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Research Article

Selection of Suppliers in Industrial Manufacturing: A Fuzzy Rough PROMETHEE Approach

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In supply chain management (SCM), the selection of suppliers plays a vital role in an efficient production process. Over the last few years, to form a trade-off between the quantitative and qualitative criteria, the selection of suppliers in SCM is considered very conclusive. The decisions generally demand different criteria to balance between every possible inconsistent parameters involving subjectivity and uncertainty in the process. The evaluation information mostly depends on the experience and knowledge of experts that are unsure and indistinct. This study introduces a novel decision-making method by integrating rough approximations with fuzzy numbers and preference ranking organization method for enrichment evaluation (PROMETHEE) to deal with subjective and objective vagueness in the assessment of decision makers. To minimize the dependency on experts' judgements, entropy weights are computed from the original data set. The preference index is then computed using entropy weights and deviations among alternatives. The alternatives are ranked using the intersection of both positive flow and negative flow. To show the significance and importance of fuzzy rough PROMETHEE method, a case study of supplier selection in industrial manufacturing is discussed in detail. The developed method is effectively used to rank the supplier alternatives under given criteria. The results are then compared with different rough numbers and fuzzy numbers based on MCDM methods. The fuzzy rough PROMETHEE method can be efficiently used for the selection of the best suppliers to reduce losses and maximize the production process.

1. Introduction

Multicriteria decision-making (MCDM) involves making preference decisions (such as evaluation, prioritization, and selection) over the available alternatives that are characterized by multiple, usually conflicting criteria. MCDM deals with assessing and choosing alternatives that provide the best results according to the requirements. There are numerous MCDM methods that were presented in the literature, and the PROMETHEE technique is one of them. In SCM, the selection of suppliers has a great feature while companies spend a minimum of 60 percent of their entire sales for obtaining things such as components, raw materials, and other components. This is what the manufacturers say up to 70 percent of product cost acquired for goods and services. In effective SCM, supplier selection should be regarded as a strategic aspect. To enhance management's competition and concentration, manufacturers tried to expand strategic collaboration during the 1990s. For the selection of suppliers, Dickson [1] identified twenty-three criteria and Weber et al. [2] studied supplier efficiency based on Dickson's criteria keeping in mind the information about the criteria such as history, restoration, financial establishment, manufacturers' position, productive capacity, transmission, standard, place, technical ability, and price. Evans [3] considered different criteria, for example, quality, cost, and transmission for the selection of suppliers. The supplier selection process gains a great attention by the marketing management literature. For this purpose, Lin and Chang [4] defined some necessary things for the selection of suppliers such as response of clients, strong communication relation, position, and standard of industry. Inexact or insufficient modeling of several situations is caused by the uncertain, imprecise, and vague information or data. The strategic sources (SSs) are considered as an essential business function. Further, to cover the buying plan, SS becomes a very important part of the firm scheme under the extended heading of logistics. Companies are attentive to discover different ways so that they quickly provide opportunities with reasonable price to the clients as compared to their competitors. Therefore, the organizers noticed that they must work with great participation in a mutual system in their organization webs including clients doubtless, manufacturing units, and warehouses. MCDM problems become very complicated because they include both the quantitative and qualitative criteria [5]. To successfully handle the issues regarding the selection of suppliers, various suitable methods exist for experts [6]. In the context of decision-making problems, applications of different types of fuzzy models were examined [7]. For the assessment and selection of suppliers, an AHP technique in a unit of Ghana named as pharmaceutical industry is described by Asamoah et al. [8]. For proper suppliers' selection, Kumar and Barman [9] proposed the qualitatively essential factors.

Wang et al. [10] proposed AHP integrated with preemptive goal programming (PGP) for the selection of suppliers. To control the obscurity of the data employing fuzzy theory, a multi-objective linear model was suggested by Amid et al. [11] explaining the issues and their successful solutions relating to the problems of supplier selection using the fuzzy weighted max-min model. Chen [12] suggested the weights of all criteria and alternatives in linguistic forms that can easily be converted into triangular fuzzy numbers (TFNs). For the selection of a satisfactory house, Chang et al. [13] proposed a new MCGP methodology to help the house buyers for the evaluation of the houses.

Other than the vagueness that occurs in experts' evaluations, one more serious problem is how to successfully aggregate these evaluations from various experts in SCM and in the group of decision-making environment. There are many operators for weighted average that have been suggested to control this issue, for instance, ordered weighted geometric aggregation operator, weighted mean method, and linguistic arithmetic averaging operator, but the weighting methods require additional information and all the experts can present parameters for criteria weights that may differ in different situations and environments.

It is a challenging task to deal with the subjectivity and vagueness for the selection of suppliers during SCM. Many techniques based on weighted averaging operators and fuzzy sets can deal with a part of such problems. A lot of subjectivity and vagueness is ignored, that is, cognitive bias among various experts, choice of the membership function, and to determine weighted averaging operators. To increase the aggregation of the evaluation data in the group decision-making, many methods used the concept of rough numbers that have been suggested to handle the imperfections in the operators named as weighted average [14–17]. Rough numbers are considered as an objective mathematical model and have no need of additional parameters. Rough numbers only depend on original assessment information. Because of its objectivity, various other mathematical models such as fuzzy sets, crisp values, and interval numbers are integrated with rough numbers to aggregate the personal evaluation data in the decision-making problem.

To eliminate the subjectivity and uncertainty for the selection of suppliers in SCM, this study suggested a well-organized approach for an MCDM problem based on fuzzy rough numbers (FRNs), PROMETHEE method, and entropy weight method (EWM). The main objective of this study was (1) to expand the applications of MCDM structure to fulfill the requirements and diminish the issues that occur in SCM, (2) to extend the objectivity of the results of assessment information using FRNs, and (3) to enhance the functionality of MCDM to enlarge the benefits of FRNs by applying the PROMETHEE method.

1.1. Related Works. For the selection of an adjustable and suitable technique, there is a great need of awareness among various existing MCDM approaches. Usually, MCDM methods are applicable for solving various problems related to certain and uncertain environments. Al-Kloub and Abu-Taleb [18] used the PROMETHEE method in the context of water resource for the project portfolios in 1998s. To enhance the effectiveness of classical MCDM techniques, researchers are extending fuzzy set-based mathematical models. The PROMETHEE method is extended to an efficient fuzzy environment known as F-PROMETHEE [19]. This method controlled the uncertainty and impreciseness that occur due to the evaluations given in linguistic terms.

For the selection of suppliers, the researchers are studying various MCDM approaches according to their interest or experience, for example, fuzzy TOPSIS, AHP, and interval type 2 fuzzy information [20–22]. The TOPSIS approach is widely implemented to select the most favorable suppliers under the green environment. Yazdani et al. [23] proposed a hybrid model for MCDA for the investigation of SCM by the analysis of the models known as stepwise weight evaluation ratio and the quality function deployment. For the green suppliers' selection, a fuzzy TOPSIS approach was suggested by Kannan et al. [24] by considering the analysis of an electronics company in Brazil. As an exemplar, Chiou et al. [25] proposed a model to study an electronics company in China based on the fuzzy AHP technique. A fuzzy VIKOR approach was used for the evaluation of suppliers by Sanayei et al. [26]. Recently, for the ordering of suppliers with the help of an optimization approach for calculating the criteria weights, a bipolar fuzzy ELECTRE II approach was suggested by Shumaiza et al. [27]. Further, Akram et al. [28] presented a new approach to select the green suppliers using bipolar fuzzy numbers in the PROMETHEE method. Akram et al.

[29] extended [28] using the AHP method for calculating the weights of criteria and m-polar numbers in the PROMETHEE approach. The methods of interval-valued fuzzy sets are more effective due to the usage of interval values instead of single fixed values in membership functions. An integrated design concept evaluation method based on the vague sets was introduced by Geng et al. [30].

Although many fuzzy and extended fuzzy decision-making approaches have been presented to reduce the uncertainties, these techniques always require membership function and some additional information. Rough approximations introduced by Pawlak [31] overcome the limitations of predefined functions and additional suppositions. The idea of lower and upper limits was given by Zhai et al. [17] to study uncertainty in MCDM approaches based on linguistic data. A rough TOPSIS method by Song et al. [32] was proposed for this purpose. Sarwar [33] provided a rough D-TOPSIS approach to improve the decision-making process and implemented this approach in agricultural farming. Moreover, Sarwar et al. [34] presented a rough ELECTRE II technique to enhance the working conditions and to reduce the loss of energy in the automatic manufacturing process. For the selection of suppliers in green supply chain implementation, a rough number MAIRCA method and hybrid rough number DEMATEL-ANP [14] were presented. In these presented techniques, the initial assessment information is directly used for generating the rough numbers. Because of the objectivity features in the subjective environment, a rough number is a powerful mathematical model for the assessment of linguistic information.

Interval rough numbers and an interval rough AHP in a method known as multi-attributive border approximation area comparison (MABAC) were developed by Pamucar et al. [15] for the assessment of Web pages of the university. For the assessment of third-party logistics, a best worst methodology (BWM), MABAC, weighted aggregated sum product assessment (WASPAS), and interval rough numbers are combined by Pamucar et al. [35]. Zhu et al. [36] suggested a TOPSIS method using fuzzy rough numbers and AHP technique. The researchers are actively working in this domain, for instance, ELECTRE I approach-based hesitant Pythagorean fuzzy information [37], uncertainty analysis in venture investment evaluation problem [38], FMEA with incomplete information based on individual semantics, heterogeneous MCDM problem based on PROMETHEE-FLP [39], suppliers' selection based on hesitant fuzzy PROMETHEE approach [40], MCDM problem based on fuzzy BWM [41], and decision-making approaches based on fuzzy type 2 technique [42].

1.2. Motivation. Based on the analysis of latest research on MCDM, the purpose and motivation of this research were to propose a novel MCDM technique to handle the uncertainty and experts' individual assessments considering the selection of suppliers. The main objectives and primary contributions of this approach are illustrated as follows:

1.3. Framework of the Paper. This paper is organized as follows:

(1) Section 1 explains the related work about the proposed study.

(2) Section 2 deals with the definitions of various types of preference functions, concept of fuzzy sets and FR numbers.

(3) The whole method for calculating the criteria weights and selection of suppliers is presented by applying an appropriate approach known as EWM and fuzzy rough PROMETHEE.

(4) Section 3 provides a novel MCDM technique for the supplier selection by integrating fuzzy rough numbers and PROMETHEE method and this is also illustrated by a flowchart diagram. EWM is used for determining the weights of all criteria.

(5) Section 4 presents a practical application of a case study for the selection of supplier in SCM, and represents the results by applying the complete method.

(6) To check the out-performance of the proposed study, Section 5 describes the comparative study of FR PROMETHEE model with different techniques such as fuzzy TOPSIS, crisp VIKOR, and fuzzy rough TOPSIS choosing different types of preference functions. Discussion about all the comparative techniques has also been illustrated in this section.

(7) Section 6 deals with advantages, limitations, conclusions, and future directions of the proposed study.

The list of abbreviations used in this study is illustrated in Table 1.

2. Preliminaries

Some basic concepts about fuzzy sets, fuzzy numbers, and fuzzy rough numbers are described in this section. Different types of preference functions that are usually used in the PROMETHEE approach regarding generalized criteria are also discussed in this section.

2.1. Fuzzy Sets and Fuzzy Numbers. Fuzzy sets are widely applicable as an effective technique to manipulate uncertainty in the depiction of information and evaluation. Generally, a fuzzy set $\widetilde{R}$ is made from a class of items, in which every item possess a specific membership degree $\mu_{\widetilde{R}}(x)$. In the literature, there are different types of membership functions; for example, Gaussian, trapezoidal, and triangular are discussed in fuzzy logic [45–47]. In real-world applications, the use of the triangular membership functions for fuzzy sets is due to their characteristics and easy calculations [48]. Further, real numbers are used to generalize the fuzzy numbers and the fuzzy numbers considering a particular case of the fuzzy sets have a normalized and convex membership degree.

Table 1: List of abbreviations.

Abbreviation	Description
TFN	Triangular fuzzy number
PF	Preference function

Definition 1 (see [49]). A *triangular fuzzy number (TFN)* $\tilde{g}$ is a fuzzy number with three triangular points as (g_1, g_2, g_3) with membership function as defined in equation (1) and illustrated in Figure 1.

$$\mu_{\tilde{g}(y)} = \begin{cases} 0, & y < g_1; \\ \dfrac{y - g_1}{g_2 - g_1}, & g_1 \le y \le g_2; \\ \dfrac{g_3 - y}{g_3 - g_2}, & g_2 \le y \le g_3; \\ 0, & y > g_3. \end{cases} \tag{1}$$

Let $\tilde{g} = (g_l, g_m, g_u)$ and $\tilde{h} = (h_l, h_m, h_u)$ be two TFNs, and then, the arithmetic operations on TFNs are given in the following equations:

$$\tilde{g} \oplus \tilde{h} = (g_l + h_l, g_m + h_m, g_u + h_u), \tag{2}$$

$$\tilde{g} \ominus \tilde{h} = (g_l - h_u, g_m - h_m, g_u - h_l), \tag{3}$$

$$\tilde{g} \otimes \tilde{h} \cong (g_l \times h_l, g_m \times h_m, g_u \times h_u), \tag{4}$$

$$\tilde{g} \oslash \tilde{h} \cong \left(\frac{g_l}{h_u}, \frac{g_m}{h_m}, \frac{g_u}{h_l}\right), \tag{5}$$

$$\tilde{g} \otimes c \cong (g_l \times c, g_m \times c, g_u \times c); \; (\because c > 0), \tag{6}$$

$$\tilde{g}^{-1} \cong \left(\frac{1}{g_u}, \frac{1}{g_m}, \frac{1}{g_l}\right). \tag{7}$$

2.2. Fuzzy Rough Numbers. Because of the advantages in the subjective evaluation process, rough numbers are extensively applied in different MCDM techniques and their applications are in various domains, for instance, supply chain management, change mode and effect evaluation, failure modes and effect analysis, remanufacturing machine tools, and design concept evaluation. Fuzzy rough numbers are introduced by Zhu et al. [36] to study subjectivity and vagueness in decision-making processes. These numbers combine the properties of fuzzy numbers and rough numbers to provide more accurate models to handle uncertainty. In fact, fuzzy rough numbers give the idea of lower and upper limits and rough boundary interval as standard rough numbers, which provide results that are much clear and stable.

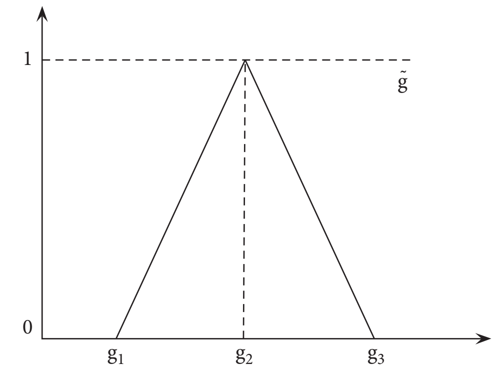

Figure 1: Triangular fuzzy number $\tilde{g}$.

Definition 2 (see [36]). Let V be the judgement set constructed from the assessment ratings composed by experts. Usually, these ratings are divided into m groups and have a sequence as $\widetilde{B}_1 \le \widetilde{B}_2 \le \cdots \le \widetilde{B_m}$. We take TFNs $\widetilde{B}_i \, (1 \le i \le m)$, where $\widetilde{B}_i = (\widetilde{B_{il}}, \widetilde{B_{im}}, \widetilde{B_{iu}})$, $\widetilde{S}$ is the collection containing $\widetilde{B}_1, \widetilde{B}_2, \ldots, \widetilde{B_m}$, and X is an arbitrary universe. The lower approximation of class $\widetilde{B}_i$ is given in the following equations:

$$\underline{Apr}\,(B_{il}) = \cup \left\{ \frac{X \in V}{\widetilde{S}(X) \le B_{il}} \right\}, \tag{8}$$

$$\underline{Apr}\,(B_{im}) = \cup \left\{ \frac{X \in V}{\widetilde{S}(X) \le B_{im}} \right\}, \tag{9}$$

$$\underline{Apr}\,(B_{iu}) = \cup \left\{ \frac{X \in V}{\widetilde{S}(X) \le B_{iu}} \right\}. \tag{10}$$

Similarly, the upper approximation of class $\widetilde{B}_i$ is given in the following equations:

$$\underline{Apr}\,(B_{il}) = \cup \left\{ \frac{X \in V}{\widetilde{S}(X) \ge B_{il}} \right\}, \tag{11}$$

$$\underline{Apr}\,(B_{im}) = \cup \left\{ \frac{X \in V}{\widetilde{S}(X) \ge B_{im}} \right\}, \tag{12}$$

$$\underline{Apr}\,(B_{iu}) = \cup \left\{ \frac{X \in V}{\widetilde{S}(X) \ge B_{iu}} \right\}. \tag{13}$$

The lower limit of class $\widetilde{B}_i$ can be computed using the following formulae:

$$\underline{Lim}\,(B_{il}) = \frac{1}{n_{Ll}} \sum_{i=1}^{n_{Ll}} X \in \underline{Apr}\,(B_{il}), \tag{14}$$

$$\underline{Lim}\,(B_{im}) = \frac{1}{n_{Lm}} \sum_{i=1}^{n_{Lm}} X \in \underline{Apr}\,(B_{im}), \tag{15}$$

$$\underline{Lim}\,(B_{iu}) = \frac{1}{n_{Lu}} \sum_{i=1}^{n_{Lu}} X \in \underline{Apr}\,(B_{iu}). \tag{16}$$

Here, n_{Ll}, n_{Lm}, and n_{Lu} are the total number of elements in $\underline{Apr}\,(B_{il}), \underline{Apr}\,(B_{im})$, and $\underline{Apr}\,(B_{iu})$. Similarly, the upper limit of class $\widetilde{B}_i$ can be computed using the following formulae:

$$\overline{Lim}\,(B_{il}) = \frac{1}{n_{Ul}} \sum_{i=1}^{n_{Ul}} X \in \overline{Apr}\,(B_{il}), \tag{17}$$

$$\overline{Lim}\,(B_{im}) = \frac{1}{n_{Um}} \sum_{i=1}^{n_{Um}} X \in \overline{Apr}\,(B_{im}), \tag{18}$$

$$\overline{Lim}\,(B_{iu}) = \frac{1}{n_{Uu}} \sum_{i=1}^{n_{Uu}} X \in \overline{Apr}\,(B_{iu}). \tag{19}$$

Here, n_{Ul}, n_{Um}, and n_{Uu} are the total number of elements in $\overline{Apr}\,(B_{il}), \overline{Apr}\,(B_{im})$, and $\overline{Apr}\,(B_{iu})$. The fuzzy rough number of $\widetilde{B}_i$ is represented as follows:

$$FRN\,(\widetilde{B}_i) = \left([\underline{Lim}\;B_{il}, \overline{Lim}B_{il}], [\underline{Lim}\;B_{im}, \overline{Lim}B_{im}], [\underline{Lim}\;B_{iu}, \overline{Lim}B_{iu}]\right). \tag{20}$$

The rough boundary interval of B_{il}, B_{im}, B_{iu} and whole $\widetilde{B}_i$ is computed in the following equations:

$$RBn\ d\,(B_{il}) = \overline{Lim}\,(B_{il}) - \underline{Lim}\,(B_{il}), \tag{21}$$

$$RBn\ d\,(B_{im}) = \overline{Lim}\,(B_{im}) - \underline{Lim}\,(B_{im}), \tag{22}$$

$$RBn\ d\,(B_{iu}) = \overline{Lim}\,(B_{iu}) - \underline{Lim}\,(B_{iu}), \tag{23}$$

$$RBn\ d\,(\widetilde{B}_i) = \overline{Lim}\,(B_{iu}) - \underline{Lim}\,(B_{il}). \tag{24}$$

The uncertainty of B_{il}, B_{im}, B_{iu}, and $\widetilde{B}_i$ is denoted by the rough boundary interval. An interval having a smaller length is considered as the more accurate, while an interval with a greater length is considered as unclear or indefinite. The fuzzy rough number is illustrated geometrically in Figure 2.

The fundamental arithmetic operations of fuzzy rough numbers are defined in [36]. It is easy to say that fuzzy rough numbers can be obtained by the evaluation of initial data given in the form of TFNs.

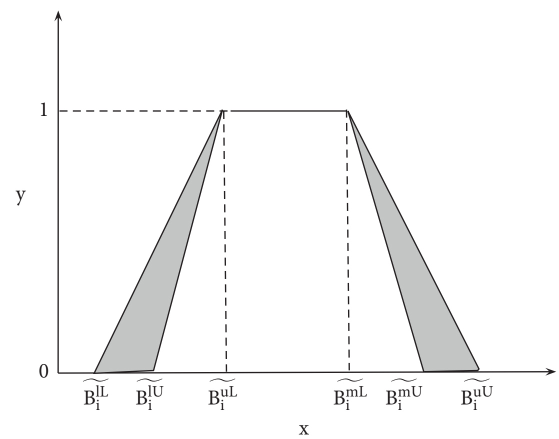

Figure 2: Fuzzy rough number $\widetilde{B}_i$.

2.3. Preference Functions (PFs). The use of a suitable and relevant preference function (PF) is an important requirement to implement the PROMETHEE approach. The PF describes the distance among alternatives related to every criterion. Brans et al. [50, 51] defined and implemented six types of PFs. These PFs can be implemented for almost all types of criteria and cover a large range of research problems.

Definition 3. The *usual criterion PF* is given as follows:

$$P\,(d) = \begin{cases} 1, & \text{whenever} d > 0; \\ 0, & \text{whenever} d \le 0. \end{cases} \tag{25}$$

Here, d is the deviation of any two alternatives. The criteria are represented by $\widehat{C}_j$, and an indifference appears between two alternatives $\widehat{A}_a$ and $\widehat{A}_b$ only when $\widehat{A}_a = \widehat{A}_b$.

Definition 4. The *quasi-criterion PF* is illustrated as a function as follows:

$$P\,(d) = \begin{cases} 1, & \text{whenever} d > c; \\ 0, & \text{whenever} d \le c. \end{cases} \tag{26}$$

Here, c is an indifference threshold value of any two alternatives. In this case, an indifference situation appears when the deviation of any two alternatives does not have a value greater than c. If not, then preference with strict value is acquired.

Definition 5. The *linear criterion PF* is defined as follows:

$$P\,(d) = \begin{cases} 1, & \text{whenever} d > k; \\ \dfrac{d}{k}, & \text{whenever} d \le k. \end{cases} \tag{27}$$

Here, $k \in [0, 1]$ is the preference value given by the experts. In this case, the experts' preference increases progressively with d as long as the alternatives' deviation has a value smaller than or equal to k. When d is greater than k, a strict preference of an alternative is obtained with respect to that criterion.

Definition 6. The *level criterion PF* is illustrated as follows:

$$P\,(d) = \begin{cases} 1, & \text{whenever} d > r + s; \\ 0.5, & \text{whenever} r < d \le r + s; \\ 0, & \text{whenever} d \le r. \end{cases} \tag{28}$$

Here, r is the preference value, s denotes an indifference value, and both are assigned by the experts and $0 \le r, s \le 1$. If the deviation within the alternatives has a value in the range from $-r$ to r, then an indifference can occur.

Definition 7. The *linear PF having indifference area* is given as follows:

$$P(d)=\begin{cases}1, & \text{whenever} d>p+q;\\[2ex] \dfrac{d-p}{q}, & \text{whenever} p<d\leq p+q;.\\[2ex] 0, & \text{whenever} d\leq p.\end{cases} \tag{29}$$

Here, p and q are the threshold values, which take values ranging from 0 to 1. In this situation of PF, two alternatives are regarded as indifferent thoroughly as soon as the distance within these alternatives does not have value greater than p. The preference values increase until the distance is equal to $p+q$. A preference with strict value occurs when the value exceeds the sum of p and q.

Definition 8. The *Gaussian criterion PF* is illustrated as follows:

$$P(d)=\begin{cases}1-\ \exp^{\frac{-d^2}{2\sigma^2}}, & \text{whenever} d>0;\\[2ex] 0, & \text{whenever} d\leq 0.\end{cases} \tag{30}$$

Here, σ shows the deviation between the origin and point of inflexion and $\sigma\in[0,1]$ is assigned according to the choice of experts.

3. The Proposed Fuzzy Rough PROMETHEE Method

PROMETHEE is a preference ranking organization method for enrichment evaluation and is an MCDM outranking approach that aims to outrank one alternative by the other regarding PFs and net outranking flow. In 1982, Brans et al. [50] developed the PROMETHEE model. The structure of this approach is based on PFs, which is the distance of any pair of alternatives relating to each criterion.

To overcome different MCDM limitations, an extended hybrid approach of PROMETHEE is implemented using fuzzy rough information, known as the fuzzy rough PROMETHEE approach. The assessment of alternatives regarding each criterion is the starting point in this approach, but this procedure requires numerical values for the evaluations and data are collected by comparing the contribution of each alternative regarding each criterion. The mathematical process of the PROMETHEE approach requires many steps as mentioned in the work of Behzadian et al. [52], Polat [53], Brans et al. [51], and Geldermann et al. [54]. The structure of fuzzy rough PROMETHEE methodology in a step-by-step diagram is shown in Figure 3. According to this structure, there is a sequence for the assessment of alternatives, choice of preference functions, the outgoing and incoming flow PROMETHEE I, and the net flow PROMETHEE II.

Step 1. Identification of the linguistic terms

Consider an MCDM problem having p alternatives $\widehat{A}_i\,(i=1,2,\ldots,p)$, which are evaluated for each criterion $\widehat{C}_j\,(j=1,2,\ldots,q)$. Suppose that r experts $\widehat{E}_k\,(k=1,2,\ldots,r)$ are selected to evaluate the alternatives regarding various criteria. For all experts $\widehat{E}_k$, an evaluation matrix $\widetilde{M^k}$ is constructed using the preference values as TFNs for alternatives $\widehat{A}_i$ regarding each criterion $\widehat{C}_j$. To evaluate the rating of an alternative regarding various criteria, experts individually give their opinion in the form of linguistic variables. It is necessary to recognize the suitable and relevant class of linguistic terms and describe the corresponding values of each linguistic term. In this approach, a group of eight linguistic terms is considered. These linguistic terms are taken as TFNs as shown in Figure 4. The numerical domain of these TFNs is taken from 1 to 10.

Step 2. Construct fuzzy rough evaluation matrix

Consider that the alternatives $\widehat{A}_i$ are assessed regarding each criterion $\widehat{C}_j$ that are estimated by each expert $\widehat{E}_k$. The fuzzy evaluation matrix $\widetilde{M^k}$ using measurement scales of Figure 4 is given as follows:

$$\widetilde{M^k}=\begin{bmatrix}\tilde{a}_{11}^k & \tilde{a}_{12}^k & \ldots & \tilde{a}_{1q}^k\\ \tilde{a}_{21}^k & \tilde{a}_{22}^k & \ldots & \tilde{a}_{2q}^k\\ \vdots & \vdots & & \vdots\\ \tilde{a}_{p1}^k & \tilde{a}_{p1}^k & \ldots & \tilde{a}_{pq}^k\end{bmatrix}, \tag{31}$$

where $\tilde{a}_{ij}^k=(a_{ijl}^k,a_{ijm}^k,a_{iju}^k)$, $i=1,2,\ldots,p$, and $j=1,2,\ldots,q$, is the evaluation value of an alternative i regarding the criterion j. In this fuzzy evaluation matrix, the triangular fuzzy ratings are converted into FRNs. Let V be the judgement set constructed from the assessment ratings composed by experts. $\widetilde{S}=$ $=\left\{\tilde{a}_{ij}^k=(a_{ijl}^k,a_{ijm}^k,a_{iju}^k)|k=1,2,\ldots,r\right\}$ is the set containing all TFN judgements, and X is an arbitrary universe. Then, the lower approximation of class $\tilde{a}_{ij}^k$ can be computed using the following equations:

$$\underline{Apr}\left(a_{ijl}^k\right)=\cup\left\{\frac{X\in V}{\widetilde{S}(X)\leq a_{ijl}^k}\right\}, \tag{32}$$

$$\underline{Apr}\left(a_{ijm}^k\right)=\cup\left\{\frac{X\in V}{\widetilde{S}(X)\leq a_{ijm}^k}\right\}, \tag{33}$$

$$\underline{Apr}\left(a_{iju}^k\right)=\cup\left\{\frac{X\in V}{\widetilde{S}(X)\leq a_{iju}^k}\right\}. \tag{34}$$

Similarly, the upper approximation of class $\tilde{a}_{ij}^k$ can be computed using the following equations:

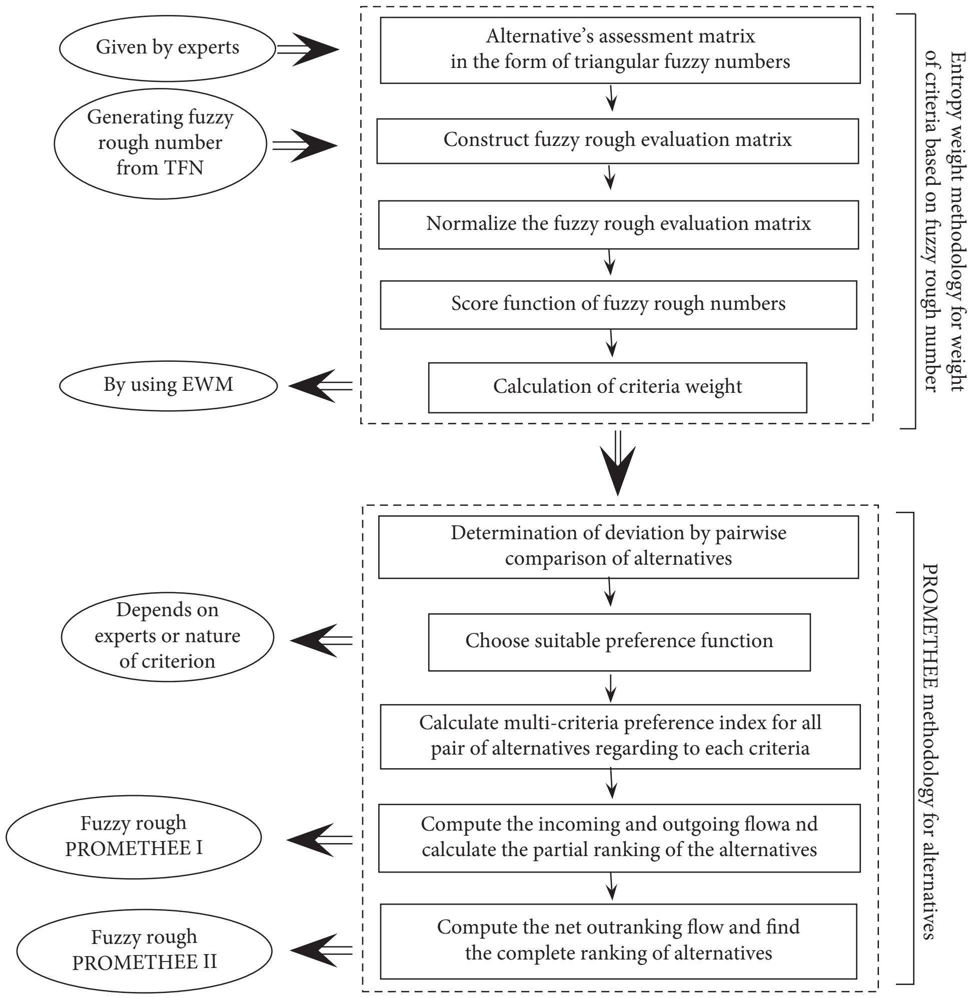

Figure 3: Flow diagram of fuzzy rough PROMETHEE approach.

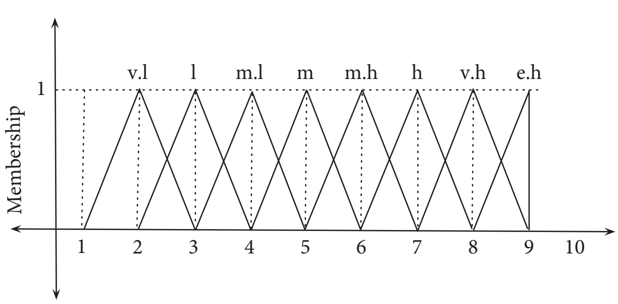

Figure 4: Linguistic terms for alternatives.

$$\overline{Apr}\left(a^k_{ijl}\right) = \cup \left\{ \frac{X \in V}{\tilde{S}(X) \ge a^k_{ijl}} \right\}, \quad (35)$$

$$\overline{Apr}\left(a^k_{ijm}\right) = \cup \left\{ \frac{X \in V}{\tilde{S}(X) \ge a^k_{ijm}} \right\}, \quad (36)$$

$$\overline{Apr}\left(a^k_{iju}\right) = \cup \left\{ \frac{X \in V}{\tilde{S}(X) \ge a^k_{iju}} \right\}. \quad (37)$$

The lower limit of class $\tilde{a}^k_{ij}$ is described in the following equations:

$$\underline{Lim}\left(a^k_{ijl}\right) = \frac{1}{n_{Ll}} \sum_{i=1}^{n_{Ll}} X \in \underline{Apr}\left(a^k_{ijl}\right), \quad (38)$$

$$\underline{Lim}\left(a^k_{ijm}\right) = \frac{1}{n_{Lm}} \sum_{i=1}^{n_{Lm}} X \in \underline{Apr}\left(a^k_{ijm}\right), \quad (39)$$

$$\underline{Lim}\left(a^k_{iju}\right) = \frac{1}{n_{Lu}} \sum_{i=1}^{n_{Lu}} X \in \underline{Apr}\left(a^k_{iju}\right), \quad (40)$$

where n_{Ll}, n_{Lm}, and n_{Lu} are the total number of elements in $\underline{Apr}\,(a^k_{ijl})$, $\underline{Apr}\,(a^k_{ijm})$, and $\underline{Apr}\,(a^k_{iju})$. Similarly, the upper limit of class $\tilde{a}^k_{ij}$ is illustrated in the following equations:

$$\overline{Lim}\left(a^k_{ijl}\right) = \frac{1}{n_{Ul}} \sum_{i=1}^{n_{Ul}} X \in \overline{Apr}\left(a^k_{ijl}\right), \quad (41)$$

$$\overline{Lim}\left(a^k_{ijm}\right) = \frac{1}{n_{Um}} \sum_{i=1}^{n_{Um}} X \in \overline{Apr}\left(a^k_{ijm}\right), \quad (42)$$

$$\overline{Lim}\left(a_{iju}^{k}\right)=\frac{1}{n_{Uu}}\sum_{i=1}^{n_{Uu}}X\in\overline{Apr}\left(a_{iju}^{k}\right),\tag{43}$$

where n_{Ul}, n_{Um}, and n_{Uu} are the total number of elements in $\overline{Apr}(a_{ijl}^{k}), \overline{Apr}(a_{ijm}^{k})$, and $\overline{Apr}(a_{iju}^{k})$. The fuzzy rough number of $\widetilde{B}_i$ is represented as follows:

$$FRN\left(\widetilde{a}_{ij}^{k}\right)=\left(\left[\underline{Lim}\left(a_{ijl}^{k}\right),\overline{Lim}\left(a_{ijl}^{k}\right)\right],\left[\underline{Lim}\left(a_{ijm}^{k}\right),\overline{Lim}\left(a_{ijm}^{k}\right)\right],\left[\underline{Lim}\left(a_{iju}^{k}\right),\overline{Lim}\left(a_{iju}^{k}\right)\right]\right).\tag{44}$$

The fuzzy rough evaluation matrix for the ranking of alternatives for kth expert is written as follows:

$$FR\left(\widetilde{M}^{k}\right)=\begin{bmatrix}\left(\left[a_{11l}^{kL},a_{11l}^{kU}\right],\left[a_{11m}^{kL},a_{11m}^{kU}\right],\left[a_{11u}^{kL},a_{11u}^{kU}\right]\right) & \cdots & \left(\left[a_{1ql}^{kL},a_{1ql}^{kU}\right],\left[a_{1qm}^{kL},a_{1qm}^{kU}\right],\left[a_{1qu}^{kL},a_{1qu}^{kU}\right]\right)\\ \left(\left[a_{21l}^{kL},a_{21l}^{kU}\right],\left[a_{21m}^{kL},a_{21m}^{kU}\right],\left[a_{21u}^{kL},a_{1u}^{kU}\right]\right) & \cdots & \left(\left[a_{2ql}^{kL},a_{2ql}^{kU}\right],\left[a_{2qm}^{kL},a_{2qm}^{kU}\right],\left[a_{2qu}^{kL},a_{2qu}^{kU}\right]\right)\\ \vdots & & \vdots\\ \left(\left[a_{p1l}^{kL},a_{p1l}^{kU}\right],\left[a_{p1m}^{kL},a_{p1m}^{kU}\right],\left[a_{p1u}^{kL},a_{p1u}^{kU}\right]\right) & \cdots & \left(\left[a_{pql}^{kL},a_{pql}^{kU}\right],\left[a_{pqm}^{kL},a_{pqm}^{kU}\right],\left[a_{pqu}^{kL},a_{pqu}^{kU}\right]\right)\end{bmatrix},\tag{45}$$

where $a_{ijl}^{kL}, a_{ijm}^{kL}, a_{iju}^{kL}, i=1,2,\ldots,p$, and $j=1,2,\ldots,q$ are, respectively, the lower limits corresponding to $a_{ijl}^{k}, a_{ijm}^{k}, a_{iju}^{k}$ and $a_{ijl}^{kU}, a_{ijm}^{kU}, a_{iju}^{kU}$ are the upper limits corresponding to $a_{ijl}^{k}, a_{ijm}^{k}, a_{iju}^{k}$, respectively.

Step 3. Aggregation and normalization of fuzzy rough evaluation matrix

The r fuzzy rough evaluation matrices $FR(\widetilde{M}^{k})$, $k=1,2,\ldots,r$, can be converted into a single aggregated fuzzy rough matrix $\widetilde{M}$ as given as follows:

$$\widetilde{M}=\begin{bmatrix}\left(\left[a_{11l}^{L},a_{11l}^{U}\right],\left[a_{11m}^{L},a_{11m}^{U}\right],\left[a_{11u}^{L},a_{11u}^{U}\right]\right) & \cdots & \left(\left[a_{1ql}^{L},a_{1ql}^{U}\right],\left[a_{1qm}^{L},a_{1qm}^{U}\right],\left[a_{1qu}^{L},a_{1qu}^{U}\right]\right)\\ \left(\left[a_{21l}^{L},a_{21l}^{U}\right],\left[a_{21m}^{L},a_{21m}^{U}\right],\left[a_{21u}^{L},a_{1u}^{U}\right]\right) & \cdots & \left(\left[a_{2ql}^{L},a_{2ql}^{U}\right],\left[a_{2qm}^{L},a_{2qm}^{U}\right],\left[a_{2qu}^{L},a_{2qu}^{U}\right]\right)\\ \vdots & \ddots & \vdots\\ \left(\left[a_{p1l}^{L},a_{p1l}^{U}\right],\left[a_{p1m}^{L},a_{p1m}^{U}\right],\left[a_{p1u}^{L},a_{p1u}^{U}\right]\right) & \cdots & \left(\left[a_{pql}^{L},a_{pql}^{U}\right],\left[a_{pqm}^{L},a_{pqm}^{U}\right],\left[a_{pqu}^{L},a_{pqu}^{U}\right]\right)\end{bmatrix},\tag{46}$$

where each fuzzy rough value $a_{ij}=([a_{ijl}^{L},a_{ijl}^{U}],[a_{ijm}^{L},a_{ijm}^{U}],[a_{iju}^{L},a_{iju}^{U}])$ can be obtained by taking the average of r fuzzy rough values a_{ij}^{k} as shown in the following formula:

$$a_{ji}=\left(\left[\frac{1}{r}\sum_{k=1}^{r}a_{ijl}^{kL},\frac{1}{r}\sum_{k=1}^{r}a_{ijl}^{kU}\right],\left[\frac{1}{r}\sum_{k=1}^{r}a_{ijm}^{kL},\frac{1}{r}\sum_{k=1}^{r}a_{ijm}^{kU}\right],\left[\frac{1}{r}\sum_{k=1}^{r}a_{iju}^{kL},\frac{1}{r}\sum_{k=1}^{r}a_{iju}^{kU}\right]\right).\tag{47}$$

To make the data comparable, the aggregated fuzzy rough evaluation matrix is normalized using the following formulae:

$$a_{ij}^{*}=\left(\left[\frac{a_{ijl}^{L}}{a_{j}^{+}},\frac{a_{ijl}^{U}}{a_{j}^{+}}\right],\left[\frac{a_{ijm}^{L}}{a_{j}^{+}},\frac{a_{ijm}^{U}}{a_{j}^{+}}\right],\left[\frac{a_{iju}^{L}}{a_{j}^{+}},\frac{a_{iju}^{U}}{a_{j}^{+}}\right]\right),\ j\in\mathbb{B},\tag{48}$$

$$a_{ij}^{*}=\left(\left[\frac{a_{j}^{-}}{a_{iju}^{U}},\frac{a_{j}^{-}}{a_{iju}^{L}}\right],\left[\frac{a_{j}^{-}}{a_{ijm}^{U}},\frac{a_{j}^{-}}{a_{ijm}^{L}}\right],\left[\frac{a_{j}^{-}}{a_{ijl}^{U}},\frac{a_{j}^{-}}{a_{ijl}^{L}}\right]\right),\ j\in\mathbb{C},\tag{49}$$

where $\mathbb{B}$ denotes the criterion of benefit, $\mathbb{C}$ represents the criterion of cost, $a_{j}^{+}=\max_{i}a_{iju}^{U}, j\in\mathbb{B}$, and $a_{j}^{-}=\min_{i}a_{ijl}^{L}, j\in\mathbb{C}$. The normalized fuzzy rough evaluation matrix $\widetilde{N}$ is represented as a matrix as follows:

$$\widetilde{N}=\begin{bmatrix}\left(\left[a_{11l}^{*L},a_{11l}^{*U}\right],\left[a_{11m}^{*L},a_{11m}^{*U}\right],\left[a_{11u}^{*L},a_{11u}^{*U}\right]\right) & \cdots & \left(\left[a_{1ql}^{*L},a_{1ql}^{*U}\right],\left[a_{1qm}^{*L},a_{1qm}^{*U}\right],\left[a_{1qu}^{*L},a_{1qu}^{*U}\right]\right)\\ \left(\left[a_{21l}^{*L},a_{21l}^{*U}\right],\left[a_{21m}^{*L},a_{21m}^{*U}\right],\left[a_{21u}^{*L},a_{1u}^{*U}\right]\right) & \cdots & \left(\left[a_{2ql}^{L},a_{2ql}^{*U}\right],\left[a_{2qm}^{*L},a_{2qm}^{*U}\right],\left[a_{2qu}^{*L},a_{2qu}^{*U}\right]\right)\\ \vdots & \ddots & \vdots\\ \left(\left[a_{p1l}^{*L},a_{p1l}^{*U}\right],\left[a_{p1m}^{*L},a_{p1m}^{*U}\right],\left[a_{p1u}^{*L},a_{p1u}^{*U}\right]\right) & \cdots & \left(\left[a_{pql}^{*L},a_{pql}^{*U}\right],\left[a_{pqm}^{*L},a_{pqm}^{*U}\right],\left[a_{pqu}^{*L},a_{pqu}^{*U}\right]\right)\end{bmatrix}. \tag{50}$$

Step 4. Score function of fuzzy rough numbers

The next steps of fuzzy rough PROMETHEE approach require crisp values; for this purpose, a fuzzy rough ranking formula (equation (51)) is applied on the normalized fuzzy rough values.

$$a'_{ij}=\frac{a_{pql}^{*L}+a_{pqm}^{*L}+a_{pqu}^{*L}+a_{pql}^{*U}+a_{pqm}^{*U}+a_{pqu}^{*U}}{6}. \tag{51}$$

These real values can be used to construct a simple evaluation matrix $\widetilde{D}=[a_{ij}]_{p\times q}$ and for the ranking of alternatives.

Step 5. Deviation of alternatives

Due to the dependency of preference structure on pairwise comparison of alternatives, deviation of alternatives is an important step in the fuzzy rough PROMETHEE approach. Regarding each criterion $\widehat{C}_j$, the deviation of alternatives can be calculated using the following formula:

$$D_j\left(\widehat{A}_i,\widehat{A}_k\right)=a_j\left(\widehat{A}_i\right)-a_j\left(\widehat{A}_k\right),\text{where} j,k=1,2,\ldots,p, \tag{52}$$

where $D_j(\widehat{A}_i,\widehat{A}_k)$ indicates the difference between any two alternatives $\widehat{A}_i$ and $\widehat{A}_k$ regarding each criterion. Here, $a_j(\widehat{A}_i)$ and $a_j(\widehat{A}_k)$ show the crisp values of alternatives $\widehat{A}_i$ and $\widehat{A}_k$, consequently, regarding the criteria j.

Step 6. Suitable choice of preference function

To calculate the preference of alternative $\widehat{A}_i$ with respect to the alternative $\widehat{A}_k$ regarding all criteria $\widehat{C}_j$, the suitable choice of preference function $P_j(\widehat{A}_i,\widehat{A}_k)=F_j[D_j(\widehat{A}_i,\widehat{A}_k)]$ plays an important role and the values lie in the unit interval. Negative values or 0 of the preference function mean that there is indifference for experts for these pairs of alternatives regarding all criteria. If the preference function has a value nearest to 1, it tells that there is a maximum preference and a preference value nearer to zero tells about the weak preference. The choice of suitable preference function is an essential step in this approach due to which ranking of alternatives can be changed. To fulfill the requirements for the selection of suppliers, this study used the definitions of preference function.

(i) $P_j(\widehat{A}_i,\widehat{A}_k)\sim 0$ expresses that there is a weak preference of $\widehat{A}_i$ over $\widehat{A}_k$.

(ii) $P_j(\widehat{A}_i,\widehat{A}_k)=0$ expresses that there is no preference of $\widehat{A}_i$ over $\widehat{A}_k$.

(iii) $P_j(\widehat{A}_i,\widehat{A}_k)\sim 1$ expresses that there is a powerful preference of $\widehat{A}_i$ over $\widehat{A}_k$.

(iv) $P_j(\widehat{A}_i,\widehat{A}_k)=1$ expresses that there is a strict preference of $\widehat{A}_i$ over $\widehat{A}_k$.

Step 7. Determination of criteria weights

To calculate the weight of each criterion, experts used different methods to give importance to each criterion $\widehat{C}_j\,(j=1,2,\ldots,q)$ using different measurement scales. These scales or values partly or completely depend on the choice of experts and also describe the corresponding significance of one criterion regarding other criteria. Various methods are utilized for calculating criteria weights. In this method, an effective technique known as the entropy weight method (EWM) by Shannon and Weaver [32] and Zhang et al. [55] was used to find the normalized weight of criteria. The benefit of using this method is that weight of criteria can be calculated from given data, and experts have no need to define any arbitrary scales or values for criteria. This method used the vagueness of the given information and diminished to depend on experts' individual thinking. Since normalized values of criteria are used in EWM, this is the reason for using the values calculated from normalized evaluation data in score matrix and calculating the projection values $\mathbb{P}(ij)$ of criterion $\widehat{C}_j$ as given as follows:

$$P(ij)=\frac{a_{ij}}{\sum_{i=1}^{p}a_{ij}}. \tag{53}$$

The projection values are utilized to get the entropy value $\mathbb{E}(j)$ for each criterion $\widehat{C}_j$ as described as follows:

$$\mathbb{E}(j)=-k\sum_{i=1}^{p}\mathbb{P}(ij)\log\mathbb{P}(ij), \tag{54}$$

where $k=(\log(p))^{-1}$ is known as constant of entropy. Then, the divergence degree $\mathbb{D}(j)$ of the given data for each criterion is calculated using the following formula:

$$\mathbb{D}(j)=1-\mathbb{E}(j),\ j=1,2,\ldots,q. \tag{55}$$

These calculated values of each divergence degree $\mathbb{D}(j)$ show the inherent contrast intensity of each criterion $\widehat{C}_j$. The greater $\mathbb{D}(j)$ value tells that the criterion $\widehat{C}_j$ is considered as much essential for that problem. The criteria weights $\mathbb{W}(j)$ can be computed using the following equation:

$$\mathbb{W}(j) = \frac{\mathbb{D}(j)}{\sum_{j=1}^{q} \mathbb{D}(j)}. \tag{56}$$

Step 8. Computation of the multicriteria preference index

After calculating the weight of all criteria using entropy weight method and describing the PF according to the nature of criterion, the next step is computing the preference index of alternatives. The preference index of all alternatives can be computed as weighted average value related to that preference function and can be depicted using the following formula:

$$\prod\left(\widehat{A}_i, \widehat{A}_k\right) = \frac{\sum_{j=1}^{q} \mathbb{W}(j) P_j\left(\widehat{A}_i, \widehat{A}_k\right)}{\sum_{j=1}^{q} \mathbb{W}(j)}, wherei \neq k, an\ di\ , k = 1, 2, \ldots, p. \tag{57}$$

Since this study proposes the normalized weights, i.e., $\sum_{j=1}^{q} \mathbb{W}(j) = 1$, equation (57) can also be written as the following formula:

$$\prod\left(\widehat{A}_i, \widehat{A}_k\right) = \sum_{j=1}^{q} \mathbb{W}(j) P_j\left(\widehat{A}_i, \widehat{A}_k\right), \text{where} i \neq k, \text{and} i, k = 1, 2, \ldots, p, \tag{58}$$

where $0 \leq \prod(\widehat{A}_i, \widehat{A}_k) \leq 1$.

(i) $\prod(\widehat{A}_i, \widehat{A}_k) \approx 0$ defines the weak preference of alternatives $\widehat{A}_i$ to $\widehat{A}_k$ relating to each criterion.

(ii) $\prod(\widehat{A}_i, \widehat{A}_k) \approx 1$ indicates the powerful preference of alternatives $\widehat{A}_i$ to $\widehat{A}_k$ regarding each criterion.

This preference index produces an outranking relation on the alternative set $\widehat{A}$, regarding each criterion, and then is depicted through an outranking graph. The alternatives are represented by the vertices in this outranking graph, and the arcs among them represent the relation among the alternatives.

Step 9. Preference ordering of alternatives

The alternatives' outranking relation calculated through the previous step is used to find the alternatives' preference ranking. The partial ranking and complete ranking are two types of ranking that are obtained using this approach. The fuzzy rough PROMETHEE I describes the incoming and outgoing flows and is used for the partial ranking of the alternatives, and fuzzy rough PROMETHEE II defines an additional step by describing the net flow and is used for the complete ranking of alternatives.

3.1. Fuzzy Rough PROMETHEE I (Partial Ranking of Alternatives). The outgoing or positive flow of each alternative $\widehat{A}_i (i = 1, 2, \ldots, p)$ is denoted by $\Upsilon^+(\widehat{A}_i)$ and is described as follows:

$$\Upsilon^+\left(\widehat{A}_i\right) = \frac{1}{p-1} \sum_{\widehat{A}_k \in \widehat{A}} \prod\left(\widehat{A}_i, \widehat{A}_k\right), (i \neq k, \text{ and } i, k = 1, 2, \ldots, p). \tag{59}$$

The outgoing or positive flow is represented by taking the average of outward arcs of alternative $\widehat{A}_i$ and shows how an alternative $\widehat{A}_i$ prevails all the other alternatives as shown in Figure 5.

Similarly, the incoming or negative flow of each alternative $\widehat{A}_i (i = 1, 2, \ldots, p)$ is denoted by $\Upsilon^-(\widehat{A}_i)$ as given as follows:

$$\Upsilon^-\left(\widehat{A}_i\right) = \frac{1}{p-1} \sum_{\widehat{A}_k \in \widehat{A}} \prod\left(\widehat{A}_i, \widehat{A}_k\right), (i \neq k, \text{ and } i, k = 1, 2, \ldots, p). \tag{60}$$

The incoming or negative flow is represented by taking the average of inward arcs of alternative $\widehat{A}_i$ and tells that how an alternative $\widehat{A}_i$ is prevailed by all the other alternatives as shown in Figure 6.

The alternative with the higher value of $\Upsilon^+(\widehat{A}_i)$ and lower value of $\Upsilon^-(\widehat{A}_i)$ is considered to be the best alternative. The preferences of alternatives regarding the positive and negative flows can be found using the following expressions and Figure 6:

$$\begin{cases} \widehat{A}_i \mathbb{P}^+ \widehat{A}_k \Leftrightarrow \Upsilon^+\left(\widehat{A}_i\right) > \Upsilon^+\left(\widehat{A}_k\right), \forall \widehat{A}_i, \widehat{A}_k \in \widehat{A}; \\ \widehat{A}_i \mathbb{I}^+ \widehat{A}_k \Leftrightarrow \Upsilon^+\left(\widehat{A}_i\right) = \Upsilon^+\left(\widehat{A}_k\right), \forall \widehat{A}_i, \widehat{A}_k \in \widehat{A}. \end{cases}, \tag{61}$$

$$\begin{cases} \widehat{A}_i \mathbb{P}^- \widehat{A}_k \Leftrightarrow \Upsilon^-\left(\widehat{A}_i\right) < \Upsilon^-\left(\widehat{A}_k\right), \forall \widehat{A}_i, \widehat{A}_k \in \widehat{A}; \\ \widehat{A}_i \mathbb{I}^- \widehat{A}_k \Leftrightarrow \Upsilon^-\left(\widehat{A}_i\right) = \Upsilon^-\left(\widehat{A}_k\right), \forall \widehat{A}_i, \widehat{A}_k \in \widehat{A}. \end{cases}. \tag{62}$$

Taking the intersection of these two preferences, the partial ranking $(\mathbb{P}_1, \mathbb{I}_1, \mathbb{R}_1)$ can be obtained using the following expression, sequentially.

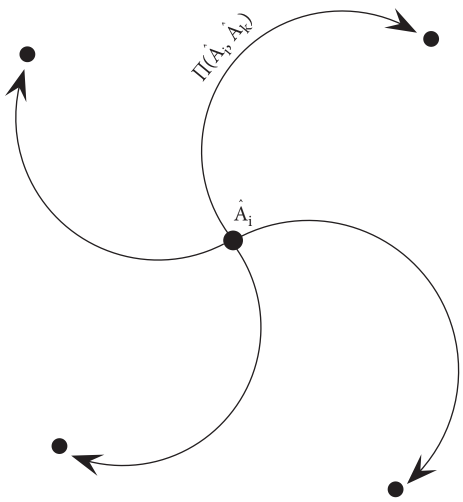

Figure 5: Positive flow of $\widehat{A}_i$.

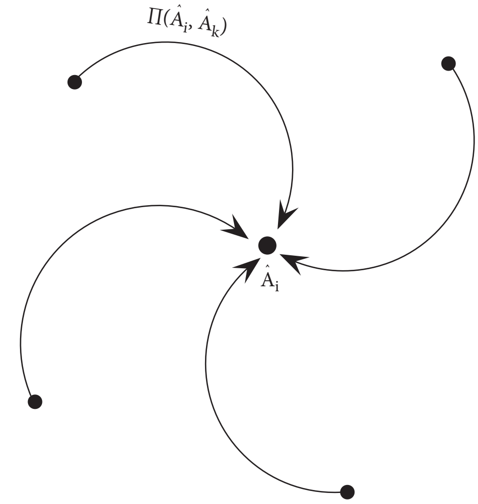

Figure 6: Negative flow of $\widehat{A}_i$.

$$\begin{cases} \widehat{A}_i \mathbb{P}_1 \widehat{A}_k \left(\widehat{A}_i\, outranks \widehat{A}_k\right), & if \widehat{A}_i \mathbb{P}^+ \widehat{A}_k an\ d \widehat{A}_i \mathbb{P}^- \widehat{A}_k; \\ & or \widehat{A}_i \mathbb{P}^+ \widehat{A}_k an\ d \widehat{A}_i \mathbb{I}^- \widehat{A}_k; \\ & or \widehat{A}_i \mathbb{I}^+ \widehat{A}_k an\ d \widehat{A}_i \mathbb{P}^- \widehat{A}_k;\ . \\ \widehat{A}_i \mathbb{I}_1 \widehat{A}_k \left(\widehat{A}_i isin\ di\ f ferenceto \widehat{A}_k\right), & i f f \widehat{A}_i \mathbb{I}^+ \widehat{A}_k an\ d \widehat{A}_i \mathbb{I}^- \widehat{A}_k; \\ \widehat{A}_i \mathbb{R}_1 \widehat{A}_k \left(incom parebilit y within \widehat{A}_i an\ d \widehat{A}_k\right), & otherwise. \end{cases} \tag{63}$$

Here, $\mathbb{P}_1, \mathbb{I}_1$, and $\mathbb{R}_1$ represent the preference, indifference, and incomparability of alternatives. Fuzzy rough PROMETHEE I cannot directly select the best alternative because this decision depends on experts. The alternatives are unable to compare in the fuzzy rough PROMETHEE I; so, to choose the best alternative, there is a need to define complete ranking method known as fuzzy rough PROMETHEE II by proceeding an additional step.

3.2. Fuzzy Rough PROMETHEE II (Complete Ranking of Alternatives). Fuzzy rough PROMETHEE II shows the difference between outgoing and incoming flows among all the alternatives. The net outranking flow of all alternatives can be figured using the following formula:

$$\Upsilon\left(\widehat{A}_i\right) = \Upsilon^+\left(\widehat{A}_i\right) - \Upsilon^-\left(\widehat{A}_i\right). \tag{64}$$

There is equity between positive and negative outranking flows, and the alternative with a higher value of net flow is considered as the best. Complete ranking $(\mathbb{P}_2, \mathbb{I}_2)$ of alternatives can be figured as given as follows:

$$\begin{cases} \widehat{A}_i P_2 \widehat{A}_k \left(\widehat{A}_i outranks \widehat{A}_k\right), & i f f \Upsilon\left(\widehat{A}_i\right) > \Upsilon\left(\widehat{A}_k\right); \\ \widehat{A}_i I_2 \widehat{A}_k \left(\widehat{A}_i isin\ di\ f ferentto \widehat{A}_k\right), & i f f \Upsilon\left(\widehat{A}_i\right) = \Upsilon\left(\widehat{A}_k\right). \end{cases} \tag{65}$$

The incomparability situation can be avoided in complete ranking. All alternatives are comparable in the fuzzy rough PROMETHEE II regarding net flows $\Upsilon(\widehat{A}_i)$.

All the MCDM techniques have a sequence of steps and calculations, in which except for the choice of calculating weights of criteria and choice of preference functions all other steps remain almost the same. To choose a suitable preference, function is dependent on the type of criterion or on the choice of experts. Furthermore, to find the multicriteria preference index, there is a need to calculate the normalized criteria weights by applying suitable techniques.

4. A Case Study in Industrial Manufacturing

A manufacturing unit of steel and iron company in Pakistan has some problems with the performance of suppliers. For raw materials, the managers want to choose the suppliers and get competitive benefits in the marketplace. There are four uniformly authorized suppliers or alternatives $\widehat{A}_1, \widehat{A}_2, \widehat{A}_3, \widehat{A}_4$ that have been considered for the selection, and a group of decision makers containing four experts $\widehat{E}_1, \widehat{E}_2, \widehat{E}_3, \widehat{E}_4$ is selected to choose the best suppliers according to the requirements. Due to the confidential policy of the concerned steel and iron manufacturing industry, names of the suppliers and steel manufacturing units are not allowed to be disclosed. To evaluate the selection of suppliers, five criteria are under consideration that are taken from the total set of criteria of steel and iron industry. The different criteria are as follows: cost ($\widehat{C}_1$), product's quality ($\widehat{C}_2$), transmission capacity ($\widehat{C}_3$), reputation ($\widehat{C}_4$), and performance ($\widehat{C}_5$).

Step 10. To examine the suppliers' selection for a small-scale steel manufacturing unit, the industry has practiced the proposed fuzzy rough PROMETHEE and EWM as its approach. All the possible suppliers $\widehat{A}_1, \widehat{A}_2, \widehat{A}_3, \widehat{A}_4$ are figured out, different assessment criteria $\widehat{C}_1, \widehat{C}_2, \widehat{C}_3, \widehat{C}_4$, and $\widehat{C}_5$ are determined, and a committee of four experts $\widehat{E}_1, \widehat{E}_2, \widehat{E}_3, \widehat{E}_4$ is formed.

Step 11. Define suitable linguistic rating as TFNs as shown in Table 2.

Step 12. Table 3 shows the evaluation matrix for different alternatives regarding each criterion as triangular fuzzy numbers.

Step 13. Table 3 is converted into TFNs and then fuzzy rough numbers using equations (32) to (43) and (47). The aggregated fuzzy rough evaluation matrix is computed in Tables 4 and 5.

Step 14. A normalized fuzzy rough evaluation matrix can be obtained using the normalization formulae of fuzzy rough numbers given in equations (48) and (49). The outcomes are given in Tables 6 and 7.

Step 15. The score function of fuzzy rough numbers can be obtained using the formula of score function of fuzzy rough numbers defined in equation (51), and the results are given in Table 8.

Step 16. The deviation of suppliers indicates the difference between any two suppliers regarding each criterion and is shown in Table 9. The choice of suitable PF depends on the nature of criteria and experts' knowledge and thinking. In the proposed fuzzy rough method, a usual criterion PF given in equation (25) is applied. The preference index of every supplier is computed using the usual criterion PF as shown in Table 10.

To calculate the weights of criteria, an EWM is applied in this study. Supplier's projection values can be obtained using equation (53), and the results are shown in Table 11. Using equations (54)–(57), entropy values, divergence values, and criteria weights can be computed as given in Table 12.

Step 17. The preference index of all the suppliers can be obtained using equations (57) and (58). The positive and negative flows of suppliers can be computed using equations (59) and (60), and partial ordering of suppliers can be obtained using equations and (63)–(64) and Figure 6. The results are given in Tables 13 and 14.

The intersection of preorders P^+ and P^- shows the partial ranking of suppliers as follows: $\widehat{A}_1\mathbb{P}_1\widehat{A}_2$, $\widehat{A}_1\mathbb{P}_1\widehat{A}_3$, $\widehat{A}_1\mathbb{P}_1\widehat{A}_4$, $\widehat{A}_3\mathbb{P}_1\widehat{A}_2$, $\widehat{A}_3\mathbb{P}_1\widehat{A}_4$, and $\widehat{A}_4\mathbb{P}_1\widehat{A}_2$. The partial ordering of suppliers is shown in Figure 7.

Table 2: Linguistic ratings for the evaluation of suppliers.

Linguistic terms	TFNs
Very low (vl)	(1, 2, 3)
Low (l)	(2, 3, 4)
Medium low (ml)	(3, 4, 5)
Medium (m)	(4, 5, 6)
Medium high (mh)	(5, 6, 7)
High (h)	(6, 7, 8)
Very high (vh)	(7, 8, 9)
Extremely high (eh)	(8, 9, 9)

Table 3: Assessment information for supplier's ranking.

Suppliers	Experts	Criteria				
		$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$
$\widehat{A}_1$	$\widehat{E}_1$	vl	vh	vh	h	h
	$\widehat{E}_2$	vl	eh	vh	vh	h
	$\widehat{E}_3$	l	vh	vh	h	h
	$\widehat{E}_4$	vl	vh	eh	vh	h
$\widehat{A}_2$	$\widehat{E}_1$	l	m	eh	vh	h
	$\widehat{E}_2$	ml	mh	vh	vh	mh
	$\widehat{E}_3$	l	me	h	vh	h
	$\widehat{E}_4$	ml	mh	eh	vh	h
$\widehat{A}_3$	$\widehat{E}_1$	l	h	eh	h	h
	$\widehat{E}_2$	l	h	eh	vh	vh
	$\widehat{E}_3$	l	h	vh	h	vh
	$\widehat{E}_4$	l	h	vh	h	vh
$\widehat{A}_4$	$\widehat{E}_1$	l	mh	h	eh	vh
	$\widehat{E}_2$	l	mh	h	eh	vh
	$\widehat{E}_3$	l	h	vh	eh	eh
	$\widehat{E}_4$	ml	h	h	eh	eh

Step 18. The complete ordering is calculated from the net flow, and the results are given in descending order. It is concluded from the outcomes of Table 15 and situation of suppliers in Figure 8 that $\widehat{A}_1$ is the best one supplier while ranking of suppliers is $\widehat{A}_1 > \widehat{A}_3 > \widehat{A}_4 > \widehat{A}_2$.

5. Comparative Study

In this section, we conduct some comparative studies in two different ways to examine the authenticity and significance of our suggested fuzzy rough PROMETHEE approach. Firstly, different combinations of preference functions are implemented and then a comparison with existing MCDM techniques is conducted. The outcomes of the suggested approach are compared with the outcomes of different combinations of preference functions, fuzzy TOPSIS, crisp VIKOR, and fuzzy rough TOPSIS techniques.

5.1. Combination of Various Types of PFs. The important benefit of the PROMETHEE approach is the choice of PFs. In this subsection, the preference functions are combined differently to provide more rational results and to check the validity of proposed method. First of all, a combination of linear and level criterion PFs is used. Then, a combination of

Table 4: Aggregated fuzzy rough assessment matrix for criteria $\widehat{C}_1$ to$\widehat{C}_3$.

Suppliers ↓	Criteria $\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$
$\widehat{A}_1$	([1.06, 1.44], [2.06, 2.44], [3.06, 3.44])	([7.06, 7.44], [8.06, 8.44], [9.00, 9.00])	([7.06, 7.44], [8.06, 8.44], [9.00, 9.00])
$\widehat{A}_2$	([2.25, 2.75], [3.25, 3.75], [4.25, 4.75])	([4.25, 4.75], [5.25, 5.75], [6.25, 6.75])	([7.56, 7.94], [8.56, 8.94], [9.00, 9.00])
$\widehat{A}_3$	([2.00, 2.00], [3.00, 3.00], [4.00, 4.00])	([6.00, 6.00], [7.00:, 7.00], [8.00, 8.00])	([7.25, 7.75], [8.25, 8.75], [9.00, 9.00])
$\widehat{A}_4$	([2.06, 2.44], [3.06, 3.44], [4.06, 4.44])	([5.25, 5.75], [6.25, 6.75], [7.25, 7.75])	([6.06, 6.44], [7.06, 7.44], [8.06, 8.44])

Table 5: Aggregated fuzzy rough assessment matrix for criteria $\widehat{C}_4$ and$\widehat{C}_5$.

Suppliers ↓	Criteria $\widehat{C}_4$	$\widehat{C}_5$
$\widehat{A}_1$	([6.25, 6.75], [7.25, 7.75], [8.25, 8.75])	([6.00, 6.00], [7.00, 7.00], [8.00, 8.00])
$\widehat{A}_2$	([7.00, 7.00], [8.00, 8.00], [9.00, 9.00])	([5.56, 5.94], [6.56, 6.94], [7.56, 7.94])
$\widehat{A}_3$	([6.06, 6.44], [7.06, 7.44], [8.06, 8.44])	([6.56, 6.94], [7.56, 7.94], [8.56, 8.94])
$\widehat{A}_4$	([8.00, 8.00], [9.00, 9.00], [9.00, 9.00])	([7.25, 7.75], [8.25, 8.75], [9.00, 9.00])

Table 6: Normalized form of fuzzy rough assessment matrix for criteria $\widehat{C}_1$ to$\widehat{C}_3$.

Suppliers ↓	Criteria $\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$
$\widehat{A}_1$	([0.31, 0.35], [0.43, 0.51], [0.75, 1.00])	([0.78, 0.83], [0.90, 0.94], [1.00, 1.00])	([0.78, 0.83], [0.90, 0.94], [1.00, 1.00])
$\widehat{A}_2$	([0.22, 0.25], [0.28, 0.33], [0.39, 0.47])	([0.47, 0.53], [0.58, 0.64], [0.69, 0.75])	([0.84, 0.88], [0.95, 0.99], [1.00, 1.00])
$\widehat{A}_3$	([0.27, 0.27], [0.35, 0.35], [0.53, 0.53])	([0.67, 0.67], [0.78, 0.78], [0.89, 0.89])	([0.81, 0.86], [0.92, 0.97], [1.00, 1.00])
$\widehat{A}_4$	([0.24, 0.26], [0.31, 0.35], [0.43, 0.51])	([0.58, 0.64], [0.69, 0.75], [0.81, 0.86])	([0.67, 0.72], [0.78, 0.83], [0.90, 0.94])

Table 7: Normalized form of fuzzy rough assessment matrix for criteria $\widehat{C}_4$ and$\widehat{C}_5$.

Suppliers ↓	Criteria $\widehat{C}_4$	$\widehat{C}_5$
$\widehat{A}_1$	([0.69, 0.75], [0.81, 0.86], [0.92, 0.97])	([0.67, 0.67], [0.78, 0.78], [0.89, 0.89])
$\widehat{A}_2$	([0.78, 0.78], [0.89, 0.89], [1.00, 1.00])	([0.62, 0.66], [0.73, 0.77], [0.84, 0.88])
$\widehat{A}_3$	([0.67, 0.72], [0.78, 0.83], [0.90, 0.94])	([0.73, 0.77], [0.84, 0.88], [0.95, 0.99])
$\widehat{A}_4$	([0.89, 0.89], [1.00, 1.00], [1.00, 1.00])	([0.81, 0.86], [0.92, 0.97], [1.00, 1.00])

Table 8: Score values of suppliers corresponding to criteria.

Suppliers	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$
$\widehat{A}_1$	0.558	0.908	0.908	0.833	0.780
$\widehat{A}_2$	0.323	0.610	0.943	0.890	0.750
$\widehat{A}_3$	0.383	0.780	0.927	0.807	0.860
$\widehat{A}_4$	0.350	0.722	0.807	0.963	0.927

Table 9: Deviation of suppliers.

	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$
$D(\widehat{A}_1 - \widehat{A}_2)$	0.235	0.298	−0.035	−0.057	0.030
$D(\widehat{A}_1 - \widehat{A}_3)$	0.175	0.128	−0.019	0.026	−0.080
$D(\widehat{A}_1 - \widehat{A}_4)$	0.208	0.186	0.101	−0.130	−0.147
$D(\widehat{A}_2 - \widehat{A}_1)$	−0.235	−0.298	0.035	0.057	−0.030
$D(\widehat{A}_2 - \widehat{A}_3)$	−0.060	−0.170	0.016	0.083	−0.110
$D(\widehat{A}_2 - \widehat{A}_4)$	−0.027	−0.112	0.136	−0.073	−0.177
$D(\widehat{A}_3 - \widehat{A}_1)$	−0.175	−0.128	0.019	−0.026	0.080
$D(\widehat{A}_3 - \widehat{A}_2)$	0.060	0.170	−0.016	−0.083	0.110
$D(\widehat{A}_3 - \widehat{A}_4)$	0.033	0.058	0.120	−0.156	−0.067
$D(\widehat{A}_4 - \widehat{A}_1)$	−0.208	−0.186	−0.101	0.130	0.147
$D(\widehat{A}_4 - \widehat{A}_2)$	0.027	0.112	−0.136	0.073	0.177
$D(\widehat{A}_4 - \widehat{A}_3)$	−0.033	−0.058	−0.120	0.156	0.067

Gaussian, quasi, and linear PF with indifference area is implemented. The results obtained using these different combinations of preference functions are given in detail.

5.1.1. Combination of Linear and Level Preference Function (Akram Et Al. [28]). The selection of different PFs is based on the experts' opinions or the nature of criteria. In this method, a comparative study by combining both linear and level PF is presented. The linear PF is implemented on cost type (quantitative) criteria $\widehat{C}_1$, whereas level criterion PF is applied on benefit type (qualitative) criteria, i.e., $\widehat{C}_2, \widehat{C}_3, \widehat{C}_4$, and $\widehat{C}_5$. The experts' evaluation matrix and method of calculating the weights of criteria are the same as given in the proposed approach.

The linear PF has a preference threshold value $k = 0.2$ and the level criterion PF has preference and indifference values $r = 0.03$ and $s = 0.2$, respectively. The partial and complete orderings of suppliers are shown in Table 16.

Table 10: Usual criterion PF.

	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$
$D(\widehat{A}_1 - \widehat{A}_2)$	1	1	0	0	1
$D(\widehat{A}_1 - \widehat{A}_3)$	1	1	0	1	0
$D(\widehat{A}_1 - \widehat{A}_4)$	1	1	1	0	0
$D(\widehat{A}_2 - \widehat{A}_1)$	0	0	1	1	0
$D(\widehat{A}_2 - \widehat{A}_3)$	0	0	1	1	0
$D(\widehat{A}_2 - \widehat{A}_4)$	0	0	1	0	0
$D(\widehat{A}_3 - \widehat{A}_1)$	0	0	1	0	1
$D(\widehat{A}_3 - \widehat{A}_2)$	1	1	0	0	1
$D(\widehat{A}_3 - \widehat{A}_4)$	1	1	1	0	0
$D(\widehat{A}_4 - \widehat{A}_1)$	0	0	0	1	1
$D(\widehat{A}_4 - \widehat{A}_2)$	1	1	0	1	1
$D(\widehat{A}_4 - \widehat{A}_3)$	0	0	0	1	1

Table 11: Supplier's projection values.

$P(i,j)$	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$
$\widehat{A}_1$	0.140	0.228	0.228	0.209	0.196
$\widehat{A}_2$	0.092	0.173	0.268	0.253	0.213
$\widehat{A}_3$	0.102	0.208	0.247	0.215	0.229
$\widehat{A}_4$	0.093	0.192	0.214	0.256	0.246

Table 12: Entropy value, divergence, and weights of criteria.

	$\widehat{C}_1$	$\widehat{C}_2$	$\widehat{C}_3$	$\widehat{C}_4$	$\widehat{C}_5$
$E(j)$	0.684	0.926	0.985	0.977	0.960
$D(j)$	0.316	0.074	0.015	0.023	0.040
$W(j)$	0.675	0.158	0.032	0.049	0.085

Table 13: Preference index of suppliers.

Suppliers	$\widehat{A}_1$	$\widehat{A}_2$	$\widehat{A}_3$	$\widehat{A}_4$
$\widehat{A}_1$	—	0.918	0.882	0.865
$\widehat{A}_2$	0.081	—	0.081	0.032
$\widehat{A}_3$	0.117	0.918	—	0.865
$\widehat{A}_4$	0.134	0.967	0.134	—

Table 14: Positive and negative flows of suppliers.

Suppliers	$\Upsilon^{+}(\widehat{A}_i)$	$\Upsilon^{-}(\widehat{A}_i)$
$\widehat{A}_1$	0.888	0.111
$\widehat{A}_2$	0.065	0.934
$\widehat{A}_3$	0.633	0.366
$\widehat{A}_4$	0.412	0.587

Figure 9 displays the outcomes of POMETHEE I using linear and level preference functions.

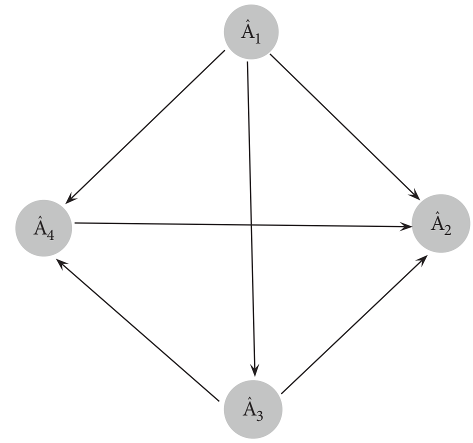

Figure 7: Partial ordering by usual criterion PF.

Table 15: Net flows and final ordering of suppliers.

Suppliers	Net flow $\Upsilon(A_i)$	Ranking
$\widehat{A}_1$	0.777	1
$\widehat{A}_2$	−0.869	4
$\widehat{A}_3$	0.267	2
$\widehat{A}_4$	−0.175	3

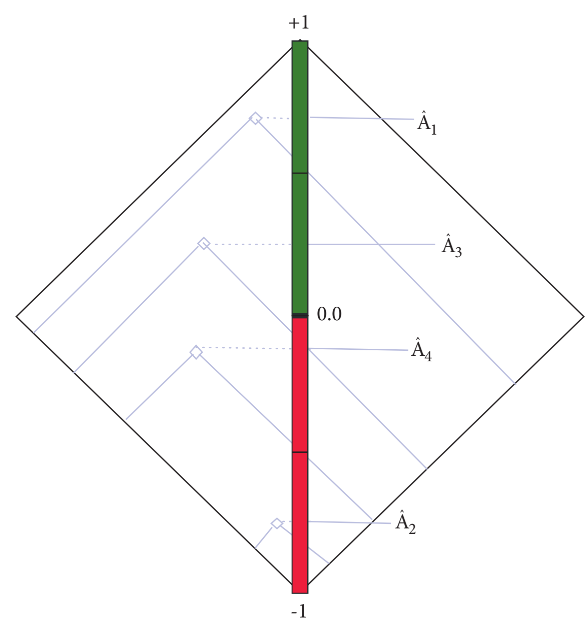

Figure 8: PROMETHEE diamond.

5.1.2. Combination of Gaussian, Quasi, and Linear with Indifference Area PF (Akram Et Al. [29]). The proposed method is compared with a combination of Gaussian PF, quasi PF, and linear PF with indifference area PF. These PFs are selected for various criteria. The Gaussian preference function is selected for $\widehat{C}_1$ cost type criteria, whereas quasi-criterion PF is implemented for benefit type criteria, i.e., $\widehat{C}_2$ and $\widehat{C}_3$. Moreover, the linear preference function with indifference area is selected for $\widehat{C}_4$ and $\widehat{C}_5$ benefit type criteria. The experts' evaluation matrix and the method of calculating the weights of criteria are the same as those given in the proposed approach.

In the Gaussian preference function, the value of σ shows the deviation within the origin and the point of inflexion. The value of σ is 0.02, which is given by the experts, whereas the quasi-criterion PF has an indifference threshold value $c = 0.02$ and linear PF with indifference area has threshold values $p = 0.01$ and $q = 0.2$. The partial and complete

Table 16: Ranking of suppliers by linear and level PF.

Suppliers	$\Upsilon^{+}(A_i)$	$\Upsilon^{-}(A_i)$	Net flow $\Upsilon(A_i)$	Ordering
$\widehat{A}_1$	0.758	0.050	0.708	1
$\widehat{A}_2$	0.027	0.465	-0.438	4
$\widehat{A}_3$	0.191	0.254	-0.063	2
$\widehat{A}_4$	0.124	0.331	-0.207	3

Table 17: Ranking of suppliers using three different PFs.

Suppliers	$\Upsilon^{+}(A_i)$	$\Upsilon^{-}(A_i)$	Net flow $\Upsilon(A_i)$	Ordering
$\widehat{A}_1$	0.848	0.054	0.794	1
$\widehat{A}_2$	0.031	0.786	−0.755	4
$\widehat{A}_3$	0.530	0.305	0.225	2
$\widehat{A}_4$	0.265	0.530	−0.265	3

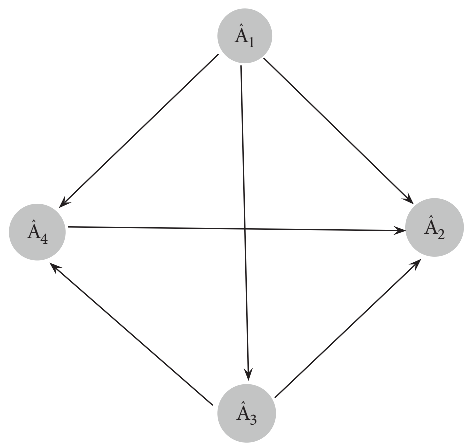

Figure 9: Partial ordering by linear and level PF.

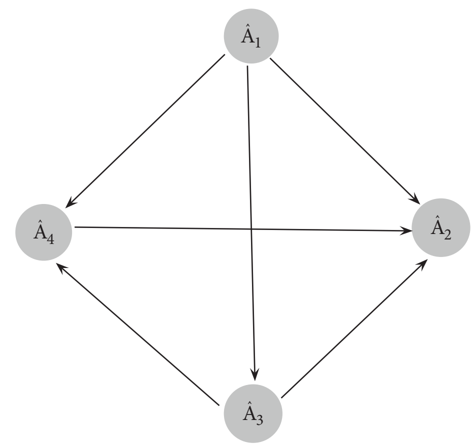

Figure 10: Partial ordering using Gaussian, quasi, and linear with indifference area PF.

orderings of suppliers are shown in Table 17. Figure 10 shows the results of PROMETHEE I using combination of Gaussian, quasi, and linear with indifference area preference functions.

5.1.3. Discussion on Utilizing Various Types of PFs. To check the behavior of the PROMETHEE approach using preference functions, a comparative study is conducted by applying various combinations of preference functions. In the proposed approach, a simple and easy type of PF known as the usual criterion PF is implemented. Then, a comparative study is organized for the ranking of suppliers, and the net flow is calculated by combining certain PFs differently. Table 18 represents the final ordering of suppliers. In all the different combinations of PFs, it is concluded that the best supplier is $\widehat{A}_1$. On the other hand, $\widehat{A}_2$ is the worst supplier.

These combinations of PFs are implemented in a case study of suppliers' selection of an iron and steel industry including four experts, four suppliers, and five criteria. Although the rankings of suppliers using these different combinations of PFs are the same, the values of positive, negative, and net flow are different as shown in Figure 11.

5.2. Comparison with Existing Techniques. A comparative study with three existing techniques namely fuzzy TOPSIS, fuzzy rough TOPSIS, and crisp VIKOR is also provided to check the importance of the proposed approach over the existing MCDM approaches.

5.2.1. Comparison with Fuzzy TOPSIS (Kumar Et Al. [9]). Fuzzy MCDM techniques use fuzzy numbers to study the uncertainty. To fuzzify the data, these methods need prior information and the membership values, which lead to the biased results. That is why, the results obtained from the fuzzy TOPSIS approach (given in Table 19) are much different from the proposed approach. In the suggested approach, the concept of lower and upper approximations is used to deal with the impreciseness of the original data without any extra information and membership functions. Moreover, the proposed approach is more beneficial by providing weights of criteria from original information and does not depend on experts' opinions.

5.2.2. Comparison with Fuzzy Rough TOPSIS (Zhu Et Al. [36]). The TOPSIS method depends on experts' opinion for weights of criteria since all experts have their own experiences, which enhances the vagueness of the information. In the suggested approach, entropy weights are obtained from original information rather than experts' opinion. The suggested approach has a lot of important aspects that are not included in the fuzzy rough TOPSIS. However, both approaches have the same results and $\widehat{A}_1$ is the best supplier. For the TOPSIS method, weights of criteria depend on experts' opinions and extra calculations are required due to which vagueness is enhanced. Therefore, the proposed approach is more preferable to fuzzy rough TOPSIS.

5.2.3. Comparison with Crisp VIKOR. The traditional MCDM approaches use crisp numbers for the assessment of alternatives against the required criteria. The values of criteria weights depend on the decision makers, due to which the uncertainty is increased. However, the suggested approach reduces the vagueness effectively and provides more

TABLE 18: Supplier's ranking by utilizing various combinations of preference functions.

	The proposed model with entropy weight	Linear and level PF (Akram et al. [28])	Gaussian, quasi, and linear with indifference area PF (Akram et al. [29])
Weights	$W1 = 0.675$, $W2 = 0.158$, $W3 = 0.032$, $W4 = 0.049$, $W5 = 0.085$,	$W1 = 0.675$ $W2 = 0.158$ $W3 = 0.032$ $W4 = 0.049$ $W5 = 0.085$	$W1 = 0.675$ $W2 = 0.158$ $W3 = 0.032$ $W4 = 0.049$ $W5 = 0.085$
Suppliers	Rank	Rank	Rank
$\widehat{A}_1$	1	1	1
$\widehat{A}_2$	4	4	4
$\widehat{A}_3$	2	2	2
$\widehat{A}_4$	3	3	3

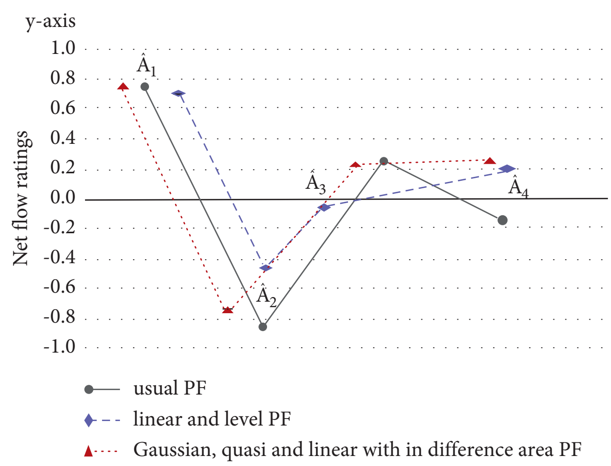

FIGURE 11: Comparison of fuzzy rough PROMETHEE with various combinations of PF.

TABLE 19: Suppliers' ranking by utilizing various techniques.

	Proposed model with entropy weights	Fuzzy TOPSIS with fuzzy weights(Kumar et al. [27])		Crisp VIKOR with fuzzy weights		Fuzzy rough TOPSIS with AHP weights with fuzzy rough weights (Zhu et al. [58])	
Weights	$W1 = 0.675$,	$W1 = (0.167,0.3,0.5)$		$W1 = 0.490$		$W1 = ([0.61, 0.72], [0.83, 0.94], [1.00, 1.00])$	
	$W2 = 0.158$,	$W2 = (0.5,0.7,0.830)$		$W2 = 0.150$		$W2 = ([0.39, 0.50], [0.61, 0.72], [0.83, 0.94])$	
	$W3 = 0.032$,	$W3 = (0.63,0.83,0.9)$		$W3 = 0.030$		$W3 = ([0.13, 0.21], [0.35, 0.43], [0.57, 0.65])$	
	$W4 = 0.049$,	$W4 = (0.43,0.63,0.8)$		$W4 = 0.250$		$W4 = ([0.11, 0.11], [0.11, 0.13], [0.15, 0.17])$	
	$W5 = 0.085$,	$W5 = (0.57,0.77,0.9)$		$W5 = 0.060$		$W5 = ([0.11, 0.11], [0.17, 0.28], [0.39, 0.50])$	
	Rank	CCIi	Rank	Qi	Rank	CCIi	Rank
$\widehat{A}_1$	1	0.4511	3	1.00	4	0.377	1
$\widehat{A}_2$	4	0.3865	4	0.20	2	0.301	4
$\widehat{A}_3$	2	0.5309	1	0.44	3	0.333	2
$\widehat{A}_4$	3	0.5053	2	0.00	1	0.320	3

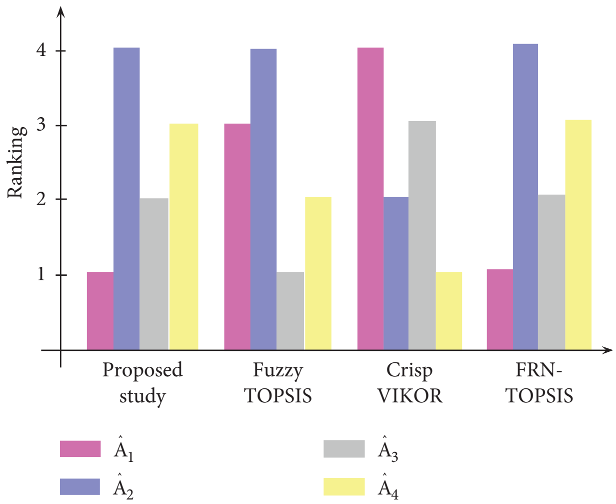

Figure 12: Comparison of fuzzy rough PROMETHEE with other MCDM techniques.

accurate results using the idea of lower and upper approximations. It also provides more choice of preference functions. Moreover, the results of the proposed method are much accurate and stable. The ranking of suppliers with various MCDM techniques is given in Table 19 and Figure 12.

5.3. Merits and Limitations of the Proposed Method. The merits of the proposed approach are as follows:

(i) The suggested approach provides the choice of selecting different types of preference functions and has a sequence of calculations and certain conditions that ensure the validity of this study.

(ii) The proposed approach study provides more precise and efficacious outcomes without additional parameters. Moreover, this study computes the uncertainty by utilizing the concept of upper and lower approximations.

(iii) The proposed approach deals with the problems of having both types of criterion, i.e., cost type criterion and benefit type criterion, and also provides the formulae to normalize the cost and benefit types of criterion.

Besides these advantages, the proposed fuzzy rough PROMETHEE approach also has some limitations:

(i) This method deals with uncertainty in linguistic data using rough approximations, but it cannot be implemented if the data have bipolar or multipolar uncertainties.

(ii) The selection of suitable PF (for the criteria having different nature) is the main limitation of this study. So, the choice of PF is completely dependent on experts' opinions.

(iii) The calculation procedure of fuzzy rough numbers becomes more complicated when there are more alternatives and criteria. Therefore, the proposed method has more calculation complexity.

6. Conclusions and Future Directions

The selection of suppliers has obtained much attention in today business for an effective production process and enhancing company benefits. In this study, the concept of fuzzy rough numbers is utilized with the PROMETHEE approach for an efficient and novel analysis of suppliers' selection. The decisions generally demand different criteria to balance between every possible inconsistent parameters involving subjectivity and uncertainty in the process to deal with subjective and objective vagueness in the assessment of decision-makers. TFN is used as initial evaluations instead of single fixed measurement scales to handle intrapersonal uncertainty in linguistic data. The evaluation data are converted into fuzzy rough numbers using rough approximations of TFNs. The criteria weights are computed from initial assessment matrix using the entropy weighting method to reduce the dependency on experts. A suitable PF is utilized to compute the preference index using entropy weights and deviations among alternatives. The alternatives are ranked using the intersection of both positive flow and negative flow. The proposed method is implemented in a case study of suppliers' selection. The outperformance and rationality of the proposed approach are discussed with two types of comparison methods, that is, PF comparison and the comparison with three existing methods. This study can further be extended to (1) integrated FRN-ELECTRE technique, (2) digraph and matrix approach based on FRNs, and (3) digraph and matrix approach based on rough numbers [54–45].

Data Availability

No data were used to support the findings of the study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by the author.

Conflicts of Interest

The authors declare no known conflicts of financial interests or personal relationships that could have appeared to influence the work reported in this study.

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