How Does the Electronic Products Retailer Transform Operational Modes Considering Consumer Preferences for Leasing?

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Received 31 May 2022; Revised 14 August 2022; Accepted 29 August 2022; Published 20 September 2022

Academic Editor: Conghu Liu

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Consumer interest in renting brand-new electronic products has increased recently. Some retailers started to lease them, but they remarkably depreciate shortly after leasing. If their rental income cannot bring in enough profit, should a retailer lease them? This paper uses consumer preferences for leasing to describe their rental utility. It develops an analytical framework to study how the retailer selects operational modes. In a supply chain consisting of a manufacturer and a retailer, three different operational modes are analyzed: pure selling (S), pure leasing (L), and hybrid selling-leasing (SL). Results mainly show that when consumer preferences for leasing exceed a certain threshold, both the manufacturer’s and the retailer’s profits can be increased if the retailer transforms the business operational mode from S to SL/L. When consumer preferences for leasing are within a certain range, the retailer’s profits can be increased if the operational mode is transformed from S/L to SL. This happens as both consumers’ lease and purchase requirements for electronic products are met after the transformation. The wholesale price, leasing price, selling price, and the manufacturer’s profit all rise in the SL mode as consumer preference for leasing rises, but the retailer’s profit falls. Only if the retailer cooperates with the manufacturer can the operational mode be transformed from S/L to SL when consumer preferences for leasing are lower, and the coordination contract can achieve Pareto improvement in the profits of both the manufacturer and the retailer.

1. Introduction

The enormous market for electronic products accelerated the growth of the retail sector. In 2020, the size of the consumer electronics market exceeded 1 trillion USD, and 8% annual growth is anticipated from 2021 to 2027 (https://www.gminsights.com/industry-analysis/consumer-electronics-market, retrieved on 27 May 2022). Ceconomy, as a retailer specializing in selling electronic products to downstream consumers, ranks ninth among German family businesses with sales of 22.1 billion euros (https://www.163.com/dy/article/F5VB842V0518WSO6.html, retrieved on 27 May 2022). Additionally, retail sales of electronic products make up a sizable portion of those of the world-renowned online retailers Amazon and JD.com. Electronic-related sales account for more than half of JD.com platform’s revenue and 44% of Amazon’s revenue (https://www.chinairn.com/hyzx/20200605/140556382.shtml, retrieved on 27 May 2022). Retailers must be responsive to market changes due to the enormous volume of sales of electronic products in order to maintain their competitive advantage and boost profits.

In recent years, leasing has gained popularity as a circular business model. Numerous rental platforms have appeared; for instance, “Renrenzu” and Alibaba’s “Zhima Credit” both introduced product leasing services that decreased the use cost and purchase risk for consumers. With the rapid replacement of electronic products, the improvement of network credit mechanisms, the application of digital technology, and the implementation of intelligent logistics systems, consumers have shown a greater interest in renting electronic products. The lease of electronic products is becoming more and more popular, and conventional electronics retailers are facing tremendous challenges.

Conventional electronics retailers can meet market challenges by undergoing rapid transformation. In addition
to selling electronic products, Dixintong, electronics retailer in China, has started to provide leasing services, offering hybrid selling and leasing of electronic products on its platform (https://www.thepaper.cn/newsDetail_forward_1720419, retrieved on 27 May 2022). JD has also launched various product leasing websites for individuals and businesses (https://www.ce.cn/cysc/tech/gd2012/201909/17/t20190917_33166817.shtml, retrieved on 27 May 2022). Retailers can also turn into pure renters. However, completely giving up the selling option of electronic goods and offering only leasing services requires a significant adjustment on the part of the retailer.

In the real world, electronic products are updated very quickly. When retailers provide leasing service, consumers who prefer using the newest electronic products can choose to rent and only use them for the duration of the leasing period. Electronic products, in contrast to other goods, have a short lifespan [1]. For instance, 80% of consumers use their phones for less than two years [2]. They have a relatively long leasing period. For example, most retailers offer new phone leases for up to a year and beyond (https://shop.ee.co.uk/, retrieved on 8 Aug. 2022). Due to its lengthy lease time, they significantly depreciate after leasing, and the renters are the ones who gain the most from their value.

Therefore, this paper mainly considers the following situation: a retailer who purchases electronic products from a manufacturer only leases new electronic products for a certain leasing term. Products are returned to the retailer who receives their residual value when the lease expires. Consumer leasing preferences are used to describe their rental utility during a certain leasing period. In view of the above-mentioned practical challenges faced by retailers of electronic products, the following questions are explored:

1. How do electronics retailers transform their operational modes considering consumer preferences for leasing? Will they transform to pure leasing or hybrid selling-leasing?
2. How do consumer preferences for leasing impact the retailer’s transformation of operational modes, prices, demand, profit, and consumer surplus of electronic products supply chains?
3. How do retailers cooperate with upstream manufacturers when they implement the operational modes transformation? What are the impacts of their cooperation on retailers’ transformation and supply chains’ efficiency?

To address the aforementioned questions, this study uses a Stackelberg game model framework that includes a manufacturer, a retailer, and consumers who have the same preferences for leasing electronic products in a given leasing period. The manufacturer provides its electronic products to the retailer, who then decides the operational mode: pure selling (S), pure leasing (L), or hybrid selling-leasing (SL). After obtaining the equilibrium solutions, the impact of consumer preferences for leasing and the residual value of the electronic product on these solutions are investigated. The retailer’s optimal choices under the three modes are then discussed, followed by a numerical analysis. Moreover, the retailer’s hybrid selling-leasing mode transformation under cooperation with the manufacturer is analyzed, and a “wholesale price discounts and revenue sharing” coordination contract is also investigated.

It was found that leasing can increase the demand for electronic products whether it is a pure leasing mode or a hybrid selling-leasing mode. Therefore, when consumer preferences for leasing exceed a certain threshold, both the manufacturer’s and the retailer’s profits can be increased if the retailer transforms its operational mode from S to SL/L. When consumer preferences for leasing are within a certain range, the retailer’s profits can be increased if the operational mode is transformed from S/L to SL. Adding a leasing/selling channel to a strictly selling/leasing mode gives consumers more options and gives retailers a new revenue stream. As consumer preference for leasing increases, selling and leasing prices as well as the manufacturer’s profits all rise in the SL mode, while the retailer’s profits fall. On the other hand, leasing prices and the manufacturer’s and the retailer’s profits all rise in the L mode. When consumer preferences for leasing are lower, only if the retailer cooperates with the manufacturer can the operational mode be transformed from S/L to SL. Using a “wholesale price discounts and revenue sharing” contract might attract the manufacturer to cooperate and achieve Pareto improvement in the profits of both the manufacturer and the retailer.

The remainder of this article is organized as follows. Section 2 reviews the related literature. Section 3 details about the assumptions and the three modes that the retailer may consider when selling the electronic products provided. In Section 4, equilibrium results of the three models are compared, and the product price, market demand, and corporate profits are analyzed. A numerical analysis is presented in Section 5. In Section 6, an integration model and a coordination contract as an extension are outlined. Section 7 concludes this article. All proofs are provided in the Appendix.

2. Literature Review

The related literature to this article includes the following three streams: (i) selling versus leasing, (ii) the product leasing channel, and (iii) consumer preferences for leasing.

2.1. Selling Versus Leasing. When consumers buy products, they pay for the right to own them, but when they rent, they only pay for the right to use them. Pure selling, pure leasing, or hybrid selling-leasing mode selection of a monopoly manufacturer are discussed in numerous academic works. Yu et al. [3] compared per-use rentals with sales from a durable goods manufacturer, where there is a vertical differentiation between leased and brand-new products. They found that the degree of vertical difference between products significantly affects the choice of selling, leasing, or hybrid selling-leasing mode. Consumers may have financial limitations and are sensitive to products’ prices. The model established by Li et al. discussed this issue and found that
manufacturers offer products to consumers at low prices through leasing, which expands the market coverage and creates price discrimination [4]. Gilbert et al. [5] analyzed how per-use rentals and sales differentiate consumers within the framework of a hybrid selling-leasing model. They found that sales allow a firm to discriminate according to consumers’ usage frequencies, and rentals allow it to discriminate according to their realized valuation. Bhaskaran and Gilbert [6] study how complementary products influence manufacturers’ choice to lease or sell their products. Another popular area of research is the sale of digital goods via subscription (leasing) or perpetual (selling) [7–9]. Liu et al. [10] considered green product design in the comparison of manufacturers’ various lease modes. In a hybrid selling-leasing mode, manufacturers combined selling and leasing to create discriminatory pricing, which attracts potential price-sensitive customers. In contrast to the above-mentioned articles on the choice of selling or leasing modes made by manufacturers, this paper shifts the research perspective to explore how retailers directly facing consumers in the downstream supply chain make selling or leasing choices.

The choices of selling or leasing modes are different in a duopoly market structure. Tang and Deo [11] analyzed the leasing price and duration competition between two retailers that only offer lease services. The model of Chau and Schulz [12] depicts two manufacturers supplying goods to the market through their own intermediaries. Manufacturers can decide whether to sell or lease their products to intermediaries, and intermediaries can also decide whether to lease or sell their goods to consumers. Desai and Purohit [13] are the first to analyze the durable goods market in which both manufacturers adopt the hybrid leasing and selling strategy. Wang et al. [14] extends this issue by exploring six scenarios in which the two manufacturers adopt pure selling, pure leasing, or hybrid selling-leasing modes. In contrast to previous studies, our analysis focuses on the increasingly popular retailer’s choice to sell or lease products and analyzes the leader-follower game relationship between a manufacturer and a retailer considering consumer preferences for leasing.

2.2. Product Leasing Channel. Direct product leasing is not only available from professional renters but also from retailers and manufacturers. Manufacturers may encounter numerous rental channel structures. Bhaskaran and Gilbert [15] studied whether producers offer goods to consumers through intermediaries and compared whether products are offered through selling or leasing. They find that manufacturers are more willing to invest in product durability when offering leasing services. Xiong et al. [16] studied the implementation of product leasing and sales in both manufacturers’ direct selling and distribution channels and found that leasing products provided by manufacturers through direct selling channels may lead to the late withdrawal of product dealer. Bhaskaran and Gilbert [17] studied the competition between dealers when a manufacturer leases and sells its goods through multiple dealers, and one of their interesting findings is that the manufacturer prefers to use leasing-brokering arrangement. Kalantari et al. [18] discussed pricing policies by a manufacturer that sells its products online and whether these policies motivate a retailer as an independent part to enter the market to provide selling and leasing options through a brick store. Additionally, numerous studies investigated the channel problem of leased used products in the secondary market [19–23]. In contrast to previous studies, this paper analyzes the reality of retailers’ leasing products while considering consumer preferences for leasing in order to analyze the selling or leasing mode selection through the model and offer recommendations for retailers to better make their decisions.

2.3. Consumers’ Preferences for Leasing. The difference between a leasing mode and a selling mode is not only the form of ownership of the product but also the extent to which consumers accept the leasing. Depicting consumers’ reactions to product leasing is an important part of the research on product leasing. Yu et al. [3] analyzed the pricing of products and rental services in a market where consumers can buy products or pay for per-use rental services. They found that leasing has three effects on consumers as compared to purchase. First, unlike a one-time payment for a purchase, a consumer is required to pay for per-use rental services. Second, the consumer obtains the consumer-side benefits from ownership (CBO) under the purchase mode and loses the negative utility of finding the lease item under the lease model. In addition, the repeated use of leased products results in vertical differences in the quality of sold products. Li et al. [4] distinguished between sold products and leased products by describing that the product for leasing is devalued compared to the unused one and the degree of condition differentiation between products for leasing and unused products for selling occurring as an exogenous variable. They held that consumers without capital constraint can freely choose between the selling or leasing channel, while those with capital constraint can only lease the product. They also mentioned two interactions between consumers and leases: the decrease in the lease product value and the restrictions on the choice of leasing and selling under capital constraint. Gilbert et al. [5] used a similar model to describe the relationship between consumer utility and usage rate. For electronic products, there are differences in consumer acceptance of product leases. The leased products are older, and the short-term lease needs to change the consumption habits of consumers. There are also troubles in data preservation. Therefore, despite the fact that some consumers prefer product leasing, this preference is less common than consumers’ directly purchasing goods. Hence, similar to the two above-mentioned studies, we describe the low utility caused by product rental as consumer preference for leasing and analyze the impact of leasing preference degree on consumer leasing behavior. Li et al. [24] believed that due to the low service level of the platform and the psychological gap caused by nonownership of products, the perceived value of lease products will undoubtedly be lower than that of retail products. This value
perception factor and owners’ maintenance costs can influence the operating mode of original equipment manufacturers. Jalili and Pangburn [25] also agreed that a rental only offers a fraction of the utility that full ownership would. When consumers face significant value uncertainty, rentals also provide a mechanism for consumers to discover whether they like a product.

Existing studies on the choice of enterprise leasing and selling modes all focus the mode selection of manufacturers who directly deal with consumers [3, 5, 24, 25], or they focus on the retailers mode selection without considering different leasing preferences of consumers [11–14]. In the electronic products industry, the rise in consumer acceptance of leasing has brought new opportunities for retailers to transform to leasing modes. There is no literature examining the effects of retailers’ pure leasing, pure selling, and hybrid selling-leasing mode in light of customers’ various leasing preferences. Therefore, we model and analyze the important challenges faced by retailers in their transition to leasing, which will provide theoretical reference for retailers’ leasing transition.

3. Problem Description and Modeling

3.1. Problem Description. Consider an electronic product supply chain consisting of a manufacturer, a retailer, and consumers with the same leasing preferences [25]. In this supply chain, the manufacturer (M) wholesales electronic products to the retailer (R), and only the retailer sells or leases new electronic products to consumers. The retailer has three alternative operational modes as shown in Figure 1: pure selling (S), pure leasing (L), and hybrid selling-leasing (SL) [16]. In the S mode, electronic products are sold to consumers directly. In the L mode, electronic products are leased to consumers directly. In the SL mode, electronic products are sold and leased to consumers who choose how to obtain the products based on their utility.

There is a Stackelberg game between the manufacturer and the retailer. The manufacturer produces the electronic products and provides them to the retailer. Assume that the unit manufacturing cost of the electronic product is $c$. The manufacturer, as the leader, decides the wholesale price of the electronic product as $w$[26, 27]. Assume that the manufacturer will be able to immediately respond to the entire product demand in the market, and there will be no shortage or overproduction of the electronic product [12]. The retailer has three options for delivering electronic goods to customers after receiving them from the manufacturer. In the mode S, the selling price of the electronic product is $p^S$. In the L mode, the leasing price of the electronic product is $r^L$. In the SL mode, the selling price of the electronic product is $p^{SL}$, and the leasing price of the electronic product is $r^{SL}$. Assume that when the retailer offers a lease service, the item is only leased for a certain leasing period [28]. When the lease expires, all the leased goods are returned, and their residual value is obtained [29]. This paper denotes the residual value as $r$, for a product that has been leased and returned.

Assume that the total market size is 1, and consumers value the purchased product as $v$, uniformly distributed over the range $[0, 1]$ [25]. It is also assumed that the leasing period of the product is less its lifespan. When consumers rent products, they cannot enjoy their full value and must return them on time, but they can take advantage of some additional rental services, such as repair and maintenance services. Therefore, there may be a discount or a raise on the value of the leased product as compared to the purchased product. Consumers value the leased product as $av$, and $a(a \geq 0)$ is used to denote consumer preferences for leasing. A similar assumption can be seen in recent studies of product leasing [3–5]. The value of $a$ is affected by numerous factors, such as manufacturer’s product replacement speed, retailer’s rental service level, consumers’ acceptance of leasing, and others [2, 8, 24]. $a = 0$ represents the situation where consumers consider the value of the leased product is zero. Therefore, according to consumer utility theory [30], the utility that a consumer obtains by purchasing a product at price $p^S$ is $U_S = v - p^S$, and the utility that a consumer obtains by leasing a product at price $r^L$ is $U_L = av - r^L$, $i \in \{S, L, SL\}$. Consumers choose to purchase or lease products according to the principle of utility maximization. Assume that $v_i = \{v_i(U_i(v) = 0), v_i = \{v_i(U_i(v) = 0)\}$, and $v_\alpha = \{v_i(U_i(v) = U_i(v)\}$, which implies that both $v_i, v_\alpha$, and $v_\beta$ are functions of $p$ and $r$. In the S mode, consumers can only choose to purchase goods or not, consumers in $v \in \{v_i, 1\}$ are buyers who purchase the product, and consumers in $v \in \{0, v_i\}$ do not purchase the product. In the L mode, consumers can only choose to lease or not to lease the product, consumers in $v \in \{v_i, 1\}$ are renters who lease, and consumers in $v \in \{0, v_i\}$ do not lease the products. In the SL mode, a consumer can decide whether to buy, lease, or do neither; consumers in $v \in \{0, v_i\}$ do not buy or lease products, consumers in $v \in \{v_i, v_\alpha\}$ choose to lease the product, and consumers in $v \in \{v_\alpha, 1\}$ choose to purchase the product. According to Refs. [31, 32], in the S mode, the consumers’ purchasing demand for the electronic product is $D^S = 1 - p^S$, and the retailer’s wholesale demand for the electronic product is $D^S = D^L$; in the L mode, the consumers’ leasing demand for the electronic product is $D^L = 1 - \left(\frac{r^L}{a}\right)$, and the retailer’s wholesale demand for the electronic product is $D^L = D^L$; in the SL mode, if $\alpha > 1$, consumers will only choose to rent, and the retailer will only choose to lease. This scenario is the same as the L mode. In the SL mode, if $0 < \alpha \leq 1$, consumers’ purchasing demand for the electronic product is $D^{SL} = 1 - \left(\frac{p^{SL} - r^{SL}}{1 - \alpha}\right)$, consumers’ leasing demand $D^{SL} = \left(\frac{p^{SL} - r^{SL}}{1 - \alpha}\right)$, and the retailer’s wholesale demand for the electronic product is $D^{SL} = D^{SL} + D^{SL}$. Figure 2 illustrates the market segmentation. The profits of the manufacturer and the retailer are $\pi^S_M, \pi^S_R, i \in \{S, L, SL\}$.

3.2. Pure Selling Mode (S). In the S mode, the manufacturer first determines the wholesale price of the electronic product, and then the retailer determines the selling price accordingly. The Stackelberg game model can be established as
3.3 Pure Leasing Mode (L). In the L mode, the manufacturer first determines the wholesale price of the electronic product, and the retailer then determines the leasing price accordingly. The Stackelberg game model can be established as

\[
\max_{w^L} \pi_M^L = (w^L - c) D^L_s, \\
\text{s.t. } \max_{p^L} \pi_R^L = (p^L - w^L) D^L_s.
\]  

(2)

Backward induction is adopted to solve this problem, and the equilibrium results are presented in Lemma 2.

**Lemma 2.** In the L mode, the equilibrium wholesale and leasing prices of the electronic product are \(w^{L*} = (a + c + r^L_s/2)\) and \(r^{L*} = (3a + c - r_e/4)\). The equilibrium wholesale and leasing demand of the electronic product are \(D^{L*} = D^{L*}_s = (a + r_e - c/4a)\). The equilibrium profit of the manufacturer and the retailer are \(\pi_M^{L*} = ((a + r_e - c)^2/8a)\) and \(\pi_R^{L*} = ((a + r_e - c)^2/16a)\).

In order to guarantee the existence of non-negative demand and equilibrium solution, the condition \(a \geq c - r_e\) needs to be satisfied. This means that when consumer preferences for leasing are large enough, the retailer’s products can be leased.

**Observation 1.** In the L mode (i) as \(a\) increases, \(D^{L*}_s\), \(w^{L*}\), \(r^{L*}\), \(\pi_M^{L*}\), and \(\pi_R^{L*}\) also increase and (ii) as \(r^L_s\) increases, \(D^{L*}_s\), \(w^{L*}\), \(r^{L*}\), \(\pi_M^{L*}\), and \(\pi_R^{L*}\) increase, but \(r^{L*}\) decreases.

Results in Observation 1 state that in the L mode, the leasing demand, wholesale price, leasing price, and profits of both the manufacturer and the retailer, all decrease as the residual value of a leased-return product decreases, but the leasing price increases; that is, the residual value of the electronic products significantly depreciates after leasing; in order to reduce the operating risk for the leasing business, the retailer should raise the lease pricing, and the leasing demand will be reduced.
3.4. Hybrid Selling-Leasing Mode (SL). In the SL mode, the retailer provides consumers with electronic products by both selling and leasing. The manufacturer first determines the wholesale price of the goods, and the retailer then determines the leasing and selling prices. The Stackelberg game model can be established as

\[
\max_{\pi_{SL}} \pi_{SL} = (w_{SL} - c)(D_{SL} - D_{SL}^L),
\]

s.t. \( \max_{p^L, p^S} \pi_{SL} = (p^L - w_{SL})D_{SL}^L + (p^S + r_e - w_{SL})D_{SL}^L. \) (3)

The equilibria result is derived by backward induction, which is summarized in Lemma 3.

**Lemma 3.** In the SL mode, the equilibrium wholesale price is \( w_{SL} = (a + c + r_e/2), \) selling and leasing prices are \( p^{SL} = (3a + c + r_e/4) \) and \( p^{SL}_L = (2a + a + c + r_e/4), \) respectively. The equilibrium wholesale, leasing demand, and purchasing demand are \( D_{SL} = (a + r_e - c/4a), D_{SL}^L = (a^2 - (1-c-r_e)\alpha - (c-r_e)/4a(1-\alpha)), \) and \( D_{SL}^L = (1-a - r_e/2(1-\alpha)), \) respectively. The profits of the manufacturers and the retailer are \( \pi_{SL}^M = ((a + r_e - c)^2/8a) \) and \( \pi_{SL}^R = (3a^3 + (2c + 6r_e - 7)a^2 + (3r_e^2 + 2r_e - 6r_e - c^2 - 2c + 4)a + (c - r_e)^2)/16a(1-\alpha), \) respectively.

To guarantee the existence of non-negative demand, the condition \( a_0 \leq a \leq a_1 (a_0 = (1 - c - r_e + \sqrt{c^2 + 2cr_e + r_e^2 + 2c - 6r_e + 1/2}), a_1 = 1 - r_e) \) needs to be satisfied. In other words, there is a range of consumer preferences for leasing. Within this range, retailers are able to attract customers to both lease and make purchases (we ignore the corner solutions of SL mode and only analyze the equilibrium solution in detail because the corner solution of SL mode will degenerate into S mode and L mode, which we have modeled in the previous two sections).

**Observation 2.** In the SL mode, as \( \alpha \) increases, (i) \( D_{SL}^L, D_{SL}^L, w_{SL}^L, r_{SL}^L, \) and \( \pi_{SL}^L \) increase, (ii) \( D_{SL}^L, D_{SL}^L, w_{SL}^L, r_{SL}^L, \) and \( \pi_{SL}^L \) decrease.

Observation 2 shows that in the SL mode, when consumer preferences for leasing increase, the leasing demand increases but the purchasing demand decreases. Additionally, the total wholesale demand, the wholesale price, the leasing price, and the manufacturer’s profits all rise, while the retailer’s profits fall. This can be taken to mean that retailers cannot directly benefit from the rise in consumer preferences for leasing once their operational mode is changed from S to SL. Therefore, they generally lack the motivation to promote the leasing business. For example, after “Jingxiaozu” and “Guomeizu” launched the mobile phone leasing business (https://prom.gome.com.cn/html/prodhtml/topics/201709/30/2461678680.html, retrieved on 27 May 2022), they did not carry out leasing publicity on their platform websites and APP clients to guide consumers to change their consumption concepts. Unlike retailers, manufacturers are always able to benefit from the increase in consumer preferences for leasing and are therefore more willing to promote leasing business.

**Observation 3.** In the SL mode, as \( r_e \) increases, (i) \( D_{SL}^L, D_{SL}^L, w_{SL}^L, p_{SL}^L, \) and \( \pi_{SL}^L \) increase, (ii) \( D_{SL}^L \) and \( r_{SL}^L \) decrease, and (iii) \( \pi_{SL}^L \) first decreases and then increases.

As shown in Observation 3, in the SL mode, the leasing demand for leased-return electronic products increases as their residual value increases, while the purchasing demand decreases. However, as the total wholesale demand increases, wholesale and sales prices increase, the lease price decreases, and the manufacturer’s profits increase. If the residual value of the leased-return electronic product is larger than a certain threshold, the retailer’s profits monotonously increase with the residual value. It is generally believed that retailers who offer mobile phones to customers through both selling and leasing will earn more profits if they lease or sell phones with higher residual values. However, since a higher wholesale price is typically associated with a larger residual value of the leased-return electronic product, retailers who offer mobile phones to consumers through both selling and leasing may experience a decline in profits if they advise customers to lease mobile phones with higher residual value unless the residual value is greater than a certain threshold.

4. Comparisons of Equilibrium Results

This section compares the equilibrium results of the retailer’s three alternative operational modes. Some analytical results are first derived by comparing the equilibrium price, demand, and profit. The equilibrium consumer surpluses are calculated and compared afterwards. The related conclusions are then analyzed and explained (These conclusions are only meaningful when \( a_0 \leq a \leq a_1 \). Because in order to guarantee that the coexistence of leasing and purchasing demand for the electronic product and the retailer’s hybrid selling-leasing business transformation make sense, condition \( a_0 \leq a \leq a_1 \) needs to be satisfied).

**Proposition 1.** The equilibrium price has the following properties: (i) \( w^{SL} \geq w^{SL} = w^{SL} \), (ii) \( p^{SL} \geq p^{SL} \geq r_{SL} = r_{SL} \), and (iii) in the SL mode, define the portion of the selling price that exceeds the leasing price as \( \Delta_0 = p_{SL}^L - p_{SL}^L \), which decreases as \( \alpha \) increases, and increases as \( r_e \) increases.

Proposition 1 (i) shows that in L and SL modes, wholesale prices of the electronic product are the same, which are lower than the wholesale prices in the S mode. For retailers only selling the electronic product, manufacturers should set relatively high wholesale prices. Proposition 1 (ii) shows that compared with the S mode, in the SL mode, the selling price is lower, which can be understood as that retailers should lower the selling price once their operational mode transforms from S to SL in order to prevent the leasing business from having a greater impact on the selling business. In S and SL modes, leasing prices are the same, and the retailer’s leasing pricing is only related to the production cost, the residual value of leased-return electronic product, and consumer preferences for leasing but has nothing to do with the operational modes. Proposition 1 (iii) shows that the larger the consumer preferences for leasing, the smaller the differences between leasing and selling prices, and the price difference increases with the rise of the residual value.
Brands such as Huawei, Apple, and other new 5G high-end smartphones have a large residual value and a significant price difference between leasing and selling prices. Therefore, these smartphones attract more customers to lease them.

**Proposition 2.** The equilibrium demand has the following properties: (i) \( D^S < D^{SL^*} = D^{L^*} \), (ii) \( D^{SL^*} > D^{L^*} \) when \( \alpha < \alpha_3 \), and \( D^{SL^*} \leq D^{L^*} \) when \( \alpha_3 \leq \alpha \), and (iii) \( D^{SL^*} \leq D^{L^*} \). \( (\alpha_2 = (1 + c - 2r_e/1 + c)) \).

Proposition 2 (i) shows that in SL and L modes, the wholesale demand for electronic products is relatively larger. The expansion of leasing business by retailers can boost the market’s wholesale demand for electronic products. Proposition 2 (ii) shows that when consumer preferences for leasing are higher than \( \alpha_3 \), the purchasing demand for the electronic product will increase after the retailer’s operational mode is transformed from S to SL. This shows that leasing has a certain promotion and erosion effect on selling. When consumer preferences for leasing are lower than a certain threshold, the promotion effect is greater than the erosion effect. Otherwise, the erosion effect will be greater than the promotion effect, and the purchasing demand for the electronic product will decrease. Proposition 2 (iii) shows that in the L mode, the leasing demand for the electronic product is larger.

**Proposition 3.** The equilibrium profits have the following properties:

(i) Comparing manufacturer profits of different modes, there are conditions such as

\[
\pi^S_M > \pi^S_{SL^*} = \pi^L_M \quad \text{when} \quad \alpha_0 \leq \alpha < \alpha_3,
\]

\[
\pi^S_M = \pi^S_{SL^*} = \pi^L_M \quad \text{when} \quad \alpha = \alpha_3,
\]

\[
\pi^S_M < \pi^S_{SL^*} = \pi^L_M \quad \text{when} \quad \alpha_3 < \alpha \leq \alpha_1.
\]

(ii) Comparing retailer profits of different modes, there are conditions such as

\[
\pi^S_R > \pi^S_{SL^*} > \pi^L_R \quad \text{when} \quad \alpha_0 \leq \alpha < \alpha_3,
\]

\[
\pi^S_R > \pi^S_{SL^*} = \pi^L_R \quad \text{when} \quad \alpha = \alpha_3,
\]

\[
\pi^S_R < \pi^S_{SL^*} > \pi^L_R \quad \text{when} \quad \alpha_3 < \alpha \leq \alpha_1.
\]

Comparison results of the manufacturer’s profits under different modes are shown in Proposition 3 (i). In S and SL modes, the manufacturer’s profits are the same. When consumer preferences for leasing are higher than \( \alpha_3 \), the manufacturer’s profit is larger in S and SL modes. Otherwise, the manufacturer’s profit is larger in the S mode. Comparison results of the retailer’s profits under different modes are shown in Proposition 3 (ii). In the SL mode, the dual needs of consumers for leasing or purchasing the electronic product are met, and the retailer has the highest profit; when consumer preferences for leasing are less than \( \alpha_3 \), the retailer’s profit is the smallest in the L mode; otherwise, the retailer’s profit is the smallest in the S mode. Results demonstrate that when consumer preferences for leasing are within a certain range, after electronic products retailers’ operational mode transforms from S/L to SL, retailer’s profits increase. However, the manufacturer’s profits also increase only when consumer preferences for leasing exceed a certain threshold within a certain range.

The equilibrium consumer surpluses in S, L, and SL modes are \( CS^{S^*} = \int_{(r^L - r)}^{1} (av - r^L) \, dv = ((1 - c)^2/32), \) \( CS^{L^*} = \int_{(r^L - r)}^{1} (av - r^L) \, dv = ((1 - c)^2/32), \) and \( CS^{SL^*} = \int_{(r^L - r)}^{1} (av - r^L) \, dv = ((1 - c)^2/32), \) respectively.

\[
\alpha_3 = \frac{c^2 - 2r_e + 1 + \sqrt{c^4 - 4c^2r_e - 2c^2 + 8r_e - 4r_e + 1}}{2}.
\]

**Proposition 4.** The equilibrium consumer surpluses have the following properties:

(i) \( CS^{SL^*} > CS^{S^*} > CS^{L^*} \) when \( \alpha_0 \leq \alpha < \alpha_3 \),

(ii) \( CS^{SL^*} > CS^{S^*} = CS^{L^*} \) when \( \alpha = \alpha_3 \),

(iii) \( CS^{SL^*} > CS^{L^*} > CS^{S^*} \) when \( \alpha_3 < \alpha \leq \alpha_1 \).

Proposition 4 shows that in the SL mode, the dual consumer demand for leasing or purchasing the electronic products are met, and the equilibrium consumer surplus is the largest. When consumer preferences for leasing are less than \( \alpha_3 \), the equilibrium consumer surplus is the smallest in the L mode. Otherwise, it is the smallest in the S mode. Therefore, after electronic products retailers’ operational mode transforms from S/L to SL, consumers can also benefit from having the choice of leasing or purchasing.

**5. Numerical Analysis**

To further understand the influence of the different parameters, the following numerical experiments that compare the equilibrium results of the above three alternative operation modes are designed.

5.1. Impact of Consumers’ Preferences for Leasing

Without loss of generality, we set the production cost \( c = 0.3 \), and the residual value of leased-returned electronic product \( r_e = 0.2[33] \). According to Lemma 3 in Section 3.4,
\( \alpha \) is defined to be in the range \([0.65, 0.8]\) (In the SL mode, when \( \alpha < 0.65 \), no customers will choose to rent the products. When \( \alpha > 0.8 \), no customers will choose to purchase the products. Only when \( 0.65 \leq \alpha \leq 0.8 \), some customers will choose to rent the products while other customers will choose to purchase the products.). The results are shown in Figures 3–7.

Figure 3 illustrates the selling, leasing, and wholesale prices of the electronic product under the three alternative operation modes. In the S mode, the wholesale price of the electronic product is higher. In SL and L modes, the wholesale and leasing prices of the electronic product are the same and increase with consumer preferences for leasing. After a retailer’s operational mode transforms from S to SL/L, the manufacturer reduces the wholesale price of the electronic product, and the retailer reduces the selling price. In the S mode, while consumer preference for leasing increases, leasing and selling prices of the electronic product increase, but the leasing and selling price gap narrows.

Figures 4 and 5 illustrate the wholesale demand, purchasing and leasing demands under the three alternative operation modes. In SL and L modes, the wholesale demand for the electronic product is relatively larger, consumer preferences for leasing increases, and the wholesale and leasing demands for the electronic product increase. When consumer preference for leasing is in the range \((0.65, 0.69]\), the purchasing demand for electronic product increases after the retailer’s operational mode transforms from S to SL/L.

Figure 6 shows that in SL and L modes, the manufacturer’s profits are the same. In the S mode, it is more profitable for the manufacturer when consumer preference for leasing is in the range \((0.65, 0.67]\). In SL and L modes, it is more profitable for the manufacturer when consumer preference for leasing is in the range \((0.67, 0.8]\). Figure 7 illustrates that in the SL mode, it is more profitable for the retailer when consumer preferences for leasing is in the range \((0.67, 0.8]\). The retailer can make maximum profits when consumer preference for leasing is 0.65. In the L mode,
the retailer’s profit is the smallest when consumer preference for leasing is in the range \([0.65, 0.67]\). In the S mode, the retailer’s profit is the smallest when consumer preference for leasing is in the range \((0.67, 0.8]\). When consumer preferences for leasing increase, retailer profits decrease in the SL mode and increase in the L mode.

5.2. Impact of the Residual Value. The residual value of the leased-returned electronic product has a significant impact on the retailer’s leasing decision, which is analyzed using numerical examples in this section. Let \(c = 0.3\) and \(\alpha = 0.7\) [25, 33]. In order to ensure the analysis is meaningful, according to Lemma 3, the condition that electronic products selling and purchasing demand exist simultaneously is \(r_e \in [0.18, 0.3]\) (In the SL mode, when \(r_e < 0.18\), no customers will choose to rent the products. When \(r_e > 0.3\), no customers will choose to purchase the products. Only when \(0.18 \leq r_e \leq 0.3\), some customers will choose to rent the products while other customers will choose to purchase the products.), then the equilibrium results under the three alternative operation modes are shown in Figures 8–13.

Figure 8 shows the relationship between the equilibrium wholesale, leasing, and selling prices with the residual value of the leased-returned electronic product. As the residual value increases, the wholesale price in the SL/L mode is also appropriately increased, but the wholesale price in the S mode is not affected. As the residual value increases, the leasing price gradually decreases, the selling prices in the S mode gradually increase, and the gap between leasing and selling prices gradually increases.

Figures 9 and 10 show the relationship between the manufacturer’s equilibrium wholesale demand and the retailer’s equilibrium leasing and purchasing demands with the residual value of leased-returned electronic product, respectively. As the residual value increases, the wholesale demand in the SL/L mode also gradually increases, but the wholesale demand in the mode S is not affected by the
As shown in Figure 13, there is a similar change trend between the consumer surplus and the retailer’s profits, and the calculation shows that the consumer surplus is always half the retailer’s profits. Therefore, the retailer’s three leasing models cannot increase the acquisition of consumer surplus, and the retailer’s increased profits after the operation model transformation depends entirely on the change of market size.

6. Extensions

The previous sections have discussed three alternative operational modes of electronic product retailers when the manufacturer and retailer make the decentralized decision. Results show that it is profitable for the retailer to transform the operational mode from S/L to SL when consumer preferences for leasing are within a certain range because both the consumer lease and purchase requirements are met after the transformation. If the retailer cooperates with the manufacturer and they make a centralized decision, the double marginal effect of the supply chain can be eliminated, the leasing and selling prices can be reduced, the conflicts in the leasing and selling market can be alleviated, and the overall profits of the supply chain can be improved. Samsung has partnered with “Xianghuanji” to launch a mobile phone leasing service in China. In Germany, Samsung and Grover conducted a mobile phone leasing pilot. The following constructs a cooperative selling-leasing model and designs a combined contract coordination pricing mechanism of “wholesale price discounts and revenue sharing” in order to achieve the cooperation between the manufacturer and retailer.

6.1. Cooperative Selling-Leasing Mode (C). In the C mode, the leasing and selling prices are determined with the objective of maximizing the profit in the supply chain. Let the total profit function of the supply chain be \( \pi^C \); thus, the following optimal decision model can be established:

\[
\max_{p^C,r^C} \pi^C = \left(p^C - c_1\right) \left(1 - \frac{p^C - c_2}{1 - \alpha}\right) \left(r^C + r_e - c_3\right) \left(\frac{p^C - r^C}{1 - \alpha} - \frac{r^C}{\alpha}\right)
\]

Lemma 4. In the C mode, when \( \alpha_4 < \alpha \leq \alpha_1 \), the leasing and purchasing markets of the electronic product coexist. The equilibrium leasing and selling prices of the electronic product are \( r^C = (\alpha + c - r_e/2) \) and \( p^C = (1 + c/2) \). The equilibrium profit of the supply chain is \( \pi^C = (2\alpha c - \alpha c^2 + 2acr_e - \alpha^2 - 2ac + c^2 - 2cr_e + r_e^2 + r_e^2 + \alpha/4(1 - \alpha\alpha)) \cdot (\alpha_4 = 1 - (r_e/c)) \).

Proposition 5. In the C mode, there are (i) \( \alpha_4 < \alpha_0 \), (ii) \( p^C < p^C \) and \( \pi^C < \pi^C \) when \( \alpha_4 \leq \alpha < \alpha_0 \) and (iii) \( p^C > p^C \), \( \pi^C > \pi^C \), \( \pi^C > \pi^C \) when \( \alpha_0 \leq \alpha \leq \alpha_1 \).

Proposition 5 states that the range of consumer preferences for leasing in the coexistence of selling-leasing
markets is larger in the C mode than it is in the S mode, which is advantageous for the transformation of the selling-leasing business in the supply chain of electronic products. When \( \alpha_4 \leq \alpha < \alpha_0 \), the leasing market does not exist in the SL mode, while the leasing and selling markets coexist in the C mode. When \( \alpha_0 \leq \alpha \leq \alpha_1 \), the leasing and selling markets coexist in the SL/C mode, the leasing and selling prices of electronic products are lower, and the supply chain’s profits are larger.

6.2. Coordination Contract. In order to make the profits of the supply chain operating in the mode SL equal to the profits when operating in the C mode, a coordinated pricing mechanism of “wholesale price discounts and revenue sharing” combination contract is designed. Let this combination contract signed by electronic products manufacturer and retailer be \( (f^*, \phi^*, w^{SC}, p^{SC}, r^{SC}) \), \( f^* \) is the manufacturer’s wholesale price discount, \( \phi^* \) is the retailer’s revenue sharing ratio, \( w^{SC} \) is the wholesale price, \( p^{SC} \) and \( r^{SC} \) are the selling and leasing prices under the contract, and the revised decision model becomes as follows:

\[
\max_{w^{SC}} \pi_M^{SC} = \left( w^{SC} f - c \right) \left( 1 - \frac{r^{SC}}{1 - \alpha} \right) + \left( p^{SC} - w^{SC} \right) \left( 1 - \frac{p^{SC} - r^{SC}}{1 - \alpha} \right) + \left( p^{SC} + r_e - w^{SC} \right) \left( \frac{p^{SC} - r^{SC}}{1 - \alpha} + r^{SC} \right) \phi^*.
\]

s.t. \( \max_{p^{SC}, r^{SC}} \pi_{R}^{SC} = \left( p^{SC} - w^{SC} \right) \left( 1 - \frac{p^{SC} - r^{SC}}{1 - \alpha} \right) + \left( r^{SC} + r_e - w^{SC} \right) \left( \frac{p^{SC} - r^{SC}}{1 - \alpha} \right) \left( 1 - \phi^* \right) \).

Backward induction is adopted to solve the equilibrium. The optimal response function of the retailer is \( p^{SC*} = (1 + w^{SC} f/2) \) and \( r^{SC*} = (a - r_e + w^{SC} f/2) \). Let \( p^{SC*} = p^{C*} \) and \( r^{SC*} = r^{C*} \), then we obtain that \( w^{SC*} = (a + c + r_e)/2 \) and \( f^* = (2c/\alpha + c + r_e) \), and the coordination of supply chain is realized.

Lemma 5. Under the coordination contract, when \( \alpha_4 \leq \alpha \leq \alpha_1 \), the leasing and purchasing markets of the electronic product coexist. The equilibrium wholesale, leasing and selling prices are \( w^{SC*} = (a + c + r_e)/2 \), \( r^{SC*} = (a + c - r_e)/2 \), and \( p^{SC*} = (1 + c)/2 \), respectively.

Proposition 6. When \( \alpha_4 \leq \alpha < \alpha_0 \) and \( \phi_0 < \phi^* < \phi_1 \), or \( \alpha_0 \leq \alpha \leq \alpha_1 \) and \( \phi_2 < \phi^* < \phi_3 \), in the coordinated pricing mechanism of portfolio contract \( (f^*, \phi^*, w^{SC*}, p^{SC*}, r^{SC*}) \), the profits of manufacturers and distributors can obtain Pareto improvement.
Proposition 6 shows that the design of the “wholesale price discounts and revenue sharing” combination contract increases the range of consumer leasing preferences that the leasing and selling markets coexist, promotes the transformation of the retailer’s rental and sale business, and realizes the Pareto improvement of manufacturer’s and retailer’s profits.

The coordination contracts between the electronic product manufacturer and retailer are analyzed. The coordination contract is further analyzed through numerical simulation. The production cost is set to $c = 0.3$, and the residual value $r_e = 0.2$ [33]. According to Proposition 6, let the consumer preferences for leasing $\alpha$ be in the range of $[0.34, 0.8]$. Under the coordination contracts, the changes of wholesale price discount and selling-leasing revenue sharing ratio are shown in Figures 14 and 15, respectively.

Figure 14 shows that the wholesale price discount parameter of the combination contract decreases as consumer preferences for leasing increase. The areas marked in grey in Figure 15 are the areas where both the manufacturer and the retailer can obtain Pareto improvement through revenue sharing, and the upper bound and interval range of revenue sharing ratio parameters increase with the increase of consumer preferences for leasing. Therefore, under this set of parameters, the coordination contract between the manufacturer and the retailer can achieve Pareto improvement when the revenue sharing ratio is roughly 0.6.

\[ \begin{align*}
\phi_0 &= \frac{\alpha(1-\alpha)(1-c)^2}{2(2\alpha^2 c - \alpha c^2 + 2acr_e - \alpha^2 - 2ac + c^2 - 2cr_e + r_e^2 + \alpha)} \\
\phi_1 &= \frac{\alpha^2 c^2 + 6\alpha^2 c - 5ac^2 + 8acr_e - 3\alpha^2 - 6ac + 4c^2 - 8cr_e + 4r_e^2 + 3a}{4(2\alpha^2 c - \alpha c^2 + 2acr_e - \alpha^2 - 2ac + c^2 - 2cr_e + r_e^2 + \alpha)} \\
\phi_2 &= \frac{(1-\alpha)(c-r_e-\alpha)^2}{2(2\alpha^2 c - \alpha c^2 + 2acr_e - \alpha^2 - 2ac + c^2 - 2cr_e + r_e^2 + \alpha)} \\
\phi_3 &= \frac{3(1-\alpha)(c-r_e-\alpha)^2}{4(2\alpha^2 c - \alpha c^2 + 2acr_e - \alpha^2 - 2ac + c^2 - 2cr_e + r_e^2 + \alpha)}
\end{align*} \]

(7)

Observation 4. (i) As $r_e$ increases, $\alpha_0$, $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ decrease; (ii) Define $\Delta_1 = \alpha_4 - \alpha_0$, and $\Delta_2 = \alpha_1 - \alpha_4$. As $r_e$ increases, $\Delta_1$ and $\Delta_2$ increase.

Observation 4 shows that when $r_e$ increases, preference thresholds ($\alpha_0$, $\alpha_1$) of retailer’s SL mode transformation decrease, preference threshold ($\alpha_2$) of a drop in purchasing demand decreases, threshold ($\alpha_3$) of an increase in the manufacturer’s profit decreases, and preference thresholds ($\alpha_4$, $\alpha_1$) of retailer’s C mode transformation decrease, and $[\alpha_0$, $\alpha_1]$, and $[\alpha_4$, $\alpha_1]$ are preference spaces of retailer’s SL and C mode selection. When $r_e$ increases, preference spaces of retailer’s SL and C mode selection ($\Delta_1, \Delta_2$) increase. In order to make more profit, Proposition 3 shows that when $a_0 \leq \alpha \leq a_1$, the retailer should choose the SL mode, whereas when $\alpha > a_1$, the retailer should choose the L mode. Combined with Proposition 3, Observation 4 can be deeply interpreted as when $r_e$ increases, retailers are more likely to lease electronic products in order to make more profit. This explains why retailers prefer to offer rental service for Apple’s new phones (because their residual value is higher). If retailers lease electronic products with higher residual value, their purchasing demand is more likely to be negatively affected. If the
7. Conclusion

Electronics retailers can turn into purely rental businesses; they can stop selling their goods and limit their services to renting them out to customers. They can also open up the leasing option to consumers while maintaining their original sales business. In this study, considering consumer preferences for leasing, electronics retailer’s three alternative operational modes were compared: pure selling, pure leasing, and hybrid selling-leasing. Our study primarily focused on how consumer preferences for leasing affect retailers’ operational selection. Additionally, the consumer surplus and upstream manufacturer profits were analyzed. Furthermore, a cooperative hybrid selling-leasing mode was examined and a “wholesale price discounts and revenue sharing” coordination contract based on the hybrid selling-leasing mode was designed.

The retailer’s selection was analyzed using a mathematical model and numerical examples. The key findings can be summarized as follows: (1) For the retailer, the pure leasing mode is better than the pure selling mode as long as the consumer preferences for leasing are not very low, but the hybrid selling-leasing mode is always the best when the consumer’s leasing preferences are within a certain range; (2) for the manufacturer, the hybrid selling-leasing mode is always the same as the pure leasing mode, and the leasing mode is always better than the selling mode, as long as the consumer preference for leasing is not too low; (3) the hybrid selling-leasing mode is always beneficial to both retailers and consumers when consumer preferences for leasing are within a certain range because it offers numerous choices to customers. Therefore, when the leasing preference and residual value are not small, the manufacturer has no incentive to prevent the retailer from transforming its operational mode; (4) in the hybrid selling-leasing mode, the wholesale, selling, and leasing prices increase, the manufacturer’s profits increase, but the retailer’s profits decrease, as the consumer preferences for leasing increase; (5) the use of “wholesale price discounts and revenue sharing” contracts can promote the retailer’s hybrid selling-leasing business transformation and achieve Pareto improvement in both the retailer’s and the manufacturer’s profits; (6) the increase of residual value, as well as supply chain cooperation, can promote the retailer’s hybrid selling-leasing business transformation.

On the other hand, it is noteworthy to mention that this study has some limitations. First, a monopoly market with only one manufacturer and one retailer was considered. This setting is in line with the practice that JD controls the majority of online market share of Apple’s iPhones. However, it may be interesting to incorporate the competition between multiple manufacturers and retailers. Second, a retailer that only leases new electronic products in a certain leasing period was considered. This setting is in line with the practice that Dixintong leases only new mobile phones for one year, but other retailers also lease second-hand electronic products and offer leasing services with varying lease terms, so future research can incorporate these factors as well. Third, this study focuses on retailer’s operational mode transformation considering consumer preferences for leasing and residual value of the leased-returned electronic product. Considering how retailers get the residual value is also an interesting problem to investigate through releasing.
through resale, or simply through recycling. Finally, faced with the risk-averse behavior of consumers and retailers, the optimal pricing strategy for retailers is also worth studying.

**Appendix**

**Proof of Lemma 1.** Backward induction is used to derive the equilibrium outcomes. In the S mode, the retailer’s profit is 

\[ \pi_R^S = (p^S - w^S)DS^S_L = (p^S - w^S)(1 - p^S). \]

\( (\partial^2 \pi_R^S / \partial p^S)^2 = -1 < 0, \) so the retailer’s profit is concave in \( p^S. \) According to the first-order conditions, we have \( p^S = (1 + w^S / 2). \)

Substituting \( p^S \) to the manufacturer’s profit function, we have \( \pi_M^S = (1 - w^S)\left(w^S - c\right) \) and \((\partial^2 \pi_M^S / \partial w^S)^2 = -1 < 0, \) so the manufacturer’s profit is concave in \( w^S. \) According to the first-order conditions, we have \( w^S = (1 + c / 2). \) Then the equilibrium outcomes in the S mode can be derived immediately.

**Proof of Lemma 2.** Backward induction is used to derive the equilibrium outcomes. In the L mode, the retailer’s profit is

\[ \pi_R^L = (r^L + r_e - w^L)DS^L = (r^L + r_e - w^L)(1 - (r^L / \alpha)). \]

\( (\partial^2 \pi_R^L / \partial r^L)^2 = -1 < 0, \) so the retailer’s profit is concave in \( r^L. \) According to the first-order conditions, we have \( r^L = (\alpha - r_e + w^L / 2). \) Substituting \( r^L \) to the manufacturer’s profit function, we have \( \pi_M^L = (\alpha + r_c - 2r^L - c) / (2\alpha), \) \((\partial^2 \pi_M^L / \partial w^L)^2 = -1 < 0, \) so the manufacturer’s profit is concave in \( w^L. \) According to the first-order conditions, we have \( w^L = (\alpha + c + r_c / 2). \) Then the equilibrium outcomes in the L mode can be derived immediately.

**Proof of Lemma 3.** Backward induction is used to derive the equilibrium outcomes. In the SL mode, the retailer’s profit is

\[ \pi_R^{SL} = (p^{SL} - w^{SL})DS^{SL}_L + (r^{SL} + r_e - w^{SL})DS^{SL}_S = (p^{SL} - w^{SL})(1 - (p^{SL} - r^{SL} / 1 - \alpha)) \]

\( (\partial^2 \pi_R^{SL} / \partial p^{SL})^2 = -2(1/\alpha - 1) < 0 \) and \( \left| H \right| = \left| \begin{array}{c} -2(1 - \alpha) \vspace{0.1cm} \end{array} \right| > 0. \) So the retailer’s profit is concave in \( p^{SL} \) and \( r^{SL}. \) According to the first-order conditions, we have \( r^{SL} = (\alpha - r_e + w^{SL} / 2) \) and \( p^{SL} = (1 + w^{SL} / 2). \) Substituting \( r^{SL} \) and \( p^{SL} \) to the manufacturer’s profit function, manufacturer’s profit is concave in \( w^{SL}. \) According to the first-order conditions, we have \( w^{SL} = (\alpha + c + r_c / 2). \) Then the equilibrium outcomes in the SL mode can be derived immediately.

**Proof of Observation 1**

\[ \frac{\partial w^*_L}{\partial \alpha} = \frac{1}{4}, \]

\[ \frac{\partial r^*_L}{\partial \alpha} = \frac{1}{4}, \]

\[ \frac{\partial D^*_L}{\partial \alpha} = \frac{1}{4} > 0, \]

\[ \frac{\partial \pi^*_M}{\partial \alpha} = \frac{\alpha + r_e - c}{4\alpha} > 0, \]

\[ \frac{\partial \pi^*_L}{\partial \alpha} = \frac{\alpha + r_e - c}{8\alpha} > 0, \]

\[ (A.1) \]

**Proof of Observation 2.** When \( \alpha_0 \leq \alpha \leq \alpha_1, \) \((\partial D^{SL*} / \partial \alpha) = (c - r_e / 4\alpha^2) > 0, \) \((\partial w^{SL*} / \partial \alpha) = (1 - \alpha / 4\alpha^2) > 0, \) \((\partial \pi^*_M / \partial \alpha) = (1 - \alpha^2 / 4\alpha^4) > 0, \) and \((\partial \pi^*_M / \partial \alpha) = (1 - \alpha^2 / 4\alpha^4) > 0, \) then \( \pi_{M*}^S = (w^{SL*} - c)D^{SL*} \), then \( w^{SL*} \) and \( D^{SL*} \) increase with \( \alpha, \) so \( \pi_{M*}^S \) increases with \( \alpha \) because \((\partial \pi_{M*}^S / \partial \alpha) = (c - r_e / 4\alpha^2) > 0, \) \((\partial w^{SL*} / \partial \alpha) = (1 - \alpha / 4\alpha^2) > 0, \) \((\partial \pi^*_M / \partial \alpha) = (1 - \alpha^2 / 4\alpha^4) > 0, \) and \((\partial \pi^*_M / \partial \alpha) = (1 - \alpha^2 / 4\alpha^4) > 0, \) then \( \pi_{M*}^S = (w^{SL*} - c)D^{SL*} \), then \( w^{SL*} \) and \( D^{SL*} \) increase with \( \alpha, \) so \( \pi_{M*}^S \) increases with \( \alpha \) because \((\partial \pi_{M*}^S / \partial \alpha) = (c - r_e / 4\alpha^2) > 0, \) \((\partial w^{SL*} / \partial \alpha) = (1 - \alpha / 4\alpha^2) > 0, \) \((\partial \pi^*_M / \partial \alpha) = (1 - \alpha^2 / 4\alpha^4) > 0, \) and \((\partial \pi^*_M / \partial \alpha) = (1 - \alpha^2 / 4\alpha^4) > 0, \) then \( \pi_{M*}^S = (w^{SL*} - c)D^{SL*} \), then \( w^{SL*} \) and \( D^{SL*} \) increase with \( \alpha, \) so \( \pi_{M*}^S \) increases with \( \alpha \).

**Proof of Observation 3**
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(i) As shown in Lemma 1 and Proposition 1, \( r_{\text{SL}}^* = \frac{(a + r_e + c/2)}{2}, w_{\text{SL}}^* - w_{\text{S}}^* = (a + r_e + c/2) - (1 + c/2) = (a + r_e - 1/2), \) and because \( a \leq a_1 = 1 - r_e, \) we have \( a + r_e - 1/2 \leq 0, \) so \( w_{\text{SL}}^* \geq w_{\text{S}}^* \).

(ii) \( \frac{\partial D_{\text{SL}}^*}{\partial r_e} = 1 > 0, \)

(iii) \( D_{\text{SL}}^* - D_{\text{S}}^* = (1 - c - 2r_e - (1 + c)a/4(1 - a), \) and let \( 1 + c - 2r_e - (1 + c)a = 0, \) so \( a_1 = (1 + c - 2r_e/1 + c), \) \( a_0 < a_2 < a_1, \) can be obtained by comparison; if \( a_0 < a < a_2, \) then \( D_{\text{SL}}^* > D_{\text{S}}^* \); if \( a = a_2, \) then \( D_{\text{SL}}^* = D_{\text{S}}^* \); if \( a_2 \leq a \leq a_1, \) then \( D_{\text{SL}}^* < D_{\text{S}}^* \).

Proof of Proposition 1

(i) \( \pi_{\text{SL}}^* - \pi_{\text{S}}^* = (a^2 + (-c^2 + 2r_e - 1)a^0 + (c - r_e)^2/8a), \) and define \( a^2 + (c^2 - 2r_e - 1)a^0 + (c - r_e)^2 = 0, \) so \( a_1 = (c^2 - 2r_e + 1 + \sqrt{c^4 - 4c^2r_e - 2c + 8r_e - 4r_e + 1/2}, \) so if \( a_0 < a < a_1, \) then \( \pi_{\text{S}}^* > \pi_{\text{SL}}^* \); if \( a = a_1, \) then \( \pi_{\text{S}}^* = \pi_{\text{SL}}^* \); and if \( a_1 \leq a \leq a_2, \) then \( \pi_{\text{S}}^* < \pi_{\text{SL}}^* = \pi_{\text{M}}^* \).

Proof of Proposition 4. The proof of Proposition 4 is the same as the proof of Proposition 3.

Proof of Proposition 5

(i) \( \pi_{\text{S}}^* - \pi_{\text{S}}^* = (3a^2 + (c^2 + 6r_e - 6)a^2 + (3r^2 + 2r_c - 6r_e - 2c^2 + 3)a + (c - r_e)^2/16a(1 - a), \) and \( \pi_{\text{S}}^* \) decreases as \( a \) increases; when \( a = 1 - r_e, \) \( \pi_{\text{S}}^* \) takes the minimum value, and when \( a = 1 - r_e, \) \( \pi_{\text{S}}^* = \pi_{\text{SL}}^* = (r_e^2(1 - c^2)/16a(1 - a), \) so \( \pi_{\text{S}}^* \geq \pi_{\text{S}}^* \); \( \pi_{\text{S}}^* - \pi_{\text{S}}^* = (a^2 + (c^2 - 2r_e - 1)a^0 + (c - r_e)^2/16a), \) define \( a^2 + (c^2 + 2r_e - 1)a^0 + (c - r_e)^2 = 0, \) so \( a_0 = (c^2 - 2r_e + 1 + \sqrt{c^4 - 4c^2r_e - 2c^2 + 8r_e - 4r_e + 1/2}, \) so when \( a_0 \leq a < a_1, \) then \( \pi_{\text{S}}^* > \pi_{\text{S}}^* > \pi_{\text{S}}^* \); when \( a = a_1, \) then \( \pi_{\text{S}}^* > \pi_{\text{S}}^* > \pi_{\text{S}}^* \), when \( a_1 < a \leq a_2, \) then \( \pi_{\text{S}}^* > \pi_{\text{S}}^* > \pi_{\text{S}}^* \).

Proof of Proposition 6. Under the coordination mechanism \((f^*, \phi^*, w_{\text{SL}}^*, p_{\text{S}}^*, r_{\text{S}}^*), \) the profit of the electronic product supply chain reaches the level of centralized decision-making. When \( a_1 < a \leq a_0, \) leasing markets that do not exist at the time of decentralized decision-making arise at the time of centralized decision-making. Therefore, in order to facilitate the selling-leasing cooperation between the retailer and the manufacturer, the combination contract shall
guaranteed $\pi_{M}^{SC} > \pi_{R}^{SC}$ and $\pi_{R}^{SC} > \pi_{R}^{SC}$. From this calculation, the value range of available $\phi$ is $\phi_{0} < \phi < \phi_{3}$. When $\alpha_{0} \leq \alpha \leq \alpha_{1}$, the portfolio contract is approximately guaranteed to $\pi_{M}^{SC} > \pi_{M}^{SC}$ and $\pi_{R}^{SC} > \pi_{R}^{SC}$. From this calculation, the value range of available $\phi$ is $\phi_{0} < \phi < \phi_{3}$.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Data Availability

The simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was funded by the National Social Science Foundation of China (Grant no. 18BJY009).

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