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Research Article

Input-to-State Stability Analysis for Stochastic Mixed Time-Delayed Neural Networks with Hybrid Impulses

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This paper investigates the mean-square exponential input-to-state stability (MEISS) of stochastic mixed time-delayed neural networks with hybrid impulses. A generalized comparison principle is introduced and a new inequality about the solution of an impulsive differential equation is established. Moreover, by utilizing the proposed inequality and average impulsive interval approach based on different kinds of impulsive sequences, some novel criteria on MEISS are established. When the external input is removed, several conclusions on mean-square exponential stability (MES) are also derived. Unusually, the hybrid impulses including destabilizing and stabilizing impulses have been taken into account in the presented system. Finally, two simulation examples are provided to demonstrate the validity of our theoretical results.

1. Introduction

Recently, neural networks have absorbed plenty of researchers' interests owing to their extensive applications in a variety of areas including system pattern identification, wireless communications, optimization problems, and machine learning [1, 2]. Actually, the majority of the applications are related to the stability of equilibrium points. Hence, it is of great significance to analyze the stability of network systems. Moreover, stochastic disturbances exist inevitably, which affect the dynamic properties of systems. Meanwhile, in view of the limited switching velocity of the amplifier in the hardware implementation, it is often encountered with time delay, which leads to the instability and oscillations of associated systems. Subsequently, abundant results about stability analysis of stochastic network systems are attained by utilizing various methods [3–7].

Since instantaneous perturbations or abrupt changes at certain moments appear unpredictably in the real environment, impulsive effects are incorporated to depict the phenomenon in neural networks. Generally, impulsive sequences can be classified into stabilizing impulses and destabilizing impulses. Impulsive sequences are called to be stabilizing impulses if they can promote the stability of differential systems, while destabilizing impulses can suppress the stability of differential systems. Stability and synchronization problems of a dynamical system with different categories of impulses have become an interesting research topic, and some results with respect to the problems have been reported in [8–16]. For instance, in [8], the asymptotic stability of impulsive recurrent neural networks with stochastic disturbances and time delays was discussed by virtue of the Lyapunov functional approach and LMI technique. In [9], the synchronization problem of stochastic memristor-based recurrent neural networks with impulsive effects was explored by utilizing the impulsive differential inequality. In the real world, some phenomena about hybrid impulses comprising stabilizing and destabilizing impulses simultaneously often occur. They can be observed in fishing management with impulsive harvesting and releasing, goods selling involving impulsive stocking and transferring, and ball motion process with impulsive accelerating and decelerating. Moreover, some stability results of neural network systems with hybrid impulses have been attained by employing comparison theory in [10, 11]. Subsequently, instead of general impulses, the exponential synchronization issue of complex networks with hybrid impulses was also tackled in [12, 13], where the approach of average impulsive gain was introduced based on the average impulsive interval method in [14].

Additionally, the external inputs have great influences on the dynamic systems. To describe accurately how external perturbations impact the asymptotical properties of the control systems, the definition of input-to-state stability (ISS) was incorporated in [17]. Noting that the external disturbances or inputs also appear in the network systems, it is significant and meaningful to investigate the ISS of neural networks. Recently, many researchers paid considerable attention to the field, and plenty of achievements on ISS have emerged [18–26]. For instance, in [18], the ISS property for dynamical neural networks was analyzed by using the Lyapunov stability theory. In [19], a nice passive weight learning rule was designed for switched Hopfield neural networks, and some asymptotic stability and ISS results were proposed. Furthermore, some results about ISS analysis were extended to stochastic neural networks. Particularly, in [21, 22], MEISS of stochastic recurrent neural networks was discussed through the Lyapunov functional method. In [23], some algebraic conditions were derived to guarantee the mean-square stability ISS of stochastic network systems with Markovian switching on the basis of vector inequality methods and stochastic analysis techniques. Furthermore, robust input-to-state stability of stochastic neural networks with Markovian switching was examined in [24] under two circumstances by means of M-matrix theory. In [25], a novel criterion about MEISS of stochastic neural networks with multiproportional delays were established applying the variable transformations approach. More recently, the ISS of delay systems with hybrid impulses was studied in light of the Razumikhin method in [26]. As far as we know, up until now, although stability and synchronization problems of neural networks with multiple impulses have been resolved, the ISS properties of stochastic network systems with hybrid impulses have not been explored. Therefore, the analysis of influence for hybrid impulses and external input on system states becomes a significant topic.

Motivated by the previous considerations, this paper focuses on the MEISS of stochastic neural networks with hybrid impulses. The essential innovations are summarized as below. First of all, a generalized comparison principle is introduced and a new inequality about the solution of the impulsive differential equation is achieved. Secondly, different from the existing works, hybrid impulses including destabilizing impulses, stabilizing impulses, and external input are taken into account simultaneously, which reflected reality more accurately and makes the addressed system more complex. Finally, some new criteria on MEISS and MES of stochastic neural networks with hybrid impulses are established by the average impulsive interval approach based on different kinds of impulsive sequences. The structure of our paper is arranged appropriately. Section 2 proposes some preliminaries including mathematical models, assumptions, definitions, and lemmas. In Section 3, based on two proposed lemmas, several criteria on MEISS and MES of neural networks with hybrid impulses are established. Some simulation examples are provided in Section 4, and conclusions are drawn in the last section.

Notations 1. Let $(\Omega, \mathscr{F}, \mathscr{F}_t, \mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq t_0}$ satisfying the usual conditions, we set $\mathbb{R}^+ = (0, +\infty)$, $\mathbb{N}^+ = \{1, 2, \ldots\}$. $PC([a,b];\mathbb{R}^n)$ represents the family of all piecewise continuous functions from $[a,b]$ to $\mathbb{R}^n$. $L^2_{\mathscr{F}_{t_0}}([t_0-\tau_0, t_0];\mathbb{R}^n)$ denotes the family of all $\mathscr{F}_{t_0}-$ measurable $PC([t_0-\tau_0, t_0];\mathbb{R}^n)$ -valued random processes $\varphi = \{\varphi(s) \mid t_0-\tau_0 \leq s \leq t_0\}$ such that $\sup_{t_0-\tau_0\leq s\leq t_0} E|\varphi(s)|^2 < \infty$. $\lambda_{\max}(\cdot)$ stands for the largest eigenvalue of a matrix. $\|u\|_\infty = \sup_{t\geq t_0} |u(t)|$ denotes the infinite norm of the input function $u(t)$. Dini-derivative $D^+(\cdot)$ is defined by $D^+ v(t) = \overline{\lim}_{\varepsilon\longrightarrow 0^+} v(t+\varepsilon) - v(t)/t$.

2. Preliminaries

Consider the following class of neural networks with stochastic disturbances and mixed delays:

$$\begin{cases} \mathrm{d}x(t) = \left[-\mathscr{A}x(t) + \mathscr{B}f(x(t)) + \mathscr{C}g(x(t-\tau(t))) + \mathscr{D}\int_{t-\tau_0}^{t} h(x(s))\mathrm{d}s\right]\mathrm{d}t \\ +\varrho(t, x(t), x(t-\tau(t))\mathrm{d}w(t), \\ x(s) = \varphi(s), t_0-\tau_0 \leq s \leq t_0 \end{cases}, \quad (1)$$

where $x(t) = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ denotes the state vector of the neurons. $\mathscr{A} = \mathrm{diag}\{a_1, a_2, \ldots, a_n\} > 0$ is the self-feedback matrix. $\mathscr{B} = (b_{ij})_{n\times n}$, $\mathscr{C} = (c_{ij})_{n\times n}$, and $\mathscr{D} = (d_{ij})_{n\times n}$ represent the connection weight strength matrices. $f(x(t)) = (f_1(x(t)), f_2(x(t)), \ldots, f(x_n(t)))^T$, $g(x(t)) = (g_1(x(t)), g_2(x(t)), \ldots, g(x_n(t)))^T$, and $h(x(t)) = (h_1(x(t)), h_2(x(t)), \ldots, h(x_n(t)))^T$ are the activation vector functions. Additionally, $w(t)$ denotes an n-dimensional independent standard Wiener process, and function $\varrho(t, x(t), x(t-\tau(t)) \in \mathbb{R}^{n\times n}$ stands for the noise intensity. We suppose that $\tau(t)$ satisfies that $0 \leq \tau(t) \leq \tau_0$. Moreover, some assumptions are imposed on the neuron activation functions and noise intensity functions.

Assumption 1. Suppose that activation functions $f_i(x), g_i(x)$, and $h_i(x)$ with $f_i(0) = 0, g_i(0) = 0, h_i(0) = 0$ satisfy the globally Lipschitz continuous condition, i.e., there exist some positive constants l_i, p_i, q_i, and $i = 1, 2, \ldots, n$ such that

$$\begin{aligned}&\left|f_i(\iota_1) - f_i(\iota_2)\right| \le l_i\left|\iota_1 - \iota_2\right|, \left|g_i(\iota_1) - g_i(\iota_2)\right| \le p_i\left|\iota_1 - \iota_2\right|,\\ &\left|h_i(\iota_1) - h_i(\iota_2)\right| \le q_i\left|\iota_1 - \iota_2\right|,\end{aligned} \tag{2}$$

hold for any $\iota_1, \iota_2 \in \mathbb{R}^n$, which indicate that

$$\begin{aligned}\operatorname{diag}\left\{\left|f_1(\iota_1) - f_1(\iota_2)\right|, \left|f_2(\iota_1) - f_2(\iota_2)\right|, \ldots, \left|f_n(\iota_1) - f_n(\iota_2)\right|\right\} &\le L\left|\iota_1 - \iota_2\right|;\\ \operatorname{diag}\left\{\left|g_1(\iota_1) - g_1(\iota_2)\right|, \left|g_2(\iota_1) - g_2(\iota_2)\right|, \ldots, \left|g_n(\iota_1) - g_n(\iota_2)\right|\right\} &\le P\left|\iota_1 - \iota_2\right|;\\ \operatorname{diag}\left\{\left|h_1(\iota_1) - h_1(\iota_2)\right|, \left|h_2(\iota_1) - h_2(\iota_2)\right|, \ldots, \left|h_n(\iota_1) - h_n(\iota_2)\right|\right\} &\le Q\left|\iota_1 - \iota_2\right|,\end{aligned} \tag{3}$$

where $L = \operatorname{diag}\{l_1, l_2, \ldots, l_n\}$, $P = \operatorname{diag}\{p_1, p_2, \ldots, p_n\}$, and $Q = \operatorname{diag}\{q_1, q_2, \ldots, q_n\}$ are the diagonal matrices.

Assumption 2. Suppose that the noise intensity function $\varrho(t, x(t), x(t-\tau(t)))$ satisfies the globally Lipschitz condition $\varrho(t, 0, 0) = 0$. Furthermore, there exist two symmetric real matrices M_1, M_2 such that

$$\begin{aligned}&\operatorname{trac}\left[\varrho^T(t, x(t), x(t-\tau(t)))\varrho(t, x(t), x(t-\tau(t))\right]\\ &\quad\le x^T(t)M_1x(t) + x^T(t-\tau(t))M_2x(t-\tau(t)).\end{aligned} \tag{4}$$

By incorporating impulsive jumps and external input function $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in PC([t_0, +\infty); \mathbb{R}^n)$, system (1) is rewritten as follows:

$$\begin{cases}\mathrm{d}x(t) = \left[-\mathscr{A}x(t) + \mathscr{B}f(x(t)) + \mathscr{C}g(x(t-\tau(t))) + \mathscr{D}\displaystyle\int_{t-\tau_0}^{t} h(x(s))\mathrm{d}s\right.\\ \left.+u(t)\right]\mathrm{d}t + \varrho(t, x(t), x(t-\tau(t))\mathrm{d}w(t), \quad t \ne t_k,\\ x(t_k) = \gamma_{1k}x(t_k^-) + \gamma_{2k}u(t_k^-), \quad t = t_k,\\ x(s) = \varphi(s), \quad t_0 - \tau_0 \le s \le t_0,\end{cases} \tag{5}$$

where γ_{1k}, γ_{2k} are bounded constants. Moreover, we assume that there exists a positive constant γ_0 satisfies $|\gamma_{2k}| \le \gamma_0$.

Definition 1. The trivial solutions of system (1) are said to be MEISS, if for arbitrary $\varphi \in L^2_{\mathscr{F}_0}([t_0 - \tau_0, t_0]; \mathbb{R}^n)$ and $u(t) \in \mathrm{PC}([t_0 - \tau_0, \infty); \mathbb{R}^n)$, there exist positive constants α_0, β_0, ε_0 such that

$$E\left[|x(t, \varphi)|^2\right] \le \alpha_0 e^{-\beta_0(t-t_0)}E\left[\|\varphi\|^2\right] + \varepsilon_0\|u\|_\infty^2. \tag{6}$$

Lemma 1. *Assume $\vartheta(t), z(t), \zeta_1(t)$, and $\zeta_2(t)$ are piecewise continuous functions. The time-varying delay $\tau(t)$ and impulsive sequences $\{t_k\}$ satisfy that $0 \le \tau(t) \le \tau_0$ and $t_{k+1} - t_k \ge \tau_0$. If there exist some constants α_j, β_k, and $j = \{1, 2, 3\}$, $k \in \mathbb{N}^+$ satisfying the following inequalities:*

$$\begin{cases}D^+(\vartheta(t)) \le \alpha_1\vartheta(t) + \alpha_2\vartheta(t-\tau(t))\\ +\alpha_3\displaystyle\int_{t-\tau_0}^{t}\vartheta(s)\mathrm{d}s + \zeta_1(t), \quad t \ne t_k; \; ,\\ \vartheta(t_k) \le \beta_k\vartheta(t_k^-) + \zeta_2(t_k^-), \quad t = t_k,\end{cases} \tag{7}$$

and

$$\begin{cases}D^+(z(t)) > \alpha_1z(t) + \alpha_2z(t-\tau(t))\\ +\alpha_3\displaystyle\int_{t-\tau_0}^{t}z(s)\mathrm{d}s + \zeta_1(t), \quad t \ne t_k;\\ z(t_k) \ge \beta_kz(t_k^-) + \zeta_2(t_k^-), \quad t = t_k,\end{cases} \tag{8}$$

then $\vartheta(t) \le z(t)$ for $t_0 - \tau_0 \le t \le t_0$ implies that $\vartheta(t) \le z(t), t > t_0$.

Proof. This proof procedure is completely parallel to Lemma 1 in [15], so the process is omitted here. □

Lemma 2. *Let α_1, $\alpha_2 \ge 0$, $\alpha_3 \ge 0$, $\beta_k \ge 0$, and $\tau_0 \ge 0$ be some constants. $\tilde{\zeta}_1(t)$ and $\tilde{\zeta}_2(t)$ are nonnegative bounded functions. Delay $\tau(t)$ and impulsive sequences $\{t_k\}$ satisfy that $0 \le \tau(t) \le \tau_0$, $t_{k+1} - t_k \ge \tau_0$. If the following impulsive differential equation with initial value $x(s) = \phi(s) \ge 0, t_0 - \tau_0 \le s \le t_0$ holds*

$$\begin{cases}z'(t) = \alpha_1z(t) + \alpha_2z(t-\tau(t))\\ +\alpha_3\displaystyle\int_{t-\tau_0}^{t}z(s)\mathrm{d}s + \tilde{\zeta}_1(t), \quad t \ne t_k,\\ z(t_k) = \beta_kz(t_k^-) + \tilde{\zeta}_2(t_k^-), \quad t = t_k, k \in \mathbb{N}^+,\end{cases} \; , \tag{9}$$

then one can derive that

$$z(t)\le\left(\prod_{t_0<t_k\le t}\beta_k\right)z(t_0)e^{\alpha_1(t-t_0)}+\left(\sum_{t_0<t_k\le t}\prod_{t_k<t_m\le t}\beta_m\right)\tilde{\zeta}_2(t_k)e^{\alpha_1(t-t_k)}$$
$$+\int_{t_0}^{t}\left(\prod_{s<t_k\le t}\beta_k\right)e^{\alpha_1(t-s)}\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s. \quad (10)$$

Proof. When $t\in[t_0,t_1)$, by (9), one gets that

$$\left[e^{-\alpha_1(t-t_0)}z(t)\right]'=e^{-\alpha_1(t-t_0)}\left[\alpha_2 z(t-\tau(t))+\alpha_3\int_{t-\tau_0}^{t}z(s)\mathrm{d}s+\tilde{\zeta}_1(t)\right]. \quad (11)$$

By integrating the above equation, it yields that

$$z(t)\le z(t_0)e^{\alpha_1(t-t_0)}+\int_{t_0}^{t}e^{\alpha_1(t-s)}\cdot\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s, \quad (12)$$

which implies that (10) holds in the time interval $[t_0,t_1)$. Moreover, we assume that for $t\in[t_0,t_k)$, the assertion (10) holds. Therefore, for $t\in[t_k,t_{k+1})$, we derive that

$$z(t)=z(t_k)e^{\alpha_1(t-t_k)}+\int_{t_k}^{t}\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s$$
$$\le\beta_k z(t_k^-)e^{\alpha_1(t-t_k)}+e^{\alpha_1(t-t_k)}\tilde{\zeta}_2(t_k^-)+\int_{t_k}^{t}\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s$$
$$\le\beta_k e^{\alpha_1(t-t_k)}\left\{\left(\prod_{t_0<t_m<t_k}\beta_m\right)z(t_0)e^{\alpha_1(t_k-t_0)}+\left(\sum_{t_0<t_m<t_k}\prod_{t_m<t_i<t_k}\beta_i\right)\tilde{\zeta}_2(t_m)e^{\alpha_1(t_k-t_m)}\right.$$
$$\left.+\int_{t_0}^{t_k}\left(\prod_{s<t_m<t_k}\beta_m\right)\times e^{\alpha_1(t_k-s)}\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s\right\}$$
$$+e^{\alpha_1(t-t_k)}\tilde{\zeta}_2(t_k^-)+\int_{t_k}^{t}\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s$$
$$\le\left(\prod_{t_0<t_k\le t}\beta_k\right)z(t_0)e^{\alpha_1(t-t_0)}+\left(\sum_{t_0<t_k\le t}\prod_{t_k<t_m\le t}\beta_m\right)\tilde{\zeta}_2(t_k)e^{\alpha_1(t-t_k)}$$
$$+\int_{t_0}^{t}\left(\prod_{s<t_k\le t}\beta_k\right)e^{\alpha_1(t-s)}\left[\alpha_2 z(s-\tau(s))+\alpha_3\int_{s-\tau_0}^{s}z(\vartheta)d\vartheta+\tilde{\zeta}_1(s)\right]\mathrm{d}s. \quad (13)$$

We can get the assertion (10).

Remark 1. In Lemma 2, when $\beta_k<1$, the impulses are called to be stabilizing impulses since the absolute value of the state is reduced, and it convergences to the equilibrium point. When $\beta_k>1$, the impulses are called to be destabilizing impulses since the absolute value of the state is enlarged and it is away from the equilibrium point. Hybrid impulses include stabilizing impulses and destabilizing impulses simultaneously. Recently, the stability and synchronization problems of neural networks or complex networks with mixed impulses have been discussed [11–14]. Furthermore, the ISS property of nonlinear delay systems with multiple impulses was examined by the Razumikhin method in the article [26]. Based on the existing results [26], this paper aims to investigate the MEISS of stochastic neural networks with hybrid impulses.

Taking the stabilizing and destabilizing impulses into account, we suppose that the values of stabilizing impulsive strengths $\{\beta_k\}$ belong to one finite set $\{\delta_i\mid\delta_i<1,i=1,2,\ldots,\bar{l}_1\}$ while the values of destabilizing impulsive strengths $\{\beta_k\}$ belong to the other finite set $\{\eta_j\mid\eta_j>1,j=1,2,\ldots,\bar{l}_2\}$. The following assumption is further introduced.

Assumption 3. Suppose that there exist positive constants $N_0\ge0$, $\lambda_i>0$, and $\mu_j>0$ $(i=1,2,\ldots,\bar{l}_1,\ j=1,2,\ldots,\bar{l}_2)$ such that $\chi_{1i}(T,t)\ge T-t/\lambda_i-N_0$, $\chi_{2j}(T,t)\le T-t/\mu_j+N_0$, where $\chi_{1i}(T,t)$ and $\chi_{2j}(T,t)$ respectively denote the amount of the stable impulsive sequence with strength δ_i and destabilizing impulsive sequence with impulsive strength η_j on the time interval (t,T). Besides, the impulsive activation times t_k satisfy that $\inf\{t_k-t_{k-1}\}=\kappa\ge\tau_0>0$.

Remark 2. From the above assumption, we can find that λ_i and μ_j stand for the average dwell-time of stabilizing and destabilizing impulsive sequences, respectively. δ_i and η_j represent the impulsive strength of stabilizing and destabilizing impulsive sequences, respectively. Let $\hat{t}_{\text{ik}}$ and $\check{t}_{\text{jk}}$ denote the activation moments of the stabilizing impulses and the destabilizing impulses. If $\max\{\hat{t}_{\text{ik}} - \hat{t}_{\text{ik}-1}\} = \lambda_i$ and $\inf\{\check{t}_{\text{jk}} - \check{t}_{\text{jk}-1}\} = \mu_j\,(i = 1, 2, \ldots, \bar{l}_1,\, j = 1, 2, \ldots, \bar{l}_2)$, where $\bar{l}_1$ and $\bar{l}_2$ represent the finite positive integers, then we have that $\chi_{1i}(T, t) \geq T - t/\lambda_i - 1$, $\chi_{2j}(T, t) \leq T - t/\mu_j + 1$.

3. Main Results

In this section, according to the proposed lemmas, several criteria on MEISS and MES of stochastic delay hybrid impulses neural networks are established by utilizing the comparison principle and stochastic Lyapunov function approach.

Theorem 1. *Let $\beta_k = 2\gamma_{1k}^2$ represent the impulsive strengths, which take values from two finite sets $\{\delta_i \mid 0 < \delta_i < 1, i = 1, 2, \ldots, \bar{l}_1\}$ and $\{\eta_j \mid \eta_j > 1, j = 1, 2, \ldots, \bar{l}_2\}$. Under Assumptions 1, 2, and 3, if there are some parameters $\epsilon_i > 0, i \in \{1, 2, 3, 4\}$ satisfying the following inequality,*

$$\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0 > 0, \tag{14}$$

then system (1) is MEISS, where $\alpha_0 = -[\sum_{i=1}^{\bar{l}_1} \ln \delta_i/\lambda_i + \sum_{j=1}^{\bar{l}_2} \ln \eta_j/\mu_j + \alpha_1]$, $\alpha_1 = \lambda_{max}[-(\mathscr{A} + \mathscr{A}^T) + \epsilon_1\mathscr{B}\mathscr{B}^T + 1/\epsilon_1 L^T L + \epsilon_2\mathscr{C}\mathscr{C}^T + \epsilon_3\mathscr{D}\mathscr{D}^T + \epsilon_4 I + M_1]$, $\alpha_2 = \lambda_{max}[1/\epsilon_2 P^T P + M_2]$, $\alpha_3 = \lambda_{\max}[\tau_0/\varepsilon_3 Q^T Q]$, and $q_0 = (\prod_{i=1}^{\bar{l}_1} 1/\delta_i \prod_{j=1}^{\bar{l}_2} \eta_j)^{N_0}$.

Proof. The following Lyapunov function is chosen:

$$\mathbf{V}(t) = x^T(t)x(t). \tag{15}$$

By the It ô formula, one can derive that

$$\begin{aligned}
\mathscr{L}\mathbf{V}(t) &= 2x^T(t)\Big[-\mathscr{A}x(t) + \mathscr{B}f(x(t)) + \mathscr{C}g(x(t-\tau(t))) + \mathscr{D}\int_{t-\tau_0}^{t} h(x(s))\mathrm{d}s \\
&\quad + u(t)\Big] + \mathrm{trac}\big(\varrho^T(t, x(t), x(t-\tau(t)))\varrho(t, x(t), x(t-\tau(t)))\big) \\
&\leq -x^T(t)\big(\mathscr{A} + \mathscr{A}^T\big)x(t) + 2x^T(t)\mathscr{B}f(x(t)) + 2x^T(t)\mathscr{C}g(x(t-\tau(t))) \\
&\quad + 2x^T(t)\mathscr{D}\int_{t-\tau_0}^{t} h(x(s))\mathrm{d}s + 2x^T(t)u(t) \\
&\quad + x^T(t)M_1^T M_1 x(t) + x^T(t-\tau(t))M_2^T M_2 x(t-\tau(t)).
\end{aligned} \tag{16}$$

By making use of the inequality $2x^T y \leq \epsilon x^T x + 1/\epsilon y^T y, x, y \in R^n, \epsilon > 0$, we can obtain that

$$\begin{aligned}
2x^T(t)\mathscr{B}f(x(t)) &\leq 2\big[\mathscr{B}^T x(t)\big]^T f(x(t)) \\
&\leq \epsilon_1 x^T(t)\mathscr{B}\mathscr{B}^T x(t) + \frac{1}{\epsilon_1}x^T(t)L^T \mathrm{L}x(t).
\end{aligned} \tag{17}$$

Similarly, one has that

$$\begin{aligned}
2x^T(t)\mathscr{C}g(x(t-\tau(t))) &\leq \epsilon_2 x^T(t)\mathscr{C}\mathscr{C}^T x(t) + \frac{1}{\epsilon_2}x^T(t-\tau(t))P^T P x(t-\tau(t)) \\
2x^T(t)\mathscr{D}\int_{t-\tau_0}^{t} h(x(s))ds &\leq \epsilon_3 x^T(t)\mathscr{D}\mathscr{D}^T x(t) + \frac{1}{\epsilon_3}\left(\int_{t-\tau_0}^{t} h(x(s))ds\right)^T \int_{t-\tau_0}^{t} h(x(s))ds \\
&\leq \epsilon_3 x^T(t)\mathscr{D}\mathscr{D}^T x(t) + \frac{\tau_0}{\epsilon_3}\int_{t-\tau_0}^{t} x^T(s)Q^T Q x(s)ds \\
2x^T(t)u(t) &\leq \epsilon_4 x^T(t)x(t) + \frac{1}{\epsilon_4}u^T(t)u(t).
\end{aligned} \tag{18}$$

Then, we derive that

$$\begin{aligned}\mathscr{L}\mathbf{V}(t) \le{}& x^T(t)\Big[-\left(\mathscr{A}+\mathscr{A}^T\right)+\epsilon_1\mathscr{B}\mathscr{B}^T+\frac{1}{\epsilon_1}L^TL+\epsilon_2\mathscr{C}\mathscr{C}^T+\epsilon_3\mathscr{D}\mathscr{D}^T+\epsilon_4 I\\ &+M_1^TM_1\Big]x(t)+x^T(t-\tau(t))\left[\frac{1}{\epsilon_2}P^TP+M_2^TM_2\right]x(t-\tau(t))\\ &+\frac{\tau_0}{\epsilon_3}\int_{t-\tau_0}^{t}x^T(s)Q^TQx(s)\mathrm{d}s+\frac{1}{\epsilon_4}u^T(t)u(t)\\ \le{}& \alpha_1\mathbf{V}(t)+\alpha_2\mathbf{V}(t-\tau(t))+\alpha_3\int_{t-\tau_0}^{t}\mathbf{V}(s)\mathrm{d}s+\zeta_1(t),\end{aligned}\tag{19}$$

where $\alpha_1=\lambda_{\max}[-(\mathscr{A}+\mathscr{A}^T)+\varepsilon_1\mathscr{B}\mathscr{B}^T+1/\varepsilon_1L^TL+\varepsilon_2\mathscr{C}\mathscr{C}^T+\varepsilon_3\mathscr{D}\mathscr{D}^T+\varepsilon_4I+M_1]$, $\alpha_2=\lambda_{\max}[1/\epsilon_2P^TP+M_2]$, and $\alpha_3=\lambda_{\max}\tau_0/\epsilon_3Q^TQ$, $\zeta_1(t)=1/\epsilon_4u^T(t)u(t)$. Moreover, for $t\ne t_k$, one can see that

$$\begin{aligned}D^+E\mathbf{V}(t)&\le E\mathscr{L}\mathbf{V}(t)\\ &\le\alpha_1E\mathbf{V}(t)+\alpha_2E\mathbf{V}(t-\tau(t))\\ &\quad+\alpha_3\int_{t-\tau_0}^{t}E\mathbf{V}(s)\mathrm{d}s+\zeta_1(t).\end{aligned}\tag{20}$$

In addition, we have that

$$E\mathbf{V}(t_k))\le 2\gamma_{1k}^2E\mathbf{V}(t_k^-)+2\gamma_{2k}^2|u(t_k^-)|^2\le\beta_kE\mathbf{V}(t_k^-)+\zeta_2(t_k^-),\tag{21}$$

where $\beta_k=2\gamma_{1k}^2,\zeta_2(t)=2\gamma_{2k}^2|u(t)|^2$. Together with (20) and (21), we construct the following comparison system:

$$\begin{cases}z'(t)=\alpha_1z(t)+\alpha_2z(t-\tau(t))\\ +\alpha_3\displaystyle\int_{t-\tau_0}^{t}z(s)\mathrm{d}s+\zeta_1(t)+\theta, & t\ne t_k,\\ z(t_k)=\beta_kz(t_k^-)+\zeta_2(t_k^-), & t=t_k,\\ z(s)=|\varphi(s)|^2, & t_0-\tau_0\le s\le t_0,\end{cases}\tag{22}$$

where θ is a sufficiently small positive constant. By virtue of Lemma 1, it can be concluded that $E\mathbf{V}(t)\le z(t),t\ge 0$. Furthermore, applying Lemma 2 to the system (22), we can derive that

$$\begin{aligned}E\mathbf{V}(t)\le z(t)\le{}&\left(\prod_{t_0<t_k\le t}\beta_k\right)H_0e^{\alpha_1(t-t_0)}+\sum_{t_0<t_k\le t}\left(\prod_{t_k<t_m\le t}\beta_m\zeta_2(t_k^-)e^{\alpha_1(t-t_k)})\right)\\ &+\int_{t_0}^{t}\left(\prod_{s<t_k\le t}\beta_k\right)e^{\alpha_1(t-s)}\left[\alpha_2E\mathbf{V}(s-\tau_1(s))+\alpha_3\int_{s-\tau_0}^{s}\mathrm{E}\mathbf{V}(u)\mathrm{d}u+\zeta_1(s)+\theta\right]\mathrm{d}s,\end{aligned}\tag{23}$$

where $H_0=\sup_{-\tau_0\le s\le 0}E|x(s)|^2$. Let $q_0=(\prod_{i=1}^{\bar{l}_1}1/\delta_i\prod_{j=1}^{\bar{l}_2}\eta_j)^{N_0}$, $\alpha_0=-[\sum_{i=1}^{\bar{l}_1}\ln\delta_i/\lambda_i+\sum_{j=1}^{\bar{l}_2}\ln\eta_j/\mu_j+\alpha_1]$. From Assumption 3, it easily follows that $t-t_0/\lambda_i-N_0\le\chi_{1i}$ and $t-t_0/\mu_j+N_0\ge\chi_{2j}$, where χ_{1i} and χ_{2j} represent the jump times of stabilizing impulses and destabilizing impulses. We have that

$$\begin{aligned}\left(\prod_{t_0<t_k\le t}\beta_k\right)e^{\alpha_1(t-t_0)}&\le\left(\prod_{i=1}^{\bar{l}_1}\delta_i^{\chi_{1i}}\prod_{j=1}^{\bar{l}_2}\eta_j^{\chi_{2j}}\right)e^{\alpha_1(t-t_0)}\\ &\le\left(\prod_{i=1}^{\bar{l}_1}\delta_i^{t-t_0/\lambda_i-N_0}\prod_{j=1}^{\bar{l}_2}\eta_j^{t-t_0/\mu_j+N_0}\right)e^{\alpha_1(t-t_0)}\\ &\le q_0e^{\left[\sum_{i=1}^{\bar{l}_1}\ln\delta_i/\lambda_i+\sum_{j=1}^{\bar{l}_1}\ln\eta_j/\mu_j+\alpha_1\right](t-t_0)}\le q_0e^{-\alpha_0(t-t_0)}.\end{aligned}\tag{24}$$

Accordingly, we acquire that

$$\left(\prod_{t_k<t_m\le t}\beta_m\right)\zeta_2(t_k^-)e^{\alpha_1(t-t_k)}\le q_0\zeta_2(t_k^-)e^{-\alpha_0(t-t_k)},$$
$$\left(\prod_{s<t_k\le t}\beta_k\right)e^{\alpha_1(t-s)}\le q_0e^{-\alpha_0(t-s)}. \tag{25}$$

Hence, it follows that

$$\begin{aligned}\mathbf{E}\mathbf{V}(t)&\le z(t)\le q_0H_0e^{-\alpha_0(t-t_0)}\\&+q_0\sum_{t_0<t_k\le t}\zeta_2(t_k^-)e^{-\alpha_0(t-t_k)}\\&+\int_{t_0}^{t}q_0e^{-\alpha_0(t-s)}\Bigg[\alpha_2\mathrm{E}\mathrm{V}(s-\tau_1(s))\\&+\alpha_3\int_{s-\tau_0}^{s}\mathbf{E}\mathbf{V}(v)\mathrm{d}v+\zeta_1(s)+\theta\Bigg]\mathrm{d}s.\end{aligned} \tag{26}$$

Considering the following equation:

$$\xi-\alpha_0+\alpha_2q_0e^{\xi\tau_0}+\alpha_3q_0\frac{e^{\xi\tau_0}-1}{\xi}=0. \tag{27}$$

Let $\Phi(\xi)=\xi-\alpha_0+\alpha_2q_0e^{\xi\tau_0}+\alpha_3q_0e^{\xi\tau_0}-1/\xi$. Since $\Phi(0^+)=-(\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0)<0$, $\Phi(+\infty)>0$, and $\Phi(\xi)$ is a continuous function in the time interval $(0,+\infty)$, there is a root satisfying (27). Besides, it is obvious that $\Phi'(\xi)>0$. Thus, there exists a unique positive root σ such that the above equation holds. Subsequently, we will claim that

$$\begin{aligned}EV(t)\le q_0H_0e^{-\sigma(t-t_0)}&+\frac{q_0\left(\|\zeta_1(t)\|_\infty+\theta\right)}{\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0}\\&+\frac{\alpha_0q_0\|\zeta_2(t)\|_\infty}{(1-e^{-\kappa\alpha_0})(\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0)},\quad t\in[-\tau_0,\infty).\end{aligned} \tag{28}$$

When $t\in[t_0-\tau_0,t_0]$, it can be easily verified that what assertion (28) holds. Let $H_1=q_0(\|\zeta_1(t)\|_\infty+\theta)/\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0+\alpha_0q_0\|\zeta_2(t)\|_\infty/(1-e^{-\kappa\alpha_0})\ (\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0)$. For $t>t_0$, if inequality (3.6) is not true, on the contrary, then there exists a $\bar t$ such that

$$EV(\bar t)\ge q_0H_0e^{-\sigma(\bar t-t_0)}+H_1, \tag{29}$$

and for $t\in[-\tau_0,\bar t)$

$$EV(t)\le q_0H_0e^{-\sigma(t-t_0)}+H_1. \tag{30}$$

Then, we obtain that

$$\begin{aligned}\mathbf{E}\mathbf{V}(\bar t)&\le q_0H_0e^{-\alpha_0(\bar t-t_0)}+q_0\sum_{t_0<t_k\le\bar t}\zeta_2(t_k^-)e^{-\alpha_0(\bar t-t_k)}\\&\quad+\int_{t_0}^{\bar t}q_0e^{-\alpha_0(\bar t-s)}\left[\alpha_2\mathrm{E}\mathrm{V}(s-\tau_1(s))+\alpha_3\int_{s-\tau_0}^{s}\mathbf{E}\mathbf{V}(u)\mathrm{d}u+\zeta_1(s)+\theta\right]\mathrm{d}s\\&\le q_0H_0e^{-\alpha_0(\bar t-t_0)}+\frac{q_0\|\zeta_2(t)\|_\infty}{1-e^{-\kappa\alpha_0}}+\int_{t_0}^{\bar t}q_0e^{-\alpha_0(\bar t-s)}\big[\alpha_2E\mathbf{V}(s-\tau_1(s))\\&\quad+\alpha_3\int_{s-\tau_0}^{s}E\mathbf{V}(u)\mathrm{d}u+\zeta_1(s)+\theta\Big]\mathrm{d}s.\end{aligned} \tag{31}$$

By means of (30), it is noted that

$$\begin{aligned}\int_{t_0}^{\bar t}q_0e^{-\alpha_0(\bar t-s)}\alpha_2E\mathbf{V}(s-\tau_1(s))\mathrm{d}s&\le q_0\int_{t_0}^{\bar t}\alpha_2e^{-\alpha_0(\bar t-s)}\left[q_0H_0e^{-\sigma(s-\tau_1(s)-t_0)}+H_1\right]\mathrm{d}s\\&\le q_0^2H_0\alpha_2e^{\sigma\tau_0}e^{\sigma t_0-\alpha_0\bar t}\int_{t_0}^{\bar t}e^{(\alpha_0-\sigma)s}\mathrm{d}s+\frac{q_0\alpha_2H_1\left[1-e^{-\alpha_0(\bar t-t_0)}\right]}{\alpha_0}\\&\le\frac{q_0^2H_0\alpha_2e^{\sigma\tau_0}}{\alpha_0-\sigma}\left[e^{-\sigma(\bar t-t_0)}-e^{-\alpha_0(\bar t-t_0)}\right]+\frac{q_0\alpha_2H_1\left[1-e^{-\alpha_0(\bar t-t_0)}\right]}{\alpha_0}.\end{aligned} \tag{32}$$

Similarly, we have that

$$\begin{aligned}
&\int_{t_0}^{\bar{t}} q_0 e^{-\alpha_0(\bar{t}-s)}\alpha_3 \int_{s-\tau_0}^{s} E\mathbf{V}(u)\mathrm{duds} \\
&\leq q_0 \int_{t_0}^{\bar{t}} \alpha_3 e^{-\alpha_0(\bar{t}-s)}\left[\int_{s-\tau_0}^{s} q_0 H_0 e^{-\sigma(\nu-t_0)} + H_1\right]\mathrm{dvds} \\
&\leq q_0^2 H_0 \alpha_3 e^{\sigma t_0-\alpha_0\bar{t}}\frac{e^{\sigma\tau_0}-1}{\sigma}\int_{t_0}^{\bar{t}} e^{(\alpha_0-\sigma)s}\mathrm{ds} + \frac{q_0\alpha_3\tau_0 H_1\left[1-e^{-\alpha_0(\bar{t}-t_0)}\right]}{\alpha_0} \\
&\leq \frac{q_0^2 H_0\alpha_3}{\alpha_0-\sigma}\frac{e^{\sigma\tau_0}-1}{\sigma}\left[e^{-\sigma(\bar{t}-t_0)}-e^{-\alpha_0(\bar{t}-t_0)}\right] + \frac{q_0\alpha_3\tau_0 H_1\left[1-e^{-\alpha_0(\bar{t}-t_0)}\right]}{\alpha_0}, \\
&\cdot \int_{t_0}^{\bar{t}} q_0 e^{-\alpha_0(\bar{t}-s)}(\zeta_1(s)+\theta)\mathrm{ds} \leq \frac{q_0\left(\|\zeta_1(s)\|_\infty+\theta\right)\left[1-e^{-\alpha_0(\bar{t}-t_0)}\right]}{\alpha_0}.
\end{aligned} \tag{33}$$

Therefore, one gets that

$$\begin{aligned}
\mathrm{EV}(\bar{t}) &\leq q_0 H_0 e^{-\alpha_0(\bar{t}-t_0)} + \frac{q_0\|\zeta_2(t)\|_\infty}{1-e^{-\kappa\alpha_0}} + q_0 H_0\left[\alpha_2 q_0 e^{\sigma\tau_0} + \alpha_3 q_0 \frac{e^{\sigma\tau_0}-1}{\sigma}\right]\left[\frac{e^{-\sigma_0(\bar{t}-t_0)}}{\alpha_0-\sigma}\right. \\
&\left.-\frac{e^{-\sigma(t-t_0)}}{\alpha_0-\sigma}\right] + \frac{\left[q_0(\alpha_2+\alpha_3\tau_0)H_1 + q_0\left(\|\zeta_1(s)\|_\infty+\theta\right)\right]\left[1-e^{-\alpha_0(\bar{t}-t_0)}\right]}{\alpha_0} \\
&< q_0 H_0 e^{-\alpha_0(\bar{t}-t_0)} + \frac{q_0\|\zeta_2(t)\|_\infty}{1-e^{-\kappa\alpha_0}} + q_0 H_0\left[\alpha_2 q_0 e^{\sigma\tau_0} + \alpha_3 q_0\frac{e^{\sigma\tau_0}-1}{\sigma}\right]\left[\frac{e^{-\sigma(\bar{t}-t_0)}}{\alpha_0-\sigma}\right. \\
&\left.-\frac{e^{-\alpha_0(\bar{t}-t_0)}}{\alpha_0-\sigma}\right] + \frac{q_0(\alpha_2+\alpha_3\tau_0)H_1 + q_0\left(\|\zeta_1(s)\|_\infty+\theta\right)}{\alpha_0}.
\end{aligned} \tag{34}$$

Noting that $\sigma - \alpha_0 + \alpha_2 q_0 e^{\sigma\tau_0} + \alpha_3 q_0 e^{\sigma\tau_0} - 1/\sigma = 0$ and $H_1 = q_0(\|\zeta_1(t)\|_\infty + \theta)/\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0 + \alpha_0 q_0\|\zeta_2(t)\|_\infty / (1-e^{-\kappa\alpha_0})(\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0)$, we have that

$$\begin{aligned}
E\mathbf{V}(\bar{t}) &< q_0 H_0 e^{-\sigma(\bar{t}-tt_0)} + \frac{q_0\|\zeta_2(t)\|_\infty}{1-e^{-\kappa\alpha_0}} \\
&+ \frac{1}{\alpha_0}\left[\alpha_0 H_1 - \frac{q_0\alpha_0\|\zeta_2(t)\|_\infty}{1-e^{-\kappa\alpha_0}}\right],
\end{aligned} \tag{35}$$

which implies that

$$E\mathbf{V}(\bar{t}) < q_0 H_0 e^{-\sigma(\bar{t}-t_0)} + H_1. \tag{36}$$

It yields a contradiction with (29). Then, we have that

$$\begin{aligned}
E\mathbf{V}(t) &\leq q_0 H_0 e^{-\sigma(t-t_0)} + \frac{q_0\left(\|\zeta_1(t)\|_\infty+\theta\right)}{\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0} \\
&+ \frac{\alpha_0 q_0\|\zeta_2(t)\|_\infty}{\left(1-e^{-\kappa\alpha_0}\right)(\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0)}.
\end{aligned} \tag{37}$$

Let $\theta \longrightarrow 0$, it yields that

$$\begin{aligned}
E\mathbf{V}(t) &\leq q_0 H_0 e^{-\sigma(t-t_0)} + \frac{q_0\left(\|\zeta_1(t)\|_\infty\right.}{\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0} \\
&+ \frac{\alpha_0 q_0\|\zeta_2(t)\|_\infty}{\left(1-e^{-\kappa\alpha_0}\right)(\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0)}.
\end{aligned} \tag{38}$$

Since $\|\zeta_1(t)\|_\infty = \|1/\epsilon_4 u^T(t)u(t)\|_\infty \leq 1/\epsilon_4\|u(t)\|_\infty^2$, $\|\zeta_2(t)\|_\infty = \|2\gamma_{2k}^2|u(t)|^2\|_\infty \leq 2\gamma_0^2\|u(t)\|_\infty^2$, $E\mathbf{V}(t) = E|x(t)|^2$, one acquires that

$$E|x(t)|^2 \leq q_0 H_0 e^{-\sigma(t-t_0)} + \frac{q_0\left[\left(1-e^{-\kappa\alpha_0}\right) + 2\gamma_0^2\alpha_0\epsilon_4\right]\|u(t)\|_\infty^2}{\epsilon_4\left(1-e^{-\kappa\alpha_0}\right)(\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0)}, \tag{39}$$

which implies that system (1) is MEISS. □

Remark 3. In [21, 22], the MEISS of stochastic recurrent neural networks and Cohen-Grossberg neural networks have been investigated. In this paper, impulsive sequences are composed of destabilizing impulses, stabilizing impulses, and external input, which makes our model complicated. Meanwhile, some novel ISS criteria are established by employing impulsive differential inequality and average impulsive interval approach based on different kinds of impulsive sequences. On the other hand, the ISS property of impulsive delay systems with multiple impulses was also analyzed by means of the Razumikhin method in [26], and some restrictive conditions are required such as $\beta_k = \beta_{N+k}$. It implies that the impulsive sequences are periodic, which is not necessary for our results. Besides, compared with the existing results [26], our results are applied to stochastic neural networks and the sufficient conditions are more easily verified.

Remark 4. It is worth pointing out that the method applied in [12, 13] is not suitable in this paper since the impulsive part includes the external input. Therefore, this paper constructs the new impulsive inequality to overcome the difficulties.

If the stochastic disturbances and distributed delays are removed (1) immediately, it yields the following system:

$$\begin{cases} dx(t) = [-\mathscr{A}x(t) + \mathscr{B}f(x(t)) + \mathscr{C}g(x(t-\tau(t))) \\ +u(t)]dt, t \neq t_k, \\ x(t_k) = \gamma_{1k}x(t_k^-) + \gamma_{2k}u(t^-), t = t_k, \\ x(s) = \varphi(s), t - \tau_0 \leq s \leq t_0. \end{cases} \quad (40)$$

Accordingly, we have the following result.

Corollary 1. *Let $\beta_k = 2\gamma_{1k}^2$ represent the impulsive strengths, which take values from two finite sets $\{\delta_i | 0 < \delta_i < 1, i = 1, 2, \ldots, \bar{l}_1\}$ and $\{\eta_j \mid \eta_j > 1, j = 1, 2, \ldots, \bar{l}_2\}$. Under Assumptions 1 and 3, if there exist positive constants $\epsilon_i, i \in \{1, 2, 3\}1, 2, 3$ such that*

$$\alpha_0 - q_0\alpha_2 > 0, \quad (41)$$

where $\alpha_0 = -[\sum_{i=1}^{N_1} \ln \delta_i/\lambda_i + \sum_{j=1}^{N_2} \ln \eta_j/\mu_j + \alpha_1]$, $\alpha_1 = \lambda_{max}[-(\mathscr{A} + \mathscr{A}^T) + \epsilon_1\mathscr{B}\mathscr{B}^T + 1/\epsilon_1 L^T L + \epsilon_2\mathscr{C}\mathscr{C}^T + \epsilon_3 I]$, $\alpha_2 = \lambda_{max}[1/\epsilon_2 P^T P]$, and $q_0 = (\prod_{i=1}^{\bar{l}_1} 1/\delta_i \prod_{j=1}^{\bar{l}_2} \eta_j)^{N_0}$, then system (40) is exponentially ISS.

In particular, when $\bar{l}_1 = \bar{l}_2 = 1$, the stabilizing impulsive strength and destabilizing impulse strength are reduced to be δ_1 and η_1, respectively. Moreover, similar to Remark 2, let $\max\{\hat{t}_{1k} - \hat{t}_{1k-1}\} = \lambda_1$ and $\inf\{\check{t}_{1k} - \check{t}_{1k-1}\} = \mu_1$. It is equivalent to $\chi_{11}(T,t) \geq T - t/\lambda_1 - 1$, $\chi_{21}(T,t) \leq T - t/\mu_1 + 1$, which leads to the following corollary.

Corollary 2. *Suppose that two constants δ_1 and η_1 satisfy $0 < \delta_1 < 1, \eta_1 > 1$. $\beta_k = 2\gamma_{1k}^2$ denotes the impulsive strengths, which can be chosen by the set $\{\delta_1, \eta_1\}$. Under Assumptions 1, 2, and 3, if*

$$\alpha_0 - q_0\alpha_2 - q_0\alpha_3\tau_0 > 0, \quad (42)$$

where $\alpha_0 = -[\ln \delta/\lambda + \ln \eta/\mu + \alpha_1]$, $\alpha_1 = \lambda_{max}[-(\mathscr{A} + \mathscr{A}^T) + \mathscr{B}\mathscr{B}^T + L^T L + \mathscr{C}\mathscr{C}^T + \mathscr{D}\mathscr{D}^T + I + M_1]$, $\alpha_2 = \lambda_{max}[P^T P + M_2]$, $\alpha_3 = \lambda_{max}[\tau_0 Q^T Q]$, and $q_0 = \prod_{i=1}^{\bar{l}_1} 1/\delta_i \prod_{j=1}^{\bar{l}_2} \eta_j$, then system (1) is MEISS.

When external input $u(t) = 0$, according to the above results, the following assertions might be derived immediately.

Theorem 2. *Let $\beta_k = \gamma_{1k}^2$ denote the impulsive strengths, which take values from two finite sets $\{\delta_i \mid 0 < \delta_i < 1, i = 1, 2, \ldots, \bar{l}_1\}$ and $\{\eta_j \mid \eta_j > 1, j = 1, 2, \ldots, \bar{l}_2\}$. Suppose that all the other conditions in Theorem 1 are on hold. If external input $u(t) = 0$, then system (1) is MES.*

Corollary 3. *Let $\beta_k = \gamma_{1k}^2$ represent the impulsive strengths, which take values from two finite sets $\{\delta_i \mid 0 < \delta_i < 1, i = 1, 2, \ldots, \bar{l}_1\}$ and $\{\eta_j | \eta_j > 1, j = 1, 2, \ldots, \bar{l}_2\}$. Suppose that all the other conditions in Corollary 1 are on hold. If external input $u(t) = 0$, then system (40) is exponentially stable.*

Remark 5. In [11], the exponential stability of delayed neural networks with hybrid impulses was discussed. This paper further explores the MEISS of mixed delayed neural networks with stochastic disturbances and hybrid impulses. Particularly, when stochastic terms, distributed delays end external input are removed, the addressed system is reduced to the system in [11]. In light of Remark 2, when $\max\{\hat{t}_{ik} - \hat{t}_{i(k-1)}\} = \lambda_i$ and $\inf\{\check{t}_{jk} - \check{t}_{j(k-1)}\} = \mu_j$, we can choose flexible constant $N_0 = 1$ in Assumption 3. Let $\epsilon_1 = \epsilon_2 = 1$. Applying Corollary 3 to this case, if $\alpha_0 = -[\sum_{i=1}^{N_1} \ln \delta_i/\lambda_i + \sum_{j=1}^{N_2} \ln \eta_j/\mu_j + \alpha_1]$, $\alpha_1 = \lambda_{\max}[-(\mathscr{A} + \mathscr{A}^T) + \mathscr{B}\mathscr{B}^T + L^T L + \mathscr{C}\mathscr{C}^T]$, $\alpha_2 = \lambda_{\max}[P^T P]$, and $q_0 = \prod_{i=1}^{\bar{l}_1} 1/\delta_i \prod_{j=1}^{\bar{l}_2} \eta_j$, then the considered system is exponentially stable, which accords with the result in [11].

Remark 6. Recently, different control strategies have been developed to deal with some nonlinear systems. For instance, in [27], a stochastic integral sliding mode control strategy for singularly perturbed Markov jump descriptor systems subject to nonlinear perturbation was proposed and a novel mode and switch-dependent integral switching surface were introduced. In [28], the problem of path following for the underactuated unmanned surface vehicles (USVs) subject to state constraints has been tackled, where one control scheme was presented by combining the backstepping technique, adaptive dynamic programming, and the event-triggered mechanism. In [29], quantized nonstationary filtering for networked Markov switching repeated scalar nonlinear systems was investigated based on a multiple hierarchical structure strategy. Moreover, these control strategies can be applied to many applicable systems in the real world. Actually, impulsive control is also one of the significant control strategies. In this paper, we establish some MEISS criteria about stochastic neural networks with hybrid impulses. In the future, we could further explore the control problems of some applicable systems combined with input-to-state stability theory.

4. Numerical Example

In this section, two examples have been provided to exhibit the validity of the theoretical results in Theorem 1 and Theorem 2, in which hybrid impulsive effects are introduced.

Example 1. Suppose that parameters of the neural network (9) meet that

$$\begin{aligned}
&\mathscr{A}=\begin{pmatrix}2.7 & 0\\ 0 & 3\end{pmatrix};\ \mathscr{B}=\begin{pmatrix}0.3 & 0.5\\ 0.8 & 0.9\end{pmatrix};\ \mathscr{C}=\begin{pmatrix}0.4 & 0.2\\ 0.1 & 0.3\end{pmatrix};\\
&\mathscr{D}=\begin{pmatrix}0.7 & 0.8\\ 0.9 & 0.5\end{pmatrix};\ \varrho(t,x(t),x(t-\tau(t)))=\begin{pmatrix}0.4x_1(t) & 0.3x_1(t-\tau(t))\\ 0.5x_2(t) & 0.2x_2(t-\tau(t))\end{pmatrix},\\
&f(x(t))=\begin{pmatrix}0.8\tanh(x_1(t))\\ 0.9\tanh(x_2(t))\end{pmatrix};\ g(x(t))=\begin{pmatrix}0.24\tanh(x_1(t))\\ 0.2\tanh(x_2(t))\end{pmatrix};\\
&h(x(t))=\begin{pmatrix}0.9\tanh(x_1(t))\\ 0.85\tanh(x_2(t))\end{pmatrix};\ u(t)=\begin{pmatrix}1.8\sin t\\ 1.8\cos t\end{pmatrix};\\
&\gamma_{1k}=\begin{cases}0.5, k=2j-1,\\ 0.8, k=2j, j\in Z,\end{cases}\ \gamma_{2k}=0.8, k\in Z.
\end{aligned} \tag{43}$$

Let $\lambda_1=0.5, \mu_1=0.5$, and time-varying $\tau(t)=0.2|\cos t|$. By choosing $\epsilon_i=1, i=1,2,3,4$, we derive $\alpha_1=\lambda_{\max}[-(\mathscr{A}+\mathscr{A}^T)+\varepsilon_1\mathscr{B}\mathscr{B}^T+1/\varepsilon_1 L^T L+\varepsilon_2\mathscr{C}\mathscr{C}^T+\varepsilon_3\mathscr{D}\mathscr{D}^T+\varepsilon_4 I+M_1]=0.2146$, $\alpha_2=\lambda_{\max}[1/\epsilon_2 P^T P+M_2]=0.1476$, and $\alpha_3=\lambda_{\max}[\tau_0/\epsilon_3 Q^T Q]=0.162$. Moreover, we calculate that $\delta=0.5, \eta=1.28, \alpha_0=0.6780, q_0=2.56$, and it can be easily verified that $\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0=0.2172>0$. Therefore, all the conditions are satisfied in Theorem 1, then system (9) is MEISS (See Figure 1). While $u(t)=0$, according to Theorem 2, the system is MES (Figure 2).

Example 2. Assume that parameters of the neural network (9) meet that

$$\begin{aligned}
&\mathscr{A}=\begin{pmatrix}2.1 & 0\\ 0 & 2.4\end{pmatrix};\ \mathscr{B}=\begin{pmatrix}0.2 & -0.15\\ 0.32 & 0.1\end{pmatrix};\ \mathscr{C}=\begin{pmatrix}0.65 & 0.25\\ -0.18 & 0.4\end{pmatrix};\\
&\mathscr{D}=\begin{pmatrix}0.12 & 0.16\\ 0.35 & -0.3\end{pmatrix};\ \varrho\left(t,x(t),x(t-\tau(t))\right)=\begin{pmatrix}0.4x_1(t) & 0.32x_2(t-\tau(t))\\ 0.6x_2(t) & 0.3x_1(t-\tau(t))\end{pmatrix}\\
&f(x(t))=\begin{pmatrix}0.36\sin(x_1(t))\\ 0.24\sin(x_2(t))\end{pmatrix};\ g(x(t))=\begin{pmatrix}0.12\sin(x_1(t))\\ 0.15\sin(x_2(t))\end{pmatrix};\\
&h(x(t))=\begin{pmatrix}0.65\sin(x_1(t))\\ 0.5\sin(x_2(t))\end{pmatrix};\ u(t)=\begin{pmatrix}5\sin t\\ 2\cos t\end{pmatrix};\ \gamma_{1k}=\begin{cases}0.9, k=2j-1,\\ 0.6, k=2j,\end{cases};\ \gamma_{2k}=0.7.
\end{aligned} \tag{44}$$

Let $\lambda_1=0.4, \mu_1=0.4$, and time-varying $\tau(t)=0.1|\cos t|+0.15$. By choosing $\varepsilon_i=1, i=1,2,3,4$, we derive $\alpha_1=\lambda_{\max}[-(\mathscr{A}+\mathscr{A}^T)+\epsilon_1\mathscr{B}\mathscr{B}^T+1/\epsilon_1 L^T L+\epsilon_2\mathscr{C}\mathscr{C}^T+\epsilon_3\mathscr{D}\mathscr{D}^T+\epsilon_4 I+M_1]=-2.3470$, $\alpha_2=\lambda_{\max}[1/\epsilon_2 P^T P+M_2]=0.1249$, and $\alpha_3=\lambda_{\max}[\tau_0/\epsilon_3 Q^T Q]=0.1056$. Moreover, we calculate that $\delta=0.72, \eta=1.62, \alpha_0=1.9622, q_0=2.25$, and it can be easily verified that $\alpha_0-q_0\alpha_2-q_0\alpha_3\tau_0=1.6218>0$. Therefore, all the conditions hold in Theorem 1, then system (9) is MEISS (shown in Figure 3). While $u(t)=0$, according to Theorem 2, the system is MES (shown in Figure 4).

Remark 7. From the above two examples, it could be seen that stabilizing impulses and destabilizing impulses coexist simultaneously. In Example 1, the stabilizing impulses are dominant, which plays an important role in stabilizing systems. While in Example 2, the destabilizing impulses are dominant, which could be seen as impulsive perturbations. Under this circumstance, the original system can tolerate the impulsive perturbations and maintain input-to-state stability. Different from the theoretical results in [26], the sufficient conditions on MEISS criteria are more easily verified by virtue of the average impulsive interval approach.

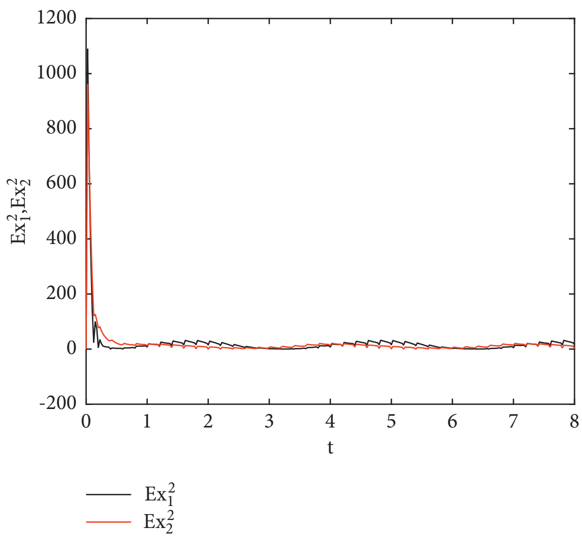

FIGURE 1: 2nd moment of the system states $x_1(t), x_2(t)$ in Example 1 with input $u(t) = (1.8\sin t, 1.8\cos t)^T$, and initial value $(-40, 36)^T$.

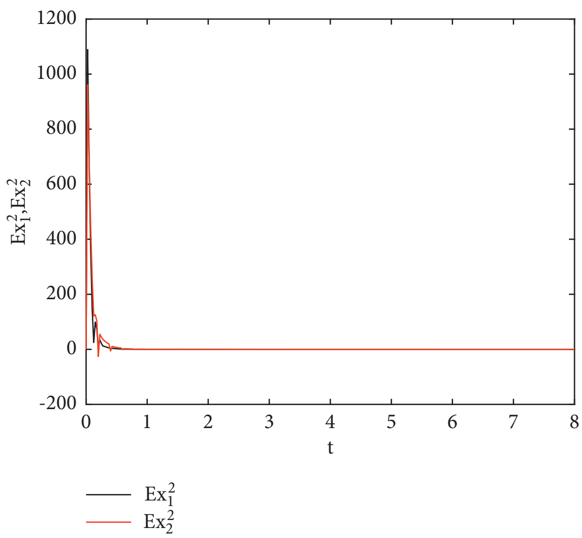

FIGURE 2: 2nd moment of the system states $x_1(t), x_2(t)$ in Example 1 input $u(t) = 0$, and initial value $(-40, 36)^T$.

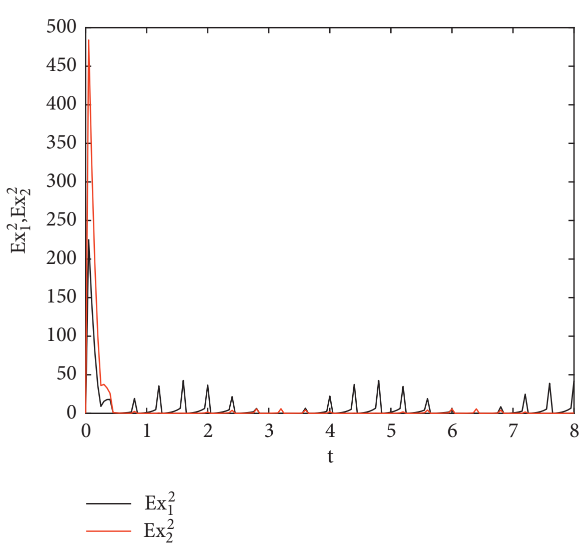

FIGURE 3: 2nd moment of the system states $x_1(t), x_2(t)$ in Example 2 with input $u(t) = (5\sin t, 2\cos t)^T$ and initial value $(18, 26)^T$.

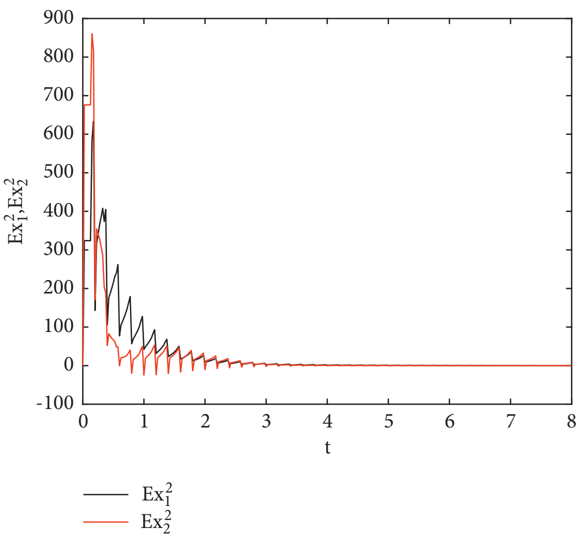

Figure 4: 2nd moment of the system states $x_1(t), x_2(t)$ in Example 2 with input $u(t) = 0$ and initial value $(18, 26)^T$.

5. Conclusions

This article investigates the issues of MEISS for stochastic neural networks with hybrid impulses. A generalized comparison principle is introduced and a new inequality about the solution of the impulsive differential equation is derived. Moreover, combining the impulsive differential inequality and average impulsive interval approach based on different kinds of impulsive sequences, some novel criteria are constructed to guarantee that the system is MEISS. When external input $u(t) = 0$, the addressed system is MES. Since hybrid impulses and external input at each impulsive moment are incorporated, our model becomes more general. Consequently, our theoretical achievements also improve the previous results. In the future, we will further explore some control problems of applicable systems by virtue of ISS theory.

Data Availability

All data, models, and code generated or used during the study appear in the submitted article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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