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Research Article

Optimal Emergency Ordering Policy for Inventory with a Random-Ending-Time Supply Disruption

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This paper considers the emergency ordering strategy for the classical economic ordering quantity inventory system with a supply disruption. For situations where the ending time of supply disruption is stochastic and the purchase price increases over time during that period, we develop an emergency ordering optimization model based on maximizing retailer's profits. Through modeling analysis in various situations, the closed-form solution of the model is obtained, and the optimal emergency ordering strategy is provided for retailers. Numerical experiments verify the effectiveness of the model and the influence of related parameters on the optimal ordering strategy.

1. Introduction

With the development of global market economy, enterprises will face various risks such as capital chain rupture, demand disruption, supply disruption, and legal risk. Among these, supply disruption is a common risk which may bring larger loss to enterprises [1, 2]. It is reported that Renesas Electronics Co., for example, is likely to suspend production for at least three months after a fire on March 19, 2021, at its semiconductor plant [3]. As a supplier of automotive chips, the Renesas Electronics Co. has a global market share of about 30% in microcontrol unit chips for vehicles. This means that the original car chip shortage will be intensified after the fire chip shortage due to the new energy vehicle production capacity increase. Recently, the global supply chain was disrupted during the COVID-19 pandemic, and by the time it subsided, it had suffered a shock and remained vulnerable [4]. Due to the large loss caused by the supply disruption to each member in the supply chain especially the downstream enterprises, the supply disruption received much attentions of researchers' [5–14].

In order to reduce the risk of supply interruption, Hendricks et al. [15] put forward some risk reduction suggestions, such as improving prediction accuracy, synchronous planning and implementation, shortening delivery cycle, working with partners, and related investments. Hopp et al. [16] elaborated on a measure that focuses on the risks that may occur throughout the supply chain structure and planned for potential disruptions from the recovery process. Skipper and Hanna [17] examined the relationship between some attributes and flexibility of the emergency response planning process and investigated a strategic approach to emergency response planning to reduce the risk of the supply disruption. Xu et al. [18] introduced the CVaR measure to hedge against the risks for the loss-averse newsvendor. Huang et al. [19] considered centralized and decentralized dual-channel supply chains in the case of demand disruptions to guarantee the robustness of optimal output in the event of demand disruptions. Sawik and Tadeusz [20] studied the supply chain with the risk of the optimal selection and protection under supply disruption, established a mixed integer programming method by combining risk neutral with risk avoidance or average risk supply, and obtained a risk conditional value to control the loss caused by supply disruption risk.

To reduce the losses caused by the supply disruption, as a remedial measure, the emergency order is taken into consideration. For an inventory system with random demand and with stochastic product supply disruption, Risa and Croix [6] offered retailers the best rush order strategy. When the occurrence time of supply interruption is subject to a certain probability distribution and the ending time of supply interruption is determined, Xu et al. [21] studied the optimal order quantity in the loss-averse newsvendor model with backordering. Huang et al. [22] established an emergency order optimization model based on minimizing the inventory cost. Xu et al. [23] studied the optimal option purchase of a loss-averse retailer under emergent replenishment. Xia et al. [24] considered the supply interruption management problem of inventory model with a loss function and provided the retailer with the optimal emergency replenishment strategy.

This paper considers the retailer's emergency ordering strategy during supply interruption such that the recovery time of supply interruption is stochastic and the purchasing price rises with the time during the period. For this problem, if the retailer places a larger rush order, the retailer would be suffered certain loss of profits if the supply disruption is ended at an earlier time; if the retailer makes a small emergency procurement, then the retailer would make a second emergency order with a higher purchase price when the supply disruption is ended at a later time. Therefore, the retailer should determine the best time to order urgent orders and the number of urgent orders to maximize his profits during supply disruption period.

The problem considered in this paper happens in reality. For instance, on March 23, 2021, an accident occurred when a super large gold class container ship named "Ever Given" passed through the Suez Canal which resulted in the total paralysis of the transportation of the Suez Canal. As the canal is the main channel between Asia, Europe, and Africa, the accident led to the supply interruption in many parts of the world. As for the time of channel restoration, it ranges from one or two days to one month from outside news. Six days later, the "Ever Given" was still in a single channel, blocked obliquely. Fortunately, on March 29, the shallowing operation of the container ship was successful, and the blockage of the Suez Canal was solved [25].

The rest of this paper is arranged as follows. Section 2 gives relevant symbols and assumptions on the problems considered in this paper. Section 3 establishes an optimization model based on retailer's profit maximization. Section 4 solves the optimization model and gives the retailer's optimal emergency order strategy. Section 5 conducts some numerical analyses and verifies the influence of relevant parameters on the model. The last section draws some conclusions.

2. Notation and Assumptions

For the concerned problem, we assume that the planning horizon of the mechanism is infinite and the demand is stable. As for the supply interruption, we assume that the end time t_e is random, but the cut off time for the end of the event is deterministic. That is, the event end time obeys a certain probability distribution within $[0, T]$. Further, we assume that shortage is not allowed during the supply disruption and the retailer's purchasing price increases with the time during the period. For the inventory system, when the supply disruption happens, the remaining inventory Q_0 will be depleted after certain period. The time is denoted by t_0. Due to the supply

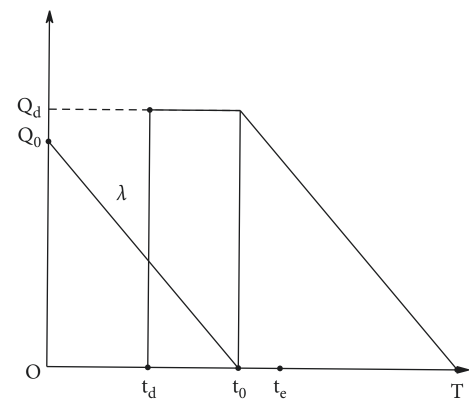

Figure 1: Make one emergency order.

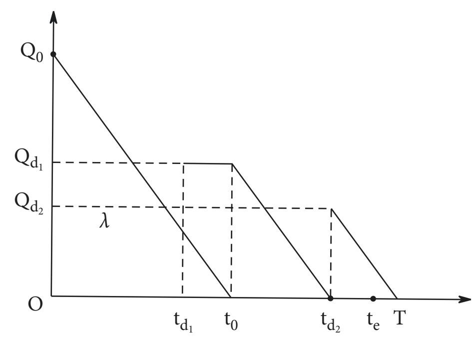

Figure 2: Second emergency order before t_e.

disruption, the retailer may place one or two emergency orders during the supply disruption period, see Figures 1 and 2. Certainly, when the event is ended, then the ordering policy would turn into the classical economic ordering quantity policy when the emergency order is depleted. In order to maximize the retailer's expected profits, the retailer should determine the optimal timing and volume of emergency replenishment. To do this, we need the following notation and associated assumptions in Table 1.

For this model, we have the following assumptions:

(1) The demand is stable throughout the planning period, and no shortage is allowed;

(2) The leading time of each emergency order is zero;

(3) At most two emergency orders are made during the supply disruption period;

(4) The ending time of the supply interruption follows the uniform distribution with probability density function $\phi(t_e) = 2(t_e - t_0)/(T - t_0)^2$ for $t_e \in [t_0, T]$;

(5) The retailer's purchasing price of the items linearly increases with time during the event period, i.e., the purchasing price is $C(t) = c_0 + \tau t$ where $c_0 > c$, $\tau > 0$ and $t \in [0, t_e]$.

3. Mathematical Formulation

For a related inventory system, due to the demand rate of $t \in [0, t_e]\lambda$, the retailer's remaining inventory Q_0 at the

Table 1: Notations.

Symbol	Description
K	Fixed order cost
λ	Demand rate
h	Holding cost per unit time per unit
b	Selling price
c	Normal purchase price
Q_0	Remaining inventory when supply disruption occurs
t_0	The time when remaining inventory Q_0 is depleted
t_e	Ending time of supply interruption
T	The deadline of ending time of supply interruption
t_d	The emergency ordering time under one-emergency-order policy
Q_d	The emergency ordering quantity under one-emergency-order policy
t_{d1}	The first emergency ordering time under two-emergency-order policy
t_{d2}	The second emergency ordering time
Q_{d1}	The first emergency ordering quantity under two-emergency-order policy
Q_{d2}	The second emergency ordering quantity under two-emergency-order policy
$*$	Indicates the optimal value

beginning will be exhausted at $t_0 = Q_0/\lambda$. As a remedy to the supply disruption, the retailer would make emergency orders during the period. Hence, we break the discussion into two cases.

Case 1. The retailer makes one emergency order.

In this case, since the shortage is not allowed and the deadline time of supply disruption is T, and the retailer should place a rush order at or before t_0 with quantity $Q_d = \lambda T - Q_0$; see Figure 1. Because the end time of this event is stochastic in $[t_0, T]$, then the retailer's inventory profit in $[0, T]$ is as follows:

$$f_1(t_d) = (b-c)Q_0 + (b - C(t_d))Q_d - \frac{hQ_0^2}{2\lambda} - K - hQ_d\left(\frac{Q_0}{\lambda} - t_d\right) - \frac{h(\lambda(T - Q_0/\lambda))^2}{2\lambda}. \tag{1}$$

where the first two terms are gross profits, the third item is the holding cost of the remaining inventory Q_0, and the remaining three items are the inventory cost of emergency replenishment [26].

For this emergency ordering policy, as the recovery time of the events obeys uniform distribution, that is, the probability density function $\phi(t_e) = 2(t_e - t_0)/(T - t_0)^2$ for $t_e \in [t_0, T]$, so the retailer's expected inventory profits in $[0, T]$ are as follows:

$$\begin{aligned} F_1(t_d) &= \int_{t_0}^{T} f_1(t_d)\phi(t_e)\mathrm{d}t_e \\ &= (b-c)Q_0 + (b - c_0 - \tau t_d)(\lambda T - Q_0) \\ &\quad - \frac{hQ_0^2}{2\lambda} - K - h(\lambda T - Q_0)\left(\frac{Q_0}{\lambda} - t_d\right) - \frac{h(\lambda T - Q_0)^2}{2\lambda}. \end{aligned} \tag{2}$$

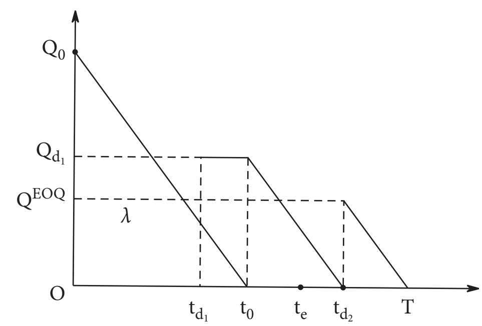

Figure 3: Second emergency order after t_e.

Case 2. The retailer makes two emergency orders.

For this strategy, if the supply interruption ending time t_e ends after t_{d2}, i.e., $t_{d2} < t_e$, and the retailer would make the first emergency replenishment at t_{d1} with size Q_{d1} and make the second emergency order at t_{d2} with size Q_{d2}, see Figure 2. According to the assumptions imposed on the model, it holds that $Q_{d1} + Q_{d2} = \lambda T - Q_0$ and $t_0 + Q_{d1}/\lambda = t_{d2}$. Thus, the retailer's inventory profit in $[0, T]$ for this case is as follows:

$$\begin{aligned} f_{21}(t_{d1}, Q_{d1}) &= (b-c)Q_0 + (b - C(t_{d1}))Q_{d1} \\ &\quad + \left(b - C\left(t_0 + \frac{Q_{d1}}{\lambda}\right)\right)(\lambda T - Q_0 - Q_{d1}) \\ &\quad - \frac{hQ_0^2}{2\lambda} - 2K - hQ_{d1}(t_0 - t_{d1}) - \frac{hQ_{d1}^2}{2\lambda} \\ &\quad - \frac{h(\lambda T - Q_0 - Q_{d1})^2}{2\lambda}, \end{aligned} \tag{3}$$

where the first three terms are the gross profits, the fourth item is the holding cost of remaining inventory Q_0, and remaining items are inventory cost of emergency replenishment.

For the emergency ordering policy, if the supply disruption is ended after t_{d1} but before t_{d2}, i.e., $t_{d1} < t_e < t_{d2}$, then one emergency order suffices and the inventory system returns to the classical EOQ model when the emergency order is depleted; see Figure 3. Thus, the retailer's inventory profit in $[0, T]$ for this case is as follows:

$$\begin{aligned} f_{22}(t_{d1}, Q_{d1}) &= (b-c)Q_0 + (b - C(t_{d1}))Q_{d1} \\ &\quad + (b-c)\lambda\left(T - t_0 - \frac{Q_{d1}}{\lambda}\right) - K \\ &\quad - \frac{hQ_0^2}{2\lambda} - hQ_{d1}(t_0 - t_{d1}) - \frac{hQ_{d1}^2}{2\lambda} \\ &\quad - \sqrt{2\lambda K h}\,\frac{T - t_0 - Q_{d1}/\lambda}{\lambda}. \end{aligned} \tag{4}$$

For this emergency ordering policy, as the recovery time of the events obeys uniform distribution, that is, the probability density function $\phi(t_e) = 2(t_e - t_0)/(T - t_0)^2$ for $t_e \in [t_0, T]$, so the retailer's expected inventory profit in $[0, T]$ is as follows:

$$
\begin{aligned}
F_2(t_{d1}, Q_{d1}) &= \int_{t_0}^{t_{d2}} f_{22}(t_{d1}, Q_{d1})\phi(t_e)\mathrm{d}t_e + \int_{t_{d2}}^{T} f_{21}(t_{d1}, Q_{d1})\phi(t_e)\mathrm{d}t_e \\
&= \left[(b-c)Q_0 + (b - c_0 - \tau t_{d1})Q_{d1} - \frac{hQ_0^2}{2\lambda} - hQ_{d1}(t_0 - t_{d1}) - \frac{hQ_{d1}^2}{2\lambda}\right] \\
&\quad + \frac{Q_{d1}^2}{\lambda^2 (T - t_0)^2}\left[(b-c)\lambda\left(T - t_0 - \frac{Q_{d1}}{\lambda}\right) - K - \sqrt{2\lambda K h}\ \frac{(T - t_0 - Q_{d1}/\lambda)}{\lambda}\right] \\
&\quad + \frac{(\lambda T - Q_0 - Q_{d1})(\lambda T - Q_0 + Q_{d1})}{\lambda^2 (T - t_0)^2}\left[-2K - \frac{h(\lambda T - Q_0 - Q_{d1})^2}{2\lambda}\right. \\
&\quad \left. + \left(b - c_0 - \tau\left(t_0 + \frac{Q_{d1}}{\lambda}\right)\right)(\lambda T - Q_0 - Q_{d1})\right].
\end{aligned} \tag{5}
$$

According to the above discussion, we can obtain the following optimization model of the concerned problem:

$$
\begin{aligned}
&\max\{F_1(t_d), F_2(t_{d1}, Q_{d1})\}, \\
&\text{s.t.}\, 0 \le t_d \le t_0, \\
&0 \le t_{d1} \le t_0, \\
&t_0 < t_{d2} < T, \\
&Q_{d1} + Q_{d2} = \lambda T - Q_0, \\
&0 \le Q_{d1} \le \lambda T - Q_0.
\end{aligned} \tag{6}
$$

In the next section, we will find the closed-form solution for the model and provide the retailer with the optimal emergency ordering policy.

4. Model Solution

For optimization problem (1), from the formation of the objective function, we know that the problem can be divided into the following two optimization problems:

$$
\begin{aligned}
&\max F_1(t_d) \\
&s.t. \quad 0 \le t_d \le t_0
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&\max F_2(t_{d1}, Q_{d1}), \\
&\text{s.t.}\, 0 \le t_{d1} \le t_0, \\
&t_0 < t_{d2} < T, \\
&Q_{d1} + Q_{d2} = \lambda T - Q_0, \\
&0 \le Q_{d1} \le \lambda T - Q_0,
\end{aligned} \tag{8}
$$

which, respectively, correspond to the ordering policy with one emergency order and that with two emergency orders.

For the ordering policy with one emergency order, for optimization model (2), we have the following conclusions.

Theorem 1. *For the emergency ordering policy with one emergency order, the optimal emergency ordering time is $t_d^* = 0$ or $t_d^* = t_0$.*

Proof. From the assumptions on the model and the discussion in Section 3, if one emergency order is made between the event period, then the retailer's expected inventory profits are as follows:

$$
\begin{aligned}
F_1(t_d) &= (b-c)Q_0 + (b - c_0 - \tau t_d)(\lambda T - Q_0) - \frac{hQ_0^2}{2\lambda} \\
&\quad - K - h(\lambda T - Q_0)\left(\frac{Q_0}{\lambda} - t_d\right) - \frac{h(\lambda T - Q_0)^2}{2\lambda}.
\end{aligned} \tag{9}
$$

which is a linear function with respect to t_d. Thus, the maximum of the profit is reached at the end points of interval $[0, t_0]$. That is, if $(h - \tau)(\lambda T - Q_0) > 0$, the objective function is monotonically increasing with respect to t_d in $[0, t_0]$, and the maximum of function is reached at t_0; otherwise, the maximum of function $F_1(t_d)$ in $[0, t_0]$ is reached at 0.

For the ordering policy with two emergency orders, for optimization model (3), we have the following conclusions. □

Theorem 2. *For the emergency ordering policy with two emergency orders, the first emergency order is either placed at $t_{d1} = 0$ with quantity*

$$
Q_{d1}^* = \begin{cases}
-\dfrac{p}{s}, & \text{if } A = B = 0, -\dfrac{p}{s} > 0, F_2\left(0, -\dfrac{p}{s}\right) > \max\left(F_2(0,0), F_2(0, \lambda T - Q_0)\right); \\
Q_{d1}, & \text{if } \Delta > 0, 0 < Q_{d1} \le \lambda T - Q_0, F_2(0,0) < F_2(0, Q_{d1}); \\
Q_{12}, & \text{if } \Delta < 0, 0 < Q_{12} \le \lambda T - Q_0, F_2(0, Q_{12}) > \max\left(F_2(0,0), F_2(0, \lambda T - Q_0)\right).
\end{cases} \tag{10}
$$

and the second emergency order is placed at $t_{d2} = t_0 + Q_{d1}^*/\lambda$ *with quantity* $Q_{d2} = \lambda T - Q_0 - Q_{d1}^*$, *or the first emergency order is placed at* $t_{d1} = t_0$ *with quantity.*

$$Q_{d1}^* = \begin{cases} -\dfrac{p}{s}, & \text{if } A = B = 0, -\dfrac{p}{s} > 0, F_2\left(t_0, -\dfrac{p}{s}\right) > \max\left(F_2(t_0, 0), F_2(t_0, \lambda T - Q_0)\right); \\ Q_{d1}, & \text{if } \Delta > 0, 0 < Q_{d1} \le \lambda T - Q_0, F_2(t_0, 0) < F_2(t_0, Q_{d1}); \\ Q_{12}, & \text{if } \Delta < 0, 0 < Q_{12} \le \lambda T - Q_0, F_2(t_0, Q_{12}) > \max\left(F_2(t_0, 0), F_2(t_0, \lambda T - Q_0)\right). \end{cases} \tag{11}$$

Moreover, the second emergency order is placed at $t_{d2} = t_0 + Q_{d1}^*/\lambda$ *with quantity* $Q_{d2} = \lambda T - Q_0 - Q_{d1}^*$. *Here,* $Q_{d1} = -s - (\sqrt[3]{Y_1} + \sqrt[3]{Y_2})/3r$, $Q_{12} = 1/3r\,(-s + \sqrt{A}\,(\cos\theta/3 - \sqrt{3}\,\sin\theta/3))$,

$$\begin{gathered} \begin{cases} r = \dfrac{2h}{\lambda^3 (T - t_0)^2}, s = -\dfrac{3\left((b - c_0)\lambda - \sqrt{2\lambda K h} - h(\lambda T - Q_0)\right)}{\lambda^3 (T - t_0)^2}, \\ p = \dfrac{4K}{\lambda^2 (T - t_0)^2} + \dfrac{2\left((b - c_0)\lambda - \sqrt{2\lambda K h}\right)}{\lambda^2 (T - t_0)} + \dfrac{2\tau}{\lambda}, q = \dfrac{h(\lambda T - Q_0)^3}{\lambda^3}(T - t_0)^2 + \dfrac{2\tau Q_0}{\lambda}, \end{cases} \\ A = s^2 - 3rp, \\ B = sp - 9rq, \\ C = p^2 - 3sq, \\ \Delta = B^2 - 4AC, \\ \theta = \arccos\left(\frac{2As - 3rB}{2\sqrt{A^3}}\right), \\ Y_1 = As + \frac{3r}{2}\left(-B + \sqrt{B^2 - 4AC}\right), \\ Y_2 = As + \frac{3r}{2}\left(-B - \sqrt{B^2 - 4AC}\right). \end{gathered} \tag{12}$$

To show the conclusion, we need the Shengjin formula for to univariate cubic equation [27].

Lemma 1. *For cubic equation* $a_3x^3 + a_2x^2 + a_1x + a_0 = 0$ *with* $a_3 > 0$, *set* $A_1 = a_2^2 - 3a_3a_1$, $A_2 = a_2a_1 - 9a_3a_0$, $A_3 = a_1^2 - 3a_2a_0$. *If* $A_1 = A_2 = 0$, *the equation has a triple real root:* $x_1 = x_2 = x_3 = -a_1/a_2$; *if* $\Delta = A_2^2 - 4A_1A_3 > 0$, *the equation has one real root:* $x = -a_2 - (\sqrt[3]{X_1} + \sqrt[3]{X_2})/3a_3$, *where* $X_1 = A_1a_2 + 3a_3/2\,(-A_2 + \sqrt{A_2^2 - 4A_1A_3})$, $X_2 = A_1a_2 + 3a_3/2\,(-A_2 - \sqrt{A_2^2 - 4A_1A_3})$; *if* $\Delta = A_2^2 - 4A_1A_3 = 0$, *it has two real roots* $x_1 = -a_2/a_3 + A_2/A_1$, $x_2 = x_3 = -A_2/2A_1$; *and if* $\Delta = A_2^2 - 4A_1A_3 < 0$, *it has three real roots* $x_1 = -1/3a_3\,(a_2 + 2\sqrt{A_1}\,\cos\theta/3)$, $x_2 = 1/3a_3\,(-a_2 + \sqrt{A_1}\,(\cos\theta/3 + \sqrt{3}\,\sin\theta/3))$, $x_3 = 1/3a_3\,(-a_2 + \sqrt{A_1}\,(\cos\theta/3 - \sqrt{3}\,\sin\theta/3))$, *where* $\theta = \arccos(2A_1a_2 - 3a_3A_2/2\sqrt{A_1^3})$.

Proof. From the assumptions on the model and the discussion above, then the retailer's expected inventory profits in $[0, T]$ are as follows:

$$\begin{aligned}
F_2(t_{d1},Q_{d1}) &= \int_{t_0}^{t_{d2}} f_{22}(t_{d1},Q_{d1})\phi(t_e)\mathrm{d}t_e + \int_{t_{d2}}^{T} f_{21}(t_{d1},Q_{d1})\phi(t_e)\mathrm{d}t_e \\
&= \left[(b-c)Q_0 + (b-c_0-\tau t_{d1})Q_{d1} - \frac{hQ_0^2}{2\lambda} - hQ_{d1}(t_0-t_{d1}) - \frac{hQ_{d1}^2}{2\lambda}\right] \\
&\quad + \frac{Q_{d1}^2}{\lambda^2(T-t_0)^2}\left[(b-c)\lambda\left(T-t_0-\frac{Q_{d1}}{\lambda}\right) - K - \sqrt{2\lambda Kh}\ \frac{(T-t_0-Q_{d1}/\lambda)}{\lambda}\right] \\
&\quad + \frac{(\lambda T-Q_0-Q_{d1})(\lambda T-Q_0+Q_{d1})}{\lambda^2(T-t_0)^2}\left[-2K - \frac{h(\lambda T-Q_0-Q_{d1})^2}{2\lambda} + \left(b-c_0-\tau\left(t_0+\frac{Q_{d1}}{\lambda}\right)\right)(\lambda T-Q_0-Q_{d1})\right].
\end{aligned} \tag{13}$$

Then, the optimization problem (3) can be written as follows:

$$\begin{aligned}
&\max F_2(t_{d1},Q_{d1}),\\
&\text{s.t.}\, 0 \le t_{d1} \le t_0,\\
&0 \le Q_{d1} \le \lambda T - Q_0.
\end{aligned} \tag{14}$$

To solve the problem, we first consider the following optimization problem with respect to t_{d1}

$$\begin{aligned}
&\max F_2(t_{d1},Q_{d1}),\\
&\text{s.t.}\, 0 \le t_{d1} \le t_0,\\
&t_0 < t_{d2} < T.
\end{aligned} \tag{15}$$

And then solve the optimization problem with respect to Q_{d1}

$$\begin{aligned}
&\max F_2(t_{d1},Q_{d1}),\\
&\text{s.t.}\, 0 \le Q_{d1} \le \lambda T - Q_0,\\
&Q_{d1} + Q_{d2} = \lambda T - Q_0.
\end{aligned} \tag{16}$$

For problem (4), since the objective is a linear function with respect to t_{d1}, we discuss it in two cases.

Case I. If $\partial F_2(t_{d1},Q_{d1})/\partial t_{d1} > 0$, then the objective function $F_2(t_{d1},Q_{d1})$ is monotonically increasing with respect to t_{d1} in $[0,t_0]$, so the maximum of the objective function in $[0,t_0]$ is reached at t_0.

Case II. If $\partial F_2(t_{d1},Q_{d1})/\partial t_{d1} < 0$, then the objective function $F_2(t_{d1},Q_{d1})$ is monotonically decreasing in $[0,t_0]$, and the maximum of the objective function is reached at 0.

Now, consider optimization problem (5). Computing the derivative of the objective function yields the following:

$$\begin{aligned}
\frac{\partial F_2(t_{d1},Q_{d1})}{\partial Q_{d1}} &= \frac{2h}{\lambda^3(T-t_0)^2}Q_{d1}^3 - \frac{3\left((b-c_0)\lambda - \sqrt{2\lambda Kh} - h(\lambda T-Q_0)\right)}{\lambda^3(T-t_0)^2}Q_{d1}^2 \\
&\quad + \left(\frac{4K}{\lambda^2(T-t_0)^2} + \frac{2\left((b-c_0)\lambda - \sqrt{2\lambda Kh}\right)}{\lambda^2(T-t_0)} + \frac{2\tau}{\lambda}\right)Q_{d1} + \frac{h(\lambda T-Q_0)^3}{\lambda^3}(T-t_0)^2 + \frac{2\tau Q_0}{\lambda} \\
&= rQ_{d1}^3 - sQ_{d1}^2 + pQ_{d1} + q,
\end{aligned} \tag{17}$$

which is a cubic function of Q_{d1}. In order to solve the root of the univariate cubic equation $\partial F_2(t_{d1},Q_{d1})/\partial Q_{d1} = 0$, that is, the derivative zero of the objective function, we discuss the following cases.

Case 1. $A = B = 0$. In this case, the equation $\partial F_2(t_{d1}, Q_{d1})/\partial Q_{d1} = 0$ has a triple real root $Q_{d1} = -p/s$. Further, if $r > 0$ and $-p/s < 0$, then function $F_2(t_{d1},Q_{d1})$ increases with the increase of Q_{d1} in $[0,+\infty)$, and the maximum of the objective function $F_2(t_{d1},Q_{d1})$ is reached at $\lambda T - Q_0$; if $r > 0$ and $-p/s > 0$, then the objective function $F_2(t_{d1},Q_{d1})$ decreases with the decrease of Q_{d1} in $(-\infty,-p/s)$ and increases with the increase of Q_{d1} in $(-p/s,+\infty)$, and the maximum of the objective function $F_2(t_{d1},Q_{d1})$ can be obtained at 0 or $\lambda T - Q_0$.

Case 2. $\Delta = B^2 - 4AC > 0$. In this case, equation $\partial F_2(t_{d1}, Q_{d1})/\partial Q_{d1} = 0$ has only one real root $Q_{d1} = -s-(\sqrt[3]{Y_1} + \sqrt[3]{Y_2})/3r$, where $Y_1 = As + 3r/2(-B +$

$\sqrt{B^2-4AC}$) and $Y_2 = As + 3r/2(-B - \sqrt{B^2-4AC})$. Further, if $Q_{d1} < 0$, then the objective function $F_2(t_{d1}, Q_{d1})$ decreases with the decrease of Q_{d1} in $(-\infty, Q_{d1}]$ and increases with the increase of Q_{d1} in $[Q_{d1}, +\infty)$. Hence, the maximum of the objective function is reached at $\lambda T - Q_0$; if $Q_{d1} > 0$, then $F_2(t_{d1}, Q_{d1})$ decreases with the decrease of Q_{d1} in $[0, Q_{d1})$ and increases with the increase of Q_{d1} in $[Q_{d1}, +\infty)$, the maximum of the objective function is be reached at 0 or $-s - (\sqrt[3]{Y_1} + \sqrt[3]{Y_2})/3r$.

Case 3. $\Delta = B^2 - 4AC = 0$. In this case, the equation $\partial F_2(t_{d1}, Q_{d1})/\partial Q_{d1} = 0$ has two real roots $Q_{11} = -s/r + B/A$, $Q_{12} = Q_{13} = -B/2A$. If $r > 0$ and $Q_{11} < 0$, the objective function $F_2(t_{d1}, Q_{d1})$ decreases with the decrease of Q_{d1} in $(-\infty, Q_{11}]$ and increases with the increase of Q_{d1} in $[Q_{11}, +\infty)$. Hence, the maximum of the objective function can be reached at $\lambda T - Q_0$; if $r > 0$ and $Q_{11} > 0$, then $F_2(t_{d1}, Q_{d1})$ decreases with the decrease of Q_{d1} in $[0, Q_{11}]$ and increases with the increase of Q_{d1} in $[Q_{11}, +\infty)$. Hence, the maximum of $F_2(t_{d1}, Q_{d1})$ can be obtained at 0 or $\lambda T - Q_0$.

Case 4. $\Delta = B^2 - 4AC < 0$. In this case, the equation $\partial F_2(t_{d1}, Q_{d1})/\partial Q_{d1} = 0$ has three real roots $Q_{11} = -1/3r(s + 2\sqrt{A}\cos\theta/3)$, $Q_{12} = 1/3r(-s + \sqrt{A}(\cos\theta/3 - \sqrt{3}\sin\theta/3))$, $Q_{13} = 1/3r(-s + \sqrt{A}(\cos\theta/3 + \sqrt{3}\sin\theta/3))$, where $\theta = \arccos(2A - 3rB/2\sqrt{A^3})$. If $Q_{12} < 0$, then function $F_2(t_{d1}, Q_{d1})$ decreases with the decrease of Q_{d1} in $(-\infty, Q_{11}]$ and $[Q_{12}, Q_{13}]$, and increases with the increase of Q_{d1} in $[Q_{11}, Q_{12}]$ and $[Q_{13}, +\infty)$. Thus, function $F_2(t_{d1}, Q_{d1})$ in $[0, +\infty]$ reaches the maximum at 0 or $\lambda T - Q_0$; if $Q_{12} > 0$, then function $F_2(t_{d1}, Q_{d1})$ decreases with the decrease of Q_{d1} in $(-\infty, Q_{11}]$ and $[Q_{12}, Q_{13}]$, and increases with the increase of Q_{d1} in $[Q_{11}, Q_{12}]$ and $[Q_{13}, +\infty)$. Hence, $F_2(t_{d1}, Q_{d1})$ in $[0, +\infty)$ reaches the maximum at Q_{12}.

Combining the discussions mentioned above, we can obtain the optimization problem of optimization problem (3) if the first emergency order is placed at $t_{d1} = 0$,

$$Q_{d1}^* = \begin{cases} \lambda T - Q_0, & \text{if } A = B = 0, -\dfrac{p}{s} < 0, F_2(0, \lambda T - Q_0) > F_2(0, 0); \\ 0, & \text{if } A = B = 0, -\dfrac{p}{s} > 0, F_2(0, 0) \geq F_2(0, \lambda T - Q_0); \\ \lambda T - Q_0, & \text{if } A = B = 0, -\dfrac{p}{s} > 0, F_2(0, 0) < F_2(0, \lambda T - Q_0); \\ -\dfrac{p}{s}, & \text{if } A = B = 0, -\dfrac{p}{s} > 0, F_2\left(0, -\dfrac{p}{s}\right) > \max(F_2(0, 0), F_2(0, \lambda T - Q_0)); \\ \lambda T - Q_0, & \text{if } \Delta > 0, Q_{d1} < 0, F_2(0, \lambda T - Q_0) > F_2(0, 0); \\ 0, & \text{if } \Delta > 0, 0 < Q_{d1} \leq \lambda T - Q_0, F_2(0, 0) \geq F_2(0, Q_{d1}); \\ Q_{d1}, & \text{if } \Delta > 0, 0 < Q_{d1} \leq \lambda T - Q_0, F_2(0, 0) < F_2(0, Q_{d1}); \\ \lambda T - Q_0, & \text{if } \Delta = 0, Q_{11} < 0, F_2(0, 0) < F_2(0, \lambda T - Q_0); \\ 0, & \text{if } \Delta = 0, 0 < Q_{11} \leq \lambda T - Q_0, F_2(0, 0) \geq F_2(0, \lambda T - Q_0); \\ \lambda T - Q_0, & \text{if } \Delta = 0, 0 < Q_{11} \leq \lambda T - Q_0, F_2(0, 0) < F_2(0, \lambda T - Q_0); \\ 0, & \text{if } \Delta < 0, Q_{12} < 0, F_2(0, 0) \geq F_2(0, \lambda T - Q_0); \\ \lambda T - Q_0, & \text{if } \Delta < 0, Q_{12} < 0, F_2(0, 0) < F_2(0, \lambda T - Q_0); \\ Q_{12}, & \text{if } \Delta < 0, 0 < Q_{12} \leq \lambda T - Q_0, F_2(0, Q_{12}) > \max(F_2(0, 0), F_2(0, \lambda T - Q_0)). \end{cases} \tag{18}$$

Or if the retailer's first emergency replenishment time is $t_{d1} = t_0$,

$$Q_{d1}^{*} = \begin{cases} \lambda T - Q_0, & \text{if } A = B = 0, -\frac{p}{s} < 0, F_2(t_0, \lambda T - Q_0) > F_2(t_0, 0); \\ 0, & \text{if } A = B = 0, -\frac{p}{s} > 0, F_2(t_0, 0) \geq F_2(t_0, \lambda T - Q_0); \\ \lambda T - Q_0, & \text{if } A = B = 0, -\frac{p}{s} > 0, F_2(t_0, 0) < F_2(t_0, \lambda T - Q_0); \\ -\frac{p}{s}, & \text{if } A = B = 0, -\frac{p}{s} > 0, F_2\left(t_0, -\frac{p}{s}\right) > \max\left(F_2(t_0, 0), F_2(t_0, \lambda T - Q_0)\right); \\ \lambda T - Q_0, & \text{if } \Delta > 0, Q_{d1} < 0, F_2(t_0, \lambda T - Q_0) > F_2(t_0, 0); \\ 0, & \text{if } \Delta > 0, 0 < Q_{d1} \leq \lambda T - Q_0, F_2(t_0, 0) \geq F_2(t_0, Q_{d1}); \\ Q_{d1}, & \text{if } \Delta > 0, 0 < Q_{d1} \leq \lambda T - Q_0, F_2(t_0, 0) < F_2(t_0, Q_{d1}); \\ \lambda T - Q_0, & \text{if } \Delta = 0, Q_{11} < 0, F_2(t_0, 0) < F_2(t_0, \lambda T - Q_0); \\ 0, & \text{if } \Delta = 0, 0 < Q_{11} \leq \lambda T - Q_0, F_2(t_0, 0) \geq F_2(t_0, \lambda T - Q_0); \\ \lambda T - Q_0, & \text{if } \Delta = 0, 0 < Q_{11} \leq \lambda T - Q_0, F_2(t_0, 0) < F_2(t_0, \lambda T - Q_0); \\ 0, & \text{if } \Delta < 0, Q_{12} < 0, F_2(t_0, 0) \geq F_2(t_0, \lambda T - Q_0); \\ \lambda T - Q_0, & \text{if } \Delta < 0, Q_{12} < 0, F_2(t_0, 0) < F_2(t_0, \lambda T - Q_0); \\ Q_{12}, & \text{if } \Delta < 0, 0 < Q_{12} \leq \lambda T - Q_0, F_2(t_0, Q_{12}) > \max\left(F_2(t_0, 0), F_2(t_0, \lambda T - Q_0)\right). \end{cases} \tag{19}$$

For the optimal solution to problem (3), if $Q_{d1} = 0$ or $Q_{d1} = \lambda T - Q_0$, then the retailer would place one emergency order; this does not satisfy the assumption of the model. Further, substituting $t_{d1} = 0$ or $t_{d1} = t_0$ into the expression of Q_{d1} yields the optimal order quantity of emergency order. The following optimal solutions are obtained under the condition of satisfying the constraints. □

5. Numerical Experiments

To verify the theoretical results given in the previous section, the following numerical experiments are carried out.

Example 1. Consider the inventory system with $c = 8$, $c_0 = 10$, $b = 48$, $Q_0 = 40$, $\tau = 3$, $\lambda = 6$, $T = 20$, $h = 2$, $K = 20$, and the retailer makes one emergency order.

By algorithm, when the retailer makes one emergency replenishment, the retailer's best emergency ordering time at the initial moment with size 80, then the retailer's expected inventory profits is 2220. If the retailer's best emergency ordering time is 6.67 with size 80, then the retailer's expected inventory profits is 1687. Therefore, the retailer's best rush order strategy is to place 80 rush orders at the initial moment. The detailed numerical results are listed in Table 2, in which we use strategies I and II to represent replenishment strategies at two different times.

Example 2. According to the inventory system considered in Example 1, we continue to consider the case where the retailer makes two emergency orders, leaving the other parameters unchanged.

By algorithm, when the retailer makes two emergency orders, and if the retailer makes the first emergency replenishment at the initial time with quantity 31, the second emergency replenishment time is 5.17 with quantity 49, then the retailer's expected inventory profits under the given strategy is 3211. If the first replenishment is at the emergency time of 6.67 with quantity 31, and make the second replenishment at 11.8 with quantity 49, then the retailer's expected inventory profits is 3773. Therefore, the retailer's best replenishment strategy in this case is that the first emergency replenishment time is $t_0 = 6.67$, the emergency replenishment quantity is 31, the second emergency replenishment time is 11.84, and the emergency replenishment quantity is 49. The detailed numerical results of this example are shown in Table 3, in which we use strategies I and II to represent replenishment strategies at two different times.

Table 2: Numerical results for Example 1.

Strategy	t_d^*	Q_d^*	Profits I	Profits II
I	0	80	2220	—
II	6.67	80	—	1687

Table 3: Numerical results for Example 2.

Strategy	t_{d1}^*	t_{d2}^*	Q_{d1}^*	Q_{d2}^*	Profits I	Profits II
I	0	5.17	31	49	3211	—
II	6.67	11.8	31	49	—	3773

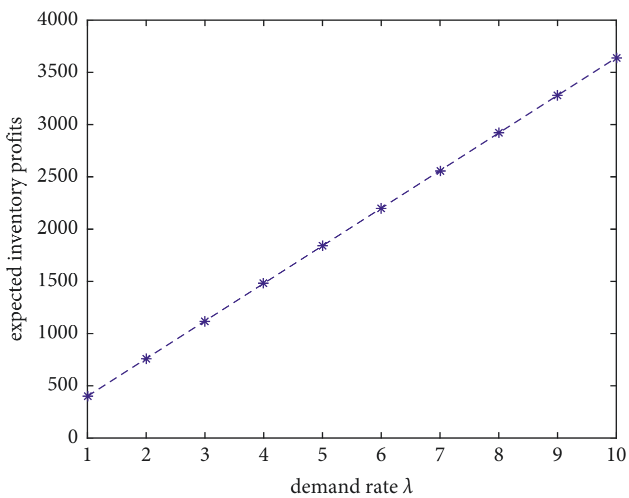

Figure 4: Effect of λ on emergency ordering time at 0 for retailer's expected inventory profits under one-emergency order.

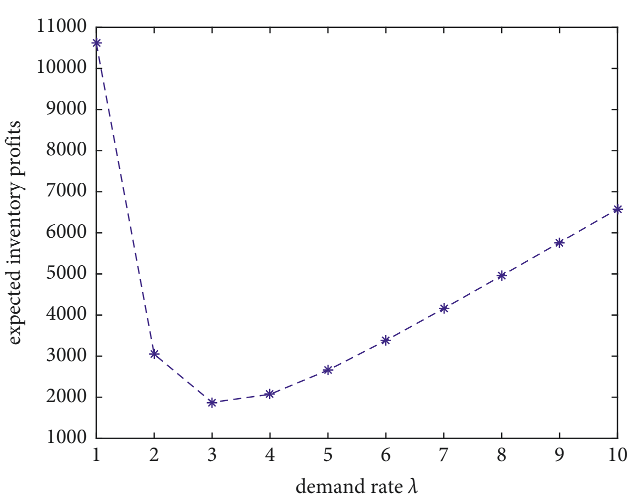

Figure 5: Effect of λ on emergency ordering time at 0 for retailer's expected inventory profits under two-emergency order.

By comparing Examples 1 with 2, we find that when the retailer chooses to make two emergency orders, their inventory profits are greater.

We conduct sensitivity analysis on the influence of relevant parameters on the model. First, the effect of the demand rate λ on retailers' expected inventory profits is

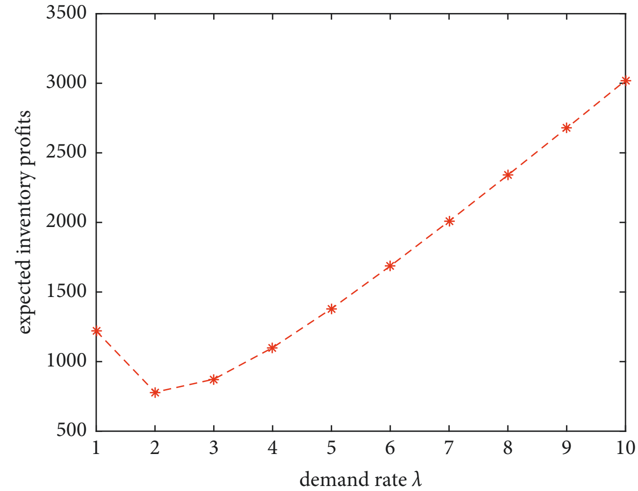

Figure 6: Effect of λ on emergency ordering time at t_0 for retailer's expected inventory profits under one-emergency order.

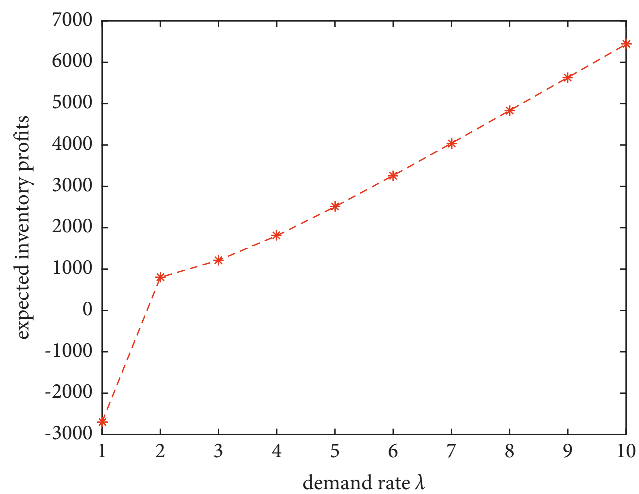

Figure 7: Effect of λ on emergency ordering time at t_0 for retailer's expected inventory profits under two-emergency order.

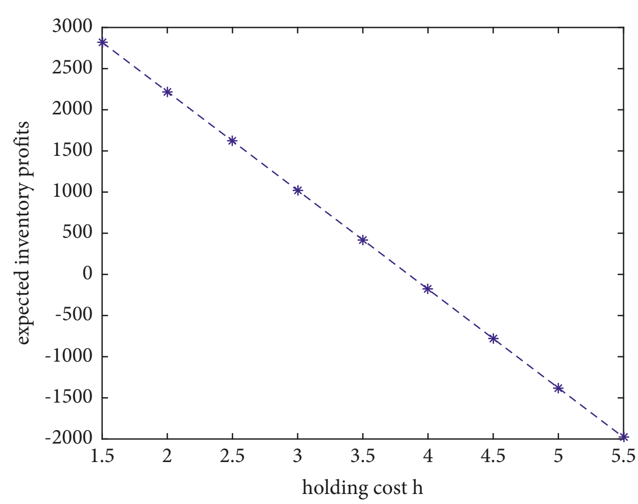

Figure 8: Effect of h on emergency ordering time at 0 for retailer's expected inventory profits under one-emergency order.

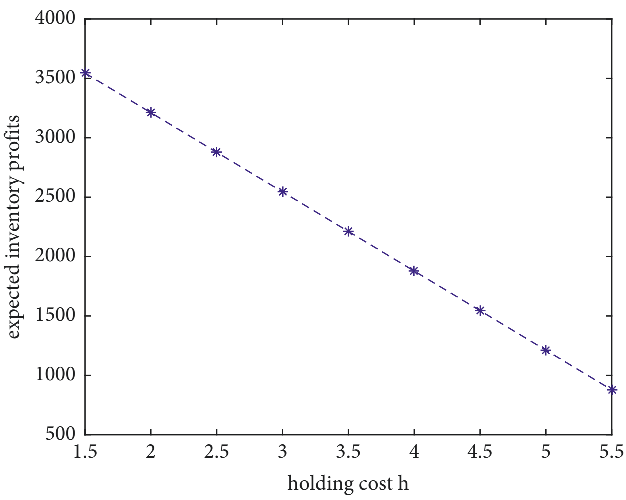

Figure 9: Effect of h on emergency ordering time at 0 for retailer's expected inventory profits under two-emergency order.

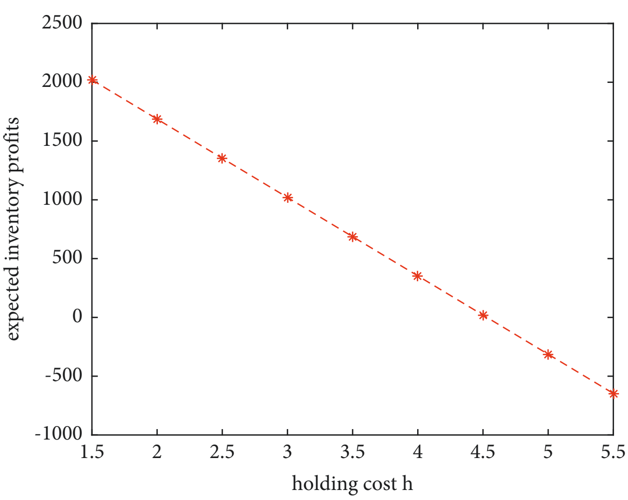

Figure 10: Effect of h on emergency ordering time at t_0 for retailer's expected inventory profits under one-emergency order.

analyzed. For the inventory system of Example 1 and Example 2, we let λ increase from 1 to 10, and other parameters remain unchanged. The numerical results are shown in Figures 4–7 from which we can see that compared with just making one emergency order, as the demand rate λ increases, the retailer chooses to make two emergency orders and place the first order at the initial time has a great impact on retailers' expected inventory profits.

Then, we analyze the effect of the holding cost h on retailer's expected inventory profits. For the emergency replenishment system given in Example 1 and Example 2, increase h from 1.5 to 5.5, leaving the other parameters unchanged. As can be seen from Figures 8–11, from which we can see that with the increase of the holding cost h has a great impact on the expected inventory profit of retailers who choose to make two emergency orders and place the first emergency order at t_0.

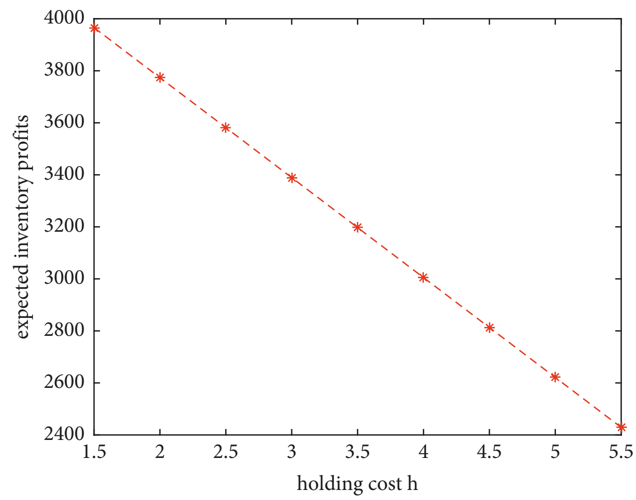

Figure 11: Effect of h on emergency ordering time at t_0 for retailer's expected inventory profits under two-emergency order.

According to the abovementioned numerical analysis, it can be seen that the changes of the demand rate λ and holding cost h have a great impact on the expected inventory profits of the retailer. Among them, the increase of the demand rate λ will have a positive impact on the retailer's profits, and the increase of holding cost h will have a negative impact on the retailer's profits, which shows that holding cost h and demand rate λ are the key factors for the retailer to choose emergency replenishment strategy. When supply interruption occurs, managers need to pay more attention to the effect of these factors on the whole inventory system.

6. Conclusions

This paper mainly studies the emergency ordering problems with the supply disruption whose ending time is random. Based on inventory profits maximization, we established an inventory model, and the optimal emergency procurement strategy of retailers was given through model analysis. Finally, the effectiveness of the optimal strategy was verified through numerical experiments. [28].

Certainly, this paper considers the emergency ordering problem of the retailer in the case of supply disruption. As a next step, we can expand on the fact that supply disruptions occur at random times, and taking into account the retailer's risk appetite, which could make the overall study more realistic.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Each author equally contributed to this paper and read and approved the final manuscript.

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