Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 6518976, 21 pages
https://doi.org/10.1155/2022/6518976

Research Article

Extended Transportation Models Based on Picture Fuzzy Sets

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Received 14 April 2022; Revised 10 May 2022; Accepted 21 May 2022; Published 27 June 2022

Academic Editor: Dragan Pamučar

Picture fuzzy set (PFS) is an extension of intuitionistic fuzzy set, and it is capable to analyze the transportation problems that contain uncertain and vague information. PFSs are applicable in situations where human decisions demand more variety of responses: yes, abstain, no, and rejection which cannot be addressed in the traditional FSs and IFSs. In this paper, we propose a technique to solve fully picture fuzzy transportation problems (FPFTPs) which are based on picture fuzzy linear programming formulation. Fully picture fuzzy transportation problems are developed by considering all the variables and parameters as nonnegative trapezoidal picture fuzzy numbers. A ranking function is used to transform picture fuzzy numbers into crisp numbers. A model is presented to explain the suggested scheme. Finally, comparison analysis of fully picture fuzzy transportation model with fully intuitionistic fuzzy transportation model and fully fuzzy transportation model is presented with pictorial illustrations.

1. Introduction

Zadeh [1] proposed the notion of fuzzy set (FS) theory to tackle the problems involving vague information. Atanassov [2] introduced intuitionistic fuzzy set (IFS), which is characterized by a membership function as well as a nonmembership function. Although IFSs have vast applications in many fields, it cannot provide all the information. In the voting process, we can vote in favor of someone, abstain, against someone, and even refuse to cost the vote, which cannot be handled by IFSs. To tackle such type of situations, Cuong [3] gave the idea of PFS, which is generalized structure of FS and IFS and further examined their basic properties and laws. In PFSs theory, we study about positive, neutral, and negative membership degrees of each element belonging to set. In practical life, PFSs theory contribute a significant role in medical diagnosis, career selection, decision making (DM), engineering, and networking.

Dubois and Prade [4] discussed basic arithmetic operations related to fuzzy numbers. Bellman and Zadeh [5] proposed the idea of decision making in fuzzy environment. Tanaka et al. [6] presented fuzzy linear programming (FLP) problems. Zimmerman [7] analyzed multiobjective functions in FLP problems. Ganesan and Veeramani [8] studied FLP problems by using trapezoidal fuzzy numbers. Lotfi et al. [9] suggested the lexicography method to solve fully FLP problems and obtained approximate solutions. Allahviranloo et al. [10] solved fully fuzzy linear system and achieved the general solutions. Kaur and Kumar [11] proposed Mehar's method to solve fully FLP problems by using *LR* fuzzy numbers. Pérez-Cañedo et al. [12] used lexicographical method and solved fully FLP problems having inequality constraints. Akram et al. [13] proposed Pythagorean FLP problems with equality constraints. Akram et al. [13] introduced *LR*-type Pythagorean fuzzy numbers and introduced a scheme to solve *LR*-type fully Pythagorean FLP problems with equality constraints. Mehmood et al. [14, 15] developed different techniques to find optimal solutions of fully bipolar FLP problems. Ahmed et al. [16] solved fully FLP problems in bipolar neutrosophic environment. Akram et al. [17] presented a scheme and obtained optimal solutions of fully FLP problems. For more information about FLP, the reader may study [18–20].

In real life, we have to minimize the transportation cost in transportation problems (TPs) because the companies have to deliver the products from different sources to numerous destinations [21]. Gani and Abbas [22] suggested a new technique to obtain optimal solution of TPs in intuitionistic fuzzy environment. TPs are being applied in many areas like networking [23], shipping [24], production [25], and shortest route problems [26]. Singh and Yadev [27] solved fuzzy transportation problems (FTPs) and obtained optimal solution by applying accuracy function. Kumar et al. [28] have proposed two algorithms to achieve initial basic solution and optimal cost of FTPs. Nagoorgani and Razak [29] developed a method to minimize the cost function by using trapezoidal fuzzy numbers (TrFNs). Kaur and Kumar [30] solved FTPs and determined optimal solution. Singh and Yadav [31] proposed a method to solve FTPs by using intuitionistic fuzzy numbers as supply and demand. Basirzadeh [32] worked on FTPs and presented three different kinds of problems. Kumar and Hussain [33] used a computational approach to tackle fully intuitionistic fuzzy real life TPs. Abhishekh and Nishad [34] solved LR-intuitionistic FTPs by using ranking approach. Kaur et al. [35] proposed different schemes to solve fully FTPs by using TrFNs. Wang et al. [36] introduced geometric operators in picture fuzzy environment and studied decision-making problems (DMPs).

PFSs are applied to handle real life TPs that involve uncertainty and have been effectively used in communication, management, and DM. Akram et al. [37] proposed the idea of complex PFS, which is generalized form of complex IFS and developed a DM model. Shit et al. [38] used harmonic operators with TrPFNs and showed its significance in multicriteria decision-making (MCDM) problems. Akram et al. [39] investigated shortest route problems by using trapezoidal picture fuzzy numbers (TrPFNs). Geetha and Selvakumari [40] solved picture fuzzy transportation problems and obtained minimum transportation cost. Mahmoodirad et al. [41] studied the existing shortcomings and proposed a method to handle TPs involving intuitionistic fuzzy numbers. Kane et al. [42] suggested a FLP scheme to solve TPs by using triangular fuzzy numbers. Veeramani et al. [43] proposed a technique regarding multiobjective fractional TP on the basis of NGP approach. Ali and Ansari [44] presented Fermatean fuzzy bipolar soft set (FFBSS) model and studied its basic properties. Tchier et al. [45] combined PFSs and soft expert sets and introduced a hybrid model which is used to analyze DMPs. Das [46] defined score function to get IBFS and solved neutrosophic TPs. Kané et al. [47] proposed two-step scheme to solve fully FTP by using all parameters as trapezoidal fuzzy numbers. Ali et al. [48] studied group DMPs and developed a hybrid model by considering bipolar soft expert sets. Ashraf et al. [49] explored MSM operator in the form of IVPFS and interpreted its applications. On the basis of CIVPFSs, Ali et al. [50] introduced Einstein operational laws by applying t-norm. Sahu et al. [51] analyzed student's career selection in PFS environment. Yildirim and Yıldırım [52] used picture fuzzy VIKOR technique and evaluated satisfaction level of people regarding municipality services. The motivation of this manuscript is described as follows:

(1) PFSs manage the problems involving uncertainty more expeditiously as compared with FSs and IFSs.
(2) No one has yet introduced this particular concept of FPFTPs which is based on picture fuzzy linear programming (PFLP) formulation.

The main contributions of this article are depicted as follows:

(1) We propose a scheme to solve FPFTPs based upon PFLP formulation
(2) We apply suggested technique to solve FPFTPs by considering all the variables as nonnegative TrPFNs
(3) We obtain picture fuzzy transportation cost/optimal value in the form of TrPFNs
(4) The supremacy of the proposed scheme is investigated by comparative analysis with existing approaches

The rest of the article is arranged as follows. Introductory concepts are depicted in Section 2. The proposed scheme is elucidated in Section 3. A model regarding FPFTPs is considered in Section 4. Comparison analysis is discussed in Section 5, and conclusion is given in the last section.

2. Preliminaries

In this section, we review some preliminary concepts regarding PFSs.

Definition 1 (see [3]). A PFS P on a universal set X is an object of the form as follows:

$$P = \{ (x, \mu_P(x), \eta_P(x), \nu_P(x)) | x \in X \}, \quad (1)$$

where $\mu_P(x) \in [0,1], \eta_P(x) \in [0,1], \nu_P(x) \in [0,1]$ denote positive, neutral, and negative membership degrees, respectively, of element $x \in P$ with $0 \le \mu_P(x) + \eta_P(x) + \nu_P(x) \le 1, \forall x \in X$, and $\Pi_P(x) = 1 - \mu_P(x) - \eta_P(x) - \nu_P(x)$ is said to be refusal degree of x in set P.

Definition 2 (see [53]). A TrPFN $P = [(u_3, u_2, u_1, s, t, v_1, v_2, v_3); (\omega, \vartheta, \zeta)]$ is a PFS defined on $\mathbb{R}$, whose positive (μ_P), neutral (η_P), and negative (ν_P) membership functions are, respectively, defined as follows:

$$\mu_P(x)=\begin{cases}\dfrac{(x-u_1)\omega}{s-u_1}, & u_1\le x\le s,\\ \omega, & s\le x\le t,\\ \dfrac{(v_1-x)\omega}{v_1-t}, & t\le x\le v_1,\\ 0, & otherwise,\end{cases}$$

$$\eta_P(x)=\begin{cases}\dfrac{(x-u_2)\vartheta}{s-u_2}, & u_2\le x\le s,\\ \vartheta, & s\le x\le t,\\ \dfrac{(v_2-x)\vartheta}{v_2-t}, & t\le x\le v_2,\\ 0, & \text{otherwise},\end{cases} \tag{2}$$

$$\nu_P(x)=\begin{cases}\dfrac{s-x+\zeta(x-u_3)}{s-u_3}, & u_3\le x\le s,\\ \zeta, & s\le x\le t,\\ \dfrac{x-t+\zeta(v_3-x)}{v_3-t}, & t\le x\le v_3,\\ 1, & \text{otherwise},\end{cases}$$

where $u_3\le u_2\le u_1\le s\le t\le v_1\le v_2\le v_3$ and the values ω, ϑ, and ζ indicate maximum degree of (μ_P), maximum degree of (η_P), and minimum degree of (ν_P), respectively, such that $\omega,\vartheta,\zeta\in[0,1]$, with $0\le\omega+\vartheta+\zeta\le 1$.

Definition 3 (see [53]). A TrPFN $P=[(u_3,u_2,u_1,s,t,v_1,v_2,v_3);(\omega,\vartheta,\zeta)]$ is said to be nonnegative (respectively, nonpositive) if $u_3\ge 0$ (respectively, $v_3\le 0$) and P is unrestricted TrPFN if u_3 is any real number.

Definition 4 (see [53]). Let $P_1=[(u_3,u_2,u_1,s,t,v_1,v_2,v_3);(\omega,\vartheta,\zeta)]$ and $P_2=[(u_3',u_2',u_1',s',t',v_1',v_2',v_3');(\omega',\vartheta',\zeta')]$ be two TrPFNs and λ be real number, then

(1) $P_1\oplus P_2=[(u_3+u_3',u_2+u_2',u_1+u_1',s+s',t+t',v_1+v_1',v_2+v_2',v_3+v_3');(\omega+\omega'-\omega\omega',\vartheta\vartheta',\zeta\zeta')]$

(2) $-P_1=[(-v_3,-v_2,-v_1,-t,-s,-u_1,-u_2,-u_3);(\omega,\vartheta,\zeta)]$

(3) $P_1\ominus P_2=[(u_3-v_3',u_2-v_2',u_1-v_1',s-t',t-s',v_1-u_1',v_2-u_2',v_3-u_3');(\omega+\omega'-\omega\omega',\vartheta\vartheta',\zeta\zeta')]$

(4) $\lambda P_1=\begin{cases}[(\lambda u_3,\lambda u_2,\lambda u_1,\lambda s,\lambda t,\lambda v_1,\lambda v_2,\lambda v_3);(\omega^{\lambda},\vartheta^{\lambda},1-(1-\zeta)^{\lambda})], & \lambda\ge 0\\ [(\lambda v_3,\lambda v_2,\lambda v_1,\lambda t,\lambda s,\lambda u_1,\lambda u_2,\lambda u_3);(\omega^{-\lambda},\vartheta^{-\lambda},1-(1-\zeta)^{-\lambda})], & \lambda<0\end{cases}$

(5) $P_1\otimes P_2=[(U_3,U_2,U_1,S,T,V_1,V_2,V_3);(\omega\omega',\vartheta\vartheta',\zeta+\zeta'-\zeta\zeta')]$

where

$$\begin{aligned}
U_1&=\begin{cases}\min\{u_1u_1',v_1u_1'\}, & u_1\ge 0,\\ \min\{u_1v_1',v_1u_1'\}, & u_1<0,v_1\ge 0,\\ \min\{u_1v_1',v_1v_1'\}, & v_1<0,\end{cases}\\
U_2&=\begin{cases}\min\{u_2u_2',v_2u_2'\}, & u_2\ge 0,\\ \min\{u_2v_2',v_2u_2'\}, & u_2<0,v_2\ge 0,\\ \min\{u_2v_2',v_2v_2'\}, & v_2<0,\end{cases}\\
U_3&=\begin{cases}\min\{u_3u_3',v_3u_3'\}, & u_3\ge 0,\\ \min\{u_3v_3',v_3u_3'\}, & u_3<0,v_3\ge 0,\\ \min\{u_3v_3',v_3v_3'\}, & v_3<0,\end{cases}\\
S&=\begin{cases}\min\{ss',ts'\}, & s\ge 0,\\ \min\{st',ts'\}, & s<0,t\ge 0,\\ \min\{st',tt'\}, & t<0,\end{cases}\\
T&=\begin{cases}\max\{st',tt'\}, & s\ge 0,\\ \max\{ss',tt'\}, & s<0,t_1\ge 0,\\ \max\{ss',ts'\}, & t_1<0,\end{cases}\\
V_1&=\begin{cases}\max\{u_1v_1',v_1v_1'\}, & u_1\ge 0,\\ \max\{u_1u_1',v_1v_1'\}, & u_1<0,v_1\ge 0,\\ \max\{u_1u_1',v_1u_1'\}, & v_1<0,\end{cases}\\
V_2&=\begin{cases}\max\{u_2v_2',v_2v_2'\}, & u_2\ge 0,\\ \max\{u_2u_2',v_2v_2'\}, & u_2<0,v_2\ge 0,\\ \max\{u_2u_2',v_2u_2'\}, & v_2<0,\end{cases}\\
V_3&=\begin{cases}\max\{u_3v_3',v_3v_3'\}, & u_3\ge 0,\\ \max\{u_3u_3',v_3v_3'\}, & u_3<0,v_3\ge 0,\\ \max\{u_3u_3',v_3u_3'\}, & v_3<0.\end{cases}
\end{aligned} \tag{3}$$

Definition 5 (see [53]). Two TrPFNs $P_1=[(u_3,u_2,u_1,s,t,v_1,v_2,v_3);(\omega,\vartheta,\zeta)]$ and $P_2=[(u_3',u_2',u_1',s',t',v_1',v_2',v_3');(\omega',\vartheta',\zeta')]$ are said to be equal if $u_3=u_3'$, $u_2=u_2'$, $u_1=u_1'$, $s=s'$, $t=t'$, $v_1=v_1'$, $v_2=v_2'$, $v_3=v_3'$, $\omega=\omega'$, $\vartheta=\vartheta'$, and $\zeta=\zeta'$.

Definition 6 (see [53]). A TrPFN $P_1=[(u_3,u_2,u_1,s,t,v_1,v_2,v_3);(\omega,\vartheta,\zeta)]$ is said to be zero if $u_3=0$, $u_2=0$, $u_1=0$, $s=0$, $t=0$, $v_1=0$, $v_2=0$, $v_3=0$, $\omega=0$, $\vartheta=0$, and $\zeta=0$.

Definition 7 (see [53]). Let $p=[(u_3,u_2,u_1,s,t,v_1,v_2,v_3);(\omega,\vartheta,\zeta)]$ be a TrPFN, then ranking of P is symbolized as $\mathfrak{R}(P)$ and defined as

$$\mathfrak{R}(P)=\frac{\omega(s+t+u_1+v_1)+\vartheta(s+t+u_2+v_2)+(1-\zeta)(s+t+u_3+v_3)}{4}. \quad (4)$$

3. Fully Picture Fuzzy Transportation Problems

In this section, we present a scheme to solve FPFTPs based on PFLP formulation. The steps are explained as follows.

3.1. Steps to Find Picture Fuzzy Optimal Solution. Consider a FPFTP containing q sources and r destinations in which cost, supply, and demand are used as TrPFNs $\widetilde{C_{kl}^P}$, $\widetilde{A}_k^P$, and $\widetilde{B}_l^P$, respectively.

$$\text{Minimize } Z=\sum_{k=1}^{q}\sum_{l=1}^{r}\widetilde{C_{kl}^P}\otimes\widetilde{X_{kl}^P}, \quad (5)$$

subject to

$$\begin{aligned}&\sum_{l=1}^{r}\widetilde{X_{kl}^P}=\widetilde{A}_k^P, \forall k=1,2,3,\dots,q,\\&\sum_{l=1}^{r}\widetilde{X_{kl}^P}=\widetilde{A}_k^P, \forall k=1,2,3,\dots,q,\\&\widetilde{X}_{kl}^P\geq 0, \forall k=1,2,3,\dots,q,\quad \forall l=1,2,3,\dots,r,\end{aligned} \quad (6)$$

where $\widetilde{C_{kl}^P}$, $\widetilde{X_{kl}^P}$, $\widetilde{A}_k^P$, and $\widetilde{B}_l^P$ are all nonnegative TrPFNs.

To solve FPFTP equation (1), we give a criterion for picture fuzzy optimal solution (PFOS).

Definition 8. A PFOS of the FPFTP equation (1) with TrPFNs will be TrPFNs $\widetilde{X}_{kl}^P$ if

(i) $\widetilde{X}_{kl}^P$ are nonnegative TrPFNs.

(ii) $\mathfrak{R}(\sum_{l=1}^{r}\widetilde{X_{kl}^P})=\mathfrak{R}(\widetilde{A}_k^P)$, $\forall k=1,2,3,\dots,q$.

(iii) $\mathfrak{R}(\sum_{k=1}^{q}\widetilde{X_{kl}^P})=\mathfrak{R}(\widetilde{B}_l^P)$, $\forall l=1,2,3,\dots,r$.

(iv) If there exists any TrPFNs $\widetilde{X}_{kl'}^P$ satisfying the above three conditions, then

$$\mathfrak{R}\left(\sum_{k=1}^{q}\sum_{l=1}^{r}\widetilde{C_{kl}^P}\otimes\widetilde{X_{kl}^P}\right)<\mathfrak{R}\left(\sum_{k=1}^{q}\sum_{l=1}^{r}\widetilde{C_{kl}^P}\otimes\widetilde{X_{kl}^P}\right). \quad (7)$$

Now, we explain the steps to determine the PFOS of FPFTP as given in equation (2).

Step 1. Calculate total picture fuzzy supply and total picture fuzzy demand.

If

$$\sum_{l=1}^{r}\widetilde{B}_l^P=\sum_{k=1}^{q}\widetilde{A}_k^P, \quad (8)$$

it is a balanced FPFTP.

If

$$\sum_{l=1}^{r}\widetilde{B}_l^P\neq\sum_{k=1}^{q}\widetilde{A}_k^P, \quad (9)$$

it is an unbalanced FPFTP.

That is,

$$\begin{aligned}&[(a,b,c,d,e,f,g,h);(\alpha,\beta,\gamma)]\\&\quad\neq[(a',b',c',d',e',f',g',h');(\alpha',\beta',\gamma')].\end{aligned} \quad (10)$$

Then, one of the following case arises:

Case (a):

$$\begin{aligned}&a\leq a',\quad b-a\leq b'-a',\quad c-b\leq c'-b',\quad d-c\leq d'-c',\quad e-d\leq e'-d',\quad f-e\leq f'-e',\\&g-f\leq g'-f',\quad h-g\leq h'-g'.\end{aligned} \quad (11)$$

Case (b):

$$\begin{aligned}&a\geq a',\quad b-a\geq b'-a',\quad c-b\geq c'-b',\quad d-c\geq d'-c',\quad e-d\geq e'-d',\quad f-e\geq f'-e',\\&g-f\geq g'-f',\quad h-g\geq h'-g'.\end{aligned} \quad (12)$$

Case (c): when the above two cases do not hold, then there may exists infinitely many nonnegative TrPFNs:

$$\begin{cases}[(a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1);(\alpha_1,\beta_1,\gamma_1)],\\[(a_1',b_1',c_1',d_1',e_1',f_1',g_1',h_1');(\alpha_1',\beta_1',\gamma_1')],\end{cases} \quad (13)$$

such that

$$[(a,b,c,d,e,f,g,h);\ (\alpha,\beta,\gamma)]\oplus[(a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1);\ (\alpha_1,\beta_1,\gamma_1)] \\ = [(a',b',c',d',e',f',g',h');\ (\alpha',\beta',\gamma')]\oplus[(a_1',b_1',c_1',d_1',e_1',f_1',g_1',h_1');\ (\alpha_1',\beta_1',\gamma_1')], \tag{14}$$

but we have to determine such nonnegative TrPFNs as

$$\begin{cases} [(a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1);\ (\alpha_1,\beta_1,\gamma_1)], \\ [(a_1',b_1',c_1',d_1',e_1',f_1',g_1',h_1');\ (\alpha_1',\beta_1',\gamma_1')], \end{cases} \tag{15}$$

which satisfy the following conditions.

(i) $[(a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1);\ (\alpha_1,\beta_1,\gamma_1)]$ and $[(a_1',b_1',c_1',d_1',e_1',f_1',g_1',h_1');\ (\alpha_1',\beta_1',\gamma_1')]$ are nonnegative TrPFNs.

(ii) It satisfies

$$[(a,b,c,d,e,f,g,h);\ (\alpha,\beta,\gamma)]\oplus[(a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1);\ (\alpha_1,\beta_1,\gamma_1)] \\ = [(a',b',c',d',e',f',g',h');\ (\alpha',\beta',\gamma')]\oplus[(a_1',b_1',c_1',d_1',e_1',f_1',g_1',h_1');\ (\alpha_1',\beta_1',\gamma_1')]. \tag{16}$$

(iii) Further, if there exists two nonnegative TrPFNs,

$$[(s_1,t_1,u_1,v_1,w_1,x_1,y_1,z_1);\ (\alpha_1,\beta_1,\gamma_1)], \\ [(s_1',t_1',u_1',v_1',w_1',x_1',y_1',z_1');\ (\alpha_1',\beta_1',\gamma_1')], \tag{17}$$

such that

$$[(a,b,c,d,e,f,g,h);\ (\alpha,\beta,\gamma)]\oplus[(s_1,t_1,u_1,v_1,w_1,x_1,y_1,z_1);\ (\alpha_1,\beta_1,\gamma_1)] \\ = [(a',b',c',d',e',f',g',h');\ (\alpha',\beta',\gamma')]\oplus[(s_1',t_1',u_1',v_1',w_1',x_1',y_1',z_1');\ (\alpha_1',\beta_1',\gamma_1')], \tag{18}$$

then

$$\mathfrak{R}[(s_1,t_1,u_1,v_1,w_1,x_1,y_1,z_1);\ (\alpha_1,\beta_1,\gamma_1)] \geq \mathfrak{R}[(a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1);\ (\alpha_1,\beta_1,\gamma_1)], \\ \mathfrak{R}[(s_1',t_1',u_1',v_1',w_1',x_1',y_1',z_1');\ (\alpha_1',\beta_1',\gamma_1')] \geq \mathfrak{R}[(a_1',b_1',c_1',d_1',e_1',f_1',g_1',h_1');\ (\alpha_1',\beta_1',\gamma_1')]. \tag{19}$$

Step 2. Suppose

$$\begin{aligned} \widetilde{C_{kl}^P} &= \left[\left(c_{kl}^1,c_{kl}^2,c_{kl}^3,c_{kl}^4,c_{kl}^5,c_{kl}^6,c_{kl}^7,c_{kl}^8\right);\ (\xi_{kl},\psi_{kl},\omega_{kl})\right], \\ \widetilde{X_{kl}^P} &= \left[\left(x_{kl}^1,x_{kl}^2,x_{kl}^3,x_{kl}^4,x_{kl}^5,x_{kl}^6,x_{kl}^7,x_{kl}^8\right);\ (\sigma_{kl},\tau_{kl},\upsilon_{kl})\right], \\ \widetilde{A}_k^P &= \left[\left(a_k^1,a_k^2,a_k^3,a_k^4,a_k^5,a_k^6,a_k^7,a_k^8\right);\ (\kappa_k,\lambda_k,\theta_k)\right], \\ \widetilde{B}_l^P &= \left[\left(b_l^1,b_l^2,b_l^3,b_l^4,b_l^5,b_l^6,b_l^7,b_l^8\right);\ (\eta_l,\epsilon_l,\chi_l)\right], \end{aligned} \tag{20}$$

then FPFTP equation (1) can be transformed as follows:

$$\text{Minimize Z} = \sum_{k=1}^{q}\sum_{l=1}^{r}\begin{bmatrix} \left(c_{kl}^1,c_{kl}^2,c_{kl}^3,c_{kl}^4,c_{kl}^5,c_{kl}^6,c_{kl}^7,c_{kl}^8\right); \\ (\xi_{kl},\psi_{kl},\omega_{kl}) \end{bmatrix} \otimes \begin{bmatrix} \left(x_{kl}^1,x_{kl}^2,x_{kl}^3,x_{kl}^4,x_{kl}^5,x_{kl}^6,x_{kl}^7,x_{kl}^8\right); \\ (\sigma_{kl},\tau_{kl},\upsilon_{kl}) \end{bmatrix}, \tag{21}$$

subject to

$$\sum_{l=1}^{r}\left[\left(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}\right); \left(\sigma_{kl}, \tau_{kl}, \upsilon_{kl}\right)\right] = \left[\left(a_{k}^{1}, a_{k}^{2}, a_{k}^{3}, a_{k}^{4}, a_{k}^{5}, a_{k}^{6}, a_{k}^{7}, a_{k}^{8}\right); \left(\kappa_{k}, \lambda_{k}, \theta_{k}\right)\right],$$
$$\sum_{k=1}^{q}\left[\left(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}\right); \left(\sigma_{kl}, \tau_{kl}, \upsilon_{kl}\right)\right] = \left[\left(b_{l}^{1}, b_{l}^{2}, b_{l}^{3}, b_{l}^{4}, b_{l}^{5}, b_{l}^{6}, b_{l}^{7}, b_{l}^{8}\right); \left(\eta_{l}, \epsilon_{l}, \chi_{l}\right)\right], \quad (22)$$

where $[(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}); (\sigma_{kl}, \tau_{kl}, \upsilon_{kl})]$ are nonnegative TrPFNs.

Step 3. By applying arithmetic operations as described in Section 2 and putting

$$\left[\left(c_{kl}^{1}, c_{kl}^{2}, c_{kl}^{3}, c_{kl}^{4}, c_{kl}^{5}, c_{kl}^{6}, c_{kl}^{7}, c_{kl}^{8}\right)\right); t\left(\xi_{kl}, \psi_{kl}, \omega_{kl}\right)\right] \otimes \left[\left(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}\right); t\left(\sigma_{kl}, \tau_{kl}, \upsilon_{kl}\right)\right]$$
$$= \left[\left(d_{kl}^{1}, d_{kl}^{2}, d_{kl}^{3}, d_{kl}^{4}, d_{kl}^{5}, d_{kl}^{6}, d_{kl}^{7}, d_{kl}^{8}\right); \left(\mu_{kl}, \delta_{kl}, \eta_{kl}\right)\right], \quad (23)$$

then the fully picture fuzzy linear programming problem (FPFLPP) equation (2) can be transformed as follows:

$$\text{Minimize } Z = \sum_{k=1}^{q}\sum_{l=1}^{r}\left[\left(d_{kl}^{1}, d_{kl}^{2}, d_{kl}^{3}, d_{kl}^{4}, d_{kl}^{5}, d_{kl}^{6}, d_{kl}^{7}, d_{kl}^{8}\right); \left(\mu_{kl}, \delta_{kl}, \eta_{kl}\right)\right], \quad (24)$$

subject to

$$\sum_{l=1}^{r}\left[\left(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}\right); \left(\sigma_{kl}, \tau_{kl}, \upsilon_{kl}\right)\right] = \left[\left(a_{k}^{1}, a_{k}^{2}, a_{k}^{3}, a_{k}^{4}, a_{k}^{5}, a_{k}^{6}, a_{k}^{7}, a_{k}^{8}\right); \left(\kappa_{k}, \lambda_{k}, \theta_{k}\right)\right],$$
$$\sum_{k=1}^{q}\left[\left(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}\right)\right); t\left(\sigma_{kl}, \tau_{kl}, \upsilon_{kl}\right)\right] = \left[\left(b_{l}^{1}, b_{l}^{2}, b_{l}^{3}, b_{l}^{4}, b_{l}^{5}, b_{l}^{6}, b_{l}^{7}, b_{l}^{8}\right); t\left(\eta_{l}, \epsilon_{l}, \chi_{l}\right)\right], \quad (25)$$

where $[(x_{kl}^{1}, x_{kl}^{2}, x_{kl}^{3}, x_{kl}^{4}, x_{kl}^{5}, x_{kl}^{6}, x_{kl}^{7}, x_{kl}^{8}); (\sigma_{kl}, \tau_{kl}, \upsilon_{kl})]$ are nonnegative TrPFNs.

Step 4. Now, by applying Definitions 5 and 7, then the FPFLPP equation (3) can be transformed as follows:

$$\text{Minimize } Z = \sum_{k=1}^{q}\sum_{l=1}^{r}\Re\left[\left(d_{kl}^{1}, d_{kl}^{2}, d_{kl}^{3}, d_{kl}^{4}, d_{kl}^{5}, d_{kl}^{6}, d_{kl}^{7}, d_{kl}^{8}\right); \left(\mu_{kl}, \delta_{kl}, \eta_{kl}\right)\right], \quad (26)$$

subject to

$$\sum_{l=1}^{r} x_{kl}^{1} = a_{k}^{1}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{2} = a_{k}^{2}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{3} = a_{k}^{3}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{4} = a_{k}^{4}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{5} = a_{k}^{5}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{6} = a_{k}^{6}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{7} = a_{k}^{7}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} x_{kl}^{8} = a_{k}^{8}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} \sigma_{kl} = \kappa_{k}, \quad \forall k = 1, 2, 3, \ldots, q,$$
$$\sum_{l=1}^{r} \tau_{kl} = \lambda_{k}, \quad \forall k = 1, 2, 3, \ldots, q,$$

$$\begin{aligned}
&\sum_{l=1}^{r} \upsilon_{kl} = \theta_k, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{k=1}^{q} x_{kl}^1 = b_l^1, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^2 = b_l^2, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^3 = b_l^3, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^4 = b_l^4, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^5 = b_l^5, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^6 = b_l^6, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^7 = b_l^7, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^8 = b_l^8, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} \sigma_{kl} = \eta_l, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} \tau_{kl} = \epsilon_l, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} \upsilon_{kl} = \chi_l, \quad \forall l = 1, 2, 3, \ldots, r,
\end{aligned} \tag{27}$$

where $[((x_{kl}^1, x_{kl}^2, x_{kl}^3, x_{kl}^4, x_{kl}^5, x_{kl}^6, x_{kl}^7, x_{kl}^8)); (\sigma_{kl}, \tau_{kl}, \upsilon_{kl})]$ are nonnegative TrPFNs.

Step 5. To obtain PFOS, solve the following crisp linear/nonlinear programming problem (LPP):

$$\text{Minimize } Z = \sum_{k=1}^{q} \sum_{l=1}^{r} \frac{1}{4} \begin{bmatrix} \mu_{kl}\left(d_{kl}^3 + d_{kl}^4 + d_{kl}^5 + d_{kl}^6\right) + \\ \delta_{kl}\left(d_{kl}^2 + d_{kl}^4 + d_{kl}^5 + d_{kl}^7\right) + \\ (1 - \eta_{kl})\left(d_{kl}^1 + d_{kl}^4 + d_{kl}^5 + d_{kl}^8\right) \end{bmatrix}, \tag{28}$$

subject to

$$\begin{aligned}
&\sum_{l=1}^{r} x_{kl}^1 = a_k^1, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^2 = a_k^2, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^3 = a_k^3, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^4 = a_k^4, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^5 = a_k^5, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^6 = a_k^6, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^7 = a_k^7, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} x_{kl}^8 = a_k^8, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} \sigma_{kl} = \kappa_k, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} \tau_{kl} = \lambda_k, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{l=1}^{r} \upsilon_{kl} = \theta_k, \quad \forall k = 1, 2, 3, \ldots, q, \\
&\sum_{k=1}^{q} x_{kl}^1 = b_l^1, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^2 = b_l^2, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^3 = b_l^3, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^4 = b_l^4, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^5 = b_l^5, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^6 = b_l^6, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^7 = b_l^7, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} x_{kl}^8 = b_l^8, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} \sigma_{kl} = \eta_l, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} \tau_{kl} = \epsilon_l, \quad \forall l = 1, 2, 3, \ldots, r, \\
&\sum_{k=1}^{q} \upsilon_{kl} = \chi_l, \quad \forall l = 1, 2, 3, \ldots, r,
\end{aligned}$$

$$x_{kl}^1 \geq 0, x_{kl}^2 - x_{kl}^1 \geq 0, x_{kl}^3 - x_{kl}^2 \geq 0, x_{kl}^4 - x_{kl}^3 \geq 0, x_{kl}^5 - x_{kl}^4 \geq 0, x_{kl}^6 - x_{kl}^5 \geq 0, x_{kl}^7 - x_{kl}^6 \geq 0, x_{kl}^8 - x_{kl}^7 \geq 0, \sigma_{kl} \geq 0, \tau_{kl} \geq 0, \upsilon_{kl} \geq 0, \sigma_{kl} + \tau_{kl} + \upsilon_{kl} \leq 1, \quad \forall k = 1, 2, \ldots, q, \forall l = 1, 2, \ldots, r. \tag{29}$$

Step 6. Solve crisp linear/non-LPP equation (28) to get optimal solution:

$$\left\{x^1 *_{kl}, x^2 *_{kl}, x^3 *_{kl}, x^4 *_{kl}, x^5 *_{kl}, x^6 *_{kl}, x^7 *_{kl}, x^8 *_{kl}, \sigma_{kl}^*, \tau_{kl}^*, \upsilon_{kl}^*\right\}. \tag{30}$$

Step 7. Find the PFOS $\widetilde{X}_{kl}^P$ of the FPFTP (1) by putting values of $x^1 *_{kl}, x^2 *_{kl}, x^3 *_{kl}, x^4 *_{kl}, x^5 *_{kl}, x^6 *_{kl}, x^7 *_{kl}, x^8 *_{kl}, \sigma_{kl}^*, \tau_{kl}^*$, and υ_{kl}^* in $\widetilde{X}_{kl}^P = [(x^1 *_{kl}, x^2 *_{kl}, x^3 *_{kl}, x^4 *_{kl}, x^5 *_{kl}, x^6 *_{kl}, x^7 *_{kl}, x^8 *_{kl}); (\sigma_{kl}^*, \tau_{kl}^*, \upsilon_{kl}^*)]$.

Step 8. Find picture fuzzy transportation cost/optimal value of FPFTP equation (1) by assigning values of $\widetilde{X}_{kl}^P$, as obtained in Step (7), in $\sum_{k=1}^q \sum_{l=1}^r \widetilde{C_{kl}^P} \otimes \widetilde{X_{kl}^P}$.

4. Numerical Examples

In this section, to explain the proposed methodology, we present a model related to FPFTPs.

Example 1. (FPFTP based on PFLP formulation). A company containing two plants produces urea fertilizer with picture fuzzy availabilities of

Proof. [(15, 35, 45, 60, 75, 95, 110, 200); (0.92, 0.02, 0.01)] *ton* and [(10, 25, 50, 70, 90, 120, 130, 190); (0.80, 0.01, 0.02)] *ton*, respectively, and supply it to two cities. The picture fuzzy demand at two cities is [(20, 40, 50, 65, 85, 100, 140, 160); (0.92, 0.01, 0.02)]*ton* and [(30, 35, 40, 50, 100, 110, 150, 170); (0.76, 0.02, 0.01)]*ton*. The price per ton by delivering the urea fertilizer at the two cities is presented in Table 1.

Find the minimum picture fuzzy transportation cost.

$$\text{Minimize}\begin{pmatrix} [(110,150,190,210,270,300,350,390);(0.7,0.1,0.1)]\otimes\tilde{x}_{11}\oplus \\ [(130,180,220,250,290,340,370,410);(0.8,0.1,0.1)]\otimes\tilde{x}_{12}\oplus \\ [(150,250,290,350,400,440,460,500);(0.6,0.1,0.2)]\otimes\tilde{x}_{21}\oplus \\ [(190,210,250,270,310,330,380,430);(0.6,0.1,0.2)]\otimes\tilde{x}_{22} \end{pmatrix}, \tag{31}$$

subject to

$$\begin{aligned} \tilde{x}_{11}\oplus\tilde{x}_{12} &= [(15,35,45,60,75,95,110,200);(0.92,0.02,0.01)], \\ \tilde{x}_{21}\oplus\tilde{x}_{22} &= [(10,25,50,70,90,120,130,190);(0.80,0.01,0.02)], \\ \tilde{x}_{11}\oplus\tilde{x}_{21} &= [(20,40,50,65,85,100,140,160);(0.92,0.01,0.02)], \\ \tilde{x}_{12}\oplus\tilde{x}_{22} &= [(30,35,40,50,100,110,150,170);(0.76,0.02,0.01)], \end{aligned} \tag{32}$$

where $\tilde{x}_{11}$, $\tilde{x}_{12}$, $\tilde{x}_{21}$, and $\tilde{x}_{22}$ are nonnegative TrPFNs.

Now,

Total supply = [(25, 60, 95, 130, 165, 215, 240, 390); (0.9840, 0.0002, 0.0002)]

Total demand = [(50, 75, 90, 115, 185, 210, 290, 330); (0.9808, 0.0002, 0.0002)]

For an unbalanced FPFTP, we add dummy row and dummy column to make a balanced picture fuzzy TP.

Dummy row = [(25, 25, 25, 25, 60, 60, 115, 115); (0.0000, 0.0000, 0.0000)]

Dummy column = [(0, 10, 30, 40, 40, 65, 65, 175); (0.1666, 0.0000, 0.0000)]

Therefore, by supposing picture fuzzy transportation cost of unit quantity of product from dummy source to all destinations and from all sources to dummy destination to be zero TrPFNs, then FPFLPP equation (31) can be transformed as follows:

Table 1: Input data for FPFTP.

	City (T_1)	City (T_2)
Plant (S_1)	[(110, 150, 190, 210, 270, 300, 350, 390); (0.7, 0.1, 0.1)]	[(130, 180, 220, 250, 290, 340, 370, 410); (0.8, 0.1, 0.1)]
Plant (S_2)	[(150, 250, 290, 350, 400, 440, 460, 500); (0.6, 0.1, 0.2)]	[(190, 210, 250, 270, 310, 330, 380, 430); (0.6, 0.1, 0.2)]

$$\text{Minimize}\begin{pmatrix} [(110,150,190,210,270,300,350,390);\ (0.7,0.1,0.1)]\otimes\widetilde{x}_{11}\oplus \\ [(130,180,220,250,290,340,370,410);\ (0.8,0.1,0.1)]\otimes\widetilde{x}_{12}\oplus \\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes\widetilde{x}_{13}\oplus \\ [(150,250,290,350,400,440,460,500);\ (0.6,0.1,0.2)]\otimes\widetilde{x}_{21}\oplus \\ [(190,210,250,270,310,330,380,430);\ (0.6,0.1,0.2)]\otimes\widetilde{x}_{22}\oplus \\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes\widetilde{x}_{23}\oplus \\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes\widetilde{x}_{31}\oplus \\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes\widetilde{x}_{32}\oplus \\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes\widetilde{x}_{33} \end{pmatrix} \tag{33}$$

$$\widetilde{x}_{11}\oplus\widetilde{x}_{12} = [(15,35,45,60,75,95,110,200);\ (0.92,0.02,0.01)],$$

subject to

$$\begin{aligned} \widetilde{x}_{11}\oplus\widetilde{x}_{12}\oplus\widetilde{x}_{13} &= [(15,35,45,60,75,95,110,200);\ (0.92,0.02,0.01)], \\ \widetilde{x}_{21}\oplus\widetilde{x}_{22}\oplus\widetilde{x}_{23} &= [(10,25,50,70,90,120,130,190);\ (0.80,0.01,0.02)], \\ \widetilde{x}_{31}\oplus\widetilde{x}_{32}\oplus\widetilde{x}_{33} &= [(25,25,25,25,60,60,115,115);\ (0.0000,0.0000,0.0000)], \\ \widetilde{x}_{11}\oplus\widetilde{x}_{21}\oplus\widetilde{x}_{31} &= [(20,40,50,65,85,100,140,160);\ (0.92,0.01,0.02)], \\ \widetilde{x}_{12}\oplus\widetilde{x}_{22}\oplus\widetilde{x}_{32} &= [(30,35,40,50,100,110,150,170);\ (0.76,0.02,0.01)], \\ \widetilde{x}_{13}\oplus\widetilde{x}_{23}\oplus\widetilde{x}_{33} &= [(0,10,30,40,40,65,65,175);\ (0.1666,0.0000,0.0000)], \end{aligned} \tag{34}$$

where $\widetilde{x}_{11}$, $\widetilde{x}_{12}$, $\widetilde{x}_{13}$, $\widetilde{x}_{21}$, $\widetilde{x}_{22}$, $\widetilde{x}_{23}$, $\widetilde{x}_{31}$, $\widetilde{x}_{32}$, and $\widetilde{x}_{33}$, are nonnegative TrPFNs.

By supposing,

$$\begin{aligned} \widetilde{x}_{11} &= [(\chi_{11},\delta_{11},\epsilon_{11},\eta_{11},\kappa_{11},\vartheta_{11},\omega_{11},\zeta_{11});\ (\alpha_{11},\beta_{11},\gamma_{11})], \\ \widetilde{x}_{12} &= [(\chi_{12},\delta_{12},\epsilon_{12},\eta_{12},\kappa_{12},\vartheta_{12},\omega_{12},\zeta_{12});\ (\alpha_{12},\beta_{12},\gamma_{12})], \\ \widetilde{x}_{13} &= [(\chi_{13},\delta_{13},\epsilon_{13},\eta_{13},\kappa_{13},\vartheta_{13},\omega_{13},\zeta_{13});\ (\alpha_{13},\beta_{13},\gamma_{13})], \\ \widetilde{x}_{21} &= [(\chi_{21},\delta_{21},\epsilon_{21},\eta_{21},\kappa_{21},\vartheta_{21},\omega_{21},\zeta_{21});\ (\alpha_{21},\beta_{21},\gamma_{21})], \\ \widetilde{x}_{22} &= [(\chi_{22},\delta_{22},\epsilon_{22},\eta_{22},\kappa_{22},\vartheta_{22},\omega_{22},\zeta_{22});\ (\alpha_{22},\beta_{22},\gamma_{22})], \\ \widetilde{x}_{23} &= [(\chi_{23},\delta_{23},\epsilon_{23},\eta_{23},\kappa_{23},\vartheta_{23},\omega_{23},\zeta_{23});\ (\alpha_{23},\beta_{23},\gamma_{23})], \\ \widetilde{x}_{31} &= [(\chi_{31},\delta_{31},\epsilon_{31},\eta_{31},\kappa_{31},\vartheta_{31},\omega_{31},\zeta_{31});\ (\alpha_{31},\beta_{31},\gamma_{31})], \\ \widetilde{x}_{32} &= [(\chi_{32},\delta_{32},\epsilon_{32},\eta_{32},\kappa_{32},\vartheta_{32},\omega_{32},\zeta_{32});\ (\alpha_{32},\beta_{32},\gamma_{32})], \\ \widetilde{x}_{33} &= [(\chi_{33},\delta_{33},\epsilon_{33},\eta_{33},\kappa_{33},\vartheta_{33},\omega_{33},\zeta_{33});\ (\alpha_{33},\beta_{33},\gamma_{33})], \end{aligned} \tag{35}$$

where $\widetilde{x}_{11}$, $\widetilde{x}_{12}$, $\widetilde{x}_{13}$, $\widetilde{x}_{21}$, $\widetilde{x}_{22}$, $\widetilde{x}_{23}$, $\widetilde{x}_{31}$, $\widetilde{x}_{32}$, and $\widetilde{x}_{33}$ are nonnegative TrPFNs, then the FPFLPP equation (33) can be transformed as follows:

$$\text{Minimize}\left(\begin{array}{c}[(110,150,190,210,270,300,350,390);\ (0.7,0.1,0.1)]\otimes\\ [(\chi_{11},\delta_{11},\epsilon_{11},\eta_{11},\kappa_{11},\vartheta_{11},\omega_{11},\zeta_{11});\ (\alpha_{11},\beta_{11},\gamma_{11})]\oplus\\ [(130,180,220,250,290,340,370,410);\ (0.8,0.1,0.1)]\otimes\\ [(\chi_{12},\delta_{12},\epsilon_{12},\eta_{12},\kappa_{12},\vartheta_{12},\omega_{12},\zeta_{12});\ (\alpha_{12},\beta_{12},\gamma_{12})]\oplus\\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes[(\chi_{13},\delta_{13},\epsilon_{13},\eta_{13},\kappa_{13},\vartheta_{13},\omega_{13},\zeta_{13});\ (\alpha_{13},\beta_{13},\gamma_{13})]\oplus\\ [(150,250,290,350,400,440,460,500);\ (0.6,0.1,0.2)]\otimes\\ [(\chi_{21},\delta_{21},\epsilon_{21},\eta_{21},\kappa_{21},\vartheta_{21},\omega_{21},\zeta_{21});\ (\alpha_{21},\beta_{21},\gamma_{21})]\oplus\\ [(190,210,250,270,310,330,380,430);\ (0.6,0.1,0.2)]\otimes\\ [(\chi_{22},\delta_{22},\epsilon_{22},\eta_{22},\kappa_{22},\vartheta_{22},\omega_{22},\zeta_{22});\ (\alpha_{22},\beta_{22},\gamma_{22})]\oplus\\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes[(\chi_{23},\delta_{23},\epsilon_{23},\eta_{23},\kappa_{23},\vartheta_{23},\omega_{23},\zeta_{23});\ (\alpha_{23},\beta_{23},\gamma_{23})]\oplus\\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes[(\chi_{31},\delta_{31},\epsilon_{31},\eta_{31},\kappa_{31},\vartheta_{31},\omega_{31},\zeta_{31});\ (\alpha_{31},\beta_{31},\gamma_{31})]\oplus\\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes[(\chi_{32},\delta_{32},\epsilon_{32},\eta_{32},\kappa_{32},\vartheta_{32},\omega_{32},\zeta_{32});\ (\alpha_{32},\beta_{32},\gamma_{32})]\oplus\\ [(0,0,0,0,0,0,0,0);\ (0.0,0.0,0.0)]\otimes[(\chi_{33},\delta_{33},\epsilon_{33},\eta_{33},\kappa_{33},\vartheta_{33},\omega_{33},\zeta_{33});\ (\alpha_{33},\beta_{33},\gamma_{33})]\end{array}\right), \tag{36}$$

subject to

$$\begin{array}{l}[(\chi_{11},\delta_{11},\epsilon_{11},\eta_{11},\kappa_{11},\vartheta_{11},\omega_{11},\zeta_{11});\ (\alpha_{11},\beta_{11},\gamma_{11})]\oplus[(\chi_{12},\delta_{12},\epsilon_{12},\eta_{12},\kappa_{12},\vartheta_{12},\omega_{12},\zeta_{12});\ (\alpha_{12},\beta_{12},\gamma_{12})]\\ \quad\oplus[(\chi_{13},\delta_{13},\epsilon_{13},\eta_{13},\kappa_{13},\vartheta_{13},\omega_{13},\zeta_{13});\ (\alpha_{13},\beta_{13},\gamma_{13})]=[(15,35,45,60,75,95,110,200);\ (0.92,0.02,0.01)]\\ \quad[(\chi_{21},\delta_{21},\epsilon_{21},\eta_{21},\kappa_{21},\vartheta_{21},\omega_{21},\zeta_{21});\ (\alpha_{21},\beta_{21},\gamma_{21})]\oplus[(\chi_{22},\delta_{22},\epsilon_{22},\eta_{22},\kappa_{22},\vartheta_{22},\omega_{22},\zeta_{22});\ (\alpha_{22},\beta_{22},\gamma_{22})]\\ \quad\oplus[(\chi_{23},\delta_{23},\epsilon_{23},\eta_{23},\kappa_{23},\vartheta_{23},\omega_{23},\zeta_{23});\ (\alpha_{23},\beta_{23},\gamma_{23})]=[(10,25,50,70,90,120,130,190);\ (0.80,0.01,0.02)]\\ \quad[(\chi_{31},\delta_{31},\epsilon_{31},\eta_{31},\kappa_{31},\vartheta_{31},\omega_{31},\zeta_{31});\ (\alpha_{31},\beta_{31},\gamma_{31})]\oplus[(\chi_{32},\delta_{32},\epsilon_{32},\eta_{32},\kappa_{32},\vartheta_{32},\omega_{32},\zeta_{32});\ (\alpha_{32},\beta_{32},\gamma_{32})]\\ \quad\oplus[(\chi_{33},\delta_{33},\epsilon_{33},\eta_{33},\kappa_{33},\vartheta_{33},\omega_{33},\zeta_{33});\ (\alpha_{33},\beta_{33},\gamma_{33})]=[(25,25,25,25,60,60,115,115);\ (0.0000,0.0000,0.0000)]\\ \quad[(\chi_{11},\delta_{11},\epsilon_{11},\eta_{11},\kappa_{11},\vartheta_{11},\omega_{11},\zeta_{11});\ (\alpha_{11},\beta_{11},\gamma_{11})]\oplus[(\chi_{21},\delta_{21},\epsilon_{21},\eta_{21},\kappa_{21},\vartheta_{21},\omega_{21},\zeta_{21});\ (\alpha_{21},\beta_{21},\gamma_{21})]\\ \quad\oplus[(\chi_{31},\delta_{31},\epsilon_{31},\eta_{31},\kappa_{31},\vartheta_{31},\omega_{31},\zeta_{31});\ (\alpha_{31},\beta_{31},\gamma_{31})]=[(20,40,50,65,85,100,140,160);\ (0.92,0.01,0.02)]\\ \quad[(\chi_{12},\delta_{12},\epsilon_{12},\eta_{12},\kappa_{12},\vartheta_{12},\omega_{12},\zeta_{12});\ (\alpha_{12},\beta_{12},\gamma_{12})]\oplus[(\chi_{22},\delta_{22},\epsilon_{22},\eta_{22},\kappa_{22},\vartheta_{22},\omega_{22},\zeta_{22});\ (\alpha_{22},\beta_{22},\gamma_{22})]\\ \quad\oplus[(\chi_{32},\delta_{32},\epsilon_{32},\eta_{32},\kappa_{32},\vartheta_{32},\omega_{32},\zeta_{32});\ (\alpha_{32},\beta_{32},\gamma_{32})]=[(30,35,40,50,100,110,150,170);\ (0.76,0.02,0.01)]\\ \quad[(\chi_{13},\delta_{13},\epsilon_{13},\eta_{13},\kappa_{13},\vartheta_{13},\omega_{13},\zeta_{13});\ (\alpha_{13},\beta_{13},\gamma_{13})]\oplus[(\chi_{23},\delta_{23},\epsilon_{23},\eta_{23},\kappa_{23},\vartheta_{23},\omega_{23},\zeta_{23});\ (\alpha_{23},\beta_{23},\gamma_{23})]\\ \quad\oplus[(\chi_{33},\delta_{33},\epsilon_{33},\eta_{33},\kappa_{33},\vartheta_{33},\omega_{33},\zeta_{33});\ (\alpha_{33},\beta_{33},\gamma_{33})]=[(0,10,30,40,40,65,65,175);\ (0.1666,0.0000,0.0000)].\end{array} \tag{37}$$

By applying arithmetic operations as described in Section 2, the FPFLPP equation (37) can be transformed as follows:

$$\text{Minimize}\left(\begin{array}{c}\left[\left(\begin{array}{c}(110\chi_{11}, 150\delta_{11}, 190\epsilon_{11}, 210\eta_{11}, 270\kappa_{11}, 300\vartheta_{11}, 350\omega_{11}, 390\zeta_{11});\\(0.7\alpha_{11}, 0.1\beta_{11}, 0.1+\gamma_{11}-0.1\gamma_{11})\end{array}\right)\right]\oplus\\\left[\left(\begin{array}{c}(130\chi_{12}, 180\delta_{12}, 220\epsilon_{12}, 250\eta_{12}, 290\kappa_{12}, 340\vartheta_{12}, 370\omega_{12}, 410\zeta_{12});\\(0.8\alpha_{12}, 0.1\beta_{12}, 0.1+\gamma_{12}-0.1\gamma_{12})\end{array}\right)\right]\oplus\\\left[\left(\begin{array}{c}(0\chi_{13}, 0\delta_{13}, 0\epsilon_{13}, 0\eta_{13}, 0\kappa_{13}, 0\vartheta_{13}, 0\omega_{13}, 0\zeta_{13});\\(0.0\alpha_{13}, 0.0\beta_{13}, 0.0+\gamma_{13}-0.0\gamma_{13})\end{array}\right)\right]\oplus\\\left[\left(\begin{array}{c}(150\chi_{21}, 250\delta_{21}, 290\epsilon_{21}, 350\eta_{21}, 400\kappa_{21}, 440\vartheta_{21}, 460\omega_{21}, 500\zeta_{21});\\(0.6\alpha_{21}, 0.1\beta_{21}, 0.2+\gamma_{21}-0.2\gamma_{21})\end{array}\right)\right]\oplus\\\left[\left(\begin{array}{c}(190\chi_{22}, 210\delta_{22}, 250\epsilon_{22}, 270\eta_{22}, 310\kappa_{22}, 330\vartheta_{22}, 380\omega_{22}, 430\zeta_{22});\\(0.6\alpha_{22}, 0.1\beta_{22}, 0.2+\gamma_{22}-0.2\gamma_{22})\end{array}\right)\right]\oplus\\\left[\left(\begin{array}{c}(0\chi_{23}, 0\delta_{23}, 0\epsilon_{23}, 0\eta_{23}, 0\kappa_{23}, 0\vartheta_{23}, 0\omega_{23}, 0\zeta_{23});\\(0.0\alpha_{23}, 0.0\beta_{23}, 0.0+\gamma_{23}-0.0\gamma_{23})\end{array}\right)\right]\oplus\\\left[\left((0\chi_{31}, 0\delta_{31}, 0\epsilon_{31}, 0\eta_{31}, 0\kappa_{31}, 0\vartheta_{31}, 0\omega_{31}, 0\zeta_{31}); (0.0\alpha_{31}, 0.0\beta_{31}, 0.0+\gamma_{31}-0.0\gamma_{31})\right)\right]\oplus\\\left[\left((0\chi_{32}, 0\delta_{32}, 0\epsilon_{32}, 0\eta_{32}, 0\kappa_{32}, 0\vartheta_{32}, 0\omega_{32}, 0\zeta_{32}); (0.0\alpha_{32}, 0.0\beta_{32}, 0.0+\gamma_{32}-0.0\gamma_{32})\right)\right]\oplus\\\left[\left((0\chi_{33}, 0\delta_{33}, 0\epsilon_{33}, 0\eta_{33}, 0\kappa_{33}, 0\vartheta_{33}, 0\omega_{33}, 0\zeta_{33}); (0.0\alpha_{33}, 0.0\beta_{33}, 0.0+\gamma_{33}-0.0\gamma_{33})\right)\right]\end{array}\right), \quad (38)$$

subject to

$$\left[\left(\begin{array}{l}\chi_{11}+\chi_{12}+\chi_{13}, \delta_{11}+\delta_{12}+\delta_{13}, \epsilon_{11}+\epsilon_{12}+\epsilon_{13}, \eta_{11}+\eta_{12}+\eta_{13},\\\kappa_{11}+\kappa_{12}+\kappa_{13}, \vartheta_{11}+\vartheta_{12}+\vartheta_{13}, \omega_{11}+\omega_{12}+\omega_{13}, \zeta_{11}+\zeta_{12}+\zeta_{13}\end{array}\right); \left(\begin{array}{c}\alpha_{11}+\alpha_{12}-\alpha_{11}\alpha_{12}\\+\alpha_{13}-\alpha_{11}\alpha_{13}-\alpha_{12}\alpha_{13}\\+\alpha_{11}\alpha_{12}\alpha_{13}, \beta_{11}\beta_{12}\beta_{13},\\\gamma_{11}\gamma_{12}\gamma_{13}\end{array}\right)\right]$$
$$= [(15, 35, 45, 60, 75, 95, 110, 200); (0.92, 0.02, 0.01)]$$

$$\left[\left(\begin{array}{l}\chi_{21}+\chi_{22}+\chi_{23}, \delta_{21}+\delta_{22}+\delta_{23}, \epsilon_{21}+\epsilon_{22}+\epsilon_{23}, \eta_{21}+\eta_{22}+\eta_{23},\\\kappa_{21}+\kappa_{22}+\kappa_{23}, \vartheta_{21}+\vartheta_{22}+\vartheta_{23}, \omega_{21}+\omega_{22}+\omega_{23}, \zeta_{21}+\zeta_{22}+\zeta_{23}\end{array}\right); \left(\begin{array}{c}\alpha_{21}+\alpha_{22}-\alpha_{21}\alpha_{22}\\+\alpha_{23}-\alpha_{21}\alpha_{23}-\alpha_{22}\alpha_{23}\\+\alpha_{21}\alpha_{22}\alpha_{23}, \beta_{21}\beta_{22}\beta_{23},\\\gamma_{21}\gamma_{22}\gamma_{23}\end{array}\right)\right]$$
$$= [(10, 25, 50, 70, 90, 120, 130, 190); (0.80, 0.01, 0.02)]$$

$$\left[\left(\begin{array}{l}\chi_{31}+\chi_{32}+\chi_{33}, \delta_{31}+\delta_{32}+\delta_{33}, \epsilon_{31}+\epsilon_{32}+\epsilon_{33}, \eta_{31}+\eta_{32}+\eta_{33},\\\kappa_{31}+\kappa_{32}+\kappa_{33}, \vartheta_{31}+\vartheta_{32}+\vartheta_{33}, \omega_{31}+\omega_{32}+\omega_{33}, \zeta_{31}+\zeta_{32}+\zeta_{33}\end{array}\right); \left(\begin{array}{c}\alpha_{31}+\alpha_{32}-\alpha_{31}\alpha_{32}\\+\alpha_{33}-\alpha_{31}\alpha_{33}-\alpha_{32}\alpha_{33}\\+\alpha_{31}\alpha_{32}\alpha_{33}, \beta_{31}\beta_{32}\beta_{33},\\\gamma_{31}\gamma_{32}\gamma_{33}\end{array}\right)\right]$$
$$= [(25, 25, 25, 25, 60, 60, 115, 115); (0.0000, 0.0000, 0.0000)]$$

$$\left[\left(\begin{array}{l}\chi_{11}+\chi_{21}+\chi_{31}, \delta_{11}+\delta_{21}+\delta_{31}, \epsilon_{11}+\epsilon_{21}+\epsilon_{31}, \eta_{11}+\eta_{21}+\eta_{31},\\\kappa_{11}+\kappa_{21}+\kappa_{31}, \vartheta_{11}+\vartheta_{21}+\vartheta_{31}, \omega_{11}+\omega_{21}+\omega_{31}, \zeta_{11}+\zeta_{21}+\zeta_{31}\end{array}\right); \left(\begin{array}{c}\alpha_{11}+\alpha_{21}-\alpha_{11}\alpha_{21}\\+\alpha_{31}-\alpha_{11}\alpha_{31}-\alpha_{21}\alpha_{31}\\+\alpha_{11}\alpha_{21}\alpha_{31}, \beta_{11}\beta_{21}\beta_{31},\\\gamma_{11}\gamma_{21}\gamma_{31}\end{array}\right)\right]$$

$$= [(20, 40, 50, 65, 85, 100, 140, 160);\ (0.92, 0.01, 0.02)]$$

$$\left[\begin{pmatrix} \chi_{12}+\chi_{22}+\chi_{32}, \delta_{12}+\delta_{22}+\delta_{32}, \epsilon_{12}+\epsilon_{22}+\epsilon_{32}, \eta_{12}+\eta_{22}+\eta_{32}, \\ \kappa_{12}+\kappa_{22}+\kappa_{32}, \vartheta_{12}+\vartheta_{22}+\vartheta_{32}, \omega_{12}+\omega_{22}+\omega_{32}, \zeta_{12}+\zeta_{22}+\zeta_{32} \end{pmatrix}; \begin{pmatrix} \alpha_{12}+\alpha_{22}-\alpha_{12}\alpha_{22} \\ +\alpha_{32}-\alpha_{12}\alpha_{32}-\alpha_{22}\alpha_{32} \\ +\alpha_{12}\alpha_{22}\alpha_{32}, \beta_{12}\beta_{22}\beta_{32}, \\ \gamma_{12}\gamma_{22}\gamma_{32} \end{pmatrix}\right]$$

$$= [(30, 35, 40, 50, 100, 110, 150, 170);\ (0.76, 0.02, 0.01)] \tag{39}$$

$$\left[\begin{pmatrix} \chi_{13}+\chi_{23}+\chi_{33}, \delta_{13}+\delta_{23}+\delta_{33}, \epsilon_{13}+\epsilon_{23}+\epsilon_{33}, \eta_{13}+\eta_{23}+\eta_{33}, \\ \kappa_{13}+\kappa_{23}+\kappa_{33}, \vartheta_{13}+\vartheta_{23}+\vartheta_{33}, \omega_{13}+\omega_{23}+\omega_{33}, \zeta_{13}+\zeta_{23}+\zeta_{33} \end{pmatrix}; \begin{pmatrix} \alpha_{13}+\alpha_{23}-\alpha_{13}\alpha_{23} \\ +\alpha_{33}-\alpha_{13}\alpha_{33}-\alpha_{23}\alpha_{33} \\ +\alpha_{13}\alpha_{23}\alpha_{33}, \beta_{13}\beta_{23}\beta_{33}, \\ \gamma_{13}\gamma_{23}\gamma_{33} \end{pmatrix}\right]$$

$$= [(0, 10, 30, 40, 40, 65, 65, 175);\ (0.1666, 0.0000, 0.0000)].$$

By applying Definition 7, the FPLPP equation (39) can be transformed as follows:

$$\text{Minimize } \mathfrak{R}\left[\begin{matrix}\begin{pmatrix} 110\chi_{11}+130\chi_{12}+150\chi_{21}+190\chi_{22}, 150\delta_{11}+180\delta_{12}+250\delta_{21}+210\delta_{22}, \\ 190\epsilon_{11}+220\epsilon_{12}+290\epsilon_{21}+250\epsilon_{22}, 210\eta_{11}+250\eta_{12}+350\eta_{21}+270\eta_{22}, \\ 270\kappa_{11}+290\kappa_{12}+400\kappa_{21}+310\kappa_{22}, 300\vartheta_{11}+340\vartheta_{12}+440\vartheta_{21}+330\vartheta_{22}, \\ 350\omega_{11}+370\omega_{12}+460\omega_{21}+380\omega_{22}, 390\zeta_{11}+410\zeta_{12}+500\zeta_{21}+430\zeta_{22} \end{pmatrix}; \\ \begin{pmatrix} 0.7\alpha_{11}+0.8\alpha_{12}-0.56\alpha_{11}\alpha_{12}+0.6\alpha_{21}-0.42\alpha_{11}\alpha_{21} \\ -0.48\alpha_{12}\alpha_{21}+0.336\alpha_{11}\alpha_{12}\alpha_{21}+0.6\alpha_{22}-0.42\alpha_{11}\alpha_{22}-0.48\alpha_{12}\alpha_{22} \\ +0.336\alpha_{11}\alpha_{12}\alpha_{22}-0.36\alpha_{21}\alpha_{22}+0.252\alpha_{11}\alpha_{21}\alpha_{22} \\ +0.288\alpha_{12}\alpha_{21}\alpha_{22}-0.2016\alpha_{11}\alpha_{12}\alpha_{21}\alpha_{22}, 0.0001\beta_{11}\beta_{12}\beta_{21}\beta_{22}, \\ (0.1+\gamma_{11}-0.1\gamma_{11})(0.1+\gamma_{12}-0.1\gamma_{12}) \\ (0.2+\gamma_{21}-0.2\gamma_{21})(0.2+\gamma_{22}-0.2\gamma_{22}) \end{pmatrix}\end{matrix}\right], \tag{40}$$

subject to

$$
\begin{aligned}
&\left[\begin{pmatrix}\chi_{11}+\chi_{12}+\chi_{13},\delta_{11}+\delta_{12}+\delta_{13},\epsilon_{11}+\epsilon_{12}+\epsilon_{13},\eta_{11}+\eta_{12}+\eta_{13},\\ \kappa_{11}+\kappa_{12}+\kappa_{13},\vartheta_{11}+\vartheta_{12}+\vartheta_{13},\omega_{11}+\omega_{12}+\omega_{13},\zeta_{11}+\zeta_{12}+\zeta_{13}\end{pmatrix};\begin{pmatrix}\alpha_{11}+\alpha_{12}-\alpha_{11}\alpha_{12}\\ +\alpha_{13}-\alpha_{11}\alpha_{13}-\alpha_{12}\alpha_{13}\\ +\alpha_{11}\alpha_{12}\alpha_{13},\beta_{11}\beta_{12}\beta_{13},\\ \gamma_{11}\gamma_{12}\gamma_{13}\end{pmatrix}\right]\\
&=[(15,35,45,60,75,95,110,200);\ (0.92,0.02,0.01)]\\
&\left[\begin{pmatrix}\chi_{21}+\chi_{22}+\chi_{23},\delta_{21}+\delta_{22}+\delta_{23},\epsilon_{21}+\epsilon_{22}+\epsilon_{23},\eta_{21}+\eta_{22}+\eta_{23},\\ \kappa_{21}+\kappa_{22}+\kappa_{23},\vartheta_{21}+\vartheta_{22}+\vartheta_{23},\omega_{21}+\omega_{22}+\omega_{23},\zeta_{21}+\zeta_{22}+\zeta_{23}\end{pmatrix};\begin{pmatrix}\alpha_{21}+\alpha_{22}-\alpha_{21}\alpha_{22}\\ +\alpha_{23}-\alpha_{21}\alpha_{23}-\alpha_{22}\alpha_{23}\\ +\alpha_{21}\alpha_{22}\alpha_{23},\beta_{21}\beta_{22}\beta_{23},\\ \gamma_{21}\gamma_{22}\gamma_{23}\end{pmatrix}\right]\\
&=[(10,25,50,70,90,120,130,190);\ (0.80,0.01,0.02)]\\
&\left[\begin{pmatrix}\chi_{31}+\chi_{32}+\chi_{33},\delta_{31}+\delta_{32}+\delta_{33},\epsilon_{31}+\epsilon_{32}+\epsilon_{33},\eta_{31}+\eta_{32}+\eta_{33},\\ \kappa_{31}+\kappa_{32}+\kappa_{33},\vartheta_{31}+\vartheta_{32}+\vartheta_{33},\omega_{31}+\omega_{32}+\omega_{33},\zeta_{31}+\zeta_{32}+\zeta_{33}\end{pmatrix};\begin{pmatrix}\alpha_{31}+\alpha_{32}-\alpha_{31}\alpha_{32}\\ +\alpha_{33}-\alpha_{31}\alpha_{33}-\alpha_{32}\alpha_{33}\\ +\alpha_{31}\alpha_{32}\alpha_{33},\beta_{31}\beta_{32}\beta_{33},\\ \gamma_{31}\gamma_{32}\gamma_{33}\end{pmatrix}\right]\\
&=[(25,25,25,25,60,60,115,115);\ (0.0000,0.0000,0.0000)]\\
&\left[\begin{pmatrix}\chi_{11}+\chi_{21}+\chi_{31},\delta_{11}+\delta_{21}+\delta_{31},\epsilon_{11}+\epsilon_{21}+\epsilon_{31},\eta_{11}+\eta_{21}+\eta_{31},\\ \kappa_{11}+\kappa_{21}+\kappa_{31},\vartheta_{11}+\vartheta_{21}+\vartheta_{31},\omega_{11}+\omega_{21}+\omega_{31},\zeta_{11}+\zeta_{21}+\zeta_{31}\end{pmatrix};\begin{pmatrix}\alpha_{11}+\alpha_{21}-\alpha_{11}\alpha_{21}\\ +\alpha_{31}-\alpha_{11}\alpha_{31}-\alpha_{21}\alpha_{31}\\ +\alpha_{11}\alpha_{21}\alpha_{31},\beta_{11}\beta_{21}\beta_{31},\\ \gamma_{11}\gamma_{21}\gamma_{31}\end{pmatrix}\right]\\
&=[(20,40,50,65,85,100,140,160);\ (0.92,0.01,0.02)]\\
&\left[\begin{pmatrix}\chi_{12}+\chi_{22}+\chi_{32},\delta_{12}+\delta_{22}+\delta_{32},\epsilon_{12}+\epsilon_{22}+\epsilon_{32},\eta_{12}+\eta_{22}+\eta_{32},\\ \kappa_{12}+\kappa_{22}+\kappa_{32},\vartheta_{12}+\vartheta_{22}+\vartheta_{32},\omega_{12}+\omega_{22}+\omega_{32},\zeta_{12}+\zeta_{22}+\zeta_{32}\end{pmatrix};\begin{pmatrix}\alpha_{12}+\alpha_{22}-\alpha_{12}\alpha_{22}\\ +\alpha_{32}-\alpha_{12}\alpha_{32}-\alpha_{22}\alpha_{32}\\ +\alpha_{12}\alpha_{22}\alpha_{32},\beta_{12}\beta_{22}\beta_{32},\\ \gamma_{12}\gamma_{22}\gamma_{32}\end{pmatrix}\right]\\
&[(30,35,40,50,100,110,150,170);\ (0.76,0.02,0.01)]\\
&\left[\begin{pmatrix}\chi_{13}+\chi_{23}+\chi_{33},\delta_{13}+\delta_{23}+\delta_{33},\epsilon_{13}+\epsilon_{23}+\epsilon_{33},\eta_{13}+\eta_{23}+\eta_{33},\\ \kappa_{13}+\kappa_{23}+\kappa_{33},\vartheta_{13}+\vartheta_{23}+\vartheta_{33},\omega_{13}+\omega_{23}+\omega_{33},\zeta_{13}+\zeta_{23}+\zeta_{33}\end{pmatrix};\begin{pmatrix}\alpha_{13}+\alpha_{23}-\alpha_{13}\alpha_{23}\\ +\alpha_{33}-\alpha_{13}\alpha_{33}-\alpha_{23}\alpha_{33}\\ +\alpha_{13}\alpha_{23}\alpha_{33},\beta_{13}\beta_{23}\beta_{33},\\ \gamma_{13}\gamma_{23}\gamma_{33}\end{pmatrix}\right]\\
&=[(0,10,30,40,40,65,65,175);\ (0.1666,0.0000,0.0000)].
\end{aligned}
\tag{41}
$$

Now, we solve the following crisp LPP:

$$\text{Minimize}\frac{1}{4}\left(\begin{array}{c}(0.7\alpha_{11}+0.8\alpha_{12}-0.56\alpha_{11}\alpha_{12}+0.6\alpha_{21}-0.42\alpha_{11}\alpha_{21}\\ -0.48\alpha_{12}\alpha_{21}+0.336\alpha_{11}\alpha_{12}\alpha_{21}+0.6\alpha_{22}-0.42\alpha_{11}\alpha_{22}-0.48\alpha_{12}\alpha_{22}\\ +0.336\alpha_{11}\alpha_{12}\alpha_{22}-0.36\alpha_{21}\alpha_{22}+0.252\alpha_{11}\alpha_{21}\alpha_{22}\\ +0.288\alpha_{12}\alpha_{21}\alpha_{22}-0.2016\alpha_{11}\alpha_{12}\alpha_{21}\alpha_{22})(210\eta_{11}+250\eta_{12}\\ +350\eta_{21}+270\eta_{22}+270\kappa_{11}+290\kappa_{12}+400\kappa_{21}+310\kappa_{22})\\ +(190\epsilon_{11}+220\epsilon_{12}+290\epsilon_{21}+250\epsilon_{22})+(300\vartheta_{11}+340\vartheta_{12}\\ +440\vartheta_{21}+330\vartheta_{22})+(0.0001\beta_{11}\beta_{12}\beta_{21}\beta_{22})\\ ((210\eta_{11}+250\eta_{12}+350\eta_{21}+270\eta_{22}+270\kappa_{11}+290\kappa_{12}+400\kappa_{21}+310\kappa_{22})\\ +(150\delta_{11}+180\delta_{12}+250\delta_{21}+210\delta_{22})+(350\omega_{11}+370\omega_{12}\\ +460\omega_{21}+380\omega_{22})+(1-(0.1+\gamma_{11}-0.1\gamma_{11})\\ (0.1+\gamma_{12}-0.1\gamma_{12})(0.2+\gamma_{21}-0.2\gamma_{21})\\ (0.2+\gamma_{22}-0.2\gamma_{22}))(210\eta_{11}+250\eta_{12}+350\eta_{21}\\ +270\eta_{22}+270\kappa_{11}+290\kappa_{12}+400\kappa_{21}+310\kappa_{22})+(110\chi_{11}+130\chi_{12}\\ +150\chi_{21}+190\chi_{22})+(390\zeta_{11}+410\zeta_{12}+500\zeta_{21}+430\zeta_{22}))\end{array}\right), \tag{42}$$

subject to

$$\begin{array}{ll}
\chi_{11}+\chi_{12}+\chi_{13}=15, & \chi_{21}+\chi_{22}+\chi_{23}=10,\\
\delta_{11}+\delta_{12}+\delta_{13}=35, & \delta_{21}+\delta_{22}+\delta_{23}=25,\\
\epsilon_{11}+\epsilon_{12}+\epsilon_{13}=45, & \epsilon_{21}+\epsilon_{22}+\epsilon_{23}=50,\\
\eta_{11}+\eta_{12}+\eta_{13}=60, & \eta_{21}+\eta_{22}+\eta_{23}=70,\\
\kappa_{11}+\kappa_{12}+\kappa_{13}=75, & \kappa_{21}+\kappa_{22}+\kappa_{23}=90,\\
\vartheta_{11}+\vartheta_{12}+\vartheta_{13}=95, & \vartheta_{21}+\vartheta_{22}+\vartheta_{23}=120,\\
\omega_{11}+\omega_{12}+\omega_{13}=110, & \omega_{21}+\omega_{22}+\omega_{23}=130,\\
\zeta_{11}+\zeta_{12}+\zeta_{13}=200, & \zeta_{21}+\zeta_{22}+\zeta_{23}=190,\\
\chi_{31}+\chi_{32}+\chi_{33}=25, & \chi_{11}+\chi_{21}+\chi_{31}=20,\\
\delta_{31}+\delta_{32}+\delta_{33}=25, & \delta_{11}+\delta_{21}+\delta_{31}=40,\\
\epsilon_{31}+\epsilon_{32}+\epsilon_{33}=25, & \epsilon_{11}+\epsilon_{21}+\epsilon_{31}=50,\\
\eta_{31}+\eta_{32}+\eta_{33}=25, & \eta_{11}+\eta_{21}+\eta_{31}=65,\\
\kappa_{31}+\kappa_{32}+\kappa_{33}=60, & \kappa_{11}+\kappa_{21}+\kappa_{31}=85,\\
\vartheta_{31}+\vartheta_{32}+\vartheta_{33}=60, & \vartheta_{11}+\vartheta_{21}+\vartheta_{31}=100,\\
\omega_{31}+\omega_{32}+\omega_{33}=115, & \omega_{11}+\omega_{21}+\omega_{31}=140,\\
\zeta_{31}+\zeta_{32}+\zeta_{33}=115, & \zeta_{11}+\zeta_{21}+\zeta_{31}=160,\\
\chi_{12}+\chi_{22}+\chi_{32}=30, & \chi_{13}+\chi_{23}+\chi_{33}=0,\\
\delta_{12}+\delta_{22}+\delta_{32}=35, & \delta_{13}+\delta_{23}+\delta_{33}=10,\\
\epsilon_{12}+\epsilon_{22}+\epsilon_{32}=40, & \epsilon_{13}+\epsilon_{23}+\epsilon_{33}=30,
\end{array}$$

$$\eta_{12} + \eta_{22} + \eta_{32} = 50, \quad \eta_{13} + \eta_{23} + \eta_{33} = 40,$$
$$\kappa_{12} + \kappa_{22} + \kappa_{32} = 100, \quad \kappa_{13} + \kappa_{23} + \kappa_{33} = 40,$$
$$\vartheta_{12} + \vartheta_{22} + \vartheta_{32} = 110, \quad \vartheta_{13} + \vartheta_{23} + \vartheta_{33} = 65,$$
$$\omega_{12} + \omega_{22} + \omega_{32} = 150, \quad \omega_{13} + \omega_{23} + \omega_{33} = 65,$$
$$\zeta_{12} + \zeta_{22} + \zeta_{32} = 170, \quad \zeta_{13} + \zeta_{23} + \zeta_{33} = 175,$$
$$\alpha_{11} + \alpha_{12} - \alpha_{11}\alpha_{12} + \alpha_{13} - \alpha_{11}\alpha_{13} - \alpha_{12}\alpha_{13} + \alpha_{11}\alpha_{12}\alpha_{13} = 0.92, \quad \alpha_{31} + \alpha_{32} - \alpha_{31}\alpha_{32} + \alpha_{33} - \alpha_{31}\alpha_{33} - \alpha_{32}\alpha_{33} + \alpha_{31}\alpha_{32}\alpha_{33} = 0.00,$$
$$\beta_{11}\beta_{12}\beta_{13} = 0.02, \quad \beta_{31}\beta_{32}\beta_{33} = 0.00,$$
$$\gamma_{11}\gamma_{12}\gamma_{13} = 0.01, \quad \gamma_{31}\gamma_{32}\gamma_{33} = 0.00,$$
$$\alpha_{21} + \alpha_{22} - \alpha_{21}\alpha_{22} + \alpha_{23} - \alpha_{21}\alpha_{23} - \alpha_{22}\alpha_{23} + \alpha_{21}\alpha_{22}\alpha_{23} = 0.80, \quad \alpha_{11} + \alpha_{21} - \alpha_{11}\alpha_{21} + \alpha_{31} - \alpha_{11}\alpha_{31} - \alpha_{21}\alpha_{31} + \alpha_{11}\alpha_{21}\alpha_{31} = 0.92,$$
$$\beta_{21}\beta_{22}\beta_{23} = 0.01, \quad \beta_{11}\beta_{21}\beta_{31} = 0.01,$$
$$\gamma_{21}\gamma_{22}\gamma_{23} = 0.02, \quad \gamma_{11}\gamma_{21}\gamma_{31} = 0.02,$$
$$\alpha_{12} + \alpha_{22} - \alpha_{12}\alpha_{22} + \alpha_{32} - \alpha_{12}\alpha_{32} - \alpha_{22}\alpha_{32} + \alpha_{12}\alpha_{22}\alpha_{32} = 0.76, \quad \alpha_{13} + \alpha_{23} - \alpha_{13}\alpha_{23} + \alpha_{33} - \alpha_{13}\alpha_{33} - \alpha_{23}\alpha_{33} + \alpha_{13}\alpha_{23}\alpha_{33} = 0.1666,$$
$$\beta_{12}\beta_{22}\beta_{32} = 0.02, \quad \beta_{13}\beta_{23}\beta_{33} = 0.00,$$
$$\gamma_{12}\gamma_{22}\gamma_{32} = 0.01, \quad \gamma_{13}\gamma_{23}\gamma_{33} = 0.00,$$
$$\delta_{11} - \chi_{11} \ge 0, \quad \epsilon_{11} - \delta_{11} \ge 0, \quad \eta_{11} - \epsilon_{11} \ge 0, \quad \kappa_{11} - \eta_{11} \ge 0, \quad \vartheta_{11} - \kappa_{11} \ge 0,$$
$$\omega_{11} - \vartheta_{11} \ge 0, \quad \zeta_{11} - \omega_{11} \ge 0, \quad \delta_{12} - \chi_{12} \ge 0, \quad \epsilon_{12} - \delta_{12} \ge 0, \quad \eta_{12} - \epsilon_{12} \ge 0,$$
$$\kappa_{12} - \eta_{12} \ge 0, \vartheta_{12} - \kappa_{12} \ge 0, \quad \omega_{12} - \vartheta_{12} \ge 0, \quad \zeta_{12} - \omega_{12} \ge 0, \quad \delta_{21} - \chi_{21} \ge 0,$$
$$\epsilon_{21} - \delta_{21} \ge 0, \quad \delta_{13} - \chi_{13} \ge 0, \quad \epsilon_{13} - \delta_{13} \ge 0, \quad \eta_{13} - \epsilon_{13} \ge 0, \quad \kappa_{13} - \eta_{13} \ge 0,$$
$$\vartheta_{13} - \kappa_{13} \ge 0, \quad \omega_{13} - \vartheta_{13} \ge 0, \quad \zeta_{13} - \omega_{13} \ge 0, \quad \eta_{21} - \epsilon_{21} \ge 0, \quad \kappa_{21} - \eta_{21} \ge 0,$$
$$\vartheta_{21} - \kappa_{21} \ge 0, \quad \omega_{21} - \vartheta_{21} \ge 0, \quad \zeta_{21} - \omega_{21} \ge 0, \quad \delta_{22} - \chi_{22} \ge 0, \quad \epsilon_{22} - \delta_{22} \ge 0,$$
$$\eta_{22} - \epsilon_{22} \ge 0, \quad \kappa_{22} - \eta_{22} \ge 0, \quad \vartheta_{22} - \kappa_{22} \ge 0, \quad \omega_{22} - \vartheta_{22} \ge 0, \quad \zeta_{22} - \omega_{22} \ge 0,$$
$$\delta_{23} - \chi_{23} \ge 0, \quad \epsilon_{23} - \delta_{23} \ge 0, \quad \eta_{23} - \epsilon_{23} \ge 0, \quad \kappa_{23} - \eta_{23} \ge 0, \quad \vartheta_{23} - \kappa_{23} \ge 0,$$
$$\omega_{23} - \vartheta_{23} \ge 0, \quad \zeta_{23} - \omega_{23} \ge 0, \quad \delta_{31} - \chi_{31} \ge 0, \quad \epsilon_{31} - \delta_{31} \ge 0, \quad \eta_{31} - \epsilon_{31} \ge 0,$$
$$\kappa_{31} - \eta_{31} \ge 0, \quad \vartheta_{31} - \kappa_{31} \ge 0, \quad \omega_{31} - \vartheta_{31} \ge 0, \quad \zeta_{31} - \omega_{31} \ge 0, \quad \delta_{32} - \chi_{32} \ge 0,$$
$$\epsilon_{32} - \delta_{32} \ge 0, \quad \eta_{32} - \epsilon_{32} \ge 0, \quad \kappa_{32} - \eta_{32} \ge 0, \quad \vartheta_{32} - \kappa_{32} \ge 0, \quad \omega_{32} - \vartheta_{32} \ge 0,$$
$$\zeta_{32} - \omega_{32} \ge 0, \quad \delta_{33} - \chi_{33} \ge 0, \quad \epsilon_{33} - \delta_{33} \ge 0, \quad \eta_{33} - \epsilon_{33} \ge 0, \quad \kappa_{33} - \eta_{33} \ge 0,$$

$$\begin{aligned}
&\vartheta_{33}-\kappa_{33}\ge 0,\quad \omega_{33}-\vartheta_{33}\ge 0,\quad \zeta_{33}-\omega_{33}\ge 0,\quad \chi_{11}\ge 0,\quad \chi_{12}\ge 0,\quad \chi_{13}\ge 0,\\
&\chi_{21}\ge 0,\quad \chi_{22}\ge 0,\quad \chi_{23}\ge 0,\chi_{31}\ge 0,\quad \chi_{31}\ge 0,\quad \chi_{32}\ge 0,\quad \chi_{33}\ge 0,\alpha_{12}\ge 0,\\
&\alpha_{13}\ge 0,\quad \alpha_{21}\ge 0,\alpha_{22}\ge 0,\quad \alpha_{23}\ge 0,\quad \beta_{11}\ge 0,\quad \beta_{12}\ge 0,\beta_{21}\ge 0,\quad \beta_{22}\ge 0,\\
&\beta_{23}\ge 0,\quad \beta_{31}\ge 0,\quad \beta_{32}\ge 0,\quad \beta_{33}\ge 0,\quad \gamma_{11}\ge 0,\gamma_{13}\ge 0,\quad \gamma_{21}\ge 0,\quad \gamma_{22}\ge 0,\\
&\gamma_{23}\ge 0,\quad \gamma_{31}\ge 0,\quad \gamma_{32}\ge 0,\quad \gamma_{33}\ge 0,\quad \alpha_{11}\ge 0,\quad \beta_{13}\ge 0,\quad \gamma_{12}\ge 0,\\
&\alpha_{11}+\beta_{11}+\gamma_{11}\le 1,\quad \alpha_{12}+\beta_{12}+\gamma_{12}\le 1,\quad \alpha_{13}+\beta_{13}+\gamma_{13}\le 1,\\
&\alpha_{21}+\beta_{21}+\gamma_{21}\le 1,\quad \alpha_{22}+\beta_{22}+\gamma_{22}\le 1,\quad \alpha_{23}+\beta_{23}+\gamma_{23}\le 1,\\
&\alpha_{31}+\beta_{31}+\gamma_{31}\le 1,\quad \alpha_{32}+\beta_{32}+\gamma_{32}\le 1,\quad \alpha_{33}+\beta_{33}+\gamma_{33}\le 1.
\end{aligned}\tag{43}$$

By using Software Maple, we get optimal solution.

$$\begin{aligned}
&\chi_{11}=15,\ \delta_{11}=35,\ \varepsilon_{11}=45,\ \eta_{11}=60,\ \kappa_{11}=75,\ \vartheta_{11}=90,\ \omega_{11}=105,\ \zeta_{11}=125,\ \alpha_{11}=0.8000,\ \beta_{11}=\\
&0.1000,\ \gamma_{11}=0.1000,\ \chi_{12}=0.0000,\ \delta_{12}=0.0000,\ \varepsilon_{12}=0.0000,\ \eta_{12}=0.0000,\ \kappa_{12}=0.0000,\ \vartheta_{12}=0.0000,\\
&\omega_{12}=0.0000,\ \zeta_{12}=20,\ \alpha_{12}=0.6000,\ \beta_{12}=0.2000,\ \gamma_{12}=0.1000,\ \chi_{13}=0.0000,\ \delta_{12}=0.0000,\ \varepsilon_{13}=0.0000,\\
&\eta_{13}=0.0000,\ \kappa_{13}=0.0000,\ \vartheta_{13}=5,\ \omega_{13}=5,\ \zeta_{13}=55,\ \alpha_{13}=0.1666,\ \beta_{13}=0.0000,\ \gamma_{13}=0.1000,\\
&\chi_{21}=0.0000,\ \delta_{21}=0.0000,\ \varepsilon_{21}=0.0000,\ \eta_{21}=0.0000,\ \kappa_{21}=0.0000,\ \vartheta_{21}=0.0000,\ \omega_{21}=0.0000\\
&,\ \zeta_{21}=0.0000,\ \alpha_{21}=0.6000,\ \beta_{21}=0.1000,\ \gamma_{21}=0.2000,\ \chi_{22}=10,\ \delta_{22}=15,\ \varepsilon_{22}=20,\ \eta_{22}=30,\ \kappa_{22}=50,\\
&\vartheta_{22}=60,\ \omega_{22}=70,\ \zeta_{22}=70,\ \alpha_{22}=0.5000,\ \beta_{22}=0.1000,\ \gamma_{22}=0.1000,\ \chi_{23}=0.0000,\ \delta_{23}=10,\ \varepsilon_{23}=30,\\
&\eta_{23}=40,\ \kappa_{23}=40,\ \vartheta_{23}=60,\ \omega_{23}=60,\ \zeta_{23}=120,\ \alpha_{23}=0.0000,\ \beta_{23}=0.0000,\ \gamma_{23}=0.0000,\ \chi_{31}=5,\\
&\delta_{31}=5,\ \varepsilon_{31}=5,\ \eta_{31}=5,\ \kappa_{31}=10,\ \vartheta_{31}=10,\ \omega_{31}=35,\ \zeta_{31}=35,\ \alpha_{31}=0.0000,\ \beta_{31}=0.0000,\ \gamma_{11}=0.0000,\\
&\chi_{32}=20,\ \delta_{32}=20,\ \varepsilon_{32}=20,\ \eta_{32}=20,\ \kappa_{32}=50,\ \vartheta_{32}=50,\ \omega_{32}=80,\ \zeta_{32}=80,\ \alpha_{32}=0.0000,\ \beta_{32}=\\
&0.0000,\ \gamma_{32}=0.0000,\ \chi_{33}=0.0000,\ \delta_{33}=0.0000,\ \varepsilon_{33}=0.0000\ \eta_{33}=0.0000,\ \kappa_{33}=0.0000,\ \vartheta_{33}=0.0000,\\
&\omega_{33}=0.0000,\ \zeta_{33}=0.0000,\ \alpha_{33}=0.0000,\ \beta_{33}=0.0000,\ \gamma_{33}=0.0000.
\end{aligned}\tag{44}$$

The PFOS is

$$\begin{aligned}
&\tilde{x}_{11}=[(15,35,45,60,75,90,105,125);\ (0.8000,0.1000,0.1000)],\\
&\tilde{x}_{12}=[(0.00,0.00,0.00,0.00,0.00,0.00,0.00,20);\ (0.6000,0.2000,0.1000)],\\
&\tilde{x}_{13}=[(0.00,0.00,0.00,0.00,0.00,5,5,55);\ (0.1666,0.0000,0.0000)],\\
&\tilde{x}_{21}=[(0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00);\ (0.6000,0.1000,0.2000)],\\
&\tilde{x}_{22}=[(10,15,20,30,50,60,70,70);\ (0.5000,0.1000,0.1000)],\\
&\tilde{x}_{23}=[(0.00,10,30,40,40,60,60,120);\ (0.0000,0.0000,0.0000)],\\
&\tilde{x}_{31}=[(5,5,5,5,10,10,35,35);\ (0.0000,0.0000,0.0000)],\\
&\tilde{x}_{32}=[(20,20,20,20,50,50,80,80);\ (0.0000,0.0000,0.0000)],\\
&\tilde{x}_{33}=[(0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00);\ (0.0000,0.0000,0.0000)].
\end{aligned}\tag{45}$$

The picture fuzzy optimal value/transportation cost of FPFTP is

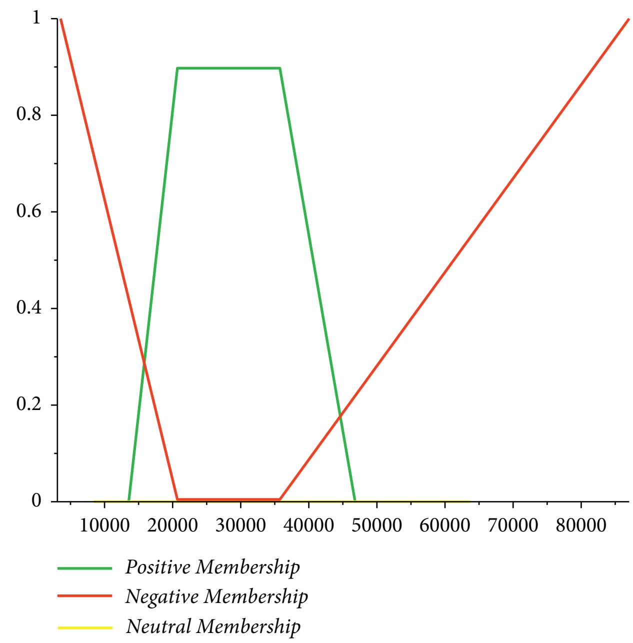

FIGURE 1: Graphical representation of picture fuzzy transportation cost.

[(3550, 8400, 13550, 20700, 35750, 46800, 63350, 87050); (0.8974976, 0.00000002, 0.00467856)] and is shown graphically in Figure 1. □

Example 2. (fully intuitionistic fuzzy transportation problem (FIFTP) based on FLP formulation).

$$\text{Minimize}\begin{pmatrix} [(190,210,270,300);\ (110,210,270,390)]\otimes\tilde{x}_{11}\oplus \\ [(220,250,290,340);\ (130,250,290,410)]\otimes\tilde{x}_{12}\oplus \\ [(0,0,0,0);\ (0,0,0,0)]\otimes\tilde{x}_{13}\oplus \\ [(290,350,400,440);\ (150,350,400,500)]\otimes\tilde{x}_{21}\oplus \\ [(250,270,310,330);\ (190,270,310,430)]\otimes\tilde{x}_{22}\oplus \\ [(0,0,0,0);\ (0,0,0,0)]\otimes\tilde{x}_{23}\oplus \\ [(0,0,0,0);\ (0,0,0,0)]\otimes\tilde{x}_{31}\oplus \\ [(0,0,0,0);\ (0,0,0,0)]\otimes\tilde{x}_{32}\oplus \\ [(0,0,0,0);\ (0,0,0,0)]\otimes\tilde{x}_{33} \end{pmatrix}, \quad (46)$$

subject to

$$\begin{aligned} \tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13} &= [(45,60,75,95);\ (15,60,75,200)],\\ \tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23} &= [(50,70,90,120);\ (10,70,90,190)],\\ \tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33} &= [(25,25,60,60);\ (25,25,60,115)],\\ \tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31} &= [(50,65,85,100);\ (20,65,85,160)],\\ \tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32} &= [(40,50,100,110);\ (30,50,100,170)],\\ \tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33} &= [(30,40,40,65);\ (0,40,40,175)], \end{aligned} \quad (47)$$

where $\tilde{x}_{11}$, $\tilde{x}_{12}$, $\tilde{x}_{13}$, $\tilde{x}_{21}$, $\tilde{x}_{22}$, $\tilde{x}_{23}$, $\tilde{x}_{31}$, $\tilde{x}_{32}$, and $\tilde{x}_{33}$, are nonnegative TrIFNs.

The intuitionistic fuzzy optimal value/transportation cost of FIFTP [41] is [(13550, 20700, 35750, 46800); (3550, 20700, 35750, 87050)] and is shown graphically in Figure 2.

Example 3. (fully fuzzy transportation problem (FFTP) based on FLP formulation).

$$\text{Minimize}\begin{pmatrix} [(190,210,270,300)]\otimes\tilde{x}_{11}\oplus[(220,250,290,340)]\otimes\tilde{x}_{12}\oplus[(0,0,0,0)]\otimes\tilde{x}_{13}\oplus \\ [(290,350,400,440)]\otimes\tilde{x}_{21}\oplus[(250,270,310,330)]\otimes\tilde{x}_{22}\oplus[(0,0,0,0)]\otimes\tilde{x}_{23}\oplus \\ [(0,0,0,0)]\otimes\tilde{x}_{31}\oplus[(0,0,0,0)]\otimes\tilde{x}_{32}\oplus[(0,0,0,0)]\otimes\tilde{x}_{33} \end{pmatrix}, \quad (48)$$

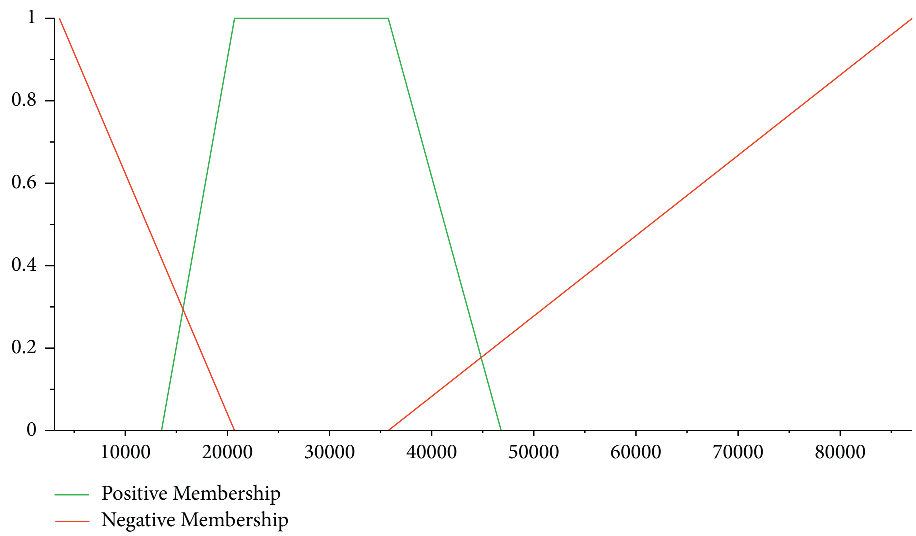

Figure 2: Graphical representation of intuitionistic fuzzy transportation cost.

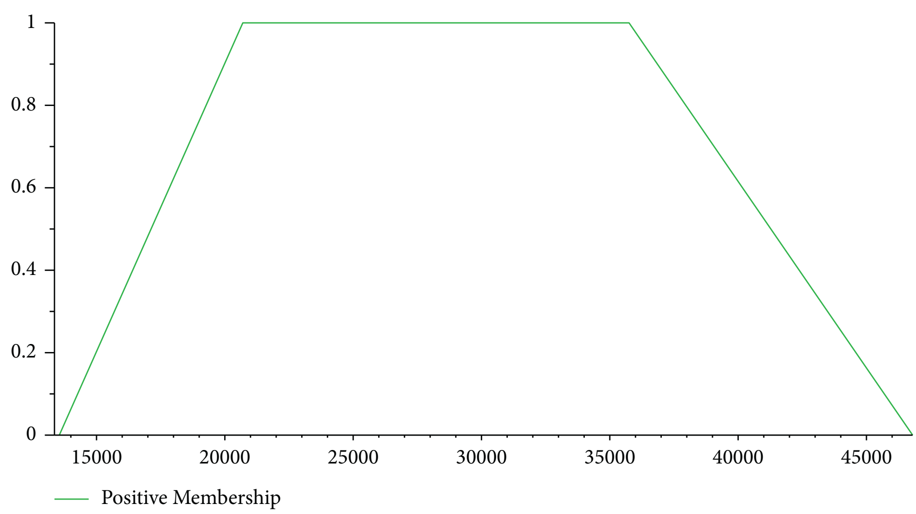

Figure 3: Graphical representation of fuzzy transportation cost.

Table 2: Comparison of optimal values.

FPFTP	[(3550, 8400, 13550, 20700, 35750, 46800, 63350, 87050)]
FIFTP [41]	[(3550, 13550, 20700, 35750, 46800, 87050)]
FFTP [35]	[(13550, 20700, 35750, 46800)]

subject to

$$\begin{aligned} &\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} = [(45, 60, 75, 95)] \\ &\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} = [(50, 70, 90, 120)] \\ &\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} = [(25, 25, 60, 60)] \\ &\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} = [(50, 65, 85, 100)] \\ &\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} = [(40, 50, 100, 110)] \\ &\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} = [(30, 40, 40, 65)], \end{aligned} \tag{49}$$

where $\tilde{x}_{11}$, $\tilde{x}_{12}$, $\tilde{x}_{13}$, $\tilde{x}_{21}$, $\tilde{x}_{22}$, $\tilde{x}_{23}$, $\tilde{x}_{31}$, $\tilde{x}_{32}$, and $\tilde{x}_{33}$ are nonnegative TrFNs.

The fuzzy optimal value/transportation cost of FFTP [35] is [(13550, 20700, 35750, 46800)] and is shown graphically in Figure 3.

5. Discussion

The picture fuzzy optimal value/transportation cost obtained in Example 1 by using the proposed method as discussed in

Section 3 is $[(3550, 8400, 13550, 20700, 35750, 46800, 63350, 87050)]$ and can be illustrated as follows:

(i) The lowest amount of picture fuzzy transportation cost is 3550 units

(ii) The feasible amount of picture fuzzy transportation cost lies in $[20700, 35750]$

(iii) The highest amount of picture fuzzy transportation cost is 87050 units

So, the least transportation cost will always be higher than 3550 units and lower than 87050 units, and the maximum chances of minimum transportation cost will belong to $[20700, 35750]$.

Furthermore, Examples 2 and 3 are considered in an intuitionistic fuzzy environment and fuzzy environment, respectively. The minimum intuitionistic fuzzy transportation cost and minimum fuzzy transportation along with picture fuzzy transportation cost are shown in Table 2.

Obviously, Table 2 demonstrates that the FPFTP gives the best optimal value/transportation cost as compared with FIFTP and FFTP.

6. Conclusion

PFS is the most comprehensive and generalized structure of FS and IFS because it is characterized by membership function, neutral membership function, and nonmembership function. In this paper, we have proposed a new scheme to discuss the fully picture fuzzy transportation problems. We have introduced FPFTPs by considering all the variables as nonnegative TrPFNs. The FPFTPs have been developed on the basis of picture fuzzy linear programming formulation. In order to transform the FPFTPs into crisp linear/non-LPPs, a ranking function is practised. The PFLP formulation technique has been applied to obtain the PFOSs in the form of TrPFNs. Further, we have investigated and compared the fully picture fuzzy transportation model with fully intuitionistic fuzzy transportation model and fully fuzzy transportation model to demonstrate that the proposed approach is more comprehensive and reliable as compared with the existing FIFTP approach [41] and FFTP approach [35].

In future, this work can be extended to

(1) *LR*-type fully picture fuzzy transportation problems

(2) *LR*-type bipolar single-valued neutrosophic transportation problems

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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