Study on Energy-Saving Optimization of Urban Rail Transit Train Timetable under Regenerative Braking

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Abstract

Energy-saving driving and regenerative braking energy utilization are two main ways to realize energy-saving optimization of urban rail transit train timetables. On the basis of the more mature energy-saving driving achievements of the predecessors, the absorption utilization rate of regenerative braking energy is improved by adjusting the dwell time of trains in the station so that while a train is braking, other trains in the same electric section are just under traction. In this paper, the overlap time between the traction and braking processes of different trains is used as a measure of the proportion of regenerative braking energy that is absorbed. In order to maximize this overlap time, an energy-saving optimization model of urban rail transit train timetable based on regenerative braking technology was established. To facilitate the solution, the nonlinear constraints are converted to linear at the time of model construction in this paper. In the solution, the spatio-temporal local rolling algorithm and the commercial optimization software ILOG CPLEX are used for the solution. The solution results show that the method in this paper can effectively improve the absorption and utilization of regenerative braking energy.

1. Background

With the development of global urbanization, the phenomenon of overconcentration of population in big cities is becoming more and more common, which also leads to frequent urban diseases in many big cities, especially in traffic congestion, traffic pollution, and other aspects. In order to solve the representative traffic problems in urban diseases, many cities have taken a lot of targeted measures, among which the large-scale development of the urban rail transit system is one of the most effective means. However, although urban rail transit has the advantages of large transportation volume and environmental protection compared with other modes of transportation, its huge energy consumption will still cause great pressure on the city’s resource carrying capacity [1]. Therefore, reducing the energy consumption of the urban rail transit system has become a hot spot. In the related research, it has become the main research direction to reduce the traction energy consumption by optimizing the driving operation strategy or reducing the train running resistance [2–6], which has been widely used in the actual work. In recent years, with the popularization of regenerative braking technology in urban rail transit systems [7], the way of effectively using regenerative braking energy to reduce the energy consumption of urban rail transit operation [8] provides a new direction for energy conservation of urban rail transit system. Regenerative braking refers to the process in which the train switches the motor to generate operation under braking conditions, uses the inertia of the train to drive the motor rotor to rotate to generate reverse torque, and converts part of kinetic energy or potential energy into electric energy for storage or utilization. Due to the short distance between urban rail transit stations and frequent braking, the electric energy generated by regenerative braking is very considerable, and may even reach more than 40% of traction energy consumption [9].
However, in the actual operation process of urban rail transit, regenerative braking energy cannot achieve the expected energy-saving effect, which is always difficult to be effectively used for various reasons [10]. According to the actual operational data, only 15–19% of regenerative braking electric energy can be effectively utilized [11], and the rest of the electric energy is finally converted into heat through braking resistance. In addition, the regenerative braking electric energy will damage the traction energy system of the train due to the large fluctuation of catenary voltage caused by its strong impact in a short time. From the perspective of actual operating results, both the use of energy storage devices and the use of grid coordinated control will encounter bottlenecks in reality. This also makes the most effective way to solve the problem of regenerative braking energy utilization at this stage by adjusting the time-space relationship between trains in the same electric section to improve the utilization ratio of regenerative braking energy and reduce the voltage fluctuation of the electric section.

In this research field, because the recovery of regenerative braking electric energy is affected by various practical factors and cannot be accurately calculated, scholars generally convert it into the following forms for estimation:

(1) Length of overlapping time of train traction process and braking process of another train [12–14].

(2) The smaller value of mechanical energy of two trains within the overlapping time of traction process and braking process [15–17].

(3) Overlapping time of mechanical energy of two trains within the overlapping time of traction process and braking process [18, 19].

According to the above estimates, scholars have established the corresponding energy-saving timetable optimization model. For example, in the literature [20], the researchers propose an integrated method of optimizing energy efficiency for multiple trains in multiple interstations with regenerative braking; and in addition, some scholars have studied how to optimize the energy-saving timetable while avoiding delay propagation in view of urban rail transit congestion [21, 22]. Some scholars formulate an optimization model incorporating energy allocation and passenger assignment based on energy-regenerative technologies and smart-card data to balance energy use and passenger travel time [23]. Furthermore, in order to minimize the net traction energy consumption (i.e., the difference between traction energy and feedback energy) of trains in an urban rail transit system, an energy-saving optimization strategy of multitrain urban rail transit train timetable based on double decision variables is proposed [24].

However, in the above research results on regenerative braking energy utilization, the model requires a high level of real-time control due to the excessive factors considered, which also makes it difficult to adjust the subsequent operation scheme once the actual operation state differs from the initial scheme. Therefore, with the widespread use of regenerative braking technology in the urban rail transit system, there is an urgent need for an energy-saving timetable method that satisfies both energy conservation and facilitates practical operation management of a multitrain energy-saving operation strategy. Based on this idea, this paper establishes a multitrain energy-saving timetable optimization model for urban rail transit based on regenerative braking conditions. The model simplifies the calculation of the utilization rate of regenerative braking energy involving multiple factors to the overlap time between the inbound traction process and the outbound braking process between trains. Although this treatment affects the accuracy of the results, it enables a direct link between the regenerative braking energy utilization and the train driving strategy. The decision variables are selected in a way that makes it easier to make practical control and scheduling decisions.

From the perspective of actual operation process, the optimization scheme of urban rail transit train timetable under regenerative braking condition must meet three conditions of "good energy-saving effect, guaranteed transportation tasks, and easy control":

(1) From the perspective of "good energy-saving effect," the running process of the train in the section shall be carried out according to the classic energy-saving operation method, without considering to improve the utilization rate of regenerative braking electric energy by adjusting the running time of the section, otherwise, it is equivalent to increasing the utilization rate of regenerative braking electric energy while increasing the energy consumption of the train in the section, and it is difficult to measure whether the final result is energy saving.

(2) From the perspective of "guaranteed transportation tasks," the optimization of the timetable cannot change the full-time operation scheme of metro trains on a large scale, but only make slight change in line with the operation specifications, otherwise, there is a risk that the transportation task cannot be guaranteed.

(3) From the perspective of "easy control," it is difficult to accurately adjust the running process of the train in the section according to the actual changes in the actual operation. Therefore, if the train wants to be "easy to control" in actual operation, it should not adjust its timetable after departure.

Therefore, the research of this paper is based on the following premises:

(1) The speed-time curve of each section is embedded in advance in the automatic train operation system, and the speed-time curve is determined by the section running time requirements and the optimal energy-saving maneuvering scheme of a single train. The speed-time curve cannot be changed during the actual operation of the train.
The initial timetable is determined by the traditional method: first, the number of trains in each hour is determined by the passenger flow; then the running interval is determined by the principle of equal time interval.

A train can only make minor adjustments to its departure time and dwell time at each station before it departs from the first station. Once the train departs from the first station, the relevant time will not be changed.

### 2. Optimization Model of Train Timetable for Urban Rail Transit

#### 2.1. Relationship between the Utilization of Regenerative Braking Energy and Train Timetables

Generally speaking, the operation of metro trains in the section from station $n$ to adjacent station $m$ can be divided into four stages according to the speed change as shown in Figure 1: traction; cruising; idle running; and braking.

Among them, the electric energy that the train needs to obtain from the grid in the process of traction is far more than the electric energy that the train needs in the other three processes. Therefore, this process is called “using extra electric energy stage” (hereinafter referred to as UES) in this paper, and the parameter $\tau_{a_{j,m}}$ is defined to represent the duration of this process. When the train $i$ leaves the station $n$ at $d_{i,n}$, this paper marks its time interval of UES as $[d_{i,n}, d_{i,n} + \tau_{d_{i,n}}]$.

In the process of braking, the train generates electrical energy through regenerative braking technology. This process is called “generating regenerative braking electric energy stage” (hereinafter referred to as GES) in this paper, and the parameter $\tau_{a_{j,m}}$ is defined to represent the duration of this process. When the train $i$ enters the station $n$ at $a_{i,m}$, this paper marks its time interval of GES as $[a_{i,m} - \tau_{a_{i,m}}, a_{i,m}]$.

These two parameters $\tau_{d_{i,n}}$ and $\tau_{a_{j,m}}$ can be expressed in Figure 2, and they also be expressed in the form of the train timetable as shown in Figure 3. Generally speaking, there is a size relationship between $\tau_{d_{i,n}}$ and $\tau_{a_{j,m}}$: $\tau_{d_{i,n}} \geq \tau_{a_{j,m}}$, but in some cases, it may also exist: $\tau_{d_{i,n}} < \tau_{a_{j,m}}$. In order to ensure the applicability of this paper, the size relationship of $\tau_{d_{i,n}}$ and $\tau_{a_{j,m}}$ is not limited.

Since the effective utilization of regenerative braking electric energy occurs only between trains in the same electric section, it is necessary to define a 0–1 parameter $\lambda_{m,n}$ to indicate the location relationship between stations and electric sections. $\lambda_{m,n}$ takes 1 if the station $n$ and the station $m$ are located in the same electric section; otherwise, it takes 0.

Under the same electric section (That is, $\lambda_{m,n} = 1$), the overlapping time of $[d_{i,n}, d_{i,n} + \tau_{d_{i,n}}]$ and $[a_{j,m} - \tau_{a_{j,m}}, a_{j,m}]$ is the effective utilization time of regenerative braking energy, which is called “effective use time of regenerative braking electric energy” (hereinafter referred to as EUES). In this paper, the duration of EUES is defined as the variable $U_{\tau_{d_{i,n}}}^{\tau_{a_{j,m}}} - \tau_{a_{j,m}}$ whose values can be analyzed in the following three categories:

1. $[d_{i,n}, d_{i,n} + \tau_{d_{i,n}}]$ and $[a_{j,m} - \tau_{a_{j,m}}, a_{j,m}]$ have no overlapping time, as shown in Figures 4 and 5. This situation can be expressed in mathematical language as follows:
   
   When $\lambda_{m,n} = 1$
   
   If
   
   $d_{i,n} + \tau_{d_{i,n}} \leq a_{j,m} - \tau_{a_{j,m}}$ or $a_{j,m} \leq d_{i,n}$, (1)
   
   then
   
   $U_{\tau_{d_{i,n}}}^{\tau_{a_{j,m}}} - \tau_{a_{j,m}} = 0$. (2)

2. $[d_{i,n}, d_{i,n} + \tau_{d_{i,n}}]$ is completely included in the scope of $[d_{i,n}, d_{i,n} + \tau_{d_{i,n}}]$, or $[a_{j,m} - \tau_{a_{j,m}}, a_{j,m}]$ is completely included in the scope of $[d_{i,n}, d_{i,n} + \tau_{d_{i,n}}]$, as shown in Figures 6 and 7. This situation can be expressed in mathematical language as follows:
   
   When $\lambda_{m,n} = 1$
   
   If
   
   $a_{j,m} - \tau_{a_{j,m}} \leq d_{i,n}$ and $d_{i,n} + \tau_{d_{i,n}} \leq a_{j,m}$, (3)
   
   then
   
   $U_{\tau_{d_{i,n}}}^{\tau_{a_{j,m}}} - \tau_{a_{j,m}} = (d_{i,n} + \tau_{d_{i,n}}) - d_{i,n} = \tau_{d_{i,n}}$. (4)

else if

   $a_{j,m} - \tau_{a_{j,m}} > d_{i,n}$ and $a_{j,m} < d_{i,n} + \tau_{d_{i,n}}$, (5)

then

   $U_{\tau_{d_{i,n}}}^{\tau_{a_{j,m}}} - \tau_{a_{j,m}} = a_{j,m} - (a_{j,m} - \tau_{a_{j,m}}) = \tau_{a_{j,m}}$. (6)
\[ d_{i,n} + \tau d_{i,n} \quad \text{and} \quad a_{j,m} - \tau a_{j,m}, a_{j,m} \] have partial time overlap, as shown in Figure 8 and 9. This situation can be expressed in mathematical language as follows:

When \( \lambda_{m,n} = 1 \)

If

\[ d_{i,n} < a_{j,m} - \tau a_{j,m} < d_{i,n} + \tau d_{i,n} < a_{j,m}, \] (7)

then

\[ U^{d(i,n)}_{\tau a(j,m)} = d_{i,n} + \tau d_{i,n} - (a_{j,m} - \tau a_{j,m}), \] (8)

else if
\[ a_{j,m} - r a_{j,m} < d_{i,n} < a_{j,m} < d_{i,n} + r d_{i,n}, \quad (9) \]

then

\[ U_{r a(j,m)}^{d(i,n)} = a_{j,m} - d_{i,n}. \quad (10) \]

In addition to the above three categories, when \( d_{i,n}, d_{i,n} + r d_{i,n} \) and \( [a_{j,m} - r a_{j,m}, a_{j,m}] \) are not in the same constraint; equation (13) is the calculation of the train's dwell time at the station; equation (14) is the dwell time range

\[
U_{r a(j,m)}^{d(i,n)} = \begin{cases} 
0 & \text{if } a_{j,m} - r a_{j,m} < d_{i,n} < a_{j,m} < d_{i,n} + r d_{i,n}, \\
0 & \text{if } d_{i,n} + r d_{i,n} \leq a_{j,m} - r a_{j,m}, \\
\min(d_{i,n} + r d_{i,n}, a_{j,m}) - \max(d_{i,n}, a_{j,m} - r a_{j,m}) & \text{otherwise.} 
\end{cases}
\]

2.2. Model Construction and Solving. The parameters and variables required for the model in this paper are shown in Table 1.

The following constraint equation is based on the logical relationship of the relevant time factors in the train timetable:

\[
\begin{align*}
&\epsilon_i^\text{min} \leq d_{i,n} \leq \epsilon_i^\text{max} \quad \forall i \in E, \\
&\delta_{j,n} = a_{j,n} - d_{i,n} \quad \forall i \in E; 2 \leq n \leq |N|, \\
&\eta_{i,n} = a_{i,n} - d_{i,n} \quad \forall i \in E; n \in N, \\
&\lambda_m^n = a_{i,n} - d_{i,n} + T_e^n \quad \forall i \in E; m, n \in N, \\
\end{align*}
\]

where equation (12) is the initial station departure time constraint; equation (13) is the calculation of the train's dwell time at the station; equation (14) is the dwell time range constraint; equation (15) is the interval running time constraint; and equation (16) is the train running interval time constraint.

In addition, for ease of solution, equation (11) can be linearized into the following six constraints:

\[
\begin{align*}
&d_{i,n} + r d_{i,n} \leq a_{j,m} - r a_{j,m}, \\
a_{j,m} \leq d_{i,n}, \\
&\lambda_m^n = 0,
\end{align*}
\]

constraint; equation (15) is the interval running time constraint; and equation (16) is the train running interval time constraint.

The objective function is to maximize the range of GES falling within the UES, i.e. to maximize the total overlap time:

\[
\max \sum U_{r a(j,m)}^{d(i,n)}. \quad (18)
\]

At this point, the model is constructed.
2.3. Model Solution. The model developed in this paper is a linear programming model that can be solved using commercial optimization software, such as ILOG CPLEX, in the case of small scale. Due to the large data size of the real background of the problem studied in this paper (in general, a subway line has more than 300 pairs of trains running in a day), it is difficult to obtain the global optimal results when solved using commercial optimization software due to microcomputer memory overflow. Therefore, we reduce the size of the solution by dividing the trains into multiple phases (called "time windows" in this paper; a "time window" can contain trains in the upward or downward direction only or in both directions) throughout the day and solving for one "time window" at a time, following the way of phasing transportation tasks during subway operations. This processing method is called spatio-temporal local rolling algorithm, and the specific steps are shown in Figure 11:

Step 1. According to the passenger flow, the initial train timetable is given.

Step 2. The trains in the initial timetable are divided according to the appropriate "time window".

Step 3. Commercial optimization software such as ILOG CPLEX is used to optimize the train in the leftmost "time window" according to the model in this paper.

Step 4. The time data of the optimized train are taken into the model as the known quantities, and the remaining "time window" is optimized to the right in turn.

Step 5. When all the time windows are optimized, the optimization process is finished.

This algorithm can be combined with the actual operation of the phase: a "time window" can also be considered as the duration of a phase of a task in actual operation. The timetable of trains in a "time window" is optimized one by one during the interval of train operation, so as to achieve the overall optimization of the full-time timetable.

3. Example

3.1. Basic Data. As shown in Figure 12, there are 16 stations in the target urban rail transit line, and each of the 4 stations is located in an electric section (the insulation point between the electric sections is in the corresponding interstation section).

The GES time is 15 seconds and the UES time is 30 seconds at each station. The standard running time of each section is shown in Table 2.

The standard dwell time of trains at each station and the variable range of dwell time are shown in Table 3.

The whole day train operation plan is shown in Table 4; in addition, in each hour, the departure time of the train at

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<th>Table 1: Notations used in the formulation.</th>
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Figure 11: Steps of the spatio-temporal local rolling algorithm.
the initial station is compiled according to the equal time interval.

In order to ensure that the optimized train operation scheme matches with the existing crew utilization plan and the EMU turnover plan, the operator requires that the optimized result should not change the departure time of the first station and the arrival time of the terminal station of each train. That is, if \( i \in E^+ \), then \( d_{i}^{16} \) and \( a_{i}^{16} \) are constants; then if \( i \in E^- \), then \( d_{i}^{1} \) and \( a_{i}^{16} \) are constants.

3.2. The Result of the Example. The data of the above example are imported into the model of this paper, and the "time windows" are divided according to the number of trains, each "time window" contains 5 up trains and 5 down trains, and the optimization timetable of the trains in a "time windows" is generally within 20 seconds. The comparison of the final optimized timetable and the initial timetable is shown in Figure 13.

After the optimization, the time of EUSE increases from 9681 s to 14916 s. If the regenerative braking energy is regarded as uniformly generated, the energy regeneration utilization rate of the operation chart will increase from 21.95% before the optimization to 33.82% after the optimization, and the utilization rate increases by 11.87%, with an obvious energy-saving optimization effect.

3.3. Discussion of the Width of the “Time Window”. The spatio-temporal local rolling algorithm proposed in this paper is essentially a greedy algorithm, and the width of the “time window” and the degree of optimization of the solution are closely related. It is difficult to guarantee the global optimality of the solution using this algorithm when the width of the “time window” does not include all trains. By trying to use ILOG CPLEX, the global optimal solution of regenerative braking power utilization between all down trains in the example is obtained within an acceptable time, and the optimal EUES between down trains is 4140 seconds. When the width of the “time window” is varied, the optimized solution is shown in Figure 14.
It can be seen from Figure 14 that when the number of trains included in a "time window" is greater than 5, the difference between the optimal value and the optimal solution will be reduced to less than 5%. In the above process, the optimization accuracy of the optimization results will not increase with the increase of the number of trains in each "time window." This shows that if the "time window" is properly selected, the optimization scheme can have better accuracy, and it does not need to blindly increase the number of trains in the "time window." This shows that when faced with some practical problems that the data is too large to directly find the global optimal solution, the results obtained by using the small-scale "time window" in this paper are also practical enough. By discussing more data, this paper suggests that the value of $C$ for the number pairs of trains included in the "time window" is chosen by the following formula:

$$ C = \max_y \left( \sum_{m,n \in y} \tau_{i,n} - n + \delta_{i,n}^{\max} + \tau d_{i,n} + \tau a_{i,n} \right) $$

(19)

4. Conclusion

An urban rail transit train energy-saving optimization model is developed in this paper to maximize the overlap between train traction and braking processes within the same electric section. On the basis of not changing the departure time of the initial station and the arrival time of the terminal station, ensuring the rolling stock scheduling and the whole-day operation plan, the coordinated utilization of regenerative braking energy absorption by multiple trains in the same electric section can be maximized by adjusting the dwell time of the train. After transforming the original model into an equivalent linear model, this paper uses the spatio-temporal local rolling algorithm and uses the commercial optimization software ILOG CPLEX to solve it. According to the example results presented in this paper, the method proposed can effectively increase the recycling rate of regenerative braking electric energy under conditions of regenerative braking technology and reduce the total energy consumption of the urban rail transit system.

There are also some problems in this paper, such as "equating overlap time with a regenerative braking rate," "the optimization scheme’s objectives are not specific enough," and "failure to effectively deal with the situation that the same deceleration process is covered by multiple acceleration processes." How to solve these problems and make the results more practical and applicable will be the focus of the next research.

Data Availability

The data used to support this study are included within the article.
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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