Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 6227794, 9 pages
https://doi.org/10.1155/2022/6227794

Research Article

A Robust DOA Tracking Method Using a Nested Array in Impulse Noise

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Received 6 June 2022; Accepted 14 July 2022; Published 9 August 2022

Academic Editor: Yanan Du

To achieve the underdetermined direction of arrival (DOA) tracking in impulse noise, we propose a robust DOA tracking method in this paper. Firstly, an infinite norm difference covariance (INDC) matrix is introduced to suppress the impulse noise. Secondly, using a nested array, we present a maximum likelihood DOA tracking equation based on the INDC matrix. Furthermore, a quantum-inspired multiverse algorithm is proposed to maximize efficiently the proposed tracking equation. The simulation results show that our method has better robustness and superiority compared to other DOA tracking methods, which can achieve underdetermined DOA tracking in impulse noise as well as in the multipath environment.

1. Introduction

In the past few decades, many high-resolution direction of arrival (DOA) estimation algorithms [1–3] have been proposed and applied in many fields such as radar and wireless communication, most of which assume that the targets are stationary, whereas the targets are usually time-varying in fact. For time-varying DOA tracking problem, many researchers have made great efforts for its development. In [4], the authors proposed a projection approximation subspace tracking (PAST) algorithm to track the subspace. In [5], a particle filter (PF) DOA tracker was proposed, which utilized a partitioned state-vector method to achieve multiple targets tracking. In [6], a fast approximated power iteration subspace tracking method was proposed, which offered a faster tracking response. In [7], on the basis of low-rank and sparse recovery, the authors proposed a novel DOA tracking method. In [8], a multisource DOA tracking approach using a superposition model was proposed, which utilized the PF to achieve the DOA tracking.

The above methods are derived in Gaussian noise, whereas in real world, there exists some impulse noises, including the lightning and the low-frequency atmospheric noise, which can be described well as symmetric α-stable (SαS) distribution [9], and these forms of impulse noise exist a long tail, which deteriorates the performance of the existing methods. Many methods, including fractional lower-order moment [10], correntropy technique [11, 12], infinite norm [13], and kernel method [14, 15], have been proposed to achieve accurate estimates in impulse noise. DOA tracking in impulse noise has also made great progress in recent years. In [16], the authors proposed a robust PAST (RPAST) algorithm for tracking subspace in impulse noise. In [17], a PF method for DOA tracking in impulse noise was proposed, which offered robustness in impulse noise but cannot obtain excellent performance in fast time-varying scenario. In [18], a correntropy method was proposed to achieve subspace tracking in impulse noise.

Furthermore, the above algorithms are based on uniform linear array (ULA), which will be invalid when the number of targets exceeds the number of antennas. Many nonuniform arrays have been proposed to achieve underdetermined DOA estimation [19, 20]. Some researchers also applied them in DOA tracking problems. In [21], a PF method using coprime array was proposed for DOA tracking, which was based on the propagator method. In [22], a spatial smoothing projection approximation subspace tracking (SSPAST) method was proposed, which utilized the difference coarray of nested array (NA) and coprime array to achieve underdetermined DOA tracking. In [23], a DOA tracking method using NA was proposed, which was based on offset compensation to obtain accurate tracking results.

The noted methods are only an improvement on one aspect of the DOA tracking problem; in order to address the above problems simultaneously, a robust DOA tracking method using nested array in impulse noise is proposed in our work. First, we propose an infinite norm difference covariance (INDC) matrix to obtain robustness in impulse noise, and on this basis, a maximum likelihood (ML) DOA tracking equation based on NA is utilized, which involves a multidimension joint optimization problem requiring enormous computational complexity. Fortunately, intelligent optimization algorithms, such as the sea lion optimization algorithm [24] and the Archimedes optimization algorithm [25], can allow for this cost function. However, the above algorithms are easy to trap in local optimum. Therefore, we propose a quantum-inspired multiverse algorithm (QMVA) to avoid this problem, which is inspired by quantum computation [26] and theory of cosmology [27]. The resulting method is termed as QMVA-INDC-ML-NA. The simulation results demonstrate that our method offers better robustness and effectiveness compared to other approaches.

The main contributions are as follows:

(1) An INDC matrix is proposed to suppress the strong impulsive noise.

(2) A ML tracking equation based on INDC, using NA, is proposed to achieve underdetermined DOA tracking in impulse noise.

(3) A quantum-inspired multiverse algorithm is proposed to solve efficiently the considered ML tracking equation.

The following is the rest of this paper. The DOA tracking model in impulse noise is introduced in Section 2. The DOA tracking method based on the QMVA is presented in Section 3. The simulation results are shown in Section 4. Finally, the conclusions of our work are given in Section 5.

2. DOA Tracking Model in Impulse Noise

Assume that a nested array consists of N ULAs and M isotropic antennas and d_m denotes the distance between the mth antenna and the first antenna, where $m = 1, 2, \ldots, M$, $d_1 = 0 < d_2 < \cdots < d_M$. The minimum spacing of the antenna elements is ε, and then the coordinates of the antenna elements are

$$\mathbf{d} = \bigcup_{n}^{N} \mathbf{d}_n = [d_1, d_2, \ldots, d_M] = \varepsilon[h_1, h_2, \ldots, h_M], \quad (1)$$

where $h_1, h_2, \ldots, h_M$ are integers, and $\mathbf{d}_n = \left\{\overline{h}\varepsilon \coprod_{\overline{n}}^{n-1} (M_{\overline{n}} + 1), \overline{h} = 1, 2, \ldots, M_n\right\}$ and $\mathbf{d}_1 = \left\{\overline{h}\varepsilon, \ \overline{h} = 1, 2, \ldots, M_1\right\}$ denote the coordinates of the nth ULA and the first ULA, respectively, where $M_n \geq 2$ and $M_1 + M_2 + \cdots + M_N = M$. A set $\dot{H} = \{h_a - h_b | a, \ b = 1, 2, \ldots, M; a > b\}$ is a continuous or nearly continuous set of natural numbers.

Consider that P narrowband signals with wavelength λ impinging on a nested array with M isotropic antenna elements thus the receiving $M \times 1$ vector is described as

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t), \quad (2)$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_P)]$ is the $M \times P$ array manifold, the pth steering vector is $\mathbf{a}(\theta_p) = [1, e^{-j2\pi d_2 \sin(\theta_p)/\lambda}, \ldots, e^{-j2\pi d_M \sin(\theta_p)/\lambda}]^T$, $p = 1, 2 \ldots, P$, $\theta = [\theta_1, \theta_2, \ldots, \theta_P]$, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_P(t)^T]$ is the $P \times 1$ signal vector, and $\mathbf{n}(t)$ is the $M \times 1$ complex impulse noise vector.

The characteristic function of zero-location SαS distribution is given by

$$\varphi(w) = \&e^{-\gamma|w|^\alpha}, \quad (3)$$

where α and γ denote the characteristic exponent and the scale. In impulse noise, one usually use generalized signal-to-noise ratio (GSNR)

$$\text{GSNR} = 10\lg\left\{\frac{E\left[\|\mathbf{s}(t)\|^2\right]}{\gamma^\alpha}\right\}, \quad (4)$$

where $E[\cdot]$ denote the expectation.

As conventional second-moment-based methods will deteriorate or even be invalid in impulse noise, we employ the infinite norm normalization preprocessing method, and on this basis, we propose an infinite norm difference covariance (INDC) matrix for the kth snapshot, and the INDC matrix is given by

$$\mathbf{R}(k) = [\mathbf{r}_1(k), \mathbf{r}_2(k), \ldots \mathbf{r}_M](k) = \begin{pmatrix} r_{11}^{(h_1-h_1)}(k) & \cdots & r_{1M}^{(h_1-h_M)}(k) \\ \vdots & \ddots & \vdots \\ r_{M1}^{(h_M-h_1)}(k) & \cdots & r_{MM}^{(h_M-h_M)}(k) \end{pmatrix}, \quad (5)$$

where

$$\mathbf{r}_m(k) = \left[r_{1m}^{(h_1-h_m)}(k), r_{2m}^{(h_2-h_m)}(k), \ldots, r_{Mm}^{(h_M-h_m)}(k)\right]^T. \quad (6)$$

The ith row and jth column element of $\mathbf{R}(k)$ is $r_{il}(k) = \overline{x}_i(k)\overline{x}_l^*(k)|\overline{x}_i(k) - \overline{x}_l^*(k)|^\sigma$, where $\overline{x}(k) = [\overline{x}_1(k), \overline{x}_2(k), \ldots, \overline{x}_M(k)]^T = \mathbf{x}(k)/\max_{1\leq m\leq M}\{|x_m(k)|\}$; σ denotes the difference constant, $k = 1, 2, \ldots, K_P$; and K_P denotes the number of snapshots.

Next, virtualize the INDC of the nested array into an extended INDC of a virtual ULA with more antenna elements, and virtualize the array manifold into a virtual array manifold. The maximum correlation delay calculated by the nested array is $\widetilde{M}$, the number of antenna elements of virtual ULA is $\widetilde{M} + 1$, the pth virtual steering vector is $\widetilde{a}(\theta_p) = [1, e^{-j2\pi\varepsilon \sin(\theta_p)/\lambda}, \ldots, e^{-j2\pi\varepsilon M \sin(\theta_p)/\lambda}]^T$, and the virtual INDC is given by

$$\widetilde{R}(k) = \left[\widetilde{r}_1(k), \widetilde{r}_2(k), \ldots \widetilde{r}_{\widetilde{M}+1}(k)\right], \quad (7)$$

where $\widetilde{r}_c(k) = [\widetilde{r}_{1c}(k), \widetilde{r}_{2c}(k), \ldots, \widetilde{r}_{(\widetilde{M}+1)c}(k)]^T$, $1 \le c \le \widetilde{M}+1$, $\widetilde{r}_{\rho\tau}(k) = E[r_{ab}^{(h_a - h_b)}(k)]$, $\rho - \tau = h_a - h_b$, $1 \le \rho, \tau \le \widetilde{M}+1$, and $1 \le a, b \le M$.

For the $(k+1)$th snapshot, the updated INDC is obtained as

$$\mathbf{R}_S(k+1) = \omega \mathbf{R}_S(k) + \widetilde{R}(k+1), \quad (8)$$

where $\mathbf{R}_S(k)$ is the updated INDC of the kth snapshot, $\widetilde{R}(k+1)$ is the virtual INDC of the $(k+1)$th snapshot, ω is the update constant, and for the first snapshot, $\mathbf{R}_S(1) = \widetilde{R}(1)$.

The maximum likelihood (ML) tracking equation can be represented by

$$\widehat{\theta} = \operatorname{argmax}_{\boldsymbol{\theta}} \operatorname{trace}\left[\mathbf{P}_{\widetilde{A}(\boldsymbol{\theta})} \mathbf{R}_S(k)\right], \quad (9)$$

where $\mathbf{P}_{\widetilde{A}(\theta)} = \widetilde{A}(\theta)[\widetilde{A}^H(\theta)\widetilde{A}(\theta)^{-1}\widetilde{A}^H(\theta)]$ denotes the projection matrix of $\widetilde{A}(\theta)$, $\widetilde{A}(\theta) = [\widetilde{a}(\theta_1), \widetilde{a}(\theta_2), \ldots, \widetilde{a}(\theta_P)]$, and $\operatorname{trace}(\cdot)$ denotes the trace of the matrix.

3. DOA Tracking Method Based on the QMVA

3.1. Quantum-Inspired Multiverse Algorithm. The quantum-inspired multiverse algorithm (QMVA) is inspired by quantum computation [26] and theory of cosmology [27]. Assume that Q denotes the number of quantum universes and G denotes the maximum number of iterations. The quantum state of the qth quantum universe at the gth iteration is $\mathbf{y}_q^g = [y_{q,1}^g, y_{q,2}^g, \ldots, y_{q,B}^g]$, where $0 \le y_{q,b}^g \le 1$ $(b = 1, 2, \ldots, B)$ with B denoting the number of variables of the optimization problems, and the actual state of the qth quantum universe $\overline{y}_q^g = [\overline{y}_{q,1}^g, \overline{y}_{q,2}^g, \ldots, \overline{y}_{q,B}^g]$ $(q = 1, 2, \ldots, Q)$ can be obtained by

$$\overline{y}_{q,b}^g = y_{q,b}^g\left(\overline{y}_b^U - \overline{y}_b^L\right) + \overline{y}_b^L, \quad (10)$$

where $\overline{y}_{q,b}^g \in [\overline{y}_b^L, \overline{y}_b^U]$, $\overline{y}_b^U$ and $\overline{y}_b^L$ denote the bth dimensional upper bound and lower bound, respectively, and $f_q^g = f(\overline{y}_q^g)$ denotes the fitness of $\overline{y}_q^g$.

In the exploration stage, quantum universes are sorted according to their fitness at each iteration, and on this basis, a quantum universe is chosen by the roulette wheel, and the specific formula is given by

$$y_{q,b}^g = \left\{ y_{\widetilde{q},b}^g, r_{q,b}^g < \overline{f}_q^g, y_{q,b}^g, r_{q,b}^g \ge \overline{f}_q^g, \right. \quad (11)$$

where $\overline{f}_q^g = f_q^g / \|\mathbf{f}^g\|_2$ denotes the normalized fitness of the qth quantum universe at the gth iteration, where $\mathbf{f}^g = [f_1^g, f_2^g, \ldots, f_Q^g]$ denotes the set of the fitness at the g th iteration, and $\|\cdot\|_2$ denotes the Euclidean norm of the vector, $r_{q,b}^g$ denotes a uniformly random number in [0, 1], and $y_{\widetilde{q},b}^g$ denotes the quantum state of the $\widetilde{q}$th quantum universe at the gth iteration selected by the roulette wheel. This roulette wheel mechanism can guarantee the exploration capacity of the QMVA.

In the exploitation stage, generate a wormhole existence probability P_g at first, which is given by

$$P^g = P_{\min} + g \times \left(\frac{P_{\max} - P_{\min}}{G}\right), \quad (12)$$

where $P_{\max}$ and $P_{\min}$ denote the user defined maximum and minimum of P_g, respectively. Then, a travelling distance rate T_r^g is given by

$$T_r^g = 1 - \frac{g^{1/v}}{G^{1/v}}, \quad (13)$$

where v denotes the exploitation constant.

For the qth quantum universe at the gth iteration, generate a uniformly random number $\widetilde{r}_{q,b}^g$ in [0, 1]. If $\widetilde{r}_{q,b}^g < P^g$, the bth quantum rotational angle of the qth quantum universe at $(g+1)$th iteration is given by

$$\widehat{\mu}_{q,b}^{g+1} = \begin{cases} r_1 \times T_r^g, \widetilde{r}_{q,b}^g < 0.5, \\ -r_1 \times T_r^g, \widetilde{r}_{q,b}^g \ge 0.5, \end{cases} \quad (14)$$

where r_1 and $\widetilde{r}_{q,b}^g$ are random numbers that obeys a uniform distribution in the range [0,1]. Then, the quantum state of the qth quantum universe is updated by

$$y_{q,b}^{g+1} = \left| y_{\text{best},b}^g \cos\left(\widehat{\mu}_{q,b}^{g+1}\right) + \sqrt{1 - \left(y_{\text{best},b}^g\right)^2} \sin\left(\widehat{\mu}_{q,b}^{g+1}\right) \right|, \quad (15)$$

where $y_{\text{best},b}^g$ denotes the bth quantum state of the quantum universe with best fitness until the gth iteration. The wormhole existence probability increases adaptively during entire iterations, which emphasizes the exploitation capacity of the QMVA. The travelling distance rate increases the accuracy of local search during entire iterations. Therefore, the balance between exploration and exploitation of the proposed QMVA is guaranteed by the effective combination of two stages.

3.2. DOA Tracking Based on the QMVA. To reduce the computational cost of DOA tracking, we propose the dynamic upper and lower bounds of the search space, which will continue to decrease as the number of snapshots increases, and the dynamic upper and lower bounds are defined as $\begin{bmatrix} u_1(k), u_2(k), \ldots, u_B(k) \\ l_1(k), l_2(k), \ldots, l_B(k) \end{bmatrix}$, where $u_b(k)$ and $l_b(k)$ denote the bth dimension upper and lower bounds of the search space for the kth snapshot, respectively.

At the first snapshot, the upper and lower bounds are the definition domain of the search space. For the kth snapshot, the upper and lower bound of the search space are updated by

$$\begin{cases} u_b(k) = \mu_b(k-1) + |u_b(k-1) - \mu_b(k-1)| \times \left(1 - \vartheta \times e^{-1/B}\right)^k + \overline{r}, \\ l_b(k) = l_b(k-1) - |l_b(k-1) - \mu_b(k-1)| \times \left(1 - \vartheta \times e^{-1/Bk}\right) - \overline{r}, \end{cases} \quad (16)$$

where ϑ denotes the convergence constant, $\mu_b(k-1)$ denotes the centre value of the bth dimension search space for the $(k-1)$th snapshot, $\mu_b(k) = \varsigma\mu_b(k-1) + (1 - \varsigma\overline{\mu})_b(k-1)$, ς denotes the genetic factor, and $\overline{\mu}_b(k-1)$ denotes the bth dimension estimated value for the $(k-1)$th snapshot.

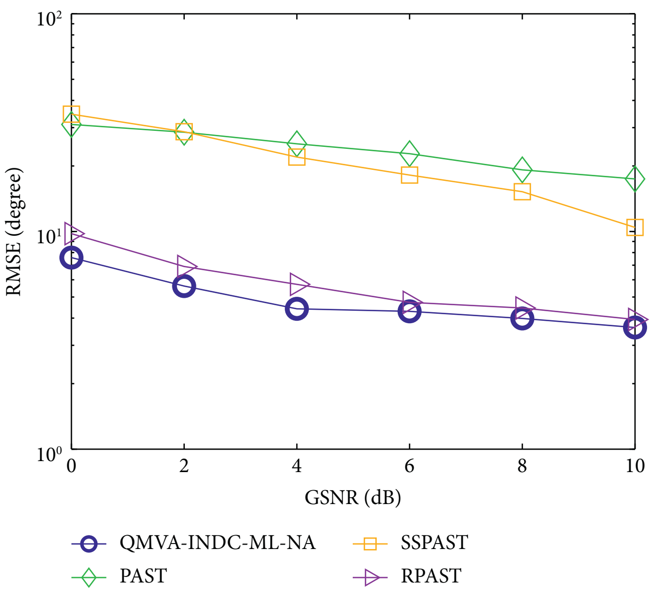

Figure 1: RMSE curves via $\alpha = 1.2$.

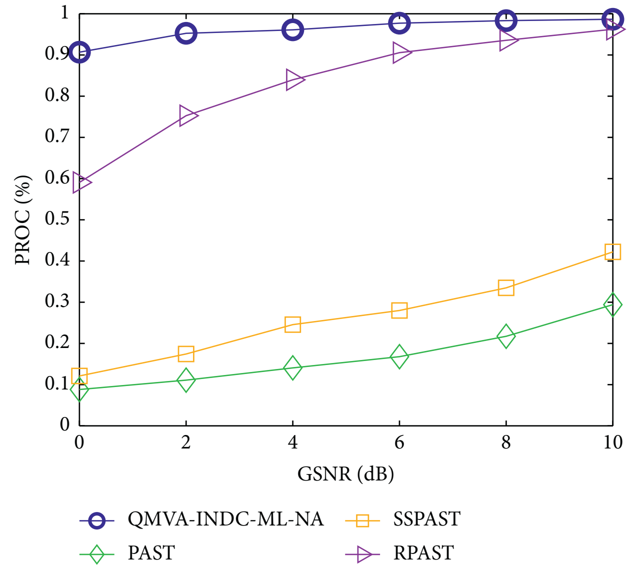

Figure 2: PROC curves via $\alpha = 1.2$.

Moreover, the maximum number of iterations is $G = \zeta \max_{1 \le p \le P}\left\{\left\lfloor u_p(k) - v_p(k) \right\rfloor\right\}$, where ζ is a positive integer, and $\lfloor \cdot \rfloor$ denotes the round down operation.

In the QMVA, the quantum states of the initial quantum universes are randomly generated in [0,1], and for the proposed DOA tracking method, the fitness function is defined as

$$f_q^g = f\left(\overline{y}_q^g\right) = \operatorname{trace}\left(\mathbf{P}_{\widetilde{A}\left(\overline{y}_q^g\right)} \mathbf{R}_S(k)\right), \quad (17)$$

where $\overline{y}_q^g = [\overline{y}_{q,1}^g, \overline{y}_{q,2}^g, \ldots, \overline{y}_{q,B}^g]$ is corresponding to the estimation of DOAs of the P moving targets; thus, $B = P$. Therefore, the proposed DOA tracking method can be summarized as follows:

Step 1. Obtain the first snapshot data, initialize $\mathbf{R}_S(1) = \widetilde{R}(1)$.

Step 2. Initialize the upper and lower bounds of the search space, and initialize the parameters of the QMVA: the number of quantum universes, the minimum and maximum of wormhole existence probability, the exploitation constant, and the maximum number of iterations G.

Step 3. Initialize randomly the quantum states of the initial quantum universes, calculate the fitness of all quantum universes and store the quantum state of the quantum universe with best fitness

Step 4. In the exploration stage, quantum universes are sorted according to their fitness at each iteration, and on this basis, a quantum universe is chosen by the roulette wheel, for details, see equation (11). In the exploitation stage, quantum universes are evolved through equations (14) and (15)

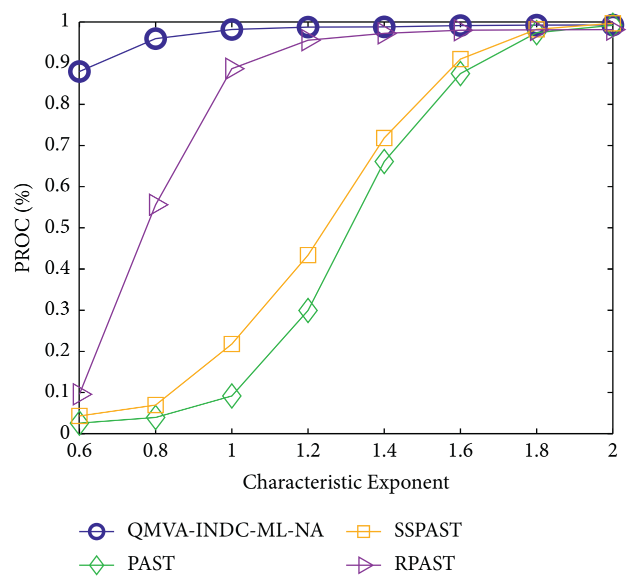

Figure 3: PROC curves via GSNR = 10 dB.

Step 5. Calculate the fitness of all quantum universes, update the quantum state of the quantum universe with best fitness

Step 6. Examine whether G is reached, if not, let $g = g + 1$, and then go back to step 4; otherwise, stop the loop iteration, output the actual state of the quantum state with best fitness for the kth snapshot and go to the next step

Step 7. Examine whether K_P is reached, if not, obtain the next snapshot data, update the INDC through (8), let $k = k + 1$, and go back to step 2; otherwise, output the DOA tracking results according to the actual state of the quantum universe with best fitness of all snapshots

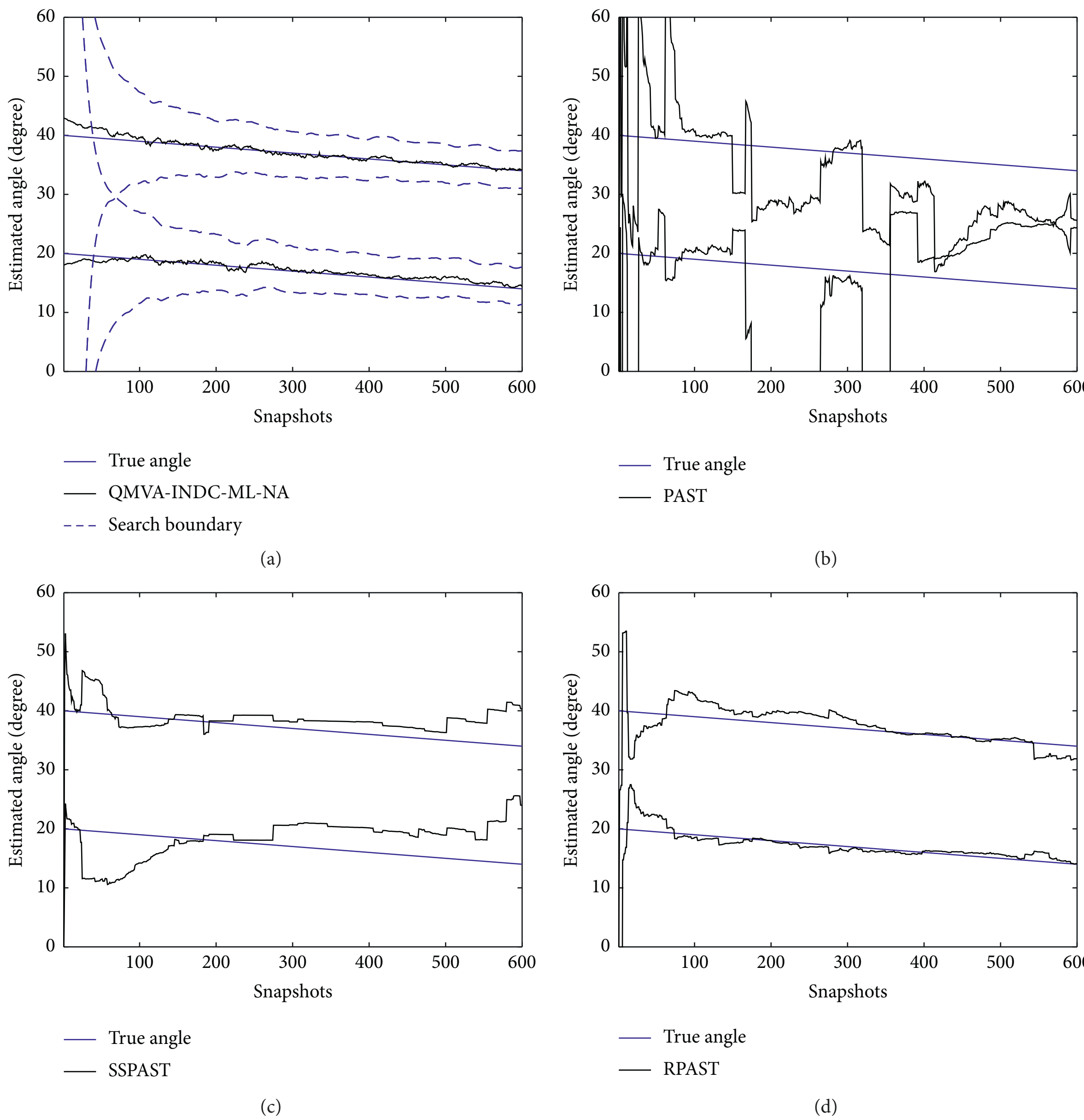

FIGURE 4: DOA tracking results via GSNR = 10 dB and $\alpha = 1.2$. (a) The proposed method. (b) The PAST method. (c) The SSPAST method. (d) The RPAST method.

4. Simulation Results

The root mean square error (RMSE) is defined as

$$\text{RMSE} = \sqrt{\sum_{k=1}^{K_P}\sum_{i=1}^{P}\sum_{\dot{n}=1}^{N_e}\frac{\left(\theta_i(k)-\widehat{\theta}_i^{\dot{n}}(k)\right)^2}{PK_PN_e}}, \quad (18)$$

where $\theta_i(k)$ denotes the ith true DOA at kth snapshot, $\widehat{\theta}_i^{\dot{n}}(k)$ denotes the ith estimated DOA at kth snapshot in the $\dot{n}$ run, and N_e denotes the number of Monte-Carlo runs.

The probability of convergence (PROC) is defined as

$$\text{PROC} = \sum_{k=1}^{K_P}\sum_{i=1}^{P}\sum_{\dot{n}=1}^{N_e}\frac{\chi_i^{\dot{n}}(k)}{PK_PN_e}, \quad (19)$$

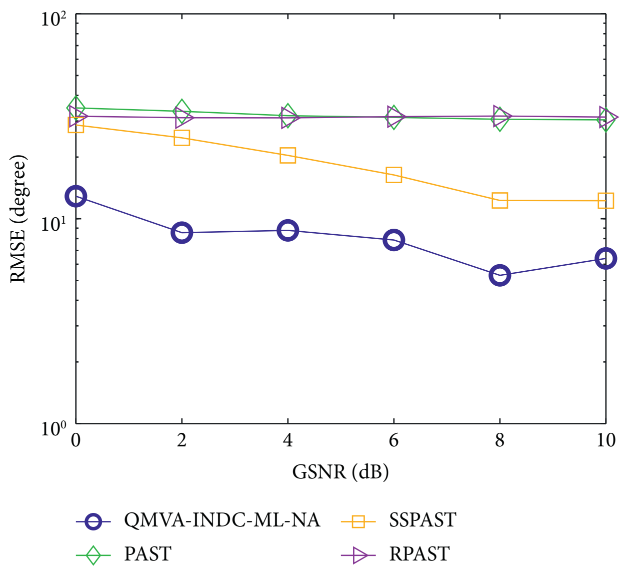

Figure 5: RMSE curves via $\alpha = 1.2$.

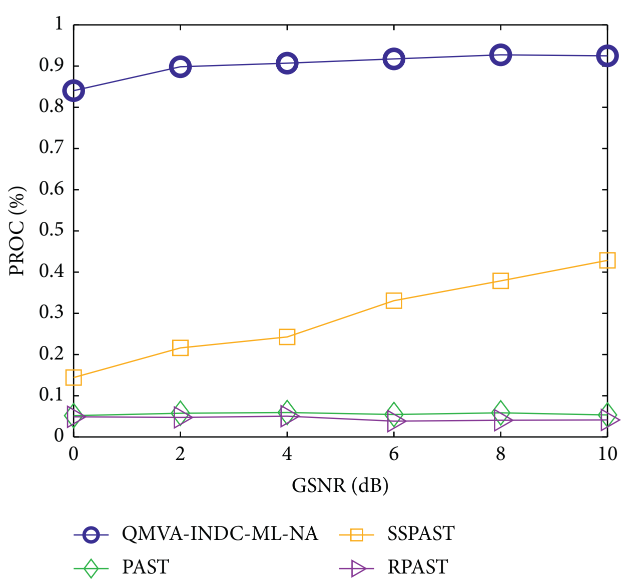

Figure 6: PROC curves via $\alpha = 1.2$.

where

$$\chi_i^{n_i}(k) = \begin{cases} 1, & \theta_i(k) - \hat{\theta}_i^{n_i}(k) < 2^\circ, \\ 0, & \text{otherwise}. \end{cases} \quad (20)$$

We consider that NA consists of two ULAs, $M_1 = M_2 = 3$, $\varepsilon = 0.5\lambda$, difference constant $\sigma = -0.5$, update constant $\omega = 0.95$, $\zeta = 2$, $\vartheta = 0.01$, $\overline{r} = 3$, and $\varsigma = 0.8$. For the QMVA, $Q = 10$, $P_{\max} = 1$, $P_{\min} = 0.2$, and $\nu = 6$.

We execute 300 Monte-Carlo runs in the numerical simulations, and the proposed method is compared with three alternative methods: PAST [4], RPAST [16], and SSPAST [22].

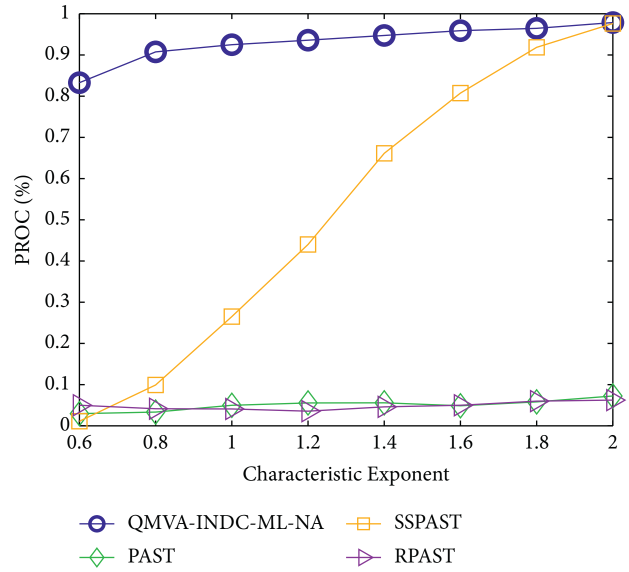

Figure 7: PROC curves via GSNR = 10 dB.

4.1. The Independent Sources Tracking Scenario. Consider that two independent time-varying sources with

$$\theta_1 = \frac{20-k}{100, k = 1, 2, \ldots, K_P}, \quad \theta_2 = \frac{40-k}{100, k = 1, 2, \ldots, K_P}, \quad (21)$$

where $K_P = 600$, and θ_1 and θ_2 are in degrees. Figures 1 and 2 show the RMSE curves and the PROC curves via $\alpha = 1.2$, respectively. Figure 3 shows the PROC curves via GSNR = 10 dB. Figure 4 plots the DOA tracking results of four methods via GSNR = 10 dB and $\alpha = 1.2$. From Figure 1, the proposed method has highest tracking accuracy in the low GSNR scenario compared with alternative algorithms. From Figures 2 and 3, the PROC of the proposed method exceeds 90% in terms of GSNR and characteristic exponent. From Figure 4, we can conclude that the tracking curve of the proposed method is smoother; in other words, the proposed method can achieve accurate tracking in the considered scenario compared with other methods.

4.2. The Coherent Sources Tracking Scenario. Consider that two coherent time-varying sources with the identical time-varying DOAs to the first simulation. Figures 5 and 6 show the RMSE curves and the PROC curves via $\alpha = 1.2$, respectively. Figure 7 shows the PROC curves via GSNR = 10 dB. Figure 8 plots the DOA tracking results of four methods via GSNR = 10 dB and $\alpha = 1.2$. From the figures, the PAST and the RPAST fail to work in the coherent sources tracking scenario. The SSPAST can achieve DOA tracking in the considered scenario, whose tracking performance is relatively weak. Furthermore, it is obvious that

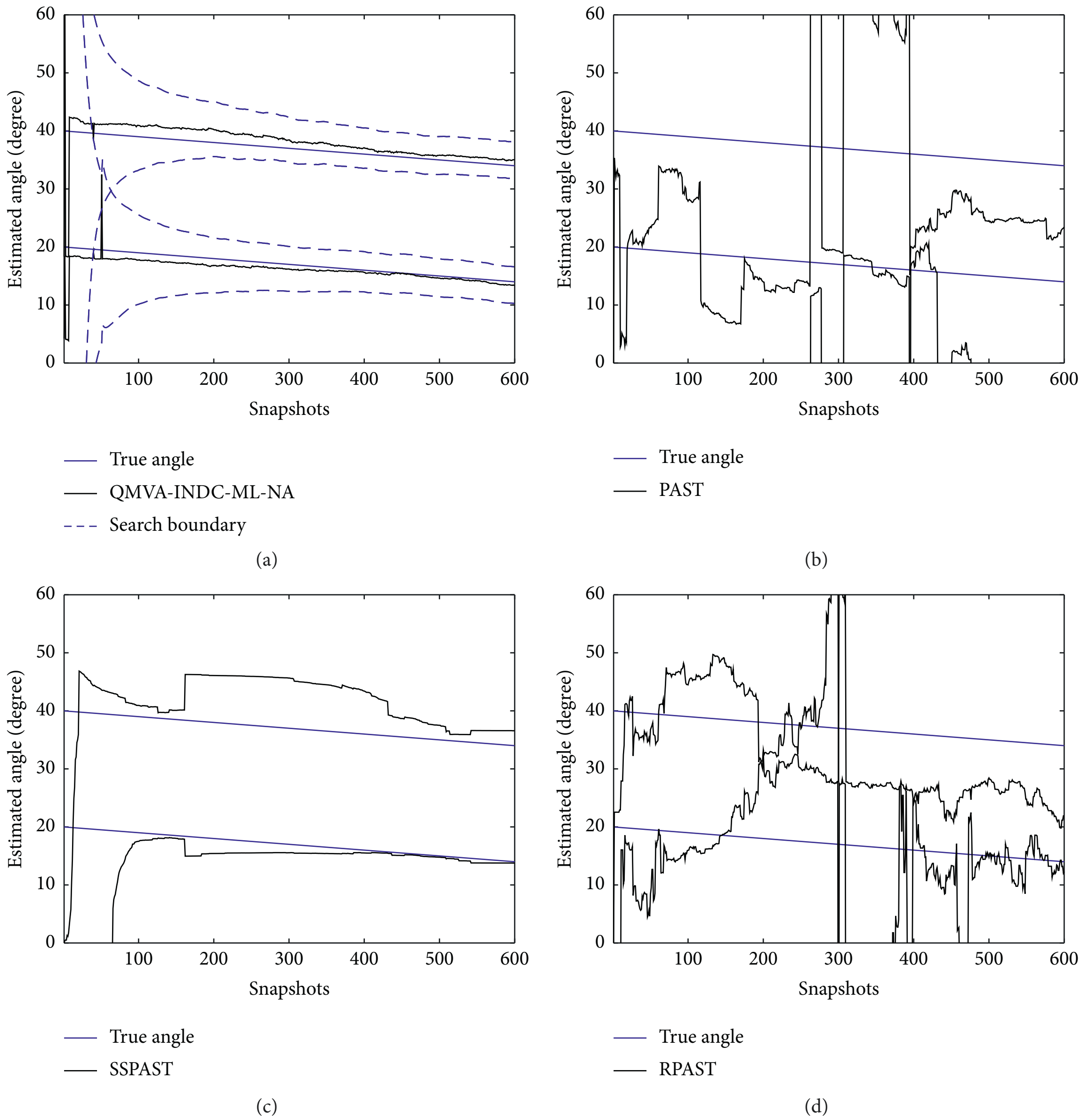

FIGURE 8: DOA tracking results via GSNR = 10 dB and $\alpha = 1.2$. (a) The proposed method. (b) The PAST method. (c) The SSPAST method. (d) The RPAST method.

the proposed method yields robust tracking performance in the coherent sources tracking scenario.

4.3. The Underdetermined DOA Tracking Scenario. Consider that seven independent time-varying sources with

$$\theta_p = \frac{\theta_p^0 - k}{100, k = 1, 2, \ldots, K_P}, \tag{22}$$

where $p = 1, 2, \cdots, 7$, and the corresponding $\theta_p^0 = -60, -40, -20, 0, 20, 40, 60$, $K_P = 600$, and θ_p are in degrees. In the underdetermined DOA tracking scenario, the PAST and the RPAST will be invalid; thus, we plot the tracking results of the proposed method and the SSPAST method. Figures 9 and 10 plot the DOA tracking results of the proposed method and the SSPAST method via GSNR = 10 dB and $\alpha = 1.2$. From the tracking results, although both the SSPAST and the proposed method can

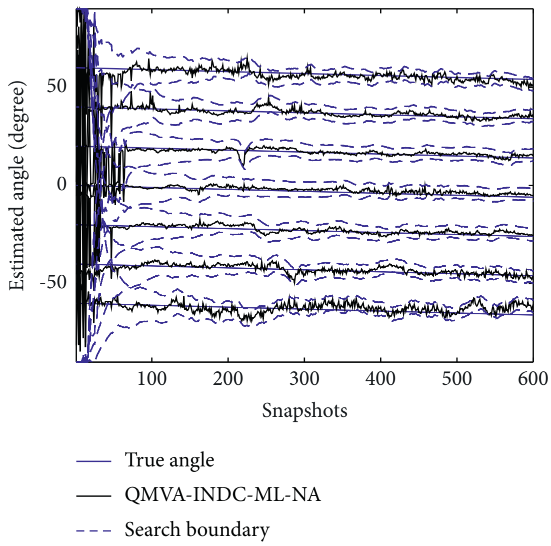

Figure 9: DOA tracking results of the proposed method.

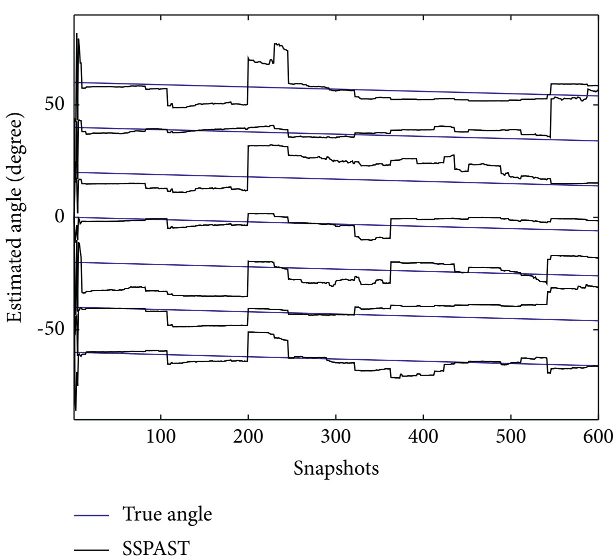

Figure 10: DOA tracking results of the SSPAST method.

achieve underdetermined DOA tracking, it is obvious that our tracking method has better DOA tracking performance in the underdetermined DOA tracking scenario.

5. Conclusions

In this paper, we propose a robust DOA tracking method using nested array for achieving the underdetermined DOA tracking in the impulse noise. Simulation results demonstrate that our method offers better robustness and effectiveness both in independent and coherent sources tracking scenarios and obtains better DOA tracking results in the underdetermined DOA tracking scenario. In the future, we will try to generalize it to other complex DOA tracking problems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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