Research Article

Dynamic Pricing and Logistics Service Decisions for Crowd Logistics Platforms with Social Delivery Capacity

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With the development of sharing economy, more and more enterprises choose crowd logistics for distribution. Because the crowd logistics platform uses social freelancers, the service quality is difficult to guarantee. Considering the reward-penalty mechanism, dynamic differential game models are constructed to study the optimal pricing and quality of crowd logistics services under stochastic demand based on the optimal control theory and Pontryagin maximum principle. The numerical simulation results show that the optimal dynamic decisions change with the fluctuation of demand dynamically. Furthermore, the platform needs to adjust the value of the reward-penalty factor to ensure the level of service quality and revenue in different situations.

1. Introduction

In recent years, the concept of sharing economy has gradually emerged, and e-commerce platforms have developed rapidly. Since “Internet + platforms” can make full use of idle social resources and reduce the waste of social resources, more and more enterprises are beginning to use Internet platforms for transactions [1]. Crowd logistics matches the logistics distribution needs of enterprises or individuals with the idle public personnel (the service provider) who voluntarily distribute through the platform, instead of delivering by full-time delivery staff. Enterprises such as Amazon Flex, JD crowdsourcing, and Flash all use crowdsourcing logistics for distribution. Due to the great flexibility and high efficiency of crowd logistics, it has played an essential role in the supermarkets and express delivery terminals [2].

The crowd logistics platform connects the demand of customers and the supply of the service providers; therefore, the crowd logistics platform is a typical bilateral platform. On the one hand, consumers can place orders at any time; at the same time, the provider can choose the working time at will. Both demand and supply are subject to great uncertainty, which is likely to lead to an imbalance between supply and demand. On the other hand, because the service providers are social idlers and lack professional training, the service quality are inconsistent and cannot meet the needs of customers, which has a negative impact on the willingness of customers to continue to use the crowd logistics platforms. Therefore, it is of practical significance for the optimization of crowd logistics system to determine the optimal pricing strategy to adjust the imbalance of supply and demand and to explore how to motivate the provider to improve the service quality level.

1.1. Pricing Problems. Many scholars have conducted research on the optimal pricing problems of bilateral platforms. Bimpikis et al. [3] discussed the spatial price discrimination strategy of crowd platform, emphasizing the impact of demand mode on platform price, profit, and consumer surplus. Sun et al. [4] determined the optimal pricing strategy for online car rental platforms, taking into account ride details and driver location. These papers
demonstrated and studied the pricing strategy of online car hailing platform in peak demand. Some scholars also considered the impact of network externalities on pricing under different demands. Shen et al. [5] proposed a pricing method based on topic model analysis (PTMA) to solve the task pricing problem in software crowdsourcing. Aiming at the problem of asymmetric information between supply and demand in the transportation service market, Wu et al. [6] established an intermediary pricing model to explore the impact of spatial differences and network externalities on the pricing mechanism of online car hailing platform. Pourrahmani and Jaller [7] focused on last mile delivery activities, provided an overview of the operational characteristics of platforms, and then identified pricing strategy. Zhang et al. [8] discussed the impact of the service efficiency and cross-platforms, and then identified pricing strategy. Zhao et al. [9] considered four different differential pricing strategies and obtained the optimal pricing strategy in different situations. The above literature studied the pricing strategy of bilateral platforms but did not consider the characteristics of market demand fluctuation and the quality level of crowd logistics services. Therefore, we will do further research on the basis of existing research.

In addition, many scholars have studied the supply and demand matching of bilateral platforms. They modeled the matching process between the driver and the customer as an unobservable queue, the arrival of passengers as a Poisson process, and the driver as a server in the queueing system. For example, Bai et al. [10], Hu and Zhou [11], Taylor [12], and Cachon et al. [13] used queuing theory to study the supply-demand matching problem of online car hailing platforms. In recent years, some scholars introduced the optimal control theory to realize the balance of supply and demand. For example, Wang et al. [14] constructed a crowd logistics dynamic pricing model considering the social supply characteristics and the loss of orders in the case of short supply. Then, the competition between two crowd service platforms is considered (Wang et al. [15], Sun and Xu [16], and Sun and Xu [17]). On this basis, Guo and Li [18] considered the cancellation behavior of passengers after booking orders in the pricing model, obtained the optimal pricing of the platform, and suggested to formulate appropriate default rules. Some scholars also introduced other economic theories for analysis. For example, Zhang et al. [19] introduced the principal-agent theory to analyze the optimal pricing of the platform, considering the uncertainty of demand side and supply side at the same time. Xu et al. [20] and Wang and Xie [21] considered the dynamic pricing under several different demand situations, respectively. Wang and Xie [22] considered both direct-network effects and cross-network effects to coordinate supply and demand balance. Compared with the existing literature, this paper will use the differential game method to consider the game between the service provider and the crowdsourcing platform and finally obtain the optimal pricing. Furthermore, the above literature only considers one or two supply and demand situations, which cannot cover all possible situations. Therefore, we will study more comprehensive supply and demand situations in this paper.

1.2. Quality Problems. In terms of product quality control, some scholars used the differential game method to study the dynamic decision-making problem of quality. For example, Pang and Tan [23] established an improved Nerlove–Arrow model to study the optimal quality decision under multi-manufacturer competition. Kogan and El Ouardighi [24] derived a quality improvement effort strategy using a differential game method for duopoly markets with partial substitute products. Li et al. [25], Ma et al. [26], and Zhou et al. [27] all considered the impact of consumers’ reference effect on quality level, and Ma et al. [26] also incorporated reciprocal altruism into the model. Wang and Hu [28] studied dynamic quality and marketing decisions with envision of brand crisis in a dual-channel supply chain. According to the above literature, the differential game method has attracted more and more attention of scholars. However, they mainly focus on solving the problem of dynamic control of product quality, and few scholars use the differential game method to study the service quality.

In fact, the quality of crowd logistics will also change with time dynamically; therefore, the differential game model can be applied to the research of crowd logistics services. Based on the dynamic perspective, some papers studied the optimal quality input of crowdsourcing logistics services. For example, Meng et al. [29] discussed the optimal quality decision making under the reward-penalty mechanism and the cost-sharing contract. Then, they explored the impact of big data technology (Meng et al. [30]) and penalty policy (Meng et al. [31]) on the service quality of the crowd logistics platforms. He et al. [32] derived the optimal service levels for the platform and the hotel in three modes, including decentralized, cost-sharing, and integrated modes. Liu et al. [33] proposed an optimal control model considering the impact of sales price and service level on immediate demand and taking the service level of the competing platform as a reference quality. Peng et al. [34] assumed that platform operators empower service providers to improve service quality, and the equilibrium conditions of the whole network are obtained based on variational inequality and equilibrium theory. Wen et al. [35] discussed the quality control behaviors of platforms and sellers in online shopping and analyzed the influence of government regulation on the evolution trend of members’ quality behavior under the situation of information asymmetry. However, in the research of these documents, the price is regarded as a fixed value, which cannot change according to the different supply and demand situations, which is not in line with the reality. Therefore, we have considered the dynamic change of price in the process of studying service quality comprehensively, so that the conclusion has higher credibility and practical significance.

1.3. Academic Contribution. Our paper differs from the above studies in the following aspects.

First, according to the above literature, most scholars only consider one or two kinds of supply and demand situations; we discuss three kinds of supply and demand
situations comprehensively. When supply exceeds demand, we consider the opportunity loss cost of providers. In contrast, we consider the cost of order delay caused by insufficient providers. When supply and demand are balanced, we do not need to consider the above two costs.

Second, although many scholars have studied the pricing strategies of bilateral platforms under stochastic demand, most of them only considered the competition scenario between two platforms, and few researchers have studied the joint decision making of platforms and service providers at the same time. We built differential game models of crowd logistics platform and service provider, respectively, and solved the optimal dynamic price of crowd logistics platform and the optimal quality level of service provider under three situations.

Third, in the case of unbalanced supply and demand, it is common to study the dynamic pricing strategy and the benefits of both parties, but few scholars have discussed the problem of dynamic quality under the imbalance of supply and demand. In fact, when an order is completed, the crowd logistics platform would pay the provider in a certain proportion. Therefore, the salary of the service provider will change with the dynamic pricing of the platform. Assuming that the quality control cost is constant, it will inevitably affect the service quality level of the provider. Therefore, it is necessary to explore the impact of dynamic prices on the service quality and the benefits of both parties. So, the provider will determine its own quality control level according to the optimal pricing of the platform to maximize its own benefits in our model.

Finally, in order to ensure the service quality, we have set up reward-penalty factor in the model, and the crowd logistics platform monitors the service quality of the provider. We also analyze the relationship between reward-penalty factor and service quality and then put forward relevant suggestions.

2. Problem Definition and Notation

The operation mode of crowd logistics is shown in Figure 1. First, consumers send requests to the crowd logistics platform according to their own needs. When the platform receives an order, it shares the order information with the platform; then, nearby social providers grab the order and deliver it offline. The profit model of platforms and service providers is as follows: the consumer pays the freight to the platform, and then the platform pays the provider in a certain percentage. At the same time, the crowd logistics platform supervises the delivery service quality of the provider.

2.1. Symbol Description. The symbols and specific meanings mentioned in this paper are shown in Table 1.

2.2. Model Assumptions. For modeling purposes, the assumptions are as follows:

(1) According to the assumption about supply chain in Meng et al. [29], the service platform and provider are considered for decentralized decision making. They established the revenue function of both parties, respectively, to explore the optimal decision of them. Therefore, assuming that the cooperation between the service platform and the provider is not considered, both parties are rational people for modeling convenience; the goal is to maximize their interests, and we play a Stackelberg differential game dominated by the service platform. In the first stage, the service platform determines the price of crowd logistics services. In the second stage, the provider determines its quality control level according to the pricing of the service platform.

(2) Referring to the idea of Meng et al. [30], the quality of crowd logistics will change with time dynamically; therefore, we propose the dynamic differential equation of service quality, and the service quality is affected by the quality control efforts of the platform and the provider. Due to the direct contact between the provider and consumers and for the convenience of calculation, we mainly consider the impact of the provider’s quality control on the service quality:

\[ \dot{q}(t) = y \cdot w(t) - \delta q(t), \]

where \( y \) is the quality influence coefficient, which represents the influence coefficient of the quality control efforts of the provider on the crowd logistics quality; \( w(t) \) refers to the quality control level of the provider (including personnel communication quality, package integrity, and so on) at time \( t \); and \( \delta \) is the attenuation coefficient of service quality. Since the range of crowd delivery is mostly 3–5 kilometers nearby, its delivery targets are mostly takeaways, fresh food, and other items that require high freshness. If quality control is not carried out, the freshness will decline naturally.

(3) Consumers are most sensitive to price and quality of service when transacting; referring to Tian et al. [36] and Liu et al. [37], we assume that the demand of crowd logistics is affected by platform’s price and provider’s quality. Furthermore, due to the consideration of optimal decision making under random demand, referring to the setting of market scale change factors by Lin and Zhang [38], we can know that the random demand function of crowd logistics is

\[ D(t) = D_0 e^{-at} - \alpha p(t) + \beta q(t), \]

where \( \alpha \) is the demand volatility factor. When \( \alpha > 0 \), it indicates the decline of demand. On the contrary, when \( \alpha < 0 \), it means that the demand increases. When \( \alpha = 0 \), the market demand remains unchanged. In addition, it is assumed that the need for crowd logistics is still positive without considering
the service quality control of provider, i.e., $D_0 e^{-\alpha t} - \alpha p(t) > 0$.

(4) In crowd logistics, most service providers are paid by a fixed commission model at this stage. Consumers pay the platform for shipping, and the platform pays the provider a certain percentage of the revenue after the order is completed, similar to the operation of Uber. Therefore, according to Cachon et al. [13], we assume a linear relationship between unit reward and price. The supply function of crowd logistics is

$$S(t) = k(1 - \phi)p(t).$$  (3)

(5) Since we only consider the impact of the service quality efforts of the provider on the quality of crowd logistics, in order to ensure the level of service quality, the platform is responsible for the supervision of the logistics service quality. According to Shi et al. [39] and Wang et al. [40], when the quality is lower than the industry standard value $q$, the platform will punish the provider $\varepsilon(q(t) - q)$. When the quality is higher than the industry standard value, $\varepsilon(q(t) - q)$ of rewards will be given. Therefore, the profit functions of the platform and the provider in the time range $[0, T]$ are

**Table 1: Description of model variables and parameters.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(t)$</td>
<td>Crowd logistics service price at time $t$</td>
</tr>
<tr>
<td>$q(t)$</td>
<td>Crowd logistics service quality</td>
</tr>
<tr>
<td>$w(t)$</td>
<td>The quality control level of the provider</td>
</tr>
<tr>
<td>$D_0$</td>
<td>Basic needs of crowd logistics services</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The market demand volatility factor</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Price sensitivity coefficient, which indicates how sensitive customers are to the price of crowd services</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Service quality sensitivity coefficient, indicating the sensitivity of customers to crowd service quality</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Reward-penalty factor of the crowd platform to the provider, $\varepsilon \geq 0$</td>
</tr>
<tr>
<td>$k$</td>
<td>Price sensitivity coefficient, which indicates the sensitivity of the provider to reward</td>
</tr>
<tr>
<td>$c$</td>
<td>The unit opportunity loss cost</td>
</tr>
<tr>
<td>$h$</td>
<td>The unit order delay cost</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Supervision cost of a crowd logistics platform to the provider</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Quality influence coefficient</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mass attenuation coefficient</td>
</tr>
<tr>
<td>$k_w$</td>
<td>Service cost coefficient of the provider</td>
</tr>
<tr>
<td>$q$</td>
<td>Crowd logistics service quality standard</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The revenue share of the platform</td>
</tr>
</tbody>
</table>

**Figure 1: Crowd logistics operation mode.**
\[
\prod_{p}(t) = \int_{0}^{T} \phi p(t)D(t) - c_s - \varepsilon (q(t) - \bar{q})dt, \\
\prod_{r}(t) = \int_{0}^{T} (1 - \phi)p(t)D(t) + \varepsilon (q(t) - \bar{q}) - \frac{1}{2}k_ww(t)^2dt,
\]

where \(c_s\) is the cost of the platform supervising the logistics service quality of the provider and \(k_w\) is the quality control cost coefficient of the provider.

3. Optimal Decision under Stochastic Demand

This section solves the optimal pricing of the crowd platform and the quality control behavior of the provider under three different supply and demand situations of decentralized decision making. The subscripts \(P\) and \(R\) represent the crowdsourcing platform and the service provider, respectively. Suppose that supply and demand are balanced at time 0. When \(a > 0\), it means oversupply. When \(a < 0\), it means that the supply is less than the demand. When \(a = 0\), it means that the state of supply and demand remains unchanged, that is, the supply-demand balance at time 0 is maintained.

3.1. Optimal Pricing with Demand Less Than Supply. When \(a > 0\), it means that the competition in the logistics supply market is fierce; that is, crowd logistics is oversupply. Therefore, the provider generates the remaining supply capacity and needs to bear the opportunity loss cost. Referring to the idea of Jørgensen and Kort [41], taking the product inventory as the state variable and building the inventory state change equation based on the replenishment rate and demand rate, we constructed the state change equations. Therefore, the provider generates the remaining supply capacity and needs to bear the opportunity loss cost. Referring to the idea of Jørgensen and Kort [41], taking the product inventory as the state variable and building the inventory state change equation based on the replenishment rate and demand rate, we constructed the state change equation based on the distribution supply rate \(S(t)\) of the provider and the customer demand rate \(D(t)\) at time \(t\):

\[
\dot{q}(t) = S(t) - D(t),\quad l(0) = 0, l(T) = l_T.
\]

The cumulative excess supply capacity at time \(t\) is

\[
p^*(t) = A_1e^{-at} + A_2q^*(t) + A_3\lambda_p^*(t),
\]

where

\[
A_1 = \frac{D_0}{2a}, \quad A_2 = \frac{\beta}{2\alpha}, \quad A_3 = \frac{k(1 - \phi) - \alpha}{2a\phi}.
\]

The optimal quality of the provider is

\[
q^*(t) = \frac{(F_2 + F_3)\gamma a + aA_1e^{-(F_2 + \delta T)}}{\delta + F_1\gamma} + q_0(F_1\gamma + \delta T)\rho\delta,
\]
where

\[ F_1 = \frac{yD_1(A_2 + E_2)}{k_w}, \]

\[ F_2 = \frac{yD_1(-A_3(E_1l_T + E_2) + D_2)}{k_w}, \]

\[ F_3 = \frac{yD_1e^{-at}}{k_w}, \]

\[ E_1 = \frac{2\alpha \phi}{T(k + \alpha - k\phi)^2}, \]

\[ E_2 = \frac{D_0e^{-at}\phi(2(e^{at} - 1)a - aT(k + \alpha) + akT\phi)}{a(k + \alpha - k\phi)^2}, \]

\[ E_3 = \frac{-\beta\phi(\alpha + k(\phi - 1))}{(k + \alpha - k\phi)^2}, \]

\[ D_1 = \frac{\beta(1 - \phi)}{\delta + \rho}, \]

\[ D_2 = \frac{\varepsilon + c\beta(T - t)}{\delta + \rho}. \]

Dynamic change track of shadow price \( \lambda_p^*(t) \) can be obtained as follows:

\[ \lambda_p^*(t) = E_1l_T + E_2 + E_3q^*(t), \]  

(14)

where

\[ l_T = \frac{(B_1 - A_1) - (B_2 - A_2)\bar{q}}{A_3E_1} + \frac{E_2}{E_1} + \frac{E_3\bar{q}}{E_1}, \]

(15)

\[ B_1 = \frac{D_0}{k(1 - \phi) + \alpha}, \]

(16)

\[ B_2 = \frac{\beta\bar{q}}{k(1 - \phi) + \alpha}. \]

The demand trajectory of crowd logistics is

\[ D^*(t) = D_0e^{-at} - ap^*(t) + \beta q^*(t). \]

(17)

The supply trajectory of crowd logistics is

\[ S^*(t) = k(1 - \phi)p^*(t). \]

(18)

Proof. According to the Pontryagin maximum principle, the necessary conditions for maximizing the revenue of crowd logistics platform are as follows:

\[ \dot{t}(t) = \frac{\partial H_{pl}}{\partial \lambda_{pl}}, \]

(19)

\[ \dot{\lambda}_{pl}(t) = -\frac{\partial H_{pl}}{\partial t}, \]

(20)

From equations (20) and (21), the optimal price of the platform is

\[ p(t) = A_1e^{-at} + A_2q(t) + A_3\lambda_p^*(t). \]

(22)

Then, we can obtain the expression of shadow price according to equation (19) and boundary conditions \( I(0) = 0 \), \( I(T) = I_T \):

\[ \lambda_p(t) = E_1l_T + E_2 + E_3q(t). \]

(23)

Since the supply and demand of the crowd platform are balanced at the initial time, we can get

\[ p(0) = D_0 + q_0\beta/k(1 - \phi) + \alpha. \]

Because \( p''(0) = A_1 + A_2q_0 + A_3(E_1l_T + E_2 + E_3q_0) \), let \( p^*(0) = p(0) \), and we can get

\[ I_T = \frac{(B_1 - A_1) - (B_2 - A_2)\bar{q}}{A_3E_1} + \frac{E_2}{E_1} + \frac{E_3\bar{q}}{E_1}. \]

(24)

Substituting it into equation (23) and substituting equation (23) into equation (22), we can get \( p^*(t) \).

In the second stage, during the service time \([0, T]\), the objective function of the maximum expected return of the provider is

\[ \prod_{\partial t} (P, t) = \int_0^T (1 - \phi)p(t)D(t) + \varepsilon(q(t) - \bar{q}) - \frac{1}{2}k_w\omega^2 \]

(25)

\[ -c(T - t)(S(t) - D(t))dt. \]

The Lagrange multiplier \( \lambda_p(t) \) is introduced to construct the Hamiltonian function of the provider:

\[ H_{11} = (1 - \phi)p(t)D(t) + \varepsilon(q(t) - \bar{q}) \]

(26)

\[ -\frac{1}{2}k_w\omega(t)^2 - c(T - t)(S(t) - D(t)) + \lambda_p(t)(\gamma w(t) - \delta q(t)), \]

where \( \lambda_p(t) \) is the covariant variable of the optimization problem, and its economic meaning refers to the marginal income that the provider can obtain from its quality control effort. Equation (26) is solved by using Pontryagin maximum principle:

According \( \frac{\partial H_{11}}{\partial w} = 0 \) we can obtain \( w(t) = \frac{y\lambda_p(t)}{k_w} \),

(27)

According \( \frac{\partial H_{11}}{\partial q(t)} = \rho \lambda_p(t) \), we can obtain \( \lambda_p(t) = D_1p(t) + D_2. \)

(28)

Substitute equation (22) into equations (27) and (28), and we can obtain

\[ \omega(t) = F_1q(t) + F_2. \]
Substitute equation (29) into equation (1) and solve the differential equation, and we can get \( q^* (t) \). \( \square \)

**Corollary 1.** When the demand of crowdsourcing market decays, the optimal dynamic price of the platform is a convex function to time and has a negative correlation with time.

**Proof.** The first partial derivative and second partial derivative of the optimal pricing with respect to time can be calculated as

\[
\frac{dP^*}{dt} = \frac{aD_0e^{-at}}{2\alpha} - \frac{e^{(D_1u + E_1)\gamma / k_w} - 1 - \delta}{k_w[(1 - \phi)k + \alpha]} \beta
\]

\[
< 0, \quad \frac{dP^*}{dt} = \frac{a^2D_0e^{-at}}{2\alpha} + \frac{e^{(D_1u + E_1)\gamma / k_w} - 1 - \delta}{k_w^2[(1 - \phi)k + \alpha]} > 0.
\]

Since the second partial derivative is larger than 0, the price of the crowd logistics is a convex function of time.

Because the first partial derivative is less than 0, the price of the crowd logistics decreases monotonically.

Corollary 1 shows that the optimal dynamic price of the crowd logistics platform will gradually decrease when the market demand decays. The price reduction can stimulate the demand for crowdsourcing logistics to a certain extent. At the same time, the reduction of price also reduces the remuneration of service provider, which could reduce the willingness of some providers to participate in crowdsourcing. In this way, the opportunity loss cost is reduced and the state of supply and demand can be adjusted to balance. However, as demand continues to decay with time, the platform’s price will stabilize to maintain basic income. \( \square \)

**Corollary 2.** During the service time \([0, T]\), when the demand of the crowdsourcing market decreases, the optimal service quality of the provider decreases with time and increases first and then decreases with the rise of reward-penalty factors.

**Proof.** The first derivative of the optimal dynamic service quality of the service provider with respect to time is:

\[
\frac{dq^*}{dt} = \frac{aD_0e^{-at}}{\gamma + \delta} \left( \frac{ae^{-at}}{k_w[\alpha(\delta + \rho)]^2} - \frac{\beta y^2(c\beta + a(\delta + \rho))(1 - \phi)}{2ak_w\delta} \right).
\]

3.2. Optimal Pricing with Demand Greater Than Supply. When \( a < 0 \), it means that the crowdsourcing market demand surges, the service platform will bear the order delay costs, and the actual crowd logistics transaction volume at this time is \( S(t) \). The state change equation of the delayed order quantity is as follows:

\[
\dot{u}(t) = D(t) - S(t)u(0) = 0, u(T) = u_T.
\]

The cumulative delayed order quantity at time \( t \) is

\[
u(t) = u(0) + \int_0^t D(s) - S(s)ds.
\]

During the service time \([0, T]\), due to the surge in crowd logistics demand, the actual supply capacity of the providers cannot meet all the requests, so the real order demand at time \( t \) is the smaller one between \( D(t) \) and \( S(t) \), i.e., \( \min(D(t), S(t)) = S(t) \), and the expected revenue objective function of the platform is

\[
\prod_{P^*} (P, T) = \int_0^T \phi pS(t) - c_x - \epsilon (q(t) - 7)
\]

\[
- h(T - t)(D(t) - S(t))dt.
\]

The constraints of equation (35) are as follows:

\[
\dot{u}(t) = D(t) - S(t)u(0) = 0, u(T) = u_T.
\]
Theorem 2. When the demand increases, the optimal price of the crowd logistics platform is
\[
p^* (t) = \frac{D_0 e^{-at} + \beta q^* (t)}{k (1 - \phi) + \alpha}.
\] (37)

The optimal quality is
\[
q^* (t) = \frac{e^{-tS} (1 + e^{\delta t}) H + q_0 \delta}{\delta},
\] (38)
where \[H = \frac{(\epsilon^2/k_w (\delta + \rho))}{\delta} \]

Therefore, the platform’s revenue is a convex function of price, and we will obtain a price that minimizes the platform revenue if we use Pontryagin principle to solve the problem, so we have to change the method. During the service time \[\int_0^T \] since \[u(t) \geq 0, u(0) = 0\]; therefore, as long as \[D(t) \geq S(t)\] is met, we can get the range of price:
\[
p(t) \leq \frac{D_0 e^{-at} + \beta q(t)}{k (1 - \phi) + \alpha}.
\] (42)

Since the expected return of the platform is a convex function of price, \[p^* (t) = \rho_{\text{max}}:\]
\[
p^* (t) = \frac{D_0 e^{-at} + \beta q(t)}{k (1 - \phi) + \alpha}.
\] (43)

In the second stage, the maximum expected return objective function of the provider is
\[
\prod_{r2} (P, t) = \int_0^T (1 - \phi) p S(t) + \epsilon (q(t) - \bar{q}) - \frac{1}{2} k_w \omega^2 dt.
\] (44)

Introduce the Lagrange multiplier \[\lambda_r (t)\] to construct the Hamiltonian function of the provider:
\[
H_{r2} = (1 - \phi) p(t) S(t) + \epsilon (q(t) - \bar{q})
\frac{1}{2} k_w \omega^2 + \lambda_r (t) (\gamma w(t) - \delta q(t)),
\] (45)

According to \[\frac{\partial H_{r2}}{\partial \omega} = 0\], we can get \[w(t) = \frac{\gamma \lambda_r (t)}{k_w} \],
\[\frac{\partial H_{r2}}{\partial q(t)} = \rho \lambda_r (t),\] we can get \[\lambda_r (t) = \frac{\epsilon}{\delta + \rho} \].
\[\frac{\partial H_{r2}}{\partial q(t)} = \rho \lambda_r (t),\] we can get \[\lambda_r (t) = \frac{\epsilon}{\delta + \rho} \].
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\[\frac{\partial H_{r2}}{\partial q(t)} = \rho \lambda_r (t),\] we can get \[\lambda_r (t) = \frac{\epsilon}{\delta + \rho} \].

Substitute equation (47) into equation (46) and substitute equation (46) into equation (1) to solve the differential equation, and we can obtain \(q^* (t)\).

The requirement of crowd logistics is
\[
D^* (t) = D_0 e^{-at} - \alpha p^* (t) + \beta q^* (t).
\] (39)

The supply of crowd logistics is
\[
S^* (t) = k (1 - \phi) p^* (t).
\] (40)

Proof. In the first stage, the Lagrange multiplier \[\lambda_r (t)\] is introduced to construct the Hamiltonian function, and the second-order partial derivative with respect to the price is obtained:
\[
\frac{\partial^2 H_{r2}}{\partial p(t)^2} = -2k (1 + \phi) \phi > 0.
\]

Corollary 3. When the crowd logistics demand increases, the optimal dynamic pricing of the platform is positively correlated with time.

Proof. The first and second partial derivatives of the optimal dynamic price \[P^* (t)\] with respect to \(t\) can be calculated as
\[
\frac{dP^* (t)}{dt} = -\frac{2k (1 + \phi) \phi}{\delta} > 0,
\]
\[\frac{d^2 P^* (t)}{dt^2} = -\frac{2k (1 + \phi) \phi}{\delta} > 0.
\]

Therefore, the optimal dynamic price of the platform is a convex function of time and is positively correlated with time.

Corollary 3 shows that the optimal dynamic price rises with time when the demand increases. We can motivate the provider to participate in crowd logistics by raising the price, the order delays will decrease as supply increases, and platform’s revenue will also increase.

Corollary 4. When the crowd logistics demand increases, the optimal dynamic quality of the provider increases with the rise of the reward-penalty factor. When the reward-penalty factor is larger than a certain threshold, the optimal dynamic quality is a monotonically increasing function of time.

Proof. The first derivative of the optimal dynamic service quality \[q^* (t)\] with respect to time is
\[
\frac{dq^* (t)}{dt} = \frac{e^{-tS} (1 + \epsilon \gamma - k_w q_0 \delta (\delta + \rho))}{k_w (\delta + \rho)}.
\]

When \[\epsilon > k_w q_0 \delta (\delta + \rho)/\gamma^2\], the optimal quality of the provider increases monotonically with time.

When \[\epsilon < k_w q_0 \delta (\delta + \rho)/\gamma^2\], the optimal quality of the provider decreases monotonically with time.
The first derivative of the optimal dynamic service quality with respect to the reward-penalty factor is
\[
\frac{dq^*(t)}{dx} = \frac{\left(1 - e^{-\delta t}\right)y^2}{k_\phi(\delta + \rho)} > 0.
\] (50)

Since \(\delta > 0\), \(e^{-\delta t} < 1\); therefore, the formula above is larger than zero. That is, the quality of crowd logistics service increases with the rise of the reward-penalty factor.

Corollary 4 shows that since the crowd logistics platform will increase the pricing, and the income of the provider will also rise when the market demand increases, so the provider has an incentive to invest more efforts in quality control. When the reward-penalty factor is significant, the service quality will be higher over time. When the reward-penalty factor is lower than a certain threshold, the reward is not enough to motivate the provider to improve the service quality. With the surge in market demand, the provider pays more attention to customer quantity rather than quality, resulting in a decrease in service quality levels over time. \(\square\)

3.3. Optimal Pricing with Balanced Supply and Demand

At this time, the demand for crowd logistics and the provider’s supply are equal roughly, and the crowd logistics will maintain a balance of supply and demand at time 0. Therefore, neither the service platform nor the provider has costs (loss of opportunity cost and order delay cost) other than quality control costs, namely, \(l(t) = u(t) = 0\).

**Theorem 3.** When the demand for crowd logistics remains unchanged, the optimal dynamic price and optimal quality control do not change with time:
\[
\begin{align*}
p^*(t) &= \frac{D_0 + \beta q_0}{k(1-\phi) + \alpha}, \\
q^*(t) &= q_0.
\end{align*}
\] (51)

**Proof.** When the demand remains unchanged, the platform will maintain the balance of supply and demand at time 0. That is, the optimal price is the price of the equilibrium state of supply and demand at time 0. Since demand and supply are equal at time 0,
\[
D_0e^{-at} - \alpha p(t) + \beta q_0 = k(1-\phi)p(t).
\] (52)

In the state of supply and demand balance, service quality will eventually tend to balance, namely, \(q(t) = q_0\).

So, we can obtain
\[
p^*(t) = \frac{D_0 + \beta q_0}{k(1-\phi) + \alpha}.
\] (53)

Theorem 3 shows that the optimal price of the crowd logistics platform is a constant when \(\alpha = 0\), and the platform will integrate a variety of parameters to maintain the price when supply and demand are balanced to maximize its income. The quality control of crowd logistics is also held at initial level. \(\square\)

**Corollary 5.** The optimal pricing is positively correlated with the service quality sensitivity coefficient \(\beta\) and the revenue share of the platform \(\phi\) and negatively correlated with the price sensitivity coefficient \(\alpha\).

**Proof.** The first derivative of the optimal pricing \(p^*(t)\) with respect to the price sensitivity coefficient \(\alpha\) is
\[
\frac{dp^*(t)}{d\alpha} = -D_0 + q_0\beta(\alpha + k(1-\phi))^2 < 0.
\]

The first derivative of the optimal price \(p^*(t)\) to the service quality sensitivity coefficient \(\beta\) and the first derivative of the optimal price to the revenue share of the platform \(\phi\) are as follows:
\[
\frac{dp^*(t)}{d\beta} = \frac{q_0}{\alpha + k(1-\phi)} > 0, \quad \frac{dp^*(t)}{d\phi} = \frac{k(D_0 + q_0\beta)}{(\alpha + k(1-\phi))^2} > 0.
\] (54)

Therefore, the optimal price decreases with the price sensitivity coefficient \(\alpha\) and increases with the service quality sensitivity coefficient \(\beta\) and the revenue share of the platform \(\phi\).

Corollary 5 shows that the more sensitive consumers are to price, the lower their pricing will be. The more sensitive consumers are to quality, the higher the proportion of revenue distribution of service providers is and the higher the corresponding pricing is. \(\square\)

4. Numerical Simulation

In this section, we will use the numerical examples to demonstrate the relevant conclusions more intuitively, analyze the relationship between optimal dynamic pricing and service quality and revenue in three scenarios, and discuss the impact of reward-penalty factors on service quality. According to Bai et al. [10], Wang et al. [14], Meng et al. [29], and Lin and Zhang [38], the main parameters are set as follows:
\[
D_0 = 10, T = 15, \alpha = 1, \beta = 2, \bar{q} = 20, k = 3, k_w = 1, c_s = 1, \delta = 0.1, c = 0.2, h = 0.2, y = 1, \rho = 0.1, \text{and } q_0 = 20.
\]

4.1. Optimal Pricing in relation to Time and Demand Variability Factor

In this section, we suppose that the range of demand volatility factor is \([0, 1]\) when the crowd logistics demand decays; on the contrary, when demand increases, the range is \([-0.08, 0]\). At the same time, assuming that \(\phi = 0.4\), \(\varepsilon = 0.5\), the trajectory of optimal pricing changing with time and market demand fluctuation factor is shown in Figures 2–4.

4.1.1. Result. First, we explore the situation that supply exceeds demand. Figure 2 shows the trajectories of the optimal price \(p^*(t)\) over time and the demand volatility factor \(a\) during service time \([0, T]\), and we can see that the optimal dynamic pricing of the crowd logistics platform decreases gradually over time; the price will stabilize when it falls to a certain value, which is consistent with the conclusion in Corollary 1. At the same time, as the degree of demand attenuation increases, the optimal pricing \(p^*(t)\) also decreases gradually when the value of demand volatility factor \(a\) increases.
that when the market demand of crowd logistics remains unchanged, the optimal pricing of the crowd logistics platform is constant and does not change with time, which is consistent with the description of Theorem 3.

Finally, Figure 5 and 6 show the relationship between the price level and the consumer’s price sensitive index and quality sensitive index in the state of supply and demand balance. It is obvious that they have positive and negative correlation, respectively, which is consistent with the conclusion of Corollary 5.

4.1.2. Discussion. In the case of declining demand, supply exceeds demand at this time because supply and demand are balanced at time 0. On the one hand, the platform needs to stimulate consumer demand and increase orders. On the other hand, the platform should reduce the number of service providers to avoid oversupply. Therefore, the platform will reduce the price $p^*(t)$ to attract consumers to participate in crowd logistics at the initial stage; at the same time, the price reduction will correspondingly reduce the remuneration level of service providers, resulting in some providers no longer participating in crowd logistics distribution. However, with the further decline of demand, if the platform continues to reduce the price when it falls below a certain threshold, the revenue of the platform $\pi_p$ will not be enough to compensate its own service cost. Therefore, the platform will maintain the price to ensure its revenue. In addition, with the intensification of demand attenuation, the order volume drops sharply, and the platform needs to increase the price reduction to further stimulate the demand for crowd logistics. Therefore, the optimal dynamic price $p^*(t)$ decreases with the increase of the market demand fluctuation factor $a$. Similarly, the price tends to stabilize when it drops to a certain value.

When the demand for crowd logistics increases, the supply in the crowdsourced logistics market is less than the demand. If consumers place an order at this time, it may not be delivered on time; as a result, order delay costs $h$ are incurred. Platforms need to adopt certain strategies to enable more service providers to participate in crowd logistics and alleviate the problem of insufficient supply. According to the supply function $S(t) = k(1 - \phi)p(t)$, the supply of service providers is positively correlated with the price $p(t)$. Therefore, under the condition that the income distribution ratio $\phi$ and the price sensitivity coefficient $k$ are constant, raising the price can increase the supply $S(t)$. At the same time, the revenue of the platform $\pi_p$ can also increase. For consumers, the increase in prices can suppress part of consumer demand and further reduce order delays. In addition, since the market demand fluctuation factor $a$ is negative at this time, the smaller the value is, the faster the crowd logistics demand increases, so the optimal dynamic pricing $p^*(t)$ increases with the decrease of the market demand fluctuation factor $a$.

When $a = 0$, the market demand for crowd logistics remains unchanged, and the balance of supply and demand at time 0 will be maintained. The crowd logistics platform has no need to adjust the price, so the optimal price is a
constant at this time. When people pay more attention to the price, a small price increase on the platform will reduce the demand for crowdsourcing logistics greatly. Therefore, a larger price sensitivity index will lead to a lower optimal price. On the contrary, when people pay more attention to the quality of services and are willing to pay higher prices for high-quality services, raising prices will not cause the decline of demand. Therefore, the larger the quality sensitive index is, the higher the optimal price will be.

4.2. Optimal Quality in relation to Time and Reward-Penalty Factor. In this section, we assume $a = 0.4$ and use $a = (0.2, -0.08)$ to represent the situation of market demand decay and increase. The trajectory of optimal quality changing with time and reward-penalty factor is shown in Figures 7–9.

4.2.1. Result. It can be seen from Figure 7 that when the demand of the crowd logistics market decays, the optimal service quality level of the service provider decreases over time during the service time $[0, T]$. Since the service quality $q^*(t)$ at the initial moment is the standard quality $q_0$, the crowd logistics platform will punish the service provider when the service quality level decreases. Let the penalty factor $\varepsilon$ increase gradually. It can be seen from Figure 7 that the optimal service quality $q^*(t)$ increases with the increase of the penalty factor $\varepsilon$ in the initial stage. When the penalty factor $\varepsilon$ reaches a certain value, the optimal service quality $q^*(t)$ begins to decline if the penalty factor continues to increase, which is consistent with Corollary 2.

When the demand for crowd logistics increases, Figure 8 shows the change of optimal service quality $q^*(t)$ with time when the value of the reward-penalty factor $\varepsilon$ satisfies a certain condition, i.e., $\varepsilon > k_\omega q_0 \delta (\delta + \rho)/\gamma^2 = 0.4$. At this time, the optimal service quality level increases with time; when a certain threshold is reached, the quality decreases with time. Similarly, since the service quality level $q_0$ at time 0 is the standard quality, the crowd logistics service platform will reward the service provider. It can be seen from Figure 8 that when the penalty factor $\varepsilon$ is small, the optimal dynamic quality also increases with the rise of the reward factor $\varepsilon$; finally, the quality tends to be stable when it reaches a certain value, which is consistent with Corollary 4.

When $a = 0$, the market demand for crowd logistics remains unchanged. Figure 9 shows the change of optimal service quality $q^*(t)$ with time. We can see that the optimal service quality remains unchanged with time, always in the initial quality level.
4.2.2. Discussion. According to the analysis in Section 4.1.2, the crowd logistics platform will reduce the price \( p^*(t) \) in order to attract more consumers to participate when the market demand declines. If the quality control cost of the service provider remains unchanged, the provider’s revenue \( \pi_r \) will decrease. Therefore, the provider will reduce the value of service quality \( q^*(t) \) to protect its own revenue, and the optimal quality of service decreases over time. When the value of the penalty factor \( \epsilon \) is small, the penalty mechanism cannot restrict the quality control behavior of the service provider effectively. As both parties have been assumed to be rational persons in Section 2.2, the provider lacks the motivation to carry out quality control, resulting in the reduction of logistics service quality \( q^*(t) \). At this point, the service provider will improve the service quality if the platform increases the penalty factor. When the penalty factor reaches a certain value, service providers will bear high quality control costs. Otherwise, they will pay high fines \( \epsilon | q(t) - \bar{q} | \) to the platform. As a result, the profit of the service provider is very low. They begin to strive for more orders rather than quality assurance. Therefore, the service quality of crowd logistics declines.

Next, we analyze the quality change in the case of increased demand. When the value of reward-penalty factor \( \epsilon \) is less than 0.4, the reward \( \epsilon | q(t) - \bar{q} | \) paid by crowd logistics platform to service providers is not enough to encourage the provider to make more quality control efforts. Moreover, due to the large demand, the service provider is more inclined to deliver more orders to obtain revenue compared with the reward of crowd platform, which will lead to the quality lower than the standard value \( \bar{q} \). Therefore, we only analyze the case that the value of the reward factor \( \epsilon \) is greater than 0.4 in the numerical example. When the value of reward factor is greater than 0.4, it can be seen from Figure 6 that the service quality of crowd logistics \( q^*(t) \) increases over time. In addition, within the range of \([0.4, 1]\), service providers can obtain more generous quality control rewards with the increase of reward factor; therefore, they have greater motivation to improve service quality.

Finally, we explore the quality level in the case of balanced supply and demand. When the supply and demand remain unchanged, if the service provider improves the service quality, it is equivalent to increasing the service cost, which leads to the reduction of its own benefits ultimately. On the contrary, if the service provider reduces the service quality, it will be lower than the industry standard service quality level. At this time, the platform will punish the service provider and also reduce its revenue. Therefore, the service quality will remain unchanged at the initial level.

4.3. Supply and Demand Rate in relation to Time. In this section, we assume \( \phi = 0.4, \epsilon = 0.5 \) and use \( a = (0.2, -0.08) \) to represent the situation of market demand decay and increase. The supply and demand trajectory in different situations is shown in Figures 10 and 11.
4.3.1. Result. Figure 10 shows the changes in supply and demand under a market demand decay scenario. The curve \( S(t) \) represents the trajectory of the crowd logistics supply when the crowd logistics platform uses the optimal dynamic pricing \( p^*(t) \) and the service provider uses the optimal dynamic quality \( q^*(t) \), and the curve \( S(0) \) represents the trajectory of the supply when the platform uses the static price \( p(0) \) at time 0 and the service provider adopts the standard quality \( q_0 \). It can be seen that the supply of dynamic decision making is significantly reduced compared with static decision making. Eventually, the supply of crowd logistics tends to stabilize. The curve \( D(t) \) represents the trajectory of the demand for crowd logistics when the platform uses the optimal dynamic pricing \( p^*(t) \) and the service provider uses the optimal dynamic quality \( q^*(t) \), and the curve \( D(0) \) represents the trajectory of the demand when the platform uses the static price \( p(0) \) at time 0 and the provider adopts the standard quality \( q_0 \). We can see that the demand when using dynamic decision making increases significantly compared with static decision making. Similarly, the demand tends to a stable state finally. In addition, optimal dynamic pricing and optimal quality control remain stable over time, so the slopes of the supply and demand curves decrease.

Figure 11 shows the changes in supply and demand when market demand increases. The meanings of the \( S(t) \), \( D(t) \), \( S(0) \), \( D(0) \) curves are the same as those described above. It can be seen that the supply of dynamic decision making increases compared with static decision making, and the demand of dynamic decision making decreases significantly compared with static decision making. Furthermore, both the supply curve and the demand curve eventually stabilize.

As we can see from Figures 10 and 11, the area between the curves \( S(t) \) and \( D(t) \) is smaller than the area between \( S(0) \) and \( D(0) \) significantly, i.e., the use of optimal dynamic decision making can narrow the gap between the supply rate and the demand rate, alleviating the state of supply and demand imbalance to a certain extent.

4.3.2. Discussion. When market demand decays, crowd logistics is in a state of oversupply. According to Theorem 1 and Section 4.1, the crowd logistics price in dynamic pricing will be lower than the pricing in static price at time 0. On the one hand, lower prices will attract more consumers to participate in crowd logistics and increase demand. On the other hand, a reduction in price will reduce the profitability of the service provider, so the supply will decrease. At the same time, according to Section 4.1, the dynamic quality \( q^*(t) \) will be lower than the standard quality \( q_\bar{T} \) at this time. In order to avoid further attenuation of crowd logistics demand due to the low level of service quality, the platform should set a higher penalty factor to restrict the service provider’s quality control actions strictly, which will slow down the rate of quality decline and the rate of demand decline.

Now we will analyze the situation when the crowd market is in short supply. According to Theorem 2 and Section 4.1, the price of crowd logistics with dynamic pricing \( p^*(t) \) will be higher than the static pricing at time 0, which will attract more service providers to join crowdsourcing logistics and increase the supply. In addition, high shipping costs will prevent some consumers from participating in crowdsourcing logistics, solving the problem of excess demand to a certain extent. Since the dynamic quality \( q^*(t) \) will be higher than the standard quality \( q_\bar{T} \) at this time, the crowd logistics platform will reward the service provider. If the reward factor \( \varepsilon \) is too high, the provider will offer consumers very high-quality services to earn rewards. The increase in customer utility will further exacerbate the problem of excess demand. Therefore, crowd logistics platforms should set a lower reward factor to slow down the increase in demand.

4.4. Revenue Comparison of the Crowd Logistics System. To explore the impact of different reward-penalty factor values on the revenue of the crowd logistics, next we will study the impact of the reward-penalty factor \( \varepsilon \) on the steady-state income of the crowd logistics platform and the provider during the time \([0, T]\). First, we assume \( \phi = 0.4 \) and use \( a = (0.2, -0.08) \) to represent the situation of market demand decay and increase, and the trajectory of the benefits of both parties with the reward-penalty factor is shown in Figures 12 and 13. Then, we assume \( \varepsilon = 0.5 \), and the value of market demand volatility factor remains unchanged; the trajectory of the benefits with the fixed commission rate is shown in Figure 14.

4.4.1. Result. When the market demand decays, it can be seen from Figure 12 that the revenue of the crowd logistics platform \( \pi_p \) increases first and then decreases slightly with the increase of the reward-penalty factor \( \varepsilon \), while the revenue of the service provider \( \pi_s \) decreases with \( \varepsilon \). When the market demand increases, Figure 13 shows that the income of the crowd logistics platform decreases with the increase of the reward-penalty factor \( \varepsilon \), and the income of the service provider increases with \( \varepsilon \).

We can see from Figure 14 that regardless of whether the market demand decays \( (a < 0) \), increases \( (a < 0) \), or unchanged \( (a = 0) \), the expected revenue of the crowd logistics platform presents a concave function that first increases and then decreases with the increase of the fixed commission rate of return \( \phi \).

4.4.2. Discussion. First, the platform will reduce prices when market demand fades, which will reduce the provider’s payment, so the provider is unwilling to pay more costs for quality control. When the value of the reward-penalty factor \( \varepsilon \) is small, the platform cannot constrain the behavior of the provider, which exacerbates the attenuation of market demand; therefore, the platform’s revenue is also reduced. With the increase of the reward-penalty factor, the provider will improve the service quality due to the pressure of fines. The cost of quality control will increase, so the provider’s income will decline, and the platform will benefit from the improvement of the provider’s service quality. However, when the reward-penalty factor increases to a specific value,
the service provider no longer participates in crowd logistics due to the low income, resulting in a decrease in the yield of the crowd logistics platform.

Next, the optimal pricing and service quality will increase when market demand increases; at this time, the platform connects with the provider for rewards. With the increase of the reward-penalty factor, the cost paid by the platform increases; therefore, the platform’s income decreases. Although increasing the reward-penalty factor at this time can improve the quality level, it will lead to a decrease in platform revenue. Therefore, the platform should reduce the value of the reward-penalty factor appropriately to ensure maximum profit. At the same time, the revenue of the provider should be taken into account to achieve a win-win situation for both parties, which can not only maintain high profits but also improve quality and meet customer demands.

Finally, the simulation results in Figure 14 are consistent with the actual operation of the crowd logistics platform. When the crowd logistics platform adopts a small fixed commission rate of return, the salary is not enough to attract more social personnel to participate in the distribution service. At this time, the expected return of the platform is small. When the crowd logistics platform adopts a large fixed commission rate of return, the revenue that the platform can obtain from each logistics order is very small; therefore, the expected income of the platform is also small at this time.

5. Conclusion

This study focuses on three kinds of supply and demand situations comprehensively. When supply exceeds demand, we consider the opportunity loss cost of providers. When the supply is insufficient, we consider the cost of order delay caused by insufficient providers. Then, we built differential game models of crowd logistics platform and service provider, respectively, and solved the optimal dynamic price of crowd logistics platform and the optimal quality level of service provider under three situations. In order to ensure the service quality, we have set up reward-penalty factor in the model, and the crowd logistics platform monitors the service quality of the provider.

Our theoretical findings suggest that when the demand of crowdsourcing logistics decreases, there is an optimal dynamic price to maximize the platform revenue, and the optimal dynamic price of the platform is a convex function to time and has a negative correlation with time. During the service time $[0, T]$, the optimal service quality of the provider decreases with time and increases first and then decreases with the rise of reward-penalty factors. The revenue of the crowd logistics platform increases first and then decreases slightly with the increase of the reward-penalty factor $\varepsilon$, while the revenue of the service provider decreases with $\varepsilon$. The platform should appropriately increase the value of the reward and punishment factor to increase its own income.
When the crowd logistics demand increases, the optimal dynamic pricing of the platform is positively correlated with time, and the optimal dynamic quality of the provider increases with the rise of the reward-penalty factor. When the reward-penalty factor is larger than a certain threshold, the optimal dynamic quality is a monotonically increasing function of time, the income of the crowd logistics platform decreases with the increase of the reward-penalty factor $\varepsilon$, and the income of the service provider increases with $\varepsilon$. The platform should appropriately reduce the value of the reward and punishment factor to ensure maximum revenue.

When the demand for crowd logistics remains unchanged, the optimal dynamic price and optimal quality control do not change with time. Similarly, crowdsourcing revenue will change with the proportion of fixed income. The platform should set an appropriate price range according to the price sensitive index and quality sensitive index of consumers.

Regardless of the supply and demand situation, the use of optimal dynamic decision making can narrow the gap between the supply rate and the demand rate, and the expected revenue of the crowd logistics platform presents a concave function that first increases and then decreases with the increase of the fixed commission rate of return $\phi$.

The research only considers the dynamic pricing and optimal dynamic quality control strategy in the case of decentralized decision making between the crowd logistics platform and the service provider. However, dynamic pricing problems under different cooperation modes are not considered. Therefore, future research can be conducted on the optimal pricing problem based on the cooperation model of both parties.

Data Availability
No data were used to support this study.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

Authors’ Contributions
DW was responsible for conceptualization, formal analysis, and resources. JC was responsible for original draft preparation, investigation, visualization, and review and editing. All authors have read and agreed to the published version of the manuscript.

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