Research Article

Study on Strength Theory Effect of Plastic Zone Distribution of Roadway Surrounding Rock

Pei Zhou1,2 and Peng Wu3,4

1Faculty of Engineering, China University of Geosciences, Wuhan 430074, China
2Engineering Research Center of Rock-Soil Drilling & Excavation and Protection, Ministry of Education, Wuhan 430074, China
3School of Electronics and Information, Yangtze University, Jingzhou 434023, China
4National Engineering Research Center for GIS, China University of Geosciences, Wuhan 430074, China

Correspondence should be addressed to Peng Wu; wupeng@yangtzeu.edu.cn

Received 16 March 2022; Revised 2 April 2022; Accepted 7 April 2022; Published 18 May 2022

Academic Editor: Hangjun Che

Copyright © 2022 Pei Zhou and Peng Wu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The plastic zone of surrounding rock is an important basis for evaluating the stability of roadway, and the distribution of plastic zone is closely related to the strength theory. The equation of boundary line of plastic zone is derived by using the approximate plastic condition method. According to specific parameters, the plastic zone is calculated. When the lateral pressure coefficient increases from 0.3 to 1, almost all the plastic zones calculated by different strength criteria have four shapes: butterfly, curved rectangle with concave horizontal direction and convex vertical direction, approximate ellipse, and circle, but the butterfly shape based on DP1 is not obvious. There are differences in the maximum plastic radius calculated by different strength criteria with the same lateral pressure coefficient from large to small: DP3 criterion, DP2 criterion, Mohr–Coulomb criterion/UST (b = 0)/DP5 criterion, UST (b = 0.25), DP4 criterion, UST (b = 0.5), UST (b = 0.75), Matsuoka–Nakai criterion, UST (b = 1), Mogi–Coulomb criterion, Lade–Duncan criterion, and DP1 criterion. With the increase in lateral pressure coefficient, the difference between the results calculated by different strength criteria is smaller. When \( K_0 = 0.3 \), the maximum plastic radius is distributed at 43°~47°. The results of this paper show that the strength theory effect of plastic zone distribution cannot be ignored, which enriches the theory of approximate plastic condition method and can provide an important reference for roadway stability evaluation and support design.

1. Introduction

The shallow resources of the earth are gradually decreasing due to high intensity mining all the year round, and the rational development and utilization of deep resources are of great significance to the development of the future society. However, a series of challenges and technical problems such as high ground stress, strong mining, and large deformation faced by deep mining need to be solved urgently [1, 2]. It is inseparable from the theoretical guidance of rock mechanics and effective engineering technical measures to mine safely and efficiently. The distribution range and size of plastic failure zone of surrounding rock are important indexes to analyze and evaluate the stability of roadway and have guiding function for effective support design.

In the early elastoplastic analysis of roadway surrounding rock, the theory represented by the Fenner equation is put forward based on the assumption of uniform distribution of initial ground stress. On this basis, scholars have developed a series of analytical calculation theories combined with the physical and mechanical characteristics of geomaterials [3–6]. Detailed stress, strain, displacement field, and plastic zone distribution can be obtained. However, under the condition that there is a great difference between horizontal in situ stress and vertical in situ stress, whether the assumption of uniform distribution of initial in situ stress field is reasonable is a question worthy of study.
Recent studies have shown that butterfly shape of the plastic zone of roadway surrounding rock may appear under the condition of nonuniform and high ground stress. The butterfly plastic failure theory obtains the distribution of plastic zone by solving the boundary equation, and the distribution and expansion law of butterfly plastic zone has been verified by numerical simulation. Moreover, the effects of physical and mechanical parameters (including rock mass gravity, friction angle, cohesion, lateral pressure coefficient, roadway radius, buried depth, and support force), roadway section shape, and deflection of principal stress direction are quantitatively calculated and discussed. The research fruits have been verified and applied in engineering fields such as rockburst, roadway roof fall, layered stratum penetration, and earthquake [7–11]. It highlights the theoretical value and practical significance of the approximate plastic condition method.

It is particularly noteworthy that the boundary line equation of plastic zone is calculated based on stress field and strength criterion. The mathematical expressions of each strength criterion are different, so the boundary lines of plastic zone are also different. Obviously, the distribution of plastic zone strongly depends on the strength theory adopted.

Because of the multiphase complexity of geotechnical materials, its strength theory involves tension-compression anisotropy (SD), hydrostatic pressure effect, stress Lode angle effect, intermediate principal stress effect, nonlinear characteristics, and so on [12, 13]. It is difficult to have a single strength criterion to fully reflect all the influencing factors, and the strength theory of rock mass mechanics is called the unsolved centennial problems [14]. Among hundreds of strength theories, there are two common problems: first, the strength theories put forward by different scholars have different definitions and symbols for some physical quantities, which lead to inconveniences and errors in the application; second, the selection of strength theory is arbitrary in the concrete research. There are many literature reports on the study of strength theory effect in the aspects of tunnel surrounding rock stability analysis, slope stability, earth pressure, foundation bearing capacity, and so on [15–17]. Studies have shown that the influence of different strength criteria on the results cannot be ignored.

At present, for the important subject of boundary line equation of plastic zone of surrounding rock of roadway, most kinds of literatures are based on the Mohr–Coulomb criterion, and a few works of literatures are based on the Drucker–Prager criterion, lacking the comparative calculation and analysis on strength theory effect. Therefore, this paper studies the influence of strength theory on the distribution of plastic zone, including the morphological characteristics of plastic zone, the maximum plastic radius, and its location.

2. Summary of Strength Criterion

The research history of strength theory is very long, and hundreds of strength criteria have been obtained, forming a rich strength theory system. In the existing literature, some of the strength theory equations are positive for tensile stress, some are positive for compressive stress, some are expressed by principal stress, and some are expressed by stress invariants. Due to the differences in algebraic sign convention and physical quantities and coefficients adopted, it is inconvenient for calculation and comparative analysis.

This article unifies the physical quantity expression of each equation based on compressive stress which is positive by selecting six classical strength criteria that are commonly utilized in engineering. For the brevity of writing expression, the unified form of stress invariants is used to sort out and summarize the six classical strength criteria. Hereinafter, $I_1$, $I_2$, and $I_3$ are the first, second, and third invariant of the stress tensor, respectively, $J_2$ is the second invariant of the deviatoric stress tensor, $\phi$ is the Lode angle of stress, $\varphi$ is the friction angle, and $c$ is the cohesion.

The stress invariants expressed by principal stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$ are as follows:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3,$$

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1,$$

$$I_3 = \sigma_1\sigma_2\sigma_3,$$

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2].$$

Haigh–Westergaard principal stress space and projection in the $\pi$ plane are shown in Figure 1.

2.1. Mohr–Coulomb Criterion. The Mohr–Coulomb criterion assumes that when the shear stress reaches a certain extreme value, the material fails. This extreme value is not a constant but is related to normal stress on the failure plane. The function expression is as follows:

$$\sqrt{J_2}\sin\left(\frac{\theta + \pi}{3}\right) - \frac{1}{\sqrt{3}}\cos\left(\frac{\theta + \pi}{3}\right)\sin\varphi - \frac{1}{3}I_1\sin\varphi - c\cos\varphi = 0. $$

This criterion is the most widely used and controversial strength theory for geomaterials, and the biggest problem of the Mohr–Coulomb criterion is that the influence of intermediate principal stress on yield and failure is not considered [18].

2.2. Lade–Duncan Criterion. The Lade–Duncan criterion is put forward according to the true triaxial test results of sand, and then it is further extended to cohesive soil and rock [19–21], which is expressed as follows:

$$\frac{I_1^3}{I_3^3} - \frac{(3 - \sin\varphi)^3}{1 - \sin\varphi - \sin^2\varphi + \sin^3\varphi} = 0. $$

When the criterion is extended to rock and soil with cohesive strength (i.e., $c \neq 0$), stress translation
2.3. Mogi–Coulomb Criterion. Mogi carried out true triaxial test of rock earlier and studied the influence of intermediate principal stress on strength [22, 23]. Al-Ajmi and Zimmerman established the Mogi–Coulomb criterion, which can reflect the effect of rock intermediate principal stress and its interval influence, based on a large number of test data and the Mogi empirical criterion combined with the Coulomb criterion [24, 25]. The mathematical expression is as follows:

\[ \sqrt{I_2} = 0 \]

in this three-dimensional stress state is called spatially moved plane (SMP) [28, 29]. The equation of the criterion is as follows:

\[ \frac{I_1 I_2}{I_3} - 8 \tan^2 \phi - 9 = 0. \]  

Similarly, when the Matsuoka–Nakai criterion is applied to geomaterials with cohesive strength, stress translation transformation is also needed as the Lade–Duncan criterion.

2.6. Unified Strength Theory (UST). Yu carried out systematic study on strength theory and put forward the unified strength theory based on the double shear stress yield criterion, double shear stress strength theory, and generalized double shear stress yield criterion [17].

When the two major principal shear stresses on the double shear element and the influence function of the corresponding normal stress on the plane reach a certain limit value, the material begins to yield or fail.

\[ \begin{align*}
& -\frac{I_1}{3} (1 - a) + (2 + a) \sqrt{\frac{I_2}{3}} \cos \theta + \frac{a (1 - b)}{1 + b} \sqrt{I_2} \sin \theta - \sigma_1 = 0 \quad 0^\circ \leq \theta \leq \theta_b \\
& -\frac{I_1}{3} (1 - a) + \left( \frac{2 - b}{1 + b} + a \right) \sqrt{\frac{I_2}{3}} \cos \theta + \left( \frac{b}{1 + b} \right) \sqrt{I_2} \sin \theta - \sigma_1 = 0 \quad \theta_b \leq \theta \leq 60^\circ
\end{align*} \]

in which \( a = (1 - \sin \varphi)/(1 + \sin \varphi) \), \( \sigma_1 = (2 \cos \varphi)/(1 + \sin \varphi) \), \( \theta_b = \arctan (\sqrt{3} (2 + \sin \varphi)/(6 - \sin \varphi)) \), and \( b \) is the influence coefficient of medium principal stress. When \( b = 0 \), it can be reduced to the Mohr–Coulomb criterion, and when \( b = 1 \), it is a double shear yield criterion.

Through derivation, calculation, and simplification, the above strength criteria under the condition of plane strain can be uniformly expressed as follows:

\[ \sigma_1 = A \sigma_3 + B. \]
Coefficients A and B in the equation are given in Table 2.

In Table 2, \( s = (3 - \sin \varphi) / \sqrt{1 - \sin \varphi + \sin^2 \varphi} \), \( t = \sqrt{\tan^2 \varphi + 9 - 1} \), \( \sin \varphi_{UST} = 2(b + 1) \sin \varphi_{UST} / (2 + b(1 + \sin \varphi_{UST})) \), and \( c_{UST} = 2(b + 1) \cos \varphi_{UST} / ((2 + b(1 + \sin \varphi_{UST})) / \cos \varphi_{UST}) \).

3. Derivation of Unified Boundary Line Equation

Mechanical model is an idealized analytical model based on practical engineering problems, grasping its mechanical

\[
\sigma_r = \frac{1}{2} \sigma_v \left( 1 + K_0 \left( 1 - \left( \frac{R_0}{r} \right)^2 \right) - \left( 1 - K_0 \right) \left( 1 - 4 \left( \frac{R_0}{r} \right)^2 + 3 \left( \frac{R_0}{r} \right)^4 \right) \cos (2\theta) \right),
\]

\[
\sigma_\theta = \frac{1}{2} \sigma_v \left( 1 + K_0 \left( 1 + \left( \frac{R_0}{r} \right)^2 \right) + \left( 1 - K_0 \right) \left( 1 + 3 \left( \frac{R_0}{r} \right)^4 \right) \cos (2\theta) \right),
\]

\[
\tau_{r\theta} = \frac{1}{2} \sigma_v \left( 1 - K_0 \left( 1 + 2 \left( \frac{R_0}{r} \right)^2 - 3 \left( \frac{R_0}{r} \right)^4 \right) \sin (2\theta) \right),
\]

in which \( \sigma_r \) is the radial stress, \( \sigma_\theta \) is the circumferential stress, \( \tau_{r\theta} \) is the shear stress, \( R_0 \) is the radius of roadway \( \sigma_r \) is the vertical in situ stress, \( \sigma_\theta \) is the horizontal in situ stress, \( K_0 \) is the lateral pressure coefficient, \( \theta \) is the angle starting horizontally to the right and increasing counterclockwise, and \( r \) represents the distance between any point in the surrounding rock and the circular roadway’s center.

Rewrite the stress component in polar coordinates \( \sigma_{ij}^{(\theta)} \) to the stress component in rectangular coordinates \( \sigma_{ij}^{(r)} \):

\[
\begin{bmatrix}
\sigma_{ij}^{(r)}
\end{bmatrix} = \mathbf{L}^T \begin{bmatrix}
\sigma_{ij}^{(\theta)}
\end{bmatrix} \mathbf{L},
\]

in which \( \mathbf{L} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta 
\end{bmatrix} \).

Solve the principal stress equation from the stress equation expressed by rectangular coordinates:

\[
\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2},
\]

\[
\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}.
\]

Substitute the principal stress equation simplified in the above steps into the unified form of yield equation (8). After a series of operations and simplification such as merging similar terms and trigonometric function conversion, the following results are obtained:

| Table 1: Parameters’ expressions of yield criteria (DP1~5). |
|----------------|----------------|-------------|
| **Yield criteria** | **Amendments** | **A** | **k** |
| DP1 | M-C exterior angle circumcircle | \( 2 \sin \varphi / (\sqrt{3} (3 - \sin \varphi)) \) | \( 2 \sqrt{3} \cos \varphi / (3 - \sin \varphi) \) |
| DP2 | M-C interior angle circumcircle | \( 2 \sin \varphi / (\sqrt{3} (3 + \sin \varphi)) \) | \( 2 \sqrt{3} \cos \varphi / (3 + \sin \varphi) \) |
| DP3 | M-C inscribed circle | \( \sin \varphi / (\sqrt{3} (3 + \sin^2 \varphi)) \) | \( \sqrt{3} \cos \varphi / (3 + \sin^2 \varphi) \) |
| DP4 | M-C equivalent area circle | \( \sqrt{2} / (3 \sin \varphi / \sqrt{\pi} (9 - \sin^2 \varphi)) \) | \( 3 / (\sqrt{2} \sqrt{3} \cos \varphi / \sqrt{\pi} (9 - \sin^2 \varphi)) \) |
| DP5 | M-C nonassociated matching circle | \( \sqrt{3} (\sin \varphi / \sqrt{\pi} (9 - \sin^2 \varphi)) \) | \( \sqrt{3} (\cos \varphi / \sqrt{\pi} (9 - \sin^2 \varphi)) \) |
Table 2: Coefficient A and B of unified form of strength theory.

<table>
<thead>
<tr>
<th>Strength criterion</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mohr–Coulomb criterion</td>
<td>((1 + \sin \varphi)/(1 - \sin \varphi))</td>
<td>(2c \cos \varphi/(1 - \sin \varphi))</td>
</tr>
<tr>
<td>Lade–Duncan criterion</td>
<td>(1/4(s + \sqrt{s^2 - 4})^2)</td>
<td>((1/4(s + \sqrt{s^2 - 4})^2 - 1)c \cot \varphi)</td>
</tr>
<tr>
<td>Mogi–Coulomb criterion</td>
<td>((\sqrt{3} + 2 \sin \varphi)/(\sqrt{3} - 2 \sin \varphi))</td>
<td>((4c \cos \varphi)/(\sqrt{3} - 2 \sin \varphi))</td>
</tr>
<tr>
<td>A series of Drucker–Prager criteria</td>
<td>((1 + 3\alpha)/(1 - 3\alpha))</td>
<td>(2k/(1 - 3\alpha))</td>
</tr>
<tr>
<td>Matsuoka–Nakai criterion</td>
<td>(1/4(t + \sqrt{t^2 - 4})^2)</td>
<td>((1/4(t + \sqrt{t^2 - 4})^2 - 1)c \cot \varphi)</td>
</tr>
<tr>
<td>Unified strength theory (UST)</td>
<td>((1 + \sin \varphi_{UST})/(1 - \sin \varphi_{UST}))</td>
<td>((2c_{UST} \cos \varphi_{UST})/(1 - \sin \varphi_{UST}))</td>
</tr>
</tbody>
</table>

Figure 2: Basic mechanical model.

\[
M_s\left(\frac{R_0}{r}\right)^6 + M_6\left(\frac{R_0}{r}\right)^6 + M_4\left(\frac{R_0}{r}\right)^4 + M_2\left(\frac{R_0}{r}\right)^2 + M_0 = 0, \quad (12)
\]

in which

\[
M_s = 9(K_0 - 1)^2 \sigma_v^2,
\]

\[
M_6 = -6(2(K_0 - 1)^2 + \cos(2\theta)(K_0^2 - 1)) \sigma_v^2,
\]

\[
M_4 = \left( (K_0 - 1)^2 \left( \cos(4\theta) \left( 6 - 2 \frac{(A - 1)^2}{(A + 1)^2} \right) + 4 - 2 \frac{(A - 1)^2}{(A + 1)^2} \right) + 4 \cos(2\theta)(K_0^2 - 1) + (K_0 + 1)^2 \right) \sigma_v^2,
\]

\[
M_2 = \left( -4 \cos(4\theta)(K_0 - 1)^2 - 2 \cos(2\theta)(K_0^2 - 1) \left( 1 - \frac{2(A - 1)^2}{(A + 1)^2} \right) \right) \sigma_v^2 + 4 \cos(2\theta)(K_0 - 1) \frac{2B(A - 1)}{(A + 1)^2} \sigma_v^2,
\]

\[
M_0 = \left( (K_0 - 1)^2 - \frac{(K_0 + 1)^2(A - 1)^2}{(A + 1)^2} \right) \sigma_v^2 - 4B(K_0 + 1) \frac{(A - 1)}{(A + 1)^2} \sigma_v - \frac{4B^2}{(A + 1)^2}.
\]

4. Calculation and Analysis

In order to analyze the distribution characteristics of plastic zone of deep roadway surrounding rock, typical parameters are selected: roadway radius of 2 m, weight of 25 kN/m³, buried depth of 800 m, internal friction angle of 30°, and cohesion of 2 MPa [31]. Considering the symmetry of the model, it is equivalent to rotating the plastic zone of 1/K₀ by 90 degrees when K₀ > 1. Thus, this paper only calculates and analyzes the distribution and morphological characteristics of the plastic zone with K₀ < 1.


Figure 3 shows the distribution of plastic zone calculated by the Mohr–Coulomb criterion (UST, b = 0) and double shear
Figure 3: Distribution of plastic zone based on UST: (a) $b = 0$ and $K_0 = 0.3$; (b) $b = 1$ and $K_0 = 0.3$; (c) $b = 0$ and $K_0 = 0.5$; (d) $b = 1$ and $K_0 = 0.5$; (e) $b = 0$ and $K_0 = 0.8$; (f) $b = 1$ and $K_0 = 0.8$. 
criterion (UST, \( b = 1 \)). It can be clearly seen that the shape of the plastic zone is basically similar regardless of the Mohr–Coulomb criterion (UST, \( b = 0 \)) or double shear criterion (UST, \( b = 1 \)).

When \( K_0 = 0.3 \), the shape of the plastic zone is butterfly, and obviously the maximum plastic radius is between the \( x \) and \( y \) axes. When \( K_0 = 0.5 \), the range of plastic zone decreases rapidly and the butterfly almost disappears, and the plastic zone becomes an approximate rectangle with curved edges, showing concave in the upper and lower horizontal directions and convex in the left and right vertical directions. At this time, the maximum plastic radius is still between the \( x \) and \( y \) axes. The outer angle of the plastic zone disappears, and the shape is approximately elliptical with the maximum plastic radius on the \( x \) axis when \( K_0 = 0.8 \).

Further calculation and analysis show that all the criteria have obvious butterfly failure zones except DP1 when \( K_0 = 0.3 \).

### 4.2. Maximum Plastic Radius and Position Analysis

In practical engineering application, the maximum plastic radius and its position are important characteristic parameters of plastic zone distribution. It can be seen from Table 3 that the maximum plastic radius value and position calculated by different strength criteria are different. When \( K_0 \) takes the same value, the maximum plastic radius calculated by strength criteria from large to small is DP3 criterion, DP2 criterion, Mohr–Coulomb criterion/UST (\( b = 0 \)), DP5 criterion, UST (\( b = 0.25 \)), Matsuoka–Nakai criterion, UST (\( b = 0.5 \)), Lade–Duncan criterion, and DP1 criterion. The smaller the value of \( K_0 \), the larger the difference of the maximum plastic radius calculated by different strength criteria. The calculation results of each strength criterion are getting closer with the increase in \( K_0 \). All the shapes of plastic zone distribution are getting closer with the increase in \( K_0 \). A closer look at the data reveals that the maximum plastic radius calculated by double shear criterion, Mogi–Coulomb criterion, Lade–Duncan criterion, and DP1 criterion is getting closer. This can be attributed to the fact that the Mohr–Coulomb criterion is a double shear criterion, and its calculation results are getting closer to the plastic zone distribution.

### Table 3: Maximum plastic radius and position calculated by different strength criteria.

<table>
<thead>
<tr>
<th>Strength theory</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mohr–Coulomb criterion</td>
<td>6.51*</td>
<td>3.90</td>
<td>3.03</td>
<td>2.78</td>
<td>2.69</td>
<td>2.66</td>
<td>2.63</td>
<td>2.61</td>
</tr>
<tr>
<td>/UST (( b = 0 ))/DP5 criterion</td>
<td>43/137/223/</td>
<td>44/136/224/</td>
<td>43/137/223/</td>
<td>37/143/217/</td>
<td>26/154/206/</td>
<td>0/180/</td>
<td>0/180/</td>
<td>0/360</td>
</tr>
<tr>
<td>UST (( b = 0.25 ))</td>
<td>31/4</td>
<td>31/6</td>
<td>31/7</td>
<td>32/3</td>
<td>33/4</td>
<td>36/0</td>
<td>36/0</td>
<td>0/360</td>
</tr>
<tr>
<td>DP1 criterion</td>
<td>5.49</td>
<td>3.52</td>
<td>2.85</td>
<td>2.67</td>
<td>2.61</td>
<td>2.58</td>
<td>2.56</td>
<td>2.55</td>
</tr>
<tr>
<td>UST (( b = 0.5 ))</td>
<td>31/6</td>
<td>31/5</td>
<td>31/4</td>
<td>31/7</td>
<td>31/6</td>
<td>33/7</td>
<td>36/0</td>
<td>36/0</td>
</tr>
<tr>
<td>DP3 criterion</td>
<td>4.95</td>
<td>3.28</td>
<td>2.73</td>
<td>2.60</td>
<td>2.55</td>
<td>2.53</td>
<td>2.52</td>
<td>2.50</td>
</tr>
<tr>
<td>DP4 criterion</td>
<td>45/135/225/</td>
<td>46/134/226/</td>
<td>43/137/223/</td>
<td>34/146/214/</td>
<td>19/161/199/</td>
<td>0/180/</td>
<td>0/180/</td>
<td>0/360</td>
</tr>
<tr>
<td>UST (( b = 0.75 ))</td>
<td>31/5</td>
<td>31/4</td>
<td>31/7</td>
<td>32/6</td>
<td>34/1</td>
<td>36/0</td>
<td>36/0</td>
<td>36/0</td>
</tr>
<tr>
<td>Double shear criterion/UST (( b = 1 ))</td>
<td>4.61</td>
<td>3.12</td>
<td>2.65</td>
<td>2.55</td>
<td>2.51</td>
<td>2.50</td>
<td>2.48</td>
<td>2.47</td>
</tr>
<tr>
<td>Lade–Duncan criterion</td>
<td>4.38</td>
<td>3.00</td>
<td>2.60</td>
<td>2.51</td>
<td>2.48</td>
<td>2.47</td>
<td>2.46</td>
<td>2.44</td>
</tr>
<tr>
<td>Matsuoka–Nakai criterion</td>
<td>4.13</td>
<td>2.86</td>
<td>2.54</td>
<td>2.47</td>
<td>2.45</td>
<td>2.43</td>
<td>2.45</td>
<td>2.43</td>
</tr>
<tr>
<td>Mogi–Coulomb criterion</td>
<td>4.31</td>
<td>3.12</td>
<td>2.65</td>
<td>2.54</td>
<td>2.51</td>
<td>2.49</td>
<td>2.48</td>
<td>2.47</td>
</tr>
<tr>
<td>DP1 criterion</td>
<td>3.90</td>
<td>3.03</td>
<td>2.78</td>
<td>2.69</td>
<td>2.66</td>
<td>2.63</td>
<td>2.61</td>
<td>2.61</td>
</tr>
<tr>
<td>DP2 criterion</td>
<td>31/6</td>
<td>31/7</td>
<td>32/3</td>
<td>33/4</td>
<td>36/0</td>
<td>36/0</td>
<td>36/0</td>
<td>36/0</td>
</tr>
<tr>
<td>DP3 criterion</td>
<td>4.76</td>
<td>4.24</td>
<td>4.19</td>
<td>4.27</td>
<td>4.29</td>
<td>4.31</td>
<td>4.34</td>
<td>4.37</td>
</tr>
</tbody>
</table>

**Note.** * Maximum plastic radius (unit: m); ** Location where the maximum plastic radius appears (unit: degree).
calculated by each criterion are circle, and value of the plastic radius is about 2.22 m to 2.66 m. 

Butterfly shape with bigger plastic radius has a great influence on the stability of roadway, which can also explain the practical engineering phenomena such as roadway damage, large deformation, and difficult support in the tectonic stress zone. Further calculation and analysis show that all the criteria have obvious butterfly failure zones except DP1 when $K_0 = 0.3$, and the maximum plastic radius is distributed near the angle bisector (43–48 degree) of major and minor principal stress direction. With the increase in $K_0$, the maximum plastic radius gradually shifts to the direction of small principal stress. The value of the maximum plastic radius calculated by DP1 is 2.27 m, which is significantly less than the results obtained by other criteria (4.13~7.69 m) when $K_0 = 0.3$.

5. Conclusion

In this paper, different strength theories are used to calculate and compare the distribution of plastic zone of surrounding rock in detail. A series of meaningful conclusions are drawn as follows:

(1) The shape of the plastic zone of roadway surrounding rock may be butterfly, horizontally concave, and vertically convex curved rectangle, ellipse, and circle. The results calculated by different strength criteria all reflect the morphological distribution characteristics of the plastic zone mentioned above, but DP1 is not applicable.

(2) When $K_0$ takes the same value, the maximum plastic radius calculated by strength criteria from large to small is DP3 criterion, DP2 criterion, Mohr–Coulomb criterion/UST ($b = 0$)/DP5 criterion, UST ($b = 0.25$), DP4 criterion, UST ($b = 0.5$), UST ($b = 0.75$), Matsuoka –Nakai criterion, UST ($b = 1$)/Double shear criterion, Mogi–Coulomb criterion, Lade–Duncan criterion, and DP1 criterion, and using the DP1 criterion is not safe in practical engineering.

(3) Strength theory has influence on the shape and size of plastic zone distribution, which cannot be ignored when carrying out support design.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References


