Research Article

Partial Randomized Response Model for Simultaneous Estimation of Means of Two Sensitive Variables

Ghulam Narjis [ID],[1] Javid Shabbir [ID],[1,2] and Ronald Onyango [ID][3]

[1]*Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan*
[2]*Department of Statistics, University of Wah, Wah Cantt, Islamabad, Pakistan*
[3]*Department of Applied Statistics, Financial Mathematics and Actuarial Science,*
Jaramogi Oginga Odinga University of Science and Technology, Bondo, Kenya

Correspondence should be addressed to Ronald Onyango; assangaronald@gmail.com

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In this study, a new partial randomized response model (RRM) has been proposed for estimating the population mean of two quantitative sensitive variables simultaneously. The utility of proposed model under stratification is also explored. The efficiency comparisons of the proposed model under simple and stratified random sampling are carried out numerically. A real data set was collected through direct questioning, proposed partial RRM and competitor randomized device from the students of statistics and animal sciences departments of Quaid-I-Azam University Islamabad, Pakistan. The performance of the proposed partial RRM is better than competitor RRM under simple and stratified random sampling.

1. Introduction

The social survey is one of leading mechanisms to obtain reliable data on attitudes, behaviors, and opinions of the human population. Sometimes, the facts about the individuals are inaccessible to the investigators due to social stigma, such facts are considered as sensitive information. When asked directly, respondents may consciously or unconsciously provide incorrect information on stigmatizing characteristics. To reduce the bias and to procure reliable data, Warner [1] developed a randomized response model (RRM) to estimate the population proportion of a sensitive attribute. In Warner [1]'s model, a randomly selected proportion P of respondents are asked the sensitive question, and the remaining proportion $(1 - P)$ of respondents are asked complement of the sensitive question. The researcher does not know whether the respondents answered the sensitive or insensitive question. Greenberg et al. [2] extended the Warner's idea for mean estimation of quantitative sensitive variables. Some other developments for the estimation of mean are due to Eichhorn and Hayre [3]; Bar-Lev et al. [4]; Gupta et al. [5]; Gupta et al. [6] Hussain et al.

[7]; Singh et al. [8]; Singh and Suman [9]; Lee and Hong [10]; Narjis and Shabbir [11, 12]; and Muneer et al. [13].

Scrambled randomized response models are built on the idea of obtaining masked rather than actual responses. Masking can be achieved by adding to, subtracting from, or multiplying a random component to actual responses. Scrambled response models may be categorized as full, partial, and optional models. In full RRM, all respondents are requested to provide the scrambled response, whereas, in partial RRM, a randomly selected group of respondents are requested to provide the truthful response and remaining are requested to provide the scrambled response. In optional RRM, respondents are requested to provide the scrambled response if he/she considers the question sensitive, and truthful response if he/she considers the question to be nonsensitive. Mangat and Singh [14] and Gupta et al. [15] introduced the partial RRM and the optional RRM, respectively. The purpose of all RRMs are to protect privacy and increase cooperation.

Researchers in the field of social, medical, and environmental sciences have well documented the situations where they may be interested to estimate the two dependent

sensitive characters at the same time. For example, a researcher may have interest in estimating the proportion of population having income greater than a specific amount weighted according to whether they are tax evaders or not. Another example may be to estimate the proportion of gamblers who are also involved in robbery. Or one may estimate the proportion of induced abortion among females who have pre-marriage sexual relations. Christofides [16] introduced the RR model to estimate the proportions of two dependent sensitive attributes at the same time. Studies on estimation of two dependent sensitive attributes have been reported by Lee et al. [17]; Batool and Shabbir [18]; Ewemooje and Amahia [19]; Ewemooje [20]; and Ewemooje et al. [21].

Similarly, the surveys related to household spending/ expenditures that present households income and expenditure on different commodities comprises sensitive questions. For example, an economist may have interest in estimating the average difference between amount spent on alcoholic beverages and tobacco items and on food and nonfood items. Or the interest may be in estimation of people's actual income and the income reported in their tax return. However, the choice of RR models are very limited in aforementioned situations where one may need to estimate the mean or average of two quantitative sensitive variables at the same time. Recently, Ahmed et al. [22] introduced full scrambled RR model for simultaneous estimation of means of two quantitative sensitive variables, Hussain and Murtaza [23] had written corrigendum on Ahmed et al. [22] and provides the correct expression of $E(S_1^3)$, $E(S_2^3)$, $\sigma_{Z_{A_2}}^2$, and $\sigma_{Z_{A_1}Z_{A_2}}$, respectively.

The notions and terminology are given in Sections 1.1 and 1.2.

1.1. Notations and Terminology under Simple Random Sampling.

Suppose a sample s of size n is drawn under simple random sampling with replacement (SRSWR) from a finite population $\Omega = \{1, 2, 3, \ldots\ldots, N\}$. Let Y_{1i} and Y_{2i} be two quantitative sensitive variables of interest with unknown mean and variance, which we wish to estimate. Assume S_1 and S_2 are two scrambling variables independent of both quantitative sensitive variables and with each other, the distribution of scrambling variables are known.

Let $E(S_1) = \theta_1 = e_{10}$, $E(S_2) = \theta_2 = e_{01}$, $V(S_1) = \delta_{20}$, $V(S_2) = \delta_{02}$, $\text{Cov}(S_1, S_2) = 0$, $\delta_{ab} = E[S_1 - \theta_1]^a [S_2 - \theta_2]^b$, $E(Y_{1i}) = \mu_{y_1}$, $E(Y_{2i}) = \mu_{y_2}$, $V(Y_{1i}) = \sigma_{y_1}^2$, $V(Y_{2i}) = \sigma_{y_2}^2$, and $\sigma_{y_1 y_2} = E[Y_{1i} - \mu_{y_1}][Y_{2i} - \mu_{y_2}]$, where, a and b are non-negative integers.

Lemma 1. *The moments of order four or less of scrambling variables S_1 and S_2, the expression of $E(S_1^a S_2^b)$, a and b being non-negative integers with $(a + b) \leq 4$, are given by*

$$E(S_1^2) = \theta_1^2 + \delta_{20} = e_{20}, \tag{1}$$

$$E(S_1^3) = \delta_{30} + 3\delta_{20}\theta_1 + \theta_1^3 = e_{30}, \tag{2}$$

$$E(S_1^4) = \delta_{40} + 4\delta_{30}\theta_1 + 6\delta_{20}\theta_1^2 + \theta_1^4 = e_{40}, \tag{3}$$

$$E(S_2^2) = \theta_2^2 + \delta_{02} = e_{02}, \tag{4}$$

$$E(S_2^3) = \delta_{03} + 3\delta_{02}\theta_2 + \theta_2^3 = e_{03}, \tag{5}$$

$$E(S_2^4) = \delta_{04} + 4\delta_{03}\theta_2 + 6\delta_{02}\theta_2^2 + \theta_2^4 = e_{04}, \tag{6}$$

$$E(S_1 S_2) = \theta_1 \theta_2 = e_{11}, \tag{7}$$

$$E(S_1^2 S_2) = (\theta_1^2 + \delta_{20})\theta_2 = e_{21}, \tag{8}$$

$$E(S_1 S_2^2) = \theta_1(\theta_2^2 + \delta_{02}) = e_{12}, \tag{9}$$

$$E(S_1^2 S_2^2) = (\theta_1^2 + \delta_{20})(\theta_2^2 + \delta_{02}) = e_{22}, \tag{10}$$

$$E(S_1^3 S_2) = (\delta_{30} + 3\delta_{20}\theta_1 + \theta_1^3)\theta_2 = e_{31}, \tag{11}$$

$$E(S_1 S_2^3) = \theta_1(\delta_{03} + 3\delta_{02}\theta_2 + \theta_2^3) = e_{13}. \tag{12}$$

1.2. Notations and Terminology under Stratified Random Sampling.

Consider a finite population $\Omega = \{1, 2, 3, \ldots\ldots, N\}$ which are partitioned into L homogeneous subgroups called strata, such that the hth stratum consists of N_h units, where $h = 1, 2, \ldots, L$ and $\sum_{h=1}^{L} N_h = N$. A sample of size n_h from N_h stratum is drawn by using SRSWR such that $\sum_{h=1}^{L} n_h = n$. Let $Y_{1_{hi}}$ and $Y_{2_{hi}}$ be the ith population values of two quantitative sensitive variables in the hth stratum, $i = 1, 2, \ldots, N_h$ and $W_h = N_h/N$ is the known proportion of population units falling in the hth stratum. Similarly, S_{1_h} and S_{2_h} are two independent scrambling variables with known means and variances.

Let $E(S_{1_h}) = \theta_{1_h} = e_{10_h}$, $E(S_{2_h}) = \theta_{2_h} = e_{01_h}$, $V(S_{1_h}) = \delta_{20_h}$, $V(S_{2_h}) = \delta_{02_h}$, $\text{Cov}(S_{1_h}, S_{2_h}) = 0$, $\delta_{ab_h} = E[S_{1_h} - \theta_{1_h}]^a [S_{2_h} - \theta_{2_h}]^b$, $E(Y_{1_{hi}}) = \mu_{y_{1_h}}$, $E(Y_{2_{hi}}) = \mu_{y_{2_h}}$, $V(Y_{1_{hi}}) = \sigma_{y_{1_h}}^2$, $V(Y_{2_{hi}}) = \sigma_{y_{2_h}}^2$ and $\sigma_{y_{1_h} y_{2_h}} = E[Y_{1_{hi}} - \mu_{y_{1_h}}][Y_{2_{hi}} - \mu_{y_{2_h}}]$, where, a and b are non-negative integers.

Lemma 2. *The moments of order four or less of scrambling variables S_{1_h} and S_{2_h}, the expression of $E(S_{1_h}^a S_{2_h}^b)$, a and b being non-negative integers with $(a + b) \leq 4$, are given by*

$$E(S_{1_h}^2) = \theta_{1_h}^2 + \delta_{20_h} = e_{20_h}, \tag{13}$$

$$E(S_{1_h}^3) = \delta_{30_h} + 3\delta_{20_h}\theta_{1_h} + \theta_{1_h}^3 = e_{30_h}, \tag{14}$$

$$E(S_{1_h}^4) = \delta_{40_h} + 4\delta_{30_h}\theta_{1_h} + 6\delta_{20_h}\theta_{1_h}^2 + \theta_{1_h}^4 = e_{40_h}, \tag{15}$$

$$E(S_{2_h}^2) = \theta_{2_h}^2 + \delta_{02_h} = e_{02_h}, \tag{16}$$

$$E(S_{2_h}^3) = \delta_{03_h} + 3\delta_{02_h}\theta_{2_h} + \theta_{2_h}^3 = e_{03_h}, \tag{17}$$

$$E\left(S_{2_h}^4\right) = \delta_{04_h} + 4\delta_{03_h}\theta_{2_h} + 6\delta_{02_h}\theta_{2_h}^2 + \theta_{2_h}^4 = e_{04_h}, \tag{18}$$

$$E\left(S_{1_h}S_{2_h}\right) = \theta_{1_h}\theta_{2_h} = e_{11_h}, \tag{19}$$

$$E\left(S_{1_h}^2 S_{2_h}\right) = \left(\theta_{1_h}^2 + \delta_{20_h}\right)\theta_{2_h} = e_{21_h}, \tag{20}$$

$$E\left(S_{1_h}S_{2_h}^2\right) = \theta_{1_h}\left(\theta_{2_h}^2 + \delta_{02_h}\right) = e_{12_h}, \tag{21}$$

$$E\left(S_{1_h}^2 S_{2_h}^2\right) = \left(\theta_{1_h}^2 + \delta_{20_h}\right)\left(\theta_{2_h}^2 + \delta_{02_h}\right) = e_{22_h}, \tag{22}$$

$$E\left(S_{1_h}^3 S_{2_h}\right) = \left(\delta_{30_h} + 3\delta_{20_h}\theta_{1_h} + \theta_{1_h}^3\right)\theta_{2_h} = e_{31_h}, \tag{23}$$

$$E\left(S_{1_h}S_{2_h}^3\right) = \theta_{1_h}\left(\delta_{03_h} + 3\delta_{02_h}\theta_{2_h} + \theta_{2_h}^3\right) = e_{13_h}. \tag{24}$$

The interest of researchers is always to investigate the true response from a population. To attain this desire, Mangat and Singh [14] proposed an ingenious partial RRT model by injecting an element of truthful responses into the Warner [1]'s model. Gupta and Thornton [24] described the partial RRT model for quantitative variables and many others also give the improvement in this area. It is important to note that almost all such types of RRT models can only estimate one quantitative sensitive variable at a time. So, in this article keeping in mind the desire of researchers, we propose an additive partial RRT model to estimate two quantitative sensitive variables simultaneously. The proposed model is an extension of Ahmed et al. [22]'s model under simple and stratified random sampling. The basic purpose of this study is to obtain truthful responses from some proportion of people and increase efficiency.

This paper is organized as follows: In Section 2, we give some existing RRT models. In Section 3, we introduce a partial randomized response model under SRSWR and numerically compare it with Ahmed et al. [22] model. In Section 4, we present a partial randomized response model under stratification and numerically compare it with stratified model of Ahmed et al. [22]. In Section 5, an application of real life data is given and comparison of proposed partial RRM is made with Ahmed et al. [22] model on basis of direct response technique. Finally, Section 6 provides a conclusion.

2. RRM in Literature

In this section, we consider the following existing RRMs.

2.1. Model under Simple Random Sampling. Ahmed et al. [22] proposed an additive and multiplicative model for estimation of mean of two quantitative sensitive variables simultaneously. Two responses are taken from each respondent, a scrambled response and fake response. The scrambled response from the ith respondent is obtained as

$$Z_{A_{1i}} = S_1 Y_{1i} + S_2 Y_{2i}. \tag{25}$$

For the second response, each respondent is requested to rotate a spinner and respond accordingly as: the respondent is asked to report the value of scrambling variable S_1 when the pointer lands in a shaded area, otherwise report the value of scrambling variable S_2. Let P be the proportion of shaded area and $(1 - P)$ be the proportion of non-shaded area of the spinner. Thus, the second response from the ith respondent, is given by

$$Z_i = \begin{cases} S_1 & \text{with probability } P, \\ S_2 & \text{with probability } (1 - P). \end{cases} \tag{26}$$

From equations (25) and (26), generate the response $Z_{A_{2i}}$ as follows:

$$Z_{A_{2i}} = Z_i Z_{A_{1i}} = \begin{cases} S_1^2 Y_{1i} + S_1 S_2 Y_{2i} & \text{with robability } P, \\ S_1 S_2 Y_{1i} + S_2^2 Y_{2i} & \text{with probability } (1 - P). \end{cases} \tag{27}$$

The unbiased estimators of the population means μ_{y_1} and μ_{y_2}, are given by

$$\widehat{\mu}_{y_{A_1}} = \frac{\{Pe_{11} + (1 - P)e_{02}\}\overline{Z}_{A_1} - e_{01}\overline{Z}_{A_2}}{(1 - P)\delta_{02}e_{10} - P\delta_{20}e_{01}}, \tag{28}$$

and

$$\widehat{\mu}_{y_{A_2}} = \frac{e_{10}\overline{Z}_{A_2} - \{Pe_{20} + (1 - P)e_{11}\}\overline{Z}_{A_1}}{(1 - P)\delta_{02}e_{10} - P\delta_{20}e_{01}}. \tag{29}$$

The variance of the proposed estimators $\widehat{\mu}_{y_{A_1}}$ and $\widehat{\mu}_{y_{A_2}}$, are given by

$$V\left(\widehat{\mu}_{y_{A_1}}\right) = \frac{\{Pe_{11} + (1 - P)e_{02}\}^2\sigma_{Z_{A_1}}^2 + e_{01}^2\sigma_{Z_{A_2}}^2 - 2\{Pe_{11} + (1 - P)e_{02}\}e_{01}\sigma_{Z_{A_1}Z_{A_2}}}{n[(1 - P)\delta_{02}e_{10} - P\delta_{20}e_{01}]^2}, \tag{30}$$

and

$$V\left(\widehat{\mu}_{y_{A_2}}\right) = \frac{e_{10}^2\sigma_{Z_{A_2}}^2 + \{Pe_{20} + (1 - P)e_{11}\}^2\sigma_{Z_{A_1}}^2 - 2e_{10}\{Pe_{20} + (1 - P)e_{11}\}\sigma_{Z_{A_1}Z_{A_2}}}{n[(1 - P)\delta_{02}e_{10} - P\delta_{20}e_{01}]^2}, \tag{31}$$

where

$$\sigma_{Z_{A_1}}^2 = \left(\mu_{y_1}^2 + \sigma_{y_1}^2\right)\delta_{20} + \left(\mu_{y_2}^2 + \sigma_{y_2}^2\right)\delta_{02} + \sigma_{y_1}^2 e_{10}^2 + \sigma_{y_2}^2 e_{01}^2 + 2\sigma_{y_1 y_2} e_{11}, \tag{32}$$

$$\sigma_{Z_{A_2}}^2 = \left(\mu_{y_1}^2 + \sigma_{y_1}^2\right)\{Pe_{40} + (1-P)e_{22}\} + \left(\mu_{y_2}^2 + \sigma_{y_2}^2\right)\{Pe_{22} + (1-P)e_{04}\}$$
$$+ 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{Pe_{31} + (1-P)e_{13}\} - \left[\{Pe_{20} + (1-P)e_{11}\}\mu_{y_1} + \{Pe_{11} + (1-P)e_{02}\}\mu_{y_2}\right]^2, \tag{33}$$

and

$$\sigma_{Z_{A_1} Z_{A_2}} = \left(\mu_{y_1}^2 + \sigma_{y_1}^2\right)\{Pe_{30} + (1-P)e_{21}\} + \left(\mu_{y_2}^2 + \sigma_{y_2}^2\right)\{Pe_{12} + (1-P)e_{03}\}$$
$$+ 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{Pe_{21} + (1-P)e_{12}\} - \left[\mu_{y_1}e_{10} + \mu_{y_2}e_{01}\right]\left[\{Pe_{20} + (1-P)e_{11}\}\mu_{y_1} + \{Pe_{11} + (1-P)e_{02}\}\mu_{y_2}\right]. \tag{34}$$

2.2. Model under Stratified Random Sampling.

From Ahmed et al. [22], in stratified random sampling, the scrambled response from the ith respondent of the hth stratum is obtained as

$$Z_{A_{1_{hi}}} = S_{1_h}Y_{1_{hi}} + S_{2_h}Y_{2_{hi}}. \tag{35}$$

The second response from the ith respondent of the hth stratum is obtained as

$$Z_{hi} = \begin{cases} S_{1_h} & \text{with probability } P_h, \\ S_{2_h} & \text{with probability } (1 - P_h). \end{cases} \tag{36}$$

The unbiased estimators of the population means μ_{y_1} and μ_{y_2} are given by

$$\widehat{\mu}_{y_{A_1 (st)}} = \sum_{h=1}^{L} W_h \frac{\{P_h e_{11_h} + (1 - P_h)e_{02_h}\}\overline{Z}_{A_{1_h}} - e_{01_h}\overline{Z}_{A_{2_h}}}{(1 - P_h)\delta_{02_h} e_{10_h} - P\delta_{20_h} e_{01_h}}, \tag{37}$$

and

$$\widehat{\mu}_{y_{A_2 (st)}} = \sum_{h=1}^{L} W_h \frac{e_{10_h}\overline{Z}_{A_{2_h}} - \{P_h e_{20_h} + (1 - P_h)e_{11_h}\}\overline{Z}_{A_{1_h}}}{(1 - P_h)\delta_{02_h} e_{10_h} - P_h\delta_{20_h} e_{01_h}}. \tag{38}$$

The variance of the proposed estimators $\widehat{\mu}_{y_{A_1 (st)}}$ and $\widehat{\mu}_{y_{A_2 (st)}}$ are given by

$$V\left(\widehat{\mu}_{y_{A_1 (st)}}\right) = \sum_{h=1}^{L} W_h^2 \frac{\sigma_{A_{1_h}}^2}{n_h}, \tag{39}$$

and

$$V\left(\widehat{\mu}_{y_{A_2 (st)}}\right) = \sum_{h=1}^{L} W_h^2 \frac{\sigma_{A_{2_h}}^2}{n_h}, \tag{40}$$

where

$$\sigma_{A_{1_h}}^2 = \frac{\{P_h e_{11_h} + (1 - P_h)e_{02_h}\}^2 \sigma_{Z_{A_{1_h}}}^2 + e_{01_h}^2 \sigma_{Z_{A_{2_h}}}^2 - 2\{P_h e_{11_h} + (1 - P_h)e_{02_h}\}e_{01_h}\sigma_{Z_{A_{1_h}} Z_{A_{2_h}}}}{\left[(1 - P_h)\delta_{02_h} e_{10_h} - P\delta_{20_h} e_{01_h}\right]^2}, \tag{41}$$

and

$$\sigma_{A_{2_h}}^2 = \frac{e_{10_h}^2 \sigma_{Z_{A_{2_h}}}^2 + \{P_h e_{20_h} + (1 - P_h)e_{11_h}\}^2 \sigma_{Z_{A_{1_h}}}^2 - 2e_{10_h}\{P_h e_{20_h} + (1 - P_h)e_{11_h}\}\sigma_{Z_{A_{1_h}} Z_{A_{2_h}}}}{\left[(1 - P_h)\delta_{02_h} e_{10_h} - P\delta_{20_h} e_{01_h}\right]^2}, \tag{42}$$

$$\sigma_{Z_{A_{1_h}}}^2 = \left(\mu_{y_{1_h}}^2 + \sigma_{y_{1_h}}^2\right)\delta_{20_h} + \left(\mu_{y_{2_h}}^2 + \sigma_{y_{2_h}}^2\right)\delta_{02_h} + \sigma_{y_{1_h}}^2 e_{10_h}^2 + \sigma_{y_{2_h}}^2 e_{01_h}^2 + 2\sigma_{y_{1_h} y_{2_h}} e_{11_h}, \tag{43}$$

$$\sigma_{Z_{A_{2_h}}}^2 = \left(\mu_{y_{1_h}}^2 + \sigma_{y_{1_h}}^2\right)\{P_h e_{40_h} + (1 - P_h)e_{22_h}\} + \left(\mu_{y_{2_h}}^2 + \sigma_{y_{2_h}}^2\right)\{P_h e_{22_h} + (1 - P_h)e_{04_h}\}$$
$$+ 2\left(\mu_{y_{1_h}}\mu_{y_{2_h}} + \sigma_{y_{1_h} y_{2_h}}\right)\{P_h e_{31_h} + (1 - P_h)e_{13_h}\} - \left[\{P_h e_{20_h} + (1 - P_h)e_{11_h}\}\mu_{y_{1_h}} + \{P_h e_{11_h} + (1 - P_h)e_{02_h}\}\mu_{y_{2_h}}\right]^2, \tag{44}$$

and

$$\sigma_{Z_{A_{1_h}} Z_{A_{2_h}}} = \left(\mu_{y_{1_h}}^2 + \sigma_{y_{1_h}}^2\right)\{P_h e_{30_h} + (1 - P_h)e_{21_h}\} + \left(\mu_{y_{2_h}}^2 + \sigma_{y_{2_h}}^2\right)\{P_h e_{12_h} + (1 - P_h)e_{03_h}\} + 2\left(\mu_{y_{1_h}}\mu_{y_{2_h}} + \sigma_{y_{1_h} y_{2_h}}\right)$$
$$\cdot \{P_h e_{21_h} + (1 - P_h)e_{12_h}\} - \left[\mu_{y_{1_h}}e_{10_h} + \mu_{y_{2_h}}e_{01_h}\right]\left[\{P_h e_{20_h} + (1 - P_h)e_{11_h}\}\mu_{y_{1_h}} + \{P_h e_{11_h} + (1 - P_h)e_{02_h}\}\mu_{y_{2_h}}\right]. \tag{45}$$

3. Proposed Partial RRM under Simple Random Sampling

In this section, we propose a partial randomized response model for simultaneous estimation of means of two quantitative sensitive variables. In the proposed partial RRM, each respondent selected in the ith sample is requested for two responses by using two randomized response (RR) devices. The RR Device I provides the scramble and true response of sensitive variables, whereas the RR Device II provides the fake response, that is free from the sensitive variables.

The RR Device I, bearing two types of statements:

(i) Report the additive true value of both sensitive variables, say $(Y_{1i} + Y_{2i})$ with probability T and

(ii) Report the scrambled response as, $(S_1 Y_{1i} + S_2 Y_{2i})$ with probability $(1 - T)$. Mathematically, each respondent is requested to report the response Z_{1i} as

$$Z_{1i} = \begin{cases} Y_{1i} + Y_{2i} & \text{with probability } T, \\ S_1 Y_{1i} + S_2 Y_{2i} & \text{with probability } (1 - T). \end{cases} \quad (46)$$

The partial randomized response Z_{1i} in the ith sample is given by

$$Z_{1i} = \beta_i (Y_{1i} + Y_{2i}) + (1 - \beta_i)(S_1 Y_{1i} + S_2 Y_{2i}). \quad (47)$$

For the second response, each respondent is requested to use RR Device 2 which is same as equation (26); thus, the fake response Z_i from equation (26) in the ith sample is given by

$$Z_i = \gamma_i S_{1i} + (1 - \gamma_i) S_{2i}, \quad (48)$$

where γ_i and β_i are Bernoulli random variables with means P and T, respectively, which are the known parameters.

From equations (47) and (48), we generate response Z_{2i} as follows:

$$Z_{2i} = Z_i Z_{1i}$$

$$= \begin{cases} S_1 Y_{1i} + S_1 Y_{2i} & \text{with probability } PT, \\ S_1^2 Y_{1i} + S_1 S_2 Y_{2i} & \text{with probability } (1 - T)P, \\ S_2 Y_{1i} + S_2 Y_{2i} & \text{with probability } T(1 - P), \\ S_1 S_2 Y_{1i} + S_2^2 Y_{2i} & \text{with probability } (1 - P)(1 - T). \end{cases} \quad (49)$$

The generated response Z_{2i} in the ith sample is given by

$$Z_{2i} = \gamma_i \beta_i (S_1 Y_{1i} + S_1 Y_{2i}) + (1 - \beta_i)\gamma_i (S_1^2 Y_{1i} + S_1 S_2 Y_{2i})$$
$$+ (1 - \gamma_i)\beta_i (S_2 Y_{1i} + S_2 Y_{2i})$$
$$+ (1 - \gamma_i)(1 - \beta_i)(S_1 S_2 Y_{1i} + S_2^2 Y_{2i}). \quad (50)$$

Taking expected values on both sides of equations (47) and (50), we have

$$E(Z_{1i}) = T(\mu_{y_1} + \mu_{y_2}) + (1 - T)(e_{10}\mu_{y_1} + e_{01}\mu_{y_2}), \quad (51)$$

and

$$\begin{aligned} E(Z_{2i}) = {} & PT\{e_{10}\mu_{y_1} + e_{10}\mu_{y_2}\} \\ & + (1 - T)P\{e_{20}\mu_{y_1} + e_{11}\mu_{y_2}\} \\ & + (1 - P)T\{e_{01}\mu_{y_1} + e_{01}\mu_{y_2}\} \\ & + (1 - P)(1 - T)\{e_{11}\mu_{y_1} + e_{02}\mu_{y_2}\}. \end{aligned} \quad (52)$$

From equations (51) and (52), by the method of moments, we have

$$A_{11}\widehat{\mu}_{y_1} + A_{12}\widehat{\mu}_{y_2} = \overline{Z}_1, \quad (53)$$

and

$$A_{21}\widehat{\mu}_{y_1} + A_{22}\widehat{\mu}_{y_2} = \overline{Z}_2, \quad (54)$$

where

$$A_{11} = T + (1 - T)e_{10}$$
$$A_{12} = T + (1 - T)e_{01}$$
$$A_{21} = PTe_{10} + (1 - T)Pe_{20} + (1 - P)Te_{01} + (1 - P)(1 - T)e_{11}$$
$$A_{22} = PTe_{10} + (1 - T)Pe_{11} + (1 - P)Te_{01} + (1 - P)(1 - T)e_{02}.$$

Solving equations (53) and (54) by using Cramer's rule, we have unbiased estimators of μ_{y_1} and μ_{y_2}, respectively, given by

$$\widehat{\mu}_{y_1} = \frac{A_{22}\overline{Z}_1 - A_{12}\overline{Z}_2}{(1 - T)[(1 - P)\delta_{02}A_{11} - P\delta_{20}A_{12}]}, \quad (55)$$

and

$$\widehat{\mu}_{y_2} = \frac{A_{11}\overline{Z}_2 - A_{21}\overline{Z}_1}{(1 - T)[(1 - P)\delta_{02}A_{11} - P\delta_{20}A_{12}]}. \quad (56)$$

Theorem 1. *The variance of proposed estimators $\widehat{\mu}_{y_1}$ and $\widehat{\mu}_{y_2}$ is, respectively, given by*

$$V(\widehat{\mu}_{y_1}) = \frac{A_{22}^2\sigma_{Z_1}^2 + A_{12}^2\sigma_{Z_2}^2 - 2A_{12}A_{22}\sigma_{Z_1 Z_2}}{n(1 - T)^2[(1 - P)\delta_{02}A_{11} - P\delta_{20}A_{12}]^2}, \quad (57)$$

and

$$V(\widehat{\mu}_{y_2}) = \frac{A_{11}^2\sigma_{Z_2}^2 + A_{21}^2\sigma_{Z_1}^2 - 2A_{11}A_{21}\sigma_{Z_1 Z_2}}{n(1 - T)^2[(1 - P)\delta_{02}A_{11} - P\delta_{20}A_{12}]^2}, \quad (58)$$

where

$$\sigma^2_{Z_1} = \left(\mu^2_{y_1} + \sigma^2_{y_1}\right)\{T + (1-T)e_{20}\} + \left(\mu^2_{y_2} + \sigma^2_{y_2}\right)\{T + (1-T)e_{02}\}$$
$$+ 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{T + (1-T)e_{11}\} - \left[A_{11}\mu_{y_1} + A_{12}\mu_{y_2}\right]^2, \tag{59}$$

$$\sigma^2_{Z_2} = \left(\mu^2_{y_1} + \sigma^2_{y_1}\right)\{PTe_{20} + (1-T)Pe_{40} + (1-P)Te_{02} + (1-P)(1-T)e_{22}\}$$
$$+ \left(\mu^2_{y_2} + \sigma^2_{y_2}\right)\{PTe_{20} + (1-T)Pe_{22} + (1-P)Te_{02} + (1-P)(1-T)e_{04}\}$$
$$+ 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{PTe_{20} + (1-T)Pe_{31} + (1-P)Te_{02} + (1-P)(1-T)e_{13}\}$$
$$- \left[A_{21}\mu_{y_1} + A_{22}\mu_{y_2}\right]^2, \tag{60}$$

and

$$\sigma_{Z_1 Z_2} = \left(\mu^2_{y_1} + \sigma^2_{y_1}\right)\{PTe_{10} + (1-P)Te_{01} + (1-T)Pe_{30} + (1-P)(1-T)e_{21}\}$$
$$+ \left(\mu^2_{y_2} + \sigma^2_{y_2}\right)\{PTe_{10} + (1-P)Te_{01} + (1-T)Pe_{12} + (1-P)(1-T)e_{03}\}$$
$$+ 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{PTe_{10} + (1-P)Te_{01} + (1-T)Pe_{21} + (1-P)(1-T)e_{12}\}$$
$$- \left[A_{11}\mu_{y_1} + A_{12}\mu_{y_2}\right]\left[A_{21}\mu_{y_1} + A_{22}\mu_{y_2}\right]. \tag{61}$$

Proof. Note that the variance expressions for two estimators $\widehat{\mu}_{y_1}$ and $\widehat{\mu}_{y_2}$ can be obtained through the formula $\mathrm{Var}(A \pm B) = \mathrm{Var}(A) + \mathrm{Var}(B) \pm 2\mathrm{Cov}(A, B)$.

Now, the variance $\sigma^2_{Z_1}$ is given by

$$\sigma^2_{Z_1} = E\left(Z^2_{1i}\right) - \{E(Z_{1i})\}^2$$
$$= TE\left(Y_{1i} + Y_{2i}\right)^2 + (1-T)E\left(S_1 Y_{1i} + S_2 Y_{2i}\right)^2$$
$$- \{E\left(\beta_i \left(Y_{1i} + Y_{2i}\right)\right)$$
$$+ (1-\beta_i)\left(S_1 Y_{1i} + S_2 Y_{2i}\right)\}^2, \tag{62}$$

or

$$\sigma^2_{Z_1} = \left(\mu^2_{y_1} + \sigma^2_{y_1}\right)\{T + (1-T)E\left(S^2_1\right)\}$$
$$+ \left(\mu^2_{y_2} + \sigma^2_{y_2}\right)\{T + (1-T)E\left(S^2_2\right)\}$$
$$+ 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{T + (1-T)E\left(S_1 S_2\right)\}$$
$$- \left[A_{11}\mu_{y_1} + A_{12}\mu_{y_2}\right]^2. \tag{63}$$

On substituting the values of $E\left(S^2_1\right)$, $E\left(S^2_2\right)$, and $E\left(S_1 S_2\right)$ in equation (63), we have equation (59).

The variance $\sigma^2_{Z_2}$ is given by

$$\sigma^2_{Z_2} = E\left(Z^2_{2i}\right) - \{E(Z_{2i})\}^2$$
$$= PTE\left(S_1 Y_{1i} + S_1 Y_{2i}\right)^2 + (1-T)PE\left(S^2_1 Y_{1i} + S_1 S_2 Y_{2i}\right)^2 + (1-P)TE\left(S_2 Y_{1i} + S_2 Y_{2i}\right)^2 + (1-P)(1-T)E\left(S_1 S_2 Y_{1i} + S^2_2 Y_{2i}\right)^2$$
$$- \left[E\{\gamma_i \beta_i \left(S_1 Y_{1i} + S_1 Y_{2i}\right) + (1-\beta_i)\gamma_i\left(S^2_1 Y_{1i} + S_1 S_2 Y_{2i}\right) + (1-\gamma_i)\beta_i\left(S_2 Y_{1i} + S_2 Y_{2i}\right) + (1-\gamma_i)(1-\beta_i)\left(S_1 S_2 Y_{1i} + S^2_2 Y_{2i}\right)\}\right]^2, \tag{64}$$

or

$$\sigma^2_{Z_2} = \left(\mu^2_{y_1} + \sigma^2_{y_1}\right)\{PTE\left(S^2_1\right) + (1-T)PE\left(S^4_1\right) + (1-P)TE\left(S^2_2\right)$$
$$+ (1-P)(1-T)E\left(S^2_1 S^2_2\right)\} + \left(\mu^2_{y_2} + \sigma^2_{y_2}\right)\{PTE\left(S^2_1\right) + (1-T)PE\left(S^2_1 S^2_2\right)$$
$$+ (1-P)TE\left(S^2_2\right) + (1-P)(1-T)E\left(S^4_2\right)\} + 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\{PTE\left(S^2_1\right)$$
$$+ (1-T)PE\left(S^3_1 S_2\right) + (1-P)TE\left(S^2_2\right) + (1-P)(1-T)E\left(S_1 S^3_2\right)\} - \left[A_{21}\mu_{y_1} + A_{22}\mu_{y_2}\right]^2. \tag{65}$$

On substituting the values of $E\left(S^2_1\right)$, $E\left(S^2_2\right)$, $E\left(S^4_1\right)$, $E\left(S^4_2\right)$, $E\left(S_1 S_2\right)$, $E\left(S^2_1 S^2_2\right)$, $E\left(S^3_1 S_2\right)$, and $E\left(S_1 S^3_2\right)$ in equation (65), we have equation (60).

The covariance $\sigma_{Z_1 Z_2}$ between Z_{1i} and Z_{2i} is given by

$$
\begin{aligned}
\sigma_{Z_1 Z_2} &= Cov\left(Z_{1i}, Z_{2i}\right) = E\left(Z_{1i} Z_{2i}\right) - E\left(Z_{1i}\right) E\left(Z_{2i}\right) \\
&= E\left(\gamma_i \beta_i \left[\beta_i \left\{\left(Y_{1i} + Y_{2i}\right)\left(S_1 Y_{1i} + S_1 Y_{2i}\right)\right\} + \left(1 - \beta_i\right)\left\{\left(S_1 Y_{1i} + S_2 Y_{2i}\right)\left(S_1 Y_{1i} + S_1 Y_{2i}\right)\right\}\right] \right. \\
&\quad + \left(1 - \beta_i\right)\gamma_i \left[\beta_i \left\{\left(Y_{1i} + Y_{2i}\right)\left(S_1^2 Y_{1i} + S_1 S_2 Y_{2i}\right)\right\} + \left(1 - \beta_i\right)\left\{\left(S_1 Y_{1i} + S_2 Y_{2i}\right)\left(S_1^2 Y_{1i} + S_1 S_2 Y_{2i}\right)\right\}\right] \\
&\quad + \left(1 - \gamma_i\right)\beta_i \left[\beta_i \left\{\left(Y_{1i} + Y_{2i}\right)\left(S_2 Y_{1i} + S_2 Y_{2i}\right)\right\} + \left(1 - \beta_i\right)\left\{\left(S_1 Y_{1i} + S_2 Y_{2i}\right)\left(S_2 Y_{1i} + S_2 Y_{2i}\right)\right\}\right] \\
&\quad \left. + \left(1 - \gamma_i\right)\left(1 - \beta_i\right)\left[\beta_i \left\{\left(Y_{1i} + Y_{2i}\right)\left(S_1 S_2 Y_{1i} + S_2^2 Y_{2i}\right)\right\} + \left(1 - \beta_i\right)\left\{\left(S_1 Y_{1i} + S_2 Y_{2i}\right)\left(S_1 S_2 Y_{1i} + S_2^2 Y_{2i}\right)\right\}\right] \right) \\
&\quad - \left[E\left\{\beta_i\left(Y_{1i} + Y_{2i}\right) + \left(1 - \beta_i\right)\left(S_1 Y_{1i} + S_2 Y_{2i}\right)\right\}\right]\left[E\left\{\gamma_i \beta_i \left(S_1 Y_i + S_1 Y_{2i}\right) + \left(1 - \gamma_i\right)\beta_i \left(S_1^2 Y_i + S_1 S_2 Y_{2i}\right)\right.\right. \\
&\quad \left.\left. + \left(1 - \gamma_i\right)\beta_i \left(S_2 Y_i + S_2 Y_{2i}\right) + \left(1 - \gamma_i\right)\left(1 - \beta_i\right) E\left(S_1 S_2 Y_i + S_2^2 Y_{2i}\right)\right\}\right],
\end{aligned}
\tag{66}
$$

or

$$
\begin{aligned}
\sigma_{Z_1 Z_2} &= \left(\mu_{y_1}^2 + \sigma_{y_1}^2\right)\left[PTE\left(S_1\right) + \left(1 - P\right)TE\left(S_2\right) + \left(1 - T\right)PE\left(S_1^3\right) + \left(1 - P\right)\left(1 - T\right)E\left(S_1^2 S_2\right)\right] \\
&\quad + \left(\mu_{y_2}^2 + \sigma_{y_2}^2\right)\left[PTE\left(S_1\right) + \left(1 - P\right)TE\left(S_2\right) + \left(1 - T\right)PE\left(S_1 S_2^2\right) + \left(1 - P\right)\left(1 - T\right)E\left(S_2^3\right)\right] \\
&\quad + 2\left(\mu_{y_1}\mu_{y_2} + \sigma_{y_1 y_2}\right)\left[PTE\left(S_1\right) + \left(1 - P\right)TE\left(S_2\right) + \left(1 - T\right)PE\left(S_1^2 S_2\right) + \left(1 - P\right)\left(1 - T\right)E\left(S_1 S_2^2\right)\right] \\
&\quad - \left[A_{11}\mu_{y_1} + A_{12}\mu_{y_2}\right]\left[A_{21}\mu_{y_1} A_{22}\mu_{y_2}\right].
\end{aligned}
\tag{67}
$$

On substituting the values of $E(S_1)$, $E(S_2)$, $E(S_1^2)$, $E(S_2^2)$, $E(S_1^3)$, $E(S_2^3)$, $E(S_1 S_2)$, $E(S_1^2 S_2)$, and $E(S_1 S_2^2)$ in equation (67), we have equation (61). $\quad\square$

Corollary 1. *The unbiased estimators for $V\left(\widehat{\mu}_{y_1}\right)$ and $V\left(\widehat{\mu}_{y_2}\right)$ are, respectively, given by*

$$
\widehat{V}\left(\widehat{\mu}_{y_1}\right) = \frac{A_{22}^2 s_{Z_1}^2 + A_{12}^2 s_{Z_2}^2 - 2A_{12} A_{22} s_{Z_1 Z_2}}{n\left(1 - T\right)^2 \left[\left(1 - P\right)\delta_{02} A_{11} - P\delta_{20} A_{12}\right]^2},
\tag{68}
$$

and

$$
\widehat{V}\left(\widehat{\mu}_{y_2}\right) = \frac{A_{11}^2 s_{Z_2}^2 + A_{21}^2 s_{Z_1}^2 - 2A_{11} A_{21} s_{Z_1 Z_2}}{n\left(1 - T\right)^2 \left[\left(1 - P\right)\delta_{02} A_{11} - P\delta_{20} A_{12}\right]^2},
\tag{69}
$$

where $s_{Z_1}^2 = (n-1)^{-1} \sum_{i=1}^{n}\left(Z_{1i} - \overline{Z}_1\right)^2$, $s_{Z_2}^2 = (n-1)^{-1} \sum_{i=1}^{n}\left(Z_{2i} - \overline{Z}_2\right)^2$ and $s_{Z_1 Z_2} = (n-1)^{-1} \sum_{i=1}^{n}\left(Z_{1i} - \overline{Z}_1\right)\left(Z_{2i} - \overline{Z}_2\right)$ are unbiased estimator of $\sigma_{Z_1}^2$, $\sigma_{Z_2}^2$, and $\sigma_{Z_1 Z_2}$, respectively.

Remark 1. When $T = 0$, the proposed partial RRM reduces to Ahmed et al. [22] model.

3.1. Percent Relative Efficiency under SRSWR. In this section, we compute the percent relative efficiency (PRE) of proposed estimators $\widehat{\mu}_{y_1}$ and $\widehat{\mu}_{y_2}$ over the estimators $\widehat{\mu}_{y_{A_1}}$ and $\widehat{\mu}_{y_{A_2}}$, respectively, as

$$
PRE\left(1\right) = \frac{V\left(\widehat{\mu}_{y_{A_1}}\right)}{V\left(\widehat{\mu}_{y_1}\right)} \times 100,
\tag{70}
$$

and

$$
PRE\left(2\right) = \frac{V\left(\widehat{\mu}_{y_{A_2}}\right)}{V\left(\widehat{\mu}_{y_2}\right)} \times 100.
\tag{71}
$$

We performed a simulation study to verify the superiority of the proposed partial RRM through FORTRAN program and showed the situation where the proposed methods might be more efficient than the method of Ahmed et al. [22]. The simulation results give large number of situations where $PRE(i)$, $i = 1, 2$ values of proposed partial RRM are more than 100. However, we presented only few values of PRE(1) and PRE(2) in Table 1, for different parameter values, $P = 0.5$, various values of T, $\mu_{y_1} = 20$, $\mu_{y_2} = 30$, two values of σ_{y_1} and σ_{y_2}, $\theta_1 = 4$, $\theta_2 = 7.2$, $\delta_{20} = 2$, $\delta_{02} = 4$, $\delta_{30} = -0.17$, $\delta_{03} = -1.65$, $\delta_{40} = 12.13$, and $\delta_{04} = 52.29$. The efficiency comparison on the values of scrambling variables that were earlier used in Ahmed et al. [22] model is also carried out, but our proposed model is less efficient on those values. Thus, we conclude that the efficiency of proposed partial RRM model can be increased or decreased by using different scrambling variables.

4. Proposed Partial RRM under Stratified Random Sampling

In this section, we present a partial randomized response model under stratification; a subsample in each stratum is drawn using a SRSWR sampling. Each sampled respondent in the hth stratum is requested for two responses by using two randomized response (RR) devices. The RR_h Device I provides the scramble and true response of sensitive variables, whereas the RR_h Device II provides the fake response, that is free from the sensitive variables.

The RR_h Device I, bearing two types of statements: (i) Report the additive true value of both sensitive variables, say

Table 1: PRE of proposed partial RRM over Ahmed et al. [22] model.

$\rho_{y_1 y_2}$	σ_{y_1}	σ_{y_2}	$T = 0.1$		$T = 0.3$		$T = 0.5$		$T = 0.7$		$T = 0.9$	
			PRE(1)	PRE(2)	PRE(1)	PRE(2)	PRE(1)	PRE(2)	PRE(1)	PRE(2)	PRE(1)	PRE(2)
−0.9	2	2	112.87	110.15	145.35	133.00	189.95	157.43	246.90	172.20	239.90	114.70
	2	6	112.87	110.15	145.37	133.02	190.01	157.47	247.05	172.31	240.38	114.93
	6	2	112.87	110.15	145.34	133.00	189.92	157.40	246.81	172.14	239.61	114.56
	6	6	112.87	110.15	145.36	133.02	190.00	157.46	247.02	172.29	240.27	114.88
−0.7	2	2	112.87	110.15	145.35	133.00	189.95	157.43	246.90	172.20	239.89	114.69
	2	6	112.87	110.15	145.36	133.02	190.00	157.47	247.04	172.30	240.36	114.92
	6	2	112.87	110.15	145.34	132.99	189.92	157.40	246.80	172.13	239.58	114.55
	6	6	112.87	110.15	145.36	133.02	189.99	157.46	246.99	172.27	240.19	114.84
−0.5	2	2	112.87	110.15	145.35	133.00	189.95	157.43	246.89	172.20	239.88	114.69
	2	6	112.87	110.15	145.36	133.02	190.00	157.47	247.03	172.30	240.33	114.90
	6	2	112.87	110.15	145.34	132.99	189.92	157.40	246.79	172.13	239.56	114.54
	6	6	112.87	110.15	145.36	133.01	189.98	157.45	246.97	172.25	240.10	114.80
−0.3	2	2	112.87	110.15	145.35	133.00	189.95	157.43	246.89	172.19	239.87	114.68
	2	6	112.87	110.15	145.36	133.02	190.00	157.46	247.03	172.29	240.30	114.89
	6	2	112.87	110.15	145.34	132.99	189.91	157.40	246.78	172.12	239.53	114.52
	6	6	112.87	110.15	145.35	133.01	189.97	157.44	246.94	172.23	240.02	114.76
−0.1	2	2	112.87	110.15	145.35	133.00	189.95	157.42	246.89	172.19	239.86	114.68
	2	6	112.87	110.15	145.36	133.02	189.99	157.46	247.02	172.28	240.27	114.88
	6	2	112.87	110.15	145.33	132.99	189.91	157.39	246.78	172.11	239.50	114.51
	6	6	112.87	110.15	145.35	133.01	189.96	157.43	246.91	172.21	239.94	114.72
0.1	2	2	112.87	110.15	145.35	133.00	189.95	157.42	246.88	172.19	239.85	114.67
	2	6	112.87	110.15	145.36	133.02	189.99	157.46	247.01	172.28	240.24	114.86
	6	2	112.87	110.15	145.33	132.99	189.91	157.39	246.77	172.11	239.48	114.50
	6	6	112.87	110.15	145.35	133.00	189.95	157.42	246.89	172.19	239.86	114.68
0.3	2	2	112.87	110.15	145.35	133.00	189.95	157.42	246.88	172.19	239.84	114.67
	2	6	112.87	110.15	145.36	133.02	189.99	157.46	247.00	172.27	240.22	114.85
	6	2	112.87	110.15	145.33	132.99	189.91	157.39	246.76	172.10	239.45	114.48
	6	6	112.87	110.15	145.34	133.00	189.94	157.42	246.86	172.17	239.78	114.64
0.5	2	2	112.87	110.15	145.35	133.00	189.95	157.42	246.88	172.19	239.83	114.67
	2	6	112.87	110.15	145.36	133.01	189.99	157.45	246.99	172.26	240.19	114.84
	6	2	112.87	110.14	145.33	132.99	189.90	157.39	246.75	172.10	239.42	114.47
	6	6	112.87	110.15	145.34	133.00	189.93	157.41	246.84	172.16	239.69	114.60
0.7	2	2	112.87	110.15	145.35	133.00	189.95	157.42	246.88	172.18	239.82	114.66
	2	6	112.87	110.15	145.36	133.01	189.98	157.45	246.98	172.26	240.16	114.82
	6	2	112.87	110.14	145.33	132.99	189.90	157.38	246.74	172.09	239.39	114.46
	6	6	112.87	110.15	145.34	133.00	189.92	157.40	246.81	172.14	239.61	114.56
0.9	2	2	112.87	110.15	145.35	133.00	189.94	157.42	246.87	172.18	239.81	114.66
	2	6	112.87	110.15	145.36	133.01	189.98	157.45	246.97	172.25	240.13	114.81
	6	2	112.87	110.14	145.33	132.99	189.90	157.38	246.73	172.08	239.37	114.44
	6	6	112.87	110.15	145.34	132.99	189.92	157.39	246.79	172.12	239.54	114.52

$(Y_{1_{hi}} + Y_{2_{hi}})$ with probability T_h, and (ii) Report the scrambled response as, $(S_{1_h} Y_{1_{hi}} + S_{2_h} Y_{2_{hi}})$ with probability $(1 - T_h)$. Mathematically, each ith respondent of the hth stratum is requested to report the response $Z_{1_{hi}}$ as:

$$Z_{1_{hi}} = \begin{cases} Y_{1_{hi}} + Y_{2_{hi}} & \text{with probability } T_h, \\ S_{1_h} Y_{1_{hi}} + S_{2_h} Y_{2_{hi}} & \text{with probability } (1 - T_h). \end{cases} \tag{72}$$

The partial randomized response $Z_{1_{hi}}$ in the ith sample of the hth stratum is given by

$$Z_{1_{hi}} = \beta_{hi}\left(Y_{1_{hi}} + Y_{2_{hi}}\right) + \left(1 - \beta_{hi}\right)\left(S_{1_h} Y_{1_{hi}} + S_{2_h} Y_{2_{hi}}\right), \tag{73}$$

and for the second response, each respondent is requested to use RR_h Device II which is same as equation (36); thus, the fake response Z_{hi} from in the ith sample of the hth stratum is given by

$$Z_{hi} = \gamma_{hi} S_{1_h} + (1 - \gamma_{hi}) S_{2_h}, \tag{74}$$

where γ_{hi} and β_{hi} are Bernoulli random variables with mean P_h and T_h, respectively, which are the known parameters.

From equations (73) and (74), we generate response $Z_{2_{hi}}$ as follows:

$$Z_{2_{hi}} = Z_{hi} Z_{1_{hi}}$$
$$= \begin{cases} S_{1_h} Y_{1_{hi}} + S_{1_h} Y_{2_{hi}} & \text{with probability } P_h T_h, \\ S_{1_h}^2 Y_{1_{hi}} + S_{1_h} S_{2_h} Y_{2_{hi}} & \text{with probability } (1 - T_h) P_h, \\ S_{2_h} Y_{1_{hi}} + S_{2_h} Y_{2_{hi}} & \text{with probability } T_h (1 - P_h), \\ S_{1_h} S_{2_h} Y_{1_{hi}} + S_{2_h}^2 Y_{2_{hi}} & \text{with probability } (1 - P_h)(1 - T_h). \end{cases}$$
$$\tag{75}$$

The generated response $Z_{2_{hi}}$ in the ith sample of the hth stratum is given by

$$Z_{2_{hi}} = \gamma_{hi}\beta_{hi}\left(S_{1_h}Y_{1_{hi}} + S_{1_h}Y_{2_{hi}}\right) + (1 - \beta_{hi})\gamma_{hi}\left(S_{1_h}^2 Y_{1_{hi}} + S_{1_h}S_{2_h}Y_{2_{hi}}\right)$$
$$+ (1 - \gamma_{hi})\beta_{hi}\left(S_{2_h}Y_{1_{hi}} + S_{2_h}Y_{2_h}\right) + (1 - \gamma_{hi})(1 - \beta_{hi})\left(S_{1_h}S_{2_h}Y_{1_{hi}} + S_{2_{hi}}^2 Y_{2_{hi}}\right).$$

(76)

Taking expected values on both sides of equations (73) and (76), we have

$$E\left(Z_{1_{hi}}\right) = T_h\left(\mu_{y_{1_h}} + \mu_{y_{2_h}}\right) + (1 - T_h)\left(e_{10_h}\mu_{y_{1_h}} + e_{01_h}\mu_{y_{2_h}}\right),$$

(77)

and

$$E\left(Z_{2_h}\right) = P_h T_h\left\{e_{10_h}\mu_{y_{1_h}} + e_{10_h}\mu_{y_{2_h}}\right\} + (1 - T_h)P_h\left\{e_{20_h}\mu_{y_{1_h}} + e_{11_h}\mu_{y_{2_h}}\right\}$$
$$+ (1 - P_h)T_h\left\{e_{01_h}\mu_{y_{1_h}} + e_{01_h}\mu_{y_{2_h}}\right\} + (1 - P_h)(1 - T_h)\left\{e_{11_h}\mu_{y_{1_h}} + e_{02_h}\mu_{y_{2_h}}\right\}.$$

(78)

From equations (77) and (78), by the method of moments, we have

$$A_{11_h}\widehat{\mu}_{y_{1_h}} + A_{12_h}\widehat{\mu}_{y_{2_h}} = \overline{Z}_{1_h},$$

(79)

and

$$A_{21_h}\widehat{\mu}_{y_{1_h}} + A_{22_h}\widehat{\mu}_{y_{2_h}} = \overline{Z}_{2_h},$$

(80)

where

$$A_{11_h} = T_h + (1 - T_h)e_{10_h}$$
$$A_{12_h} = T_h + (1 - T_h)e_{01_h}$$
$$A_{21_h} = P_h T_h e_{10_h} + (1 - T_h)P_h e_{20_h} + (1 - P_h)T_h e_{01_h} + (1 - P_h)(1 - T_h)e_{11_h}$$
$$A_{22_h} = P_h T_h e_{10_h} + (1 - T_h)P_h e_{11_h} + (1 - P_h)T_h e_{01_h} + (1 - P_h)(1 - T_h)e_{02_h}.$$

Solving equations (79) and (80) by using Cramer's rule, we have unbiased estimators of μ_{y_1} and μ_{y_2}, respectively, given by

$$\widehat{\mu}_{y_1(st)} = \sum_{h=1}^{L} W_h \frac{A_{22_h}\overline{Z}_{1_h} - A_{12_h}\overline{Z}_{2_h}}{(1 - T_h)\left[(1 - P_h)\delta_{02_h}A_{11_h} - P_h\delta_{20_h}A_{12_h}\right]},$$

(81)

and

$$\widehat{\mu}_{y_2(st)} = \sum_{h=1}^{L} W_h \frac{A_{11_h}\overline{Z}_{2_h} - A_{21_h}\overline{Z}_{1_h}}{(1 - T_h)\left[(1 - P_h)\delta_{02_h}A_{11_h} - P_h\delta_{20_h}A_{12_h}\right]}.$$

(82)

Theorem 2. *The variance of proposed estimators $\widehat{\mu}_{y_1(st)}$ and $\widehat{\mu}_{y_2(st)}$ is, respectively, given by*

$$V\left(\widehat{\mu}_{y_1(st)}\right) = \sum_{h=1}^{L} W_h^2 \frac{\sigma_{1_h}^2}{n_h},$$

(83)

and

$$V\left(\widehat{\mu}_{y_2(st)}\right) = \sum_{h=1}^{L} W_h^2 \frac{\sigma_{2_h}^2}{n_h},$$

(84)

where

$$\sigma_{1_h}^2 = \frac{A_{22_h}^2\sigma_{Z_{1_h}}^2 + A_{12_h}^2\sigma_{Z_{2_h}}^2 - 2A_{12_h}A_{22_h}\sigma_{Z_{1_h}Z_{2_h}}}{(1 - T_h)^2\left[(1 - P_h)\delta_{02_h}A_{11_h} - P_h\delta_{20_h}A_{12_h}\right]^2},$$

(85)

and

$$\sigma_{2_h}^2 = \frac{A_{11_h}^2\sigma_{Z_{2_h}}^2 + A_{21_h}^2\sigma_{Z_{1_h}}^2 - 2A_{11_h}A_{21_h}\sigma_{Z_{1_h}Z_{2_h}}}{(1 - T_h)^2\left[(1 - P_h)\delta_{02_h}A_{11_h} - P_h\delta_{20_h}A_{12_h}\right]^2},$$

(86)

$$\sigma_{Z_{1_h}}^2 = \left(\mu_{y_{1_h}}^2 + \sigma_{y_{1_h}}^2\right)\left\{T_h + (1 - T_h)e_{20_h}\right\} + \left(\mu_{y_{2_h}}^2 + \sigma_{y_{2_h}}^2\right)\left\{T_h + (1 - T_h)e_{02_h}\right\}$$
$$+ 2\left(\mu_{y_{1_h}}\mu_{y_{2_h}} + \sigma_{y_{1_h}y_{2_h}}\right)\left\{T_h + (1 - T_h)e_{11_h}\right\} - \left[A_{11_h}\mu_{y_{1_h}} + A_{12_h}\mu_{y_{2_h}}\right]^2,$$

(87)

$$\sigma_{Z_{2_h}}^2 = \left(\mu_{y_{1_h}}^2 + \sigma_{y_{1_h}}^2\right)\left\{P_h T_h e_{20_h} + (1 - T_h)P_h e_{40_h} + (1 - P_h)T_h e_{02_h} + (1 - P_h)(1 - T_h)e_{22_h}\right\}$$
$$+ \left(\mu_{y_{2_h}}^2 + \sigma_{y_{2_h}}^2\right)\left\{P_h T_h e_{20_h} + (1 - T_h)P_h e_{22_h} + (1 - P_h)T_h e_{02_h} + (1 - P_h)(1 - T_h)e_{04_h}\right\}$$
$$+ 2\left(\mu_{y_{1_h}}\mu_{y_{2_h}} + \sigma_{y_{1_h}y_{2_h}}\right)\left\{P_h T_h e_{20_h} + (1 - T_h)P_h e_{31_h} + (1 - P_h)T_h e_{02_h} + (1 - P_h)(1 - T_h)e_{13_h}\right\} - \left[A_{21_h}\mu_{y_{1_h}} + A_{22_h}\mu_{y_{2_h}}\right]^2,$$

$$(88)$$

and

$$\sigma_{Z_{1_h}Z_{2_h}} = \left(\mu_{y_{1_h}}^2 + \sigma_{y_{1_h}}^2\right)\left\{P_h T_h e_{10_h} + (1 - P_h)T_h e_{01_h} + (1 - T_h)P_h e_{30_h} + (1 - P_h)(1 - T_h)e_{21_h}\right\}$$
$$+ \left(\mu_{y_{2_h}}^2 + \sigma_{y_{2_h}}^2\right)\left\{P_h T_h e_{10_h} + (1 - P_h)T_h e_{01_h} + (1 - T_h)P_h e_{12_h} + (1 - P_h)(1 - T_h)e_{03_h}\right\}$$
$$+ 2\left(\mu_{y_{1_h}}\mu_{y_{2_h}} + \sigma_{y_{1_h}y_{2_h}}\right)\left\{P_h T_h e_{10_h} + (1 - P_h)T_h e_{01_h} + (1 - T_h)P_h e_{21_h} + (1 - P_h)(1 - T_h)e_{12_h}\right\}$$
$$- \left[A_{11_h}\mu_{y_{1_h}} + A_{12_h}\mu_{y_{2_h}}\right]\left[A_{21_h}\mu_{y_{1_h}} + A_{22_h}\mu_{y_{2_h}}\right].$$

$$(89)$$

Proof is simple, so omitted.

Corollary 2. *The variance of proposed estimators $\widehat{\mu}_{y_1(st)}$ and $\widehat{\mu}_{y_2(st)}$ under different methods of sample allocation are as:*

(i) Equal Allocation:

$$V\left(\widehat{\mu}_{y_1(st)}\right)_{EA} = \frac{L}{n}\sum_{h=1}^{L} W_h^2 \sigma_{1_h}^2, \qquad (90)$$

and

$$V\left(\widehat{\mu}_{y_2(st)}\right)_{EA} = \frac{L}{n}\sum_{h=1}^{L} W_h^2 \sigma_{2_h}^2. \qquad (91)$$

(ii) Proportional allocation:

$$V\left(\widehat{\mu}_{y_1(st)}\right)_{PA} = \frac{1}{n}\sum_{h=1}^{L} W_h \sigma_{1_h}^2, \qquad (92)$$

and

$$V\left(\widehat{\mu}_{y_2(st)}\right)_{PA} = \frac{1}{n}\sum_{h=1}^{L} W_h \sigma_{2_h}^2. \qquad (93)$$

(iii) Optimum Allocation:

$$V\left(\widehat{\mu}_{y_2(st)}\right)_{OA} = \frac{1}{n}\left\{\sum_{h=1}^{L} \frac{W_h \sigma_{2_h}^2}{\sqrt{\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2/C_h}}\right\}\left\{\sum_{h=1}^{L} W_h \sqrt{\frac{\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2}{C_h}}\right\}, \qquad (94)$$

and

$$V\left(\widehat{\mu}_{y_1(st)}\right)_{OA} = \frac{1}{n}\left\{\sum_{h=1}^{L} \frac{W_h \sigma_{1_h}^2}{\sqrt{\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2/C_h}}\right\}\left\{\sum_{h=1}^{L} W_h \sqrt{\frac{\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2}{C_h}}\right\}, \qquad (95)$$

where $n_h = nW_h\sqrt{\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2/C_h}/$
$\sum_{h=1}^{L} W_h\sqrt{\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2/C_h}$,
which are estimated by using linear cost function such as $C = C_0 + \sum_{h=1}^{L} n_h C_h$ *(where C_0 is fixed cost, and C_h is the variable cost in each stratum) and the compromised variance as:*

$V_{comp} = \sum_{h=1}^{L} W_h^2(\eta\sigma_{1_h}^2 + (1 - \eta)\sigma_{2_h}^2)/n_h$, *whereas η is known constant.*

Corollary 3. *The unbiased estimators for $V(\widehat{\mu}_{y_1(st)})$ and $V(\widehat{\mu}_{y_2(st)})$ are, respectively, given by*

$$\widehat{V}\left(\widehat{\mu}_{y_1(st)}\right) = \sum_{h=1}^{L} W_h^2 \frac{s_{1_h}^2}{n_h}, \qquad (96)$$

and

$$\widehat{V}\left(\widehat{\mu}_{y_2(st)}\right) = \sum_{h=1}^{L} W_h^2 \frac{s_{2_h}^2}{n_h}, \qquad (97)$$

where

$$s_{1_h}^2 = \frac{A_{22_h}^2 s_{Z_{1_h}}^2 + A_{12_h}^2 s_{Z_{2_h}}^2 - 2A_{12_h}A_{22_h}s_{Z_{1_h}Z_{2_h}}}{(1 - T_h)^2 \left[(1 - P_h)\delta_{02_h}A_{11_h} - P_h\delta_{20_h}A_{12_h}\right]^2}, \quad (98)$$

and

$$s_{2_h}^2 = \frac{A_{11_h}^2 s_{Z_{2_h}}^2 + A_{21_h}^2 s_{Z_{1_h}}^2 - 2A_{11_h}A_{21_h}s_{Z_{1_h}Z_{2_h}}}{(1 - T_h)^2 \left[(1 - P_h)\delta_{02_h}A_{11_h} - P_h\delta_{20_h}A_{12_h}\right]^2}, \quad (99)$$

$s_{Z_{1_h}}^2 = (n_h - 1)^{-1} \sum_{i=1}^{n_h} (Z_{1_{hi}} - \overline{Z}_{1_h})^2, s_{Z_{2_h}}^2 = (n_h - 1)^{-1} \sum_{i=1}^{n_h} (Z_{2_{hi}} - \overline{Z}_{2_h})^2$ and
$s_{Z_{1_h}Z_{2_h}} = (n_h - 1)^{-1} \sum_{i=1}^{n_h} (Z_{1_{hi}} - \overline{Z}_{1_h})(Z_{2_{hi}} - \overline{Z}_{2_h})$ are unbiased estimator of $\sigma_{Z_{1_h}}^2$, $\sigma_{Z_{2_h}}^2$ and $\sigma_{Z_{1_h}Z_{2_h}}$, respectively.

4.1. Percent Relative Efficiency under Stratification. In this section, we compute the percent relative efficiency (PRE) of proposed partial RRM over Ahmed et al. [22] model under stratification using proportional allocation method. For numerical comparison, we use real data set that is taken from Rosner [25]; the childhood respiratory disease study of Boston. We consider $Y_1 = AGE$ of a child and $Y_2 = FEV$ (forced expiratory volume) both as sensitive variables as earlier used by Ahmed et al. [22]. The population is subdivided into two strata on the basis of gender. The PREs of proposed estimators $\widehat{\mu}_{y_1(st)}$ and $\widehat{\mu}_{y_2(st)}$ with respect to Ahmed et al. [22] estimators $\widehat{\mu}_{y_{S1}(st)}$ and $\widehat{\mu}_{y_{S2}(st)}$, respectively, are defined as:

$$PRE(1) = \frac{V\left(\widehat{\mu}_{y_{A_1}(st)}\right)_{PA}}{V\left(\widehat{\mu}_{y_1(st)}\right)_{PA}} \times 100$$
$$= \frac{\sum_{h=1}^{L} W_h \sigma_{A_{1_h}}^2}{\sum_{h=1}^{L} W_h \sigma_{h1}^2} \times 100, \quad (100)$$

and

$$PRE(2) = \frac{V\left(\widehat{\mu}_{y_{A_2}(st)}\right)_{PA}}{V\left(\widehat{\mu}_{y_2(st)}\right)_{PA}} \times 100$$
$$= \frac{\sum_{h=1}^{L} W_h \sigma_{A_{2_h}}^2}{\sum_{h=1}^{L} W_h \sigma_{h2}^2} \times 100. \quad (101)$$

The results are presented in Table 2, the scrambling variable are same as in Ahmed et al. [22] for both strata, that is, $\theta_{1_h} = 2.6$, $\theta_{2_h} = 4.5$, $\delta_{20_h} = 2.3$, $\delta_{02_h} = 1.2$, $\delta_{30_h} = 2.2$, $\delta_{03_h} = 2.4$, $\delta_{40_h} = 3.2$, and $\delta_{04_h} = 3.2$. The $PRE(1)$ and $PRE(2)$ are free from the sample size. The numerical comparison shows that the efficiency of proposed partial RRM may be increased by choosing appropriate values of design parameters (P_h, T_h). We also observe that when $T_h = 0$, the proposed model reduces to Ahmed et al. [22] model.

We also compute the PRE of proposed partial RRM under SRSWR over proposed partial RRM under stratification to observe the gain in efficiency due to stratification. For both estimators, the *PRE* is almost 100 at different values of design parameters. This is because the variation between strata is almost same, and the randomization devices are also identical.

In the next section, we consider an application of a real data set.

5. Application of Real Data Set

Hussain et al. [26] estimated the average total number of classes that were missed by the students and Gjestvang and Singh [27] considered the problem of estimating the average GPA of students. In this study, we simultaneously estimate the average total number of classes that were missed by the students and average GPA of the students, by using the proposed partial RRM under SRSWR.

We took a sample of 80 students from Stat 317, Stat 629, and Zoo 203 classes to estimate the average GPA and average total number of missed classes by the students, at Quaid-i-Azam University, Islamabad. We generated 20 random numbers of S_1 and S_2 separately from Poisson distribution with means 5 and 2, respectively. To collect the data through proposed RRM, two decks of cards were used: Deck I, a deck of yellow cards and Deck II, a deck of blue cards. A deck of yellow cards bearing two different types of statements by setting $T = 0.5$ to get the scrambled and true responses, whereas deck of blue cards bearing values of two scramble variables by setting $P = 0.7$ to get fake responses.

In Deck I or deck of yellow cards, 20 cards, on 10 cards, we wrote the statements:

$$\textbf{PleaseReport}: \begin{pmatrix} GPA \\ scored\ in \\ last\ semester \end{pmatrix} + \begin{pmatrix} Total\ classes \\ missed\ during \\ last\ semester \end{pmatrix}, \quad (102)$$

and, on remaining 10 cards, we wrote the statements:

$$PleaseReport: \begin{pmatrix} GPA \\ scored\ in \\ last\ semester \end{pmatrix}\begin{pmatrix} S_1 \\ random \\ number \end{pmatrix}$$
$$+ \begin{pmatrix} Total\ classes \\ missed\ during \\ last\ semester \end{pmatrix}\begin{pmatrix} S_2 \\ random \\ number \end{pmatrix}. \quad (103)$$

In this process, 20 values of S_i, $i = 1, 2$ were written on each card.

The Deck II or deck of blue cards, consists of 20 cards, out of which 14 cards had the values of S_1 and the remaining six cards had values of S_2, with statement "kindly report one selected random number."

Before starting the data collection process, we highlighted the importance of randomized response methods

TABLE 2: PRE of proposed partial RRM with respect to Ahmed et al. [22] model under stratification.

(T_1, T_2)	$(P_1, P_2) = 0.2$		$(P_1, P_2) = 0.4$		$(P_1, P_2) = 0.6$		$(P_1, P_2) = 0.8$	
	PRE(1)	PRE(2)	PRE(1)	PRE(2)	PRE(1)	PRE(2)	PRE(1)	PRE(2)
0.0	100	100	100	100	100	100	100	100
0.1	130	126	104	101	107	104	108	105
0.2	172	160	108	100	113	106	115	109
0.3	229	203	110	98	119	107	123	112
0.4	307	257	109	92	124	106	130	112
0.5	413	323	105	83	125	101	134	110
0.6	552	396	94	69	122	91	133	102
0.7	713	458	76	50	109	74	122	86
0.8	826	458	48	28	81	48	95	59
0.9	642	290	16	8	36	18	46	24

TABLE 3: Data obtained through Ahmed et al. [22]; proposed partial RRM and direct responses.

Data obtained through RR device I of Ahmed et al. [22] model
45, 38.5, 28.6, 7.2, 31.1, 51.2, 42.9, 30.6, 8.4, 38, 29.2, 21.5, 32.2, 15.6, 15, 18.9, 27.7, 32, 36.5, 16.5, 35.4, 28, 24.2, 5.6, 16.6, 24.5, 16.4, 39.3, 34.8, 13.8, 20.6, 25, 26.8, 38.7, 36.4, 10.2, 27, 21.5, 25, 28.2, 43, 28.8, 34.7, 32, 31.4, 8.8, 63.8, 9.2, 15.8, 61.5, 27.4, 30.5, 29.9, 22.5, 14.2, 25, 14.6, 28.1, 18.8, 28.4, 14.3, 27.5, 38.5, 30.6, 9, 34.7, 13.4, 19.5, 14.5, 33.6, 54, 25, 17.6, 6.3, 41, 28.4, 37, 10.4, 27.5, 22.6
Data obtained through RR device I of proposed partial RRM by setting $T = 0.5$
10.5, 38.5, 28.6, 7.2, 31.1, 9.8, 10.9, 30.6, 6.7, 38, 7.6, 8.3, 8.4, 11.8, 15, 10.3, 27.7, 7.2, 36.5, 16.5, 35.4, 6.4, 24.2, 5.6, 10.2, 24.5, 7.1, 39.3, 34.8, 13.8, 12.2, 25, 11.2, 38.7, 36.4, 7.6, 9, 21.5, 7, 6.7, 43, 28.8, 34.7, 32, 8.3, 8.8, 11.1, 9.2, 15.8, 61.5, 8.2, 6.7, 7.7, 10.1, 10.1, 8.6, 14.6, 28.1, 18.8, 28.4, 14.3, 27.5, 8.9, 9.1, 9, 34.7, 7.8, 19.5, 14.5, 7.6, 54, 7.5, 17.6, 6.3, 41, 28.4, 5.5, 7.2, 27.5, 22.6
Data obtained through RR device II for fake responses by setting $P = 0.7$
6, 5, 6, 1, 9, 4, 9, 4, 2, 5, 7, 1, 1, 2, 5, 3, 7, 4, 7, 5, 3, 5, 6, 1, 3, 2, 1, 3, 3, 1, 3, 2, 1, 9, 4, 1, 2, 3, 7, 6, 5, 6, 1, 4, 1, 2, 9, 4, 2, 5, 7, 5, 1, 5, 2, 5, 3, 7, 3, 7, 3, 5, 5, 6, 1, 9, 3, 5, 1, 6, 6, 8, 1, 1, 1, 6, 6, 1, 2, 5, 4
GPA on last semester from direct response
3.4, 2.5, 3.2, 3.33, 3.4, 3.6, 3.05, 2.5, 3, 4.2, 4.1, 3.25, 3.3, 3, 3, 3.5, 2.9, 2.6, 3.1, 2.9, 3.9, 2.1, 3.5, 3.9, 3.4, 3, 2.5, 2.6, 3, 2.5, 2.6, 3, 3, 3, 3.7, 4, 4, 3.2, 3.5, 3.8, 4, 3, 3.8, 3.4, 2.8, 2.7, 3.5, 2.02, 3.6, 3.2, 3.2, 3.2, 1.9, 4, 3.5, 2.5, 3.65, 2.15, 3, 3.9, 4, 3.7, 3.3, 3.86, 3, 3.4, 2.9, 3.6, 3.63, 3.4, 2.8, 3, 2.94, 2.7, 2.6, 3.7, 3.5, 2.6, 2.3, 3
Total classes missed during last semester from direct response
6, 7, 2, 9, 4, 10, 5, 3, 6, 0, 0, 5, 12, 6, 0, 8, 4, 2, 5, 3, 3, 4, 3, 3, 2, 12, 12, 6, 6, 7, 2, 4, 5, 4, 0, 6, 10, 4, 3, 4, 3, 9, 4, 6, 5, 12, 2, 5, 5, 5, 6, 6, 3.3, 8, 5, 8, 4, 0, 6, 0, 0, 7, 3, 5, 10, 4, 7, 9, 5, 9, 6, 3, 10, 0, 5, 2, 6, 5, 6

and explained the proposed partial RRM and Ahmed et al. [22] model to the students. Each student was requested to draw one card from the Deck I and report the response as requested on the card, on yellow color paper. Similarly, repeat the process for Deck II and report the response as requested on the card, on blue color paper. Then, each student was provided a pink color card written on Ahmed et al. [22] model which was similar to equation (103) and requested to provide the scramble response on pink color paper. After marking the three responses on three different color papers, the students were advised to staple these papers together and put them into the box lying on the table. Finally, all students wrote the actual GPA and true total number of classes which they had missed during last semester on white color paper without disclosing their identity. Table 3 presents the responses obtained from the students.

Table 4 presents results of the survey. The estimates of the means μ_{y_1} and μ_{y_2} of proposed partial RRM are close to the estimates based on true responses as compared to the estimates obtained from Ahmed et al. [22] model. We noted that the standard error values are large due to small sample

TABLE 4: Results of the survey.

Model	Estimated mean		Standard error		95% confidence interval	
	$\widehat{\mu}_{y_1}$	$\widehat{\mu}_{y_2}$	$\widehat{\mu}_{y_1}$	$\widehat{\mu}_{y_2}$	$\widehat{\mu}_{y_1}$	$\widehat{\mu}_{y_2}$
Proposed partial RRM	3.18	6.17	0.57	1.12	[2.06, 4.30]	[3.97, 8.37]
Ahmed et al. [22]	2.33	7.53	0.47	1.15	[1.41, 3.25]	[5.26, 9.80]
Direct questioning	3.19	5.05	0.06	0.34	[3.07, 3.31]	[4.38, 5.72]

sizes; thus, we suggest that a large scale sample survey should be conducted in future to reach more realistic outcomes. The estimates of randomized response models reflect that the students are more reluctant to admit the total number of missed classes to do something unrelated to university study through direct questioning, whereas GPA is a less sensitive question for students. In conclusion, one can see that the proposed method of collecting scrambled data on sensitive issues can be used safely and securely.

6. Conclusion

In the present paper, we have suggested a partial randomized response model for estimating two population means simultaneously. Through simulation study and real life data application, it is observed that the proposed partial RRM performs better than the Ahmed et al. [22] model. The superiority of suggested partial RRM under stratification revealed through numerical comparison and it is observed that the proposed partial RRM under stratified random sampling performs better as compared to stratified model of Ahmed et al. [22]. Moreover, we also observed that design parameters play an important role in increasing or decreasing the efficiency of suggested models. The main advantage of proposed partial RRM is that it enables researchers to collect truthful responses at least from some proportion of people. Thus, the proposed partial randomized response model are therefore recommended for its use in practice as an alternative to Ahmed et al. [22] randomized response model.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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