Research Article

Optimal Replenishment Policies and Trade Credit for Integrated Inventory Problems in Fuzzy Environment

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In this highly volatile, uncertain, complex, and ambiguous (VUCA) environment, a fuzzy inventory optimization problem is a hot topic that received the most attention from both academicians and practitioners. In this paper, the authors have studied the integrated inventory problems under trade credit in fuzzy environments, where the demand rate and deterioration rate are taken here as triangular fuzzy numbers. Accordingly, three types of fuzzy programming models as fuzzy expected cost minimization model, fuzzy \( \alpha \)-cost minimization model, and credibility maximization model are built according to different management goals. Since the problem is shown to be NP-hard, this research focuses on proposing a hybrid intelligent algorithm by integrating fuzzy simulation, neural network, and genetic algorithm for solving these models. Then, some numerical examples are presented to illustrate the benefits of the models and evaluate the efficiency of the algorithms. The computational experiments show that the trade credit strategy has a positive impact on improving the supply chain’s performance and reducing its total cost. In addition, the maximum relative error of the objective values is less than 1%, which implies that the hybrid intelligent algorithm is robust to the parameter settings and effective in solving our models. Finally, the practical implications are discussed and useful managerial insights are given.

1. Introduction

Inventory problems have always been an important topic in supply chain management due to the increasing competition in today’s global marketplace and the pressure to reduce costs. The research of inventory problems has drawn much attention from decision-makers and researchers. In fact, the inventory problem is to determine an optimal inventory policy so as to minimize (maximize) the total cost (profit) under resource constraints such as budgetary cost, limited storage space, and the number of orders.

Since the 1920s, inventory problems have been studied extensively in the literature. Harris is among the first ones who analyzed an inventory model to obtain the classical lot-size formula. Since then, many other research papers (cf. Van Eijs [1], Chen [2], Zhang et al. [3], Sun and Queyranne [4], Jana et al. [5]) have dealt with inventory models with various optimization methods.

The above-mentioned inventory models almost assumed that each inventory parameter is known with certainty and there is no ambiguity in them. Sometimes, the optimal inventory policy is infeasible because the above authors neglect the uncertainties which exist widely in the real world. In conventional inventory models, uncertainties are usually treated as randomness and handled by appealing to probability theory. In 1970, Kaplan [6] considered the inventory problem by developing a dynamic inventory model with stochastic lead times. Subsequently, Cohen and Lee [7] devised a stochastic model which focused on measuring costs, service level, and the flexibility obtained under varying conditions and settings. Lee and Billington [8] established a stochastic heuristic model to determine inventory levels under desired service level constraints. Assuming that demands in different periods are random variables and their distributions depend on the product price, Chen and Simchi-Levi [9] analyzed a finite horizon, single-product,
periodic review model in which pricing and production-inventory decisions are made simultaneously. In the same direction, Yan and Kulkarni [10] developed a single-stage production-inventory model with stochastic production and demand rates. Moreover, by establishing a profit maximization model and profit margin maximization model with stochastic demand and order decisions, Hu et al. [11] proposed optimal solutions for distributors under risk-neutral and risk-averse conditions, respectively. Recently, incorporating stochastic demand and suppliers’ imperfect quality, Thomy et al. [12] proposed an integrated model for supplier selection with considering inventory management and inbound transportation. Chiu et al. [13] explored the combined effects of overtime, stochastic breakdowns, and rework on the inventory replenishment decision. Johari et al. [14] analyzed coordinated joint pricing and periodic review ordering choices for a supply chain with stochastic price-dependent demand. Most recent studies on stochastic inventory problems include Claudio et al. [15], Luo et al. [16], Achin and Sunil [17], Raza [18], Mario et al. [19], Malek et al. [20], Tai et al. [21], Momeni and Afshar-Nadjafi [22], and Li et al. [23].

However, due to the various sources of uncertainty and complex interrelationships among the different actors in the supply chain, it is very difficult to determine the optimal inventory policy via stochastic models because of no available historical data. For instance, it is impossible to precisely predict the demand rate and deterioration rate of new products, because we cannot get enough real sales data for the new products. In addition, in front of various emergencies, making a scientific decision is a challenge for many managers; for example, the outbreak and spread of COVID-19 caused the discontinuities and deficiency of many critical parameters in the supply chain. Due to the variability of market conditions and customer demands, there also exist some impreciseness and ambiguousness in existing products. In either case, fuzzy set theory [24] and possibility theory [25] can provide an alternative approach for dealing with inventory uncertainties. Sommer [26] first introduced fuzzy dynamic programming in an inventory and production scheduling problem in which the management wishes to fulfill a contract for providing a product and then withdraw from the market. Next, Kacprzyk and Stanieski [27] considered the determination of the optimal inventory policy of firms with fuzzy constraints imposed on replenishment. Sark [28] reexamined the economic order quantity (EOQ) formula in a fuzzy-set-theoretic perspective associating fuzziness with cost parameters. Petrovic and Sweeney [29] developed an inventory control model in fuzzy manufacturing environments by fuzzy if-then rules. After 2000, there were several interesting and relevant papers related to fuzzy inventory problems. For example, assuming that shortage cost is a fuzzy variable, Katagiri and Ishii [30] clarified the difference between the optimal solution of the nonfuzzy shortage cost case and that of the fuzzy shortage cost case; furthermore, they also discussed the possibilities of inventory problems with other fuzzy costs. In order to find the optimal decision with fuzzy demand under different situations, Wang et al. [31] proposed and adopted two decision methods, i.e., finding a minimum value of the expected annual total cost and maximizing the credibility of an event that the total cost in the planning periods does not exceed a certain budget level. Several other related papers concerning inventory models with different fuzzy variables and situations are worthy to mention: Roy [32], Sarkar and Mahapatra [33], Khatua et al. [34], Kumar et al. [35], Rajeswari et al. [36], and Maiti [37]. In addition, there is another kind of uncertain inventory problem with “small samples” and “poor information” in reality; some researchers (cf. Dasu et al. [38], Olgun et al. [39], Yazdi et al. [40], and Zhang et al. [41]) have attempted to solve the inventory decision problems in this case with grey system theory.

All the aforementioned research almost assumed that retailers must pay for the consignments as soon as the consignments are received. However, in these highly competitive marketing situations, the supplier is willing to grant the retailer an amount of trade credit with a certain term to encourage sales, promote market shares, reduce on-hand stock, and escalate business. Thus, the trade credit policy approach is becoming one of the most popular policies in inventory research. Goyal [42] first developed an EOQ model under the conditions of permissible delay in payments. Chand and Ward [43] studied the same problem by using the classical EOQ model. However, they finally drew a quite different conclusion. Anggarwal and Jaggi [44] extended Goyal’s model to determine the EOQ for deteriorating items. Furthermore, Jamal et al. [45] extended the permissible delay-in-payments concepts with allowable shortage and deterioration. Chen and Chung [46] formulated a mathematical inventory model of a light buyer under trade credit. Teng [47] amended Goyal’s model to discuss an economic order quantity under the conditions of permissible delay in payments by considering the difference between the unit price and unit cost. Recently, Lei et al. [48] built the Stackelberg game with the supplier as the leader and divided the retailer’s initial inventory and capital into different wealth regions. The above-mentioned models are several major contributions to the inventory problems under trade credit whereas it should be noted that the previous inventory models on trade credit are from the perspective of the retailer or supplier only. As a matter of fact, there have been a handful of review papers on integrated inventory problems under trade credit. Jaber and Osman [49] proposed a two-echelon supply chain model, in which the permissible delay in payments was considered a decision variable, and a trade credit scenario was adopted to coordinate the order quantity between supplier and retailer. Luo [50] studied and analyzed the benefit of coordinating supply chain inventories through the use of credit periods. Huang [51] developed an integrated inventory model to determine the optimal policy under the conditions of order processing cost reduction and permissible delay in payments. Yadav et al. [52] formulated a multi-item integrated supply chain inventory model under trade credit. Mohanty et al. [53] modeled a vendor-buyer integrated production-inventory system by considering issues of imperfect quality of the item,
trade credit finance, setup cost reduction, and shortages including partial backlogging and lost sales. In another paper, Ebrahimi et al. [54] proposed a coordination inventory model under the condition of delay in payments with the stochastic demand influenced by the retailer’s promotional effort; they proved that the proposed coordination scheme considerably improves the profitability of both supply chain members and the whole supply chain in comparison with the decentralized setting.

It is clear that many inventory models under trade credit have been discussed widely in the literature. However, as far as we know, the integrated inventory problem under trade credit in fuzzy environments has not drawn much attention in inventory literature. In spite of this fact, few studies (cf. Mahata and Goswami [55], Ouyang et al. [56], Chakraborty et al. [57], and Pramanik et al. [58]) addressed this problem so far. With these implementations, the fuzzy inventory models under trade credit are analyzed using the graded mean integration value approach, generalized reduced gradient technique, the signed distance method of defuzzification approach, and Particle Swarm Optimization approach. However, these methods are not very effective for discrete complex fuzzy inventory problems.

Summarizing the above, effective inventory management can reduce inventory, improve inventory turnover, increase cash flow, improve customer satisfaction, and ensure the sustainable development of enterprises. As market competition intensifies, the competition between enterprises becomes the competition between supply chains. In the digital age, Industry 4.0 has promoted the transformation of the industrial model from large-scale assembly line production to customized large-scale production. However, because of the complexity and instability of the supply chain, the customization production model is particularly urgent to solve the inventory optimization problem of the supply chain in an uncertain environment. The purpose of this paper is to overcome these issues by developing comprehensive fuzzy inventory models. The main contributions of this paper are summarized as follows.

1. We propose three decision-making criteria. On this basis, we establish some fuzzy integrated inventory models under trade credit with credibility theory [59] where both demand rate and deterioration rate are fuzzy variables.

2. We design a hybrid intelligent algorithm by integrating fuzzy simulation, neural network, and genetic algorithm to help determine the optimal solution.

3. We present three numerical examples to illustrate the benefits of the models and show the effectiveness of the algorithms.

The remainder of this paper is organized as follows: Section 2 formulates the problem based on certain basic notations and assumptions. Section 3 builds three fuzzy models after introducing the preliminaries of credibility theory. Section 4 integrates fuzzy simulations, neural network, and genetic algorithm to design a hybrid intelligent algorithm. Section 5 gives three numerical examples. Finally, Section 6 concludes this paper with some practical implications and useful managerial insights.

2. Fuzzy Integrated Inventory Problem under Trade Credit

2.1. Problem Description. For simplicity, in this paper, we consider the integrated inventory models consisting of a single supplier who provides one type of product to a single retailer, where the demand rate and deterioration rate are assumed to be fuzzy variables. Consequently, the integrated supply chain’s total cost per unit time consists of the supplier’s total cost per unit time and the retailer’s total cost per unit time. In addition, the supplier offers a trade credit period to the retailer. We analyze the following three possible cases, in which \( T_1 \) is replenishment cycle length and \( T_2 \) is trade credit period, as Figure 1 shows.

Meanwhile, both the storage capacity and production capacity are considered constraints. The objective is to determine the optimal inventory policies to meet different management requirements subject to limitations on storage capacity and production capacity.

2.2. Notation and Assumptions. In the subsequent analysis, we use the following notations.

2.2.1. Parameters

- \( C_{sp} \): Supplier’s preparation cost
- \( C_{ro} \): Retailer’s ordering cost
- \( C_{rp} \): Retailer’s purchase cost per unit item
- \( C_{sp} \): Supplier’s production cost per unit item
- \( C_{srp} \): Retailer’s/supplier’s inventory carrying cost per unit item
- \( D_r / D_s \): Customer’s/retailer’s annual demand rate (fuzzy variable)
- \( \theta_{rd} / \theta_{sd} \): Retailer’s/supplier’s inventory deterioration rate (fuzzy variable)
- \( P \): Retailer’s sale price per unit item
- \( I_{ro} / I_{so} \): Retailer’s/supplier’s opportunity cost per unit time
- \( I_{ri} \): Retailer’s interest income per unit time
- \( Q_{ro} \): Retailer’s order quantity
- \( Q_{sp} \): Supplier’s production
- \( TC^{so} \): Supply chain’s budget
- \( PC^{so} \): Supply chain’s available production capacity
- \( SC^{so} \): Supply chain’s available inventory capacity
- \( \alpha \): The quotiety of production capacity per unit time
- \( \beta \): The quotiety of storage capacity per unit time
2.2.2. Decision Variables

- **T_1**: Retailer’s replenishment intervals
- **T_2**: Trade credit period

2.2.3. Functions

- \( I_r(t)/I_s(t) \): Retailer’s/supplier’s inventory level at time \( t \), \( 0 \leq t < T_1 \)
- \( TC(T_1,T_2|\bar{D}_c,\bar{D}_r,\bar{\theta}_{rd},\bar{\theta}_{sd}) \): The integrated supply chain’s total cost
- \( PC(T_1,T_2|\bar{D}_c,\bar{D}_r,\bar{\theta}_{rd},\bar{\theta}_{sd}) \): Production capacity
- \( SC(T_1,T_2|\bar{D}_c,\bar{D}_r,\bar{\theta}_{rd},\bar{\theta}_{sd}) \): Storage capacity

2.2.4. Assumptions. The following assumptions are made in the models:

(a) There is a single supplier and a single retailer for a single product
(b) Demand rate and deterioration rate are fuzzy variables
(c) The supplier offers a credit period opportunity for his/her retailers
(d) Shortages are not allowed
(e) Lead-time is negligible and the time horizon is infinite
(f) The initial inventory level is zero

2.3. Problem Formulation. We begin by assuming that the retailer’s inventory level \( I_r(t) \) is depleted by the effects of customer’s demand and deterioration. Therefore, we get the variation of \( I_r(t) \) with respect to \( t \) as follows:

\[
\frac{dI_r(t)}{dt} = -\bar{D}_r - \bar{\theta}_{rd}I_r(t), \quad 0 \leq t < T_1,
\]

with the boundary condition \( I_r(0) = 0 \), so we have the retailer’s inventory level \( I_r(t) \), initial order quantity, and numbers of deteriorating items during a replenishment cycle given by

\[
I_r(t) = \frac{\bar{D}_r}{\bar{\theta}_{rd}} \left[ e^{\bar{\theta}_{rd}(T_1-t)} - 1 \right], \quad 0 \leq t < T_1,
\]

\[
Q_{t_0} = I_r(0) = \frac{\bar{D}_r}{\bar{\theta}_{rd}} \left[ e^{\bar{\theta}_{rd}T_1} - 1 \right], \quad (2)
\]

Likewise, we can get the supplier’s inventory level \( I_s(t) \), initial production quantity, and the number of deteriorating items by assuming that the supplier’s inventory level is depleted by the effects of the retailer’s demand and deterioration.

\[
I_s(t) = \frac{\bar{D}_s}{\bar{\theta}_{sd}} \left[ e^{\bar{\theta}_{sd}(T_1-t)} - 1 \right], \quad 0 \leq t < T_1,
\]

\[
Q_{t_0} = I_s(0) = \frac{\bar{D}_s}{\bar{\theta}_{sd}} \left[ e^{\bar{\theta}_{sd}T_1} - 1 \right], \quad (3)
\]

\[
Q_{t_0} = D_rT_1 = \frac{\bar{D}_s \left( e^{\bar{\theta}_{sd}T_1} - \bar{\theta}_{sd}T_1 - 1 \right)}{\bar{\theta}_{sd}},
\]

At first, we suppose that the retailer’s total cost per unit time consists of the following elements:

(1) Ordering cost:

\[
\frac{C_{rd} T_1}{T_1} \int_0^{T_1} I_r(t) dt = C_{rd} \frac{\bar{D}_r}{\bar{\theta}_{rd}} \frac{\bar{\theta}_{sd}T_1 - 1}{\bar{\Theta}_{sd}}. \quad (4)
\]

(3) Deterioration cost:

\[
\frac{C_{rd} \bar{D}_s \left( e^{\bar{\theta}_{sd}T_1} - \bar{\theta}_{sd}T_1 - 1 \right)}{\bar{\theta}_{sd} T_1}. \quad (5)
\]
(4) Opportunity cost: we now consider three possible cases, namely, (a) \( T_2 = 0 \); (b) \( 0 < T_2 \leq T_1 \); (c) \( T_2 > T_1 \). We get the opportunity cost:

\[
\begin{align*}
C_{op} \frac{I_{rn}}{2} &= \frac{C_{op}I_{rn}}{2}, & \text{if } T_2 = 0, \\
\frac{C_{op}I_{rn}}{T_1} \int_{T_2}^{T_1} D_r (t - T_2) dt &= \frac{C_{op}I_{rn}D_r (T_1 - T_2)^2}{2T_1}, & \text{if } 0 < T_2 \leq T_1, \\
0, & & \text{if } T_2 > T_1.
\end{align*}
\]

(5) Interest income:

\[
\begin{align*}
0, & & \text{if } T_2 = 0, \\
\frac{PI_{ri}}{T_1} \int_{0}^{T_2} D_r t dt &= \frac{PI_{ri}D_r T_2^2}{2T_1}, & \text{if } 0 < T_2 \leq T_1, \\
\frac{1}{T_1} \left[ \frac{PI_{ri}}{T_1} \int_{0}^{T_1} D_r t dt + PI_{ri}D_c (T_2 - T_1) \right] &= D_c PI_{ri} \left( T_2 - \frac{T_1}{2} \right), & \text{if } T_2 > T_1.
\end{align*}
\]

Here, we suppose that the supplier's total cost per unit time includes preparation cost, inventory carrying cost, deterioration cost, and opportunity cost.

(1) Preparation cost:

\[
\frac{C_{pr}}{T_1}
\]

(2) Inventory carrying cost:

\[
\frac{C_{ri}}{T_1} \int_{0}^{T_1} I_r (t) dt = \frac{C_{ri}D_c}{\theta_{rd}T_1} \left( e^{\theta_{rd}T_1} - 1 \right) - \frac{C_{ri}D_c}{\theta_{rd}}
\]

(3) Deterioration cost:

\[
\frac{C_{sp}D_c \left( e^{\theta_{rd}T_1} - \theta_{rd}T_1 - 1 \right)}{\theta_{rd}T_1}
\]

(4) Opportunity cost: we will get the supplier's opportunity cost under the above three possible cases.

\[
\begin{align*}
0, & & \text{if } T_2 = 0, \\
\frac{C_{op}I_{rn}}{T_1} \int_{0}^{T_2} D_r t dt &= \frac{C_{op}I_{rn}D_r T_2^2}{2T_1}, & \text{if } 0 < T_2 \leq T_1, \\
\frac{C_{op}I_{rn}}{T_1} \int_{0}^{T_2} D_r t dt &= \frac{C_{op}I_{rn}D_r T_2^2}{2T_1}, & \text{if } T_2 > T_1.
\end{align*}
\]

Consequently, the integrated supply chain's total cost per unit time is
TC(T₁, T₂|Dc, Dₙ, τd, τad).

\[
\begin{align*}
\frac{Cr_p}{T_1} + \frac{Cs}{T_1} \int_0^{T_1} I_r(t) dt + \frac{Cr_p D_c e^{\theta_d T_1} - \theta_d T_1 - 1}{\theta_d T_1} + \frac{Cs}{T_1} \int_0^{T_1} I_s(t) dt, \\
\frac{Cs}{T_1} \int_0^{T_1} I_r(t) dt + \frac{Cr_p D_c e^{\theta_d T_1} - \theta_d T_1 - 1}{\theta_d T_1} + \frac{Cs}{T_1} \int_0^{T_1} I_s(t) dt, \\
\frac{Cr_p}{T_1} D_c (t - T_2) dt + \frac{Cr_p I_{ro} Q_{ro}}{T_1} \int_0^{T_1} I_d(t) dt,
\end{align*}
\]

\((12)\)

In addition, for the integrated supply chain, we say that an inventory policy \((T₁, T₂)\) is feasible if it satisfies \(PC(T₁, T₂|D_c, Dₙ, τd, τad) \leq PC₀\), \(SC(T₁, T₂|D_c, Dₙ, τd, τad) \leq SC₀\), and three kinds of trade credit period conditions. Here, we set \(\alpha = 2, \beta = 4\), for simplicity, a truncated Taylor series expansion for the exponential term: \(e^{\theta T} = 1 + \theta T + 1/2\theta^2 T^2\) is used in this paper; then, production capacity and storage capacity are demonstrated as equations (13) and (14), respectively.

\[
\begin{align*}
PC(T₁, T₂|D_c, Dₙ, τd, τad) &= Q_{rp} \alpha \\
&= 2D_c T₁ + D_c \theta_d T₁^2, \\
SC(T₁, T₂|D_c, Dₙ, τd, τad) &= Q_{rp} \beta \\
&= 4D_c T₁ + 2D_c \theta_d T₁^2.
\end{align*}
\]

(13)

(14)

3. Fuzzy Models

3.1. Preliminaries. The fuzzy set, which was first introduced by Zadeh [24] in 1965, is the extension of the conventional set. In 1978, Zadeh [60] proposed the concept of possibility measure to measure a fuzzy event. However, it has no self-duality property. In order to refine this problem and study the behavior of fuzzy phenomena, Liu and Liu [61–63] introduced the concept of credibility measure and founded credibility theory.

Let \(\Theta\) be a nonempty set, and \(P(\Theta)\) the power set of \(\Theta\) (i.e., all subsets of \(\Theta\)). In order to present an axiomatic definition of credibility measure, it is necessary to assign to each event \(A\) a number \(Cr(A)\) which indicates the belief degree that \(A\) will occur; then, the number \(Cr(A)\) should satisfy the following axioms.

Axiom 1 (Normality). \(Cr(\Theta) = 1\) for the universal set \(\Theta\).

Axiom 2 (Monotonicity). \(Cr(A) \leq Cr(B)\) whenever \(A \subset B\).

Axiom 3 (Self-Duality). \(Cr(A) + Cr(A^c) = 1\) for any \(A \in P(\Theta)\). (\(A^c\) is the complement of \(A\)).

Axiom 4 (Maximality). \(Cr(\cup_i A_i) \leq 0.5 = \sup_i Cr(A_i)\) for any \(\{A_i\}\) with \(Cr(A_i) \leq 0.5\), where the symbol “\(\wedge\)” means take the minimum.
**Axiom 5** (Product Measure). Let \( \Theta k \) be nonempty sets on which \( Cr_k \) satisfy the first four axioms, \( k = 1, 2, \ldots, n \), respectively, and let \( \Theta = \Theta_1 \times \Theta_2 \times \ldots \times \Theta_n \); then,

\[
Cr[(\Theta_1, \Theta_2, \ldots, \Theta_n)] = Cr[\Theta_1] \land Cr[\Theta_2] \land \cdots \land Cr[\Theta_n],
\]

for each \( (\Theta_1, \Theta_2, \ldots, \Theta_n) \in \Theta \).

**Definition 1.** The set function \( Cr \) is called a credibility measure if it satisfies the first four axioms.

**Definition 2.** Let \( \Theta \) be a nonempty set, \( P(\Theta) \) the power set of \( \Theta \), and \( Cr \) a credibility measure. Then, the triplet \((\Theta, P(\Theta), Cr)\) is called a credibility space.

**Definition 3.** A fuzzy variable is a function from a credibility space \((\Theta, P(\Theta), Cr)\) to the set of real numbers.

**Definition 4.** Let \( \xi \) be a fuzzy variable with membership function \( \mu \). Then, for any set \( B \) of real numbers, we have

\[
Cr[\xi \in B] = \frac{1}{2} \left( \sup_{x \in B} \mu(x) + 1 - \sup_{x \in B} \mu(x) \right).
\]

This formula is also called the credibility inversion theorem.

**Definition 5.** Let \( \xi \) be a fuzzy variable. Then, the expected value of \( \xi \) is defined by

\[
E[\xi] = \int_{-\infty}^{\infty} Cr[\xi \geq r] \, dr - \int_{-\infty}^{0} Cr[\xi < r] \, dr,
\]

provided that at least one of the two integrals is finite.

### 3.2. Fuzzy Expected Cost Minimization Model

Assume that the decision-maker believes the expected value criterion; that is to say, if the decision-maker is risk-averse, he/she may want to make a decision with optimized expected objectives subject to some expected constraints, which just shows the philosophy of the expected value model. Therefore, we have the following fuzzy expected cost minimization model for the integrated inventory problem under storage capacity constraint and production capacity constraint:

\[
\begin{align*}
\min & \ E[T\!C(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] , \\
\text{subject to} & : \\
E[PC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] & \leq PC^0 , \\
E[SC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] & \leq SC^0 , \\
T_2 = 0, T_1 & \geq 0 \text{ if } T_2 = 0 , \\
0 < T_2 & \leq T_1 \text{ if } 0 < T_2 \leq T_1 , \\
T_2 > T_1 & \geq 0 \text{ if } T_2 > T_1 ,
\end{align*}
\]

where \( TC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd} \), \( PC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd} \), and \( SC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd} \) are defined by equations (12)–(14), respectively.

### 3.3. Fuzzy \( \alpha \)-Cost Minimization Model

Taking into account the risk, another decision-making criterion is to optimize some critical value with at least some given confidence levels subject to some chance constraints. Based on chance-constrained programming, which was initialized by Charnes and Cooper [64] as a powerful means for modeling stochastic decision systems traditionally, Liu and Iwamura [65, 66] extended the chance-constrained programming from stochastic decision systems to fuzzy decision systems. With this criterion, the fuzzy chance-constrained programming for the integrated inventory problem can be obtained by introducing Definition 6 about \( \alpha \)-cost.

**Definition 6.** The \( \alpha \)-cost of a fuzzy integrated inventory problem is defined as \( \min \{ TC^0 | Cr[TC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] \leq TC^0 \} \geq \alpha \), where \( \alpha \) is the predetermined confidence level.

If the decision-maker wants to minimize the \( \alpha \)-cost subject to the production capacity chance constraint and storage capacity chance constraint with the predetermined confidence levels, we can present the fuzzy \( \alpha \)-cost minimization model as follows:

\[
\begin{align*}
\min & \ TC^0 , \\
\text{subject to} & : \\
Cr[TC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] & \leq TC^0 \geq \alpha , \\
Cr[PC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] & \leq PC^0 \geq \beta , \\
Cr[SC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd}] & \leq SC^0 \geq \gamma , \\
T_2 = 0, T_1 & \geq 0 \text{ if } T_2 = 0 , \\
0 < T_2 & \leq T_1 \text{ if } 0 < T_2 \leq T_1 , \\
T_2 > T_1 & \geq 0 \text{ if } T_2 > T_1 ,
\end{align*}
\]

where \( TC^0 \) is the \( \alpha \)-cost, \( \alpha, \beta, \) and \( \gamma \) are predetermined confidence levels, and \( TC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd} \), \( PC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd} \), and \( SC(T_1, T_2)\!D_\!r, \bar{D}_\!r, \theta_{rd}, \bar{\theta}_{rd} \) are defined by equations (12)–(14), respectively.

### 3.4. Credibility Maximization Model

Although the above two types of fuzzy expected cost minimization model and fuzzy \( \alpha \)-cost minimization model can work quite well in dealing with the integrated inventory problem, they indeed...
oversimplify the decision system. For example, in many practical problems, there usually exist multiple tasks in a complex fuzzy decision system. For this multi-task case, the decision-maker who concerns the risk of some unfavorable event occurring may believe the chance criterion. In order to model this type of fuzzy decision system, Liu [67] provided the third type of fuzzy programming, called dependence-chance programming, in which the underlying philosophy is based on selecting the decision with maximal credibility to meet the task. Therefore, if the management goal is given in advance so that the credibility of the total cost does not exceed the given budget, \(TC^0\) under some fuzzy constraints should be as large as possible. The fuzzy credibility maximization model is given in the following form:

\[
\begin{align*}
\max & \; C_r\{TC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd}) \leq TC^0\}, \\
\text{subject to} & \
PC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd}) \leq PC^0, \\
SC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd}) \leq SC^0, \\
T_2 & = 0, T_1 \geq 0 \text{ if } T_2 = 0, \\
0 & < T_2 \leq T_1 \text{ if } 0 < T_2 < T_1, \\
T_2 & > T_1 \geq 0 \text{ if } T_2 > T_1, \\
\end{align*}
\]

where \(TC^0\) is the integrated supply chain’s budget, which should be ideally provided by the decision-maker. \(TC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd})\), \(PC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd})\), and \(SC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd})\) are defined by equations (12)–(14), respectively.

Among these three models, the fuzzy expected cost minimization model is the most widely used decision-making criterion in practice. However, one of the key features of this model is indifference to risk. In fact, it is more reasonable for decision-makers to consider the potential risks in the decision process. Hence, from both a practical and theoretical standpoint, the idea of building a fuzzy \(\alpha\)-cost minimization model and credibility maximization model for the integrated inventory problem should be well received by many decision-makers because these two models provide them with more effective and feasible decisions.

### 4. Hybrid Intelligent Algorithm

From the mathematical viewpoint, there is no difference between deterministic mathematical programming and uncertain programming except for the fact that there exist uncertain functions in the latter. By uncertain functions, here we mean the functions with fuzzy variables. It is clear that the uncertain functions in the above models in Section 3 fall into the following three types:

\[
\begin{align*}
U_1: (T_1, T_2) & \rightarrow E[TC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd})], \\
U_2: (T_1, T_2) & \rightarrow Cr[TC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd}) \leq TC^0], \\
U_3: (T_1, T_2) & \rightarrow \min TC^0[Cr\{TC(T_1, T_2 | D_r, D_r, \tilde{r}_{sd}, \tilde{r}_{sd}) \leq TC^0\} \geq \alpha].
\end{align*}
\]

Apparently, the value of these functions cannot be computed directly since these functions are uncertain functions. Furthermore, reaching an analytical solution (if any) to each of the above fuzzy programming is difficult. Fortunately, based on the Monte Carlo simulation method (i.e., stochastic discretization algorithm), a fuzzy simulation approach was proposed; see Liu and Iwamura [65] and Liu [59]. However, this method is time-consuming. In order to overcome this shortage, researchers begin to explore and use heuristic algorithms and metaheuristic algorithms, which are very effective in solving complex combinatorial optimization problems, especially for NP-hard problems (cf. Tirkolae et al. [68], Alinaghian and Goli [69], Goli and Malmir [70]). The commonly used heuristic algorithms and metaheuristic algorithms mainly include Local Search Algorithm (LS), Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithms (GA), and Artificial Neural Network (ANN). LS is a kind of heuristic optimization algorithm based on a greedy idea. Its idea is to continuously search for the current solution until no better solution can be found, but it is easy to fall into local optimum. SA is a metaheuristic algorithm based on physics, which has a strong ability to escape from local optima but a slow convergence speed. TS is a “variation” of LS, which performs a global search by limiting the movement that has already occurred to occur again. However, the algorithm has a strong dependence on the initial solution and neighborhood construction and has no ability for parallel search. GA is a stochastic global search and optimization approach based on biological natural selection and evolution theory [71]. One advantage of a genetic algorithm is that it has fast convergence to or near the global optimum without requiring any gradient information and inherent parallelism in searching the design space, thus making it a robust adaptive optimization technique. GA has been applied to optimize the fuzzy inventory models, see, for instance, Chakraborty et al. [72] and Jana et al. [73]. ANN is made up of many simple neurons, which is a highly complex nonlinear dynamic system. It can deal with the imprecise and fuzzy information problem that many variables should be considered at the same time. Therefore, in this paper, we integrate fuzzy simulations, neural networks, and genetic algorithms to produce an effective hybrid intelligent algorithm for solving fuzzy programming models. The combined method mainly includes three stages. The first stage is to generate input-output data for fuzzy functions by adopting fuzzy simulations. The second stage is to train a neural network on the set of input-output data, and the third stage is to embed the trained neural network into a genetic
Stage 1. Generate a training data set for uncertain functions by adopting fuzzy simulations.

In order to train a neural network to approximate the fuzzy functions, we should generate a training data set (i.e., input-output data) by fuzzy simulation first. In what follows, we address the calculation of these uncertain functions by designing some fuzzy simulations. For simplicity, we shall not go into details and only show the main idea in the form of listing simulation steps.

(1) Fuzzy Simulation Algorithm I (for the first type of uncertain function):

Step 1. Set $\epsilon = 0$.
Step 2. Randomly generate $\omega_1, \omega_2, \ldots, \omega_N$ form $\Theta$ such that $\text{Pos}\{\omega_k\} \geq \epsilon$, and denote $\nu_k = \text{Pos}\{\omega_k\}$, where $\epsilon$ is a sufficiently small positive number.
Step 3. Set

\[
a = TC(\bar{D}_c(\omega_1), \bar{D}_r(\omega_1), \bar{\theta}_{rd}(\omega_1), \bar{\theta}_{ad}(\omega_1)) \land \cdots \land TC(\bar{D}_c(\omega_N), \bar{D}_r(\omega_N), \bar{\theta}_{rd}(\omega_N), \bar{\theta}_{ad}(\omega_N)).
\]

\[
b = TC(\bar{D}_c(\omega_1), \bar{D}_r(\omega_1), \bar{\theta}_{rd}(\omega_1), \bar{\theta}_{ad}(\omega_1)) \lor \cdots \lor TC(\bar{D}_c(\omega_N), \bar{D}_r(\omega_N), \bar{\theta}_{rd}(\omega_N), \bar{\theta}_{ad}(\omega_N)).
\]

Step 4. Randomly generate $r$ from $[a, b]$.
Step 5. If $r \geq 0$, then

\[
e \leftarrow e + Cr\{TC(T_1, T_2)\bar{D}_c, \bar{D}_r, \bar{\theta}_{rd}, \bar{\theta}_{ad}) \geq r\}.
\]

Step 6. If $r < 0$, then

\[
e \leftarrow e - Cr\{TC(T_1, T_2)\bar{D}_c, \bar{D}_r, \bar{\theta}_{rd}, \bar{\theta}_{ad}) \leq r\}.
\]

Step 7. Repeat the fourth to sixth steps for $N$ times.
Step 8. $E[TC(T_1, T_2)\bar{D}_c, \bar{D}_r, \bar{\theta}_{rd}, \bar{\theta}_{ad}] = a\nu_0 + b\land 0 + e \times (b - a)/N$.

(2) Fuzzy Simulation Algorithm II (for the second type of uncertain function):

Step 1. Randomly generate $\omega_1, \omega_2, \ldots, \omega_N$ from $\Theta$ such that $\text{Pos}\{\omega_k\} \geq \epsilon$, where $\epsilon$ is a sufficiently small positive number, and denote $\nu_k = \text{Pos}\{\omega_k\}$.
Step 2. Set $\nu_k = \text{Pos}\{\omega_k\}$ for $k = 1, 2, \ldots, N$.
Step 3. Return $L$, where

\[
L = \frac{1}{2} \left( \max_{1 \leq k \leq N} \{\nu_k\} \land TC(\bar{D}_c(\omega_k), \bar{D}_r(\omega_k), \bar{\theta}_{rd}(\omega_k), \bar{\theta}_{ad}(\omega_k)) \leq TC^0 \} 
\]

\[
+ \min_{1 \leq k \leq N} \{1 - \nu_k\} \land TC(\bar{D}_c(\omega_k), \bar{D}_r(\omega_k), \bar{\theta}_{rd}(\omega_k), \bar{\theta}_{ad}(\omega_k)) > TC^0 \} \right).
\]

(3) Fuzzy Simulation Algorithm III (for the third type of uncertain function):

Step 1. Randomly generate $\omega_1, \omega_2, \ldots, \omega_N$ from $\Theta$ such that $\text{Pos}\{\omega_k\} \geq \epsilon$, and denote $\nu_k = \text{Pos}\{\omega_k\}$, where $\epsilon$ is a sufficiently small positive number.

Step 2. Find the maximal value $r$ satisfying $L(r) \geq \alpha$, where

\[
L(r) = \frac{1}{2} \left( \max_{1 \leq k \leq N} \{\nu_k\} \land TC(\bar{D}_c(\omega_k), \bar{D}_r(\omega_k), \bar{\theta}_{rd}(\omega_k), \bar{\theta}_{ad}(\omega_k)) \leq r \}
\]

\[
+ \min_{1 \leq k \leq N} \{1 - \nu_k\} \land TC(\bar{D}_c(\omega_k), \bar{D}_r(\omega_k), \bar{\theta}_{rd}(\omega_k), \bar{\theta}_{ad}(\omega_k)) > r \} \right).
\]

Step 3. Return $r$.

Stage 2. Train a neural network to approximate the uncertain functions.

The training process of the BP neural network is as follows.

According to the research results of Hornik et al. [74] and other scholars, three-layer feedforward neural network, that is, the neural network containing only one hidden layer can approximate any nonlinear function relation with arbitrary precision. In this paper,
we train a single-layer BP neural network to approximate the fuzzy functions.

**Step 2.** Select an initialized column pattern from the training data set randomly as the input signal to the network.

\[
\{(T^{(k)}_1, T^{(k)}_2, Y^{(k)})|k = 1, 2, \ldots, N\},
\]

where \(T_1\) and \(T_2\) are decision variables, and \(Y(k)\) are the corresponding fuzzy function values that are calculated by fuzzy simulations.

**Step 3.** Calculate the input and output of neurons in the hidden layer.

**Step 4.** Calculate the input and output of neurons in the output layer.

**Step 5.** Calculate the general error of neurons in the output layer; if the result is feasible, proceed to Step 8; otherwise, proceed to Step 6.

**Step 6.** Calculate the general error of neurons in the hidden layer.

**Step 7.** Correct weights and thresholds.

**Step 8.** Take the next column pattern as the input signal; repeat Step 3 to Step 7, until the total error \(E\) reaches the predetermined accuracy; the learning is terminated; otherwise, update learning times and return to training.

\[
E = \frac{1}{2} \sum_{k=1}^{N} (Y^{r}(k) - Y(k))^2,
\]

where \(Y^{r}(k)\) are the output values of the BP neural network, and \(Y(k)\) are the goals output. The training process of the BP neural network will be finished until the total error \(E\) reaches the expected accuracy.

**Stage 3.** Embed the trained neural network into a genetic algorithm.

In the integrated inventory problem, we use a non-negative vector \(V = (T_1, T_2)\) as a chromosome to represent an integrated inventory policy. The embedding process is as follows.

(1) **Initialize** \(\text{pop\_size}\) (the population size) chromosomes in the following way.

**Step 1.** Randomly generate \(\text{pop\_size}\) chromosomes \(V^m, m = 1, 2, \ldots, \text{pop\_size}\).

**Step 2.** Check the feasibility of each chromosome \(V^m\) by adopting the trained BP neural network. If \(V^m\) is feasible, we take it as an initial chromosome. If not, we repeat Step 1 and Step 2 until \(V^m\) is examined to be feasible.

**Step 3.** Repeat Step 1 and Step 2 \(\text{pop\_size}\) times to obtain \(\text{pop\_size}\) feasible chromosomes \(V^m\).

(2) **Renew** the chromosomes by crossover operations with respect to crossover rate \(P_c\).

**Step 1.** Repeat the following process from \(m = 1\) to \(\text{pop\_size}\) to choose \(\text{pop\_size} \times P_c\) parents: randomly generating a number \(r\) from the interval \([0, 1]\), the chromosome \(V^m\) will be selected as a parent if \(r < P_c\).

**Step 2.** Produce children \(XL\) and \(YL\), \(L = 1, 2, \ldots, \text{pop\_size}/2\) by crossover operations on the above parents as follows:

Randomly grouping the above \(\text{pop\_size} \times P_c\) parents to the pairs \((V^1, V^2), (V^3, V^4), \ldots, (V^{\text{pop\_size}-1}, V^{\text{pop\_size}})\), let us mention that if the \(\text{pop\_size} \times P_c\) is odd, simply remove one of the parents. Then, we can produce the children by doing a crossover operation for each pair:

\[
X_1 = c \cdot V_1^{r} + (1-c) \cdot V_2^{r},
\]

\[
Y_1 = (1-c) \cdot V_1^{r} + c \cdot V_2^{r},
\]

\[
X_2 = c \cdot V_1^{r} + (1-c) \cdot V_2^{r},
\]

\[
Y_2 = (1-c) \cdot V_1^{r} + c \cdot V_2^{r},
\]

where \(c\) is a random number generated from the open interval \((0, 1)\).

**Step 3.** Check the feasibility of these new chromosomes; that is, we replace the parents with them if they are feasible; else, we redo the crossover operation until we get \(\text{pop\_size}\) new chromosomes \(V^m\).

(3) **Update** the chromosomes by mutation operations with respect to mutation rate \(P_m\).

**Step 1.** Repeat the following process from \(m = 1\) to \(\text{pop\_size}\) to choose \(\text{pop\_size} \times P_m\) parents: randomly generating a number \(r\) from the interval \([0, 1]\), the chromosome \(V^m\) will be selected as a parent provided that \(r < P_m\).

**Step 2.** Generate the new chromosomes \(V^m\) by mutating the parents \(V^m\) in the following way. We randomly choose a mutation direction \(d\) in \(Rn\); then the new chromosome \(X\) can be expressed as

\[
V''^m = V^m + M \cdot d,
\]

where \(M\) is an appropriate large positive number.

**Step 3.** Check the feasibility of these new chromosomes. If they are feasible, we keep them as new chromosomes; if not, we set \(M\) as a random number between 0 and \(M\), and repeat Step 2 to regenerate new chromosomes until they are feasible or a given number of iterations is finished.

(4) **Evaluate** the new chromosomes \(V''^m\) in the following way.

**Step 1.** Calculate the objective function values of these new chromosomes.
Step 2. Rearrange these chromosomes from good to bad by comparing their corresponding objective values. In addition, we define the rank-based evaluation function as follows:

\[
\text{eval}(V^n_m) = a \cdot (1 - a)^{k-1},
\]

\[
m = 1, 2, \ldots, \text{pop}_z\text{size},
\]

where \(a \in (0, 1)\) is a parameter in the genetic system given in advance.

(5) Select the chromosomes by spinning the roulette wheel \(\text{pop}_z\text{size}\) times to generate a new generation.

Step 1. Calculate the cumulative probability \(q_m\) for each chromosome \(V^n_m\) according to the following form:

\[
\begin{align*}
q_0 &= 0, \\
q_m &= \sum_{j=1}^{m} \text{eval}(V_j), m = 1, 2, \ldots, \text{pop}_z\text{size}
\end{align*}
\]

Step 2. Select the \(m\)-th chromosome \(V^n_m\) if \(q_m - 1 < r \leq q_m\).

Step 3. Repeat Step 2 \(\text{pop}_z\text{size}\) times to obtain \(\text{pop}_z\text{size}\) copies of chromosomes.

(6) After repeating (2) to (5) for a given number of cycles, we use the best chromosomes as the optimal solution.

5. Numerical Examples

In this section, we present three numerical examples which aim at illustrating the benefits of fuzzy integrated inventory models developed in Section 3 and evaluating the efficiency of the hybrid intelligent algorithm proposed in Section 4. In Example 1, we study special case 1 (i.e., \(T_2 = 0\)) by developing the fuzzy expected cost minimization model. In Example 2, we consider case 2 (i.e., \(0 < T_2 \leq T_1\)) and develop its corresponding fuzzy \(\alpha\)-cost minimization model. Then, we study case 3 (i.e., \(T_2 > T_1\)) in Example 3 by developing its corresponding credibility maximization model. Some possible combinations of the hybrid intelligent algorithm parameters (\(\text{Pc}, \text{Pm},\) and \(\text{pop}_z\text{size}\)) among their chosen values are employed. In addition, we introduce two indexes, called absolute error and relative error, respectively. The absolute error is calculated by the formula: actual value – optimal value, and the relative error is calculated by the formula: absolute error/optimal value \(\times 100\%\), where the optimal value is the best objective value of all the objective values obtained.

Example 1. Let us start by considering the following fuzzy expected cost minimization model for case 1. In this case, the supplier does not offer trade credit to the retailer. In fact, the case of no-credit then is just the special case of the general model. Now suppose \(C_0 = \$10/\text{unit}, \ C_{z} = \$5/\text{unit}, \ C_{\text{set}} = \$350/\text{setup}, \ C_{m} = \$40/\text{order}, \ C_{\text{r}} = \$1/\text{unit/\text{year}}, \ C_{\text{a}} = \$0.5/\text{unit/\text{year}}, \ P = \$16/\text{unit}, \ I_{\text{r}} = 0.05, \ I_{\text{a}} = 0.07, \ I_{\text{o}} = 0.03, \alpha = 2, \) and \(\beta = 4.\) Besides, four fuzzy variables \(\bar{D}_i, \ \bar{D}_r, \ \bar{\theta}_\text{rd}, \) and \(\bar{\theta}_\text{ad}\) (triangular fuzzy numbers) are \(\bar{D}_i = (1200, 1500, 1800), \ \bar{D}_r = (1400, 1800, 2100), \ \bar{\theta}_\text{rd} = (0.05, 0.07, 0.08), \) and \(\bar{\theta}_\text{ad} = (0.03, 0.05, 0.06),\) respectively. Further, the total available production capacity \(\text{PC}\) and storage capacity \(\text{SC}\) of the integrated supply chain are 3000 and 6000, respectively. In order to minimize the expected cost of the integrated supply chain, we have the following fuzzy expected cost minimization model:

\[
\begin{align*}
\min E[TC(T_1, T_2 | D_e, D_r, \bar{\theta}_\text{rd}, \bar{\theta}_\text{ad})], \\
\text{subject to:} \\
E[PC(T_1, T_2 | D_e, D_r, \bar{\theta}_\text{rd}, \bar{\theta}_\text{ad})] &\leq 3000, \\
E[\text{SC}(T_1, T_2 | D_e, D_r, \bar{\theta}_\text{rd}, \bar{\theta}_\text{ad})] &\leq 6000, \\
T_1 &> 0, T_2 = 0,
\end{align*}
\]

In order to solve model equation (31), following the former discussed hybrid intelligent algorithm, we designed a computer program to find the optimal solution. As a result, after a run of the hybrid intelligent algorithm (2 input neurons, 6 hidden neurons, 3 output neurons, 5000 cycles in simulation, and 2000 data in the neural network; 1000 generations in the genetic algorithm) when we adopt different genetic algorithm settings, we obtain the computational results shown in Table 1.

From the results shown in Table 1, it is interesting to note that, in general, as the replenishment cycle length \(T_1\) increases, the expected cost increases. These results agree with the facts. On the other hand, for each combination of genetic algorithm parameters, both the optimal solutions and the objective values differ little from each other. The last column in Table 1 shows that the maximum relative error of the objective values does not exceed 0.2\%, which implies that the proposed hybrid intelligent algorithm performs well in solving the fuzzy expected cost minimization model.

Example 2. Let us continue by developing the fuzzy \(\alpha\)-cost minimization model for case 2. In this case, the supplier offers the retailer a credit period of length \(T_2\); this credit period expires on or before the inventory is depleted completely. In this example, we are required to minimize the 0.90-cost while the total production capacity and storage capacity should not exceed 3000 and 6000 with the credibility of 0.95 and 0.95, respectively. Using the same data as in Example 1, we have the following fuzzy 0.90-cost minimization model:

\[
\begin{align*}
\min TC^\alpha, \\
\text{subject to:} \\
Cr[TC(T_1, T_2 | D_e, D_r, \bar{\theta}_\text{rd}, \bar{\theta}_\text{ad})] &\leq TC^\alpha \geq 0.90, \\
Cr[PC(T_1, T_2 | D_e, D_r, \bar{\theta}_\text{rd}, \bar{\theta}_\text{ad})] &\leq 3000 \geq 0.95, \\
Cr[SC(T_1, T_2 | D_e, D_r, \bar{\theta}_\text{rd}, \bar{\theta}_\text{ad})] &\leq 6000 \geq 0.95, \\
0 < T_2 &\leq T_1,
\end{align*}
\]
compared with Example 1, as the trade credit period \( T \) chain’s budget is 1560. Therefore, according to the

Another numerical example for case 3 will be used to verify the proposed model 3. The data is the same as Example 1 except we suppose that the integrated supply chain’s budget is 1560. Therefore, according to the philosophy of dependent-chance programming, if the manager wants to maximize the credibility

\[
\text{TC}(T_1, T_2|D_c, D_r, \bar{\theta}_{rd}, \bar{\theta}_{sd}) = 390/T_1 + T_1 D_r/4 + 17T_1 D_r/20 + 5T_1 D_r \bar{\theta}_{rd} + 5T_1 D_r \bar{\theta}_{sd}/2 + 3T_2^2 D_r/20 T_1 - T_2^2 D_r/20 T_1 - 7T_2 D_r/10.
\]

Likewise, we designed a computer program to solve model equation (32) according to the former discussed hybrid intelligent algorithm. Consequently, we obtain the computational results in Table 2.

From Table 2, we can see that the maximal relative error is only 0.18% when different values of the genetic algorithm parameters are used, which indicates that the hybrid intelligent algorithm is robust to the parameter settings and effective in solving our model equation (32).

To show and analyze the effects of trade credit strategy, the optimal solutions of Examples 1 and 2 under the same combinations of genetic algorithm parameters are listed in Table 3. Besides, we present another two indexes, called cost savings and cost reduction in percentage. Here, we let cost savings = cost without trade credit–cost with trade credit and cost reduction in percentage = (cost savings/cost without trade credit) × 100%. Considering the results in Table 3, it is clear that, compared with Example 1, as the trade credit period \( T_2 \) increases, there is a significant increase in replenishment cycle length \( T_1 \) but there is a strictly decrease in the integrated supply chain’s total cost. In fact, the average cost savings is 1508.8457, while the maximal cost reduction in percentage is 47.19% (i.e., the total cost is almost reduced by half). Hence, our computational results identify that, in an integrated inventory model, the trade credit strategy has a positive impact on reducing the total cost.

Example 3. Another numerical example for case 3 will be used to verify the proposed model 3. The data is the same as Example 1 except we suppose that the integrated supply chain’s budget is 1560. Therefore, according to the

<table>
<thead>
<tr>
<th>Order</th>
<th>pop_size</th>
<th>( P_c )</th>
<th>( P_m )</th>
<th>Optimal solution</th>
<th>Cost</th>
<th>Absolute error</th>
<th>Relative error (%)</th>
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<td>1</td>
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<td>0.4</td>
<td>0.2</td>
<td>0.2462</td>
<td>3202.0399</td>
<td>0.0000</td>
<td>0.00</td>
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<td>0.3</td>
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</tr>
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<td>3</td>
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<td>0.4</td>
<td>0.2411</td>
<td>3204.2447</td>
<td>2.2048</td>
<td>0.07</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2411</td>
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<td>2.2048</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
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<td>0.5</td>
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<td>0.4</td>
<td>0.2415</td>
<td>3204.6179</td>
<td>2.5780</td>
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Table 2: Comparative solutions of Example 2.

<table>
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<th>( P_m )</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>Cost</th>
<th>Absolute error</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
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<td>0.3</td>
<td>0.2</td>
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<td>0.4561</td>
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<td>0.7741</td>
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</tr>
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<td>0.3</td>
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<td>0.4605</td>
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<td>0.4496</td>
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<td>1.3548</td>
<td>0.08</td>
</tr>
<tr>
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Table 3: Comparative solutions of Example 1.
6. Conclusions

In this paper, we consider an integrated inventory problem. Contrary to previous models, we relax the classical assumptions that some parameters (e.g., the customer demand rate, the deterioration rate) are deterministic and the retailers must settle for the account when purchasing the items. In response, we explore the integrated inventory policy in fuzzy environments under trade credit; that is, here, we assume that the customer demand rate, retailer demand rate, retailer’s inventory deterioration rate, and supplier’s inventory deterioration rate follow a triangular distribution. Moreover, the supplier offers the retailer a certain credit period. We first formulate this fuzzy integrated inventory problem; then, by analyzing the total cost function, we develop three types of fuzzy programming models for three cases of the credit period. A hybrid intelligent algorithm by integrating fuzzy simulations, neural networks, and genetic algorithms is further presented for solving the above fuzzy programming models. Furthermore, the efficiency of the proposed hybrid intelligent algorithm is verified in the illustrative examples. The computational experiments explicitly characterized the optimal inventory control and trade credit strategies in a fuzzy environment. That is, as the trade credit period $T_2$ increases, there is a significant increase in replenishment cycle length $T_1$ but there is a strictly decrease in the integrated supply chain’s total cost. As can be clearly seen from Table 3, through adopting the trade credit

<table>
<thead>
<tr>
<th>pop_size</th>
<th>$P_c$</th>
<th>$P_m$</th>
<th>Optimal solution</th>
<th>Cost</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2462 0.4534 0</td>
<td>3202.0399 1696.8947</td>
<td>1505.1452 47.01</td>
</tr>
<tr>
<td>30</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2443 0.4458 0</td>
<td>3206.7054 1696.0922</td>
<td>1510.6132 47.11</td>
</tr>
<tr>
<td>40</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2411 0.4773 0</td>
<td>3204.2447 1697.5710</td>
<td>1506.6737 47.02</td>
</tr>
<tr>
<td>40</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2411 0.4562 0</td>
<td>3204.2447 1695.5063</td>
<td>1508.7384 47.09</td>
</tr>
<tr>
<td>50</td>
<td>0.3</td>
<td>0.5</td>
<td>0.2497 0.4461 0</td>
<td>3204.9772 1695.2401</td>
<td>1509.7371 47.11</td>
</tr>
<tr>
<td>50</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2415 0.4480 0</td>
<td>3204.6179 1692.4479</td>
<td>1512.1700 47.19</td>
</tr>
</tbody>
</table>

Note: ($T_1$, $T_2$) and cost$_1$ are the optimal solution and objective value of Example 1. Similarly, ($T_1'$, $T_2'$) and cost$_2$ are the optimal solution and objective value of Example 2.

<p>| Table 4: Comparative solutions of Example 3. |</p>
<table>
<thead>
<tr>
<th>Order</th>
<th>pop_size</th>
<th>$P_c$</th>
<th>$P_m$</th>
<th>Optimal solution</th>
<th>Credibility</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>0.1</td>
<td>0.2</td>
<td>0.5012 0.8858</td>
<td>0.9237</td>
<td>0.0072 0.77</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5069 0.9465</td>
<td>0.9179</td>
<td>0.0130 1.40</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5128 0.9182</td>
<td>0.9297</td>
<td>0.0012 0.13</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4940 0.8786</td>
<td>0.9309</td>
<td>0.0000 0.00</td>
</tr>
<tr>
<td>5</td>
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<td>0.3</td>
<td>0.2</td>
<td>0.5174 0.9206</td>
<td>0.9293</td>
<td>0.0016 0.17</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
<td>0.4</td>
<td>0.6</td>
<td>0.4901 0.8416</td>
<td>0.9222</td>
<td>0.0087 0.93</td>
</tr>
</tbody>
</table>

Figure 2: The impact of trade credit policy.
strategy, the replenishment cycle length $T_1$ was prolonged by about 86%, and the total cost was reduced by about 47%.

The findings of this study can provide the following managerial insights to administrative personnel in decision-making:

(1) According to different manager’s risk appetites and management goals, we propose three types of decision-making criteria: expected value criterion, chance-constrained criterion, and credibility maximization criterion. Specifically, if the decision-maker is risk-averse, he/she may choose the first criterion to make a decision. In contrast, taking into account the risk, the second decision-making criterion is to optimize some critical value with at least some given confidence levels subject to some chance constraints, and the third decision-making criterion is especially suitable for the complex fuzzy decision system with multiple tasks.

(2) Managers sometimes cannot make scientific inventory decisions in a VUCA world, in part because of the paucity of historical data. When this happens, it is helpful for managers to deal with inventory uncertainties by using the fuzzy inventory models and hybrid intelligent algorithm proposed in this paper.

(3) In the homogenization of products and services tend to today, adopting trade credit strategies is an advantageous way to reduce the integrated supply chain’s cost and enhance competitive power.

For further research, some possible extensions of this work are as follows:

(1) We will incorporate more realistic assumptions and constraints into our models, such as multisupplier, multiretailer, multiproduction, imperfect items, allowable shortages, and raw material constraint

(2) Also, it is interesting to consider a two-level trade credit policy in the proposed models

(3) Compared with fuzzy simulation, the hybrid intelligent algorithm is less time-consuming. However, it turns out to be a higher algorithmic complexity, and unavoidably, a lower efficiency. Therefore, some other uncertainty approaches such as the new fuzzy simulation algorithms (Liu et al. [75]), robust optimization (Tirkolaee et al. [76]), grey systems (Ju et al. [77]), and other metaheuristic algorithms (Goli et al. [78]) may be explored and employed to solve the uncertain nature of the problem

(4) As one may consider the model parameters fuzzy, it is natural and reasonable to assume them as random variables, fuzzy random variables, random fuzzy variables, grey variables, etc.

Data Availability

All relevant data are included within the paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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