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Research Article

Consolidation of a Certain Discrete Probability Distribution with a Subclass of Bi-Univalent Functions Involving Gegenbauer Polynomials

Ala Amourah,[1] **Mohammad Alomari**,[1] **Feras Yousef**,[2] **and Abdullah Alsoboh**[3]

[1]*Department of Mathematics, Irbid National University, Irbid, Jordan*
[2]*Department of Mathematics, School of Science, University of Jordan, Amman 11942, Jordan*
[3]*Umm-Alqura University, P.O Box 715, Mecca, Saudi Arabia*

Correspondence should be addressed to Mohammad Alomari; alomari@inu.edu.jo

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In this work, we introduce and investigate a new subclass of analytic bi-univalent functions based on subordination conditions between the zero-truncated Poisson distribution and Gegenbauer polynomials. More precisely, we will estimate the first two initial Taylor–Maclaurin coefficients and solve the Fekete–Szegö functional problem for functions belonging to this new subclass.

1. Introduction

Let $\mathscr{A}$ denote the class of all analytic functions f defined on the open unit disk $\mathbb{U} = \{z \in \mathbb{C}: |z| < 1\}$ and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. Thus, each $f \in \mathscr{A}$ has a Taylor–Maclaurin series expansion of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \ (z \in \mathbb{U}). \tag{1}$$

Let the functions f and g be in the class $\mathscr{A}$. We say that the function f is subordinate to g, written as $f \prec g$, if there exists a Schwarz function w, which is analytic in $\mathbb{U}$ with

$$\begin{aligned} &w(0) = 0, \\ &|w(z)| < 1 \, (z \in \mathbb{U}), \end{aligned} \tag{2}$$

such that

$$f(z) = g(w(z)). \tag{3}$$

Further, let $\mathscr{S}$ denote the class of all functions $f \in \mathscr{A}$ that are univalent in $\mathbb{U}$. It is worth to mention that if the function g is univalent in $\mathbb{U}$, then the following equivalence holds (see [1]):

$$f(z) \prec g(z) \quad \text{if and only if} \quad f(0) = g(0), \tag{4}$$

and

$$f(\mathbb{U}) \subset g(\mathbb{U}). \tag{5}$$

It is well known [2] that every function $f \in \mathscr{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \ (z \in \mathbb{U}), \tag{6}$$

$$f^{-1}(f(w)) = w\left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right), \tag{7}$$

where

$$\begin{aligned} f^{-1}(w) = w - a_2 w^2 &+ \left(2a_2^2 - a_3\right) w^3 \\ &- \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots. \end{aligned} \tag{8}$$

A function is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). For interesting examples of functions in the class Σ, see [3–10].

In probability theory, the zero-truncated Poisson distribution is a certain discrete probability distribution whose support is the set of positive integers, that is, the zero-truncated Poisson distribution is the same as the Poisson distribution without the zero count [11]. This distribution is also known as the conditional Poisson distribution [12] or the positive Poisson distribution [13]. The probability density function of the zero-truncated Poisson distribution is given by

$$P_m(X=s)=\frac{m^s}{(e^m-1)s!}, s=1,2,3,\ldots,m>0. \tag{9}$$

Here, let us consider a power series whose coefficients are probabilities of the zero-truncated Poisson distribution, that is,

$$\mathbb{P}(m,z)=z+\sum_{n=2}^{\infty}\frac{m^{n-1}}{(e^m-1)(n-1)!}z^n, z\in\mathbb{U}, \tag{10}$$

where $m>0$. By the ratio test, the radius of convergence of this series is infinity.

Define the linear operator $\chi\colon \mathscr{A}\longrightarrow\mathscr{A}$ by

$$\begin{aligned}\chi_m f(z)&=\mathbb{P}(m,z)*f(z)\\&=z+\sum_{n=2}^{\infty}\frac{m^{n-1}}{(e^m-1)(n-1)!}a_nz^n, z\in\mathscr{U},\\&=z+\frac{m}{(e^m-1)}a_2z^2+\frac{m^2}{2(e^m-1)}a_3z^3+\cdots,\end{aligned} \tag{11}$$

where * denotes the convolution or Hadamard product of two series (see [14]).

Orthogonal polynomials have been studied extensively as early as they were discovered by Legendre in 1784 [15]. In mathematical treatment of model problems, orthogonal polynomials arise often to find solutions of ordinary differential equations under certain conditions imposed by the model. The importance of the orthogonal polynomials for the contemporary mathematics, as well as for wide range of their applications in the physics and engineering, is beyond any doubt. Recently, many researchers have been exploring bi-univalent functions associated with orthogonal polynomials [16–24]. Very recently, Amourah et al. [25, 26] considered the generating function for Gegenbauer polynomials $H_\alpha(x,z)$, which is given by

$$H_\alpha(x,z)=\frac{1}{\left(1-2xz+z^2\right)^\alpha}, \tag{12}$$

where $x\in[-1,1]$, $\alpha>-1/2$, and $z\in\mathbb{U}$. For fixed x, the function H_α is analytic in $\mathbb{U}$, so it can be expanded in a Taylor series as

$$H_\alpha(x,z)=\sum_{n=0}^{\infty}C_n^\alpha(x)z^n. \tag{13}$$

where $C_n^\alpha(x)$ is Gegenbauer polynomial of degree n (see [27]).

Obviously, H_α generates nothing when $\alpha=0$. Therefore, the generating function of the Gegenbauer polynomial is set to be

$$H_0(x,z)=1-\log\left(1-2xz+z^2\right)=\sum_{n=0}^{\infty}C_n^0(x)z^n, \tag{14}$$

for $\alpha=0$. Moreover, it is worth to mention that a normalization of α to be greater than $-1/2$ is desirable [28, 29]. Gegenbauer polynomials can also be defined by the following recurrence relations:

$$C_n^\alpha(x)=\frac{1}{n}\left[2x(n+\alpha-1)C_{n-1}^\alpha(x)-(n+2\alpha-2)C_{n-1}^\alpha(x)\right], \tag{15}$$

with the initial values

$$C_0^\alpha(x)=1, C_1^\alpha(x)=2\alpha x \text{ and } C_2^\alpha(x)=2\alpha(1+\alpha)x^2-\alpha. \tag{16}$$

Special cases of Gegenbauer polynomials $C_n^\alpha(x)$ are Chebyshev polynomials when $\alpha=1$ and Legendre polynomials when $\alpha=1/2$.

2. The Class $\zeta_\Sigma(x,\alpha,\mu)$

In this section, we introduce a new subclass of Σ involving the new constructed series (10) and Gegenbauer polynomials.

Definition 2.1. A function $f\in\Sigma$ given by (1) is said to be in the class $\zeta_\Sigma(x,\alpha,\mu)$ if the following subordinations are satisfied:

$$(1-\mu)\frac{\chi_m f(z)}{z}+\mu\left(\chi_m f(z)\right)'\prec H_\alpha(x,z), \tag{17}$$

$$(1-\mu)\frac{\chi_m g(w)}{w}+\mu\left(\chi_m g(w)\right)'\prec H_\alpha(x,w), \tag{18}$$

where $\alpha>0$, $\mu\geq 0$, $x\in(1/2,1]$, and the function $g=f^{-1}$ is given by (8).

Upon specializing the parameter μ, one can get the following new subclass of Σ as illustrated in the following example.

Example 2.1. If $\mu=1$, then we have $\zeta_\Sigma(x,\alpha,1)=\zeta_\Sigma(x,\alpha)$, in which $\zeta_\Sigma(x,\alpha)$ denotes the class of functions $f\in\Sigma$ given by (1) and satisfying the following conditions.

$$\left(\chi_m f(z)\right)'\prec H_\alpha(x,z), \tag{19}$$

$$\left(\chi_m g(w)\right)'\prec H_\alpha(x,w). \tag{20}$$

3. Estimates of the Class $\zeta_\Sigma(x,\alpha,\mu)$

First, we give the coefficient estimates for the class $\zeta_\Sigma(x,\alpha,\mu)$ given in Definition 2.1.

Theorem 3.1. *Let $f\in\Sigma$ given by (1) belong to the class $\zeta_\Sigma(x,\alpha,\mu)$. Then,*

$$|a_2| \le \frac{2\alpha x(e^m-1)\sqrt{2\alpha x}}{m\sqrt{\left|\left[2\alpha^2(1+2\mu)(e^m-1)-2\alpha(1+\mu)^2(1+\alpha)\right]x^2+\alpha(1+\mu)^2\right|}}, \tag{21}$$

$$|a_3| \le \frac{4\alpha^2x^2(e^m-1)^2}{m^2(1+\mu)^2}+\frac{4\alpha x(e^m-1)}{m^2(1+2\mu)}. \tag{22}$$

Proof. Let $f \in \zeta_\Sigma(x,\alpha,\mu)$. From Definition 2.1, for some analytic functions w, v such that $w(0)=v(0)=0$ and $|w(z)|<1$, $|v(w)|<1$ for all $z,w \in \mathbb{U}$, then we can write

$$(1-\mu)\frac{\chi_m f(z)}{z}+\mu(\chi_m f(z))' = H_\alpha(x,w(z)), \tag{23}$$

$$(1-\mu)\frac{\chi_m g(w)}{w}+\mu(\chi_m g(w))' = H_\alpha(x,v(w)). \tag{24}$$

From the equalities (23) and (24), we obtain that

$$\begin{aligned}&(1-\mu)\frac{\chi_m f(z)}{z}+\mu(\chi_m f(z))'\\ &\quad= 1+C_1^\alpha(x)c_1z+\left[C_1^\alpha(x)c_2+C_2^\alpha(x)c_1^2\right]z^2+\cdots,\end{aligned} \tag{25}$$

$$\begin{aligned}&(1-\mu)\frac{\chi_m g(w)}{w}+\mu(\chi_m g(w))'\\ &\quad= 1+C_1^\alpha(x)d_1w+\left[C_1^\alpha(x)d_2+C_2^\alpha(x)d_1^2\right]w^2+\cdots.\end{aligned} \tag{26}$$

It is fairly well known that if

$$|w(z)| = \left|c_1z+c_2z^2+c_3z^3+\cdots\right|<1,\ (z\in\mathbb{U}), \tag{27}$$

$$|v(w)| = \left|d_1w+d_2w^2+d_3w^3+\cdots\right|<1,\ (w\in\mathbb{U}), \tag{28}$$

then (see [30])

$$|c_j|\le 1 \text{ and } |d_j|\le 1 \text{ for all } j\in\mathbb{N}. \tag{29}$$

Thus, upon comparing the corresponding coefficients in (25) and (26), we have

$$\frac{(1+\mu)m}{e^m-1}a_2 = C_1^\alpha(x)c_1, \tag{30}$$

$$\frac{(1+2\mu)m^2}{2(e^m-1)}a_3 = C_1^\alpha(x)c_2+C_2^\alpha(x)c_1^2, \tag{31}$$

$$-\frac{(1+\mu)m}{e^m-1}a_2 = C_1^\alpha(x)d_1, \tag{32}$$

$$\frac{(1+2\mu)m^2}{2(e^m-1)}\left[2a_2^2-a_3\right] = C_1^\alpha(x)d_2+C_2^\alpha(x)d_1^2. \tag{33}$$

It follows from (30) and (32) that

$$c_1=-d_1, \tag{34}$$

$$\frac{2(1+\mu)^2m^2}{(e^m-1)^2}a_2^2 = \left[C_1^\alpha(x)\right]^2\left(c_1^2+d_1^2\right). \tag{35}$$

If we add (31) and (33), we get

$$\frac{(1+2\mu)m^2}{(e^m-1)}a_2^2 = C_1^\alpha(x)(c_2+d_2)+C_2^\alpha(x)\left(c_1^2+d_1^2\right). \tag{36}$$

Substituting the value of $(c_1^2+d_1^2)$ from (35) in the right hand side of (36), we deduce that

$$\begin{aligned}&\left[(1+2\mu)-\frac{2(1+\mu)^2}{(e^m-1)}\frac{C_2^\alpha(x)}{\left[C_1^\alpha(x)\right]^2}\right]\frac{m^2}{(e^m-1)}a_2^2\\ &\quad= C_1^\alpha(x)(c_2+d_2).\end{aligned} \tag{37}$$

Moreover, using (16), (29), and (37), we find that

$$|a_2| \le \frac{2\alpha x(e^m-1)\sqrt{2\alpha x}}{m\sqrt{\left|\left[2\alpha^2(1+2\mu)(e^m-1)-2\alpha(1+\mu)^2(1+\alpha)\right]x^2+\alpha(1+\mu)^2\right|}}. \tag{38}$$

Now, if we subtract (33) from (31), we obtain

$$\frac{(1+2\mu)m^2}{(e^m-1)}\left(a_3-a_2^2\right) = C_1^\alpha(x)(c_2-d_2)+C_2^\alpha(x)\left(c_1^2-d_1^2\right). \tag{39}$$

Then, in view of (16) and (35), (39) becomes

$$a_3 = \frac{(e^m-1)^2\left[C_1^\alpha(x)\right]^2}{2m^2(1+\mu)^2}\left(c_1^2+d_1^2\right)+\frac{(e^m-1)C_1^\alpha(x)}{m^2(1+2\mu)}(c_2-d_2). \tag{40}$$

Thus, applying (16) and (29), we conclude that

$$|a_3| \le \frac{4\alpha^2x^2(e^m-1)^2}{m^2(1+\mu)^2}+\frac{4\alpha x(e^m-1)}{m^2(1+2\mu)}. \tag{41}$$

This completes the proof of Theorem 3.1. □

Making use of the values of a_2^2 and a_3, we prove the following Fekete–Szegö inequality for functions in the class $\zeta_\Sigma(x,\alpha,\mu)$.

Theorem 3.2. *Let $f \in \Sigma$ given by (1) belong to the class $\zeta_\Sigma(x,\alpha,\mu)$. Then,*

$$|a_3 - \eta a_2^2| \le \begin{cases} 4\alpha x(e^m-1)/m^2(1+2\mu), \\ 8\alpha^2 x^3(e^m-1)^2(1-\eta)/|m^2 & |\eta-1|\le\delta \\ ([2\alpha(1+2\mu)(e^m-1) & |\eta-1|\ge\delta, \\ -2(1+\mu)^2(1+\alpha)]x^2+(1+\mu)^2)|, \end{cases}$$

where

$$\delta = \left|1 - \frac{(1+\mu)^2\left(2(1+\alpha)x^2-1\right)}{2\alpha x^2(e^m-1)(1+2\mu)}\right|. \quad (42)$$

Proof. From (37) and (39),

$$\begin{aligned} a_3 - \eta a_2^2 &= (1-\eta)\frac{(e^m-1)^2[C_1^\alpha(x)]^3(c_2+d_2)}{m^2\left[(e^m-1)(1+2\mu)[C_1^\alpha(x)]^2 - 2(1+\mu)^2C_2^\alpha(x)\right]} + \frac{(e^m-1)C_1^\alpha(x)}{m^2(1+2\mu)}(c_2-d_2) \\ &= C_1^\alpha(x)\left[h(\eta)+\frac{(e^m-1)}{m^2(1+2\mu)}\right]c_2 + C_1^\alpha(x)\left[h(\eta)-\frac{(e^m-1)}{m^2(1+2\mu)}\right]d_2, \end{aligned} \quad (43)$$

where

$$h(\eta) = \frac{(e^m-1)^2[C_1^\alpha(x)]^2(1-\eta)}{m^2\left[(e^m-1)(1+2\mu)[C_1^\alpha(x)]^2 - 2(1+\mu)^2C_2^\alpha(x)\right]}. \quad (44)$$

Then, in view of (16), we conclude that

$$|a_3 - \eta a_2^2| \le \begin{cases} \dfrac{2(e^m-1)|C_1^\alpha(x)|}{m^2(1+2\mu)} & 0 \le |h(\eta)| \le \dfrac{(e^m-1)}{m^2(1+2\mu)}, \\ 2|C_1^\alpha(x)||h(\eta)| & |h(\eta)| \ge \dfrac{(e^m-1)}{m^2(1+2\mu)}, \end{cases} \quad (45)$$

which completes the proof of Theorem 3.2. □

Corresponding essentially to Example 2.1, Theorems 3.1 and 3.2 yield the following consequence.

Corollary 3.3. *Let $f \in \Sigma$ given by (1) belong to the class $\zeta_\Sigma(x,\alpha,0)$. Then,*

$$\begin{aligned} |a_2| &\le \frac{2\alpha x(e^m-1)\sqrt{2\alpha x}}{m\sqrt{\left|\left[2\alpha^2(e^m-1)-2\alpha(1+\alpha)\right]x^2+\alpha\right|}}, \\ |a_3| &\le \frac{4\alpha^2x^2(e^m-1)^2}{m^2} + \frac{4\alpha x(e^m-1)}{m^2}, \end{aligned} \quad (46)$$

$$|a_3 - \eta a_2^2| \le \begin{cases} \dfrac{4\alpha x(e^m-1)}{m^2}, & |\eta-1| \le \left|1 - \dfrac{\left(2(1+\alpha)x^2-1\right)}{2\alpha x^2(e^m-1)}\right| \\ \dfrac{8\alpha^2x^3(e^m-1)^2(1-\eta)}{\left|m^2\left(\left[2\alpha(e^m-1)-2(1+\alpha)\right]x^2+1\right)\right|}, & |\eta-1| \ge \left|1 - \dfrac{\left(2(1+\alpha)x^2-1\right)}{2\alpha x^2(e^m-1)}\right|. \end{cases} \quad (47)$$

4. Conclusions

In our present investigation, we have introduced a new subclass $\zeta_\Sigma(x,\alpha,\mu)$ of normalized analytic and bi-univalent functions associated with the zero-truncated Poisson distribution and Gegenbauer polynomials. For functions belonging to this class, we have derived the estimates of the Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$ and the Fekete–Szegö functional problem. Furthermore, by suitably specializing the parameter μ, one can deduce the result for the subclass $\zeta_\Sigma(x,\alpha)$ which is defined in Example 2.1.

The results presented in this paper would lead to various other new results for the classes $\zeta_\Sigma(x,1,\mu)$ for Chebyshev polynomials and $\zeta_\Sigma(x,1/2,\mu)$ for Legendre polynomials.

The special families examined in this research paper and linked with zero-truncated Poisson distribution and Gegenbauer polynomials could inspire further research related to other aspects, such as families using q–derivative operator [31, 32] and bi-univalent function families associated with differential operators [33].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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