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Research Article

On the Supply Chain Coordination with Trade Credit and Partial Backorder under CVaR Criterion

Gaoke Wu,[1] **Bo Feng**,[2] **and Chengfei Wang**[1]

[1]*School of Management Science, Qufu Normal University, Shandong, Rizhao 276800, China*
[2]*Business and Law Faculty, Deakin University, Locked Bag 20000, Geelong, VIC 3220, Australia*

Correspondence should be addressed to Chengfei Wang; chengfei-wang@163.com

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For the secondary supply chain composed of a supplier with sufficient funds and a retailer with limited funds and risk aversion, to enhance the supply chain stability and comprehensively improve benefits, the supply chain members may choose to implement a supply chain's coordination strategy. Thus, we study the following supply chain coordination problems, i.e., the supplier provides a trade credit policy and signs a buyback contract for the surplus products with the retailer. For this, based on maximizing the benefits of supply chain members, the supply chain coordination model has been established. By analyzing two participants' benefit, the supply chain's optimum coordination mechanism is obtained. The theoretical results are verified by numerical experiments.

1. Introduction

With the growth of the economy system and Internet retail, supply chain (SC) has become further lengthened and more and more complicated. Under this circumstance, to strengthen the connection between enterprises and increase the benefit value, SC members should coordinate with each other to resist risks and complexity together. Thus, the issue of supply chain coordination (SCC) receives much attention [1–3]. It is well known that the buyback contract is a significant strategy to implement SCC which can coordinate operations' performance of each member and spur retailers to make larger orders by the supplier's commitment to buy back the residual goods after the sale is completed. For this, Wu [4] showed that buyback contracts might bring greater benefits than other strategies in the increasingly competitive environment, Lin et al. [5] constructed the operation decision and coordination strategy model in the influenza vaccine SC and obtained the optimal coordination method, and so on [6–9].

The enterprises need many funds to coordinate the total SC. Therefore, the retailer will encounter the situation of insufficient funds. Trade credit is the form of deferred payment in intercompany transactions, which can relieve financial pressure and make enterprises operate smoothly [10, 11]. As for the trade credit problem under SCC, Phan et al. [12] showed that when the trade credit interest rate is comparable, SC contracts can coordinate the profits of all parties. Silaghi et al. [13] studied trade credit within the framework of real options and proved that trade credit is the coordination means within the framework of real options. Tsao et al. [14] considered the SCC strategy under credit risk based on information effort and used the compound contract to coordinate channel revenue. For more literatures on this, see [15–20].

Due to the long-term depressed financial crisis, the risk aversion awareness of SC members has been significantly improved [21]. For instance, clothing-related enterprises become more risk-averse in the decision to deal with the instability brought by COVID-19 [22]. Hence, the retailer's risk aversion is taken into account in the SCC model. For this, Yang et al. [23] showed that buyback strategy can coordinate the risk-averse SC. Ye et al. [24] developed a new mechanism for the sake of coordinating the SC related to agriculture with risk aversion agents in complex situations.

Backorder is also an essential part of SCC. In the operation process, some customers with unmet needs will give

up the purchase, while some customers with unmet needs will be willing to wait for the replenishment of the enterprise to purchase, which is the backorder. The backorder strategy in SCC management also receives much attention in recent years; see [25, 26] and references therein.

Upstream enterprises of some seasonal commodities or fast-moving products in the SC, such as Nike and Mengniu Group [27], generally cooperate with downstream enterprises through buyback contracts, and enterprises will invest excess funds or provide trade credit to obtain additional income. Due to the randomness of demand, enterprises are usually accompanied by risks and shortages. Therefore, we take risk aversion and partial backorder into account to meet the actual business situation. Relevant papers rarely discuss the above contents together, and some papers do not consider the interest of trade credit strategy while others do not study the significance of backorder in decision-making and profits. Therefore, our research work is of specific significance. In actual business operations, trade credit, risk aversion, partial backorder, and buyback contracts tend to coexist throughout the SC, and thus it is necessary to consider them simultaneously. From this, we consider the following SCC problem: the supplier offers a trade credit policy and two participants make a buyback contract for the surplus products. For this, based on maximizing the SC members' profits, we establish a SCC model with trade credit policy, partial backorder, and buyback contract. By analyzing the SC members' benefit, we obtain the optimal SCC mechanism. The effects of the trade credit, Conditional Value at Risk (CVaR) criterion, buyback contract, and partial backorder on the decision variables and profit are analyzed. Numerical experiments verify the theoretical achievements. The above content is the innovation and contribution of this paper.

The rest of this paper is arranged as follows. Section 2 describes the model framework of this paper. Section 3 discusses SC decisions and coordination under trade credit. Numerical experiments are carried out in Section 4. Section 5 gives conclusions and extensions.

2. The Model Framework of Supply Chain

2.1. Problem Description, Notations, and Assumptions. Consider the SC with a well-funded supplier and a capital-constrained retailer. As the starter of a replenishment cycle, the supplier sets a wholesale price and provides him with trade credit. To this, the retailer should determine the order quantity. Meanwhile, the retailer will adopt trade credit financing in case of insufficient funds, the retailer can use the initial capital to pay partial payment to the supplier and shall pay the arrears and interest to supplier at the finish of the replenishment cycle. Under the decentralized decision, the supplier needs to choose the wholesale price for retailer to encourage them to order more products. Furthermore, the retailer decides his ordering quantity based on the wholesale price and trade credit strategy. This is regarded as a supplier-led Stackelberg game. Under the centralized decision, the buyback contract is used for SCC. To solve the above research problems, the following assumptions are required. The notations are shown in Table 1.

The required assumptions are as follows:

(1) Supplier is risk-neutral and well funded, while the retailer is risk-averse and capital constraints

(2) The market demand of D is random, its probability density function is $f(x)$, and its distribution function is $F(x)$

(3) When retailer is shortage, the retailer will replenish the goods to nearby enterprises

The variables have the following relationships: $r_s > r_f$ and $p > f > w > b > c > \varepsilon > 0$. When $\beta(p-f) < (1-\beta)k$, the optimal order quantity's solution will be the sum of inverse functions of the two distribution functions, so it is difficult to solve the wholesale price further and find the sensitivity of each variables on decision variables and profits. Therefore, suppose $\beta(p-f) > (1-\beta)k$.

2.2. The Retailer's General Financing Model. In the process of operation, the retailer can obtain sales revenue of $p\min(Q,D)$ and recovery revenue of $\varepsilon(Q-D)^+$. Backorder can make the retailer gain revenue, i.e., $\beta(p-f)(D-Q)^+$. The penalty cost of shortage is $(1-\beta)k(D-Q)^+$. Since the retailer is capital constrained, he needs to pay interest payment $(wQ-y_0)^+ r_s$ at the end. Then, the retailer's benefit is

$$\begin{aligned}\pi(Q) = {} & p\min(Q,D) - wQ - (wQ-y_0)^+ r_s + \varepsilon(Q-D)^+ \\ & + \beta(p-f)(D-Q)^+ - (1-\beta)k(D-Q)^+.\end{aligned} \tag{1}$$

2.3. Some Preliminaries about the CVaR Measure. In order to better measure retailers' risk aversion, we use the CVaR criterion proposed by Kahneman and Tversky [28, 29]. The CVaR criterion focuses on profits that do not meet a given target level. This criterion is commonly used in the risk management [30, 31]. Thus, the risk-averse retailer's VaR benefit $\pi(Q)$ is

$$\mathrm{VaR}_\alpha[\pi(Q)] = \sup\{v \in R \mid \Pr[\pi(Q) \geqslant v] \geqslant \alpha\}, \tag{2}$$

where $\Pr[\pi(Q) \geqslant v]$ represents the probability of the $\pi(Q)$ more than v and $\mathrm{VaR}_\alpha[\pi(Q)]$ represents the maximum profit of the retailer with risk aversion α. At this time, we regard $\mathrm{VaR}_\alpha[\pi(Q)]$ as the targeted benefit. Therefore, the CVaR criterion for benefit $\pi(Q)$ is

$$\mathrm{CVaR}_\alpha[\pi(Q)] = \frac{1}{1-\alpha} E\{\pi(Q) \mid \pi(Q) \leqslant \mathrm{VaR}_\alpha[\pi(Q)]\}. \tag{3}$$

Because this formula is hard to calculate, the researcher usually converts it into the following formula [30, 31]:

$$\max_{Q,v} H_\alpha(Q,v) = v - \frac{1}{1-\alpha} E[v-\pi(Q)]^+. \tag{4}$$

Table 1: Notations.

Symbol	Description	Symbol	Description
Q	Order quantity	r_s	Interest rate of trade credit
w	Wholesale price	r_f	Risk-free interest rate
y_0	Retailer's initial funds	ε	Residual value of remaining goods
p	Selling price	α	Risk aversion level
c	Unit production cost	β	Backorder rate
f	Unit backorder cost	k	Unit penalty cost of shortage
b	Buyback price		

3. The Supply Chain Decision and Coordination under Trade Credit

Trade credit is a financing method to enhance the retailer's purchasing power and increase his profits when the retailer is short of funds in which the supplier first provides a certain amount of goods to the retailer and then receives payment and interest after selling the goods.

3.1. The Retailer's Optimal Decision under Trade Credit. In this subsection, we consider the circumstance such that the supplier sets the product's wholesale price, and the retailer determines its optimum order quantity. The retailer is short of funds and applies for deferment of payment to the supplier.

According to the above description and (1), we can obtain the retailer's benefit function.

$$\pi(Q) = p\min(Q,D) - wQ - (wQ - y_0)r_s + \varepsilon(Q-D)^+ + \beta(p-f)(D-Q)^+ - (1-\beta)k(D-Q)^+. \tag{5}$$

The supplier can invest the received payment and obtain financing income. Thus, the supplier's benefit is

$$\Pi(w) = (w-c)Q + (wQ - y_0)r_s + (y_0 - cQ)r_f. \tag{6}$$

Theorem 1. *Under the decentralized decision, the optimum order quantity when the retailer adopts trade credit is* $Q^* = F^{-1}(((1-\alpha)[p - w(1+r_s) - \beta(p-f) + (1-\beta)k])/(p - \varepsilon - \beta(p-f) + (1-\beta)k))$.

Proof. Based on this expression that $\min(Q,D) = Q - (Q-D)^+$ and $(D-Q)^+ = D - Q + (Q-D)^+$, one has

$$\begin{aligned}\pi(Q) &= p\min(Q,D) - wQ - (wQ - y_0)r_s + \varepsilon(Q-D)^+ \\ &\quad + \beta(p-f)(D-Q)^+ - (1-\beta)k(D-Q)^+ \\ &= [p - w - \beta(p-f) + (1-\beta)k]Q + [\beta(p-f) - (1-\beta)k]D - (wQ - y_0)r_s \\ &\quad - [p - \varepsilon - \beta(p-f) + (1-\beta)k](Q-D)^+.\end{aligned} \tag{7}$$

Then, the benefit function (4) can be rewritten as

$$\begin{aligned}H_\alpha(Q,v) &= v - \frac{1}{1-\alpha}E(v - \pi(Q))^+ \\ &= v - \frac{1}{1-\alpha}\int_0^{+\infty}\{v - [p - w - \beta(p-f) + (1-\beta)k]Q - [\beta(p-f) - (1-\beta)k]x \\ &\quad + (wQ - y_0)r_s + [p - \varepsilon - \beta(p-f) + (1-\beta)k](Q-x)^+\}^+ \mathrm{d}F(x) \\ &= v - \frac{1}{1-\alpha}\int_0^{Q}[v - \varepsilon Q + wQ - (p-\varepsilon)x + (wQ - y_0)r_s]^+ \mathrm{d}F(x) \\ &\quad - \frac{1}{1-\alpha}\int_Q^{+\infty}\{v - [p - w - \beta(p-f) + (1-\beta)k]Q - [\beta(p-f) - (1-\beta)k]x + (wQ - y_0)r_s\}^+ \mathrm{d}F(x).\end{aligned} \tag{8}$$

To facilitate derivation, the inner layer optimization problem is considered first, i.e., $\max_v H_\alpha(Q, v)$ for fixed Q in two cases. □

Case 1. $v < pQ - wQ - (wQ - y_0)r_s$. In this case, it is obtained by equation (8):

$$H_\alpha(Q, v) = v - \frac{1}{1-\alpha}\int_0^{\frac{v-\varepsilon Q+wQ+(wQ-y_0)r_s}{p-\varepsilon}} [v - \varepsilon Q + wQ - (p-\varepsilon)x + (wQ - y_0)r_s]\mathrm{d}F(x), \tag{9}$$

and

$$\frac{\partial H(Q, v)}{\partial v} = 1 - \frac{1}{1-\alpha}F\left(\frac{v - \varepsilon Q + wQ + (wQ - y_0)r_s}{p-\varepsilon}\right). \tag{10}$$

Certainly, $\partial H(Q, v)/\partial v|_{v=\varepsilon Q - wQ - (wQ - y_0)r_s} = 1 > 0$. Thus, if $\partial H(Q, v)/\partial v|_{v=pQ-wQ-(wQ-y_0)r_s} = 1 - F(Q)/(1-\alpha) \leqslant 0$, i.e., $Q \geqslant F^{-1}(1-\alpha)$, then the optimum solution to $\max_{v\geqslant 0} H(Q, v)$ is its stationary point, which means that

$$v^* = (p-\varepsilon)F^{-1}(1-\alpha) - wQ + \varepsilon Q - (wQ - y_0)r_s. \tag{11}$$

On the other hand, if $\partial H(Q, v)/(\partial v)|_{v=pQ-wQ-(wQ-y_0)r_s} = 1F(Q)/(1-\alpha) > 0$, then $H(Q, v)$ is incremented with respect to v, and it is classified into the next case discussion.

Case 2. $v \geqslant pQ - wQ - (wQ - y_0)r_s$. Under this condition, it is concluded from equation (8) that

$$\begin{aligned} H_\alpha(Q, v) = {} & v - \frac{1}{1-\alpha}\int_0^Q [v - \varepsilon Q + wQ - (p-\varepsilon)x + (wQ - y_0)r_s]\mathrm{d}F(x) \\ & - \frac{1}{1-\alpha}\int_Q^{\frac{v-[p-w-\beta(p-f)+(1-\beta)k]Q+(wQ-y_0)r_s}{\beta(p-f)-(1-\beta)k}} \{v - [p - w - \beta(p-f) + (1-\beta)k]Q - [\beta(p-f) - (1-\beta)k]x + (wQ - y_0)r_s\}^+ \mathrm{d}F(x), \end{aligned} \tag{12}$$

and

$$\frac{\partial H(Q, v)}{\partial v} = 1 - \frac{1}{1-\alpha}F\left(\frac{v - [p - w - \beta(p-f) + (1-\beta)k]Q + (wQ - y_0)r_s}{\beta(p-f) - (1-\beta)k}\right). \tag{13}$$

Then, there exists sufficiently large v_0 such that $\partial H(Q, v)/\partial v|_{v=v_0} = 1 - F((v_0 - [p - w - \beta(p-f) + (1-\beta)k]Q + (wQ - y_0)r_s)/(\beta(p-f) - (1-\beta)k))/(1-\alpha) \leq 0$. If $\partial H(Q, v)/\partial v|_{v=pQ-wQ-(wQ-y_0)r_s} = 1 - F(Q)/(1-\alpha) > 0$, i.e., $Q < F^{-1}(1-\alpha)$, then the optimum solution v^* to $\max_{v\geqslant 0} H(Q, v)$ satisfies that

$$1 - \frac{1}{1-\alpha}F\left(\frac{v - [p - w - \beta(p-f) + (1-\beta)k]Q + (wQ - y_0)r_s}{\beta(p-f) - (1-\beta)k}\right) = 0, \tag{14}$$

which means that $v^* = [\beta(p-f) - (1-\beta)k]F^{-1}(1-\alpha) + [p - w - \beta(p-f) + (1-\beta)k]Q - (wQ - y_0)r_s$. Otherwise, $\partial H(Q, v)/\partial v|_{v=pQ-wQ-(wQ-y_0)r_s} = 1 - F(Q)/(1-\alpha) > 0$, and $H(Q, v)$ decreases with the increase in v, which will be classified as Case 1.

From the above discussion, for any fixed Q in $\max_{v\geqslant 0} H(Q, v)$, we can obtain:

$$v^* = \begin{cases} (p-\varepsilon)F^{-1}(1-\alpha) + \varepsilon Q - wQ - (wQ - y_0)r_s, \text{if } Q \geqslant F^{-1}(1-\alpha), \\ [\beta(p-f) - (1-\beta)k]F^{-1}(1-\alpha) - (wQ - y_0)r_s \\ +[p - w - \beta(p-f) + (1-\beta)k]Q, if\ Q < F^{-1}(1-\alpha). \end{cases} \tag{15}$$

Next, we solve the outer layer optimization problem of the $\max_{Q\geqslant 0}\max_{v\geqslant 0} H(Q,v)$, i.e., $\max_{Q\geqslant 0} H(Q,v^*)$. In this regard, we also discuss it in two cases.

Case 3. $Q\geqslant F^{-1}(1-\alpha)$. Under this condition, from the result of $v^* = (p-\varepsilon)F^{-1}(1-\alpha)+\varepsilon Q - wQ - (wQ-y_0)r_s$, $H(Q,v^*)$ can be rewritten into

$$H(Q,v^*) = (p-\varepsilon)F^{-1}(1-\alpha)+\varepsilon Q - wQ - (wQ-y_0)r_s - \frac{1}{1-\alpha}\int_0^{F^{-1}(1-\alpha)}\left[(p-\varepsilon)\left(F^{-1}(1-\alpha)-x\right)\right]\mathrm{d}F(x). \tag{16}$$

Since $\partial H(Q,v^*)/\partial Q = \varepsilon - w(1+r_s) < 0$, the optimal value point is not in this region. Thus, it is classified as the next case for discussion.

Case 4. $Q < F^{-1}(1-\alpha)$. In this case, $v^* = [\beta(p-f)-(1-\beta)k]F^{-1}(1-\alpha)+[p-w-\beta(p-f)+(1-\beta)k]Q-(wQ-y_0)r_s$. Hence,

$$\begin{aligned} H(Q,v^*) = {} & [\beta(p-f)-(1-\beta)k]F^{-1}(1-\alpha)+[p-w-\beta(p-f)+(1-\beta)k]Q-(wQ-y_0)r_s \\ & -\frac{1}{1-\alpha}\int_0^{Q}\left\{[\beta(p-f)-(1-\beta)k]F^{-1}(1-\alpha)+[p-\varepsilon-\beta(p-f)+(1-\beta)k]Q-(p-\varepsilon)x\right\} \\ & \mathrm{d}F(x)+\frac{1}{1-\alpha}\int_{Q}^{F^{-1}(1-\alpha)}\left\{[\beta(p-f)-(1-\beta)k]\left(F^{-1}(1-\alpha)-x\right)\right\}\mathrm{d}F(x), \end{aligned} \tag{17}$$

and $\partial H(Q,v^*)/\partial Q = p - w(1+r_s) - \beta(p-f) + (1-\beta)k - [p-\varepsilon-\beta(p-f)+(1-\beta)k]F(Q)/(1-\alpha)$.

Then, we can obtain the optimum solution Q to $\max_{Q\geqslant 0} H(Q,v^*)$, i.e.,

$$Q^* = F^{-1}\left(\frac{(1-\alpha)[p-w(1+r_s)-\beta(p-f)+(1-\beta)k]}{p-\varepsilon-\beta(p-f)+(1-\beta)k}\right). \tag{18}$$

For the above optimum order quantity, the following corollary can be obtained:

Corollary 1. *For the parameters β, f, k, and α involved in the model, it holds that*

(1) The optimum order quantity Q^* decreases with β and α

(2) The optimum order quantity Q^* increases with f and k

Proof. According to Theorem 1, computing the derivative Q^* w.r.t β, f, k, and α, respectively, yields

$$\begin{aligned} \frac{\partial Q^*}{\partial \beta} &= \frac{-(1-\alpha)(p-f+k)(w(1+r_s)-\varepsilon)}{f(Q^*)(p-\varepsilon-\beta(p-f)+(1-\beta)k)^2}<0, \quad \frac{\partial Q^*}{\partial \alpha} = \frac{-[p-w(1+r_s)-\beta(p-f)+(1-\beta)k]}{f(Q^*)(p-\varepsilon-\beta(p-f)+(1-\beta)k)^2}<0, \\ \frac{\partial Q^*}{\partial f} &= \frac{\beta(1-\alpha)(w(1+r_s)-\varepsilon)}{f(Q^*)(p-\varepsilon-\beta(p-f)+(1-\beta)k)^2}>0, \quad \frac{\partial Q^*}{\partial k} = \frac{(1-\beta)(1-\alpha)(w(1+r_s)-\varepsilon)}{f(Q^*)(p-\varepsilon-\beta(p-f)+(1-\beta)k)^2}>0. \end{aligned} \tag{19}$$

This proves the assertion.

From Corollary 1 (1), when the backorder rate β increases, it means that some excess demand can be satisfied, and the retailer will make fewer orders. The optimal ordering quantity Q decreases with respect to the risk aversion level α. Thus, to reduce the risk of loss caused by over-ordering, the retailer will make a lower order decision. From Corollary 1 (2), the increase in unit backorder cost f will lead to increased cost, while when the unit penalty cost of shortage k increases, the retailer will have a higher penalty cost of shortage. From this, to reduce the backorder cost and penalty cost of shortage, the retailer will increase orders.

For the optimum wholesale price, the following conclusion can be obtained. □

Theorem 2. *Under the decentralized decision, the optimum wholesale price is*

$$\frac{(p-\varepsilon-\beta(p-f)+(1-\beta)k)Q^* f(Q^*)+c(1+r_f)(1-\alpha)}{(1+r_s)(1-\alpha)}. \tag{20}$$

By $Q^* = F^{-1}(((1-\alpha)[p - w(1+r_s) - \beta\ (p-f) + (1-\beta)k])/(p - \varepsilon - \beta(p-f) + (1-\beta)k))$ *from Theorem 1, we have*

$$\frac{\partial Q^*}{\partial w} = \frac{-(1+r_s)(1-\alpha)}{(p-\varepsilon-\beta(p-f)+(1-\beta)k)f(Q^*)} < 0. \tag{21}$$

Using the fact from (6) that

$$\frac{\partial \Pi(w)}{\partial w} = Q^* + \left(w - c\left(1+r_f\right)\right)\frac{\partial Q^*}{\partial w} + \left(Q^* + w\frac{\partial Q^*}{\partial w}\right)r_s, \tag{22}$$

the optimum wholesale price is

$$w = \frac{(p-\varepsilon-\beta(p-f)+(1-\beta)k)Q^* f(Q^*) + c\left(1+r_f\right)(1-\alpha)}{(1+r_s)(1-\alpha)}. \tag{23}$$

For the above optimal wholesale price, there are the following properties.

Corollary 2. *For the parameters* α, r_s, β, *and* f *involved in the model, one has*

(1) Increase in the optimum wholesale price w^* *with* α, f

(2) Decrease in the optimum wholesale price w^* *with* r_s, β

Proof. According to Theorem 2, computing the derivative w^* with respect to α, r_s, β, and f, respectively, we can get

$$\begin{aligned}
\frac{\partial w^*}{\partial \alpha} &= \frac{(p-\varepsilon-\beta(p-f)+(1-\beta)k)Q^* f(Q^*)}{(1+r_s)(1-\alpha)} > 0,\\
\frac{\partial w^*}{\partial r_s} &= \frac{-(1-\alpha)\left[(p-\varepsilon-\beta(p-f)+(1-\beta)k)Q^* f(Q^*) + c\left(1+r_f\right)(1-\alpha)\right]}{(1+r_s)^2(1-\alpha)^2} < 0.\\
\frac{\partial w^*}{\partial \beta} &= \frac{-(p-f+k)Q^* f(Q^*)}{(1+r_s)(1-\alpha)} < 0, \frac{\partial w^*}{\partial f} = \frac{\beta Q^* f(Q^*)}{(1+r_s)(1-\alpha)} > 0.
\end{aligned} \tag{24}$$

The desired results are as follows.

From Corollary 2 (1), the increase in the risk aversion level α will prevent retailer from making a fewer order. In order to maximize profits, the supplier's optimum wholesale price w will increase with respect to the risk aversion level α. Moreover, to avoid shortage, the order quantity Q increases with respect to the unit backorder cost f. According to the game equilibrium, the supplier will raise the wholesale price w to return the order quantity Q to the most favorable one. From Corollary 2 (2), when the interest rate of trade credit r_s increases, the supplier will reduce the wholesale price w to impel the retailer to choose the trade credit strategy. Moreover, the order quantity Q decreases with respect to the backorder rate β. According to the game equilibrium, the supplier will reduce the wholesale price w to return the order quantity Q to the most favorable one. □

3.2. The Supply Chain Coordination under Trade Credit. This subsection considers the circumstance such that the supplier buys back the remaining goods after the sale is completed to encourage retailers to order more goods.

Under this circumstance, from (1), the retailer's benefit is

$$\begin{aligned}
\pi_c(\overline{Q}) = {} & p\min(\overline{Q}, D) - w\overline{Q} - (w\overline{Q} - y_0)r_s + b(\overline{Q} - D)^+\\
& + \beta(p-f)(D-\overline{Q})^+ - (1-\beta)k(D-\overline{Q})^+.
\end{aligned} \tag{25}$$

As the supplier can obtain risk-free investment income from the payment received, thus the supplier's benefit is

$$\Pi_c(w) = (w-c)\overline{Q} + (w\overline{Q} - y_0)r_s + (y_0 - c\overline{Q})r_f - (b-\varepsilon)(\overline{Q} - D)^+. \tag{26}$$

According to the research of Yang et al. [23] and Chen et al. [32], the total SC can be considered as the newsvendor under the centralized decision, i.e., the risk-neutral supplier sells their products directly to the market. Then, the total supply chain's benefit is

$$\begin{aligned}
W_c(Q_c) = {} & p\min(Q_c, D) - cQ_c + (y_0 - cQ_c)r_f + \varepsilon(Q_c - D)^+\\
& + \beta(p-f)(D-Q_c)^+ - (1-\beta)k(D-Q_c)^+.
\end{aligned} \tag{27}$$

Theorem 3. *For the concerned model, the supplier's optimum buyback price is* $b^* = p - \beta(p-f) + (1-\beta)k - (((1-\alpha)[p - w\ \ (1+r_s) - \beta(p-f) + (1-\beta)k][p - \varepsilon - \beta(p-f) + (1-\beta)]\ k)/(p - c(1+r_f) - \beta(p-f) + (1-\beta)k))$, *and the optimum order quantity of SC is* $Q_c^* = F^{-1}((p - c(1+r_f) - \beta\ (p-f) + (1-\beta)k)/(p - \varepsilon - \beta(p-f) + (1-\beta)k))$.

Proof. According to equation (27), by $\min(Q, D) = Q - (Q-D)^+$, $(D-Q)^+ = D - Q + (Q-D)^+$, for the benefit of SC, one has

$$\begin{aligned} W_c(Q_c) &= p\min(Q_c, D) - cQ_c + (y_0 - cQ_c)r_f + \varepsilon(Q_c - D)^+ \\ &\quad + \beta(p-f)(D-Q_c)^+ - (1-\beta)k(D-Q_c)^+ \\ &= \left[p - c\left(1+r_f\right) - \beta(p-f) + (1-\beta)k\right]Q_c + [\beta(p-f) - (1-\beta)k]D + r_f y_0 \\ &\quad - [p - \varepsilon - \beta(p-f) + (1-\beta)k](Q_c - D)^+, \end{aligned} \tag{28}$$

$$\begin{aligned} E[W_c(Q_c)] &= \left[p - c\left(1+r_f\right) - \beta(p-f) + (1-\beta)k\right]Q_c + [\beta(p-f) - (1-\beta)k]\int_0^\infty x\mathrm{d}F(x) \\ &\quad + r_f y_0 - [p - \varepsilon - \beta(p-f) + (1-\beta)k]\int_0^{Q_c}(Q_c - x)\mathrm{d}F(x). \end{aligned}$$

By computing the stationary point of the function, we can obtain the total supply chain's optimum order quantity

$$Q_c^* = F^{-1}\left(\frac{p - c\left(1+r_f\right) - \beta(p-f) + (1-\beta)k}{p - \varepsilon - \beta(p-f) + (1-\beta)k}\right). \tag{29}$$

By similar argument to Theorem 1, the retailer's optimum order quantity is

$$\overline{Q}^* = F^{-1}\left(\frac{(1-\alpha)[p - w(1+r_s) - \beta(p-f) + (1-\beta)k]}{p - b - \beta(p-f) + (1-\beta)k}\right). \tag{30}$$

To implement SCC, by setting $Q_c^* = \overline{Q}^*$, the optimum buyback price is

$$\begin{aligned} b^* &= p - \beta(p-f) + (1-\beta)k \\ &\quad - \frac{H}{p - c\left(1+r_f\right) - \beta(p-f) + (1-\beta)k}. \end{aligned} \tag{31}$$

Among them, $H = (1-\alpha)[p - w(1+r_s) - \beta(p-f) + (1-\beta)k][p - \varepsilon - \beta(p-f) + (1-\beta)k]$.

According to the conclusion, the wholesale price is proportional to buyback price, i.e., when the wholesale price increases, the buyback price also increases. Further, by the fact that $0 < \alpha < 1$ and $w > b > \varepsilon$, one can obtain the range of α. □

Corollary 3. *The buyback contract can realize the SCC if the supplier's wholesale price satisfies*

$$\begin{aligned} &\frac{cQ_c^* + y_0 r_s - (y_0 - cQ_c^*)r_f + (b^* - \varepsilon)\int_0^{Q_c^*}(Q_c^* - x)\mathrm{d}F(x) + E(\Pi(Q^*, w^*))}{Q_c^*((1+r_s))} < w \\ &< \frac{\left(\begin{array}{l} [p - \beta(p-f) + (1-\beta)k]Q_c^* + [\beta(p-f) - (1-\beta)k]\int_0^\infty x\mathrm{d}F(x) + y_0 r_s \\ -[p - b^* - \beta(p-f) + (1-\beta)k]\int_0^{Q_c^*}(Q_c^* - x)\mathrm{d}F(x) - E(\pi(Q^*, w^*)) \end{array}\right)}{Q_c^*(1+r_s)}. \end{aligned} \tag{32}$$

Proof. According to Theorem 1, under decentralized decision, the benefit of both sides is a certain value. Therefore, we make a comparison to the relative benefit.

$$\begin{aligned} \Delta\pi &= E(\pi_c - \pi(Q^*, w^*)) = [p - w - \beta(p-f) + (1-\beta)k]Q_c^* + [\beta(p-f) - (1-\beta)k]\int_0^\infty x\mathrm{d}F(x) \\ &\quad - (wQ_c^* - y_0)r_s - [p - b^* - \beta(p-f) + (1-\beta)k]\int_0^{Q_c^*}(Q_c^* - x)\mathrm{d}F(x) - E(\pi(Q^*, w^*)) \\ \Delta\Pi &= E(\Pi_c - \Pi(Q^*, w^*)) = (w-c)Q_c^* + (wQ_c^* - y_0)r_s + (y_0 - cQ_c^*)r_f \\ &\quad - (b^* - \varepsilon)\int_0^{Q_c^*}(Q_c^* - x)\mathrm{d}F(x) - E(\Pi(Q^*, w^*)). \end{aligned} \tag{33}$$

The range of wholesale prices is obtained from $\Delta\pi > 0$:

$$w < \frac{\left(\begin{array}{l}[p-\beta(p-f)+(1-\beta)k]Q_c^* + [\beta(p-f)-(1-\beta)k]\int_0^\infty x\mathrm{d}F(x) + y_0 r_s \\ -[p-b^*-\beta(p-f)+(1-\beta)k]\int_0^{Q_c^*}(Q_c^*-x)\mathrm{d}F(x) - E(\pi(Q^*,w^*))\end{array}\right)}{Q_c^*(1+r_s)}. \tag{34}$$

Then, the range of wholesale prices is obtained from $\Delta\Pi > 0$:

$$w > \frac{cQ_c^* + y_0 r_s - (y_0 - cQ_c^*)r_f + (b^*-\varepsilon)\int_0^{Q_c^*}(Q_c^*-x)\mathrm{d}F(x) + E(\Pi(Q^*,w^*))}{Q_c^*(1+r_s)}. \tag{35}$$

By making the profits of both sides of the supply chain under the centralized decision greater than those under the decentralized decision, we get the wholesale price range that can coordinate the supply chain. □

Corollary 4. *The buyback contract can realize the SCC if the retailer risk aversion level satisfies*

$$\alpha_c \epsilon \left(0, 1 - \frac{[p-w-\beta(p-f)+(1-\beta)k]\left[p-c\left(1+r_f\right)-\beta(p-f)+(1-\beta)k\right]}{[p-w(1+r_s)-\beta(p-f)+(1-\beta)k][p-\varepsilon-\beta(p-f)+(1-\beta)k]}\right). \tag{36}$$

Under the restriction of individual rationality, to ensure that the SC members agree to provide and accept the buyback contract, we must first ensure that both parties' profits are not less than the profits obtained under the decentralized decision. Through the method similar to Corollary 3, we get the range of risk aversion level that can coordinate the supply chain.

4. Numerical Experiments

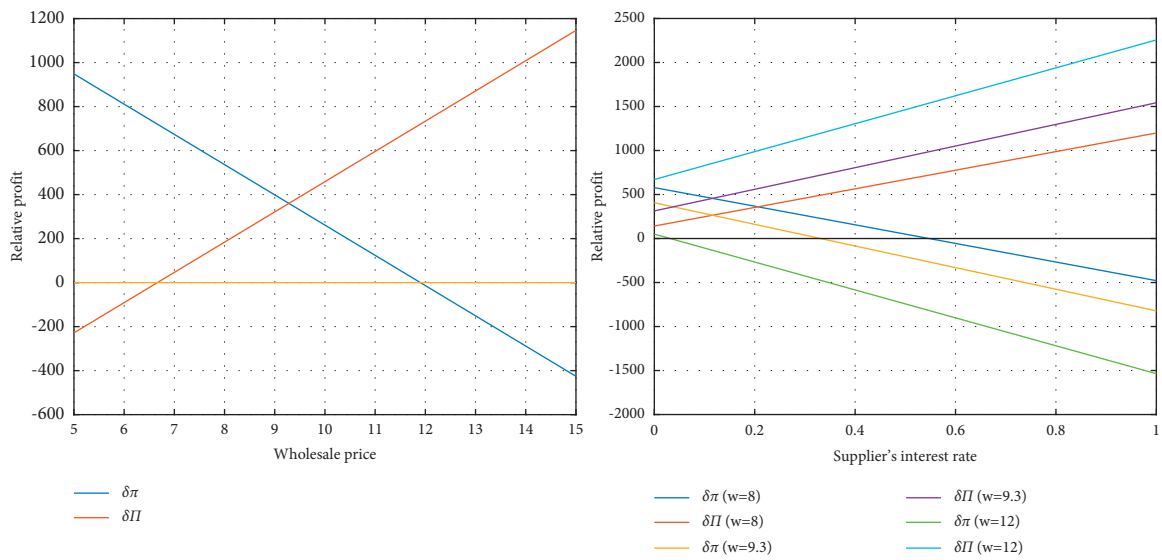
This section makes numerical experiments to test the model's effectiveness. First, the market demand D of the model obeys the uniform distribution in $[10, 200]$, and other variables are such that $p = 15, w = 10, f = 11, c = 5, \varepsilon = 2, k = 3, r_f = 0.02$, $r_s = 0.04, y_0 = 300, \alpha = 0.15$, and $\beta = 0.5$. By computation, we can obtain that under the decentralized decision, the optimum order quantity $Q^* = 55.724$ and the optimum wholesale price $w^* = 9.0509$.

From Figure 1(a), when the SCC under the buyback contract is adopted, the increased profits of both sides are linear functions of the wholesale price w. The retailer's profit decreases with the increase in wholesale price w, while the profits of the supplier increase with respect to wholesale price w. For $w\varepsilon[6.6, 11.9]$, the buyback contract can implement SCC, so that both sides of the SC can reach mutually beneficial results. For the sake of fairness, the most accessible point to implement is $w = 9.3$, where both parties increase their profits equally.

To increase comparability, we take $w = 8, w = 9.3$, and $w = 12$, respectively. Figure 1(b) shows that with the increase in interest rate r_s, the relative benefit of the supplier is greater than zero and increasing, while the relative benefit greater than zero of the retailer is $\underset{w=8}{r_s}\ \varepsilon(0, 0.55]$, $\underset{w=9.3}{r_s}\ \varepsilon(0, 0.33]$, $\underset{w=12}{r_s}\ \varepsilon(0, 0.03]$. This indicates that the repurchase contract can reach a mutually beneficial situation in SCC within the above range. With the increase in wholesale prices, the range of interest rates r_s that can achieve a win-win situation is gradually narrowing. Further, if we take $w = 9.3$, then the easiest point to implement is $r_s = 0.04$.

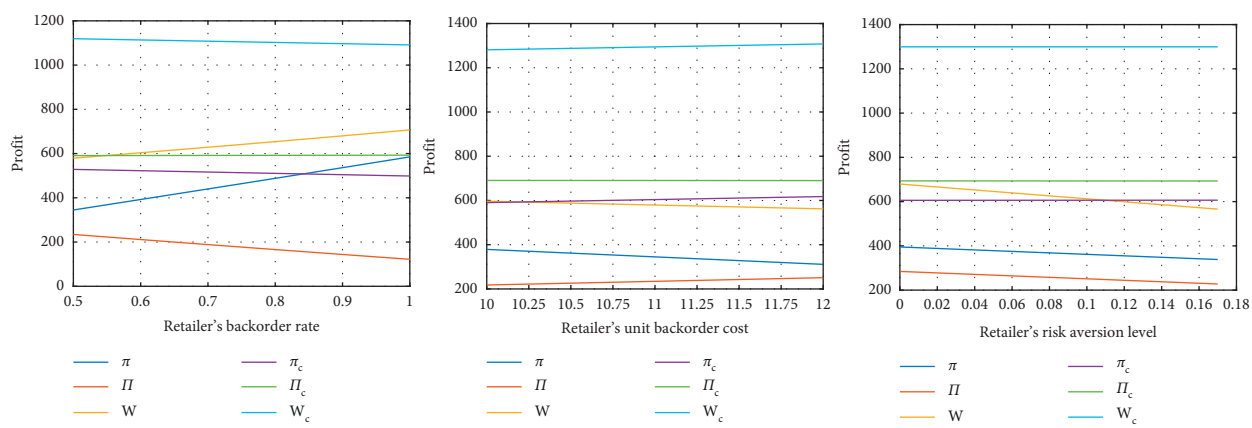
From Figure 2(a), under the decentralized decision, the retailer's benefit increases with respect to backorder rate β. Backorder is the income item of retailer's benefit. Although the increase in backorder rate β in Corollary 1 will reduce the order quantity Q, the increase in backorder rate β in Corollary 2 will reduce the wholesale price w which is another reason. The supplier's benefit decreases with the increase in backorder rate β which is the same as above. The reason why the benefit of the SC increases with the backorder rate β is that backorder is the revenue item, and the retailer's benefit increases faster than the supplier's benefit decreases. Under the centralized decision, the benefit of the retailer and SC decreases with respect to backorder rate β because the rise of backorder rate β will reduce the order quantity Q.

Figure 2(b) shows that the retailer's benefit decreases with unit backorder cost f under decentralized decision.



(a) w and relative benefit (b) rs and relative benefit

Figure 1: The impact of some parameters on the relative benefit.



(a) β and benefit (b) f and benefit (c) α and benefit

Figure 2: The impact of some parameters on the benefit.

One reason is that the increase in unit backorder cost f leads to decreased backorder revenue. Although the unit backorder cost f increases with respect to the order quantity Q in Corollary 1, another reason is that the increase in unit backorder cost f will increase the wholesale price w in Corollary 2. The reason why the supplier's benefit increases with the increase in unit backorder cost f is the same as above. The SC benefit decreases with the unit backorder cost f because the revenue of backorder decreases, and the retailer's benefit decreases faster than the increase in supplier's benefit. Under the centralized decision, the benefit of the retailer and SC increases with respect to unit backorder cost f because the rise of unit backorder cost f will increase the order quantity Q.

From Figure 2(c), under decentralized decision-making, since the retailer is risk-averse and the supplier is risk-neutral, the influence of risk aversion level α on the order quantity Q is greater than that on the wholesale price w. This is why profits on both sides decrease as risk aversion level α increases. Under the centralized decision, the risk aversion level α has little effect on both sides.

From the above numerical experiments, the SCC with buyback contract can not only achieve a win-win situation but also perform more robust than decentralized decision-making when each element changes.

5. Conclusions

This paper investigated the capital-constrained SCC with buyback contracts and partial backorder under the CVaR criterion. For these scenarios, two models were established to maximize the supplier and retailer's benefit. According to analysis of the two parties' benefit, the SC members' optimal decisions, and the impact analysis of the trade credit, CVaR

criterion and backordering to the retailer's decision were obtained. From the obtained results, we can see that as long as the risk aversion level, wholesale price and trade credit interest rate fall within a "Pareto zone," the whole SC can be coordinated, and both sides can reach mutually beneficial results, which provides management insights for the SC members. Under the decentralized decision, the retailer and the whole SC tend to have a higher backorder rate, while suppliers tend to have a lower backorder rate. Under the centralized decision, the retailer and SC tend to have a low backorder rate. The impact of unit backorder cost on the SC is just contrary to the above conclusion. The specific explanation is given in the previous section. Further, the benefit of both sides of the SC will decrease with respect to risk aversion level, which means that excessive risk aversion is not desirable. To better analyze the results and make the paper go on, we assume that the return on backorder is greater than the penalty cost of shortage, which is also one of the limitations of this paper. The extension of our research is to consider that the supplier is capital constrained. Meanwhile, the supplier can also be risk-averse, and the paper can also be combined with carbon policy to meet the real business environment.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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