

## Research Article

# A Multi-attribute Two-Sided Matching Decision Method Based on Multi-granularity Probabilistic Linguistic MARCOS

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With the complexity of the matching environment, individual differences in matching objects and the uncertainty of evaluation information should be considered. The probabilistic linguistic term set (PLTS) is a useful tool to describe the uncertainty and limited cognition of matching objects. Thus, this paper proposes a multi-attribute two-sided matching method based on multi-granularity probabilistic linguistic MARCOS. First, we use a probabilistic linguistic term set with different granularities to express the evaluation information of different matching objects. Then, a conversion function is used to unify different granularity probabilistic linguistic terms. Second, a linguistic scale function is introduced to improve the expectation function, deviation degree, and distance of PLTS. The processed probabilistic linguistic evaluation information is transformed into accurate utility values through the transformation function. The evaluation attribute weights are determined by PLTS distance entropy. Based on this, this paper proposes a multi-granularity probabilistic linguistic MARCOS method to obtain the two-sided satisfaction degree. Finally, an optimization model which aims to maximize the overall satisfaction degree of matching objects by considering the stable matching condition is then established and solved to determine the matching between matching objects. The multi-objective two-sided matching model is constructed with the objective of maximizing the two-sided satisfaction degree. A case study of the service outsourcing matching is presented to validate the proposed method. The comparative analyses and discussions are also provided to demonstrate its effectiveness and scientific character.

## 1. Introduction

Two-sided matching involves considering the preference information of two-sided matching objects at the same time to obtain reasonable and satisfactory matching results to maximize the satisfaction degrees (SDs) of both parties, so as to improve the matching efficiency and reduce the matching cost. The earliest two-sided matching research mainly included the male and female marriage matching problem and the university admission problem [1]. The literature on the two-sided matching problem has been growing steadily and it is now widely used in such domains as human-post matching [2], matching of buyers and sellers [3], technological knowledge matching [4], environmental protection matching [5], medical treatment service matching [6], and the matching of supply and demand of logistics services [7]. Thus, we can see that

the two-sided matching problem has a wide range of practical significance and value.

The multi-attribute two-sided matching (MATSM) problem is an important branch of the two-sided matching problem, which usually refers to the multi-attribute preference information provided by the matching parties. Yang et al. [8] focused on the MATSM problem of 2-tuple preference and proposed a multi-stage matching framework composed of the matching process and the feedback process. Liang et al. [9] proposed a new MATSM decision method from the perspective of prospect theory and constructed a multi-stage dynamic matching model for talent sharing based on the attribute priorities. Liang et al. [10] discussed a strict two-sided matching problem based on multi-attribute interval-valued preference ordinal information. They introduced the ranking method of probability degree to deal with the information of various interval numbers.

Due to the complexity of the two-sided matching environment and the limitations of human rationality, evaluation information is often presented in fuzzy form, which is why two-sided matching in a fuzzy environment has always been the focus of scholarly research. Yue et al. [5] developed a decision method for two-sided matching with triangular intuitionistic fuzzy numbers and applied it to smart environmental protection. Yu et al. [11] proposed some novel intuitionistic fuzzy Choquet integral aggregation operators to describe the correlations between the evaluated attributes. Li et al. [12] transformed the dual hesitant fuzzy preference information into the SDs by projection technology and proposed a novel two-sided matching model to solve the fuzziness and uncertainty of preference information in the matching process of complex product manufacturing tasks on a cloud manufacturing platform. Qi et al. [13] provided a new score function of HFN and established a two-sided matching decision model on the basis of HFNs. Zhang et al. [14] utilized the log least squares method (LLSM) to derive priority vectors from FPR-SC and developed a new method for two-sided matching of FPRs-SC based on LLSM and the proposed consistency improvement algorithm. Zhang et al. [15] developed an approach to TSMDM with multi-granular HFLTSs.

There may be ambiguities and differences in semantic descriptions between the two parties in the evaluation process. To improve the flexibility of preference expression, linguistic distribution has become a popular tool in decision-making [16]. Probability linguistic term sets [17] are a combination of linguistic terms and probabilities. The combination of qualitative and quantitative evaluation can reflect the real thoughts of the evaluator as intuitively as possible. In terms of measure theory and methods, in addition to the classical ensemble operators proposed by Pang et al. [17], Guo et al. [18] proposed a series of new algorithms for PLTS based on linguistic terms and membership conversion relations. Based on the existing theoretical tools, scholars have successively proposed the GSM operator [19], Muirhead mean operator [20], and others in the probabilistic linguistic environment. In terms of preference relationships, Zhang et al. [21] improved the existing correction algorithms of additive consistency and discordant preference relations, and proposed a group consensus process under a group decision-making environment. Gao et al. [22] defined the correction algorithm of product preference relation, consistency index, and acceptable and unacceptable product preference relations in a probabilistic linguistic environment. Decision-making methods can be mainly divided into two categories: (1) ranking-based decision methods, such as the ELECTRE III method [23], PROMETHEE method [24], and GLDS method [25], and (2) utility-based decision-making methods, such as the TOPSIS method [26], TODIM method [27], and MULTIMOORA method [28]. Compared to these, PLTS has significant advantages in describing evaluation information. Thus, some scholars have also begun to extend PLTS to the field of two-sided matching. Li et al. [29] were the first to introduce the PLTSs into a two-sided matching environment. They provided a novel MATSM method under the probabilistic

linguistic environment with unknown attribute weights by providing the PL-DEMATEL method and the PL-MABAC ranking method [6].

Although great progress has been made for MATSM problems and PLTS, respectively, there remain some research gaps.

- (1) In the research on MATSM, in view of the uncertainty of evaluation or preference information, it is rare to describe evaluation information through probabilistic linguistic terms. As a matching object, individual differences such as cognitive ability, experience knowledge, and evaluation style are given less consideration. While measuring and processing probabilistic linguistic evaluation information during the matching process, we can see how necessary it is to reflect the matching individual differences of objects.
- (2) In the process of processing probabilistic linguistic information, the traditional method is to quantify through the score function. However, this method is relatively simple and will result in information loss. How to choose a better method is an urgent problem to be considered.
- (3) When measuring the SDs of matching objects, the more traditional method is to introduce regret theory, prospect theory, and others to consider psychological perception and perception level. However, the above methods are not suitable for the study of MATSM problems. There is an urgent need to construct a new multi-attribute decision-making method for probabilistic linguistic information to measure two-sided SDs.

In summary, the contributions of this paper are summarized as follows:

- (1) For the evaluation information, we use multi-granularity probabilistic linguistic term sets to describe them. Individual differences among evaluators were discriminated using linguistic scale functions. The expectation function and the deviation function are improved by integrating the linguistic scale function. Then, we use transformation function to quantify the evaluation information and obtain accurate utility values. It retains more original evaluation information and reduces information loss.
- (2) We combine the probabilistic linguistic distance measure with the information entropy to obtain the distance entropy and determine the attribute weight accordingly. We obtain the two-sided SDs by providing probabilistic linguistic Measurement of Alternatives and Ranking according to the Compromise Solution (PL-MARCOS) method.
- (3) We construct a two-sided matching model considering matching stability based on the obtained two-sided SDs. We also apply the proposed method to solve the problem of matching service-supplying companies and service-demanding companies.

The remainder of this paper is organized as follows. Section 2 introduces the basic concepts of PLTS and MATSM. Section 3 presents our approach to processing the multi-granularity probabilistic linguistic evaluation information. Section 4 proposes the multi-granularity PL-MARCOS to obtain the SDs. Afterward, we construct a two-sided matching model to determine the matching result in Section 5. In Section 6, a numerical example is provided to illustrate the proposed approach. Some comparative analyses and discussions are also conducted in this section. Finally, the main conclusions are drawn in Section 7.

## 2. Preliminaries

### 2.1. Linguistic Term Sets (LTS)

*Definition 1* (see [17]). Let  $S = \{s_0, s_1, \dots, s_\tau\}$  be a set consisting of an odd number of linguistic term elements with

each  $s_i$  representing a different degree of linguistic terminology. If the set  $S$  satisfies the following characteristics:

- (1) The set is ordered:  $s_i > s_j$ , when  $i > j$
- (2) The negative operator is defined:  $s_i = \text{neg}(s_j)$ ,  $i + j = \tau$
- (3) If  $s_i > s_j$ , then  $\max(s_i, s_j) = s_i$ ,  $\min(s_i, s_j) = s_j$

then the set  $S$  is called the linguistic term set (LTS). If  $T_m = \tau + 1$ , then  $T_m$  is the granularity of the LTS.

### 2.2. Probabilistic Linguistic Term Sets (PLTS)

#### 2.2.1. Definition of PLTS

*Definition 2* (see [17]). Let  $S = \{s_\alpha | \alpha = -\tau, -\tau + 1, \dots, 0, \dots, \tau - 1, \tau\}$  be an LTS; then, a probabilistic linguistic term set (PLTS) is defined as

$$L(p) = \left\{ L^{(l)}(p^{(l)}) | L^{(l)} \in S, \quad 0 \leq p^{(l)} \leq 1, l = 1, 2, \dots, \#L(p), \sum_{l=1}^{\#L(p)} p^{(l)} \leq 1 \right\}, \quad (1)$$

where  $L^{(l)}(p^{(l)})$  is the  $l$ -th linguistic term  $L^{(l)}$  with its probabilistic information  $p^{(l)}$ , and  $\#L(p)$  denotes the number of the probabilistic linguistic elements (PLEs) in  $L(p)$ .

*2.2.2. Normalization of PLTS.* The value of  $\sum_{l=1}^{\#L(p)} p^{(l)}$  in PLTS indicates the sum of all PLEs' probabilistic information of the PLTS. If  $\sum_{l=1}^{\#L(p)} p^{(l)} < 1$ , it indicates that the evaluation information given by the experts is an incomplete probability distribution and needs to be normalized.

*Definition 3* (see [17]). Let  $L(p)_1$  and  $L(p)_2$  be any two probabilistic linguistic term sets,  $L_1(p) = \{L_1^{(l)}(p_1^{(l)}) | l = 1, 2, \dots, \#L(p)_1\}$  and  $L_2(p) = \{L_2^{(l)}(p_2^{(l)}) | l = 1, 2, \dots, \#L(p)_2\}$ , and let  $\#L(p)_1$  and  $\#L(p)_2$  be the numbers of PLEs in  $L(p)_1$  and  $L(p)_2$ , respectively. If  $\#L(p)_1 > \#L(p)_2$ , then we will add  $\#L(p)_1 - \#L(p)_2$  linguistic term elements to  $\#L(p)_2$ , so that the number of linguistic term elements in both PLTSs are the same, and the added linguistic term elements are the smallest linguistic term elements in  $L(p)_2$  with zero probabilities.

*Definition 4* (see [17]). Given a PLTS  $L(p)$  with  $\sum_{l=1}^{\#L(p)} p^{(l)} < 1$ , then the normalized PLTS  $\bar{L}(p)$  can be expressed as  $\bar{L}(p) = \{L^{(l)}(\bar{p}^{(l)}) | 0 \leq p^{(l)} \leq 1, l = 1, 2, \dots, \#L(p)\}$ , where  $\bar{p}^{(l)} = p^{(l)} / \sum_{l=1}^{\#L(p)} p^{(l)}$ . Let  $r^{(l)}$  be the subscript of the probabilistic linguistic term  $L^{(l)}$ .  $L(p)$  is called an ordered PLTS if all PLEs are arranged according to the values of  $r^{(l)} p^{(l)}$  in ascending order. If the values of  $r^{(l)} p^{(l)}$  are equal, the PLEs are arranged according to the values of  $r^{(l)}$  in ascending order.

*2.3. Multi-attribute Two-Sided Matching.* Let  $M = \{1, 2, \dots, m\}$ ,  $N = \{1, 2, \dots, n\}$ ,  $F = \{1, 2, \dots, f\}$ , and  $G = \{1, 2, \dots, g\}$ . Let  $A = \{A_1, A_2, \dots, A_m\}$  and  $B = \{B_1, B_2, \dots, B_n\}$  be two sets of matching objects,  $m \geq 2$ ,  $n \geq 2$ .  $A_i$  represents the  $i$ -th object of set  $A$  and  $B_j$  represents the  $j$ -th object of set  $B$ ,  $i \in M$ ,  $j \in N$ . Without loss of generality, let us set  $m \leq n$ .  $B$  is evaluated by  $A$  under the attribute set  $C^A = \{C_1^A, C_2^A, \dots, C_f^A\}$ .  $C_k^A$  is the  $k$ -th attribute that the matching objects  $A$  pay attention to,  $k \in F$ .  $A$  is evaluated by  $B$  under the attribute set  $C^B = \{C_1^B, C_2^B, \dots, C_g^B\}$ .  $C_h^B$  is the  $h$ -th attribute that the matching objects  $B$  pay attention to,  $h \in G$ .

*Definition 5.* Let  $\mu: A \cup B \rightarrow A \cup B$  be a one-to-one mapping. If  $\forall A_i \in A, \forall B_j \in B$  satisfies (i)  $\mu(A_i) \in B$ , (ii)  $\mu(B_j) \in A \cup \{B_j\}$ , and (iii)  $\mu(A_i) = B_j$  if and only if  $\mu(B_j) = A_i$ , then  $\mu$  is called two-sided matching.

*Note 1.* In Definition 1,  $\mu(A_i) = B_j$  indicates that  $A_i$  matches  $B_j$  and  $\mu(B_j) = B_j$  indicates that  $B_j$  does not find a matching object.

*Definition 6.* If  $\mu(A_i) = B_j$ , then  $(A_i, B_j)$  is called a matching pair. All matching pairs consist of a matching scheme (result).

According to Definitions 5 and 6, if  $(A_i, B_j)$  is a matching pair, then  $(B_j, A_i)$  is also a matching pair. No matching object is found, such as  $\mu(B_o) = B_o$ , which can be recorded as  $(B_o, B_o)$ . Therefore, two-sided matching can be recorded as  $\mu = \mu_t \cup \mu_s$ ,  $\mu_t = \{(A_i, B_{f(i)}) | i = 1, 2, \dots, m\}$ ,  $\mu_s = \{(B_o, B_o) | o = \{1, 2, \dots, n\} \setminus \{f(1), f(2), \dots, f(m)\}\}$  where  $B_{f(i)} = \mu(A_i)$ ,  $f(i) \in N$ , and  $\forall i, i' \in I, i \neq i'$ , exists  $f(i) \neq f(i')$ .

### 3. Multi-granularity Probabilistic Linguistic Evaluation Information Processing

In this section, we assembled the original evaluation information from evaluation experts. The original probabilistic linguistic information with different granularities is standardized and converted into accurate utility values by a conversion function.

#### 3.1. Multi-granularity PLTS Consistent Conversion Function.

Evaluation experts differ in professional background, experience level, and expression methods. At the same time, due to differences in the understanding of the evaluation object, they may choose probabilistic linguistic term sets with different granularities when evaluating the same evaluation object. Experts with strong information analysis capabilities will use higher-precision granular linguistic term sets. Alternatively, experts who have relatively poor information analysis abilities tend to use linguistic term sets with a coarser granularity. The granular linguistic terms of the evaluation information are different. To perform subsequent operations, it is necessary to uniformly process linguistic term sets with different granularities. We select a probabilistic linguistic term set with one granular as the basic probabilistic linguistic term set of this evaluation information. The probabilistic linguistic term information of other granularities is converted into the basic probabilistic linguistic term set, so as to obtain the linguistic evaluation matrix with the same granularity.

*Definition 7* (see [30]). Let  $s_k^T \in s_{[0-T]}^T$ . The subscript value  $k$  of the linguistic term  $s_k^T$  can be obtained by the function  $I: s_{[0,T]}^T \rightarrow [0, T]$ :

$$I_T(s_k^T) = k, \quad s_k^T \in s_{[0,T]}^T. \quad (2)$$

*Definition 8* (see [30]). Let  $s_k^T \in s_{[0-T]}^T$ . The linguistic term  $s_k^T$  of subscript value  $k$  can be obtained by function  $I^{-1}: [0, T] \rightarrow s_{[0,T]}^T$ :

$$I_T^{-1}(k) = s_k^T, \quad k \in [0, T]. \quad (3)$$

At present, few studies have explored the uniformity of multi-granularity probabilistic language evaluation sets. Based on [31, 32], this paper constructed a conversion function to realize the conversion between probabilistic linguistic term sets with different granularities.

*Definition 9.* Let  $S^H = \{s_\alpha^H | \alpha = -H, \dots, 0, \dots, H\}$  and  $S^G = \{s_\beta^G | \beta = -G, \dots, 0, \dots, G\}$  be two linguistic term sets with different granularities. Let us set the probabilistic linguistic term set with granularity  $s^H$  as the basic probabilistic linguistic term set; then, we convert  $s^G$  to the corresponding form of the basic probabilistic linguistic term set through the conversion function as follows:

$$s_\alpha^H = \left\{ I_H^{-1} \left( \Delta_{\min} \left( \frac{I_G(s_\beta^G) 2H}{2G} \right) \right) (1-t), I_H^{-1} \left( \Delta_{\max} \left( \frac{I_G(s_\beta^G) 2H}{2G} \right) \right) (t) \right\}, \quad (4)$$

where  $\Delta_{\min}(z)$  and  $\Delta_{\max}(z)$  denote downward and upward rounding of the value of  $z$ , respectively. Let  $z = I_G(s_\beta^G) 2H / 2G$ ; when  $z$  is an integer,  $\Delta_{\min}(z) = \Delta_{\max}(z)$ ; the original probabilistic linguistic term needs two linguistic terms to be represented, and now only one probabilistic linguistic term is sufficient. Let  $t = z - \Delta_{\min}(z)$ . The converted probabilistic linguistic term sets can be expressed as

$$L_S^{T_H}(p) = \{s_\alpha^H(p_i^H) | s_\alpha^H \in S^H, p_i^H = \omega_\alpha^{(l)} p_i^G\}, \quad (5)$$

where  $\omega_\alpha^{(l)}$  represents the probability weight of the linguistic term element  $s_\alpha^H$ , which can be expressed as

$$\omega_\alpha^{(l)} = \begin{cases} t, & s_\alpha^H = I_H^{-1}(\Delta_{\min}(z)), \\ 1-t, & s_\alpha^H = I_H^{-1}(\Delta_{\max}(z)). \end{cases} \quad (6)$$

*3.2. Linguistic Scale Function.* Different evaluation experts have various feelings about the interval distance between linguistic term variables. According to their evaluation preference type, they can be distinguished by the change in the distance between their adjacent linguistic variables. If the evaluation expert is a risk-seeking type, the distance between the adjacent linguistic variables of his evaluation

information increases monotonically. Otherwise, it decreases monotonically. The linguistic scale functions can assign different semantic values to linguistic terms under different circumstances. As the subscript value of linguistic terms increases, the absolute deviation between adjacent linguistic terms may increase or decrease. Based on this, the following linguistic scale functions can give more definite results.

*Definition 10* (see [33]). Let  $S = \{s_\alpha | \alpha = -\tau, -\tau + 1, \dots, 0, \dots, \tau - 1, \tau\}$  be an LTS, where  $\tau$  is a positive integer; then, the standard linguistic scale function is

$$G(s_\alpha) = \left( \frac{1}{2} \left( \frac{\alpha}{\tau} + 1 \right) \right)^\theta = v_\alpha, \quad v_\alpha \in [0, 1], \quad (7)$$

where  $G(s_{-\tau}) = 0, G(s_\tau) = 1$ . According to the numerical distribution of linguistic variables, through the linguistic scale parameter  $\theta$ , the linguistic scale function can be simply classified as follows:

(i) Neutral linguistic scale function ( $\theta = 1$ ).

$$G_1(s_\alpha) = \frac{1}{2} \left( \frac{\alpha}{\tau} + 1 \right). \quad (8)$$

(ii) Optimistic linguistic scale function ( $\theta = \tau$ ).

$$G_2(s_\alpha) = \left(\frac{1}{2}\left(\frac{\alpha}{\tau} + 1\right)\right)^\tau. \quad (9)$$

(iii) Pessimistic linguistic scale function ( $\theta = 1/\tau$ ).

$$G_3(s_\alpha) = \left(\frac{1}{2}\left(\frac{\alpha}{\tau} + 1\right)\right)^{1/\tau}. \quad (10)$$

The ratings and the corresponding values of linguistic variables for the different types of linguistic scale functions are shown in Figure 1 and Table 1. When the absolute deviation between two adjacent linguistic terms is constant, we use the neutral linguistic scale function to describe it; when the absolute deviation between two adjacent linguistic terms increases, we use the optimistic linguistic scale function to describe it; when the absolute deviation between two adjacent linguistic terms decreases, we use the pessimistic linguistic scale function to describe it. In this paper, we consider the three types of linguistic scale functions mentioned above. For the convenience of calculation, we use  $G(s_\alpha)$  to represent the linguistic scale function in the subsequent paragraphs.

**3.3. PLTS Expectation Function and Variance Function Based on Linguistic Scale Function.** Most research uses the score function and the degree of deviation to quantify and compare PLTSs. However, these two methods only use the weighted summation of subscripts of linguistic terms and probabilities without considering the influence of linguistic scales. Since PLTSs cause information loss during the quantification process, this paper improved the expectation function and variance function by fusing the linguistic scale function to process the probabilistic linguistic information.

*Definition 11.* Let  $L(p) = \{L^{(l)}(p^{(l)}) | l = 1, 2, \dots, \#L(p)\}$  be a PLTS; then, the improved expected function and variance function of  $L(p)$  are formulated as follows:

$$\widehat{E}(L(p)) = \frac{\sum_{l=1}^{\#L(p)} (G(L^{(l)})P^{(l)})}{\sum_{l=1}^{\#L(p)} P^{(l)}}. \quad (11)$$

$$\widehat{\sigma}(L(p)) = \frac{\left(\sum_{l=1}^{\#L(p)} ((G(L^{(l)}) - E(L(p)))P^{(l)})^2\right)^{1/2}}{\sum_{l=1}^{\#L(p)} P^{(l)}}. \quad (12)$$

Let  $L_1(p)$  and  $L_2(p)$  be any two PLTSs. The comparison rules between the two are as follows:

- (1) If  $\widehat{E}(L_1(p)) > \widehat{E}(L_2(p))$ , then  $L_1(p) > L_2(p)$
- (2) If  $\widehat{E}(L_1(p)) < \widehat{E}(L_2(p))$ , then  $L_1(p) < L_2(p)$
- (3) If  $\widehat{E}(L_1(p)) = \widehat{E}(L_2(p))$ ,  $\widehat{\sigma}(L_1(p)) > \widehat{\sigma}(L_2(p))$ , then  $L_1(p) < L_2(p)$
- (4) If  $\widehat{E}(L_1(p)) = \widehat{E}(L_2(p))$ ,  $\widehat{\sigma}(L_1(p)) < \widehat{\sigma}(L_2(p))$ , then  $L_1(p) > L_2(p)$
- (5) If  $\widehat{E}(L_1(p)) = \widehat{E}(L_2(p))$ ,  $\widehat{\sigma}(L_1(p)) = \widehat{\sigma}(L_2(p))$ , then  $L_1(p) \sim L_2(p)$

*Example 1.* Consider two PLTSs,  $L_1(p) = \{s_0(0.4), s_1(0.4), s_3(0.2)\}$  and  $L_2(p) = \{s_0(0.3), s_1(0.7)\}$  of Example 4 in [34]. Using (11) with a neutral linguistic scale function, we find that  $\widehat{E}(L_1(p)) = 0.6667$ ,  $\widehat{E}(L_2(p)) = 0.6667$  and  $\widehat{E}(L_1(p)) = 0.6167$ . According to comparison rules, the ranking order is  $L_1(p) > L_2(p)$ , which is the same as the result of Example 4 in [34].

*Example 2.* Consider three PLTSs,  $L_1(p) = \{s_1(0.4), s_2(0.6)\}$ ,  $L_2(p) = \{s_0(0.2), s_2(0.8)\}$ , and  $L_3(p) = \{s_1(0.3999), s_2(0.6001)\}$  of Example 3 in [35]. By Equation (11), one has  $\widehat{E}(L_1(p)) = \widehat{E}(L_2(p)) = 0.76667$  and  $\widehat{E}(L_3(p)) = 0.76668$ , considering the neutral linguistic scale function. According to the comparison rules,  $L_3(p)$  is a little larger than  $L_1(p)$  and  $L_2(p)$ . Using Equation (12) to compare  $L_1(p)$  and  $L_2(p)$ , we find that  $\widehat{\sigma}(L_1(p)) = 0.0566$ ,  $\widehat{\sigma}(L_2(p)) = 0.0566$  and  $\widehat{\sigma}(L_3(p)) = 0.0754$ , which means  $L_1(p) > L_2(p)$ . The ranking order of these three PLTSs is  $L_3(p) > L_1(p) > L_2(p)$ . The ranking order between  $L_1(p)$  and  $L_3(p)$  is the same as the result of Example 3 in [35], but the rest of the obtained results are not. The method in [35] mainly calculates according to the relationship between linguistic terms. The method proposed in this paper pays more attention to the distance between linguistic terms and the value of linguistic variables. It considers the position of linguistic term elements in the overall linguistic term set. It can also reflect the influence of linguistic scale differences on the results. Thereby, the comparison rules for ranking PLTSs are valuable to some extent.

**3.4. PLTS Transformation Function.** Based on the uncertainty, hesitation, and inconsistency of evaluation experts in the evaluation process reflected by probabilistic linguistic terms, one study [36] proposed the calculation formula of hesitation degree of PLTS according to the score function and deviation degree. On this basis, this paper considers the influence of linguistic scale on hesitation and integrates the expectation and variance function to propose an improved transformation function, which converts the PLTS information into accurate values through the above process for calculation.

*Definition 12.* Let  $S = \{s_\alpha | \alpha = -\tau, -\tau + 1, \dots, 0, \dots, \tau - 1, \tau\}$  be an LTS and let  $L(p) = \{L^{(l)}(p^{(l)}) | l = 1, 2, \dots, \#L(p)\}$  be a PLTS. Then, the hesitancy of  $L(p)$  is

$$H(L(p)) = \frac{1/\#L(p) \sum_{l=1}^{\#L(p)} (P^{(l)}(G(L^{(l)}) - E(L(p))))^2}{2\tau + 1}. \quad (13)$$

When  $\#L(p) = 1$  means that the decision maker does not hesitate to give an evaluation, it can be expressed as  $H(L(p)) = 0$ .

*Definition 13.* Let  $L(p) = \{L^{(l)}(p^{(l)}) | l = 1, 2, \dots, \#L(p)\}$  be a PLTS; then, the transformation function for converting  $L(p)$  to an accurate utility value can be expressed as

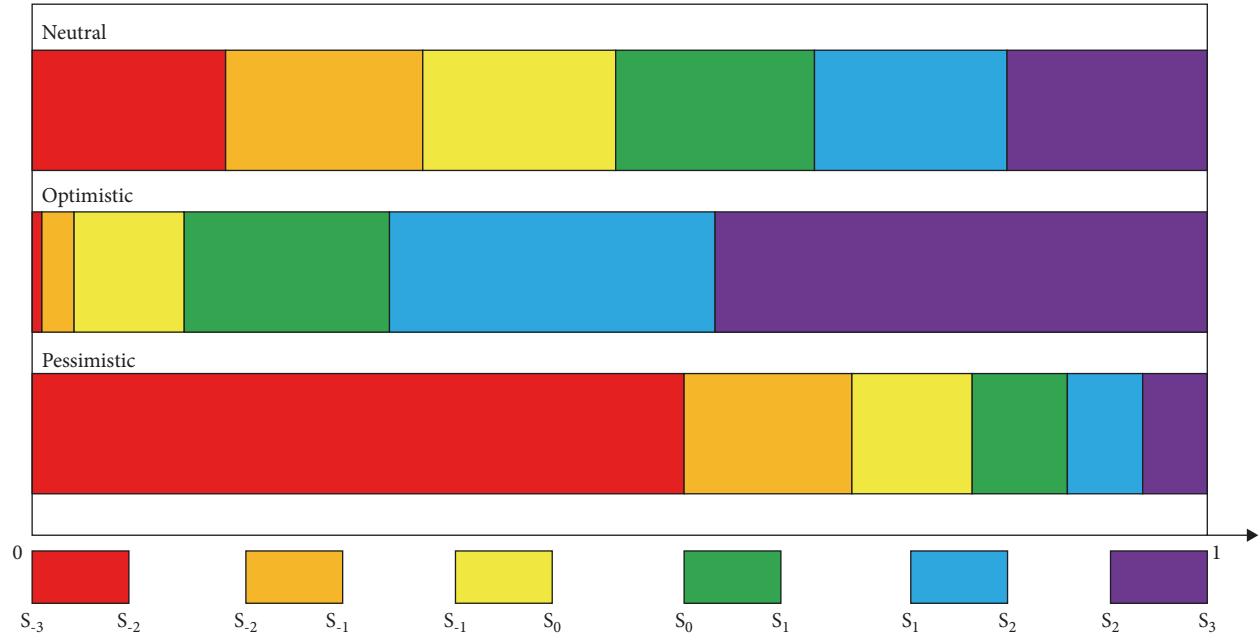


FIGURE 1: Rating of the linguistic variables by using different types of linguistic scale functions.

TABLE 1: Corresponding values of linguistic variables for different types of linguistic scale functions.

	$S_{-3}$	$S_{-2}$	$S_{-1}$	$S_0$	$S_1$	$S_2$	$S_3$
Neutral	0	0.167	0.333	0.500	0.667	0.833	1
Optimistic	0	0.005	0.037	0.125	0.296	0.579	1
Pessimistic	0	0.550	0.693	0.794	0.874	0.941	1

$$T(L(p)) = \hat{E}(L(p)) - \hat{\sigma}(L(p)) - H(L(p)), \quad (14)$$

where the larger  $\hat{E}(L(p))$  is, the smaller  $\hat{\sigma}(L(p))$  is, the smaller the hesitancy is, and the better the set of probabilistic linguistic terms  $L(p)$  is.

**3.5. Determination of Attribute Weights Based on Multi-granularity PLTS Distance Entropy.** Information entropy is a measure of information content, which is proportional to the degree of change in the index. This concept has now been introduced into various domains and used to determine attribute weights. In the process of MATSM, different evaluators have different access to the information of the evaluation object and their own understanding and analytical ability of the information, so the attribute evaluation of different evaluation objects is different. To solve the attribute weight problem of two-sided matching in a multi-granularity probabilistic linguistic term set environment, this paper uses the concept of multi-granularity probabilistic linguistic distance entropy [37] to solve the attribute weight by combining the distance between probabilistic linguistic terms and information entropy. The greater the distance entropy of the evaluation object attribute, the less the influence of the attribute, and the smaller the weight of the attribute.

The formula in the literature [17] for the distance between two PLTSs is the difference between a weighted sum of the subscripts and the probabilities of the corresponding linguistic terms. In this paper, the distance formula is improved by combining the linguistic scale functions.

*Definition 14.* Let  $L_1(p) = \{L_1^{(l)}(p_1^{(l)}) | l = 1, 2, \dots, \#L(p)_1\}$  and  $L_2(p) = \{L_2^{(l)}(p_2^{(l)}) | l = 1, 2, \dots, \#L(p)_2\}$  be two distinct PLTSs; then, the distance  $d(L_1(p), L_2(p))$  between  $L_1(p)$  and  $L_2(p)$  is presented as follows:

$$d(L_1(p), L_2(p)) = \sqrt{\frac{\sum_{l=1}^{\#L_1(p)} (p_1^{(l)} G(L_1^{(l)}) - p_2^{(l)} G(L_2^{(l)}))^2}{\#L_1(p)}}. \quad (15)$$

*Property 1.* (i)  $0 \leq d(L_1(p), L_2(p)) \leq 1$ ; (ii)  $d(L_1(p), L_2(p)) = d(L_2(p), L_1(p))$ ; (iii) if  $L_1(p) = L_2(p)$ , then  $d(L_1(p), L_2(p)) = 0$ .

The specific calculation steps of the probabilistic linguistic distance entropy are as follows:

- (1) The initial multi-granularity probabilistic evaluation information is assembled. The probabilistic linguistic terms for different granularities are aligned by Equations (2)–(6) to form the initial evaluation matrix  $Y_i = [y_{jk}^i]_{n \times f}$ , where  $y_{jk}^i = (L(p))_{jk}^i$  denotes the evaluation information of the  $i$ -th evaluator on the  $j$ -th attribute of the evaluation object  $k$ .
- (2) Combined with the information entropy theory, the probabilistic linguistic distance entropy  $E_k$  of attribute  $k$  is calculated based on Equations (15)–(19) as follows:

$$D_{jk} = \sum_{h=1, h \neq j}^n d(y_{jk}^i, y_{hk}^i) \quad j, h = 1, 2, \dots, n. \quad (16)$$

$$D_k = \sum_{j=1}^n \sum_{h=1, h \neq j}^n d(y_{jk}^i, y_{hk}^i) \quad j, h = 1, 2, \dots, n. \quad (17)$$

$$E_k = \frac{1}{In(n)} \sum_{j=1}^n \frac{D_{jk}}{D_k} In\left(\frac{D_{jk}}{D_k}\right). \quad (18)$$

- (3) The attribute weights are determined by the results of the above distance entropy calculation with the following equation :

$$w_k = \frac{1 - E_k}{\sum_{k=1}^f (1 - E_k)}. \quad (19)$$

#### 4. Multi-granularity PL-MARCOS-Based Satisfaction Degree Measurement Model

The Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) method was proposed by Stevic et al. in 2020 [38]. This method defines the relationship between alternatives and reference values (ideal and anti-ideal solutions), determines the utility function of the alternatives according to the defined relationship, and performs a compromise ranking of ideal and anti-ideal solutions. The decision preferences are defined based on the utility function. The utility function represents the distance of the alternatives from the ideal and anti-ideal solutions. The best alternative is the one closest to the ideal solution and at the same time farthest from the anti-ideal solution. The advantage of the MARCOS method is that the ideal and anti-ideal solutions are considered at the beginning of the initial matrix formation, allowing a more precise determination of the utility degree of the evaluation object associated with the two solutions while proposing a new method for determining the utility function. The method considers a large number of criteria and the possibility of evaluating the object while maintaining the stability of the method. The algorithm is simple and does not become more complicated by adding attribute indicators or alternatives; the results of the ratio method and the reference point ranking method are integrated, making the results obtained by the MARCOS method more reasonable. The article proposes the multi-granularity PL-MARCOS method based on the combination of multi-granularity probabilistic linguistic term sets and MARCOS, by which the SDs of the evaluation object are obtained.

The specific steps are as follows:

- (1) Formation of the initial evaluation matrix: the evaluation experts evaluate the evaluation objects according to their attribute indexes and aggregate the resulting evaluation matrix into an initial evaluation matrix  $L = [L_{ij}(p)]_{m \times n}$  where  $m$  denotes the evaluation objects and  $n$  denotes their attribute indices.

- (2) Extending the initial evaluation matrix: the initial evaluation matrix is extended by defining the ideal solution (AI) and the anti-ideal solution (AII). The specific form is as follows:

$$\tilde{L} = \begin{matrix} C_1 & C_2 & \dots & C_n \\ AAI & \left[ \begin{matrix} L_{aa1}(p) & L_{aa2}(p) & \dots & L_{aan}(p) \\ A_1 & L_{11}(p) & L_{12}(p) & \dots & L_{1n}(p) \\ A_2 & L_{21}(p) & L_{22}(p) & \dots & L_{2n}(p) \\ \dots & \dots & \dots & \ddots & \dots \\ A_m & L_{m1}(p) & L_{m2}(p) & \dots & L_{mn}(p) \\ AI & L_{ai1}(p) & L_{ai2}(p) & \dots & L_{ain}(p) \end{matrix} \right] \end{matrix}, \quad (20)$$

where  $C_j$  represents the attribute index of each evaluation object,  $A_i$  represents the evaluation object,  $x_{ij}$  is the evaluation value,  $AI$  is the best evaluation object value under the attribute, and  $AAI$  is the worst evaluation object value under the attribute, which can be expressed as

$$L_{aa_j}(p) = \min_i L_{ij}(p) \text{ if } j \in B \text{ and } \max_i L_{ij}(p) \text{ if } j \in C, \quad (21)$$

$$L_{ai_j}(p) = \max_i L_{ij}(p) \text{ if } j \in B \text{ and } \min_i L_{ij}(p) \text{ if } j \in C, \quad (22)$$

where  $B$  denotes a benefit-based indicator and  $C$  denotes a cost-based indicator.

- (3) Normalize the extended initial matrix: the evaluation information is converted into an accurate utility value  $T(L_{ij}(p))$  through a transformation function, thereby forming an initial matrix. The normalized matrix is obtained by processing the elements in the extended initial matrix by the following equation:

$$n_{ij} = \frac{T(L_{aij}(p))}{T(L_{ij}(p))} \text{ if } j \in C, \quad (23)$$

$$n_{ij} = \frac{T(L_{ij}(p))}{T(L_{aij}(p))} \text{ if } j \in B. \quad (24)$$

- (4) Determine the weighting matrix: the weighting matrix is obtained by multiplying the normalized matrix with the attribute index weight coefficients by the following equation:

$$v_{ij} = n_{ij} \times w_j, \quad (25)$$

where  $w_j$  is the weight coefficient of the attribute index.

- (5) Calculate the alternative solution utility degree: the utility degrees of the alternatives relative to the ideal and anti-ideal solutions are calculated as follows:

$$K_i^+ = \frac{S_i}{S_{ai}}, \quad (26)$$

$$K_i^- = \frac{S_i}{S_{aai}}, \quad (27)$$

where  $S_i (i = 1, 2, \dots, m)$  denotes the sum of the attribute elements in the weighting matrix, as given by

$$S_i = \sum_{i=1}^n v_{ij}. \quad (28)$$

- (6) Determine the utility function of the evaluation object: the utility function is the compromise embodiment of the alternatives to the ideal and anti-ideal solutions, and the utility function of each alternative is defined by the following equation:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + 1 - f(K_i^+)/f(K_i^+) + 1 - f(K_i^-)/f(K_i^-)}, \quad (29)$$

where  $f(K_i^+)$  denotes the utility function associated with the ideal solution and  $f(K_i^-)$  denotes the utility function associated with the anti-ideal solution, as expressed by the following equation:

$$f(K_i^+) = \frac{K_i^-}{K_i^+ + K_i^-}, \quad (30)$$

$$f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^-}. \quad (31)$$

The final utility value of the evaluation object obtained based on the above utility function is the evaluation expert's SD with the evaluation object, and the SD is proportional to the possibility of the expected alternative.

## 5. Construction of the Two-Sided Matching Model

**5.1. Problem Description.** The article considers the one-to-one MATSM problem under multi-granularity probabilistic linguistic preference information. The basic problem description is given below.

Let  $S = \{S^{T_1}, S^{T_2}, \dots, S^{T_m}\}$  denote the set of different granularity linguistic terms, where  $T_m$  represents their granularity.

$R^i = [R_{jk}^i]_{n \times g} (R_{ij} \in L(p), i \in M, j \in N, k \in G)$  denotes the multi-granularity probabilistic linguistic preference evaluation information given by object  $A_i$  about attribute  $k$  of object  $B_j$ ;  $T^j = [T_{ih}^j]_{m \times f} (T_{ij} \in L(p), i \in M, j \in N, h \in F)$  denotes the multi-granularity probabilistic linguistic preference evaluation information given by object  $B_j$  about attribute  $h$  of object  $A_j$ .

The problem solved in this paper is to obtain the accurate utility value of evaluation and the weight of evaluation

attribute based on the evaluation preference information  $R^i$  and  $T^j$  of probabilistic linguistic terms with different granularities given by two-sided objects while considering the satisfaction, fairness, and stability of two-sided matching. The proposed multi-granularity PL-MARCOS method was used to calculate the SDs of each evaluation object, which was then aggregated into the overall SD matrices  $P = [p_{ij}]_{m \times n}$  and  $Q = [q_{ij}]_{m \times n}$  of the evaluation object. The two-sided matching model was established to obtain the two-sided matching result.

The multi-attribute matching process framework is shown in Figure 2.

**5.2. Model Construction.** Based on the SDs of both sides ( $P = [p_{ij}]_{m \times n}$  represents the SDs of party A to party B;  $Q = [q_{ij}]_{m \times n}$  represents the SDs of party B to party A), the variable  $x_{ij}$  (valued 0–1) is introduced, and the multi-objective two-sided matching model is constructed based on the satisfaction, fairness, and stability of two-sided matching, with the objective of maximizing the SDs of both sides.

Model 1

$$\text{Max } Z_1 = \sum_{i=1}^m \sum_{j=1}^n p_{ij} x_{ij}. \quad (32a)$$

$$\text{Max } Z_2 = \sum_{i=1}^m \sum_{j=1}^n q_{ij} x_{ij}. \quad (32b)$$

$$\text{s.t. } \sum_{j=1}^n x_{ij} \leq 1, \quad j \in N. \quad (32c)$$

$$\sum_{i=1}^m x_{ij} \leq 1, \quad i \in M. \quad (32d)$$

$$x_{ij} + \sum_{p_{ih} > p_{ij}} x_{ih} + \sum_{q_{kj} > q_{ij}} x_{kj} \geq 1 \quad (32e)$$

$$x_{ij} \in \{0, 1\}, \quad i \in M, j \in N. \quad (32f)$$

In Model 1, Equations (32a) and (32b) are the objective functions, indicating that the maximum SD is achieved for both objects; Equations (32c)–(32f) are the matching constraints, and (32c) and (32d) indicate that both parties can only match with one other object; Equation (32e) is the constraint for stable matching of bilateral objects. In Equation (32f),  $x_{ij} = 1$  indicates that both objects can be matched and  $x_{ij} = 0$  indicates that both objects cannot be matched.

**5.3. Model Solving.** Model 1 is a multi-objective 0–1 integer programming model. This paper uses the linear weighted sum method to transform the above model into a single-objective programming model. Let  $\omega_1$  and  $\omega_2$  be the weights of  $Z_1$  and  $Z_2$ , and  $0 \leq \omega_1, \omega_2 \leq 1, \omega_1 + \omega_2 = 1$ . Let  $\omega_1 = \omega_2 = 0.5$  to ensure the fairness of the matching results of bilateral objects. Model 1 is transformed from a multi-objective



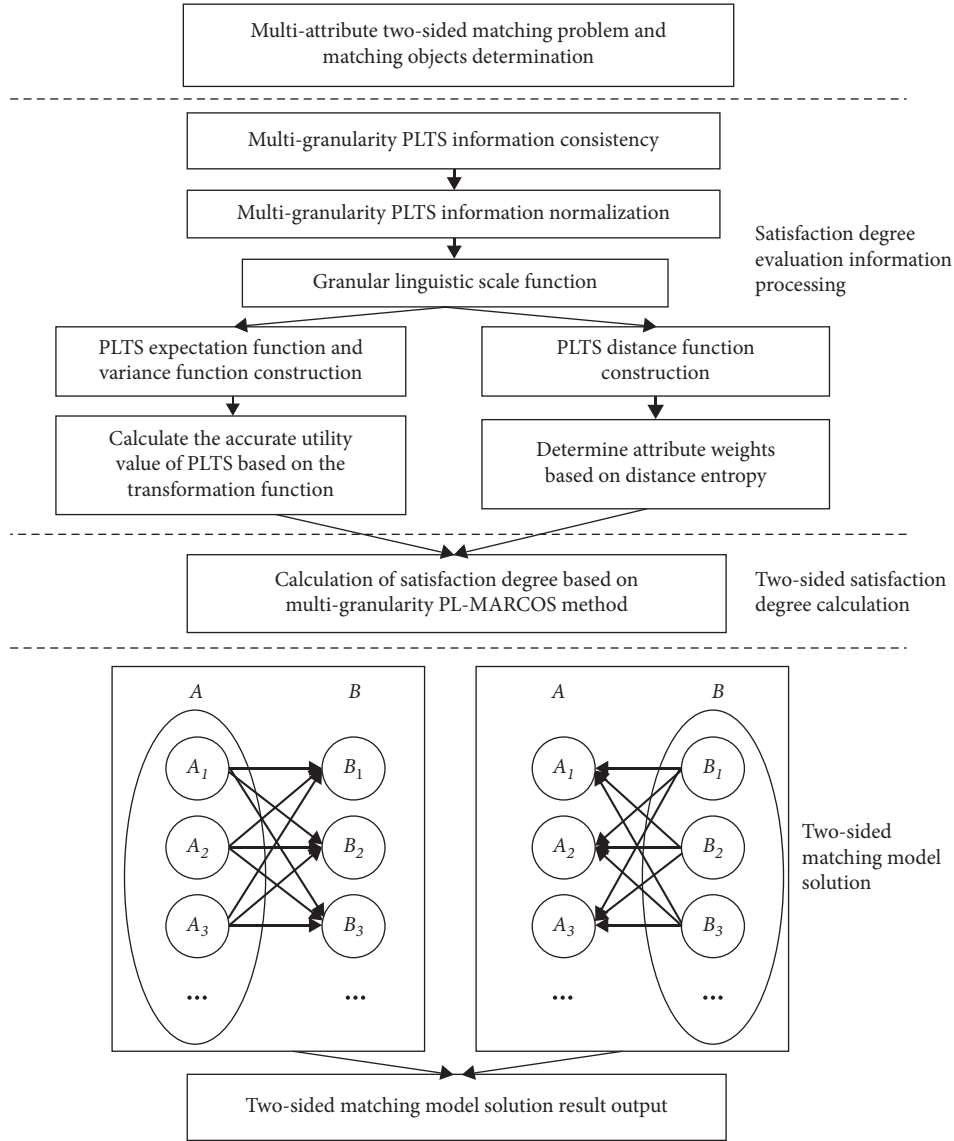


FIGURE 2: Flowchart of multi-attribute two-sided matching for multi-granularity probabilistic linguistic terms.

programming model to a single-objective programming model by weighting:

Model 2

$$\text{Max } Z^* = \omega_1 Z_1 + \omega_2 Z_2. \quad (33a)$$

$$\text{s.t. } \sum_{j=1}^n x_{ij} \leq 1, \quad j \in N. \quad (33b)$$

$$\sum_{i=1}^m x_{ij} \leq 1, \quad i \in M. \quad (33c)$$

$$x_{ij} + \sum_{p_{ih} > p_{ij}} x_{ih} + \sum_{q_{kj} > q_{ij}} x_{kj} \geq 1. \quad (33d)$$

$$x_{ij} \in \{0, 1\}, \quad i \in M, j \in N \quad (33e)$$

The specific steps of the MATSM decision method based on multi-granularity probabilistic linguistic term sets proposed in the article are as follows:

*Step 1.* Aggregate the original evaluation information and use Equations (2)–(6) to unify the probabilistic linguistic information of different granularities through the conversion function.

*Step 2.* Normalize the probabilistic linguistic information of the same granularity, and use Equations (8)–(14) to obtain the accurate utility value of the probabilistic linguistic term.

*Step 3.* Use Equations (15)–(19) to determine the evaluation object attribute weight.

*Step 4.* Based on the data and weights obtained in Steps 2 and 3, the SD matrix of evaluation experts for each evaluation object is obtained through Equations (20)–(31).

*Step 5.* Construct a multi-objective two-sided matching model (32a)–(32f).

*Step 6.* Use the linear weighted sum method to transform the multi-objective programming model into a single-objective programming model (33a)–(33e).

*Step 7.* Solve model (33a)–(33e) to obtain two-sided matching results.

## 6. Analysis of Calculation Examples and Results

*6.1. Calculation Results and Analysis.* A service outsourcing matching platform, specializing in matching services between service-demanding companies and service-supplying companies, matches through the relevant information provided by both parties. In a quarter, the platform received service demand information from 4 companies and service supply information from 5 companies. Through the service information released by the platform, the internal evaluation experts of the service-demanding companies evaluate the service supply enterprises in terms of historical service quality  $C_1^B$ , service quotation  $C_2^B$ , estimated completion time  $C_3^B$ , and business management ability  $C_4^B$ . The internal evaluation experts of the service-supplying companies evaluate the service-demanding companies from the perspective of business service demand  $C_1^A$ , required service technology level  $C_2^A$ , service mode  $C_3^A$ , and enterprise reputation  $C_4^A$ . The platform optimizes the two-sided matching based on the preference information given by both parties. Due to the different amount of information given by each company and the differences in evaluation capabilities of each company's internal evaluation expert team, the raw evaluation data are expressed in the form of probabilistic linguistic terms with different granularity as shown in Tables 1 and 2. The PLTS with 3-granularity, 5-granularity, and 7-granularity are used for these evaluation teams.

The original probabilistic linguistic evaluation information of different granularities is processed uniformly through the transformation function. Taking the service-supplying company  $B_1$  to the service-demanding companies  $A$  as an example,  $B_1$  uses three probabilistic linguistic term sets of different granularities ( $S^3, S^5, S^7$ ) for evaluation. The granular linguistic term set with high precision is selected as the basic evaluation set (in this case,  $S^7$  is selected as the basic probabilistic linguistic term evaluation set), and Equations (2)–(6) are used for consistent transformation. The results are shown in Table 3.

According to Table 3, the probabilistic linguistic information is standardized. Based on the linguistic scale function (taking the neutral linguistic scale function as an example), the accurate utility value of the probabilistic linguistic term is obtained by using the transformation function of

Equations (8)–(14) through the expectation function, variance function, and hesitancy degree as shown in Table 4.

Based on Table 3, by calculating the probabilistic linguistic distance combined with the information entropy, Equations (15)–(19) are also used to determine the attribute weight of the evaluation object. Similarly, the evaluation attribute weights of all companies to each other can be obtained according to the above method, as shown in Tables 5 and 6.

Based on the data and attribute weights obtained above, the  $B_1$  evaluation is still taken as an example. According to Table 4, the initial evaluation matrix is extended by Equations (20)–(22) as shown in Table 7. Based on the attribute weight of  $B_1$  to  $A$  in Table 6, Table 8 shows the extended evaluation matrix. The SDs of  $B_1$  to  $A$  are calculated by using Equations (23)–(31) as shown in Table 9.

The overall SD matrix of two-sided  $P = [p_{ij}]_{4 \times 5}$  and  $Q = [q_{ij}]_{4 \times 5}$  can be obtained from the above process.

$$P = [p_{ij}]_{4 \times 5} = \begin{bmatrix} 0.1546 & 0.1979 & 0.1787 & 0.1710 & 0.2355 \\ 0.1197 & 0.1697 & 0.1596 & 0.1896 & 0.1802 \\ 0.1045 & 0.1004 & 0.2126 & 0.1982 & 0.1722 \\ 0.1610 & 0.1496 & 0.1981 & 0.1598 & 0.1966 \end{bmatrix},$$

$$Q = [q_{ij}]_{4 \times 5} = \begin{bmatrix} 0.2156 & 0.2618 & 0.2298 & 0.1735 & 0.1727 \\ 0.1815 & 0.0941 & 0.2547 & 0.1527 & 0.1312 \\ 0.1706 & 0.1583 & 0.2375 & 0.2152 & 0.1784 \\ 0.2360 & 0.1388 & 0.1771 & 0.2220 & 0.2263 \end{bmatrix}. \quad (34)$$

A multi-objective two-sided matching model (32a)–(32f) is constructed and converted using formula (33a) into a single-objective programming model (33a)–(33e). The model is solved by LINGO 17.0. Set  $\omega_1 = \omega_2 = 0.5$  to obtain  $x_{12} = x_{24} = x_{33} = x_{45} = 1$ , and the rest  $x_{ij} = 0$ . According to the solution results in Table 10, the final matching result is  $\mu(A_1) = B_2, \mu(A_2) = B_4, \mu(A_3) = B_3, \mu(A_4) = B_5$ .  $B_1$  has no matching objects.

### 6.2. Comparisons and Discussions

*6.2.1. Comparison of Quantitative Methods.* This subsection compares the quantitative methods of multi-granularity probabilistic linguistic evaluation information. Consider the matching results in two situations: Situation 1 is a matching result that considers the degree of deviation and hesitancy. Situation 2 is a matching result that measures the probabilistic linguistic evaluation information only by the expectation function without considering the degree of deviation and hesitancy. Setting the same value  $\omega_1 = \omega_2 = 0.5$ , we obtained different results based on the two situations. From Table 11 and Figure 3, we can see the following:

- (1) The different situations affected the matching results and the values of  $Z_1, Z_2$ , and  $Z^*$ .

TABLE 2: The evaluation information of service-demanding companies to service-supplying companies.

		$C_1^B$	$C_2^B$	$C_3^B$	$C_4^B$
$A_1$	$B_1$	$s_0^5(0.3), s_2^5(0.5)$	$s_1^5(0.2), s_2^5(0.8)$	$s_{-1}^5(0.6), s_0^5(0.2)$	$s_{-1}^5(0.5), s_0^5(0.5)$
	$B_2$	$s_1^5(0.8)$	$s_1^5(0.6), s_2^5(0.2)$	$s_1^5(0.3), s_2^5(0.5)$	$s_0^5(0.4), s_1^5(0.6)$
	$B_3$	$s_0^7(0.1), s_1^7(0.9)$	$s_{-1}^7(0.3), s_0^7(0.3)$	$s_0^7(0.3), s_1^7(0.7)$	$s_1^7(0.9), s_2^7(0.1)$
	$B_4$	$s_2^7(0.6), s_3^7(0.4)$	$s_{-3}^7(0.7), s_{-1}^7(0.3)$	$s_2^7(1)$	$s_{-2}^7(0.5), s_{-1}^7(0.5)$
	$B_5$	$s_2^7(1)$	$s_2^7(0.65), s_3^7(0.35)$	$s_{-3}^7(0.2), s_{-1}^7(0.6)$	$s_3^7(1)$
$A_2$	$B_1$	$s_1^7(0.5), s_2^7(0.5)$	$s_{-2}^7(0.45), s_{-1}^7(0.45), s_{-3}^7(0.1)$	$s_3^7(1)$	$s_{-2}^7(0.5), s_{-1}^7(0.5)$
	$B_2$	$s_{-1}^5(0.5), s_0^5(0.5)$	$s_1^5(0.5), s_2^5(0.5)$	$s_1^5(0.3), s_2^5(0.5)$	$s_0^5(1)$
	$B_3$	$s_{-1}^7(0.3), s_0^7(0.5)$	$s_0^7(0.7), s_2^7(0.3)$	$s_2^7(0.7), s_3^7(0.3)$	$s_2^7(0.6), s_3^7(0.2)$
	$B_4$	$s_2^7(0.6), s_3^7(0.4)$	$s_3^7(1)$	$s_{-1}^7(0.6), s_0^7(0.4)$	$s_0^7(0.5), s_1^7(0.3)$
	$B_5$	$s_1^7(0.3), s_2^7(0.7)$	$s_1^7(0.5), s_2^7(0.5)$	$s_0^7(0.2), s_2^7(0.6)$	$s_2^7(0.2), s_3^7(0.8)$
$A_3$	$B_1$	$s_0^5(0.5), s_1^5(0.5)$	$s_1^5(0.8), s_2^5(0.2)$	$s_{-1}^5(0.8), s_0^5(0.2)$	$s_{-2}^5(0.6), s_{-1}^5(0.4)$
	$B_2$	$s_{-1}^5(0.8)$	$s_1^5(0.4), s_2^5(0.6)$	$s_{-1}^5(1)$	$s_{-1}^5(0.3), s_0^5(0.5)$
	$B_3$	$s_0^7(0.2), s_1^7(0.6)$	$s_2^7(0.5), s_3^7(0.5)$	$s_1^7(0.5), s_2^7(0.5)$	$s_2^7(0.3), s_3^7(0.5)$
	$B_4$	$s_1^7(0.1), s_2^7(0.9)$	$s_{-1}^7(0.4), s_0^7(0.6)$	$s_2^7(0.2), s_3^7(0.8)$	$s_{-1}^7(0.7), s_0^7(0.3)$
	$B_5$	$s_3^7(1)$	$s_{-2}^7(0.2), s_{-1}^7(0.8)$	$s_0^7(0.6), s_2^7(0.2)$	$s_2^7(0.9), s_3^7(0.1)$
$A_4$	$B_1$	$s_1^5(0.2), s_2^5(0.6)$	$s_1^5(0.1), s_2^5(0.9)$	$s_0^5(1)$	$s_0^5(0.5), s_1^5(0.5)$
	$B_2$	$s_1^7(0.125), s_2^7(0.125), s_3^7(0.75)$	$s_1^7(0.15), s_2^7(0.15), s_2^7(0.7)$	$s_0^7(0.1), s_1^7(0.45), s_2^7(0.45)$	$s_{-2}^7(0.1), s_{-1}^7(0.1), s_0^7(0.8)$
	$B_3$	$s_1^7(0.7), s_2^7(0.3)$	$s_3^7(0.9)$	$s_1^7(0.9), s_2^7(0.1)$	$s_2^7(0.3), s_3^7(0.7)$
	$B_4$	$s_2^7(0.8), s_3^7(0.2)$	$s_1^7(0.9), s_2^7(0.1)$	$s_2^7(0.6), s_3^7(0.4)$	$s_0^7(0.5), s_2^7(0.5)$
	$B_5$	$s_2^7(0.5), s_3^7(0.5)$	$s_2^7(1)$	$s_1^7(0.6), s_2^7(0.4)$	$s_3^7(1)$

TABLE 3: The evaluation information of service-supplying companies to service-demanding companies.

		$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$
$B_1$	$A_1$	$s_1^5(0.2), s_2^5(0.6)$	$s_0^5(0.8)$	$s_0^5(0.3), s_2^5(0.7)$	$s_1^5(0.4), s_2^5(0.4)$
	$A_2$	$s_1^3(0.8)$	$s_0^3(0.4), s_1^3(0.6)$	$s_{-1}^3(0.4), s_0^3(0.4)$	$s_0^3(0.8)$
	$A_3$	$s_2^7(0.4), s_3^7(0.6)$	$s_1^7(0.7)$	$s_{-2}^7(0.2), s_{-1}^7(0.6)$	$s_{-1}^7(0.6), s_0^7(0.4)$
	$A_4$	$s_2^7(0.5), s_3^7(0.5)$	$s_2^7(1)$	$s_1^7(0.3), s_2^7(0.7)$	$s_1^7(0.3), s_3^7(0.5)$
$B_2$	$A_1$	$s_0^5(0.5), s_1^5(0.5)$	$s_1^5(0.3), s_2^5(0.5)$	$s_2^5(0.7)$	$s_1^5(0.4), s_2^5(0.4)$
	$A_2$	$s_0^3(0.6), s_1^3(0.2)$	$s_0^3(0.4), s_1^3(0.6)$	$s_{-1}^3(0.3), s_0^3(0.7)$	$s_{-1}^3(0.3), s_0^3(0.5)$
	$A_3$	$s_2^7(0.4), s_3^7(0.6)$	$s_2^7(0.4), s_3^7(0.4)$	$s_{-1}^7(0.6)$	$s_0^7(0.6), s_2^7(0.4)$
	$A_4$	$s_1^7(0.6), s_2^7(0.2)$	$s_{-2}^7(0.6), s_0^7(0.4)$	$s_0^7(0.5), s_1^7(0.5)$	$s_1^7(0.3), s_2^7(0.5)$
$B_3$	$A_1$	$s_0^5(0.5), s_1^5(0.5)$	$s_1^5(0.8)$	$s_0^5(0.4), s_1^5(0.6)$	$s_0^5(0.3), s_1^5(0.5)$
	$A_2$	$s_0^3(0.25), s_1^3(0.75)$	$s_1^3(0.8)$	$s_{-1}^3(0.2), s_0^3(0.8)$	$s_0^3(0.6), s_1^3(0.2)$
	$A_3$	$s_0^7(0.6), s_2^7(0.4)$	$s_2^7(0.7), s_3^7(0.3)$	$s_0^7(1)$	$s_1^7(0.75), s_2^7(0.25)$
	$A_4$	$s_1^7(0.5), s_2^7(0.5)$	$s_{-1}^7(0.8), s_0^7(0.2)$	$s_{-1}^7(0.3), s_0^7(0.7)$	$s_{-1}^7(0.3), s_0^7(0.5)$
$B_4$	$A_1$	$s_{-2}^7(0.2), s_{-1}^7(0.2), s_0^7(0.6)$	$s_0^7(0.2), s_3^7(0.8)$	$s_1^7(0.5), s_2^7(0.5)$	$s_1^7(0.1), s_2^7(0.1), s_3^7(0.8)$
	$A_2$	$s_0^3(0.5), s_1^3(0.3)$	$s_0^3(0.4), s_1^3(0.6)$	$s_0^3(1)$	$s_0^3(0.6), s_1^3(0.4)$
	$A_3$	$s_1^7(0.5), s_2^7(0.5)$	$s_2^7(0.7), s_3^7(0.3)$	$s_1^7(0.2), s_3^7(0.6)$	$s_1^7(0.25), s_2^7(0.75)$
	$A_4$	$s_2^7(0.5), s_3^7(0.5)$	$s_0^7(1)$	$s_0^7(0.25), s_2^7(0.75)$	$s_2^7(0.3), s_3^7(0.7)$
$B_5$	$A_1$	$s_{-2}^7(0.1), s_{-1}^7(0.1), s_0^7(0.8)$	$s_1^7(0.15), s_2^7(0.15), s_3^7(0.7)$	$s_3^7(1)$	$s_1^7(0.3), s_2^7(0.3), s_3^7(0.4)$
	$A_2$	$s_{-1}^3(0.1), s_0^3(0.9)$	$s_0^3(0.4), s_1^3(0.6)$	$s_0^3(0.2), s_1^3(0.8)$	$s_{-1}^3(0.2), s_0^3(0.6)$
	$A_3$	$s_1^7(0.7), s_2^7(0.3)$	$s_2^7(1)$	$s_0^7(0.3), s_1^7(0.5)$	$s_2^7(0.35), s_3^7(0.65)$
	$A_4$	$s_2^7(0.6), s_3^7(0.2)$	$s_2^7(0.3), s_3^7(0.7)$	$s_2^7(0.25), s_3^7(0.75)$	$s_3^7(0.7)$

TABLE 4: The consistent results of evaluation information of service-supplying companies  $B_1$  to service-demanding companies  $A$ .

		$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$
$B_1$	$A_1$	$s_1^7(0.125), s_2^7(0.125), s_3^7(0.75)$	$s_0^7(1)$	$s_0^7(0.3), s_3^7(0.7)$	$s_1^7(0.25), s_2^7(0.25), s_3^7(0.5)$
	$A_2$	$s_3^7(1)$	$s_0^7(0.4), s_3^7(0.6)$	$s_{-3}^7(0.5), s_0^7(0.5)$	$s_0^7(1)$
	$A_3$	$s_2^7(0.4), s_3^7(0.6)$	$s_1^7(1)$	$s_{-1}^7(0.75), s_{-2}^7(0.25)$	$s_{-1}^7(0.6), s_0^7(0.4)$
	$A_4$	$s_2^7(0.5), s_3^7(0.5)$	$s_2^7(1)$	$s_1^7(0.3), s_2^7(0.7)$	$s_1^7(0.375), s_3^7(0.625)$

TABLE 5: The accurate utility value of evaluation information of service-supplying companies  $B_1$  to service-demanding companies  $A$ .

		$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$
$B_1$	$A_1$	0.8779	0.5000	0.6999	0.7927
	$A_2$	1.0000	0.6271	0.0710	0.5000
	$A_3$	0.8765	0.6667	0.2473	0.3432
	$A_4$	0.8575	0.8333	0.7337	0.7752

TABLE 6: The weight of the evaluation attribute of service-demanding companies  $A$  to service-supplying companies  $B$ .

		$C_1^B$	$C_2^B$	$C_3^B$	$C_4^B$
$A_1$		0.4698	0.1015	0.1188	0.3099
$A_2$		0.1013	0.5015	0.2975	0.0997
$A_3$		0.2069	0.1979	0.4357	0.1595
$A_4$		0.0694	0.3550	0.2409	0.3347

TABLE 7: The weight of the evaluation attribute of service-supplying companies  $B$  to service-demanding companies  $A$ .

		$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$
$B_1$		0.4600	0.2445	0.1829	0.1126
$B_2$		0.0359	0.2660	0.5206	0.1776
$B_3$		0.3052	0.4127	0.2387	0.0434
$B_4$		0.4105	0.1533	0.1743	0.2620
$B_5$		0.4152	0.1950	0.2634	0.1264

TABLE 8: Extended evaluation matrix of service-supplying companies  $B_1$ .

		$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$
$B_1$	$AAI$	0.8575	0.5000	0.0710	0.3432
	$A_1$	0.8779	0.5000	0.6999	0.7927
	$A_2$	1.0000	0.6271	0.0710	0.5000
	$A_3$	0.8765	0.6667	0.2473	0.3432
	$A_4$	0.8575	0.8333	0.7337	0.7752
	$AI$	1.0000	0.8333	0.7337	0.7926

TABLE 9: The SD of service-supplying companies  $B_1$  to service-demanding companies  $A$ .

		$S_i$	$K_i^+$	$K_i^-$	$f(K_i^+)$	$f(K_i^-)$	$f(K_i)$
$B_1$	$AAI$	1.7717					
	$A_1$	0.8557	0.2547	0.4830	0.6547	0.3453	0.2155
	$A_2$	0.7247	0.2157	0.4090	0.6547	0.3453	0.1825
	$A_3$	0.6794	0.2022	0.3835	0.6547	0.3453	0.1711
	$A_4$	0.9369	0.2789	0.5288	0.6547	0.3453	0.2359
	$AI$	3.3597					

TABLE 10: Two-sided matching result for service outsourcing companies.

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
$A_1$	0	1	0	0	0
$A_2$	0	0	1	0	0
$A_3$	0	0	0	1	0
$A_4$	0	0	0	0	1

TABLE 11: Two-sided matching results based on two considering situations.

	Two-sided matching results	$Z_1$	$Z_2$	$Z^*$
Situation 1	$A_1 \sim B_2, A_2 \sim B_3, A_3 \sim B_4, A_4 \sim B_5, B_1 \sim B_1$	0.7521	0.9581	0.8551
Situation 2	$A_1 \sim B_2, A_2 \sim B_4, A_3 \sim B_3, A_4 \sim B_5, B_1 \sim B_1$	0.7676	0.8220	0.7948

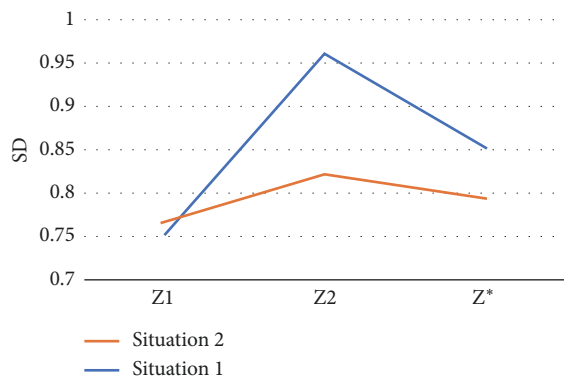


FIGURE 3: The overall SD of two-sided matching results based on two situations.

- (2) Some matching pairs have changed. The matching objects of  $A_2$  and  $A_3$  are different in situations 1 and 2.
- (3) The SDs of two-sided  $Z_1$  and  $Z_2$  are shifted. However, the value of overall SD  $Z^*$  in situation 1 is better than that in situation 2.

It can be seen that ignoring the deviation and hesitation information in the evaluation information will lead to information loss and deviations in the measurement of object SDs. The method proposed in this paper more accurately measures the SDs of the objects, and the matching result obtained is more reasonable and effective.

6.2.2. Comparison of Different Linguistic Scale Functions. To compare and analyze the influence of different linguistic scale functions on the results, the two-sided matching results are calculated separately through the three linguistic scale functions cited in this paper. According to the data in Tables 2 and 3, we calculate the attribute weights and accurate utility values of the evaluation information based on the optimistic and pessimistic linguistic scale function, respectively. Taking the evaluation information of service-supplying companies  $B_1$  to the service-demanding companies  $A$  as an example, the results are shown in Tables 12 and 13. Tables 14 and 15 show the attribute weights of the service-demanding companies  $A$  to the service-supplying companies  $B$  under different linguistic scale functions.

TABLE 12: Optimistic linguistic scale function. The accurate utility value of evaluation information of service-supplying companies  $B_1$  to service-demanding companies  $A$ .

	$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$	
$B_1$	$A_1$	0.7261	0.1250	0.4728	0.5379
	$A_2$	1.0000	0.3399	0.0182	0.1250
	$A_3$	0.6870	0.2963	0.0203	0.0423
	$A_4$	0.6388	0.5787	0.4096	0.5107

TABLE 13: Pessimistic linguistic scale function. The accurate utility value of evaluation information of service-supplying companies  $B_1$  to service-demanding companies  $A$ .

	$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$	
$B_1$	$A_1$	0.9547	0.7937	0.8766	0.9228
	$A_2$	1.0000	0.8470	0.1106	0.7937
	$A_3$	0.9564	0.8736	0.6196	0.6994
	$A_4$	0.9496	0.9410	0.9007	0.9217

TABLE 14: Optimistic linguistic scale function. The weight of the evaluation attribute of service-demanding companies  $A$  to service-supplying companies  $B$ .

	$C_1^B$	$C_2^B$	$C_3^B$	$C_4^B$
$A_1$	0.0524	0.1513	0.0664	0.7300
$A_2$	0.0822	0.3491	0.3009	0.2678
$A_3$	0.3261	0.0545	0.4507	0.1687
$A_4$	0.1006	0.2857	0.4136	0.2001

TABLE 15: Pessimistic linguistic scale function. The weight of the evaluation attribute of service-demanding companies  $A$  to service-supplying companies  $B$ .

	$C_1^B$	$C_2^B$	$C_3^B$	$C_4^B$
$A_1$	0.5431	0.1593	0.1528	0.1448
$A_2$	0.1057	0.4203	0.2895	0.1846
$A_3$	0.1554	0.2193	0.3929	0.2324
$A_4$	0.0996	0.3863	0.2020	0.3121

By comparing Tables 5, 12, and 13, we can see the difference between the three linguistic scale functions. The size relationship of their corresponding data is Optimistic < Neutral < Pessimistic, which is in line with the

description in Figure 1 and Table 1. Applying the linguistic scale functions to the actual evaluation style of experts can be expressed as follows: The comments given by some experts are unbiased without any obvious “praise” or “suppression” and can be inscribed with a neutral linguistic scale function. Some experts do not particularly care about one or more aspects of the service, and even if there are some flaws in those aspects, the experts will give them a higher rating even though they may not actually be that good; but if they think it is bad, their score may be worse than if it were bad on a neutral linguistic scale. The evaluation style of such experts can be portrayed by a positive linguistic scale function, such as the “tight on the left, loose on the right” pattern shown in Figure 2. There are also some experts who pay special attention to one or several aspects of the service. If there are slight deficiencies in these aspects, the experts will give a very low evaluation where the quality may not be so bad in reality; what they think is good may be better than it in fact is. The evaluation style of such experts can be characterized by a pessimistic scale function, such as the “loose on the left, tight on the right” pattern shown in Figure 1.

According to Tables 6, 14, and 15, the difference in linguistic scale functions has a significant effect on the weights of the evaluation attributes even if the influence of linguistic scale functions on distance entropy calculation is not a simple linear influence in the process of calculating attribute weights. Distance entropy is related to the absolute deviation between linguistic terms. In contrast to the neutral linguistic scale function, the sum of distance calculated by the optimistic and pessimistic linguistic scale functions is greater when performing the distance calculation, which ultimately leads to the difference in the attribute weights. The two-sided SD matrices are then obtained by the multi-granularity PL-MARCOS method. Finally, three results are obtained by solving the two-sided matching model, as shown in Table 16 and Figure 4.

From the data in Table 16 and Figure 4, we can see that the matching result is changed. In all three cases,  $B_1$  has no matching object. The matching objects for  $A_1$  and  $A_4$  are  $B_2$  and  $B_5$ . This situation proves the stability of the proposed method in this paper. The difference between  $A_2$  and  $A_3$  matching objects is also due to the influence of different linguistic scale functions on the results. This shows that differences in expert evaluation styles have an impact on the results of two-sided matching.

**6.2.3. Comparison of Different Methods.** To demonstrate the effectiveness of the proposed method, we compare the matching results and overall SDs of the proposed method with the method in [39]. The matching model adopts Model 2 and both  $\omega_1$  and  $\omega_2$  are set to be 0.5. The matching results of the two methods are shown in Table 17.

According to Table 17, we can see that some matching pairs are different between the two matching results. The matching objects of  $A_2$  and  $A_3$  are different and  $B_1$  still has no matching object. The overall SD  $Z^*$  obtained by the proposed method is higher than that of the method in [39].

TABLE 16: Two-sided matching results based on different types of linguistic scale functions.

	Two-sided matching result	$Z^*$
Neutral	$A_1 \sim B_2, A_2 \sim B_3, A_3 \sim B_4, A_4 \sim B_5, B_1 \sim B_1$	0.8551
Optimistic	$A_1 \sim B_2, A_2 \sim B_3, A_3 \sim B_4, A_4 \sim B_5, B_1 \sim B_1$	0.9065
Pessimistic	$A_1 \sim B_2, A_2 \sim B_4, A_3 \sim B_3, A_4 \sim B_5, B_1 \sim B_1$	0.7780

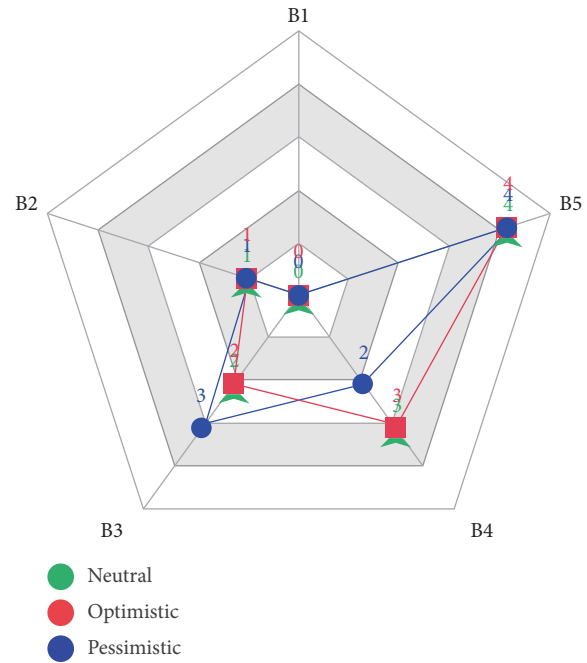


FIGURE 4: Two-sided matching results based on different types of linguistic scale functions.

TABLE 17: Two-sided matching results based on different methods.

	Two-sided matching result	$Z^*$
Proposed method	$A_1 \sim B_2, A_2 \sim B_3, A_3 \sim B_4, A_4 \sim B_5, B_1 \sim B_1$	0.8551
Other method [39]	$A_1 \sim B_2, A_2 \sim B_4, A_3 \sim B_3, A_4 \sim B_5, B_1 \sim B_1$	0.2544

In terms of matching results, there is little difference between the two matching results, indicating that the proposed method is reasonable and effective. However, the reasons for the difference are as follows. (1) In the attribute weight part, the method in [39] obtains the weight through the expected value combined with the information entropy. The method proposed in this paper takes into account the influence of the probabilistic linguistic distance before determining the weight through the distance entropy. (2) The method in [39] uses the PL-TOPSIS method for decision-making, while this paper uses the PL-MARCOS method. The two ranking methods are different and the obtained results are also distinct. TOPSIS considers the closeness of the distance between positive and negative ideal solutions. The MARCOS method considers the ideal solution and the anti-ideal solution at the beginning of the initial matrix and more

accurately determines the utility of the evaluation object related to the two solutions. The results obtained by the MARCOS method are more reasonable due to the fusion of the results of the ratio approach and reference point sorting approach [39].

## 7. Conclusion and Future Research

This paper considers the uncertainty of the evaluation environment and proposes the multi-granularity PL-MARCOS method to transform the evaluation information into a two-sided matching model of SDs based on the form of multi-granularity probabilistic linguistic evaluation. In the evaluation information processing part, we first considered describing the evaluation information through different granular linguistic term sets. The expectation function and the variance function are improved by integrating the linguistic scale function, which can better reflect the individual differences of the evaluators. Second, the multi-granularity probabilistic linguistic information is converted into accurate utility values by combining the hesitation degree. The concept of probabilistic linguistic distance is combined with information entropy, and the weight of the evaluation information attribute is obtained by calculating the distance entropy. Then, the multi-granularity PL-MARCOS method is proposed by combining the probabilistic linguistic term sets and the MARCOS method, which determines the evaluation SDs of both parties. Finally, with the goal of maximizing the SDs of both parties, a multi-objective two-sided matching model is constructed. The method proposed in this paper complements the research on two-sided matching in an uncertain environment (considering the granularity of evaluation information and individual differences between evaluators) so that the evaluation information can retain as much original information as possible. The proposed multi-granularity PL-MARCOS method calculates the information utility based on the distance between the compromise solution and the evaluation object so that the obtained results are scientific. The constructed two-sided matching model comprehensively considers satisfaction, fairness, and stability to improve matching efficiency.

In future research work, we will focus on the following topics:

- (1) Different people may have different understandings of the same linguistic information [40]. How to integrate personalized individual semantics to extend linguistic scaling functions in MATSM problems is a promising direction for future research.
- (2) Considering the complexity and diversity of the two-sided matching decision-making problems, we will study the multi-stage and dynamic matching processes during MATSM problems.

## Data Availability

All the data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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