω_5	$(1, 0.07e^{2\pi(0.07)}, 0.08e^{2\pi(0.08)}, 0.09e^{2\pi(0.07)})$	$(4, 0.46e^{2\pi(0.53)}, 0.58e^{2\pi(0.44)}, 0.54e^{2\pi(0.47)})$	$(5, 0.62e^{2\pi(0.71)}, 0.68e^{2\pi(0.61)}, 0.77e^{2\pi(0.70)})$	$(2, 0.12e^{2\pi(0.16)}, 0.14e^{2\pi(0.19)}, 0.15e^{2\pi(0.17)})$

TABLE 11: Score matrix.

Φ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	1.9067	1.0767	1.6167	1.7207
ω_2	1.98	1.14	1.3867	2.03
ω_3	2.11	0.7067	1.66	2.0433
ω_4	1.13	0.8767	0.8333	1.4367
ω_5	0.3533	1.8067	2.3642	0.9067

TABLE 12: C3PF6S-PIS and C3PF6S-NIS.

Criteria	F	\sqsupset
ϕ_1	$(1, 0.07e^{i2\pi(0.07)}, 0.08e^{i2\pi(0.09)}, 0.09e^{i2\pi(0.07)})$	$(5, 0.47e^{i2\pi(0.54)}, 0.60e^{i2\pi(0.50)}, 0.60e^{i2\pi(0.62)})$
ϕ_2	$(4, 0.46e^{i2\pi(0.53)}, 0.58e^{i2\pi(0.44)}, 0.54e^{i2\pi(0.47)})$	$(2, 0.11e^{i2\pi(0.17)}, 0.18e^{i2\pi(0.12)}, 0.18e^{i2\pi(0.16)})$
ϕ_3	$(5, 0.62e^{i2\pi(0.71)}, 0.68e^{i2\pi(0.67)}, 0.71e^{i2\pi(0.70)})$	$(2, 0.18e^{i2\pi(0.21)}, 0.19e^{i2\pi(0.25)}, 0.26e^{i2\pi(0.21)})$
ϕ_4	$(2, 0.10e^{i2\pi(0.12)}, 0.11e^{i2\pi(0.15)}, 0.13e^{i2\pi(0.14)})$	$(5, 0.56e^{i2\pi(0.66)}, 0.59e^{i2\pi(0.65)}, 0.62e^{i2\pi(0.70)})$

TABLE 13: Normalized Euclidean distance from ideal solutions.

Equipments	$d(\omega_j, F)$	$d(\omega_j, \sqsupset)$
ω_1	0.55	0.23
ω_2	0.62	0.18
ω_3	0.66	0.19
ω_4	0.54	0.31
ω_5	0.12	0.73

TABLE 14: Revised closeness index (δ) of each medical equipment.

Alternatives	ω_1	ω_2	ω_3	ω_4	ω_5
δ	27.1849	30.7534	32.7397	26.5753	0

TABLE 15: Ranking of medical equipments.

Alternatives	ω_1	ω_2	ω_3	ω_4	ω_5
Ranking	3	4	5	2	1

most preferable one. The ranking is given in Table 29.

7. Discussion

In this section, we involve a detailed comparative discussion between the working mechanism of both CmPFNS-TOPSIS and mFNS-TOPSIS techniques, their working principle on the case study of selection of best surgical equipment in the Shaukhat Khanum Hospital, Lahore (Pakistan) and their final results in ranking of the alternatives.

- (i) A comparison of our proposed technique with the mFNS-TOPSIS [1] is being provided to prove the credibility of the CmPFNS-TOPSIS. Although the formulas used for evaluation of the normalized Euclidean distance, score matrix and the revised closeness index are different for both of the techniques, but they evaluate the same final results. Thus, the same surgical equipment, abdominal

perineal resection instruments set, is predicted as the most suitable one by both methods.

- (ii) An illustrative bar-chart is being displayed in Figure 4, depicting the comparison between the revised closeness index of the CmPFNS-TOPSIS and of the mFNS-TOPSIS by taking the alternatives on the x -axis and the revised closeness index on the y -axis. The bar-chart in Figure 4 clearly shows that the ranking of the alternatives is same evaluated by both of the methods.
- (iii) Both of the techniques, CmPFNS-TOPSIS and mFNS-TOPSIS, works on the same principle of finding the solution nearest to the positive ideal solution and farthest from the negative ideal solution, but there are some differences in their implication representing in section 3 and section 5, including the normalized Euclidean distance, formula to evaluate the score matrix and the revised closeness index.

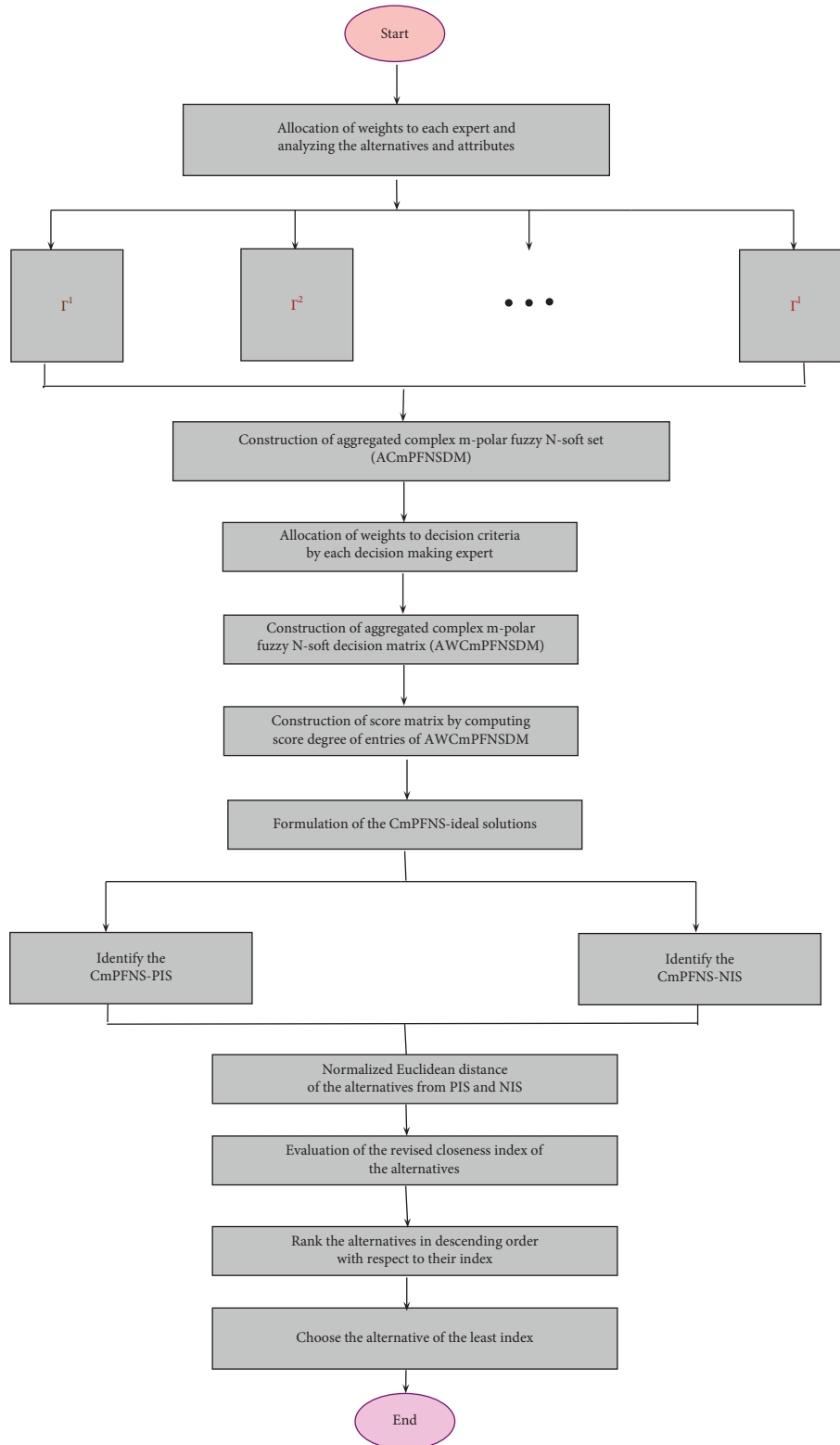


FIGURE 3: Pictorial representation of the algorithm.

(iv) No doubt, mFNS-TOPSIS makes its way through all the hurdles of dealing with the multipolar information, but this technique is of no use while having

a complex two-dimensional domain. It fails to help us when we have a complex two-dimensional data to deal with.

TABLE 16: Weightage of experts.

Δ/Ψ	Ψ_1	Ψ_2	Ψ_3	Ψ_4
Δ_{η}	0.21	0.24	0.37	0.18

TABLE 17: 3F6SDM of the project supervisor (ψ_1).

Γ^1	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(4, 0.62, 0.68, 0.79)	(2, 0.29, 0.22, 0.33)	(3, 0.42, 0.54, 0.59)	(4, 0.75, 0.63, 0.69)
ω_2	(5, 0.89, 0.92, 0.99)	(2, 0.21, 0.24, 0.34)	(3, 0.52, 0.49, 0.57)	(5, 0.82, 0.93, 0.95)
ω_3	(5, 0.92, 0.89, 0.87)	(2, 0.22, 0.28, 0.34)	(3, 0.45, 0.56, 0.49)	(5, 0.97, 0.93, 0.95)
ω_4	(2, 0.21, 0.35, 0.29)	(3, 0.42, 0.48, 0.47)	(1, 0.17, 0.15, 0.18)	(3, 0.43, 0.57, 0.48)
ω_5	(0, 0.02, 0.05, 0.03)	(4, 0.62, 0.69, 0.65)	(5, 0.88, 0.97, 0.87)	(1, 0.11, 0.15, 0.13)

TABLE 18: 3F6SDM of the finance supervisor (ψ_2).

Γ^2	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(3, 0.42, 0.47, 0.55)	(3, 0.43, 0.47, 0.53)	(4, 0.62, 0.68, 0.64)	(4, 0.71, 0.66, 0.73)
ω_2	(4, 0.62, 0.67, 0.72)	(3, 0.55, 0.45, 0.58)	(3, 0.44, 0.45, 0.49)	(5, 0.82, 0.85, 0.87)
ω_3	(4, 0.77, 0.79, 0.69)	(2, 0.22, 0.35, 0.29)	(4, 0.69, 0.76, 0.79)	(5, 0.85, 0.89, 0.99)
ω_4	(3, 0.43, 0.55, 0.44)	(1, 0.15, 0.16, 0.11)	(2, 0.25, 0.27, 0.35)	(3, 0.47, 0.55, 0.56)
ω_5	(1, 0.17, 0.15, 0.16)	(5, 0.88, 0.96, 0.84)	(4, 0.62, 0.69, 0.77)	(2, 0.21, 0.25, 0.27)

TABLE 19: 3F6S DM of the health supervisor (ψ_3).

Γ^3	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(4, 0.75, 0.67, 0.63)	(3, 0.42, 0.46, 0.58)	(2, 0.22, 0.34, 0.26)	(4, 0.69, 0.75, 0.72)
ω_2	(4, 0.65, 0.76, 0.77)	(3, 0.40, 0.55, 0.52)	(3, 0.52, 0.54, 0.48)	(4, 0.65, 0.75, 0.63)
ω_3	(5, 0.84, 0.86, 0.89)	(2, 0.22, 0.33, 0.36)	(3, 0.45, 0.52, 0.54)	(4, 0.65, 0.71, 0.65)
ω_4	(3, 0.51, 0.43, 0.55)	(1, 0.14, 0.15, 0.18)	(2, 0.24, 0.25, 0.36)	(2, 0.29, 0.35, 0.33)
ω_5	(1, 0.16, 0.11, 0.15)	(5, 0.84, 0.92, 0.96)	(4, 0.75, 0.62, 0.68)	(1, 0.12, 0.14, 0.13)

TABLE 20: 3F6S DM of the technical supervisor (ψ_4).

Γ^4	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(5, 0.98, 0.82, 0.85)	(1, 0.18, 0.12, 0.17)	(3, 0.52, 0.55, 0.47)	(4, 0.71, 0.73, 0.65)
ω_2	(4, 0.68, 0.67, 0.75)	(3, 0.42, 0.44, 0.52)	(2, 0.22, 0.38, 0.31)	(4, 0.66, 0.77, 0.68)
ω_3	(5, 0.82, 0.88, 0.87)	(0, 0.02, 0.05, 0.07)	(1, 0.15, 0.15, 0.18)	(5, 0.82, 0.95, 0.87)
ω_4	(3, 0.48, 0.53, 0.54)	(1, 0.12, 0.15, 0.18)	(2, 0.24, 0.28, 0.25)	(4, 0.66, 0.74, 0.63)
ω_5	(1, 0.12, 0.15, 0.18)	(4, 0.72, 0.71, 0.65)	(5, 0.82, 0.94, 0.95)	(2, 0.22, 0.27, 0.29)

TABLE 21: Aggregated 3-polar fuzzy 6-soft decision matrix (AC3P6FS DM).

Γ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(5, 0.73, 0.67, 0.70)	(3, 0.36, 0.37, 0.47)	(4, 0.44, 0.52, 0.48)	(4, 0.71, 0.70, 0.71)
ω_2	(5, 0.73, 0.78, 0.87)	(3, 0.48, 0.45, 0.50)	(3, 0.46, 0.48, 0.48)	(5, 0.75, 0.82, 0.81)
ω_3	(5, 0.84, 0.86, 0.85)	(2, 0.19, 0.28, 0.30)	(4, 0.48, 0.56, 0.57)	(5, 0.77, 0.83, 0.88)
ω_4	(3, 0.39, 0.40, 0.44)	(3, 0.20, 0.24, 0.24)	(2, 0.23, 0.24, 0.30)	(4, 0.44, 0.54, 0.48)
ω_5	(1, 0.13, 0.12, 0.13)	(5, 0.80, 0.89, 0.87)	(5, 0.78, 0.85, 0.83)	(2, 0.16, 0.20, 0.21)

(v) Our proposed technique, CmPFNS-TOPSIS, is the most general and effective decision-making strategy that not only deals with the multipolarity of the present day as mFNS-TOPSIS does but also helps us in addressing the problems related to the complex two-dimensional domain.

(vi) In our proposed technique, CmPFNS-TOPSIS, we benefit with both complex data and the multipolar information. But when we consider the complex part zero, CmPFNS-TOPSIS converts to the mFNS-TOPSIS which comprises of the absence of the complex part and equipped with only multipolarity.

TABLE 22: Weightage of criteria assign by the experts.

$\Upsilon_k^{(m)}$	Ψ_1	Ψ_2	Ψ_3	Ψ_4
ϕ_1	(3, 0.41, 0.57, 0.44)	(5, 0.82, 0.93, 0.84)	(2, 0.23, 0.33, 0.37)	(4, 0.64, 0.73, 0.75)
ϕ_2	(4, 0.48, 0.52, 0.58)	(4, 0.64, 0.74, 0.50)	(3, 0.54, 0.58, 0.49)	(4, 0.64, 0.75, 0.73)
ϕ_3	(5, 0.84, 0.92, 0.98)	(5, 0.92, 0.84, 0.87)	(4, 0.64, 0.72, 0.72)	(4, 0.75, 0.65, 0.72)
ϕ_4	(4, 0.63, 0.67, 0.62)	(3, 0.47, 0.49, 0.47)	(5, 0.87, 0.84, 0.82)	(4, 0.64, 0.66, 0.69)

TABLE 23: Aggregated weightage of criteria.

Φ/Υ	<i>C3PF6S weights</i>
ϕ_1	(5, 0.55, 0.70, 0.70)
ϕ_2	(4, 0.57, 0.65, 0.62)
ϕ_3	(5, 0.80, 0.80, 0.87)
ϕ_4	(5, 0.73, 0.72, 0.70)

TABLE 24: Aggregated weighted 3-polar fuzzy 6-soft decision matrix (*AWC3PF6S DM*).

Γ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	(5, 0.40, 0.47, 0.50)	(3, 0.21, 0.24, 0.29)	(4, 0.35, 0.42, 0.42)	(4, 0.52, 0.51, 0.49)
ω_2	(5, 0.40, 0.55, 0.61)	(3, 0.24, 0.29, 0.31)	(3, 0.37, 0.39, 0.41)	(5, 0.55, 0.59, 0.57)
ω_3	(5, 0.47, 0.60, 0.60)	(2, 0.11, 0.18, 0.18)	(4, 0.39, 0.45, 0.49)	(5, 0.56, 0.59, 0.62)
ω_4	(3, 0.22, 0.28, 0.31)	(3, 0.12, 0.15, 0.15)	(2, 0.18, 0.19, 0.26)	(4, 0.32, 0.39, 0.34)
ω_5	(1, 0.07, 0.08, 0.09)	(4, 0.46, 0.58, 0.54)	(5, 0.62, 0.68, 0.77)	(2, 0.12, 0.14, 0.15)

TABLE 25: Score matrix.

Φ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
ω_1	1.46	0.85	1.20	1.31
ω_2	1.52	0.88	0.99	1.57
ω_3	1.56	0.56	1.24	1.59
ω_4	0.87	0.74	0.61	1.15
ω_5	0.28	1.33	1.67	0.54

TABLE 26: *C3PF6S – PIS* and *C3PF6S – NIS*.

Criteria	F	\sqsupset
ϕ_1	(1, 0.07, 0.08, 0.09)	(5, 0.47, 0.60, 0.60)
ϕ_2	(4, 0.46, 0.58, 0.54)	(2, 0.11, 0.18, 0.18)
ϕ_3	(5, 0.62, 0.68, 0.71)	(2, 0.18, 0.19, 0.26)
ϕ_4	(2, 0.10, 0.11, 0.13)	(5, 0.56, 0.59, 0.62)

TABLE 27: Normalized Euclidean distance from ideal solutions.

Equipments	$d(\omega_j, F)$	$d(\omega_j, \sqsupset)$
ω_1	0.43	0.19
ω_2	0.49	0.14
ω_3	0.51	0.16
ω_4	0.42	0.24
ω_5	0.01	0.57

TABLE 28: Revised closeness index (δ) of each medical equipment.

Alternatives	ω_1	ω_2	ω_3	ω_4	ω_5
δ	42.6667	48.7544	50.7193	41.5789	0

TABLE 29: Ranking of medical equipments.

Alternatives	ω_1	ω_2	ω_3	ω_4	ω_5
Ranking	3	4	5	2	1

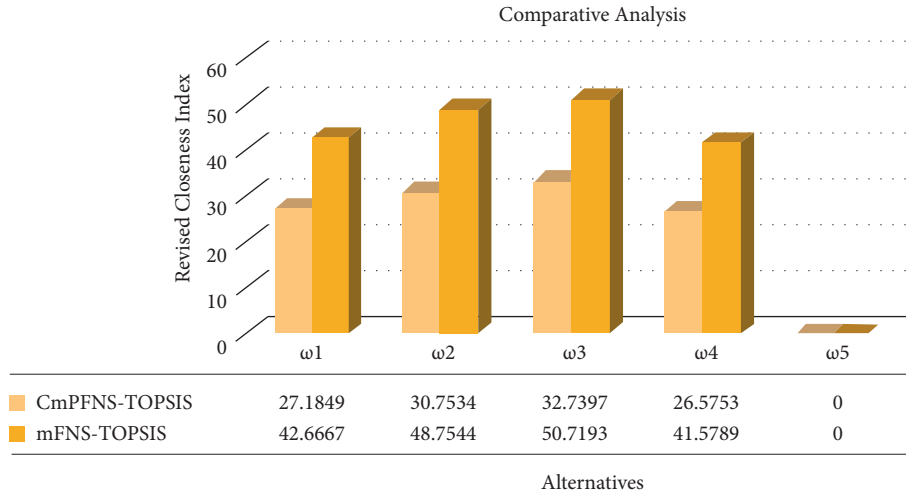


FIGURE 4: Graphical representation of comparative analysis of CmPFNS-TOPSIS and mFNS-TOPSIS.

TABLE 30: Comparative analysis.

Method	Ranking	Optimal alternative
CmPFNS-TOPSIS	$\omega_3 > \omega_2 > \omega_1 > \omega_4 > \omega_5$	ω_5
mFNS-TOPSIS [1]	$\omega_3 > \omega_2 > \omega_1 > \omega_4 > \omega_5$	ω_5

The comparison Table 30 shows that both the MCGDM techniques a give the same ranking of the alternatives.

8. Conclusion

This study has eradicated all the hurdles, like the multipolar information and the complex two-dimensional data, faced in making the apt decisions in daily routine tasks. The hinderance of encountering the multipolarity of the modern era and the complexity of the present time all together has been abolished by introducing a very novel hybrid decision making strategy, namely, CmPFNS-TOPSIS that shares the burden of choosing the best alternative in accordance with our requirements. To play with this astonishing technique, first we have developed its working mechanism and formulated the CmPFNSWD operator and the normalized Euclidean distance for the CmPFNS and then we have exemplified the case study of choosing the best surgical equipment for the oncology department in Shaukat Khanum Hospital, Lahore. To verify the authenticity of CmPFNS-TOPSIS, a comprehensive algorithm of working principle of mFNS-TOPSIS has been explained and the similar case study has been taken under consideration. After this, a comparison between CmPFNS-TOPSIS and mFNS-TOPSIS has been displayed by using a bar-chart to prove its effectiveness and reliability. The CmPFNS-TOPSIS has been proved very productive in this multipolar complex modern

era. Thus, in near future we are intended to boost up our research work by establishing the more generalized framework and extending our approach to the additional MCGDM techniques as VIKOR method, ELECTRE method and PROMETHE method.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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References

- [1] A. Adeel, M. Akram, and A. N. A. Koam, "Group decision-making based on m-polar fuzzy linguistic TOPSIS method," *Symmetry Plus*, vol. 11, no. 6, p. 735, 2019.
- [2] M. Akram, A. Adeel, and J. C. R. Alcantud, "Group decision-making methods based on hesitant N-soft sets," *Expert Systems with Applications*, vol. 115, pp. 95–105, 2019.

- [3] M. Akram, F. Wasim, and A. N. Al-Kenani, "Complex q -rung orthopair fuzzy N -soft sets: a new model with applications," *Complexity*, vol. 2021, pp. 1–29, Article ID 3690597, 2021.
- [4] M. Akram, G. Ali, and J. C. R. Alcantud, "Hybrid multi-attribute decision-making model based on (m, N) -soft rough sets -soft rough sets," *Journal of Intelligent and Fuzzy Systems*, vol. 36, no. 6, pp. 6325–6342, 2019.
- [5] M. Akram, A. Adeel, and J. C. R. Alcantud, "Fuzzy N -soft sets: a novel model with applications," *Journal of Intelligent and Fuzzy Systems*, vol. 35, no. 4, pp. 4757–4771, 2018.
- [6] M. Akram, M. Shabir, A. N. Al-Kenani, and J. C. R. Alcantud, "Hybrid decision-making frameworks under complex spherical fuzzy N -soft sets," *Journal of Mathematics*, vol. 2021, pp. 1–46, 2021.
- [7] M. Akram and M. Sultan, "Complex m -polar fuzzy N -soft model," *Multiple-Valued Logic and Soft Computing*, vol. 39, 2022.
- [8] D. Ramot, R. Milo, M. Friedman, and A. Kandel, "Complex fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 171–186, 2002.
- [9] J. C. R. Alcantud, F. Feng, and R. R. Yager, "An N -Soft set approach to rough sets," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 11, pp. 2996–3007, 2020.
- [10] J. Alcantud, S. Rambaud, and M. Torrecillas, "Valuation fuzzy soft sets: a flexible fuzzy soft set based decision making procedure for the valuation of assets," *Symmetry Plus*, vol. 9, no. 11, p. 253, 2017.
- [11] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
- [12] R. Benayoun, B. Roy, and N. Sussman, "Manual de reference du programme ELECTRE," *Note de Synthèse et Formation*, vol. 25, p. 79, 1966.
- [13] J. P. Brans and P. Vincke, "Note-A preference ranking organisation method," *Management Science*, vol. 31, no. 6, pp. 647–656, 1985.
- [14] C.-T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, vol. 114, no. 1, pp. 1–9, 2000.
- [15] J. Chen, S. Li, S. Ma, and X. Wang, " m -Polar fuzzy sets: an extension of bipolar fuzzy sets," *The Scientific World Journal*, vol. 2014, pp. 1–8, Article ID 416530, 2014.
- [16] T. Chu and M. Kysely, "Ranking objectives of advertisements on Facebook by a fuzzy TOPSIS method," *Electronic Commerce Research*, vol. 21, pp. 881–916, 2021.
- [17] S. Eraslan, "A decision making method via TOPSIS on soft sets," *Journal of New Results in Science*, vol. 4, no. 8, pp. 57–70, 2015.
- [18] S. Eraslan and F. Karaaslan, "A group decision making method based on TOPSIS under fuzzy soft environment," *Journal of New Theory*, vol. 2015, no. 3, pp. 30–40, 2015.
- [19] F. Fatimah and J. C. R. Alcantud, "The multipolar fuzzy N -soft set and its applications to decision-making," *Neural Computing & Applications*, vol. 33, pp. 11437–11446, 2021.
- [20] F. Fatimah, D. Rosadi, R. B. F. Hakim, and J. C. R. Alcantud, " N -soft sets and their decision making algorithms," *Soft Computing*, vol. 22, no. 12, pp. 3829–3842, 2018.
- [21] C.-L. Hwang and K. Yoon, "Methods for multiple attribute decision making," *Multiple Attribute Decision Making*, Springer, vol. 186, pp. 58–191, 1981.
- [22] J. Li, X. Xu, Z. Yao, and Y. Lu, "Improving service quality with the fuzzy TOPSIS method: a case study of the Beijing rail transit system," *IEEE Access*, vol. 7, pp. 114271–114284, 2019.
- [23] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.
- [24] T. Mahmood, U. ur Rehman, and Z. Ali, "A novel complex fuzzy N -soft sets and their decision-making algorithm," *Complex & Intelligent Systems*, vol. 7, no. 5, pp. 2255–2280, 2021.
- [25] D. Molodtsov, "Soft set theory-first results," *Computers & Mathematics With Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.
- [26] S. Opricovic, "Multicriteria optimization of civil engineering systems," *Faculty of Civil Engineering*, vol. 2, no. 1, pp. 5–21, 1998.
- [27] T. L. Saaty, "Axiomatic foundation of the analytical hierarchy process," *Management Science*, vol. 32, no. 7, pp. 773–907, 1986.
- [28] G. Torlak, M. Sevkli, M. Sanal, and S. Zaim, "Analyzing business competition by using fuzzy TOPSIS method: an example of Turkish domestic airline industry," *Expert Systems with Applications*, vol. 38, no. 4, pp. 3396–3406, 2011.
- [29] Z. Yang, J. Chang, L. Huang, and A. Mardani, "Digital transformation solutions of entrepreneurial SMEs based on an information error-driven T-spherical fuzzy cloud algorithm," *International Journal of Information Management*, vol. 102384, pp. 102–384, 2021.
- [30] Z. Yang, M. Lin, Y. Li, W. Zhou, and B. Xu, "Assessment and selection of smart agriculture solutions using an information error-based Pythagorean fuzzy cloud algorithm," *International Journal of Intelligent Systems*, vol. 36, no. 11, pp. 6387–6418, 2021.
- [31] Z. Yang, Y. Gao, and X. Fu, "A decision-making algorithm combining the aspect-based sentiment analysis and intuitionistic fuzzy-VIKOR for online hotel reservation," *Annals of Operations Research*, pp. 1–17, 2021.
- [32] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.