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Research Article

Investigation of Atom-Bond Connectivity Indices of Line Graphs Using Subdivision Approach

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A topological index is a numerical measure that characterises the whole structure of a graph. Based on vertex degrees, the idea of an atom-bond connectivity (ABC) index was introduced in chemical graph theory. Later, different versions of the ABC index were created, and some of these indices were recently designed. In this paper, we present the edge version of the atom-bond connectivity (ABC_e) index, edge version of the multiplicative atom-bond connectivity ($ABCII_e$) index, and atom-bond connectivity temperature ($ABCT$) index for the line graph of subdivision graph of tadpole graph $(T_{n,k})$, ladder graph (L_n), and wheel graph (W_{n+1}). Numerical simulation has also been shown for some novel families of atom-bond connectivity index comparing the three types of indices which can be useful for QSAR and QSPR studies.

1. Introduction

In this article, we have considered simple graphs, which are unweighted, undirected graphs that have no loops and multiple edges attached. Let G be a simple graph, with vertex set $V(G)$ and edge set $E(G)$. Suppose e is an edge of G, which connects the vertices u and v, then we denote $e = uv$ and state that "u and v are adjacent." The degree d_u of a vertex u is the number of edges that are incident to it. Topological indices are the mathematical measures that correspond to the structure of any simple finite graph. They are invariant under the graph isomorphism. There are some famous degree-based topological indices, which are introduced and applied in chemical engineering, for instance, the Randic index (refer to Ali & Du [1], Li & Shi [2], and Shi [3] for more details). These indices are also significant in quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) (see [4, 5]).

The subdivision graph [6, 7] $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2. In graph G, if the corresponding edges share a vertex in G, the line graph $L(G)$ of a graph G is considered as a graph with vertices of the edges in G. Two vertices e and f are incident if and only if they have a common end vertex in G. Estrada et al. [8] put forward a topological index named atom-bond connectivity index (briefly, ABC) as

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}}, \quad (1)$$

where d_u and d_v represent the degrees of the vertices u and v, respectively. Recent advances on ABC index can be referred to in Das et al. [9], Lin et al. [10], Gao & Shao [11], and Bianchi et al. [12]. Referring to the end vertex degree d_e and d_f of edges e and f in a line graph of G, Farahani [13] proposed the edge version of atom-bond connectivity, ABC_e index. This idea is described in the following:

$$ABC_e(G) = \sum_{ef\in E(G)} \sqrt{\frac{d_e + d_f - 2}{d_e \times d_f}}, \quad (2)$$

where d_e and d_f are the degree of the edge e and f, respectively. The reader can find more information about ABC_e index in References [14–18].

The multiplicative atom-bond connectivity index was introduced by Kulli in 2016 [19]. More information regarding the multiplicative atom-bond connectivity index may be found in references [20, 21]. Later, the edge version of the multiplicative atom-bond connectivity index [22] of a graph G was introduced, and it is defined as

$$ABCII_e(G) = \prod_{ef \in E(L(G))} \sqrt{\frac{d_e + d_f - 2}{d_e \cdot d_f}}, \quad (3)$$

where d_e is the degree of the edge e in $L(G)$.

The temperature of a vertex u of a connected graph G is defined by Fajtlowicz [23] as

$$T(u) = \frac{d_u}{n - d_u}, \quad (4)$$

where d_u is the degree of a vertex u and n is the size of a graph G. Recently, Kahasy et al. [24] introduced a new index, known as the atom-bond connectivity temperature index. This index is defined as follows:

$$ABCT(G) = \prod_{uv \in E(G)} \sqrt{\left|\frac{T_u + T_v - 2}{T_u T_v}\right|}, \quad (5)$$

where T_u and T_v are the temperature of the vertex u and v, respectively.

2. Main Results

In 2011, Ranjini et al. calculated the explicit expression for the Shultz indices of the subdivision graphs of the tadpole, wheel, helm, and ladder graphs [25]. They also studied the Zagreb indices of line graphs of tadpole, wheel, and ladder graphs with subdivision in [26]. In 2015, Su & Xu calculated the general sum-connectivity indices and coindices of line graphs of tadpole, wheel, and ladder graphs with subdivision in [27]. In [28], Nadeem et al. computed ABC_4 and GA_5 indices of the line graphs of these graphs by using the notion of subdivision. They also studied the ABC_4 and GA_5 of these graphs [28]. Other studies on these include Rajasekar & Nagarajan [29] research on the location domination number of the line graph. Recently, Li &. Taylor [30] also studied the first Zagreb index and some Hamilton properties of the line graph.

A tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle of n vertices with a path of length k. A ladder L_n is obtained by taking the Cartesian product of two paths $P_n \times P_2$. A wheel graph W_n or order n composed of a vertex is called the hub, adjacent to all vertices of a cycle of the order n.

Motivated by the results of [26, 27, 31], we studied the line graph of the subdivision graph $T_{n,k}$, L_n, and W_{n+1} and derived an expression for the edge version of atom-bond connectivity, multiplicative atom-bond connectivity indices, and atom-bond connectivity temperature index of the graphs $L(S(T_{n,k}))$, $L(S(W_{n+1}))$, and $L(S(L_n))$.

Theorem 1. *The edge version of the atom-bond connectivity index of $L(S(T_{n,k}))$ is*

$$ABC_e(G) = \begin{cases} \sqrt{2}\,n + \sqrt{2}\,k + 2 - \sqrt{2}, \text{when } k > 1; \\ \sqrt{2}\,n + 2 + \sqrt{\frac{2}{3}} - \frac{3\sqrt{2}}{2}, \text{when } k = 1. \end{cases} \quad (6)$$

Proof. Let G be the line graph of the subdivision graph $L(S(T_{n,k}))$, seeing Figure 1. It contained $2(n+k)$ edges of the subdivision graph of $S(T_{n,k})$, and then, in the graph of G, it contained $2(n+k)$ vertices. It consists of three types of degree of edge e, such as 1, 2, and 3. Out of which, 3 vertices are of the degree 3, one vertex of degree 1 and the remaining $2(n+k-2)$ vertices are of the degree 2. The graph of G contains path of length $2k-1$. Let V_1 be the vertex of degree 3 which is attached to this path. Let V_1' and V_2' be the neighbor of V_1 which are of degree 3 in the $L(G)$. The vertices V_1' and V_2' have two neighbors of degree 3 and one neighbor of degree 2 in $L(S(C_n)+e)$, where e is the edge adjacent to $S(C_n)$. The vertex V_1 has 2 adjacent vertices of degree 3 and one vertex of degree 2 in the path. Let we derive an expression for the edge version of topological indices of the graph G, for $k=1$. In graph G, it contains a path of length 1 which attached with V_1. Hence, $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f}$ with respect to the path is $\sqrt{2/3}$. For $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f}$ corresponding to the vertices V_1, V_1', and V_2' in $L(S(C_n)+e)$, hence, we have $4+\sqrt{2}$. Since one edge in G is shared between pairs of vertices, $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f} = 2 + \sqrt{1/2}$. Among the remaining, $2n-4$ vertices, for $2n-5$ vertices, have neighbors of degree 2, and one vertex has neighbor of degree 3. Hence, $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f}$ with respect to $2n-4$ vertices is $\sqrt{2}n - 2\sqrt{2}$. Adding all these number together, the edge version of atom-bond connectivity index of G is found as $ABC_e(G) = \sqrt{2}n + 2 + \sqrt{2/3} - 3\sqrt{2}/2$.

If $k>1$, then the graph G contains path of length $2k-1$ which attached with V_1. Hence, $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f}$ with respect to the path is $\sqrt{1/2}(2k-1)$. For $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f}$ corresponding to the vertices V_1, V_1' and V_2' in $L(S(C_n)+e)$, hence, we have $4+\sqrt{2}$. Since one edge in G is shared between pairs of vertices, $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f} = 2 + \sqrt{1/2}$. Out of $2n-2$ vertices, for $2n-3$ vertices have neighbors of degree 2 and one vertex has neighbor of degree 3. Hence, $\sum \sqrt{d_e + d_f - 2/d_e \cdot d_f}$ with respect to $2n-2$ vertices is $\sqrt{2}n - \sqrt{2}$. Adding all these number together, the edge version of atom-bond connectivity index of G is found as $ABC_e(G) = \sqrt{2}n + \sqrt{2}k + 2 - \sqrt{2}$. This completes the proof. □

Theorem 2. *The edge version of the multiplicative atom-bond connectivity index of $L(S(T_{n,k}))$ is*

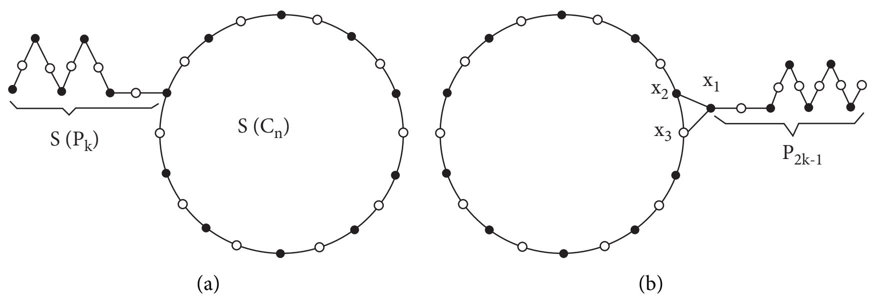


FIGURE 1: [27] (a) The subdivision graph $S(T_{n,k})$ of the tadpole graph $T_{n,k}$. (b) The line graph $L(S(T_{n,k}))$.

$$ABCII_e(G) = \begin{cases} 3\sqrt{2}\,(n+k-3), \text{when } k>1; \\ 2\sqrt{\frac{2}{3}}\,(2n-5), \text{when } k=1. \end{cases} \tag{7}$$

Proof. After adopting the induction method, it is clear that overall speaking, this line graph of subdivision graph possesses of $2(n+k)$ vertices and $2(n+k)+1$ edges. If d_e and d_f are the degree of edge e, then there are 1 edge of type $d_e = 1, d_f = 3$, $2n-5$ edges with $d_e = d_f = 2$, 2 edges of type $d_e = 2, d_f = 3$, 3 edges with $d_e = d_f = 3$. Hence, for graph G with $k = 1$, we have $ABCII_e(G) = 2\sqrt{2/3}(2n-5)$. For $k > 1$, we have 1 edge of type $d_e = 1, d_f = 2$, $2n+2k-6$ edges with $d_e = d_f = 2$, 3 edges of type $d_e = 2$, $d_f = 3$, 3 edges with $d_e = d_f = 3$. Hence, we deduce

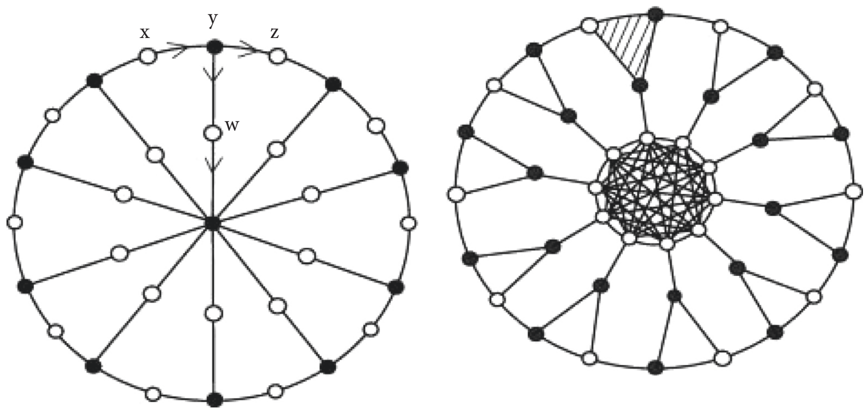
$$ABCII_e(G) = (1)\sqrt{\frac{1+2-2}{1\times 2}} \times (2n+2k-6)\sqrt{\frac{2+2-2}{2\times 2}} \times (3)\sqrt{\frac{2+3-2}{2\times 3}} \times (3)\sqrt{\frac{3+3-2}{3\times 3}} = 3\sqrt{2}(n+k-3). \tag{8}$$

This completes the proof. □

Theorem 3. *The atom-bond connectivity temperature index of $L(S(T_{n,k}))$ is*

$$ABCT(G) = \begin{cases} \sqrt{\left|-4n^2+9n-8nk+9k-4k^2-4\right|} + (2n+2k-6)\sqrt{\left|(-2n-2k+4)(n+k-1)\right|}, & \\ \qquad\qquad \text{when } k>1; & \\ +3\sqrt{\left|\frac{-4n^2-8nk+15n-4k^2+15k-12}{3}\right|} + 3\sqrt{\left|\frac{(-4n-4k+12)(2n+2k-3)}{9}\right|}, & \\ (2n-5)\sqrt{\left|-2n^2+2n\right|} + \sqrt{\left|\frac{-8n^2+8n+4}{3}\right|} + 2\sqrt{\left|\frac{-4n^2+7n-1}{3}\right|}, & \\ \qquad\qquad \text{when } k=1. & \\ +2\sqrt{\left|-2n^2+5n-2\right|}, & \end{cases} \tag{9}$$

Proof. In G, there are total $2(n+k)$ vertices, among which 3 vertices are of the degree 3, one vertex of degree 1 and the remaining $2(n+k-2)$ vertices are of the degree 2. The total number of edges of G is $2(n+k)+1$. For $k = 1$, we have $2(n+1)$ vertices, which one vertex of degree 1, 3 vertices of degree 3 and $2(n-1)$ vertices of degree 2. Therefore, after adopting the induction trick, we have 4 edge partition based on the temperature. If T_u and T_v are the temperature of the vertex u and v, then there are 1 edge of type $T_1 = 1/2n+1$, $T_3 = 3/2n-1$, $2n-5$ edges with $T_2 = T_2 = 1/n$, 2 edges of type $T_2 = 1/n$, $T_3 = 3/2n-1$, 3 edges with $T_3 = T_3 = 3/2n-1$. Hence, for graph G with $k = 1$, we have



(a) (b)

Figure 2: [27] (a) The subdivision graph $S(W_n)$ of the tadpole graph W_n. (b) The line graph $L(S(W_n))$.

$$\begin{aligned} ABCT(G) &= (1)\sqrt{\left|\frac{1/2n+1+3/2n-1-2}{1/2n+1\times 3/2n-1}\right|} + (2n-5)\sqrt{\left|\frac{1/n+1/n-2}{1/n\times 1/n}\right|} + (2)\sqrt{\left|\frac{1/n+3/2n-1-2}{1/n\times 3/2n-1}\right|} + (3)\sqrt{\left|\frac{3/2n-1+3/2n-1-2}{3/2n-1\times 3/2n-1}\right|} \\ &= (2n-5)\sqrt{\left|-2n^2+2n\right|} + \sqrt{\left|\frac{-8n^2+8n+4}{3}\right|} + 2\sqrt{\left|\frac{-4n^2+7n-1}{3}\right|} + 2\sqrt{\left|-2n^2+5n-2\right|}. \end{aligned} \tag{10}$$

For $k>1$, we have 1 edge of type $T_1 = 1/2n+2k-1$, $T_2 = 2/2n+2k-2$, $2n+2k-6$ edges with $T_2 = T_2 = 2/2n+2k-2$, 3 edges of type $T_2 = 2/2n+2k-2$, $T_3 = 3/2n+2k-3$, 3 edges with $T_3 = T_3 = 3/2n+2k-3$. Therefore, we get

$$\begin{aligned} BCT(G) &= (1)\sqrt{\left|\frac{1/2n+2k-1+2/2n+2k-2-2}{1/2n+2k-1\times 2/2n+2k-2}\right|} + (2n+2k-6)\sqrt{\left|\frac{2/2n+2k-2+2/2n+2k-2-2}{1/2n+2k-2\times 3/2n+2k-2}\right|} \\ &\quad + (3)\sqrt{\left|\frac{2/2n+2k-2+3/2n+2k-3-2}{2/2n+2k-2\times 3/2n+2k-3}\right|} + (3)\sqrt{\left|\frac{3/2n+2k-3+3/2n+2k-3-2}{3/2n+2k-3\times 3/2n+2k-3}\right|} \\ &= \sqrt{\left|-4n^2+9n-8nk+9k-4k^2-4\right|} + (2n+2k-6)\sqrt{\left|(-2n-2k+4)(n+k-1)\right|} \\ &\quad + 3\sqrt{\left|\frac{-4n^2-8nk+15n-4k^2+15k-12}{3}\right|} + 3\sqrt{\left|\frac{(-4n-4k+12)(2n+2k-3)}{9}\right|}. \end{aligned} \tag{11}$$

This completes the proof. □

Theorem 4. *The edge version of the atom-bond connectivity index of $L(S(W_{n+1}))$ is*

$$ABC_e(G) = \frac{8n}{3} + n\sqrt{\frac{n+1}{3n}} + \frac{(n-1)\sqrt{2(n-1)}}{2}. \tag{12}$$

Proof. Let G be the line graph of the subdivision graph $L(S(T_{n,k}))$, seeing Figure 2. It contains $4n$ vertices are of degree 3 and n vertices of degree n. Out of $n^2+9n/2$ edges, the $4n$ edges of degree 3 have neighbor of degree 3. Hence, $\sum\sqrt{d_e+d_f-2/d_e\cdot d_f}$ corresponding to these $4n$ edges which have only neighbor of degree 3 is $8n/3$. The remaining n vertices of degree 3 are adjacent to vertices of degree n. Hence, $\sum\sqrt{d_e+d_f-2/d_e\cdot d_f}$ with respect to these n vertices is $n\sqrt{n+1/3n}$. Also remaining $n(n\text{-}1)/2$ edges of degree n have neighbor of degree n. Hence, $\sum\sqrt{d_e+d_f-2/d_e\cdot d_f}$ with respect to all these degrees of n is $(n-1)\sqrt{2(n-1)}/2$. Adding all these number together, the edge version of atom-bond connectivity index of G is found as $ABC_e(G) = 8n/3 + n\sqrt{n + 1/3n} + (n-1)\sqrt{2(n-1)}/2$. This completes the proof. □

Theorem 5. *The edge version of the multiplicative atom-bond connectivity index of* $L(S(W_{n+1}))$ *is*

$$ABCII_e(G) = \frac{4n^2(n-1)\sqrt{3+n-2/3n}\sqrt{2n-2}}{3}. \quad (13)$$

Proof. After adopting the induction technology, it is clear to find that, roughly speaking, this line graph of subdivision graph has contained $4n$ vertices and $n^2 + 9n/2$ edges. Also, there are $4n$ edges of type $d_e = d_f = 3$, n edges of type $d_e = 3$, $d_f = n$, $n(n-1)/2$ edges with $d_e = d_f = n$. As a result, we infer

$$ABCII_e(G) = (4n)\sqrt{\frac{3+3-2}{3\times 3}} \times (n)\sqrt{\frac{3+n-2}{3\times n}} \times \left(\frac{n(n-1)}{2}\right)\sqrt{\frac{n+n-2}{n\times n}} = \frac{4n^2(n-1)\sqrt{3+n-2/3n}\sqrt{2n-2}}{3}. \quad (14)$$

This completes the proof. □

Theorem 6. *The atom-bond connectivity temperature index of* $L(S(W_{n+1}))$ *is*

$$ABCT(G) = \frac{4}{3}n\sqrt{|(4n-3)(-8n+12)|} + n\sqrt{\left|\frac{-20n+24}{3}\right|} + \sqrt{3}n(n-1). \quad (15)$$

Proof. In G, there are total $4n$ vertices are of degree 3 and n vertices of degree n. The total number of edges of G is $n^2 + 9n/2$. After adopting the induction method, we have 3 edge partition based on the temperature. If T_u and T_v are the temperature of the vertex u and v, then there are $4n$ edge of type $T_3 = T_3 = 3/4n - 3$, n edges with $T_3 = 3/4n - 3$, $T_n = 1/3$ and $n(n-1)/2$ edges of type $T_n = T_n = 1/3$. Hence, we deduce

$$\begin{aligned} ABCT(G) &= (4n)\sqrt{\left|\frac{3/4n-3+3/4n-3-2}{3/4n-3\times 3/4n-3}\right|} + (n)\sqrt{\left|\frac{3/4n-3+1/3-2}{3/4n-3\times 1/3}\right|} \\ &+ \left(\frac{n(n-1)}{2}\right)\sqrt{\left|\frac{1/3+1/3-2}{1/3\times 1/3}\right|} = \frac{4}{3}n\sqrt{|(4n-3)(-8n+12)|} + n\sqrt{\left|\frac{-20n+24}{3}\right|} + \sqrt{3}n(n-1). \end{aligned} \quad (16)$$

This completes the proof. □

Theorem 7. *The edge version of the atom-bond connectivity index of* $L(S(L_n))$ *is*

$$ABC_e(G) = 5\sqrt{2} + \frac{2(9n-20)}{3}. \quad (17)$$

Proof. Let G be the line graph of subdivision graph $L(S(L_n))$, seeing Figure 3. The number of vertices in G is $6n-4$ among which 8 vertices are of degree 2 and the remaining $6n-12$ vertices are of degree 3. The number of edges in G is $9n-10$ among which 6 edges are of degree 2 with itself, 4 edges are of degree 2 and 3, and the remaining $9n-20$ edges are of degree 3 with itself. Adding all these numbers together, the edge version of the atom-bond connectivity index of G is found as $ABC_e(G) = 5\sqrt{2} + 2(9n-20)/3$. This completes the proof. □

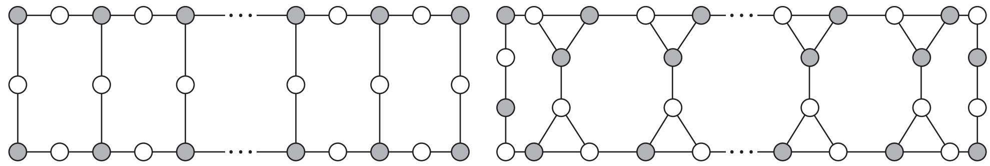
Theorem 8. *The edge version of multiplicative atom-bond connectivity index of* $L(S(L_n))$ *is*

$$ABCII_e(G) = 8(9n-20). \quad (18)$$

Proof. After adopting the induction trick, we can find that, in general, this line graph of subdivision graph has $6n-4$ vertices and $9n-10$ edges. At the same, there are 6 edges of type $d_e = d_f = 2$, 4 edges of type $d_e = 2$, $d_f = 3$, $9n-20$ edges with $d_e = d_f = 3$. Therefore, we get

$$\begin{aligned} ABCII_e(G) &= (6)\sqrt{\frac{2+2-2}{2\times 2}} \times (4)\sqrt{\frac{2+3-2}{2\times 3}} \\ &\times (9n-20)\sqrt{\frac{3+3-2}{3\times 3}} = 8(9n-20). \end{aligned} \quad (19)$$

This completes the proof. □



(a) (b)

FIGURE 3: [27]: (a) The subdivision graph $S(L_n)$ of the tadpole graph L_n. (b) The line graph $L(S(L_n))$.

TABLE 1: Comparison between $\mathbf{ABC_e}$, $\mathbf{ABC_e}$, and **ABCT** of $\mathbf{L(S(T_{n,k}))}$.

(n, k)	$\mathbf{ABC_e}$	$\mathbf{ABCII_e}$	ABCT
(1, 1)	2.1093898	−4.898979486	4.7876937
(2, 2)	6.242640687	4.242640687	23.98539667
(3, 3)	9.071067812	12.72792206	72.10624376
(4, 4)	11.89949494	21.21320344	142.5586461
(5, 5)	14.72792206	29.69848481	235.5731609
(6, 6)	17.55634919	38.18376618	351.1898721
(7, 7)	20.38477631	46.66904756	489.4215973
(8, 8)	23.21320344	55.15432893	650.2737199
(9, 9)	26.04163056	63.63961031	833.7488996
(10, 10)	28.87005769	72.12489168	1039.848602

Theorem 9. *The atom-bond connectivity temperature index of $L(S(L_n))$ is*

$$ABCT(G) = 6\sqrt{|(3n-3)(-6n+8)|} + 4\sqrt{\left|\frac{-36n^2+93n-58}{3}\right|} + \frac{(9n-20)\sqrt{|(6n-7)(-12n+20)|}}{3}. \quad (20)$$

Proof. In G, there are total $6n-4$ vertices in which 8 vertices are of degree 2 and the remaining $6n-12$ vertices are of degree 3. The total number of edges of G is $9n-10$. After adopting the induction technology, we have 3 edge partition based on the temperature. If T_u and T_v are the temperature of the vertex u and v, then there are 6 edge of type $T_2 = T_2 = 1/3n-3$, 4 edges with $T_2 = 1/3n-3$, $T_3 = 3/6n-7$ and $9n-20$ edges of type $T_3 = T_3 = 3/6n-7$. As a result, we infer

$$\begin{aligned} ABCT(G) &= (6)\sqrt{\left|\frac{1/3n-3+1/3n-3-2}{1/3n-3\times 1/3n-3}\right|} + (4)\sqrt{\left|\frac{1/3n-3+3/6n-7-2}{1/3n-3\times 3/6n-7}\right|} + (9n-20)\sqrt{\left|\frac{3/6n-7+3/6n-7-2}{3/6n-7\times 3/6n-7}\right|} \\ &= 6\sqrt{|(3n-3)(-6n+8)|} + 4\sqrt{\left|\frac{-36n^2+93n-58}{3}\right|} + \frac{(9n-20)\sqrt{|(6n-7)(-12n+20)|}}{3}. \end{aligned} \quad (21)$$

This completes the proof. □

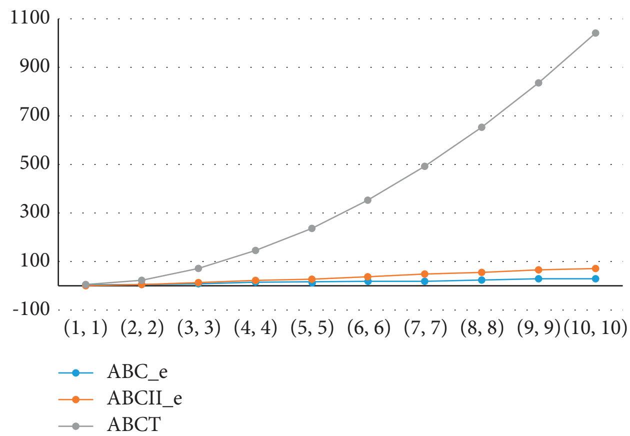


FIGURE 4: Comparison between ABC_e, ABC_e, and $AABC_e$ of $L(S(T_{n,k}))$.

TABLE 2: Comparison between $\mathbf{ABC_e}$, $\mathbf{ABCII_e}$, and **ABCT** of $\mathbf{L(S(W_{n+1}))}$.

n	$\mathbf{ABC_e}$	$\mathbf{ABCII_e}$	ABCT
1	3.483163248	0	3.821367205
2	7.454653677	5.333333333	20.00859965
3	12	32	62.35382907
4	16.92289018	101.1928851	124.0640009
5	22.15246524	238.5139176	205.2566258
6	27.64735154	473.2863826	305.8662831
7	33.37946531	838.1312546	425.8255499
8	39.32811367	1368.662608	565.0772788
9	45.47722558	2103.254621	723.5738722
10	51.81385047	3082.855819	901.2753388

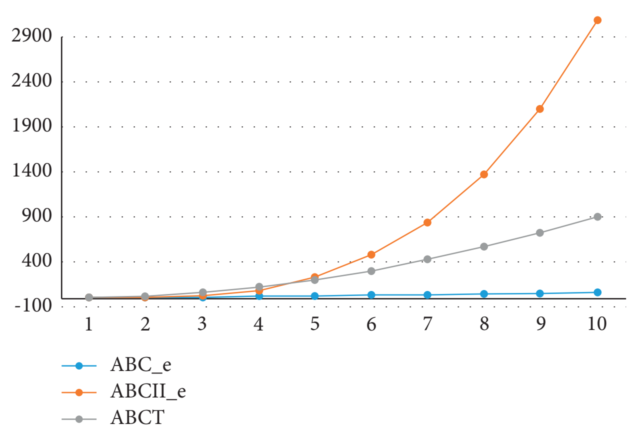


FIGURE 5: Comparison between ABC_e, $ABCII_e$, and $ABCT$ of $L(S(W_{n+1}))$.

TABLE 3: Comparison between $\mathbf{ABC_e}$, $\mathbf{ABCII_e}$, and **ABCT** of $\mathbf{L(S(L_n))}$.

n	$\mathbf{ABC_e}$	$\mathbf{ABCII_e}$	ABCT
1	−0.2622655215	−88	−8.061498381
2	5.737734479	−16	27.04079003
3	11.73773448	56	100.868826
4	17.73773448	128	225.7405179
5	23.73773448	200	401.5280011
6	29.73773448	272	628.2266894
7	35.73773448	344	905.8363724
8	41.73773448	416	1234.357204
9	47.73773448	488	1613.789329
10	53.73773448	560	2044.132853

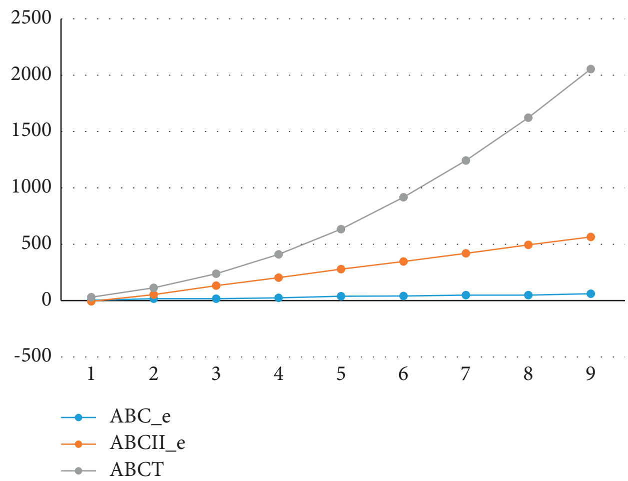


FIGURE 6: Comparison between ABC_e, $ABCII_e$, and $ABCT$ of $L(S(L_n))$.

3. Numerical Simulation and Conclusion

In this paper, we propose some novel families of atom-bond connectivity index. Now, the results of these indices will be compared. The comparison between ABC_e, $ABCII_e$, and $ABCT$ of $L(S(T_{n,k}))$ is shown in Table 1. The graphical representation of Table 1 is illustrated in Figure 4.

Similarly, the results for ABC_e, $ABCII_e$, and $ABCT$ of $LS(W_{n+1})$ are compared in Table 2. Table 2 is given in Figure 5. Finally, the comparison of ABC_e, $ABCII_e$, and $ABCT$ of $L(S(L_n))$ is shown in Table 3. The illustration of the results for ABC_e, $ABCII_e$, and $ABCT$ is shown as Figure 6.

In this paper, certain degree-based topological indices, namely, ABC indices, were studied for the case of the line graphs of the subdivision graphs. It is anticipated that this computational study will encourage the researchers to have a firm grasp on the index framework they have chosen. The computational technique presented here can be useful for analysing the physicochemical features of the specified network, as well as being cost-effective and time-efficient. Future work includes the investigation of new classes of line graph of subdivision graphs and their topological indices which is useful in QSAR and QSPR studies.

Data Availability

All data required for this paper are included within these papers.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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