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Research Article

Dynamic Pricing Strategy for Two-Generation Products under Different Trade-in Subsidy Strategies

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Trade-in is one of the most widely used recycling methods in practice. A key problem in the trade-in process is the dynamic pricing strategy under different trade-in subsidy strategies. Consumers are divided into myopic and strategic consumers. Considering three situations, namely, (1) no trade-in subsidy strategy, (2) the firm's trade-in subsidy strategy, and (3) the government's trade-in subsidy strategy, the dynamic game equilibrium between firms and consumers is analyzed. The optimal dynamic pricing strategy is obtained by constructing a two-stage dynamic game model. The results show that (1) the number of consumers who purchase next-generation new products is not impacted by the trade-in subsidy strategy. However, the number of consumers who engage in trade-ins is impacted by the trade-in subsidy strategy. (2) The number of consumers who engage in trade-ins with the trade-in subsidy strategy is larger than that without the trade-in subsidy strategy. (3) Compared with the government's trade-in subsidy strategy, the firm's trade-in subsidy strategy can more effectively motivate consumers to engage in trade-in activity.

1. Introduction

1.1. Background. Trade-in program is one of the most widely used recycling methods in practice. For example, mobile phone, air conditioners, and computers can be recycled through trade-in programs (Altug and Aydinliyim [1]). An important problem faced by firms during trade-ins is the product's pricing problem. To retain old consumers and encourage consumers to buy products many times, firms often offer a discount price for first-generation products (Klemperer [2]), which we call the trade-in rebate. Due to the product life cycle and product upgrades during trade-ins, firms are often faced with the dynamic pricing problem (Cachon and Feldman [3]; Correa et al. [4]). A dynamic pricing strategy means that the enterprise determines the selling price for each period. Many firms have used the dynamic pricing strategy during their sales process. For instance, many producers, such as Huawei and Samsung, make pricing decisions for each generation's products dynamically. Under the dynamic pricing strategy, in the first period, customers purchase first-generation products and firms determine the selling price of these first-generation products. In the second period, customers choose to purchase second-generation products using trade-ins, and enterprises decide the selling price of new-generation products.

In general, when enterprises determine to adopt the dynamic pricing strategy, they should know the effect of customers' strategic choice behavior. The strategic customer can estimate the price of future products and strategically choose when to buy. Customers' strategic behavior mainly expresses two aspects, that is, on the one hand, customers can get the retail price of next-generation products in many ways. On the other hand, when firms offer trade-ins, customers will select whether and when to engage in trade-ins.

In view of the important influence of trade-ins in improving demand for new products and the circular economy, many governments have pointed out a series of trade-in policies to improve customers to engage in trade-ins. For example, the United States offers a subsidy of \$3500–\$4500 to customers. The Chinese government subsidizes 4000–6000 yuan for customers who engage in vehicle trade-in activities. In addition to the government providing trade-in subsidies, some enterprises also provide trade-in subsidies to encourage repeat purchases and sales of new products. For example, Huawei offers a subsidy of 300 yuan for customers. Apple offers a maximum subsidy of 1000 yuan for customers. The dynamic pricing strategy for successive-generation products considering consumer choice behavior and different trade-in subsidy strategies should be worthy of research. In this paper, the following questions are addressed: (a) What are the optimal pricing strategies and trade-in strategies for firms under different trade-in subsidy strategies. (b) How would the different trade-in subsidy strategies influence consumers' choice behavior and the firm's optimal dynamic pricing strategy.

First, the two-stage game model was built based on myopic customers, strategic customers, and different trade-in subsidy strategies. It is assumed that the enterprise sells two-generation products. First, the enterprise sells the first-generation products to myopic customers and strategic customers. In the second period, the enterprise sells the second-generation products and offers trade-ins for myopic and strategic consumers under different trade-in subsidy strategies. Second, myopic and strategic customers' choice behavior and pricing strategy within each period are analyzed. Third, the impact of different trade-in subsidy strategies on two kinds of customers' choice behavior and the firm's optimal dynamic pricing strategy is analyzed.

The main results are summarized as follows: From the perspective of consumers, (1) in the first period, some consumers would like to purchase the second-generation new products, some consumers prefer to purchase the first-generation new products at a discount price, and others would like to engage in trade-in activity. (2) In the second period, many customers who engage in the trade-in activity are impacted by the trade-in subsidy strategy. The number of consumers who engage in trade-ins with the trade-in subsidy strategy is larger than that without it. From the view of the enterprise, (1) the enterprise's profit in the dynamic pricing strategy is decreasing when the discount factor and the innovation incremental value are at a higher level, (2) the firm can obtain much more profit when two kinds of consumers exist in the market, and (3) with the increase in trade-in subsidies, the trade-in rebate decreases.

1.2. Contribution Statement. First, this study discusses the optimal dynamic pricing strategy under different trade-in subsidies which includes no trade-in subsidy strategy, the firm's trade-in subsidy strategy, and the government's trade-in subsidy strategy. Second, this study is the first to characterize the impact of myopic consumers and strategic consumers' choice behavior on the dynamic pricing strategy under different trade-in subsidy strategies.

The remainder of this study is organized as follows: Section 2 points out the literature review. Section 3 provides the model and the notation. Section 4 analyzes the enterprise's dynamic pricing strategy during two periods. In Section 5, the results under three decision-making models are compared, and Section 6 concludes the paper.

2. Literature Review

Many scholars have studied pricing decision issues in trade-ins. Some papers discussed the static pricing strategy problem in trade-ins. Ray et al. [5] discussed the optimal static pricing and the trade-in strategy. Huang et al. [6] considered the static pricing strategy in an automobile supply chain. Correa et al. [4] pointed out that the static pricing policy could reduce the waiting behavior of strategic customers and increase the enterprise's profit. Han et al. [7] discussed the situations when enterprises should provide a trade-old for remanufactured (TOR) program. Chen et al. [8] analyzed three different choices of the enterprise. Ma et al. [9] analyzed one enterprise's static pricing strategy and obtained the thresholds that determined whether the enterprise provides "trade-old for new" and "trade-old for remanufactured" programs. Miao et al. [10] analyzed three static pricing decision models with trade-ins. Cao et al. [11] analyzed the static pricing strategy in trade-in programs in B2C platforms. Cao et al. [12] discussed the optimal static pricing strategy and the optimal trade-in strategy of an enterprise subject to carbon tax policies. Hu et al. [13] discussed the enterprise's static pricing strategy considering the trade-in duration. Sheu and Choi [14] used one multimethodological approach to explore trade-in-upgrade-related pricing decisions.

Some scholars have discussed the dynamic pricing strategy problem in trade-ins. Zhu et al. [15] discussed the effect of a competitive environment on dynamic pricing strategy during the trade-in program. Xiao et al. [16] considered a firm that offered both trade-in options and discussed the dynamic pricing strategy during two periods. Liu et al. [17] compared the difference between static pricing strategies and dynamic pricing strategies. In the previous sections, some researchers analyzed the influence of customers' strategic behavior on both static pricing strategy and dynamic pricing strategy. Huang et al. [6], Zhu et al. [15], Chen and Hsu [8], Xiao and Zhou [16], and Sheu and Choi [14] considered the effect of customers' strategic behavior on pricing strategies. Moreover, they discussed the advantages and usage situations of different trade-in strategies. Some other papers that considered trade-in rebate decisions are as follows: Ray et al. [5] discussed optimal pricing and trade-in rebate decisions. Considering the strategic consumer and trade-in rebate, Liu et al. [17] discussed the difference between the static pricing strategy and the dynamic pricing strategy. Some papers explore the trade-in subsidy strategy offered by enterprises or governments. Levinthal and Purohit [18] pointed out that the enterprise uses trade-in rebates to improve product quality. Fudenberg and Tirole [19] discussed the optimal pricing and trade-in rebate decisions for a firm. Kim et al. [20] built one analytical model to discuss customers' choices during trade-in program. Sana [21] investigated a reduction-inventory model where regular preventive maintenance starts at the end of production for smooth functioning in the next cycle.

As is known to us, only a few scholars discuss the effect of the trade-in subsidy on the pricing strategy. For example, Huang et al. [6] considered one automobile supply chain and analyzed the impact of subsidy strategy on stimulating consumers' trade-in transactions. Cao et al. [12] explored the optimal static pricing strategy and third-party collection authorization strategies for the manufacturer. Meng et al.

[22] investigated the optimal government subsidy and its impact on the operation of the CLSC. He et al. [23] discussed the pricing decisions for the firm and government's subsidy policy.

This study contributes to the literature in following aspects: Some studies discuss the effect of myopic customers or strategic customers on the static pricing strategy or the dynamic pricing strategy in trade-ins. Others consider the optimal trade-in rebate decision problem during trade-ins. However, they do not analyze the impact of different trade-in subsidy on consumers' choice and on firms' pricing strategy. To fill the gap in the literature, this study discusses the optimal dynamic pricing strategy and the optimal trade-in rebate strategy. Moreover, this study considers the effect of myopic customers and strategic customers' choice on the pricing strategy under the situations of different trade-in subsidy strategies and dual rollover strategies. Then, the effect of different trade-in subsidy strategies on consumers' choice and the enterprise's pricing strategy is compared.

3. The Model

Considering the two-stage game model between firms and customers, firms sell products by means of a dual rollover strategy; in period 1, the enterprise sells the first-generation new products X_1 at the retail price p_1. The manufacturing cost is c_1 (i.e., $c_1 \le p_1$). The customers' valuation of first-generation new products X_1 is v_1 (i.e., $p_1 \le v_1$), which follows a [0,1] uniform distribution. We assume that it has D (i.e., = 1) potential consumers in the market. Based on the consumer's purchase behavior, $\alpha (0 \le \alpha \le 1)$ myopic consumers and $1-\alpha$ strategic consumers were considered. Strategic customers analyze the opportunity to purchase products in period 2. However, myopic customers will not think to purchase products; when the retail price of the first-generation products is no larger than its valuation, myopic consumers will make purchase decisions. Assuming that the utility of the consumer decreases with the usage time, the discount factor $\delta \in [0,1]$ is introduced.

In period 2, the enterprise sells the second-generation new products X_2 at price p_2, and the enterprise sells the first-generation products X_1 in the market at discount price $p_d (0 \le p_d \le p_1, 0 \le p_d \le p_2)$. To retain the old consumers, we motivate consumers to buy repeatedly and stabilize market share, and the firm provides the trade-in service. The customers who have purchased the products X_1 returns the old products X_1 to the enterprise and get the certain trade-in rebate $p_t (0 \le p_t \le p_2)$ when the customers purchase X_2, and the manufacturing cost is c_2 (i.e., $c_2 \le p_2$). The customers' valuation of the second-generation new products is v_2 (i.e., $p_2 \le v_2$), where $v_2 = (\theta+1)v_1$, Each old product has a salvage value $s (0 \le s \le \theta)$. Figure 1 illustrates the time sequence of events.

In Figure 1, at the beginning of the period 1, based on the number of consumers waiting for the second period, the firm decides the unit retail price p_1. After that, two kinds of customers will determine whether to purchase them. At the beginning of the period 2, based on the unit retail price p_1 and the expectation of the market demand, the firm determines the unit retail price p_2, the discount price p_d, and trade-in rebate p_t.

We consider the following decision-making models (see Figure 2).

(a) Model (N), neither the firm nor the government provides the trade-in subsidy strategy (Figure 2(a)).

(b) Model (R), the firm provides the trade-in subsidy strategy (Figure 2(b)). Any customers who purchase the next generation new products and return his or her old generation products can obtain a specific trade-in subsidy s_r from the firm.

(c) InModel (G), the government provides the trade-in subsidy strategy (Figure 2(c)). Any customers who purchase the next generation new products and return his or her old generation products can obtain a specific trade-in subsidy s_g from the government.

Moreover, for easier analysis, the notations are summarized in Table 1.

4. Decision-Making Models

Because the government or the firm can provide different trade-in subsidy strategies for customers, the impact of different trade-in subsidy strategies on customers' choice and enterprises' dynamic pricing strategies will be discussed. We characterize the outcomes of each decision-making model as shown in Figure 2.

4.1. Benchmark: Without Trade-in Subsidies (Model N)

4.1.1. Consumers' Strategic Choice Behavior. First, myopic consumers' choice behavior is analyzed. When the retail price of the first-generation products is no higher than its valuation, myopic consumers will make purchase decisions. When myopic customers with a valuation higher than τ_1^m purchase X_1 in period 1, myopic consumers with a valuation lower than τ_1^m choose to wait. In period 2, myopic customers include X_1 -holders and non- X_1 -holders. The X_1-holders decide whether to trade-in X_1 for X_2 (the utility is $(1+\theta)v_1 - p_2 + p_t$) or to keep using X_1 (the utility is v_1). The non- X_1 -holders decide whether to buy X_1 at the discount price (the utility is $v_1 - p_d$) or buy X_2 at the regular price (the utility is $(1+\theta)v_1 - p_2$). Comparing the myopic consumers' surplus, their purchasing decision can be obtained in the second period. The X_1 -holders with a valuation higher than τ_2^m choose to trade-in X_1 for X_2. For the non-X_1 -holders, it exists two different selections. In case I(II), the customers would like to buy $X_2 (X_1)$, the non- X_1 -holders with a valuation between τ_3^m and τ_1^m will purchase $X_2 (X_1)$, and the non- X_1 -holders with a valuation between τ_4^m and τ_3^m will purchase $X_1 (X_2)$, while those with a valuation lower than τ_4^m will buy nothing.

Next, strategic customer choice behavior is analyzed. Strategic customers can analyze the choice to purchase products in period 2, and the customers can compare the utility gotten from the first period with the delayed period to utilise and choose the best purchase chance. When strategic

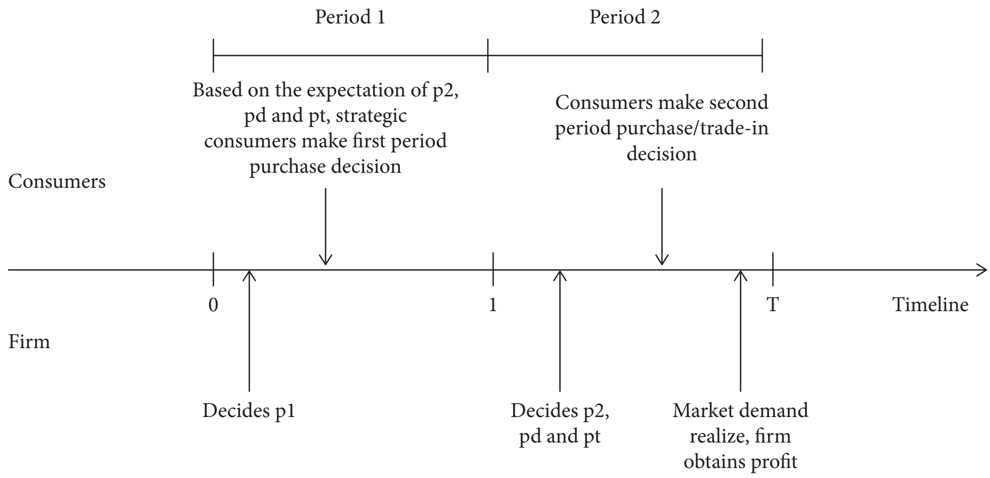


Figure 1: Sequence of events under a dynamic pricing strategy.

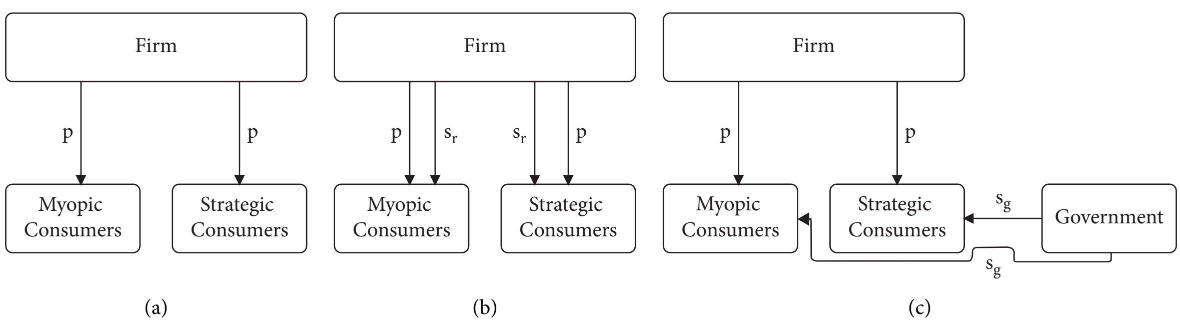


Figure 2: Decision-making models. (a) Model N. (b) Model R. (c) Model G.

Table 1: Model parameters.

Parameters	Definitions
q_i^{2T}	Number of customers who engage in trade-in program in period 2, where, $i \in \{m, s\}$, m means myopic consumers, s means strategic consumers
$q_i^{2\mathrm{NT}}$	Number of customers who do not engage in trade-ins in period 2
q_i^{2B}	Number of customers who want to buy the second-generation new products in period 2
q_i^{2NB}	Number of customers who do not want to buy the second-generation new products in period 2
q_i^{2Bd}	Number of customers who want to buy first-generation new products in period 2 at a discount price
q_i^{1B}	Number of customers who want to buy first-generation new products in period 1
p_1	Retail price
p_2	Retail price
p_d	Discount price
p_t	Firm offers a trade-in rebate
s_r	The trade-in subsidy provided by the firm
s_g	The trade-in subsidy provided by the government
δ	Discount factor
θ	Innovation incremental value
c_1	The manufacturing cost of the first-generation new products
c_2	The manufacturing cost of the next generation new products
v_1	Customers' valuation of the first-generation new products
v_2	Customers' valuation of the next generation new products
α	Proportion of the myopic consumer
π_2	Firm's profit in the second period
π_t	Firm's total profit in the two periods

consumers with the valuation larger than τ_1^s purchase X_1 in period 1, strategic customers with a valuation lower than τ_1^s choose to wait. In period 2, strategic customers include X_1-holders and non-X_1-holders. The X_1-holders decide whether to trade-in X_1 for X_2 (the utility is $(1+\theta)v_1 - p_2 + p_t$) or to keep using X_1 (the utility is v_1). The non-X_1-holders decide whether to buy X_1 with the discount price (the utility is $v_1 - p_d$) or to buy X_2 with the regular price (the utility is $(1+\theta)v_1 - p_2$).

4.1.2. Firm's Dynamic Pricing Decisions. Based on the consumer's market demand under different choice conditions, the optimal dynamic pricing decision model in period 2 is constructed, π_2^N is the enterprise's profit in period 2, and the optimization problem is as follows:

$$\begin{aligned}\max_{p_2,p_t,p_d} \pi_2^N = {} & (p_2 - c_2 - p_t)\left(q_m^{2T} + q_s^{2T}\right) \\ & + (p_2 - c_2)\left(q_m^{2B} + q_s^{2B}\right) \\ & + (p_d - c_1)\left(q_m^{2Bd} + q_s^{2Bd}\right),\end{aligned} \tag{1}$$

$$\text{s.t.} \quad 0 \le q_m^{2B} \le \alpha - q_m^{1B}, \tag{2}$$

$$0 \le q_m^{2Bd} \le \alpha - q_m^{1B}, \tag{3}$$

$$q_m^{2Bd} + q_m^{2B} \le \alpha - q_m^{1B}, \tag{4}$$

$$q_m^{2T} \le q_m^{1B}, \tag{5}$$

$$0 \le p_t \le p_2, \tag{6}$$

$$0 \le p_d \le p_1, \tag{7}$$

$$0 \le q_s^{2B} \le 1 - \alpha - q_s^{1B}, \tag{8}$$

$$0 \le q_s^{2Bd} \le 1 - \alpha - q_s^{1B}, \tag{9}$$

$$q_s^{2Bd} + q_s^{2B} \le 1 - \alpha - q_s^{1B}, \tag{10}$$

$$q_s^{2T} \le q_s^{1B}. \tag{11}$$

Constraints (2)–(4) guarantee that myopic consumers who will not hold X_1 buy X_2 at p_2 or purchase X_1 with p_d in the second period. Constraint (5) makes sure that the X_1-holders could trade-in X_1 for X_2. Constraints (8)–(10) guarantee that strategic consumers will not hold X_1, buy X_2 at p_2, or purchase X_1 with p_d in the second period. Constraint (11) makes sure that the X_1-holders will trade-in X_1 for X_2.

Next, based on customer's demand under different choice conditions, the firm should decide the retail price p_1 in the first period to maximize the total profit π_t^N of two periods.

$$\max \pi_t^N = (p_1 - c_1)\left(q_m^{1B} + q_s^{1B}\right) + \pi_2^{N*}, \tag{12}$$

$$\text{s.t.} \quad 0 \le q_m^{1B} \le \alpha, \tag{13}$$

$$0 \le q_s^{1B} \le 1 - \alpha, \tag{14}$$

$$0 \le p_1 \le 1, \tag{15}$$

where the first item expresses the profit gained by the firm from selling the products X_1, and the second item expresses the enterprise's optimal profit in period 2. Constraints (13)–(15) make sure that the first-period quantity and price are non-negative. Based on the previous analysis, the equilibrium solution, when the firm uses the dynamic pricing strategy, can be obtained in Proposition 1 (See Appendix for the proof of Proposition 1).

Proposition 1. *When the enterprise uses the dynamic pricing strategy, the equilibrium results are derived as follows:*

(i) In case I, the equilibrium prices and trade-in rebate are as follows:

$$\begin{aligned} p_1^* &= \frac{(3\theta-1)(2\delta\theta-\theta-5)-(1+\theta)^2}{2\theta+1} + \frac{c_1}{4}, \\ p_2^* &= \frac{c_2}{2} + \frac{(1+\theta)}{2} + \frac{c_1(4\theta+1)(1+\theta)}{4(3\theta-1)(4\delta\theta-\theta-1)-4(1+\theta)^2}, \\ p_t^* &= \frac{1-\theta}{8} + \frac{c_1(9\theta+1)(1+\theta)}{4(3\theta-1)(4\delta\theta-\theta-5)-4(1+\theta)^2}, \\ p_d^* &= \frac{1}{8} + \frac{c_1(4\theta+1)}{4(3\theta-1)(4\delta\theta-\theta-1)-4(1+\theta)^2} + \frac{c_1}{2}. \end{aligned} \tag{16}$$

(ii) In case I, the equilibrium quantities are as follows:

$$\begin{aligned} q_m^{2B*} &= \frac{(\theta+1)}{2(9\theta+1)} + \frac{(8\theta+3)}{2(3\theta-1)(4\delta\theta-\theta-5)-4(1+\theta)^2} - \frac{c_2}{2\theta} - \frac{1}{8}, \\ q_s^{2B*} &= \frac{\theta}{(9\theta+1)} + \frac{(7\theta-1)}{4\left((3\theta-1)(4\delta\theta-\theta-5)-(1+\theta)^2\right)} - \frac{c_2}{2\theta} - \frac{1}{8}, \\ q_m^{1B*} &= \alpha - \frac{2(\theta+1)(2p_1-c_1)}{(3\theta-1)(6\delta\theta-\theta-1)-(1+\theta)^2}, \\ q_s^{2Bd*} &= \frac{\theta(1-\delta)+c_1(\theta+1)}{(2\theta+1)(3+\delta)} + \frac{c_1(2+\theta)}{\theta}, \\ q_m^{2Bd*} &= \frac{(1+\delta)+2c_1(2\theta+1)}{(2\theta+1)} + \frac{c_1(1+\theta)}{2}, \\ q_s^{2T*} &= 1-\alpha-\frac{c_2}{2\theta}, \\ q_m^{2T*} &= \alpha - \frac{c_2}{2\theta}, \\ q_s^{1B*} &= 1-\alpha-\frac{8\theta(2p_1-c_1)}{(3\theta-1)(6\delta\theta-\theta-5)-(1+\theta)^2}. \end{aligned} \tag{17}$$

(iii) In case II, the equilibrium prices and trade-in rebate are as follows:

$$p_1^* = \frac{\theta + c_1(2\theta+1)}{(2\theta+1)(3+\delta)},$$
$$p_2^* = \frac{2c_2}{4} + \frac{\theta(1+\theta) + 2c_1(2\theta+1)(1+\theta)}{(2\theta+1)(3+\delta)} - \frac{c_1(1+\theta)}{4},$$
$$p_d^* = \frac{\theta + 2c_1(2\theta+1)}{(2\theta+1)(3+\delta)},$$
$$p_t^* = \frac{\theta(1+\theta) + 2c_1(2\theta+1)(1+\theta)}{(2\theta+1)(3+\delta)} - \frac{2c_1(1+\theta)}{4} - \frac{\theta}{4}. \quad (18)$$

(iv) In case II, the equilibrium quantities are as follows:

$$q_m^{1B*} = \frac{\alpha(\theta+1) - \theta}{(2\theta+1)},$$
$$q_m^{2T*} = \alpha - \frac{2c_2}{4\theta} - \frac{1}{4},$$
$$q_s^{1B*} = \frac{(1-\alpha)(2\theta+1)(3+\delta) - \theta(1-\delta) - 2c_1(2\theta+1)(1-\delta)}{(2\theta+1)(3+\delta)},$$
$$q_s^{2Bd*} = \frac{\theta(1-\delta) + 2c_1(2\theta+1)(1-\delta)}{(2\theta+1)(3+\delta)} + \frac{c_1(1+\theta)}{4\theta},$$
$$q_m^{2Bd*} = \frac{\theta(2+\delta) + 2c_1(2\theta+1)(2+\delta)}{(2\theta+1)(3+\delta)} - 2c_1 + \frac{c_1(1+\theta)}{4\theta},$$
$$q_m^{2B*} = \frac{c_2}{2\theta} - \frac{(1+\theta)}{2\theta} - \frac{c_2}{2(1+\theta)},$$
$$q_s^{2B*} = \frac{c_2}{2\theta} - \frac{(1+\theta)}{2\theta} - \frac{c_2}{2(1+\theta)},$$
$$q_s^{2T*} = 1 - \alpha - \frac{2c_2}{4\theta} - \frac{1}{4}. \quad (19)$$

Lemma 1. *Under the situation of without trade-in subsidy strategy, (i) the retail price is larger than the discount price, and (ii) the customers who have bought the first-generation's old products is larger than the customers who engage in trade-in program.*

We can get the following conclusions:

① *In the case without the trade-in subsidy strategy, in period 2, the retail price of the second-generation new products is larger than the discount price.*

② *Comparing with the choice behavior of different types of consumers, it was found that* $q_m^{1B*} > q_m^{2T*}$; $q_s^{1B*} > q_s^{2T*}$. *This means that customers who have purchased old products in period 1 do not necessarily engage in trade-ins in period 2. Some customers can choose to buy the second-generation new products; in case I, strategic customers select to buy the new-generation products directly.*

4.2. The Firm Provides Trade-in Subsidies (Model R). To retain old consumers, we motivate consumers to buy products frequently, reduce the encroachment effect from the secondary market, and promote product upgrading; the firm provides trade-in subsidies s_r to customers who engage in trade-ins.

4.2.1. Consumers' Strategic Choice Behavior. First, when the retail price is no higher than its valuation, myopic consumers will make purchase decisions. When myopic customers with a valuation larger than τ_1^m buy X_1 in period 1, myopic customers with a valuation lower than τ_1^m choose to wait. In period 2, myopic customers include X_1-holders and non-X_1-holders. The X_1-holders decide whether to trade-in X_1 for X_2 (the utility is $(1+\theta)v_1 - p_2 + p_t + s_r$) or to keep using X_1 (the utility is v_1). The non-X_1-holders decide whether to buy X_1 at a discount price (the utility is $v_1 - p_d$) or buy X_2 at a regular price (the utility is $(1+\theta)v_1 - p_2$). Comparing the myopic customers' utility, the X_1-holders with the valuation larger than τ_2^m choose to trade-in X_1 for X_2, and the others will keep using X_1. For the non-X_1-holders, in case I(II), the customers would like to buy $X_2(X_1)$, the non-X_1-holders with a valuation between τ_3^m and τ_1^m will purchase $X_2(X_1)$, and the non-X_1-holders with the valuation between τ_4^m and τ_3^m buys $X_1(X_2)$.

Next, strategic customer choice behavior is analyzed. When strategic customers with the valuation larger than τ_1^s buy X_1 in the first period, strategic consumers with a valuation lower than τ_1^s choose to wait. In the second period, strategic customers include X_1-holders and the non-X_1-holders. The X_1-holders decide whether to trade-in X_1 for X_2 (the utility is $(1+\theta)v_1 - p_2 + p_t + s_r$) or to keep using X_1 (the utility is v_1). The non-X_1-holders decide whether to buy X_1 at a discount price (the utility is $v_1 - p_d$) or to buy X_2 at the regular price (the utility is $(1+\theta)v_1 - p_2$).

4.2.2. Firm's Dynamic Pricing Decisions. Based on the consumer's market demand under different choice conditions, the optimal dynamic pricing decision model in period 2 is constructed, π_2^R is the enterprise's profit in the second period, and the optimization problem is that

$$\max_{p_2,p_t,p_d} \pi_2^R = (p_2 - c_2 - p_t - s_r)\left(q_m^{2T} + q_s^{2T}\right) + (p_2 - c_2)\left(q_m^{2B} + q_s^{2B}\right) + (p_d - c_1)\left(q_m^{2Bd} + q_s^{2Bd}\right), \quad (20)$$

$$\text{s.t.} \quad 0 \le q_m^{2B} \le \alpha - q_m^{1B}, \quad (21)$$

$$0 \le q_m^{2Bd} \le \alpha - q_m^{1B}, \quad (22)$$

$$q_m^{2Bd} + q_m^{2B} \le \alpha - q_m^{1B}, \quad (23)$$

$$q_m^{2T} \le q_m^{1B}, \quad (24)$$

$$0 \le p_t \le p_2, \quad (25)$$

$$0 \le p_d \le p_1, \quad (26)$$

$$0 \le q_s^{2B} \le 1 - \alpha - q_s^{1B}, \quad (27)$$

$$0 \le q_s^{2Bd} \le 1 - \alpha - q_s^{1B}, \quad (28)$$

$$q_s^{2Bd} + q_s^{2B} \leq 1 - \alpha - q_s^{1B}, \quad (29)$$

$$q_s^{2T} \leq q_s^{1B}. \quad (30)$$

Constraints (21)–(23) guarantee that myopic consumers will not hold X_1 purchase X_2 at p_2 or purchase X_1 with p_d in period 2. Constraint (24) makes sure that the X_1-holders trade-in X_1 for X_2. Constraints (27)–(29) guarantee that strategic consumers will not hold X_1 purchase X_2 at p_2 or purchase X_1 with p_d in period 2. Constraint (30) makes sure that the X_1-holders trade-in X_1 for X_2.

Next, based on the consumer's demand under different choice conditions, the firm should decide the retail price p_1 in the first period to maximize the total profit π_t^R of the two periods. π_2^{R*} is the firm's optimal profit in the second period.

$$\max_{p_1} \pi_t^R = (p_1 - c_1)\left(q_m^{1B} + q_s^{1B}\right) + \pi_2^{R*}, \quad (31)$$

$$\text{s.t.} \quad 0 \leq q_m^{1B} \leq \alpha, \quad (32)$$

$$0 \leq q_s^{1B} \leq 1 - \alpha, \quad (33)$$

$$0 \leq p_1 \leq 1, \quad (34)$$

where the first item expresses the profit gained by the firm from selling the products X_1, and the second item expresses the enterprise's optimal profit in period 2. Constraints (32)–(34) make sure that the first-period quantity and price are non-negative. Based on the previous analysis, the equilibrium solution when the firm uses the dynamic pricing strategy can be obtained in Proposition 2 (See Appendix for the proof of Proposition 2).

Proposition 2. *If the firm provides a trade-in subsidy, under a dynamic pricing strategy, it has one perfect equilibrium.*

(i) In case I, the equilibrium solutions are as follows:

$$\begin{aligned}
p_1^{**} &= \frac{(3\theta-1)(2\delta\theta-\theta-5)-(1+\theta)^2}{2\theta+1} + \frac{c_1}{4}, \\
p_d^{**} &= \frac{1}{8} + \frac{c_1(4\theta+1)}{4(3\theta-1)(4\delta\theta-\theta-1)-4(1+\theta)^2} + \frac{c_1}{2}, \\
p_t^{**} &= \frac{1-\theta}{8} + \frac{c_1(9\theta+1)(1+\theta)}{4(3\theta-1)(4\delta\theta-\theta-5)-4(1+\theta)^2} - s_r, \\
p_2^{**} &= \frac{c_2}{2} + \frac{(\theta+1)}{2} + \frac{c_1(\theta+1)(4\theta+1)}{4(3\theta-1)(4\delta\theta-\theta-1)-4(1+\theta)^2}.
\end{aligned} \quad (35)$$

(ii) In case II, the equilibrium quantities are as follows:

$$\begin{aligned}
q_m^{2B**} &= \frac{(\theta+1)}{2(9\theta+1)} + \frac{(8\theta+3)}{2(3\theta-1)(4\delta\theta-\theta-5)-4(1+\theta)^2} - \frac{c_2}{2\theta} - \frac{1}{8}, \\
q_m^{2Bd**} &= \frac{(1+\delta)+2c_1(2\theta+1)}{(2\theta+1)} + \frac{c_1(1+\theta)}{2}, \\
q_s^{2B**} &= \frac{\theta}{(9\theta+1)} + \frac{(7\theta-1)}{4\left((3\theta-1)(4\delta\theta-\theta-5)-(1+\theta)^2\right)} - \frac{c_2}{2\theta} - \frac{1}{8}, \\
q_s^{2Bd**} &= \frac{\theta(1-\delta)+c_1(\theta+1)}{(2\theta+1)(3+\delta)} + \frac{c_1(2+\theta)}{\theta}, \\
q_s^{1B**} &= 1 - \alpha - \frac{8\theta(2p_1-c_1)}{(3\theta-1)(6\delta\theta-\theta-5)-(1+\theta)^2}, \\
q_m^{1B**} &= \alpha - \frac{2(\theta+1)(2p_1-c_1)}{(3\theta-1)(6\delta\theta-\theta-1)-(1+\theta)^2}, \\
q_s^{2T**} &= \frac{3}{4} - \alpha - \frac{c_2}{2\theta} + \frac{s_r}{\theta}, \\
q_m^{2T**} &= \alpha - \frac{c_2}{2\theta} - \frac{1}{4} + \frac{s_r}{\theta}.
\end{aligned} \quad (36)$$

(iii) In case I, the equilibrium solutions are as follows:

$$\begin{aligned} p_2^{**} &= \frac{2c_2}{4} + \frac{\theta(1+\theta) + 2c_1(2\theta+1)(1+\theta)}{(2\theta+1)(3+\delta)} - \frac{c_1(1+\theta)}{4}, \\ p_t^{**} &= \frac{\theta(\theta+1) + 2c_1(2\theta+1)(\theta+1)}{(2\theta+1)(3+\delta)} - \frac{2c_1(\theta+1)}{4} - \frac{\theta}{4} - \frac{s_r}{2}, \\ p_1^{**} &= \frac{\theta + c_1(2\theta+1)}{(2\theta+1)(3+\delta)}, \\ p_d^{**} &= \frac{\theta + 2c_1(2\theta+1)}{(2\theta+1)(3+\delta)}. \end{aligned} \tag{37}$$

(iv) In case II, the equilibrium quantities are as follows:

$$\begin{aligned} q_s^{1B**} &= \frac{(1-\alpha)(2\theta+1)(3+\delta) - \theta(1-\delta) - 2c_1(2\theta+1)(1-\delta)}{(2\theta+1)(3+\delta)}, \\ q_s^{2B**} &= \frac{c_2}{2\theta} - \frac{(1+\theta)}{2\theta} - \frac{c_2}{2(1+\theta)}, \\ q_m^{2T**} &= \alpha - \frac{2c_2}{4\theta} - \frac{1}{4} + \frac{s_r}{2\theta}, \\ q_m^{2Bd**} &= \frac{\theta(2+\delta) + 2c_1(2\theta+1)(2+\delta)}{(2\theta+1)(3+\delta)} - 2c_1 + \frac{c_1(1+\theta)}{4\theta}, \\ q_s^{2T**} &= 1 - \alpha - \frac{2c_2}{4\theta} - \frac{1}{4} + \frac{s_r}{2\theta}, \\ q_m^{1B**} &= \frac{\alpha(\theta+1) - \theta}{(2\theta+1)}, \\ q_s^{2Bd**} &= \frac{\theta(1-\delta) + 2c_1(2\theta+1)(1-\delta)}{(2\theta+1)(3+\delta)} + \frac{c_1(1+\theta)}{4\theta}, \\ q_m^{2B**} &= \frac{c_2}{2\theta} - \frac{(1+\theta)}{2\theta} - \frac{c_2}{2(1+\theta)}. \end{aligned} \tag{38}$$

Lemma 2. *When the enterprise uses the trade-in subsidy strategy, (i) the retail price is larger than the discount price and (ii) number of customers who have purchased the first-generation old products is larger than number of customers who engage in trade-ins in period 2.*

Based on Lemma 2, the following conclusions are obtained as follows:

① *With the enterprise's trade-in subsidy strategy, under two cases, the retail price of the second-generation new products is larger than the discount price. Some myopic consumers and strategic customers prefer to purchase first-generation new products with discount prices, while others prefer to buy second-generation new products.*

② *Comparing with the choice behavior of different types of consumers, it was found that $q_m^{1B**} > q_m^{2T**}$; $q_s^{1B**} > q_s^{2T**}$. This means that customers who have purchased old products in period 1 do not necessarily engage in trade-ins in period 2. Some customers will choose to purchase the second-generation new products, and other consumers prefer to purchase the first-generation new products at a discounted price.*

4.3. The Government Provides Trade-in Subsidies (Model G). To stimulate the demand for new products and play an important role in the circular economy, the government provides trade-in subsidies s_g to consumers who engage in trade-ins.

4.3.1. Consumers' Strategic Choice Behavior. While analyzing myopic consumers' choice behavior, the retail price of the first-generation products is no higher than its valuation, and myopic consumers make purchase decisions. When myopic consumers with the valuation larger than τ_1^m buys X_1 in the first period, myopic consumers with the valuation smaller than τ_1^m select to wait. In period 2, myopic customers include X_1-holders and non-X_1-holders. The X_1-holders decide whether to trade-in X_1 for X_2 (the utility is $(1+\theta)v_1 - p_2 + p_t + s_g$) or to keep using X_1 (the utility is v_1). The non-X_1-holders determine whether to purchase X_1 at a discount price (the utility is $v_1 - p_d$) or to purchase X_2 at a regular price (the utility is $(1+\theta)v_1 - p_2$). Comparing the myopic consumers' utility, their purchasing decision in period 2 can be obtained. The X_1-holders with the valuation larger than τ_2^m choose to trade-in X_1 for X_2, and the others will keep using X_1. The non-X_1-holders have two different selections. In case I(II), the customers would like to buy $X_2\,(X_1)$, the non-X_1-holders between τ_3^m and τ_1^m will buy $X_2\,(X_1)$, and the non-X_1-holders between τ_4^m and τ_3^m would like to buy $X_1\,(X_2)$.

Next, strategic customer choice behavior is analyzed. When strategic customers with the valuation larger than τ_1^s buy X_1 in period 1, strategic customers with the valuation lower than τ_1^s choose to wait. Then, in the second period, strategic customers include X_1-holders and the non-X_1-holders. The X_1-holders decide whether to trade-in X_1 for X_2 (the utility is $(1+\theta)v_1 - p_2 + p_t + s_g$) or to keep using X_1 (the utility is v_1). The non-X_1-holders decide whether to buy X_1 at a discount price (the utility is $v_1 - p_d$) or to buy X_2 at a regular price (the utility is $(1+\theta)v_1 - p_2$).

4.3.2. Firm's Dynamic Pricing Decisions. Based on the consumer's market demand under different choice conditions, the optimal dynamic pricing decision model in period 2 is constructed as follows:

$$\max_{p_2,p_t,p_d} \pi_2^G = (p_2 - c_2 - p_t)\left(q_m^{2T} + q_s^{2T}\right) + (p_2 - c_2)\left(q_m^{2B} + q_s^{2B}\right) + (p_d - c_1)\left(q_m^{2Bd} + q_s^{2Bd}\right), \quad (39)$$

$$\text{s.t.}\, 0 \le q_m^{2B} \le \alpha - q_m^{1B}, \quad (40)$$

$$0 \le q_m^{2Bd} \le \alpha - q_m^{1B}, \quad (41)$$

$$q_m^{2Bd} + q_m^{2B} \le \alpha - q_m^{1B}, \quad (42)$$

$$q_m^{2T} \le q_m^{1B}, \quad (43)$$

$$0 \le p_t \le p_2, \quad (44)$$

$$0 \le p_d \le p_1, \quad (45)$$

$$0 \le q_s^{2B} \le 1 - \alpha - q_s^{1B}, \quad (46)$$

$$0 \le q_s^{2Bd} \le 1 - \alpha - q_s^{1B}, \quad (47)$$

$$q_s^{2Bd} + q_s^{2B} \le 1 - \alpha - q_s^{1B}, \quad (48)$$

$$q_s^{2T} \le q_s^{1B}. \quad (49)$$

Constraints (40)–(42) guarantee that myopic consumers will not hold X_1 buy X_2 at p_2 or purchase X_1 with p_d in the second period. Constraint (43) makes sure that the X_1-holders trade-in X_1 for X_2. Constraints (46)–(48) guarantee that strategic consumers will not hold-X_1 buy X_2 at p_2 or purchase X_1 with p_d in the second period. Constraint (49) makes sure that the X_1-holders trade-in X_1 for X_2.

Next, based on consumer's demand under different choice conditions, the firm should decide the retail price p_1 to maximize the total profit π_t^G of two periods. π_2^{G*} is the enterprise's optimal profit in period 2.

$$\max_{p_1} \pi_t^G = (p_1 - c_1)\left(q_m^{1B} + q_s^{1B}\right) + \pi_2^{G*}, \quad (50)$$

$$\text{s.t.} \quad 0 \le q_m^{1B} \le \alpha, \quad (51)$$

$$0 \le q_s^{1B} \le 1 - \alpha, \quad (52)$$

$$0 \le p_1 \le 1, \quad (53)$$

where the first item expresses the profit gained by the firm from selling the products X_1, and the second item expresses the enterprise's optimal profit in period 2. Constraints (51)–(53) make sure that the first-period quantity and price are non-negative. Based on the previous analysis, the equilibrium solution when the firm uses the dynamic pricing strategy can be obtained in Proposition 3 (See Appendix for the proof of Proposition 3).

Proposition 3. *If the government provides a trade-in subsidy, under a dynamic pricing strategy, it shows perfect equilibrium.*

(i) In case I, the equilibrium solutions are as follows:

$$\begin{aligned}
p_2^{***} &= \frac{c_2}{2} + \frac{(\theta+1)}{2} + \frac{c_1(4\theta+1)(\theta+1)}{4(3\theta-1)(4\delta\theta-\theta-1) - 4(1+\theta)^2}, \\
p_t^{***} &= \frac{(1-\theta)}{8} + \frac{c_1(9\theta+1)(\theta+1)}{4(3\theta-1)(4\delta\theta-\theta-5) - 4(1+\theta)^2} - s_g, \\
p_1^{***} &= \frac{(3\theta-1)(2\delta\theta-\theta-5) - (1+\theta)^2}{2\theta+1} + \frac{c_1}{4}, \\
p_d^{***} &= \frac{c_1}{2} + \frac{1}{8} + \frac{c_1(4\theta+1)}{4(3\theta-1)(4\delta\theta-\theta-1) - 4(1+\theta)^2}.
\end{aligned} \quad (54)$$

(ii) In case I, the equilibrium quantities are as follows:

$$
\begin{aligned}
q_m^{2B***} &= \frac{(\theta+1)}{2(9\theta+1)} + \frac{(8\theta+3)}{2(3\theta-1)(4\delta\theta-\theta-5)-4(1+\theta)^2} - \frac{c_2}{2\theta} - \frac{1}{8},\\
q_m^{2Bd***} &= \frac{(1+\delta)+2c_1(2\theta+1)}{(2\theta+1)} + \frac{c_1(1+\theta)}{2},\\
q_m^{1B***} &= \alpha - \frac{2(\theta+1)(2p_1-c_1)}{(3\theta-1)(6\delta\theta-\theta-1)-(1+\theta)^2},\\
q_s^{1B***} &= 1-\alpha - \frac{8\theta(2p_1-c_1)}{(3\theta-1)(6\delta\theta-\theta-5)-(1+\theta)^2},\\
q_s^{2B***} &= \frac{\theta}{(9\theta+1)} + \frac{(7\theta-1)}{4\left((3\theta-1)(4\delta\theta-\theta-5)-(1+\theta)^2\right)} - \frac{c_2}{2\theta} - \frac{1}{8},\\
q_s^{2Bd***} &= \frac{\theta(1-\delta)+c_1(\theta+1)}{(2\theta+1)(3+\delta)} + \frac{c_1(2+\theta)}{\theta},\\
q_s^{2T***} &= 1-\alpha - \frac{2c_2}{4\theta} - \frac{1}{4} + \frac{s_g}{2\theta},\\
q_m^{2T***} &= \alpha - \frac{2c_2}{4\theta} - \frac{1}{4} + \frac{s_g}{2\theta}.
\end{aligned}
\tag{55}
$$

(iii) In case II, the equilibrium solutions are as follows:

$$
\begin{aligned}
p_2^{***} &= \frac{2c_2}{4} + \frac{\theta(1+\theta)+2c_1(2\theta+1)(1+\theta)}{(2\theta+1)(3+\delta)} - \frac{c_1(1+\theta)}{4},\\
p_t^{***} &= \frac{\theta(\theta+1)+2c_1(2\theta+1)(\theta+1)}{(2\theta+1)(3+\delta)} - \frac{2c_1(\theta+1)}{4} - \frac{\theta}{4} - \frac{s_g}{2},\\
p_1^{***} &= \frac{\theta+c_1(2\theta+1)}{(2\theta+1)(3+\delta)},\\
p_d^{***} &= \frac{\theta+2c_1(2\theta+1)}{(2\theta+1)(3+\delta)}.
\end{aligned}
\tag{56}
$$

(iv) In case II, the equilibrium quantities are as follows:

$$
\begin{aligned}
q_s^{1B***} &= \frac{(1-\alpha)(2\theta+1)(3+\delta)-\theta(1-\delta)-2c_1(2\theta+1)(1-\delta)}{(2\theta+1)(3+\delta)},\\
q_m^{2B***} &= \frac{c_2}{2\theta} - \frac{(1+\theta)}{2\theta} - \frac{c_2}{2(1+\theta)},\\
q_s^{2Bd***} &= \frac{\theta(1-\delta)+2c_1(2\theta+1)(1-\delta)}{(2\theta+1)(3+\delta)} + \frac{c_1(1+\theta)}{4\theta},\\
q_m^{2Bd***} &= \frac{\theta(2+\delta)+2c_1(2\theta+1)(2+\delta)}{(2\theta+1)(3+\delta)} - 2c_1 + \frac{c_1(1+\theta)}{4\theta},\\
q_s^{2B***} &= \frac{c_2}{2\theta} - \frac{(1+\theta)}{2\theta} - \frac{c_2}{2(1+\theta)},\\
q_m^{2T***} &= \alpha - \frac{2c_2}{4\theta} - \frac{1}{4} + \frac{s_g}{2\theta},\\
q_s^{2T***} &= 1-\alpha - \frac{2c_2}{4\theta} - \frac{1}{4} + \frac{s_g}{2\theta},\\
q_m^{1B***} &= \frac{\alpha(\theta+1)-\theta}{(2\theta+1)}.
\end{aligned}
\tag{57}
$$

Lemma 3. *When the government provides trade-in subsidy strategy, (i) the retail price of the second-generation new products is larger than the discount price, (ii) the number of consumers who have bought the first-generation old products is larger than the customers who engage in trade-ins in period 2, and (iii) the trade-in rebate will decrease with an increase in the government's trade-in subsidy.*

Based on Lemma 3, the following conclusions are obtained:

① *Under two cases, in the second period, the retail price of the second-generation new products is larger than the discount price of the first-generation new products. Some myopic customers and strategic customers prefer to purchase first-generation new products with discount prices.*

② *Comparing the choice behavior of different types of customers, it was found that $q_m^{1B***} > q_m^{2T***}$; $q_s^{1B***} > q_s^{2T***}$. This means that customers who have purchased the first-generation old products in the first period do not necessarily engage in trade-ins. Some consumers will select to purchase the second-generation new products, and other consumers prefer to purchase the first-generation new products with the discount price.*

From Propositions 1 to 3, the equilibrium results in two cases under Model N, Model R, and Model G can be obtained. The following Lemma 4 is obtained:

Lemma 4. *For different models, (i) in the first period, no myopic customers purchase the first-generation product when $c_1 \geq c^{(1)*}$; (ii) in the first period, no strategic customers purchase the first-generation product when $c_1 \geq c^{(2)*}$, and (iii) $c^{(1)*} > c^{(2)*}$.*

In the first period, Lemma 4 points out the thresholds under which customers do not buy the first-generation products. The first threshold implies that the manufacturing cost of the first-generation new product should be less than $c^{(1)*}$; this suggests that the sales quantity from myopic customers is positive. Similarly, when $c_1 \geq c^{(2)*}$, it ensures that the sales quantity from the strategic consumer is positive. Of course, the thresholds $c^{(1)*}$ and $c^{(2)*}$ mean the consumers' trade-off between the valuation of the first-generation new product (v_1) and the retail price (p_1) should pay for the first-generation new product.

Lemma 5. *For different models, (i) in the second period, no myopic customers purchase the second-generation new product when $c_2 \geq c^{(1)**}$; (ii) in the second period, no strategic customers purchase the second-generation new product when $c_2 \geq c^{(2)**}$; and (iii) $c^{(1)**} = c^{(2)**}$.*

In the second period, Lemma 5 suggests the thresholds under which customers do not buy the second-generation products. The first threshold implies that the manufacturing cost of the second-generation product is less than $c^{(1)**}$, which suggests that when $c_2 \geq c^{(1)**}$, the sales quantity from the myopic consumer is positive. When $c_2 \geq c^{(2)**}$, it means that the sales quantity from the strategic consumer is positive. However, $c^{(1)**} = c^{(2)**}$ implies that sales quantity from the strategic consumer is equivalent to that from the myopic consumer. Therefore, the purchase behavior of consumers in two selling periods was considered. The second selling period is the last period, and there, it is not meaningful for the strategic consumer to choose to wait. Thus, some strategic consumers choose to purchase the second-generation product to meet needs. The number of strategic customers who purchase the second-generation product is equivalent to the number of myopic customers who purchase the second-generation product.

5. Analysis of Results

5.1. Optimal Dynamic Pricing Decisions and Demand. We can get the following propositions by comparing the equilibrium results:

Proposition 4. *Comparing the equilibrium results in previous models, the following is obtained:*

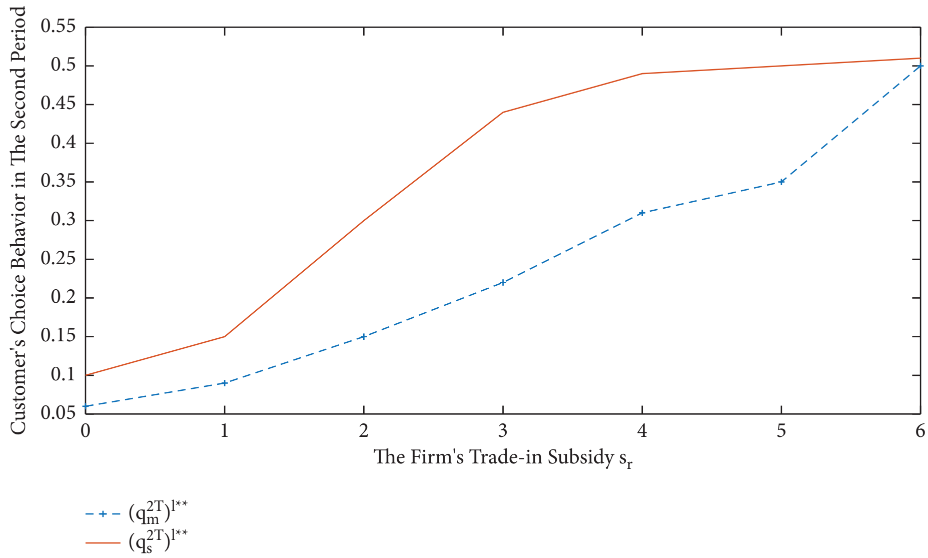
(i) *In case I, in the first period, $p_1^* = p_1^{**} = p_1^{***}$, $q_m^{1B*} = q_m^{1B**} = q_m^{1B***}$, $q_s^{1B*} = q_s^{1B**} = q_s^{1B***}$; in the second period, $p_2^* = p_2^{**} = p_2^{***}$, $p_d^* = p_d^{**} = p_d^{***}$, $q_m^{2B*} = q_m^{2B**} = q_m^{2B***}$, $q_s^{2B*} = q_s^{2B**} = q_s^{2B***}$, $q_m^{2Bd*} = q_m^{2Bd**} = q_m^{2Bd***}$, $q_s^{2Bd*} = q_s^{2Bd**} = q_s^{2Bd***}$; if $s_r > s_g > \theta/2$, $p_t^{**} < p_t^{***} < p_t^*$, $q_m^{2T*} < q_m^{2T***} < q_m^{2T**}$, $q_s^{2T*} < q_s^{2T***} < q_s^{2T**}$.*

(ii) *In case II, in the first period, $p_1^* = p_1^{**} = p_1^{***}$ $q_m^{1B*} = q_m^{1B**} = q_m^{1B***}$, $q_s^{1B*} = q_s^{1B**} = q_s^{1B***}$; in the second period, $p_2^* = p_2^{**} = p_2^{***}$, $p_d^* = p_d^{**} = p_d^{***}$, $q_m^{2B*} = q_m^{2B**} = q_m^{2B***}$, $q_s^{2B*} = q_s^{2B**} = q_s^{2B***}$, $q_m^{2Bd*} = q_m^{2Bd**} = q_m^{2Bd***}$, $q_s^{2Bd*} = q_s^{2Bd**} = q_s^{2Bd***}$; if $s_r > s_g$, $p_t^{**} < p_t^{***} < p_t^*$, $q_m^{2T*} < q_m^{2T***} < q_m^{2T**}$, $q_s^{2T*} < q_s^{2T***} < q_s^{2T**}$.*

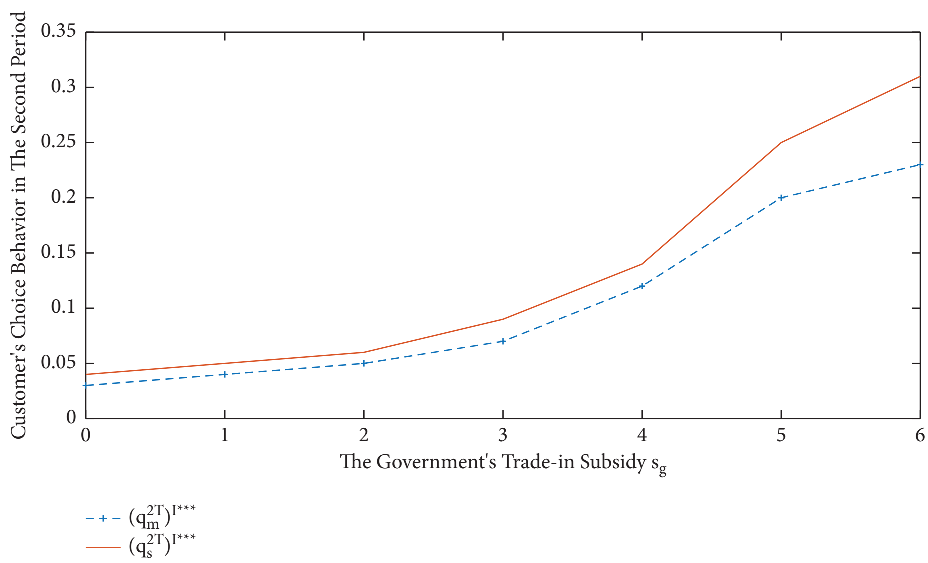
First, the previous result shows that in period 1, in two cases, the retail price of the first-generation products is equivalent in three decision-making models. The sales quantities to two kinds of consumers are equivalent in three decision-making models. Neither the firm nor the government provides the trade-in subsidy in the first period.

Second, in the second period, in two cases, the retail price of the second-generation products is equivalent in three decision-making models. The discount price of the first-generation products is also equivalent in three decision-making models. This also leads to the result that the sales quantities of the second-generation products are equivalent.

Third, in the second period, since providing trade-ins obtains the residual value, the firm has an incentive to provide trade-ins. Regardless of whether the firm or the government provides trade-in subsidies, the customers who engage in trade-ins under Model R and Model G is higher than that under Model N. This means that trade-in subsidies can effectively motivate customers to engage in trade-ins. In addition, a very interesting phenomenon was found, in



(a)



(b)

Figure 3: (a) The impact of a firm's trade-in subsidy strategy on consumers' choice behavior in case I. (b) The impact of the government's trade-in subsidy strategy on consumers' choice behavior in case I.

which the trade-in rebate under Model N is higher than that under Model R and Model G.

Then, the impact of the innovation incremental value θ and the discount factor δ on the equilibrium results is analyzed, and some management insights are obtained.

Proposition 5

(i) *In case I, under three decision-making models, there is* $(\partial p_2^*/\partial\theta) > 0$, $(\partial p_2^{**}/\partial\theta) > 0$, $(\partial p_2^{***}/\partial\theta) > 0$, $(\partial p_d^*/\partial\theta) > 0$, $(\partial p_d^{**}/\partial\theta) > 0$, $(\partial p_d^{***}/\partial\theta) > 0$, $(\partial p_t^*/\partial\theta) > 0$, $(\partial p_t^{**}/\partial\theta) > 0$, $(\partial p_t^{***}/\partial\theta) > 0$.

(ii) *In case II, under three decision-making models, there is* $(\partial p_2^*/\partial\theta) > 0$, $(\partial p_2^{**}/\partial\theta) > 0$, $(\partial p_2^{***}/\partial\theta) > 0$, $(\partial p_d^*/\partial\theta) > 0$, $(\partial p_d^{**}/\partial\theta) > 0$, $(\partial p_d^{***}/\partial\theta) > 0$, $(\partial p_t^*/\partial\theta) > 0$, $(\partial p_t^{**}/\partial\theta) > 0$, $(\partial p_t^{***}/\partial\theta) > 0$.

Proposition 5 shows that the retail price of the second-generation new products and the trade-in rebate increases with increasing innovation incremental value. The larger the innovation incremental value, the larger the value of the second-generation new products, and the higher the retail price of the second-generation new products. At the same time, the surplus value of new products is very high; when

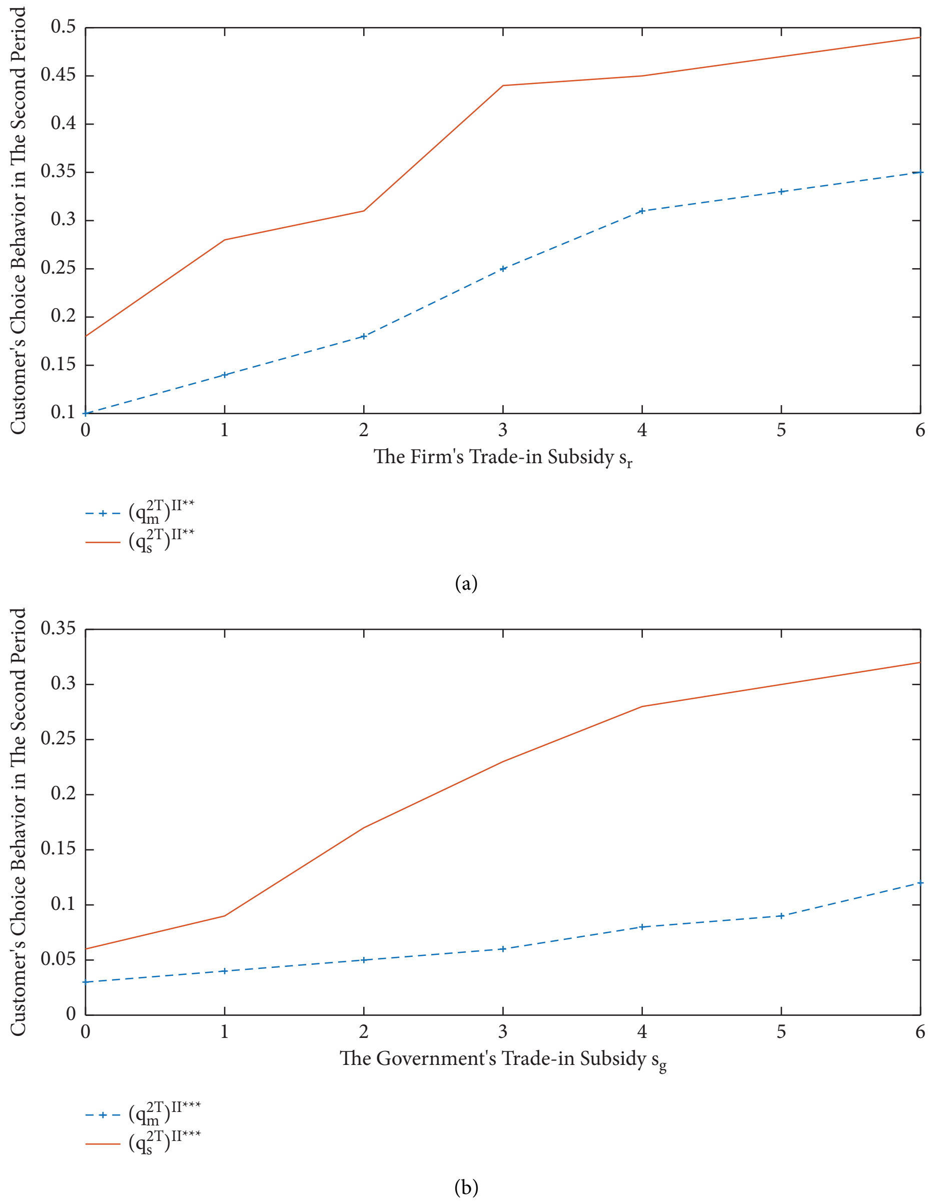


FIGURE 4: (a) The impact of a firm's trade-in subsidy strategy on consumers' choice behavior in case II. (b) The impact of the government's trade-in subsidy strategy on consumers' choice behavior in case II.

consumers engage in trade-ins, the consumer will obtain a relatively high discount price.

Proposition 6

(i) In case I, under three decision-making models, there is $(\partial p_1^{*}/\partial\delta) > 0$, $(\partial p_1^{**}/\partial\delta) > 0$, $(\partial p_1^{***}/\partial\delta) > 0$, $(\partial p_2^{*}/\partial\delta) > 0$, $(\partial p_2^{**}/\partial\delta) > 0$, $(\partial p_2^{***}/\partial\delta) > 0$, $(\partial p_d^{*}/\partial\delta) > 0$, $(\partial p_d^{**}/\partial\delta) > 0$, $(\partial p_d^{***}/\partial\delta) > 0$.

(ii) In case II, under three decision-making models, there is $(\partial p_1^{*}/\partial\delta) > 0$, $(\partial p_1^{**}/\partial\delta) > 0$, $(\partial p_1^{***}/\partial\delta) > 0$, $(\partial p_2^{*}/\partial\delta) > 0$, $(\partial p_2^{**}/\partial\delta) > 0$, $(\partial p_2^{***}/\partial\delta) > 0$, $(\partial p_d^{*}/\partial\delta) > 0$, $(\partial p_d^{**}/\partial\delta) > 0$, $(\partial p_d^{***}/\partial\delta) > 0$.

Proposition 6 shows that in case I, consumers prefer to buy the second-generation new products X_2, the pricing of the first-generation new products increases with the increase in the discount factor. Because the consumer prefers to purchase the second-generation new products, the enterprise should reduce the retail price of the second-generation new products to meet the needs of consumers and obtain much more profit.

5.2. The Different Trade-in Subsidy Strategies. In this subsection, the impact of trade-in subsidy strategy on the consumer's choice behavior and the firm's dynamic pricing strategy is analyzed. The equilibrium solution under three decision-making models is very complex, and the numerical example is used in this subsection. The relevant parameters are as follows: $c_1 = 2$, $c_2 = 3$, $t_m = 6$, $t \in [0, 6]$, $k = 1$, $D = 1$, $\theta = 0.5$, $\alpha = 0.5$, $\delta = 0.6$, $s_g \in [1, 6]$, $s_r \in [1, 6]$.

From Figures 3 and 4, it can be seen that in the second period, with the increase in the firm's or the government's trade-in subsidy, the customers who engage in trade-in activity increases. This can reflect that the firm's or the government's trade-in subsidy strategy can effectively motivate consumers to engage in trade-in activity. Moreover, compared with the government's trade-in subsidy, the firm's trade-in subsidy can more effectively motivate consumers to participate in the trade-in activity.

6. Conclusions

Trade-ins are one of the most widely used recycling methods in practice and can effectively motivate consumers to return their old products. One of the key problems in the trade-in process is the pricing strategy in the presence of different trade-in subsidy strategies and trade-in rebates. Thus, from the angle of game theory, in the market, consumers are divided into myopic consumers and strategic consumers, considering three situations of no trade-in subsidy strategy (Model N), the firm's trade-in subsidy strategy (Model R), and the government's trade-in subsidy strategy (Model G); the dynamic game equilibrium between firms and consumers is analyzed. The optimal dynamic pricing strategy is obtained by constructing a two-stage dynamic game model. The effect of the trade-in subsidy strategy on myopic customers' and strategic customers' choice behavior and on the enterprise's dynamic pricing strategy is analyzed. Finally, the following important conclusions are obtained:

For pricing and quantity, (i) in the first period, in two cases, the retail price of the first-generation products is equivalent in three decision-making models. The sales quantities of two kinds of consumers are equivalent in three decision-making models. Neither the firm nor the government provides the trade-in subsidy in the first period; thus, there are no changes in the retail price of the first-generation products and the sales quantities to customers. (ii) In the second period, in two cases, the retail price of the second-generation products is equivalent in three decision-making models. The discount price of the first-generation products is also equivalent in three decision-making models. This also leads to the result that the sales quantities of the second-generation products are equivalent.

For consumer choice behavior, (i) on the one hand, in two cases, under three decision-making models, the customers who purchase two generations products are not impacted by the trade-in subsidy strategy. On the other hand, the customers who participate in trade-ins are impacted by the trade-in subsidy strategy. (ii) The customers who participate in trade-ins with the trade-in subsidy strategy are larger than that without the trade-in subsidy strategy.

For firm profit, (i) the enterprise's profit in the dynamic pricing strategy decreases when the discount factor and the innovation incremental value are at a higher level. (ii) The enterprise's profit increases in the dynamic pricing strategy when the discount factor is at a low level. (iii) The firm uses the dynamic pricing strategy, and the firm can obtain much profit when two different kinds of customers exist in the market.

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(1) Customers should carefully consider whether to participate in trade-ins based on the relationship between the residual value of the old product and the trade-in rebate.

(2) To encourage consumers to repeat purchases and retain old consumers, the enterprise provides a trade-in subsidy to customers. The firm should determine an appropriate amount of trade-in subsidy when they implement the dynamic pricing strategy.

(3) To improve the recycling and reuse of old products, the government can provide trade-in subsidies to consumers.

In this study, we only discuss the effect of different trade-in subsidy strategies on the enterprise's dynamic pricing strategy. However, we do not discuss the impact of different trade-in subsidy strategies on the firm's static pricing strategy. Moreover, we do not compare the difference between the dynamic pricing and the static pricing under different trade-in subsidy strategies. Some interesting questions can be analyzed in the future. First, the impact of different trade-in subsidy strategies on the firm's static pricing strategy could be explored. Second, when it has asymmetric information, how customers make the selection is of interest. How should firms make decisions accordingly is discussed. Third, we can discuss the difference between two pricing strategies under different trade-in subsidy strategies.

Appendix

Proof of Proposition 1. Without trade-in subsidy strategy, when the firm follows dynamic pricing strategy, it has two possible cases. Thus, it needs to solve two optimization pricing problems. In the first period, the myopic consumer's demand for X_1 which the firm is faced with is $q_m^{1B} = (\alpha - \tau_1^m)^+$. The strategic consumer's demand for X_1 which the firm is faced with is $q_s^{1B} = (1 - \alpha - \tau_1^s)^+$.

(1) In case I, the indifference condition of the myopic consumer in the second period is $(1+\theta)\tau_2^m - p_2 + p_t = \tau_2^m$, $(1+\theta)\tau_3^m - p_2 = \tau_3^m - p_d$, $\tau_4^m - p_d = 0$. Thus, the threshold value is $\tau_2^m = (p_2 - p_t/\theta)$, $\tau_3^m = (p_2 - p_d/\theta)$, $\tau_4^m = p_d$. The $q_m^{2T} = (\alpha - \tau_2^m)^+$, $q_m^{2B} = (\tau_1^m - \tau_3^m)^+$, $q_m^{2Bd} = (\tau_3^m - \tau_4^m)^+$. Similarly, the indifference condition of the strategic consumer in the second period is $(1+\theta)\tau_2^s - p_2 + p_t = \tau_2^s$, $(1+\theta)\tau_3^s - p_2 = \tau_3^s - p_d$, $\tau_4^s - p_d = 0$. Thus, the threshold value is $\tau_2^s = (p_2 - p_t/\theta)$, $\tau_3^s = (p_2 - p_d/\theta)$, $\tau_4^s = p_d$. The $q_s^{2T} = (1 - \alpha - \tau_2^s)^+$, $q_s^{2B} = (\tau_1^s - \tau_3^s)^+$, $q_s^{2Bd} = (\tau_3^s - \tau_4^s)^+$.

The firm's pricing problem in the second period is that

$$\max_{p_2,p_t,p_d} \pi_2^N$$
$$+\left(p_d - c_1\right)\left(\frac{2p_2 - 2p_d}{\theta} - 2p_d\right),$$
$$\text{s.t.} \quad 0 \le \tau_1^m - \frac{p_2 - p_d}{\theta} \le \tau_1^m,$$
$$0 \le \frac{p_2 - p_d}{\theta} - p_d \le \tau_1^m, \tag{A.1}$$
$$0 \le p_t \le p_2,$$
$$0 \le \tau_1^s - \frac{p_2 - p_d}{\theta} \le \tau_1^s,$$
$$1 - \alpha - \frac{p_2 - p_t}{\theta} \le 1 - \alpha - \tau_1^s.$$

The authors can get that the Hessian matrix is negative; thus, the function π_2^N is joint concave, the unconstrained solution is $p_2^* = (2c_2 + (1+\theta)\tau_1^m + (1+\theta)\tau_1^s/4)$, $p_t^* = ((1+\theta)\tau_1^m + (1+\theta)\tau_1^s - \theta/4)$, $p_d^* = (\tau_1^m + \tau_1^s + 2c_1/4)$. In case I, when myopic consumers with a valuation higher than the threshold value τ_1^m purchases X_1 in the first period, myopic consumers with a valuation lower than τ_1^m chooses to wait, i.e., $0 + (1+\theta)\tau_1^m - p_2^* = \tau_1^m - p_1$; moreover, when strategic consumers with a valuation higher than the threshold value τ_1^s purchases X_1 in period 1, strategic consumers with a valuation lower than τ_1^s chooses to wait, i.e., $0 + \delta[(1+\theta)\tau_1^s - p_2^*] = (\tau_1^s - p_1) + \delta(\tau_1^s)$.

Thus, the authors can get that $\tau_1^{m*} = (2(5\theta+1)(2p_1 - c_1)/(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2)$ and $\tau_1^{s*} = (8\theta(2p_1 - c_1)/(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2)$. In order to satisfy the constraints of the pricing strategy in the second period, the p_1 should satisfy the conditions that $(\theta/2(1+\theta)) \le \tau_1^{m*}(p_1) \le (\theta/2\theta+1)$ and $(\theta/\theta+2) \le \tau_1^{s*}(p_1) \le (\theta/\theta+1)$. The previous constraints are added to the optimization pricing problem in the first period. The authors can get that

$$\max_{p_1} \pi_t^N$$
$$\text{s.t.} \quad 0 \le \alpha - \frac{(5\theta+1)(4p_1 - 2c_1)}{(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2} \le \alpha, \tag{A.2}$$
$$0 \le p_1 \le 1.$$

The second derivative is $(\partial^2 \pi_t^N/\partial p_1^2) = -(8(9\theta+1)/(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2) \le 0$. Thus, the function is concave. The authors can get that $p_1^* = ((3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2/8(9\theta+1)) + (3c_1/4)$. Moreover, in case I, the authors can obtain the equilibrium prices and trade-in rebate.

The proof process in Case II is the same as in Case I, and we have omitted it. □

Proof of Proposition 2. With the firm's trade-in subsidy strategy, when the firm follows dynamic pricing strategy, it has two possible cases. It needs to solve two optimization pricing problems. In the first period, the myopic consumer's demand for X_1 which the firm is faced with is $q_m^{1B} = (\alpha - \tau_1^m)^+$. The strategic consumer's demand for X_1 which the firm is faced with is $q_s^{1B} = (1 - \alpha - \tau_1^s)^+$.

(1) In case I, the indifference condition of the myopic consumer in the second period is $(1+\theta)\tau_2^m - p_2 + p_t + s_r = \tau_2^m$, $(1+\theta)\tau_3^m - p_2 = \tau_3^m - p_d$, $\tau_4^m - p_d = 0$. Thus, the threshold value is $\tau_2^m = (p_2 - p_t - s_r/\theta)$, $\tau_3^m = (p_2 - p_d/\theta)$, $\tau_4^m = p_d$. The $q_m^{2T} = (\alpha - \tau_2^m)^+$, $q_m^{2B} = (\tau_1^m - \tau_3^m)^+$, Similarly, the indifference condition of the strategic consumer in the second period is
1 + θτ2s − p2 + pt + sr = τ2s,
1 + θτ3s − p2 = τ3s − pd, τ4s − pd = 0. Thus, the threshold value is τ2s = p2 − pt − srθτ3s = p2 − pdθ,

τ4s = pd. The qs2T = 1 − α − τ2s+, qs2B = τ1s − τ3s + qs2Bd = τ3s − τ4s+. The firm's pricing problem in the second period is that

$$\max_{p_2,p_t,p_d} \pi_2^R$$
$$+ (p_d - c_1)\left(\frac{2p_2 - 2p_d}{\theta} - 2p_d\right),$$
$$\text{s.t.}\, 0 \le \tau_1^m - \frac{p_2 - p_d}{\theta} \le \tau_1^m,$$
$$0 \le \frac{p_2 - p_d}{\theta} - p_d \le \tau_1^m, \tag{A.3}$$
$$\tau_1^s - p_d \le \tau_1^s,$$
$$1 - \alpha - \frac{p_2 - p_t - s_r}{\theta} \le 1 - \alpha - \tau_1^s.$$

The authors can get that the Hessian matrix is negative, thus, the function π_2^R is joint concave, and the unconstrained solution is $p_2^{**} = (2c_2 + (\tau_1^m + \tau_1^s)(\theta + 1)/4)$, $p_t^{**} = ((\tau_1^m + \tau_1^s)(\theta + 1) - \theta - 4s_r/4)$, $p_d^{**} = (\tau_1^m + \tau_1^s + 2c_1/4)$.

In case I, when myopic consumers with a valuation higher than the threshold value τ_1^m purchases X_1 in the first period, myopic consumers with a valuation lower than τ_1^m chooses to wait, i.e., $0 + (1 + \theta)\tau_1^m - p_2^{**} = \tau_1^m - p_1$; moreover, when strategic consumers with a valuation higher than the threshold value τ_1^s purchases X_1 in the first period, strategic consumers with a valuation lower than τ_1^s chooses to wait, i.e., $0 + \delta[(1 + \theta)\tau_1^s - p_2^{**}] = (\tau_1^s - p_1) + \delta(\tau_1^s)$. Thus, the authors can get that $\tau_1^{m\,**} = (2(5\theta + 1)(2p_1 - c_1)/(3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2)$ and $\tau_1^{s\,**} = (8\theta(2p_1 - c_1)/(3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2)$. In order to satisfy the constraints of the pricing strategy in the second period, the p_1 should satisfy the conditions that $(\theta/2(1+ \theta)) \le \tau_1^{m\,**}(p_1) \le (\theta/2\theta + 1)$ and $(\theta/\theta + 2) \le \tau_1^{s\,**}(p_1) \le (\theta/\theta + 1)$. The previous constraints are added to the optimization pricing problem in the first period. The authors can get that

$$\max_{p_1} \pi_t^R$$
$$\text{s.t.}\, 0 \le \alpha - \frac{2(5\theta + 1)(2p_1 - c_1)}{(3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2} \le \alpha,$$
$$0 \le 1 - \alpha - \frac{8\theta(2p_1 - c_1)}{(3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2} \le 1 - \alpha, \tag{A.4}$$
$$\frac{\theta}{2(1 + \theta)} \le \frac{2(5\theta + 1)(2p_1 - c_1)}{(3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2} \le \frac{\theta}{2\theta + 1},$$
$$0 \le p_1 \le 1.$$

The second derivative is $\partial^2\pi_t^N/\partial p_1^2 = -(8(9\theta + 1)/(3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2) \le 0$, thus, the function is concave. The authors can get that $p_1^{**} = ((3\theta - 1)(4\delta\theta - \theta - 5) - (1 + \theta)^2/8(9\theta + 1)) + (3c_1/4)$. Moreover, in case I, I authors can obtain the equilibrium prices and trade-in rebate.

The proof process in Case II is the same as in Case I, and we have omitted it. □

Proof of Proposition 3. With the government's trade-in subsidy strategy, when the firm follows dynamic pricing strategy, it has two possible cases. It needs to solve two optimization pricing problems. In the first period, the myopic consumer's demand for X_1 which the firm is faced with is $q_m^{1B} = (\alpha - \tau_1^m)^+$. The strategic consumer's demand for X_1 which the firm is faced with is $q_s^{1B} = (1 - \alpha - \tau_1^s)^+$.

(1) In case I, the indifference condition of the myopic consumer in the second period is $(1+\theta)\tau_2^m - p_2 + p_t + s_g = \tau_2^m$, $(1+\theta)\tau_3^m - p_2 = \tau_3^m - p_d$, $\tau_4^m - p_d = 0$. Thus, the threshold value is $\tau_2^m = (p_2 - p_t - s_g/\theta)$, $\tau_3^m = (p_2 - p_d/\theta)$, $\tau_4^m = p_d$. The $q_m^{2T} = (\alpha - \tau_2^m)^+$, $q_m^{2B} = (\tau_1^m - \tau_3^m)^+$, $q_m^{2Bd} = (\tau_3^m - \tau_4^m)^+$. Similarly, the indifference condition of the strategic consumer in the second period is $(1+\theta)\tau_2^s - p_2 + p_t + s_g = \tau_2^s$, $(1+\theta)\tau_3^s - p_2 = \tau_3^s - p_d$, $\tau_4^s - p_d = 0$. Thus, the threshold value is $\tau_2^s = (p_2 - p_t - s_g/\theta)$, $\tau_3^s = (p_2 - p_d/\theta)$, $\tau_4^s = p_d$. The $q_s^{2T} = (1 - \alpha - \tau_2^s)^+$, $q_s^{2B} = (\tau_1^s - \tau_3^s)^+$, $q_s^{2Bd} = (\tau_3^s - \tau_4^s)^+$.

The firm's pricing problem in the second period is that

$$\max_{p_2,p_t,p_d} \pi_2^G$$

$$+ (p_d - c_1)\left(\frac{2p_2 - 2p_d}{\theta} - 2p_d\right),$$

$$\text{s.t.}\, 0 \le \tau_1^m - \frac{p_2 - p_d}{\theta} \le \tau_1^m, \tag{A.5}$$

$$\frac{p_2 - p_d}{\theta} - p_d + \tau_1^s - \frac{p_2 - p_d}{\theta} \le \tau_1^s,$$

$$1 - \alpha - \frac{p_2 - p_t - s_g}{\theta} \le 1 - \alpha - \tau_s^1.$$

The authors can get that the Hessian matrix is negative, thus, the function π_2^R is joint concave, and the unconstrained solution is $p_2^{***} = 2c_2 + (\theta+1)(\tau_1^m + \tau_1^s)/4$, $p_t^{***} = (\theta+1)(\tau_1^m + \tau_1^s) - \theta/4 - s_g/2$, $p_d^{***} = 2c_1 + (\tau_1^m + \tau_1^s)/4$. In case I, when myopic consumers with a valuation higher than the threshold value τ_1^m purchases X_1 in the first period and myopic consumers with a valuation lower than τ_1^m chooses to wait, i.e., $0 + (1+\theta)\tau_1^m - p_2^{***} = \tau_1^m - p_1$; moreover, when strategic consumers with a valuation higher than the threshold value τ_1^s purchases X_1 in the first period and strategic consumers with a valuation lower than τ_1^s chooses to wait, i.e., $0 + \delta[(1+\theta)\tau_1^s - p_2^{***}] = (\tau_1^s - p_1) + \delta(\tau_1^s)$. Thus, the authors can get that $\tau_1^{m***} = 2(5\theta+1)(2p_1 - c_1)/(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2$ and $\tau_1^{s***} = 8\theta(2p_1 - c_1)/(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2$. In order to satisfy the constraints of the pricing strategy in the second period, the p_1 should satisfy the conditions that $\theta/2(1+\theta) \le \tau_1^{m***}(p_1) \le \theta/2\theta + 1$ and $\theta/\theta + 2 \le \tau_1^{s***}(p_1) \le \theta/\theta + 1$. The previous constraints are added to the optimization pricing problem in the first period. The authors can get that

$$\max_{p_1} \pi_t^G$$

$$\text{s.t.}\, 0 \le \alpha - \frac{2(5\theta+1)(2p_1 - c_1)}{(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2} \le \alpha,$$

$$0 \le 1 - \alpha - \frac{8\theta(2p_1 - c_1)}{(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2} \le 1 - \alpha, \tag{A.6}$$

$$\frac{\theta}{2(1+\theta)} \le \frac{2(5\theta+1)(2p_1 - c_1)}{(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2} \le \frac{\theta}{2\theta+1}.$$

The second derivative is $\partial^2\pi_t^G/\partial p_1^2 = -8(9\theta+1)/(3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2 \le 0$, thus, the function is concave. The authors can get that $p_1^{***} = (3\theta-1)(4\delta\theta-\theta-5) - (1+\theta)^2/8(9\theta+1) + 3c_1/4$.

Moreover, the authors can obtain the equilibrium prices and trade-in rebate.

The proof process in Case II is the same as in Case I. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that he has no conflicts of interest.

Authors' Contributions

J.Y.W. conceptualized the study and wrote the original study and edited the article.

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