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Research Article

Cubic Planar Graph and Its Application to Road Network

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In this research article, we present the notion of a cubic planar graph and investigate its related properties. The cubic graphs are more effective than both interval-valued and fuzzy graphs as it represents the level of participation (membership degree) of vertices and edges both in interval form and as a fuzzy number. Moreover, it handles the uncertainty and vagueness more efficiently than both interval-valued fuzzy graph and fuzzy graph. The interval indicates a continuous process, whereas the point indicates a specific process. We introduce the terms cubic multigraph, cubic strong and weak edges, and degree of planarity for cubic planar graphs. Some fundamental theorems based on these concepts are also elaborated. We also propose the idea of a cubic strong and weak fuzzy faces and cubic dual graph. Some results related to these concepts are also established. Comparison with the existing method shows the worth of our proposed work.

1. Introduction

Zadeh et al. [1] started basic work on fuzzy sets in 1965. Kaufmann [2] inspected fuzzy sets and fuzzy relations thoroughly. Fuzzy graphs were introduced by Rosenfeld [3], ten years after Zadeh et al.'s [1] achievement paper "Fuzzy Sets." Bhattacharya [4] introduced a couple of thoughts on fuzzy graphs. Bhutani [5] focused on automorphism of fuzzy graphs. Intuitionistic fuzzy set were introduced by Atanassov [6]. Then, after the development of intuitionistic fuzzy set, intuitionistic fuzzy graphs were also presented by Shannon and Atanassov [7]. After the development of the intuitionistic fuzzy set and graph, Pythagorean fuzzy sets were introduced by Yager [8] which removed the weakness of the intuitionistic fuzzy set as it was more adaptable. Fuzzy graphs theory has various applications in present-day science and development especially in the fields of data hypothesis, neural organizations, clinical determination, control hypothesis, and so on. Fuzzy modeling is a fundamental instrument in all parts of science, design, and medication. Fuzzy models give more exactness, flexibility, and likeness to the system when diverged from the excellent models. It is used in evaluation of human heart work, fuzzy neural organizations, and so on. A fuzzy graph in same manner used to handle traffic light issue, time table preparation, etc.

Graph theory has immense applications in information mining, picture division, picture catching, plan organizing, correspondence problems, electric circuits, road network, and so forth. For instance, an information construction can be planned as a tree, which uses vertices and edges. Likewise, demonstration of organization geographies can be possible by utilizing the idea of graphs. Furthermore, paths and circuits are utilized to tackle numerous issues, viz., mobile sales rep, information base plan, asset organizing, and so on. There are numerous graph networks in which crossing between edges can lead to a problem, such as plan issues for electrical circuits, metros, and utility corridors. The crossing point of 2 affiliations customarily shows the correspondence lines ought to be run at different heights. For electrical wires, this is authentically not a significant issue, yet it makes extra expenses for particular sorts of lines, e.g., covering 1 metro tunnel under another. This prompts us to utilize the possibility of the planar graph to beat these kinds of issues. Planar graphs are utilized to address the crossing point of lines in the circuits. Cuts or intersections of the planar graphs are utilized in various computational hardships including picture division or shape matching.

This idea can likewise be utilized in the road network. In the event that roads have crossing, a few streets or roads can be developed underground as underground streets decrease the mishap yet costs to develop them are higher. We can likewise see that the mishap proportion in congested regions is higher than in the noncongested areas so we can make the crossing of congested and noncongested regions since, in such a case that two congested regions having crossing then, at that point, chances of mishaps are higher. Also, we can see congested words have no particular importance and it is a linguistic term since the region can be very highly congested, highly congested, congested, low congested, and very low congested. The congested regions lead to the idea of strong edge and noncongested regions prompts the idea of weak edges. In this way, from the above conversation, we can say that the intersection among two strong edges is more wasteful than crossing areas of strength among weak edges.

Rosenfeld [3] developed the fuzzy graph theory which prompts numerous applications. The fuzzy graph theory is extended with innumerable branches. Fuzzy tolerance graphs were introduced by Pal et al. [9]. Graphs for the analysis of bipolar fuzzy information were investigated by Akram et al. [10]. Fuzzy intersection graphs were depicted by McAllister [11]. Fuzzy planar graphs were analyzed by Samanta and Pal [12]. Special fuzzy planar graphs were presented by Nirmala and Dhanabal [13]. The planar graph hypothesis is a critical assessment area. In 1930, Kuratowski [14] fostered a couple of huge outcomes on planar graphs. Pal et al. [15] and Samanta et al. [16] described the fuzzy planar graph with another thought, where the crossing of edges is allowed. They also focused on the different properties of a fuzzy planar graph. Zadeh [17] presented that interval-valued uncertainties are depicted all the more impeccably or productively different types of interval-valued fuzzy graphs were discussed by Akram et al. [18, 19]. Pramanik et al. [20] presented introduced interval-valued fuzzy planar graphs. A short time later, Abdul et al. [21] presented fuzzy dual graph. Numerous new ideas connected with planarity of graphs are presented by Akram et al. [22] and Alshehri and Akram [23] which included planar graphs under Pythagorean fuzzy environment and intuitionistic fuzzy planar graphs.

One of the genuine speculations of fuzzy sets is the cubic set given by Jun et al. [24] during the most recent 5 years. Mappings of cubic sets were concentrated by Kang and Kim [25]. Stable cubic sets were presented by Muhiuddin et al. [26]. Cubic graphs were introduced by Rashid et al. [27].

The motivation of our work is a cubic set which contained two sets (fuzzy set and interval-valued fuzzy set) together. One of the important parameters of the cubic set is to present a specific and continuous process at the same time. So far, a lot of work has been performed on the cubic set, but a little effort has been made on cubic graphs. Persuaded by the possibility of a fuzzy planar graph and interval-valued planar graph, we present the idea of the cubic planar graph (CPG) which consolidates the two thoughts together in one design. Cubic planar graphs focused on the planarity, in both specific and continuous time as in the present and future.

The proposed research work deals with the idea of CPG and is organized as follows: Section 2 contains a few fundamental primers and definitions of CPG, cubic multiset, cubic multigraph, strength of an edge, and strong and weak edges with examples and related results. Section 3 includes definitions and theorems on planarity, considerable number, and strong and weak faces. Section 4 deals with dual graphs of cubic planar graphs. Section 5 contains a genuine application connected with the road network and toward the end whole work and course for additional work is depicted.

From now onwards, $\mathfrak{B}$ denotes a nonempty set.

2. Cubic Planar Graph

Yager [28] introduced fuzzy multiset which is stated as a component of a set which might appear at least a couple of times with the same or different membership values. A (crisp) multiset is essentially a mapping $O\colon \mathfrak{B} \longrightarrow \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers. Characteristic speculation of this set prompts the idea of a fuzzy multiset defined as follows:

Definition 1. Let $\varpi\colon \mathfrak{B} \longrightarrow [0,1]$ be a mapping. If multi-membership values of $\dot{x} \in \mathfrak{B}$ are $\varpi^b(\dot{x}), b = 1,2,\ldots,l$ where $l = \max\{b\colon \varpi^b(\dot{x}) \neq 0\}$. Then, the fuzzy multiset (FMS) is $M = \{(\dot{x}, \varpi^b(\dot{x})), b = 1,2,\ldots,l|v \in \mathfrak{B}\}$.

Definition 2. Let $\varpi_p^-\colon \mathfrak{B} \longrightarrow [0,1]$ and $\varpi_p^+\colon \mathfrak{B} \longrightarrow [0,1]$ be the mappings such that $\varpi_p^-(\dot{x}) \leq \varpi_p^+(\dot{x})$ for all $\dot{x} \in \mathfrak{B}$ and $p = 1,2,\ldots,i$, where $i = \max\left\{p\colon \varpi_p(\dot{x}) \neq 0\right\}$. The interval-valued fuzzy multiset (IVFMS) on $\mathfrak{B}$ represented as $A = (\mathfrak{B}, [\varpi_p^-, \varpi_p^+])$ and can be defined as $A = (\mathfrak{B}, [\varpi_p^-, \varpi_p^+]) = \left\{\dot{x}, [\varpi_p^-, \varpi_p^+]|\dot{x} \in \mathfrak{B}, p = 1,2,\ldots,i\right\}$.

Definition 3. Let $\varpi_p^-\colon \mathfrak{B} \longrightarrow [0,1]$ and $\varpi_p^+\colon \mathfrak{B} \longrightarrow [0,1]$ be the mappings. Then, the IVFMS on $\mathfrak{B}$ is given as $A = \left\{(\mathfrak{B}, [\varpi_p^-, \varpi_p^+]), p = 1,2,\ldots,i\right\}$. Let $\chi_q^-\colon \mathfrak{B} \times \mathfrak{B} \longrightarrow [0,1], \chi_q^+\colon \mathfrak{B} \times \mathfrak{B} \longrightarrow [0,1]$ be the mappings. Then, the IVFMG on $\mathfrak{B} \times \mathfrak{B}$ is given as $B = \left\{(\mathfrak{B} \times \mathfrak{B}, [\chi_q^-, \chi_q^+]), q = 1,2,\ldots,j\right\}$. The pair (A,B) is said to be IVFMG if $\chi_q^-(\dot{x},\dot{y}) \leq \min\left\{\varpi_p^-(\dot{x}), \varpi_p^-(\dot{y})\right\}$, $\chi_q^+(\dot{x},\dot{y}) \leq \min\left\{\varpi_p^+(\dot{x}), \varpi_p^+(\dot{y})\right\} p = \left\{1,2,\ldots,i_{\dot{x}\dot{y}}\right\}, q = \left\{1,2.\ldots,j_{\dot{x}\dot{y}}\right\}$ for all $\dot{x},\dot{y} \in \mathfrak{B}$, where $j_{\dot{x}\dot{y}} = \max \left\{q\colon \chi_q(\dot{x},\dot{y}) \neq 0\right\}$.

Definition 4. The cubic multiset over a nonempty set $\mathfrak{B}$ is defined as

$$C = \{\dot{x}, A(\dot{x}), M(\dot{x})|\dot{x} \in \mathfrak{B}\}, \tag{1}$$

where $A(\dot{x})$ is IVFMS and $M(\dot{x})$ is FMS, that is,

$$C = \left\{\mathfrak{B}, \left[\varpi_p^-, \varpi_p^+\right], \varpi_r^*|p = 1,2\ldots,i\, r = 1,2\ldots,s\right\}. \tag{2}$$

Definition 5. Let $\varpi_p^-\colon \mathfrak{B} \longrightarrow [0,1], \varpi_p^+\colon \mathfrak{B} \longrightarrow [0,1]$ and $\varpi_q^*\colon \mathfrak{B} \longrightarrow [0,1]$ be the mappings and $D = \{(\dot{x},\dot{y}), [\chi_q^+(\dot{x},\dot{y}), \chi_q^-(\dot{x},\dot{y})], \chi_k(\dot{x},\dot{y}), q = 1,2,\ldots,j\, k = 1,2,\ldots,$

$o| (\dot{x}, \dot{y}) \in \mathfrak{B}\}$ be a FMS on $\mathfrak{B} \times \mathfrak{B}$ such that $\chi_q^- (\dot{x}, \dot{y}) \le \min\{\varpi_p^- (\dot{x}), \varpi_p^- (\dot{y})\}$, $\chi_q^+ (\dot{x}, \dot{y}) \le \min\{\varpi_p^+ (\dot{x}), \varpi_p^+ (\dot{y})\}$, and $\chi_k (\dot{x}, \dot{y}) \le \min\{\varpi_r^* (\dot{x}), \varpi_r^* (\dot{y})\}$, $j = 1, 2, \ldots, p$ and $k = 1, 2, \ldots, q$ for all $\dot{x}, \dot{y} \in \mathfrak{B}$. Then, $G = (C, D)$ is called cubic multigraph (CMG) such that C and D are the set of cubic vertices and cubic multiedges, respectively.

Example 1. Let $\mathfrak{B} = \{a, b, c\}$, then $\varpi^- (a) = 0.2$, $\varpi^+ (a) = 0.5, \varpi (a) = 0.4, \varpi^- (b) = 0.3, \varpi^+ (b) = 0.6, \varpi (b) = 0.3$ and $\varpi^- (c) = 0.7, \varpi^+ (c) = 0.9, \varpi (c) = 0.6$. Now, $\chi_1^- (a, b) = 0.1$, $\chi_1^+ (a, b) = 0.4$, $\chi_1 (a, b) = 0.2$, $\chi_2^- (a, b) = 0.2$, $\chi_2^+ (a, b) = 0.5, \chi_2 (a, b) = 0.3, \chi_3^- (a, c) = 0.1, \chi_3^+ (a, c) = 0.5, \chi_3 (a, c) = 0.1, \chi_4^- (b, c) = 0.2, \chi_4^+ (b, c) = 0.5$ and $\chi_4 (b, c) = 0.2$, then,

$$\mathfrak{B} = \{(a, [0.2, 0.5], 0.4), (b, [0.3, 0.6], 0.3), (c, [0.7, 0.9], 0.6)\}, \tag{3}$$

and

$$E = \{((a, b), [0.1, 0.4], 0.2), ((a, b), [0.2, 0.5], 0.3), ((b, c), [0.2, 0.5], 0.2) ((c, a), [0.1, 0.5], 0.1)\}. \tag{4}$$

So, this is CMG as there are two edges between vertex a and b.

Definition 6. Let $G = (C, D)$ be CMG, then strength of an edge is defined by $I_{mn} = ([I_{mn}^-, I_{mn}^+], I_{mn}^*)$, where $I_{mn}^- = \chi^- (m, n)/\min\{\varpi^+ (m), \varpi^+ (n)\}$, $I_{mn}^+ = \chi^+ (m, n)/ \min\{\varpi^+ (m), \varpi^+ (n)\}$ and $I_{mn}^* = \chi (m, n)/\min\{\varpi (m), \varpi (n)\}$, $mn \in D$, and $m, n \in C$. If $I_{mn} \ge ([0.5, 0.5], 0.5)$, then the cubic edge is called strong edge. An edge which is not strong is obviously a weak edge. In CMG, a specific value is given to the point where two edges cut themselves, which is known as cubic-valued number and is calculated in the following manner.

Suppose there are two edges (r, s) and (t, u) which have cutting points, so we calculate I_{rs} and I_{tu} for the corresponding edges. The cutting number at a point P is defined as

$$I_{\mathrm{P}} = \frac{I_{rs} + I_{tu}}{2}. \tag{5}$$

It is easy to see that $I_{\mathrm{P}} \in [0, 1]$.

3. Planarity

Cutting of edges does not exist in cubic crisp planar graphs. So, these types of graphs have full planarity. Therefore, the planarity is decreased if the number of cutting edges in a CMG increased. So, for CMG, planarity is inversely proportional to the I_{P} which leads to a new concept of degree of planarity for CMG as introduced below:

Definition 7. Let $G = (C, D)$ be a CMG and for a specific geometrical representation, $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_k (k \in \mathbb{Z})$ be the cutting points for the edges of G. Then, the graph G is said to be cubic planar graph (CPG) containing the degree of planarity $\mathsf{F} = ([\mathsf{F}^-, \mathsf{F}^+], \mathsf{F}^*)$, where

$$\mathsf{F}^- = \frac{1}{1 + \left(I_{\mathrm{P}_1}^+ + I_{\mathrm{P}_2}^+, \ldots, I_{\mathrm{P}_k}^+\right)}, \tag{6}$$

$$\mathsf{F}^+ = \frac{1}{1 + \left(I_{\mathrm{P}_1}^- + I_{\mathrm{P}_2}^-, \ldots, I_{\mathrm{P}_k}^-\right)}, \tag{7}$$

$$\mathsf{F}^* = \frac{1}{1 + \left(I_{\mathrm{P}_1}^* + I_{\mathrm{P}_2}^*, \ldots, I_{\mathrm{P}_k}^*\right)}. \tag{8}$$

Assuming that there is no crossway for a particular mathematical representation of a CPG. Then, its level of planarity is considered as $([1, 1], 1)$. In this case, CPG is the crisp cubic planar graph.

Moreover, w the ordering relation defined on the degree is given as:

$\mathsf{F}_1 = ([\mathsf{F}_a^-, \mathsf{F}_b^+], \mathsf{F}_c^*) \ge \mathsf{F}_2 = ([\mathsf{F}_d^-, \mathsf{F}_e^+], \mathsf{F}_f^*)$ if and only if $\mathsf{F}_a^- \ge \mathsf{F}_d^-, \mathsf{F}_b^+ \ge \mathsf{F}_e^+$ and similarly $\mathsf{F}_b^+ > \mathsf{F}_b^+$.

Example 2. Consider a crisp graph $G^* = (\mathfrak{B}, E)$ such that $\mathfrak{B} = \{x, y, z\}$ and $E = \{xy, yz, zx\}$. Then, we defined CPG $G = (C, D)$ as shown in Figure 1.

$\mathfrak{V}$	a	b	c	d
	([0.2, 0.3], 0.6)	([0.5, 0.7], 0.8)	([0.1, 0.4], 0.4)	([0.5, 0.6], 0.3)
E	ab	bd	cd	ac
	([0.1, 0.2], 0.5)	([0.1, 0.3], 0.4)	([0.1, 0.4], 0.1)	([0.2, 0.3], 0.2)

Here, the cutting point is between the edges (b, d) and (a, c). Now, we calculate I_{bd} and I_{ac} which are given as

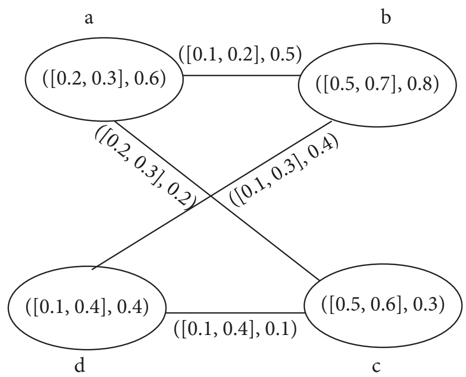

FIGURE 1: Example of CPG.

$$I^-_{ac} = \frac{0.2}{\min(0.3, 0.6)} = \frac{0.2}{0.3} = 0.66,$$
$$I^+_{ac} = \frac{0.3}{\min(0.3, 0.6)} = \frac{0.3}{0.3} = 1, \tag{9}$$
$$I^*_{ac} = \frac{0.2}{\min(0.6, 0.3)} = \frac{0.2}{0.3} = 0.66.$$

For edge (b, d),

$$I^-_{bd} = \frac{0.1}{\min(0.4, 0.7)} = \frac{0.1}{0.4} = 0.25,$$
$$I^+_{bd} = \frac{0.3}{\min(0.4, 0.7)} = \frac{0.3}{0.4} = 0.75, \tag{10}$$
$$I^*_{bd} = \frac{0.4}{\min(0.4, 0.8)} = \frac{0.4}{0.4} = 1.$$

Now,

$$\begin{aligned} I_{\mathrm{P}} &= \left(\left[\frac{0.66 + 0.25}{2}, \frac{1 + 0.75}{2}\right], \frac{0.66 + 1}{2}\right) \\ &= ([0.45, 0.87], 0.83), \\ \mathsf{F} &= \frac{1}{1 + I_{\mathrm{P}}}, \\ \mathsf{F} &= \left(\left[\frac{1}{1 + 0.87}, \frac{1}{1 + 0.45}\right], \frac{1}{1 + 0.83}\right) \\ &= ([0.53, 0.68], 0.54). \end{aligned} \tag{11}$$

Here, we can clearly see that $\mathsf{F} > ([0.5, 0.5], 0.5)$.

Theorem 1. *Let $G = (C, D)$ be a complete cubic planar graph. Then, the cubic planarity value F of G is given by $\mathsf{F} = ([\mathsf{F}^-, \mathsf{F}^+], \mathsf{F}^*)$, where $\mathsf{F}^- = 1/1 + N_{\mathrm{P}}$, $1/1 + N_{\mathrm{P}} \le \mathsf{F}^+ \le 1$, $\mathsf{F}^* = 1/1 + N_{\mathrm{P}}$, and N_{P} is the number of cutting points for the edges in G.*

Proof. For complete multigraph, we have,

$$\begin{aligned} \chi^-(m, n) &= \min\{\varpi^-(m), \varpi^-(n)\}, \\ \chi^+(m, n) &= \min\{\varpi^+(m), \varpi^+(n)\}, \\ \chi^*(m, n) &= \min\{\varpi^*(m), \varpi^*(n)\}. \end{aligned} \tag{12}$$

For all, $m, n \in \mathfrak{B}$. Let $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_k$ be the cutting of points along the edges in G, $k \in \mathbb{Z}$. For any edge (r, s) in complete CPG,

$$\begin{aligned} I^-_{rs} &= \frac{\chi^-(r, s)}{\min\{\varpi^+(r), \varpi^+(s)\}} \le 1, \\ I^+_{rs} &= \frac{\chi^+(r, s)}{\min\{\varpi^+(r), \varpi^+(s)\}} = 1, \\ I^*_{rs} &= \frac{\chi(r, s)}{\min\{\varpi(r), \varpi(s)\}}. \end{aligned} \tag{13}$$

In this way, for the point P, the cutting point along the edges (r, s) and (t, u), $I^+_{\mathrm{P}_1} = 1 + 1/2 = 1$, $I^-_{\mathrm{P}_1} \le 1 + 1/2 = 1$, and $I^*_{\mathrm{P}_1} = 1 + 1/2 = 1$; hence, $I^+_{\mathrm{P}_a} = 1$, $I^-_{\mathrm{P}_a} \le 1$ and $I^*_{\mathrm{P}_a} = 1$ where $a = 1, 2, \ldots, k$. Then,

$$\begin{aligned} \mathsf{F}^- &= \frac{1}{1 + I^+_{\mathrm{P}_1} + I^+_{\mathrm{P}_2}, \ldots, I^+_{\mathrm{P}_k}} \\ &= \frac{1}{1 + (1 + 1, \ldots, 1)} \\ &= \frac{1}{1 + N_{\mathrm{P}}}, \\ \mathsf{F}^* &= \frac{1}{1 + I^*_{\mathrm{P}_1} + I^*_{\mathrm{P}_2}, \ldots, I^*_{\mathrm{P}_k}} \\ &= \frac{1}{1 + (1 + 1, \ldots, 1)} \\ &= \frac{1}{1 + N_{\mathrm{P}}}. \end{aligned} \tag{14}$$

Here, the number of edges having intersection in G is shown by N_{P}. So, F is given as $\mathsf{F} = ([\mathsf{F}^-, \mathsf{F}^+], \mathsf{F}^*)$, where $\mathsf{F}^- = 1/1 + N_{\mathrm{P}}$, $1/1 + N_{\mathrm{P}} \le \mathsf{F}^+ \le 1$, and $\mathsf{F}^* = 1/1 + N_{\mathrm{P}}$. □

Theorem 2. *Let G be CMG with $\mathsf{F} \ge ([0.5, 0.5], 0.5)$. Then, cubic-valued strong edges in G containing the number of cutting points is at most 1.*

Proof. Let $G = (C, D)$ be a CMG with $\mathsf{F} = ([\mathsf{F}^-, \mathsf{F}^+], \mathsf{F}^*)$ where $\mathsf{F}^- > 0.5$, $\mathsf{F}^+ > 0.5$, and $\mathsf{F}^* > 0.5$. Suppose that P_1 and P_2 corresponding to 2 strong cubic-valued edges are two cutting points in G. For a strong edge $\langle (r, s), \langle [\chi^-(r, s), \chi^+(r, s)], \chi^*(r, s)\rangle\rangle$, $I^-_{rs} \ge 0.5$, $I^+_{rs} \ge 0.5$, and $I^*_{rs} \ge 0.5$. Accordingly, for two intersecting cubic-valued strong edges $\langle (r, s), \langle [\chi^-(r, s), \chi^+(r, s)], \chi^*(r, s)\rangle\rangle$ and $\langle (t, u), \langle [\chi^-(t, u), \chi^+(t, u)], \chi^*(t, u)\rangle\rangle$.

$$\frac{I_{rs}^{-}+I_{tu}^{-}}{2}\geq 0.5,$$
$$\frac{I_{rs}^{+}+I_{tu}^{+}}{2}\geq 0.5, \tag{15}$$

and

$$\frac{I_{rs}^{*}+I_{tu}^{*}}{2}\geq 0.5. \tag{16}$$

That is, $I_{P_1}^{-}\geq 0.5$, $I_{P_1}^{+}\geq 0.5$, $I_{P_1}^{*}\geq 0.5$, $I_{P_2}^{-}\geq 0.5$, $I_{P_2}^{+}\geq 0.5$, and $I_{P_2}^{*}\geq 0.5$. Then, $1+I_{P_1}^{-}+I_{P_2}^{-}\geq 2$, $1+I_{P_1}^{+}+I_{P_2}^{+}\geq 2$, and $1+I_{P_1}^{*}+I_{P_2}^{*}\geq 2$; therefore, $\digamma^{-}=1/1+I_{P_1}^{+}+I_{P_2}^{+}\leq 0.5$, $\digamma^{+}=1/1+I_{P_1}^{-}+I_{P_2}^{-}\leq 0.5$, and $\digamma^{*}=1/1+I_{P_1}^{*}+I_{P_2}^{*}\leq 0.5$, which is a contradiction as, $\digamma\geq([0.5,0.5],0.5)$. So, the number of cutting points among cubic strong edges can never be 2. It is evident that by increasing the number of cutting places of cubic strong edges, the level of planarity diminishes. Moreover, if the quantity of cutting point of cubic-valued strong edges is 1, then in this case, the level of planarity $\digamma$ is assumed as $\digamma=([0.5,0.5],0.5)$. Accordingly, we found that cubic value strong edges in G containing the number of cutting points is at most 1. □

Example 3. Consider two cubic planar graphs as shown in Figure 2. In Figure 2(a), a cubic planar graph is considered with 1 crossing among two strong edges (b,f) and (a,c). The cubic planarity of the graph is $([0.5,0.502],0.502)$. Hence, this planar cubic graph is strong and the number of cutting points is 1. In Figure 2(b), a CPG is considered with 2 crossing among strong-edged $(b,f)(a,c)$ and $(a,c)(b,d)$. The cubic planarity of this graph is $([0.5,0.5],0.5)$, and hence, this cubic graph is not strong. Also, we can observe that if there is no crossing, then the CPG must be strong.

Theorem 3. *Let G be a CPG having a degree of planarity* $\digamma$*. If* $\digamma\geq([0.67,0.67],0.67)$*, then, two cubic multivalued strong edges in G does not have any cutting point between them.*

Proof. Let $G=[C,D]$ be a CPG with $\digamma=([0.67,0.67],0.67)$. Consider a place P where 2 cubic-valued strong edges $((r,s),([\chi^{-}(r,s),\chi^{+}(r,s)],\chi^{*}(r,s)))$ and $((t,u),([\chi^{-}(t,u),\chi^{+}(t,u)]\chi^{*}(t,u)))$ intersect. For any cubic-valued strong edge $((r,s),([\chi^{-}(r,s),\chi^{+}(r,s)],\chi^{*}(r,s)))$, $I_{rs}^{-}\geq 0.5$, $I_{rs}^{+}\geq 0.5$, and $I_{rs}^{*}\geq 0.5$. For the minimum value of I_{rs}^{-}, I_{rs}^{+}, I_{rs}^{*}, I_{tu}^{-}, I_{tu}^{+}, and I_{tu}^{*}. $I_{P}^{-}=0.5$, $I_{P}^{+}=0.5$, and $I_{P}^{*}=0.5$. Then, the degree of planarity is $\digamma^{-}=1/1+0.5\leq 0.67$, $\digamma^{+}=1/1+0.5\leq 0.67$, and $\digamma^{*}=1/1+0.5\leq 0.67$. This leads to the inconsistency. Consequently, G contains no cutting point between cubic-valued strong edges.

The above theorem motivated to define term strong planar graph. □

Definition 8. A cubic planar graph is known as a strong planar graph if $\digamma\geq([0.67,0.67],0.67)$.

Theorem 4. *A CMG having a complete* K_5 *or* $K_{3,3}$ *cubic graph is not a strong cubic graph.*

Proof. Let $G=(C,D)$ be complete cubic graph corresponding to the crisp graph $G^{*}=(\mathfrak{B},E)$ with 5 vertices such that $\mathfrak{B}=\{a,b,c,d,e\}$ and $E=\{(r,s),([\chi^{-}(r,s),\chi^{+}(r,s)],\chi^{*}(r,s))|r,s\in\mathfrak{B}\}$. For all $r,s\in\mathfrak{B}\chi^{-}(r,s)=\min\{\varpi^{-}(r)\varpi^{-}(s)\}$, $\chi^{+}(x,y)=\min\{\varpi^{+}(r),\varpi^{+}(s)\}$, and $\chi^{*}(r,s)=\min\{\varpi^{*}(r),\varpi^{*}(s)\}$. From Theorem 1, the degree of planarity of complete cubic graph is the number of cutting points of edges in G. The geometric representation of crisp graph of K_5 is shown in the Figure 3.

Conclusively, it has only one cutting point and it cannot be avoided. So, $\digamma^{-}=1/1+N_P=1/1+1=0.5$. Hence, G is not strong. □

Example 4. Let G be a graph with five vertices, and there is one cutting point between (a,c) and (b,d) as shown in Figure 4. So, the value of the cutting point is $I_{P}^{-}=I_{ac}^{-}+I_{bd}^{-}/2=0.5+0.3/2=0.4$, $I_{P}^{+}=I_{ac}^{+}+I_{bd}^{+}/2=1+1/2=0.5$ and $I_{P}^{*}=I_{ac}^{*}+I_{bd}^{*}/2=1+1/2=0.5$. So, $\digamma=([0.4,0.5],0.5)$. By using Theorem 4, we can see that a complete CG containing five vertices is not a strong CPG.

Definition 9. Let G be a CPG and $0\leq c\leq 0.5$ be a rational number. An edge (r,s) is known as a considerable edge if $([\chi^{-}(r,s)/\min\{\varpi^{+}(r),\varpi^{+}(s)\},\chi^{+}(r,s)/\min\{\varpi^{+}(r),\varpi^{+}(s)\}],\chi^{*}(r,s)/\min\{\varpi^{*}(r),\varpi^{*}(s)\})\geq([c,c],c)$. In case an edge is not considerable, it will be known as nonconsiderable edge.

Clearly, one specific value of c gives a set of considerable edges and different values give a different set of considerable edges. In short, we fix the value of c for particular application. If we consider a social network of CPG where people, organization, etc., are used as vertex and the relationship between these social units are used as edges. The membership degree of edges is used to show the quantity of relationship. The set of considerable edges S obtained if we used a specific value of $c=0.25$ for social network. Considerable amount of relationship among a group of people is provided by this set. In this case, amount of relationship will be 0.25.

The following theorem states the maximum number of cutting points between considerable edges.

Theorem 5. *Let c be the considerable number, where* $G=(C,D)$ *be a strong CPG. Then, considerable edges in G have at most* $[0.49/c]$ *cutting points (here* $[x]$ *is the greatest integer not exceeding x).*

Proof. Let $\digamma$ be the degree of planarity and $0\leq c\leq 0.5$. For a considerable edge (r,s), it is seen that $\chi^{-}(r,s)/\min\{\varpi^{+}(r),\varpi^{+}(s)\}\geq c$ so $\chi^{-}(r,s)\geq c\times\min\{\varpi^{+}(r),\varpi^{+}(s)\}$. In this case, I_{rs}^{-}, I_{rs}^{+}, and $I_{rs}^{*}\geq c$. Let $P_1,P_2,\ldots,P_k$ be k cutting points among the considerable edges. Let P_1 be the cutting point among the considerable edges (r,s) and (t,u). Then,

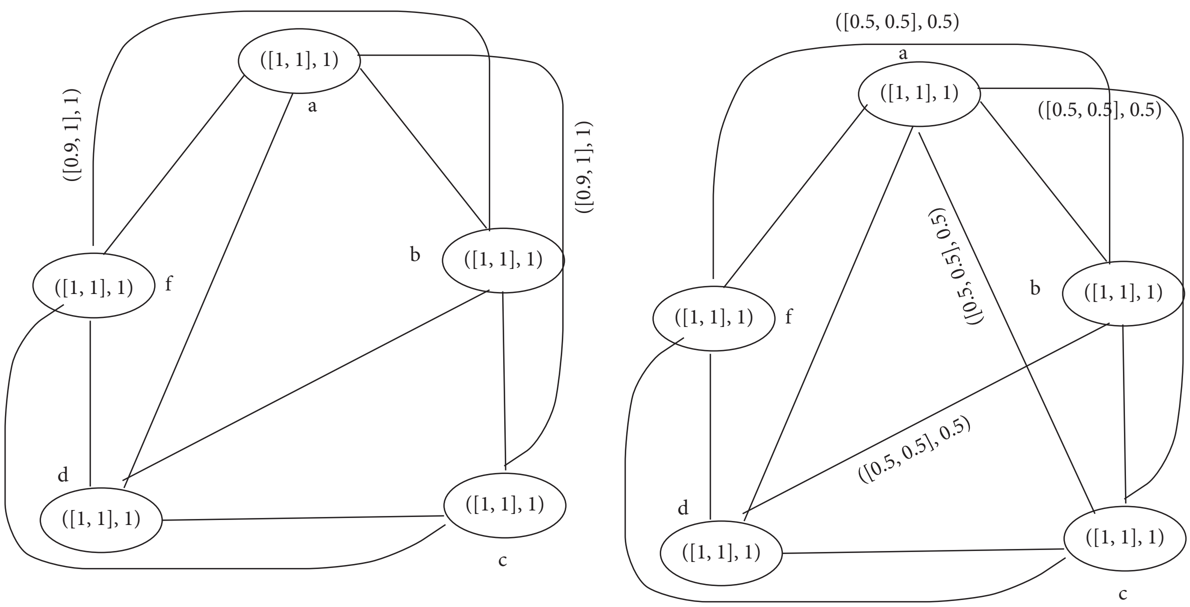

Figure 2: Example of strong and not strong CPG.

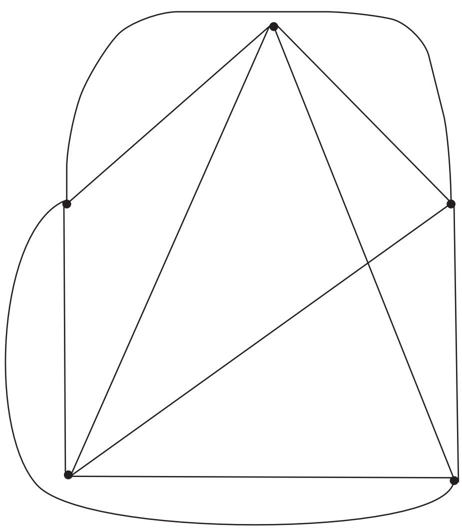

Figure 3: K_5 graph.

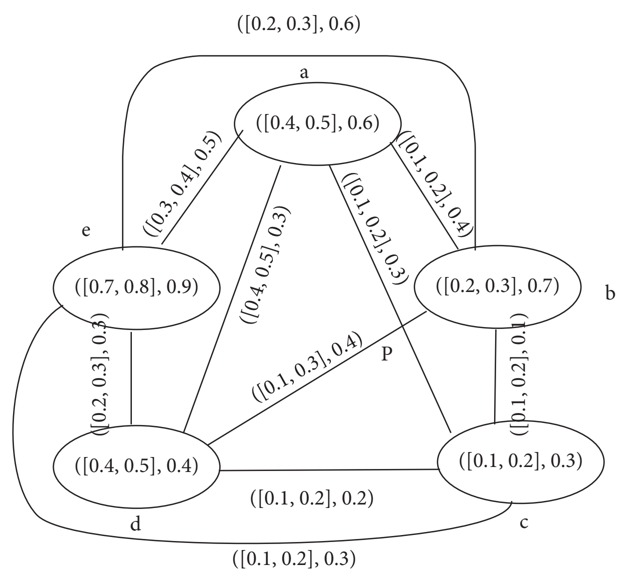

Figure 4: Not a strong complete cubic graph.

$I^-_{P_k} = I^-_{rs} + I^-_{tu}/2$. So, $\sum_{i=1}^{k} I^-_{P_i} \geq kc$, $\sum_{i=1}^{k} I^+_{P_i} \geq kc$, and $\sum_{i=1}^{k} I^*_{P_i} \geq kc$. Hence,

$$\mathsf{F}^-, \mathsf{F}^+ \text{and } \mathsf{F}^* \leq \frac{1}{1+kc}. \tag{17}$$

Since G is a strong CPG, we have

$$([0.67, 0.67], 0.67) \leq \mathsf{F} \leq \left(\left[\frac{1}{1+kc}, \frac{1}{1+kc} \right], \frac{1}{1+kc} \right). \tag{18}$$

Hence,

$$\begin{aligned} 0.67 &\leq \frac{1}{1+kc}, \\ k &\leq \left[\frac{0.49}{c} \right], \\ k &= \left[\frac{0.49}{c} \right]. \end{aligned} \tag{19}$$

□

3.1. Faces of Cubic Planar Graph. The faces of the graphs are the region bounded by the edges. For faces of the cubic planar graph, we consider a graph with the degree of planarity $([1,1],1)$ which shows clearly that the graph is planar, i.e., it has no crossing of edges. If we remove an edge, the membership value of the edge is $([0,0],0)$, then clearly the corresponding face is also removed as the area of the region is now not surrounded by an edge and two faces merge into one face. So, the number of faces depends on the strength of edges. A graph has two types of faces given as

(1) Outer face having an infinite region and not surrounded by edges.
(2) Inner face region surrounded by a finite number of edges.

Definition 10. Let $G = (C, D)$ be a strong CPG, with $\digamma = ([1,1],1)$ on $\mathfrak{B}$. The area, bounded by the arrangement of edges E, is referred to as the face of G with the strength of the face is

$$\{([\min\{I^-_{rs}\}, \min\{I^+_{rs}\}], \min\{I^*_{rs}\}) | (r,s) \in E\}. \tag{20}$$

A face having strength >rbin$([0.5,0.5],0.5)$ is called a cubic-valued strong fuzzy face; otherwise, a cubic-valued weak fuzzy face.

Example 5. Consider a cubic planar graph with a set of vertices $\mathfrak{B} = \{a,b,c,d,e\}$ and edges $\{ab, ac, bc, cd, ce, de\}$ as shown in Figure 5.

Clearly, 3 faces of the graph are f_1, f_2, and f_3. The area surrounded by the edges $\{ab, ac, bc\}$ is the face f_1, f_2 is surrounded by the edges $\{cd, ce, ed\}$, and f_3 is outer face or infinite region. Now, we compute the strength of fuzzy faces.

$$\begin{aligned}\text{Strength of } f_1 &= \{[\min\{I^-_{ab}, I^-_{bc}, I^-_{ac}\}, \min\{I^+_{ab}, I^+_{bc}, I^+_{ac}\}], \min\{I^*_{ab}, I^*_{bc}, I^*_{ac}\}\}\\ &= \{[\min\{1,1,0.5\}, \min\{1,0.83,0.66\}], \min\{0.5,0.57,0.25\}\}\\ &= \{[0.5,0.66],0.25\}.\end{aligned} \tag{21}$$

So, f_1 is not a strong face. By continuing the same process, we find strength for faces f_2 and f_3.

$$\begin{aligned}\text{Strength of } f_2 &= \{[\min\{I^-_{cd}, I^-_{ce}, I^-_{de}\}, \min\{I^+_{cd}, I^+_{ce}, I^+_{de}\}], \min\{I^*_{cd}, I^*_{ce}, I^*_{de}\}\}\\ &= \{[\min\{0.75,0.5,0.5\}, \min\{1,1,1\}], \min\{1,1,1\}\}\\ &= \{[0.5,1],1\},\\ \text{Strength of } f_3 &= \{[\min\{I^-_{ab}, I^-_{bc}, I^-_{ac}, I^-_{cd}, I^-_{ce}, I^-_{de}\}, \min\{I^+_{ab}, I^+_{bc}, I^+_{ac}, I^+_{cd}, I^+_{ce}, I^+_{de}\}], \min\{I^*_{ab}, I^*_{bc}, I^*_{ac}, I^*_{cd}, I^*_{ce}, I^*_{de}\}\}\\ &= \{[\min\{1,1,0.5,0.75,0.5,0.5\}, \min\{1,0.83,0.66,1,1,1\}], \min\{0.5,0.57,0.25,1,1,1\}\}\\ &= \{[0.5,0.6],0.25\},\end{aligned} \tag{22}$$

which shows that every face is a weak face.

E	ab	bc	cd	da	bd
	$([0.5,0.6],0.1)$	$([0.2,0.3],0.2)$	$([0.3,0.5],0.3)$	$([0.3,0.7],0.3)$	$([0.4,0.6],0.2)$
E	de	ef	fd	ae	
	$([0.4,0.7],0.4)$	$([0.1,0.2],0.5)$	$([0.1,0.2],0.3)$	$([0.6,0.7],0.3)$	

3.2. Dual Graph. The dual graph can be constructed if the graph is planar. A cubic graph has no intersection between strong edges if $\digamma \geq ([0.67,0.67],0.67)$. Thus, we can make a dual graph of a cubic graph if it is a 0.67-cubic planar graph.

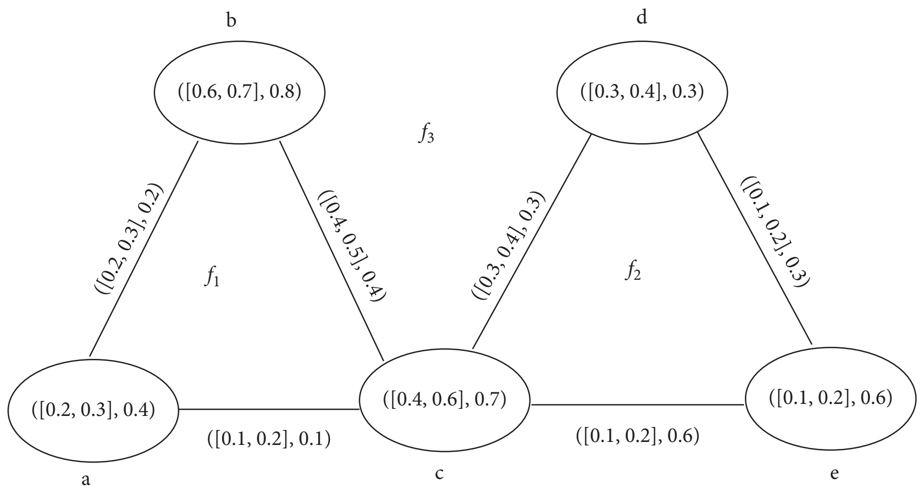

Figure 5: Cubic planar graph.

Faces of the graph correspond to vertices and edges between vertices are corresponding to edges in the boundary of faces.

Definition 11. Let $G = (C, D)$ be a cubic planar graph (means edges having no crossing) and the faces of G surrounded by the edges are $f_1, f_2, \ldots, f_k$, where $k \in \mathbb{Z}$. The dual graph of $G = (C, D)$ is $G' = (C', D')$, where C' is the vertex set and every vertex $m_i \in C\prime$ is a face in G surrounded by edges, then,

$$\nu(m_i) = \{[\max\{\chi^-(r,s)\}, \max\{\chi^+(r,s)\}], \max\{\chi^*(r,s) \mid (r,s) \text{is the edge of the boundary of the face}\}\}. \tag{23}$$

If a graph has a pendant vertex, then its dual graph contains a loop and the membership value of the loop is the same as the membership value of this pendant vertex. Every edge of the dual graph $G\prime$ is obtained if it cuts the edge of G and the membership value of the edge in $G\prime$ is the same as the membership value of edges in G. The membership value of the edges of the dual graph is represented by λ.

Example 6. Consider a cubic planar graph with the set of vertices $\{a, b, c, d, e, f\}$ and edges $\{ab, bc, cd, da, bd, de, ef, fd, ae\}$ and their membership values is given in Table 1. Now, we calculate the membership values for the dual graph by using the membership values given in Figure 6. Since, face f_1 is surrounded by the edge set $\{(a,b), (b,d), (d,a)\}$, f_2 is surrounded by the edge set $\{(b,c), (b,d), (d,c)\}$, f_3 is surrounded by the edge set $\{(a,e), (e,d), (d,a)\}$, f_4 is surrounded by the edge set $\{(d,f), (d,e), (e,f)\}$, and f_5 is surrounded by the edge set $\{(a,b), (b,c), (c,d), (d,f), (f,e), (e,a)\}$ which is the outer face. We insert a vertex in every face. Let the set of inserted vertices be $\mathfrak{B} = \{\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5\}$.

$$\begin{aligned}
\nu^-(\dot{x}_1) &= \max\{0.5, 0.4, 0.3\} = 0.5,\\
\nu^+(\dot{x}_1) &= \max\{0.6, 0.6, 0.7\} = 0.7,\\
\nu^*(\dot{x}_1) &= \max\{0.1, 0.2, 0.3\} = 0.3,\\
\nu^-(\dot{x}_2) &= \max\{0.4, 0.2, 0.3\} = 0.4,\\
\nu^+(\dot{x}_2) &= \max\{0.6, 0.3, 0.5\} = 0.6,\\
\nu^*(\dot{x}_2) &= \max\{0.2, 0.2, 0.3\} = 0.3,\\
\nu^-(\dot{x}_3) &= \max\{0.6, 0.4, 0.3\} = 0.6,\\
\nu^+(\dot{x}_3) &= \max\{0.7, 0.7, 0.7\} = 0.7,\\
\nu^*(\dot{x}_3) &= \max\{0.3, 0.4, 0.3\} = 0.4,\\
\nu^-(\dot{x}_4) &= \max\{0.1, 0.4, 0.1\} = 0.4,\\
\nu^+(\dot{x}_4) &= \max\{0.2, 0.7, 0.2\} = 0.7,\\
\nu^*(\dot{x}_4) &= \max\{0.3, 0.4, 0.5\} = 0.5,\\
\nu^-(\dot{x}_5) &= \max\{0.5, 0.2, 0.3, 0.1, 0.1, 0.6\} = 0.6,\\
\nu^+(\dot{x}_5) &= \max\{0.6, 0.3, 0.5, 0.2, 0.2, 0.7\} = 0.7,\\
\nu^*(\dot{x}_5) &= \max\{0.2, 0.2, 0.3, 0.3, 0.5, 0.3\} = 0.5.
\end{aligned} \tag{24}$$

Table 1: Membership values of vertices of Figure 6.

$\mathfrak{B}$	a	b	c	d	e	f
	$([0.6, 0.7], 0.3)$	$([0.7, 0.8], 0.2)$	$([0.3, 0.5], 0.3)$	$([0.4, 0.7], 0.4)$	$([0.8, 0.9], 0.6)$	$([0.1, 0.2], 0.5)$

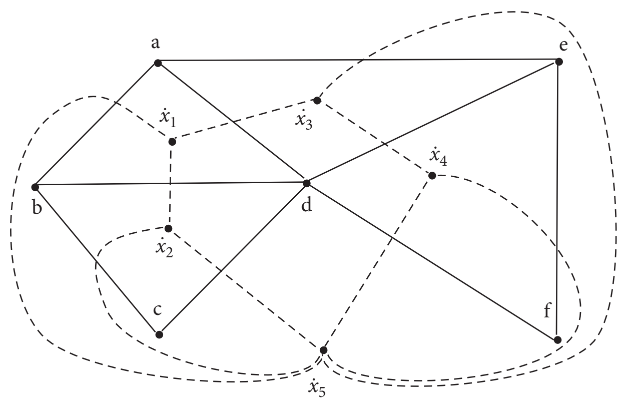

Figure 6: Example of CDG.

So, $\dot{x}_1 = ([0.5, 0.7], 0.3)\dot{x}_2 = ([0.4, 0.6], 0.3)$, $\dot{x}_3 = ([0.6, 0.7], 0.4)$, $\dot{x}_4 = ([0.4, 0.7], 0.5)$, and $\dot{x}_5 = ([0.6, 0.7], 0.5)$. Now, we calculate the edge value. The edges occur 2 times between the faces 2 and 5. $\lambda^-(\dot{x}_2, \dot{x}_5) = \chi^-(c, d) = 0.3$, $\lambda^+(\dot{x}_2, \dot{x}_5) = \chi^+(c, d) = 0.5$, $\lambda^*(\dot{x}_2, \dot{x}_5) = \chi^*(c, d) = 0.3$, $\lambda^-(\dot{x}_2, \dot{x}_5) = \chi^-(b, c) = 0.2$, $\lambda^+(\dot{x}_2, \dot{x}_5) = \chi^+(b, c) = 0.3$, and $\lambda^*(\dot{x}_2, \dot{x}_5) = \chi^*(b, c) = 0.2$.

Similarly, the edges occur 2 times between the faces 4 and 5. $\lambda^-(\dot{x}_4, \dot{x}_5) = \chi^-(d, f) = 0.1$, $\lambda^+(\dot{x}_4, \dot{x}_5) = \chi^+(d, f) = 0.2$, $\lambda^*(\dot{x}_4, \dot{x}_5) = \chi^*(d, f) = 0.3$, $\lambda^-(\dot{x}_4, \dot{x}_5) = \chi^-(f, e) = 0.1$, $\lambda^+(\dot{x}_4, \dot{x}_5) = \chi^+(f, e) = 0.2$, and $\lambda^*(\dot{x}_4, \dot{x}_5) = \chi^*(f, e) = 0.5$.

The membership value of other edges are $\lambda(\dot{x}_1, \dot{x}_2) = \chi(b, d) = ([0.4, 0.6], 0.2)$, $\lambda(\dot{x}_1, \dot{x}_3) = \chi(a, d) = ([0.3, 0.7], 0.3)$, $\lambda(\dot{x}_1, \dot{x}_5) = \chi(a, b) = ([0.5, 0.6], 0.1)$, $\lambda(\dot{x}_3, \dot{x}_4) = \chi(d, e) = ([0.4, 0.7], 0.4)$, and $\lambda(\dot{x}_3, \dot{x}_5) = \chi(a, e) = ([0.6, 0.7], 0.4)$.

Theorem 6. *Let G be a strong CPG such that e, v, and f are the number of edges, vertices, and strong faces, respectively. Let G′ be the CDG of G. Then,*

(1) The cardinality of the vertex set in G′ = f in G

(2) The number of edges in G′ = e in G

(3) The number of faces in G′ = v in G

Theorem 7. *Let $G = (C, D)$ be a strong cubic planar graph which contains no weak edge and $G' = (C', D')$ be its dual graph. Then, G and G′ have equal membership degrees of edges.*

Proof. The cubic dual planar graph of G is G' which is also a strong graph having no crossing of edges. The set of strong faces of G is consider to be $\{f_1, f_2, \ldots, f_k\}$. By definition, $\lambda_l(x_1, x_2) = \chi_l(a, b)$, where (x_1, x_2) is an edge between two strong faces f_u and f_o and $l = 1, 2, \ldots, m$ where m represents the number of common edges in the boundary of the faces f_u and f_o. The number of edges in G and G' is same such that both contain no weak edges. So, for every edge in G, there is also an edge in G' which has same membership value as in G. □

4. Application

An accident on a one-way road blocks the traffic on that road. Due to this, the flow of traffic on this road is diverted to other roads which causes a bottleneck on the other road as well. Nowadays, street mishaps occur frequently. The investigation shows that street mishaps occur due to multifactors such as street condition, number of vehicles, overspeeding, careless driving, and construction on roads. Street mishaps happen at the crossing point zone. So, if we can construct underpasses or overhead bridges over crossing points, then mishaps can be reduced. Thus, the reduction of this zone may help to settle this problem. This can be elaborated by the following example. Consider a system containing 5 vertices, each vertex represents the city and the edges represent the road between them. There is a directed road between cities, then we draw an edge between them as shown in Figure 7.

Presently, we consider the crowdness of the streets associated with urban communities. Clearly, the crowdness of a street is a fuzzy amount. We address the estimation of traffic in future by an interval and the present measure of the crowd is addressed by a fuzzy number. The strength of mobs on the roads in the future is estimated from the present and past year data. Also, during estimation of the strength of the mob, we also set some parameters in our mind like whether in the future, roads will be in the same condition or not, the number of vehicles on roads will increase or not. Moreover, there are also chances that we get an alternative road which is good in condition (like motorway or new short path). The strength of the future mob is presented by the interval but the present mob strength is shown by a fuzzy number. Presently, we relegate worth to the edges implying we are seeing the crowd on streets in the present and future. The strength of the crowdness on the roads is given in Table 2.

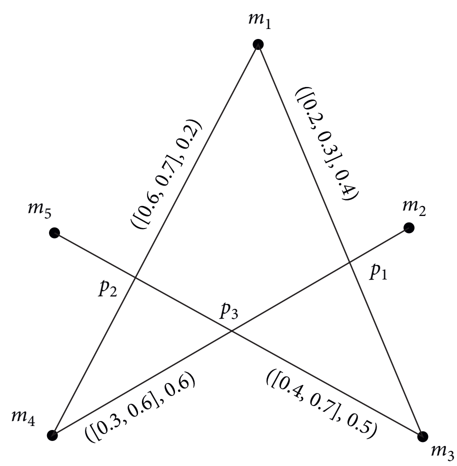

FIGURE 7: Road network.

TABLE 2: Strength of the crowdness of a road network given in Figure 7.

Roads	(m_1, m_3)	(m_1, m_4)	(m_2, m_3)	(m_3, m_5)
Mob	$([0.2, 0.3], 0.4)$	$([0.6, 0.7], 0.2)$	$([0.3, 0.6], 0.6)$	$([0.4, 0.7], 0.5)$

We can see that where a mob of two streets combined there will be more crowd and chances of mishap increases. As we can find in Figure 7, there are three places where streets have crossing and here, the chance of mishap is higher. We will assign $([1, 1], 1)$ a value to every vertex. To find the degree of planarity, we use the procedure given in following Algorithm 1.

$$
\begin{aligned}
I^-_{m_1m_3} &= \frac{\chi^-(m_1, m_3)}{\min\{\varpi^+(m_1), \varpi^+(m_3)\}} = \frac{0.2}{1} = 0.2,\\
I^+_{m_1m_3} &= \frac{\chi^+(m_1, m_3)}{\min\{\varpi^+(m_1), \varpi^+(m_3)\}} = \frac{0.3}{1} = 0.3,\\
I^*_{m_1m_3} &= \frac{\chi^*(m_1, m_3)}{\min\{\varpi^*(m_1), \varpi^*(m_3)\}} = \frac{0.4}{1} = 0.4.
\end{aligned}
\tag{25}
$$

Similarly, we can find $I^-_{m_1m_4} = 0.6$, $I^+_{m_1m_4} = 0.7$, $I^*_{m_1m_4} = 0.2$, $I^-_{m_2m_4} = 0.3$, $I^+_{m_2m_4} = 0.6$, $I^*_{m_2m_4} = 0.6$, $I^-_{m_3m_5} = 0.4$, $I^+_{m_3m_5} = 0.7$, and $I^*_{m_3m_5} = 0.5$.

We can see that it is the same as the edge value. Now, we will calculate the cutting point:

$$
\begin{aligned}
I_{P_1} &= \left(\left[\frac{I^-_{m_1m_3} + I^-_{m_2m_4}}{2}, \frac{I^+_{m_1m_3} + I^+_{m_2m_4}}{2}\right], \frac{I^*_{m_1m_3} + I^*_{m_2m_4}}{2}\right)\\
&= \left(\left[\frac{0.2 + 0.3}{2}, \frac{0.3 + 0.6}{2}\right], \frac{0.4 + 0.6}{2}\right)\\
&= ([0.25, 0.45], 0.5),\\
I_{P_2} &= \left(\left[\frac{I^-_{m_1m_4} + I^-_{m_3m_5}}{2}, \frac{I^+_{m_1m_4} + I^+_{m_3m_5}}{2}\right], \frac{I^*_{m_1m_4} + I^*_{m_3m_5}}{2}\right)\\
&= \left(\left[\frac{0.6 + 0.4}{2}, \frac{0.7 + 0.7}{2}\right], \frac{0.2 + 0.5}{2}\right)\\
&= ([0.5, 0.7], 0.35),\\
I_{P_3} &= \left(\left[\frac{I^-_{m_2m_4} + I^-_{m_3m_5}}{2}, \frac{I^+_{m_2m_4} + I^+_{m_3m_5}}{2}\right], \frac{I^*_{m_2m_4} + I^*_{m_3m_5}}{2}\right)\\
&= \left(\left[\frac{0.3 + 0.4}{2}, \frac{0.6 + 0.7}{2}\right], \frac{0.6 + 0.5}{2}\right)\\
&= ([0.37, 0.65], 0.55).
\end{aligned}
\tag{26}
$$

The degree of planarity will be as

$$
\begin{aligned}
\mathsf{F}^- &= \frac{1}{1 + 0.45 + 0.7 + 0.65} = \frac{1}{2.8} = 0.35,\\
\mathsf{F}^+ &= \frac{1}{1 + 0.25 + 0.5 + 0.35} = \frac{1}{2.11} = 0.47,\\
\mathsf{F}^* &= \frac{1}{1 + 0.5 + 0.35 + 0.55} = \frac{1}{2.4} = 0.41.
\end{aligned}
\tag{27}
$$

We observe that the degree of planarity is $([0.35, 0.47], 0.41)$ which is not close to $([0.67, 0.67], 0.67)$; therefore, roads will be a mob. If the intersection of roads is removed, then the ratio of the mob will be decreased automatically, so the accident ratio will also be decreased. So, we concluded that we need to build an underpass or flyover at the intersecting points. We can also see that number of vehicles on the roads (in short, crowd) in the future will almost be equal to the number of vehicles at the present time.

If we get the degree of planarity $\geq ([0.67, 0.67], 0.67)$, the intersection of roads will not be a reason for mishaps and we do not need to remove the intersection of roads.

Step 1. Consider a rough fuzzy road network of developing countries.
Step 2. Find the direction of flow in the considered network.
Step 3. Notice the intersection or cutting point for the traffic flow.
Step 4. Disregard the noncrossing traffic stream from the street network for additional conversation.
Step 5. Find the value of the cutting point by using the equation (5).
Step 6. Similarly, find the degree of planarity by using the equations (6)–(8).

Algorithm 1: Method to determine planarity in a road network.

Table 3: Comparison of $\digamma$.

Method	Degree of planarity	Result
FPG [15]	$\digamma = 0.5$	Cutting of strong edges in G is at most one
IVPG [29]	$\digamma < [0.5, 0.5]$	Cutting of strong edges in G can be more than one
CPG	$\digamma < \langle [0.5, 0.5], 0.5 \rangle$	Cutting of strong edges in G can be more than one

4.1. Comparative Analysis. In this part, we compare our new portrayed technique with the already existing strategies which are fuzzy planar graphs and interval-valued planar graphs. We can see that if the level of planarity or degree of planarity for a certain fuzzy planar graph is ≥ 0.5, the level of planarity for the corresponding interval-valued planar graph can be ≤ 0.5 as well as the other way around. This situation can be settled by using the cubic planar graph model. Thus, in the cubic planar graph, the level of planarity for a graph is more precise than the level of planarity for FPG or IVPG. The level of planarity for Example 4 is given in Table 3 using the model of FPG [15], model of IVPG [29], and our proposed model CPG.

This shows that using PFG when it has cutting point at most 1 and it has more than 1 cutting point using IVPG. We combine them and see that graph has cutting point among strong edges at most 1 if and only if the cutting point among strong edges of both PFG and IVPG is at most 1 which shows its efficiency. In this way, it is more efficient together as though the level of planarity in both cases ≥ 0.67 then is considered as a strong cubic planar graph.

4.2. Limitations and Advantages. The proposed technique of cubic planar graphs is restricted to undirected graphs only. This technique is more beneficial as we merge two types of strategies together in one model. Due to this, it covers the advantages of both the FPG and IVPG models. Moreover, weakness in one method can be overcome by other methods in one model. Cubic planar graphs can deal with two types of processes, i.e, continuous and discrete. For example, the future and past process can be considered as a continuous and present value can be taken as discrete. Additionally, a level of planarity exists in each cubic planar graph regardless of whether it has a cutting point or not.

5. Conclusion

Planar graphs are useful in the designing of circuits and road networks. We had proposed the notion of cubic planar graphs which is a combination of interval-valued planar graphs and fuzzy planar graphs. We came up here with the notions of strong and weak cubic edges, degree of planarity, considerable number, and considerable edges. We similarly inspected some huge results connected with planarity. We likewise discuss strong and weak cubic faces and dual graphs for a cubic planar graph. We additionally prove some results related to faces and dual graphs of a cubic planar graph. We moreover give a concise use of a cubic planar graph to enhance its worth and significance. The future directions of our work are to investigate the planarity of cubic intuitionistic fuzzy graphs and cubic Pythagorean fuzzy graphs.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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