Research Article

Energy Saving via a Minimal Structure

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1. Introduction

The current era is distinguished by its ability to collect and store large amount of data on any topic we want, to make, and take a decision regarding it. Uncertainty is a characteristic of almost all real-life problems and mathematics plays a vital role in processing these data to obtain the best decisions.

The analysis process has many mathematical methods that we can use. Since the current statement of a rough set theory, topology and decision making have had a long history together. The theory of rough set is a useful tool for analyzing unclear data. Pawlak first introduced the rough set model in 1982 [1] and constructed a topological space using the equivalent class of equivalence relations on the items as its foundation [2–4]. These equivalence classes is a base for a special type of topological structure in which every open set is closed. This condition is a constrain which limits the range of application. Some extensions under other relations, such as 1998 [5, 6], have been proposed in an attempt to solve such unreasonableness. New types of neighborhoods were added for various goals, including improving the set’s accuracy values, such as intersection neighborhoods, which contained some topology expansions such as the minimal structure [2, 7]. The manuscript contributes to this trend by employing a simple structure to generalize rough set approximation. The data are formatted as an information system (Σ) and are processed in several stages. The first of which is the selection of effective features (reduction) and the second is an approximation. This method was used for many medical applications such as those in [7, 8] and engineering applications such as those in [9, 10], and others such as those in [2, 11–15].

Many researchers all over the world attempted to overcome the shortcomings that the original model of rough set theory may lose in the classification or have a discrete classification in which every class has a single element. This paper aims to construct lower and upper approximations based on a similarity degree between objects of Σ and demonstrate that a minimal structure framework can be used to solve a variety of real-world challenges. Consequently, we apply these concepts to obtain reducts and core of Σ using the accuracy measure of decision sets.
We use a variable precision to improve the decision accuracy in our approach. Also, we provide a comparison between our strategy and Pawlak’s strategy.

The following is the structure of this paper:

We go over the fundamental ideas in the second section. In the third section, we describe our proposed method for reducing the characteristics. In the fourth section, there are some examples that explain the method used.

2. Preliminaries

Definition 1. [16] Information system

An information system (Σ) can be written as a (N, h, V, f) where N is the universe of finite objects, N = \{N_1, N_2, \ldots, N_n\}, V attribute values, and h is the set of attributes \{h_1, h_2, \ldots, h_m\}, f: N x h → V.

(1) These objects, for example: patients, electric generators, students, . . .

(2) The attributes can be features or characteristics of objects.

Example 1. An information system for an exam of five students in Table 1, where N = \{N_1, N_2, N_3, N_4, N_5\}, h = \{M, C, P, E\} are four courses, mathematics M, chemistry C, physics P, and English language E. The entries of the table represent the set V of the scores for the students.

Adding the column D values B, C, B, and D is an example of the decision table [Table 2].

Definition 2 [17, 18]. A minimal structure (short for \mathcal{M}-structure on a set \mathcal{N} defined as \mathcal{M} \subseteq \mathcal{P}(\mathcal{N}) (the collection of all \mathcal{N} subsets) containing (\emptyset, \mathcal{N}). A minimal structure space is defined by the pair (\mathcal{N}, \mathcal{M}) (simplify, \mathcal{M}-space). Members of \mathcal{M}'s are known as open sets, and the complement of \mathcal{M}'s is known as closed sets.

Definition 3 [17, 18]. Let \mathcal{M} be an \mathcal{M}-structure on \mathcal{N} and \mathcal{N} be a nonempty set. Let \mathcal{M}_K be an \mathcal{M}_K-structure on \mathcal{N} and \mathcal{N} be a nonempty set.

(i) \mathcal{M}(\mathcal{N}) = \cap\{\mathcal{F}: \mathcal{N} \subseteq \mathcal{F}, \mathcal{N}/\mathcal{F} \in \mathcal{M}\},

(ii) \mathcal{M}(\mathcal{N}) = \cup\{\emptyset: \emptyset \subseteq \mathcal{D}, \emptyset \in \mathcal{M}_K\}.

Definition 4 [3, 19]. If \mathcal{N} is a set of objects that is not empty, and an equivalence relation on \mathcal{N} is named \mathcal{R}, the approximation space is named after the pair (\mathcal{N}, \mathcal{R}). Let \mathcal{N} \subseteq \mathcal{N}, the rough set approximation is as follows:

(i) The set of all objects that can be categorized as \mathcal{N} concerning \mathcal{R} is called upper approximation of \mathcal{N} concerning \mathcal{R} and is denoted by \mathcal{R}(\mathcal{N}). That is, \mathcal{R}(\mathcal{N}) = \{n \in \mathcal{N} : \mathcal{R}(n) \cap \mathcal{N} \neq \emptyset\}, where n determines the equivalence class and \mathcal{R}(n) is the equivalence class.

(ii) The lower approximation of \mathcal{N} concerning \mathcal{R} is \mathcal{R}^-(\mathcal{N}), which denotes the set of all objects that may absolutely be classified as \mathcal{N} concerning \mathcal{R}. To put it another way, \mathcal{R}^-(\mathcal{N}) = \{n \in \mathcal{N} : \mathcal{R}(n) \subseteq \mathcal{D}\}, where \mathcal{R}(n) denotes the equivalence class determined by n.

(iii) \mathcal{R}(\mathcal{N}) \subseteq \mathcal{D} boundary region of \mathcal{N} for \mathcal{R}. That is,

\begin{equation}
\mathcal{R}(\mathcal{N}) = \mathcal{R}(\mathcal{N}) - \mathcal{R}^-(\mathcal{N}),
\end{equation}

which is the collection of all elements that cannot be categorized as either \mathcal{N} or \mathcal{N}/\mathcal{R}. The set \mathcal{N} is said to be rough concerning \mathcal{R} if \mathcal{R}(\mathcal{N}) \neq \mathcal{R}(\mathcal{N}). That is, \mathcal{N} \neq \mathcal{R}(\mathcal{N}) \neq \emptyset. Then, the set \mathcal{N} is called rough to \mathcal{R}.

Definition 5 [3, 19]. Let (\mathcal{N}, \mathcal{R}, \mathcal{D}) be a decision approximation space, with (\mathcal{N}, \mathcal{R}) being an approximation space and \mathcal{D} being a decision equivalence relation on \mathcal{N}. Hence, \mathcal{N}/\mathcal{D} = \{Z_1, Z_2, \ldots, Z_n\} is a partition on \mathcal{N}. In terms of, the lower and upper approximations of \mathcal{N}/\mathcal{D} with respect to \mathcal{R} denoted as follows:

\begin{equation}
\mathcal{R}^-(\mathcal{N}/\mathcal{D}) = \{\mathcal{R}(Z_1), \mathcal{R}(Z_2), \ldots, \mathcal{R}(Z_n)\},
\end{equation}

\begin{equation}
\mathcal{R}^+(\mathcal{N}/\mathcal{D}) = \{\mathcal{R}(Z_1), \mathcal{R}(Z_2), \ldots, \mathcal{R}(Z_n)\}.
\end{equation}

2.1. Variable Precision Model

(1) For all \mathcal{C}, \mathcal{D} \subseteq \mathcal{N} such that \mathcal{N} denotes a non-null finite universe set and \mathcal{C} \subseteq \mathcal{D} that all elements of \mathcal{C} are contained in \mathcal{D}.

(2) \mathcal{C} \subseteq \mathcal{D} with an error ratio \beta based on the value of measure \mathcal{F}(\mathcal{C}, \mathcal{D}) is discussed in [4, 7], i.e., \mathcal{C} \subseteq \mathcal{D} if and only if \mathcal{F}(\mathcal{C}, \mathcal{D}) \leq \beta, where \mathcal{F}(\mathcal{C}, \mathcal{D}) = 1 - \text{card}(\mathcal{C} \cap \mathcal{D})/\text{card}(\mathcal{C}) if \text{card}(\mathcal{C}) > 0, \mathcal{F}(\mathcal{C}, \mathcal{D}) = 0 if \text{card}(\mathcal{C}) = 0, where the allowable error must be 0 < \beta \leq 0.5.
3. Similarity Approach for Reduction

We demonstrate the usefulness of a minimal structural space in engineering science for use in decision-making situations in this section. Therefore, we have decided to use it to solve a technical problem. This data collection contains the results of four attributes for eight different generators. The research study was carried out at the Princess Nourah bint Abdulrahman University’s Mathematical Department in Riyadh, Saudi Arabia. The study includes eight different generators presenting to this university with different presenting features, where the first two are two different transmission lines and the second two for self-generation and self-load. 1 denotes low, 2 denotes medium, and 3 denotes high. Finally, the heart failure diagnosis is normal or rest.

The equivalence relation is a condition that sometimes generates a discrete equivalence classes, that is, each class contains one object, and this makes all concepts exactly as in [20].

The following example for an \( \Sigma \) when applying Pawlak methods gives equivalence classes each of which contains singleton.

Example 3. Consider the table of \( \Sigma \) (Table 3).

Using the original model, the classes are \( \{ [a], [b], [c], [d], [e] \} \), but the similarity degrees are as follows:

\[
\mu(a,a) = 1, \mu(a,b) = \frac{1}{5}, \mu(a,c) = \frac{1}{5}, \mu(a,d) = \frac{2}{5}, \mu(a,e) = \frac{2}{5}
\]

which produce no discrete classes that give nontrivial approximation.

Definition 6. Attribute similarity.

In an \( \Sigma = (\mathcal{N}, h = D \cup C, V, f) \), for an attribute \( h_i \), the similarity value for two objects \( x, y \) is denoted by \( \delta_i(x, y) \),

\[
\delta_i(x, y) = \begin{cases} 
1, & h_i(x) = h_i(y), \\
0, & h_i(x) \neq h_i(y).
\end{cases}
\]

The similarity degree among two objects \( x, y \) is as follows:

\[
\mu(x, y) = \frac{\sum_{i=1}^{n} \delta_i(x, y)}{n} \quad \forall a_i \in h,
\]

where \( n \) is \( \mathcal{N} \)’s cardinality.

3.1. Discussion of the Results of the Experiment. We explain the experimental results in this paragraph by introducing a preparatory study that was conducted on four features for eight generators. The study was conducted at the Mathematical department, Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia. The remaining 8 records were offered to this university with comparable presenting qualities, such as comprehensive history, date of manufacture, volume, power, and wren done. Due to similar features, the data in the information system for just eight features, as shown in Table 4, discuss energy conservation.

The following example is taken from an engineering problem.

Example 3. In Table 4, our universe contains eight different generators, \( \mathcal{N} = \{ N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8 \} \). Moreover, there are four features and a decision. The feature set is \( h = \{ h_1, h_2, h_3, h_4 \} \) where the first two are two different transmission lines and the second two are for self-generation and self-load. 1 denotes low, 2 denotes medium, and 3 denotes high.

Now, using the connection that is relevant to the nature of the examined problem, we design a minimal structure space. It is worth noting that we define the relationship in each case based on the expert’s specifications. In this instance, \( \mathcal{A}B \iff \mu(a, b) > 4/5 \), where \( \mu(a, b) \) is the sum of the similar features between \( a \) and \( b \) divided on the number of features.

From Table 5, we have \( \mathcal{A}(N_1) = \{ N_1, N_2, N_3, N_4 \} \), \( \mathcal{A}(N_2) = \{ N_2, N_3, N_4 \} \), \( \mathcal{A}(N_3) = \{ N_4 \} \), \( \mathcal{A}(N_4) = \{ N_1, N_3 \} \), \( \mathcal{A}(N_5) = \{ N_5 \} \), \( \mathcal{A}(N_6) = \{ N_6, N_7 \} \), \( \mathcal{A}(N_7) = \{ N_8 \} \). Then, the minimal structure space on \( \mathcal{N} \) is as follows:

(i) \( \mathcal{M}_R = \{ \mathcal{N}, \emptyset, \{ N_1 \}, \{ N_2, N_3 \}, \{ N_4 \}, \{ N_3 \}, \{ N_6, N_8 \}, \{ N_7 \} \} \) and

(ii) \( \mathcal{M}_R = \{ \mathcal{N}, \emptyset, \{ N_1 \}, \{ N_2, N_3 \}, \{ N_4 \}, \{ N_3 \}, \{ N_6, N_8 \}, \{ N_7 \} \} \).

We can calculate the accuracy of decision making for two groups of features using the rough set theory:

1. \( \mathcal{D}_N = \{ N_1, N_3, N_6, N_8 \} \), the set of normal generators.
2. \( \mathcal{D}_R = \{ N_2, N_4, N_5, N_7 \} \), the set of rest generators.

Therefore, the lower, the upper, and the accuracy of \( \mathcal{D}_N \) are calculated as follows:

<table>
<thead>
<tr>
<th>l</th>
<th>m</th>
<th>n</th>
<th>o</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Information system table.

<table>
<thead>
<tr>
<th>EX</th>
<th>h_1</th>
<th>h_2</th>
<th>h_3</th>
<th>h_4</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>N</td>
</tr>
<tr>
<td>N_2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>R</td>
</tr>
<tr>
<td>N_3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>N</td>
</tr>
<tr>
<td>N_4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>R</td>
</tr>
<tr>
<td>N_5</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>R</td>
</tr>
<tr>
<td>N_6</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>N_7</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>R</td>
</tr>
<tr>
<td>N_8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>N</td>
</tr>
</tbody>
</table>

Table 4: Table of the attributes.
3.2. Reduction of Attributes. Here begins the reduction for the attributes by constructing similarity matrices by removing one of the features and noticing how that will affect the accuracy of the decision set.

The results in Table 7 denote the values by removing $h_4$, while Table 8 shows the accuracy:

$$\mathcal{M}_N = \{N, \emptyset, \{N_1\}, \{N_2, N_3\}, \{N_4\}, \{N_5\}, \{N_6, N_7, N_8\}\} \text{ and,}$$

$$\mathcal{M}_N' = \{N, \emptyset, \{X\}, \{X_1\}, \{X_2\}, \{X_3\}, \{X_4\}, \{X_5\}, \{X_6, X_7, X_8\}\}.$$

3.2.1. For $h = \{h_1, h_2, h_3\}$. The $h_4$ variable is not superfluous and $\{h_1, h_2, h_3\}$ is not a reduction. We repeat the same steps for all possible subsets of $h$.

3.2.2. For $h = \{h_1, h_2, h_4\}$. The $h_3$ variable is superfluous and $\{h_1, h_2, h_4\}$ is a reduction.

3.2.3. For $h = \{h_1, h_3, h_4\}$. The $h_2$ variable is superfluous and $\{h_1, h_3, h_4\}$ is a reduction.

3.2.4. For $h = \{h_2, h_3, h_4\}$. The $h_1$ variable is not superfluous and $\{h_2, h_3, h_4\}$ is not a reduction.

We conclude that $\{\{h_1, h_2, h_3\}, \{h_1, h_3, h_4\}\}$ is the class of 2 effective feature subsets and the core of effective features is $\{h_1, h_4\}$.

3.3. Using a Variable Precision Rough Set Model to Improve Accuracy. Because decision making is so important in engineering situations, we can use a variable precision $\beta$ as suggested in [21, 22] to try to improve accuracy. We may see the inconsistency when we compare the definitions of $\mathcal{M}$-interior and $\mathcal{M}$-closure with a majority $\beta$ to traditional definitions. As a result, we present a novel $\mathcal{M}$-interior and $\mathcal{M}$-closure generalization concept as follows:

### Table 5: Similarities between features of eight generators.

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
<th>$N_6$</th>
<th>$N_7$</th>
<th>$N_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>3/4</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
<td>1</td>
<td>1/2</td>
<td>1/4</td>
<td>1/2</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td>1</td>
<td>1</td>
<td>1/2</td>
<td>1/4</td>
<td>1/2</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td>1</td>
<td>1/4</td>
<td>1/2</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td>3/4</td>
<td>1/4</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1</td>
<td>1/4</td>
<td>1/4</td>
</tr>
<tr>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1</td>
<td>1</td>
<td>3/4</td>
<td>1</td>
</tr>
<tr>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1/4</td>
<td>1</td>
<td>3/4</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 6: Accuracy table of the similarity matrix.

<table>
<thead>
<tr>
<th>Decision set $D_N = {N_1, N_2, N_6, N_8}$</th>
<th>$D_R = {N_2, N_4, N_5, N_7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{R}$</td>
<td>${N_1, N_6, N_8}$</td>
</tr>
<tr>
<td>$\mathcal{R}'$</td>
<td>${N_4, N_5, N_7}$</td>
</tr>
</tbody>
</table>

### Table 7: Table of $h = \{h_1, h_2, h_3\}$.

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$N_5$</th>
<th>$N_6$</th>
<th>$N_7$</th>
<th>$N_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/3</td>
<td>1</td>
<td>3/4</td>
<td>3/4</td>
<td>3/4</td>
<td>1</td>
<td>3/4</td>
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<tr>
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<td>3/4</td>
<td>3/4</td>
<td>3/4</td>
<td>1</td>
<td>3/4</td>
</tr>
</tbody>
</table>

### Table 8: Accuracy table of $h = \{h_1, h_2, h_4\}$.

<table>
<thead>
<tr>
<th>Decision set $D_N = {N_1, N_2, N_3, N_8}$</th>
<th>$D_R = {N_2, N_4, N_5, N_7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{R}$</td>
<td>${N_1, N_3, N_8}$</td>
</tr>
<tr>
<td>$\mathcal{R}'$</td>
<td>${N_4, N_5}$</td>
</tr>
</tbody>
</table>

Accuracy

$$\frac{1}{6}, \frac{2}{7}$$
Definition 7. Let \( \mathcal{N} \) be a nonempty set and \( \mathcal{M} \) a \( \mathcal{M} \)-structure on \( \mathcal{N} \). For a subset \( \mathcal{A} \) of \( \mathcal{N} \) and variable precision \( 0 < \beta \leq 0.5 \),

1. \( \mathcal{M}_\beta^{-}(\mathcal{A}) = \cap \{ \mathcal{F} \subseteq \mathcal{N} : \mathcal{C}(\mathcal{F},\mathcal{A}) < 1 - \beta \} \),
2. \( \mathcal{M}_\beta^+(\mathcal{A}) = \cup \{ \mathcal{O} \cap \mathcal{D} : \mathcal{C}(\mathcal{O},\mathcal{A}) \leq \beta, \mathcal{O} \in \mathcal{M} \} \).

Example 4. Comparison between our improved method and Pawlak’s approach.

From Table 9, at the variable \( \beta = 0.5 \) in the original information system, then

1. \( \mathcal{R}_{0.5}^{-}(\mathcal{D}_\mathcal{N}) = \{N_1, N_3, N_6, N_8\} \), \( \mathcal{R}_{0.5}^+(\mathcal{D}_\mathcal{N}) = \{N_1, N_2, N_3, N_5, N_6, x_3\} \), and \( \Gamma_{0.5}(\mathcal{D}_\mathcal{N}) = 4/5 \).
2. \( \mathcal{R}_{0.5}^{-}(\mathcal{D}_\mathcal{R}) = \{N_2, N_4, N_5, N_7\} \), \( \mathcal{R}_{0.5}^+(\mathcal{D}_\mathcal{R}) = \{N_2, N_3, N_4, N_5, x_3\} \), and \( \Gamma_{0.5}(\mathcal{D}_\mathcal{R}) = 4/5 \).

Based on a variable precision rough set model, we can see that the accuracy is improving in Example 4. Furthermore, when it comes to removing \( h_3 \) and \( h_2 \), the accuracy does not change. Table 9 illustrates that our method is improving in comparison to Pawlak’s method [1].

4. Conclusion

The suggested approach helps in selecting effective features in some type of information system for which we cannot get reducts using the original rough set model.

The approach is suitable for researchers from application fields since it does not need lot of mathematics. The subsequent work will consider control strategies (related to medicine) and mathematical models (related to fuzzy with a minimal structure) in the incipient stage of cancer [23–25].

Abbreviations

\[ \Sigma: \ \text{Information system} \]
\[ \mathcal{M}: \ \text{A minimal structure} \]
\[ 2^\mathcal{N}: \ \text{The collection of all } \mathcal{N} \text{ subsets} \]
\[ (\mathcal{M}_\mathcal{N} - \text{CL}(\mathcal{N})) : \text{The } \mathcal{M}_\mathcal{N} \text{-closure of } \mathcal{N} \]
\[ (\mathcal{M}_\mathcal{N} - \text{INT}(\mathcal{N})) : \text{The } \mathcal{M}_\mathcal{N} \text{-interior of } \mathcal{N} . \]

Data Availability

The data sets generated during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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