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Research Article

The Analytical Solutions of the Stochastic Fractional Kuramoto–Sivashinsky Equation by Using the Riccati Equation Method

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In this work, we consider the stochastic fractional-space Kuramoto–Sivashinsky equation using conformable derivative. The Riccati equation method is used to get the analytical solutions to the space-fractional stochastic Kuramoto–Sivashinsky equation. Because this equation has never been examined with space-fractional and multiplicative noise at the same time, we generalize some previous results. Moreover, we display how the multiplicative noise influences on the stability of obtained solutions of the space-fractional stochastic Kuramoto–Sivashinsky equation.

1. Introduction

Fractional differential equations (FDEs) have become the subject of many investigations due to their widespread occurrence in numerous applications in finance, chemistry, control theory, engineering, biology, physics, systems' identification, and signal processing. In the literature, fractional derivatives and integrals are discussed in numerous ways, for instance, the Caputo, Riesz, Grunwald–Letnikov, and Riemann–Liouville. The majority of them are differentiated by fractional integrals, and as a result, they obtain nonlocal properties from integrals. Heredity and nonlocality are well-known features of these ideas [1], which are important in many domains and are not precisely equal to old style Newton–Leibniz calculus. These derivatives no longer follow the Chain Rule, the Product Rule, or the Quotient Rule for derivative operations. Recently, a few authors introduced the concept of local fractal derivatives (LFD). Kolwankar and Gangal [2] developed a kind of LFD by allowing an upper or lower limit for the Riemann–Liouville derivative technique. While, Khalil et al. [3] proposed a new fundamental idea and kind of fractal derivative LFD called conformable local fractal derivative (CFD), whose major attributes are similar to Newton derivative and can be used to solve local fractal-type differential equations more efficiently.

Many papers have been written about some features of fractional differential equations, such as methods for explicit and numerical solutions, the existence and uniqueness of solutions, and solution stability [4–6]. One of the most significant topics in FDEs is the search for the exact solutions of FDEs. Therefore, many efficient and powerful approaches have been presented to get the exact solutions of FDEs, such as the fractional Riccati subequation method, the Adomian decomposition method, the (G'/G)-expansion method, the $\exp(-\phi(\varsigma))$-expansion method, the tanh-sech method, the modified Kudryashov method, the fractional modified trial equation method, the Jacobi elliptic function method, and the sine-cosine method [7–29].

On the contrary, stochastic differential equations (SDEs) are very important for modeling many physical phenomena in different areas including environmental sciences, physics, oceanography, engineering, and biology [30–32]. In particular, SDEs are used to explain all dynamical systems in which quantum effects are either ignored or can be considered as perturbations. They can be thought of as an extension of dynamical systems theory to model with noise. This is a significant extension since real systems cannot be entirely isolated from their environments, and therefore, this external stochastic effect is always present.

The importance of investigating FPDE models with stochastic impacts appears to be greater. To the best of my knowledge, very little study has been done to find exact solutions to fractional SPDEs, for example, [33–37]. Therefore, it is very important to consider FDEs with some random force. Here, we consider the following the space-fractional stochastic Kuramoto–Sivashinsky equation (SFSKSE) in the sense of Atangana's conformable derivative:

$$d\psi + \left[\psi D_x^{\alpha}\psi + pD_x^{2\alpha}\psi + rD_x^{4\alpha}\psi\right]dt = \rho\psi\, d\beta, \tag{1}$$

where $\psi(x,t)$ is a real stochastic function, D_x^{α} is a conformable derivatives [3] of order α, p and r are nonzero real constants, $\beta(t)$ is the Brownian motion and it relies only on t, and ρ is a noise intensity.

The Kuramoto–Sivashinsky (KS) (1), with $\rho = 0$ and $\alpha = 1$, can be used to demonstrate long waves at the interface between two viscous fluids and unstable drift waves in plasmas, as well as Benard convection in an elongated box in one space dimension. Also, it is used to control surface roughness in sputtering-grown thin solid films, amorphous film generation, and step dynamics in epitaxy. Many authors have been obtained the exact solutions of KS via various methods such as the truncated expansion method [38], the modified polynomial expansion method [39–42], the tanh method, and the extended tanh method [43], the (G'/G)-expansion [44], the perturbation method [45], the tanh-coth method [46, 47], the homotopy analysis method [48], and the Painlevé expansions methods [49]. The exact solutions of SFSKSE (1) have been discussed in [50, 51].

Our aim of this study is to employ the Riccati equation method to establish the analytical solutions of SFSKSE (1). The obtained solutions given here generalize previous research, such as those discussed in [43, 47]. The influence of multiplicative noise on these solutions is also explored. This is the first publication that we are aware of which has found the exact solution to SFSKSE (1) in the sense of Atangana's conformable derivative.

This study will be formatted as follows. We give the definition of conformable fractal derivative (CFD) and Brownian motion in Section 2. In Section 3, the wave equation for SFSKSE (1) is attained, while in Section 4, we use the Riccati equation method to get the analytical stochastic solutions of SFSKSE (1). In Section 5, we show several graphs to observe the influence of the multiplicative noise on the SFSKSE solutions. Finally, we give the conclusions of this study.

2. Preliminaries

In this section, we state the definition of the CFD and Brownian motion. Also, we state some features of the CFD. First, we define the CD as follows.

Definition (cf. [3]). Define the CFD of $\phi\colon (0,\infty) \longrightarrow \mathbb{R}$ of order $\alpha \in (0,1]$ as

$$\mathscr{D}_x^{\alpha}\phi(x) = \lim_{\kappa\longrightarrow 0}\frac{\phi\left(x+\kappa x^{1-\alpha}\right)-\phi(x)}{\kappa}. \tag{2}$$

Theorem 1. *Let $\phi, g\colon (0,\infty) \longrightarrow \mathbb{R}$ be differentiable and, also, α be differentiable functions; then, the next rule holds:*

$$\mathscr{D}_x^{\alpha}(\phi^{\circ} g)(x) = x^{1-\alpha}g'(x)\phi'(g(x)). \tag{3}$$

In the following some features of the CFD,

(1) $\mathscr{D}_x^{\alpha}[c_1\phi(x) + c_2 g(x)] = c_1\mathscr{D}_x^{\alpha}\phi(x) + c_2\mathscr{D}_x^{\alpha}g(x)$, c_1, $c_2 \in \mathbb{R}$

(2) $\mathscr{D}_x^{\alpha}[C] = 0$, *$C$ is a constant*

(3) $\mathscr{D}_x^{\alpha}[x^{\gamma}] = \gamma x^{\gamma-\alpha}$, $\gamma \in \mathbb{R}$

(4) $\mathscr{D}_x^{\alpha}g(x) = x^{1-\alpha}dg/dx$

In the next definition, we define Brownian motion $\beta(t)$.

Definition 1. Stochastic process $\{\beta(t)\}_{t\geq 0}$ is called a Brownian motion if it satisfies

(1) $\beta(0) = 0$

(2) $\beta(t)$, $t \geq 0$, is continuous function of t

(3) $\beta(t) - \beta(s)$ is independent for $s < t$

(4) $\beta(t) - \beta(s)$ has a Gaussian distribution with mean 0 and variance $t - s$

3. Wave Equation for SFSKSE

To get the wave equation of SFSKSE (1), we utilize the following wave transformation:

$$\psi(x,t) = \phi(\eta)e^{\left(\rho\beta(t)-1/2\rho^2 t\right)},\ \eta = \frac{\lambda}{\alpha}x^{\alpha} + ct, \tag{4}$$

where ϕ is the deterministic function. Differentiating (4) with regards to t and x, we obtain

$$\begin{aligned} d\psi &= \left(c\phi' + \frac{1}{2}\rho^2\phi - \frac{1}{2}\rho^2\phi\right)e^{\left(\rho\beta(t)-1/2\rho^2 t\right)}dt \\ &\quad + \rho\phi e^{\left(\rho\beta(t)-1/2\rho^2 t\right)}d\beta\,, \\ D_x^{\alpha}\psi &= \lambda\phi' e^{\left[\rho\beta(t)-\rho^2 t\right]},\ D_x^{2\alpha}\psi = \lambda^2\phi'' e^{\left[\rho\beta(t)-\rho^2 t\right]}, \\ D_x^{3\alpha}\psi &= \lambda\phi''' e^{\left(\rho\beta(t)-1/2\rho^2 t\right)},\ \mathrm{D}_{\mathrm{x}}^{4\alpha}\psi = \lambda^4\phi''' \mathrm{e}^{\left(\rho\beta(t)-1/2\rho^2 t\right)}, \end{aligned} \tag{5}$$

where $+1/2\rho^2\phi$ is the Itô correction term. Inserting (4) into (1) and using (5), we have

$$\widetilde{c}\phi' + \phi\phi' e^{\left(\rho\beta(t)-1/2\rho^2 t\right)} + \widetilde{p}\phi'' + \widetilde{r}\phi''' = 0, \tag{6}$$

where we put $\widetilde{c} = c/\lambda$, $\widetilde{p} = \lambda p$, and $\widetilde{r} = \lambda^3 r$. Taking expectation on both sides, we have

$$\widetilde{c}\phi' + \phi\phi' e^{-1/2\rho^2 t}\mathbb{E}\left(e^{\rho\beta(t)}\right) + \widetilde{p}\phi'' + \widetilde{r}\phi''' = 0, \tag{7}$$

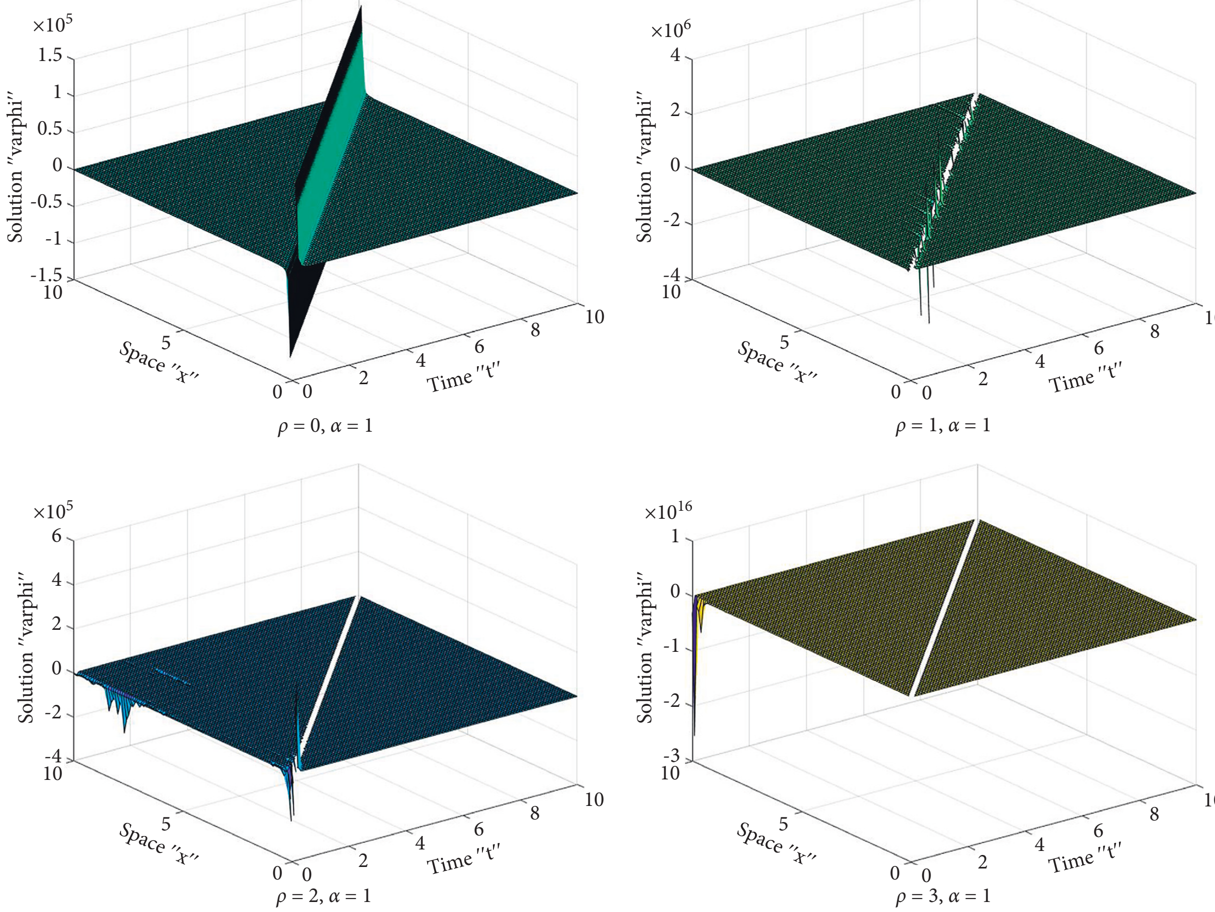

Figure 1: The plots of (27) with $\alpha = 1$.

where ϕ is the deterministic function. We note that $\mathbb{E}(e^{\rho\beta(t)}) = e^{\rho^2/2t}$, where $\beta(t)$ is Normal standard distribution and ρ is a real constant. Now, (7) has the form

$$-\tilde{c}\phi' + \tilde{r}\phi\phi' + \tilde{p}\phi'' + \tilde{q}\phi''' = 0. \quad (8)$$

Integrating (8) and putting the constant of integration equal zero, we obtain

$$\tilde{r}\phi''' + \tilde{p}\phi' + \frac{1}{2}\phi^2 + \tilde{c}\phi = 0. \quad (9)$$

4. The Analytical Solutions

To find the solutions of (9), we apply the Riccati equation method. Consequently, we acquire the analytical solutions of SFSKSE (1). Assume that the solution of SFSKSE (9) is

$$\phi = \sum_{\ell=0}^{N} a_\ell \chi^\ell, \quad (10)$$

where χ solves

$$\chi' = \chi^2 + b, \quad (11)$$

where b is a constant will be determine later. Balancing ϕ^2 with ϕ''' in (9), we can calculate the parameter N as follows:

$$2N = N + 3. \quad (12)$$

Hence,

$$N = 3. \quad (13)$$

From (13), we can rewrite (10) as

$$\phi = a_0 + a_1\chi + a_2\chi^2 + a_3\chi^3. \quad (14)$$

Inserting (14) into (9) and using (11), we have the next polynomial with degree 6 of χ as

$$\begin{aligned}
&\left(60\tilde{r}a_3 + \frac{1}{2}a_3^2\right)\chi^6 + \left(24\tilde{r}a_2 + a_2a_3\right)\chi^5, \\
&+\left(6\tilde{r}a_1 + 3\tilde{p}a_3 + 54b\tilde{r}a_3 + a_1a_3 + \frac{1}{2}a_2^2\right)\chi^4, \\
&+\left(2\tilde{p}a_2 + 40b\tilde{r}a_2 + a_1a_2 + \tilde{c}a_3 + a_0a_3\right)\chi^3, \\
&+\left(\tilde{p}a_1 + 8b\tilde{r}a_1 + 63a_3\tilde{r}b^2 + \frac{1}{2}a_1^2 + \tilde{c}a_2 + a_0a_2 + 3a_1a_3b\right)\chi^2, \\
&+\left(16a_2b^2\tilde{r} + 2a_2b\tilde{p} + a_0a_1 + \tilde{c}a_1\right)\chi, \\
&+\left(\tilde{c}a_0 + 2a_1b^2\tilde{r} + 6a_3b^3\tilde{r} + a_1b\tilde{p} + a_0^2\right) = 0.
\end{aligned} \quad (15)$$

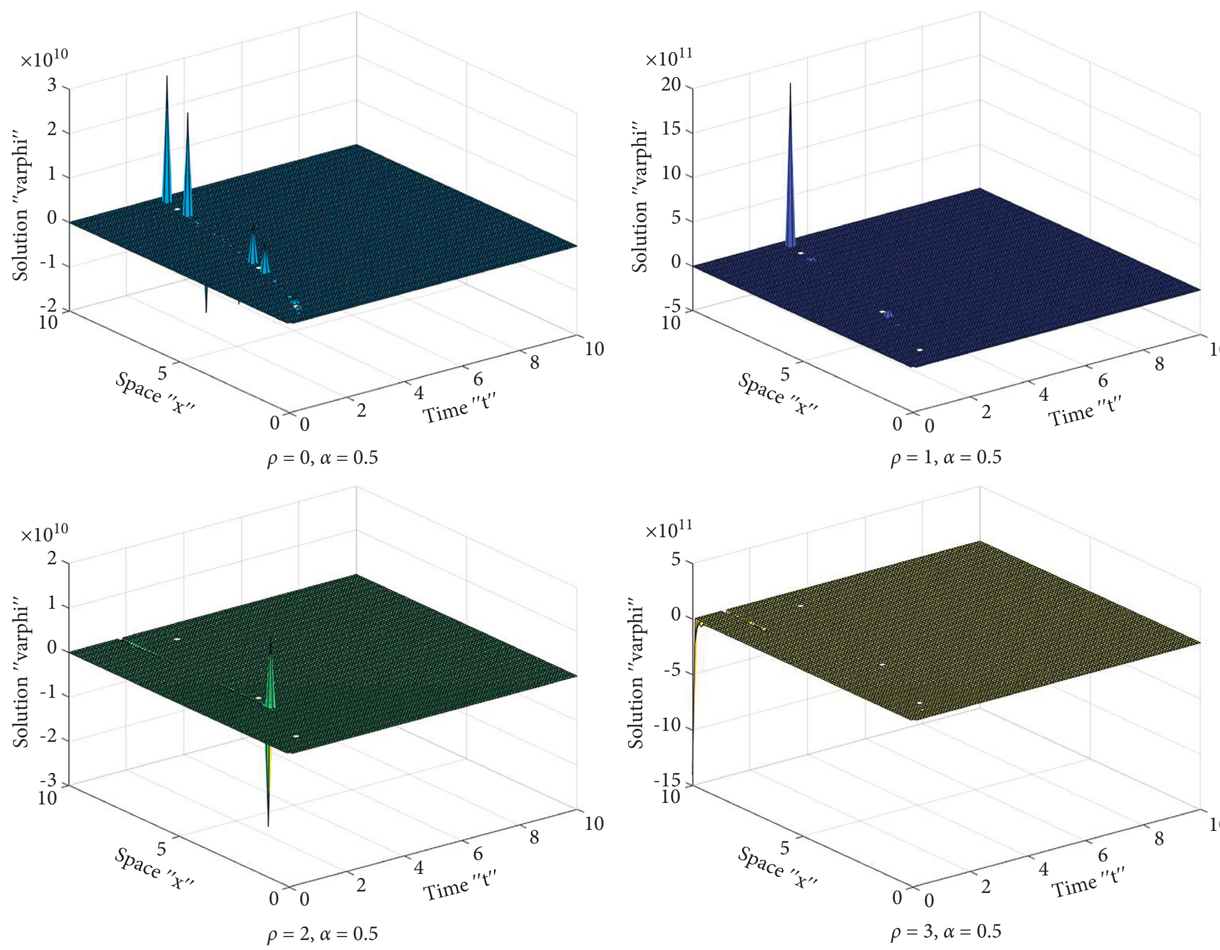

Figure 2: The plots of (27) with $\alpha = 0.5$.

Equating each coefficient of χ^k $(k = 6, 5, 4, 3, 2, 1, 0)$ by zero, we get the following set of algebraic equations:

$$\begin{aligned} 60\tilde{r}a_3 + \frac{1}{2}a_3^2 &= 0, \\ 24\tilde{r}a_2 + a_2a_3 &= 0, \\ 6\tilde{r}a_1 + 3\tilde{p}a_3 + 54b\tilde{r}a_3 + a_1a_3 + \frac{1}{2}a_2^2 &= 0, \\ 2\tilde{p}a_2 + 40b\tilde{r}a_2 + a_1a_2 + \tilde{c}a_3 + a_0a_3 &= 0, \\ \tilde{p}a_1 + 8b\tilde{r}a_1 + 63a_3\tilde{r}b^2 + \frac{1}{2}a_1^2 + \tilde{c}a_2 + a_0a_2 + 3a_1a_3b &= 0, \\ 16a_2b^2\tilde{r} + 2a_2b\tilde{p} + a_0a_1 + \tilde{c}a_1 &= 0, \end{aligned} \tag{16}$$

$$\tilde{c}a_0 + 2a_1b^2\tilde{r} + 6a_3b^3\tilde{r} + a_1b\tilde{p} + a_0^2 = 0. \tag{17}$$

Solving these equations by using Mathematica, we get the following cases.

First case is

$$a_0 = -\tilde{c},\; a_1 = -\frac{60\tilde{p}}{19},\; a_2 = 0,\; a_3 = -120\tilde{r}, \text{ and } b = 0. \tag{18}$$

Since $b = 0$, then the solution of (11) is

$$\chi(\eta) = \frac{-1}{\eta}. \tag{19}$$

According to (14), the corresponding solution of travelling wave (9) is

$$\phi(\eta) = -\tilde{c} + \frac{60\tilde{p}}{19}\eta^{-1} + 120\tilde{r}\eta^{-3}. \tag{20}$$

Hence, the analytical solution of SFSKSE (1) is

$$\psi_1(x,t) = e^{(\rho\beta(t) - 1/2\rho^2 t))} \cdot \left[-\tilde{c} + \frac{60\tilde{p}}{19}\left(\frac{\lambda}{\alpha}x^{\alpha} + ct\right)^{-1} + 120\tilde{r}\left(\frac{\lambda}{\alpha}x^{\alpha} + ct\right)^{-3}\right]. \tag{21}$$

Second case is

$$a_0 = -\tilde{c},\; a_1 = -\frac{60\tilde{p}}{19},\; a_2 = 0,\; a_3 = -120\tilde{r}, \text{ and } b = \frac{\tilde{p}}{76\tilde{r}}. \tag{22}$$

Since $b = \tilde{p}/76\tilde{r}$, then the solution of (11) is

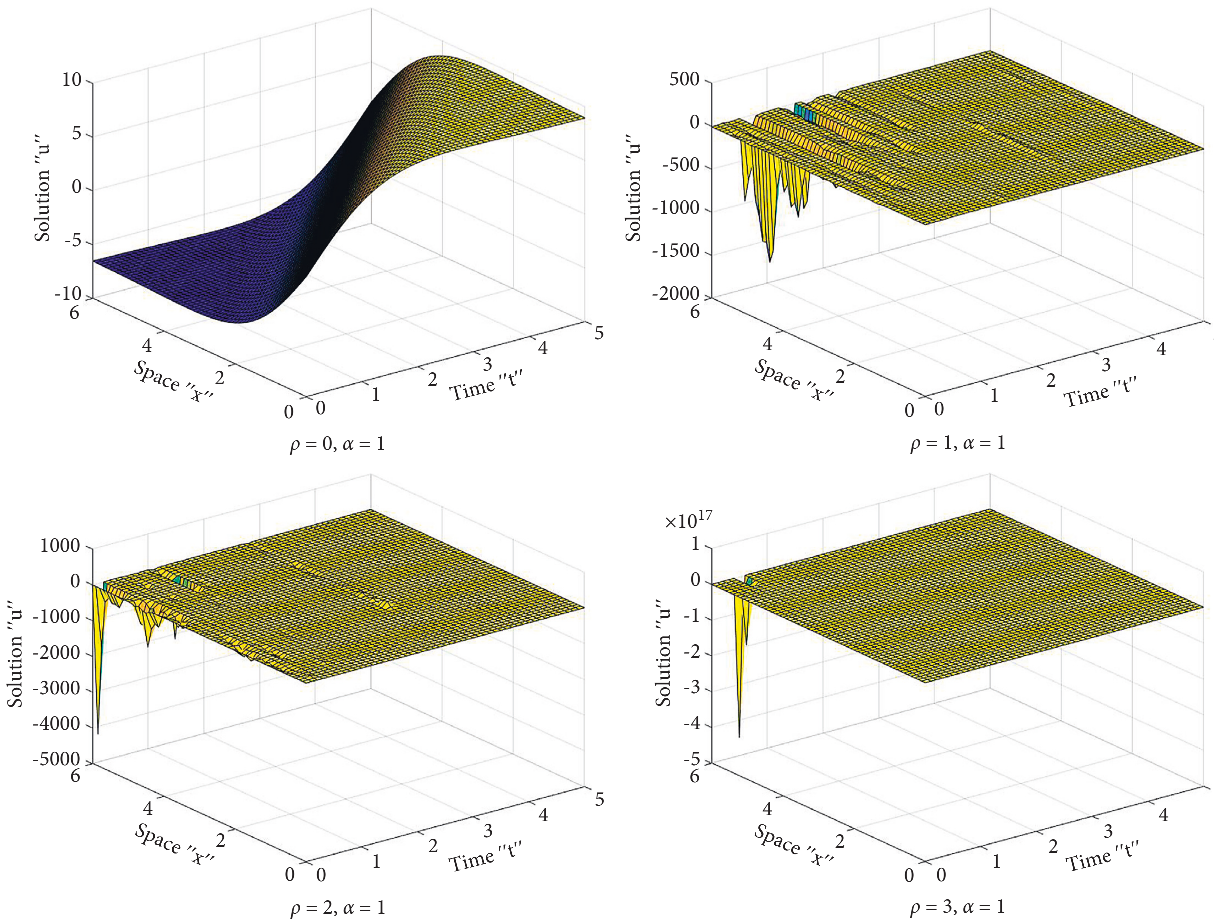

Figure 3: The plots of (35) with $\alpha = 0.5$.

$$\chi(\eta) = \sqrt{b}\tan(\sqrt{b}\,\eta) \text{if } b = \frac{\tilde{p}}{76\tilde{r}} > 0 \tag{23}$$

or

$$\chi(\eta) = -\sqrt{-b}\tanh(\sqrt{-b}\,\eta) \text{ if } b = \frac{\tilde{p}}{76\tilde{r}} < 0. \tag{24}$$

Then, the solution of (9) in this case is

$$\phi(\eta) = -\tilde{c} - \frac{60\tilde{p}}{19}\sqrt{b}\tan(\sqrt{b}\,\eta) - 120\tilde{r}(\sqrt{b})^3\tan^3(\sqrt{b}\,\eta) \tag{25}$$

if $b = \tilde{p}/76\tilde{r} > 0$ or

$$\phi(\eta) = -\tilde{c} + \frac{60\tilde{p}}{19}\sqrt{-b}\tanh(\sqrt{-b}\,\eta) + 120\tilde{r}(\sqrt{-b})^3\tanh^3(\sqrt{-b}\,\eta) \tag{26}$$

if $b = \tilde{p}/76\tilde{r} < 0$.

Therefore, by using (4), the analytical solution of SFSKSE (1) is

$$\psi_2(x,t) = e^{(\rho\beta(t)-1/2\rho^2 t)}\left[-\tilde{c} - \frac{60\tilde{p}}{19}\sqrt{b}\tan\left(\sqrt{b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right) - 120\tilde{r}(\sqrt{b})^3\tan^3\left(\sqrt{b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right)\right] \tag{27}$$

if $b = \tilde{p}/76\tilde{r} > 0$ i.e., $\tilde{p}/\tilde{r} > 0$) or

$$\psi_3(x,t) = e^{(\rho\beta(t)-1/2\rho^2 t)} \cdot \left[-\tilde{c} + \frac{60\tilde{p}}{19}\sqrt{-b}\tanh\left(\sqrt{-b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right) + 120\tilde{r}(\sqrt{-b})^3\tanh^3\left(\sqrt{-b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right)\right] \tag{28}$$

if $b = \tilde{p}/76\tilde{r} < 0$ (*i.e* $\tilde{p}/\tilde{r} < 0$).

Third case is

$$a_0 = -\tilde{c},\ a_1 = \frac{270\tilde{p}}{19},\ a_2 = 0,\ a_3 = -120\tilde{r}, \text{ and } b = \frac{-11\tilde{p}}{76\tilde{r}}. \tag{29}$$

Since $b = -11\tilde{p}/76\tilde{r}$, then the solution of (11) is

$$\chi(\eta) = \sqrt{b}\tan(\sqrt{b}\,\eta) \text{if } b = \frac{-11\tilde{p}}{76\tilde{r}} > 0 \tag{30}$$

or

$$\chi(\eta) = -\sqrt{-b}\tanh(\sqrt{-b}\,\eta) \text{ if } b = \frac{-11\tilde{p}}{76\tilde{r}} < 0. \tag{31}$$

Hence, the solution of (9) is

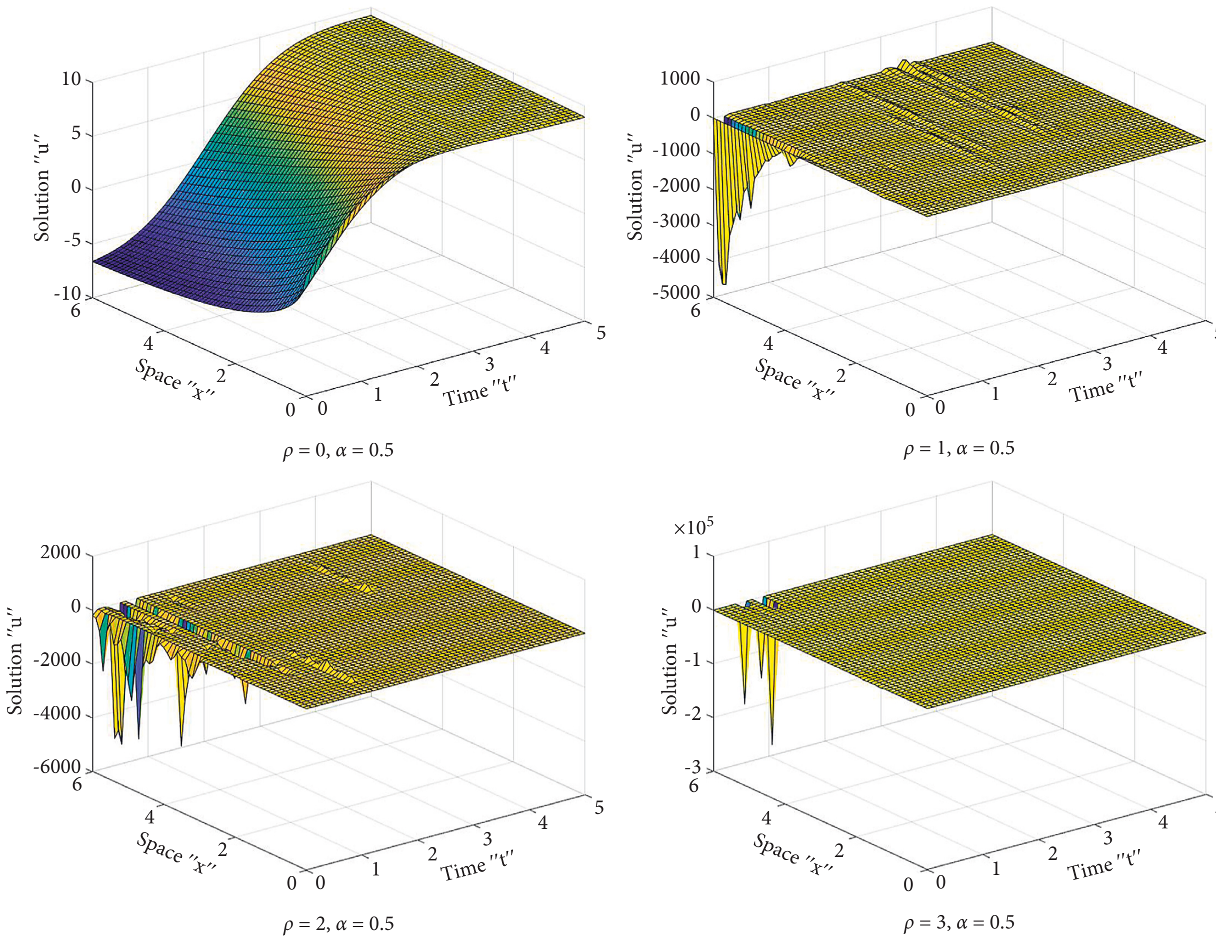

FIGURE 4: The plots of (35) with $\alpha = 0.5$.

$$\phi(\eta) = -\tilde{c} + \frac{270\tilde{p}}{19}\sqrt{b}\tan(\sqrt{b}\,\eta) - 120\tilde{r}(\sqrt{b})^3\tan^3(\sqrt{b}\,\eta) \tag{32}$$

if $b = -11\tilde{p}/76\tilde{r} > 0$ (i.e., $\tilde{p}/\tilde{r} < 0$) or

$$\phi(\eta) = -\tilde{c} - \frac{270\tilde{p}}{19}\sqrt{-b}\tanh(\sqrt{-b}\,\eta) + 120\tilde{r}(\sqrt{-b})^3\tanh^3(\sqrt{-b}\,\eta) \tag{33}$$

if $b = -11\tilde{p}/76\tilde{r} < 0$ i.e., $\tilde{p}/\tilde{r} > 0$).

Thus, the analytical solution of SFSKSE (1), by using (2), is

$$\psi_4(x,t) = e^{(\rho\beta(t) - 1/2\rho^2 t)}\left[-\tilde{c} - \frac{60\tilde{p}}{19}\sqrt{b}\tan\left(\sqrt{b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right) - 120\tilde{r}(\sqrt{b})^3\tan^3\left(\sqrt{b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right)\right] \tag{34}$$

if $b = -11\tilde{p}/76\tilde{r} > 0$ or

$$\psi_5(x,t) = e^{(\rho\beta(t) - 1/2\rho^2 t)} \cdot \left[-\tilde{c} + \frac{60\tilde{p}}{19}\sqrt{-b}\tanh\left(\sqrt{-b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right) + 120\tilde{r}(\sqrt{-b})^3\tanh^3\left(\sqrt{-b}\left(\frac{\lambda}{\alpha}x^\alpha + ct\right)\right)\right] \tag{35}$$

if $b = -11\tilde{p}/76\tilde{r} > 0$.

Remark 2. If we put $\rho = 0$ and $\alpha = 1$ in equations (27), (28), (34), and (35), then we get the same solutions of equation (1) that are stated in [43, 47].

5. The Influence of Noise on the SFSKSE Solutions

Here, we show the influence of multiplicative noise on the solutions of SFSKSE (1). Fix the parameters $\tilde{c} = \tilde{r} = \tilde{p} = -1$. We display some of a graphical representation for various values of ρ (noise intensity). To plot the solution $\psi_1(t,x)$ and

$\psi_2(t, x)$ defined in equations (27) and (35), we use the MATLAB tool as follows.

In Figures 1–4, the surface is not flat when the noise intensity is equal to zero, as shown in the first graph in the tables. When noise appears and its strength grows ($\rho = 1, 2, 3$), the surface is becoming more and more flat after minor transit behaviors. Due to the multiplicative noise effects, this implies that the solutions of SFSKSE (1) are stable.

6. Conclusions

We have provided different analytical fractional-space stochastic solutions of SFSKSE (1) via the Riccati equation method in this study. In addition, some results such as those presented in [43, 47] were expanded and improved. These types of space-fractional stochastic solutions can be employed to describe a wide range of fascinating and difficult scientific phenomena. In the end, we applied the MATLAB package to produce some graphical representations to show how the stochastic term affects SFSKSE (1) solutions.

Data Availability

All data used to support the findings of the study are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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