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Research Article

Robust Fault-Tolerant Control of Continuous Lurie Networked Control Systems Based on the Observer with the Event-Triggered Mechanism

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For a class of uncertain continuously networked Lurie control systems with both sensor-to-controller time-delay and controller-to-actuator time-delay, the problem of codesigning its observer and fault-tolerant controller based on an event-triggered mechanism under actuator failure is investigated. Firstly, considering that the state of the system cannot be measured directly, an observer is constructed on the controller node. Secondly, to reduce the waste of network bandwidth resources and improve the performance of the network control system, a network control system approach based on event-triggered mechanism is proposed. By introducing the event-triggered mechanism and the actuator fault indication matrix, the Lurie networked control system is modeled as a Lurie system with time-delay using the state augmentation technique to obtain a model of the closed-loop system. Finally, based on Lyapunov stability theory, sufficient condition for the stability of the closed-loop system is obtained, and the design method of the fault-tolerant controller and the observer is given. The obtained results are given in the form of linear matrix inequalities, which are easy to be solved by using the linear matrix inequality toolbox. Finally, the feasibility and effectiveness of the method are illustrated by a simulation example.

1. Introduction

With the rapid development and wide application of computer, communication and control technologies, networked control systems (NCSs), and control systems, in which nodes (sensors, controllers, actuators, and controlled objects) transmit data through a shared communication network, have attracted much attention [1]. Compared with the traditional point-to-point control system, the NCS has the advantages of low installation cost, easy maintenance, high flexibility, information resource sharing, and easy remote operation. It has been widely used in intelligent medical, intelligent transportation, and intelligent manufacturing fields [2–5]. However, due to the problems of time-delay, packet loss, and wasted communication resources derived from the application of NCS, the actual production process will be affected by many uncertain factors, such as equipment wear and tear and production line aging, and the system will inevitably fail [6]. Once the system fails, it may cause catastrophic losses or even damage to personal safety, so it is of great economic and social significance to improve the reliability and stability of NCS operation with the help of fault-tolerant control.

Fault-tolerant control is the most common and effective control method [7, 8], which has received a lot of attention from academics and has achieved some results. The existing research literature on fault-tolerant control of NCS with time-delay can be divided into two categories. The first category of literature focuses on fault-tolerant control of NCS with time-delay between the sensor and the controller. In Ref. [9], a fault-tolerant controller design method based on observer estimation is proposed for the time-delay between the sensor and the controller, and a closed-loop NCS is made asymptotically stable and satisfies the given control performance by combining Lyapunov stability theory and the linear matrix inequality method. The literature [10] models the time-delay from the sensor to the controller as a Markov chain with incomplete transfer probability and uses the state augmentation technique to transform the closed-loop system into a Markov jump system, gives sufficient conditions to guarantee the stability of the NCS, and designs fault-tolerant controllers. The second type of literature focuses on fault-tolerant control of NCS with time-delay between both sensors to controllers and controllers to actuators. The literature [11] combined the time-delay in the sensor-to-controller network and the time-delay in the controller-to-actuator network into a total time-delay, established a robust fault-tolerant control method for discrete NCS with time-delay, and combined Lyapunov stability theory and switching system theory, for the design of fault-tolerant controllers. The literature [12] describes the sum of the time-delay from the sensor to the controller and from the controller to the actuator in terms of random variables conforming to the Bernoulli distribution. A stability criterion for systems with stochastic time-varying time-delay is proposed to ensure that the designed fault-tolerant controller can satisfy the asymptotic stability conditions and operability requirements when the state trajectory enters the slip surface.

With the development and integration of control science and computer technology, the demand for data and information transmission is also increasing. Due to the limitation of network communication bandwidth, communication resources are limited, and if the amount of data transmitted by the system exceeds the threshold value that can be carried, it will lead to network congestion or even system collapse. To address this problem, some scholars have conducted research. The traditional time-triggered mechanism is simple and easy to implement, but the selected sampling period is small, and frequent data transmission and update will lead to the waste of communication resources and easily cause network congestion [13, 14]. In order to overcome the shortcomings of the time-triggered mechanism, many scholars have successively adopted the event-triggered mechanism to save network resources. The implementation method is to let the sampled signal reach an event generator, then the event generator decides whether the signal is transmitted to the network according to the triggering conditions. This mechanism of transmitting only necessary signals and reducing unnecessary signal transmission determined by the triggering conditions effectively reduces the occupation of limited communication resources and improves the efficiency of information transmission [15–17]. With the continuous development of NCS, many innovative results have been achieved for event-triggered mechanisms. For example, the adaptive event-triggered mechanism proposed in the literature [18, 19] has attracted much attention. Compared with the traditional event-triggered mechanism, its triggering threshold is a time-variable related to the system state, so it can flexibly adjust the triggering conditions according to the system state to further reduce the data transmission and maintain a better system performance, which is significant for NCS. Scholars have introduced different kinds of event-triggered mechanisms into NCS to replace the traditional time-based periodic transmission method, which has become an important research element in NCS.

In recent years, some results have been achieved in the research about fault-tolerant control of NCS with time-delay under the event-triggered mechanism. For example, an event-triggered mechanism with the fault-tolerant control strategy is proposed for continuous uncertain NCS with time-delay from the sensor to thecontroller, which enables the system to remain stable in case of actuator failure [20]. The literature [21] investigated the asynchronous H_∞ fault-tolerant control of discrete Markov jump systems with actuator failure and time-delay, proposed a pattern-dependent event-triggered mechanism method based on the pattern-dependent Lyapunov–Krasovskii function, and obtained the performance of the closed-loop system with stochastic mean-square stability under actuator failure and the joint design method of the asynchronous fault-tolerant controller and event-triggered mechanism. However, all the above-mentioned literatures are studies done based on state feedback in the case of measurable system states. For the case where the system state is not measurable, literature [22] proposes a delay-based event-triggered fault estimation observer for a class of continuous NCSs with time-delay and faults, gives a fault-tolerant control method based on fault estimation to compensate the effects generated by system faults, and designs fault-tolerant controller gains and event-triggered parameters to ensure the required performance of the faulty system and to reduce the communication resources wastage. However, only the time-delay in the sensor-to-controller network is considered in the paper. In the literature [23], a fault-tolerant control model for linear NCS with time-delay under the dynamic event-triggered mechanism is developed, an observer design method for simultaneously estimating the state vector and unknown actuator faults is proposed, and an observer-based fault-tolerant controller is designed, but the time-delay from sensor-to-controller and controller-to-actuator is combined in the paper to deal with them.

Lurie systems are a class of nonlinear systems with typical structural features that can be more accurately modeled to reflect objective reality, and they represent many essential features of nonlinear systems. The absolute stability of Lurie systems has received much attention. However, few literature have been devoted to the problem of fault-tolerant control under actuator failure as an object of study.

It is well known that time-delay is the main cause of system instability in NCS. Most of the existing techniques have only studied the controller-to-actuator time-delay or have combined the sensor-to-controller and controller-to-actuator time-delay into one treatment. Based on the above discussion, the following questions naturally arise: Is it possible to build a unified model of NCS with bilateral delays based on event-triggered mechanisms? How can the joint design of event-triggered mechanism, observer, and fault-tolerant controller under this unified model make the closed-loop system with time-delay, uncertainty, and actuator failure stable?

In this paper, we address the codesign of its event-triggered mechanism and fault-tolerant controller under actuator failure for a continuously networked Lurie control system with time-delay and uncertainties, with the following main contributions:

(1) The influence of the time-delay from the sensor to the controller and from the controller to the actuator on the control system is taken into account, which is more interpretable and mechanistic

(2) Also considering the time-delay, uncertainty of system parameters, and actuator failure, an observer is constructed on the controller node to establish the Lurie NCS model under the event-triggered mechanism, which enables the impact of time-delay, uncertainty, event-triggered mechanism, and actuator failure on system performance to be analyzed in the same framework

(3) According to Lyapunov stability theory, the stability conditions of the closed-loop system are obtained, and the design methods for solving the event-triggered weight matrix, robust fault-tolerant controller, and observer are given

The remainder of this paper is organized as follows. The modeling of a continuous Lurie NCS with actuator failures under an event-triggered mechanism is shown in Section 2. In Section 3, the system stability analysis and the codesign of the observer, fault-tolerant controller, and event-triggered mechanism are provided. Section 4 exhibits the construction of the simulation model and the verification of the algorithm. Section 5 summarizes the contents of the full paper.

Notations are as follows: R^n represents the n-dimensional Euclidean space; $*$ represents the transpose of the corresponding matrix block; if the matrix A is invertible, A^{-1} represents the inverse of A; A^{T} and A^{+} represent the transpose and Moore–Penrose inverse of the matrix, respectively, and the real positive definite matrix X is represented as $X > 0$; I represents the unit matrix of appropriate dimensions; and $\mathrm{diag}\{a, b \ldots\}$ represents the diagonal matrix with a, b as the main diagonal.

2. Problem Description

Consider the following Lurie system with uncertainty and actuator failure:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)Fu(t) + (D + \Delta D)\omega(t), \\ y(t) = Cx(t), \\ \omega(t) = -\psi(y(t)). \end{cases} \quad (1)$$

where $x(t) \in R^n, u(t) \in R^m, y(t) \in R^q$ are the system state vector, input vector, and output vector, respectively; A, B, C, D are real constant matrices of appropriate dimensions; $\Delta A, \Delta B, \Delta D$ are uncertainty matrices with bounded parametrization and satisfy $[\Delta A \;\; \Delta B \;\; \Delta D] = U\Xi(t)[H_1 \;\; H_2 \;\; H_3]$, where U, H_1, H_2, H_3 are known matrices with appropriate dimensions, and $\Xi(t)$ is an unknown matrix satisfying $\Xi(t)^T \Xi(t) < I$; F is the fault indication matrix which we define under actuator failure and satisfies $F = \mathrm{diag}\{f_1, f_2, \ldots f_m\}, f_j \in [0, 1], \quad j = 1, 2, \ldots m$, where $f_j = 0$ means the jth actuator is completely failed, $f_j = 1$ indicates normal operation of the j th actuator, and $f_j \in (0, 1)$ indicates partial failure of the j th actuator. $\psi(\cdot)$: $R^q \longrightarrow R^q$ is a memoryless nonlinear function, satisfying the local Lipschitz condition, $\psi(0) = 0$, and for any $y(t) \in R^q$ satisfies, the sector condition is

$$\psi^T(y(t))[\psi(y(t)) - \Theta y(t)] \leq 0, \quad (2)$$

where Θ is the real diagonal matrix. Such a nonlinear function $\psi(\cdot)$ is usually said to belong to the sector $[0, \Theta]$, denoted as $\psi(\cdot) \in \ell[0, \Theta]$.

Remark 1. For the more general case $\psi(\cdot) \in \ell[\Theta_1, \Theta_2]$, it can always be transformed into the case of $\psi(\cdot) \in \ell[0, \Theta]$ by loop transformation, so only the case of $\psi(\cdot) \in \ell[0, \Theta]$ is considered in this paper.

The NCS structure diagram shown in Figure 1 consists of an actuator, a controlled object, a sensor, a sampler, a zero-order retainer, an observer, an event generator, and a controller. The sampler acquires the output signal of the system with a fixed period h. The signal collected by the sampler is directly transmitted to the event generator connected to it. Then, whether the event generator can send the sampled data $y(t_k h + jh)$ to the controller needs to be determined according to the following conditions:

$$\begin{aligned} &[y(t_k h + jh) - y(t_k h)]^T V [y(t_k h + jh) - y(t_k h)] \\ &\quad > \sigma y^T(t_k h + jh) V y(t_k h + jh), \end{aligned} \quad (3)$$

where V is a positive definite matrix and $j \in Z^+, \sigma \in [0, 1]$.

Remark 2. Only the output sampled data $y(t_k h + jh)$ satisfying equation (3) will be transmitted, and obviously this event-triggered mechanism will reduce the communication load in the network and reduce the computation of the controller. There is a special case that if $\sigma = 0$ in Eq. (3) holds for all sampled output data $y(t_k h + jh)$, then the event-triggered mechanism will degrade to a time-triggered mechanism. In addition, the triggering instant of the event-triggered mechanism used in this paper is based on the system sampling period h, which can be triggered only at the sampling instant, ensuring that the minimum event interval time is positive and avoiding Zeno behavior.

After the event generator has released the current sample signal, the sample signal is transmitted directly to the zero-order keeper via the network and then to the actuator via the observer. In this process, the time-delay from the sensor to the controller is denoted by $\tau_{i_k}^{sc}$, and the time-delay from the controller to the actuator is denoted by $\tau_{i_k}^{ca}$.

To facilitate the following analysis, the following hypotheses are proposed:

(a) The matrix C is a row-full rank matrix

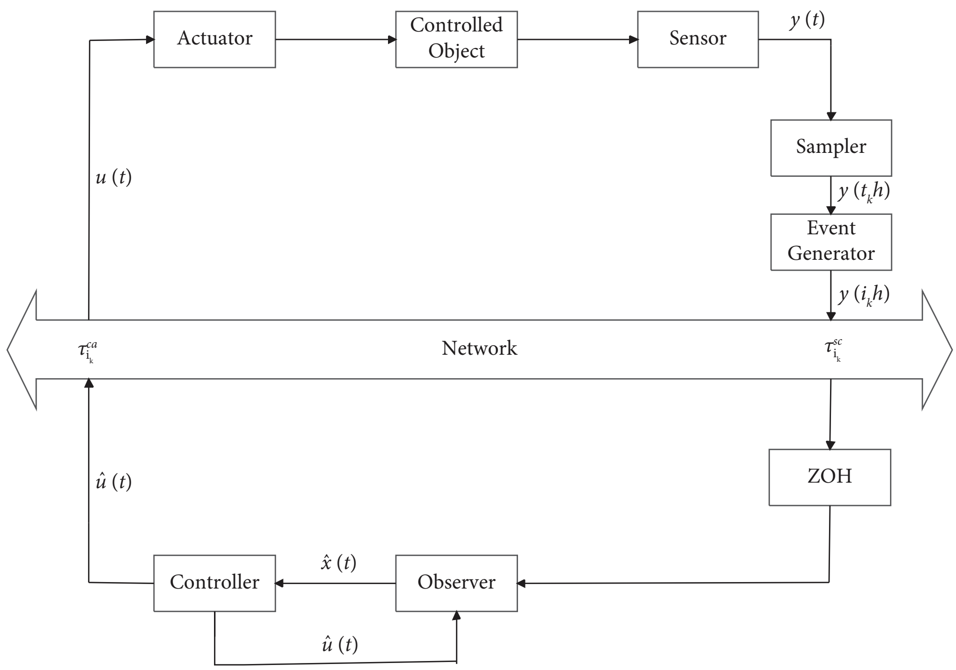

Figure 1: Structure of observer-based NCS under the event-triggered mechanism.

(b) The sensor is time-driven, the controller and actuator are event-driven, and the data is single packet transmission

(c) Time-delay $\tau^{sc}_{i_k}$ and $\tau^{ca}_{i_k}$ are both bounded for any $t \geq 0$, $0 < \tau^{sc}_m \leq \tau^{sc}_{i_k} \leq \tau^{sc}_M$, $0 < \tau^{ca}_m \leq \tau^{ca}_{i_k} \leq \tau^{ca}_M$

The instants when the event generator sends data are noted as $i_0h, i_1h, i_2h, \ldots$, where $i_0h = 0$ is the initial triggering instant. Due to the network time-delay, the instants when those released signals reach the observer are $i_0h + \tau^{sc}_{i_0}, i_1h + \tau^{sc}_{i_1}, i_2h + \tau^{sc}_{i_2}, \ldots$. In addition, the period of the event generator sending data can be expressed as $\lambda_k h = i_{k+1}h - i_kh$.

Based on the above analysis of the system output received at the observer side, we get

$$\tilde{y}(t) = y(i_kh), \quad t \in \left[i_kh + \tau^{sc}_{i_k}, i_{k+1}h + \tau^{sc}_{i_{k+1}}\right). \tag{4}$$

For $t \in [i_kh + \tau^{sc}_{i_k}, i_{k+1}h + \tau^{sc}_{i_{k+1}})$ and $\lambda_k h = i_{k+1}h - i_kh$, two cases are discussed:

(a) If $\lambda_k h \leq h + \tau^{sc}_M - \tau^{sc}_{i_{k+1}}$, we define

$$\rho(t) = t - i_kh, \quad t \in \left[i_kh + \tau^{sc}_{i_k}, i_{k+1}h + \tau^{sc}_{i_{k+1}}\right). \tag{5}$$

We can obtain

$$\tau^{sc}_{i_k} \leq \rho(t) \leq i_{k+1}h - i_kh + \tau^{sc}_{i_{k+1}} \leq h + \tau^{sc}_M. \tag{6}$$

Accordingly, define an error vector as

$$\vartheta_k(t) = 0. \tag{7}$$

(b) If $\lambda_k h > h + \tau^{sc}_M - \tau^{sc}_{i_{k+1}}$, since $0 \leq \tau^{sc}_{i_k} \leq \tau^{sc}_M$, it is easy to prove the existence of a positive integer $\varepsilon \geq 1$ such that

$$\varepsilon h + \tau^{sc}_M - \tau^{sc}_{i_{k+1}} < \lambda_k h \leq (\varepsilon + 1)h + \tau^{sc}_M - \tau^{sc}_{i_{k+1}}. \tag{8}$$

To simplify the expression, let

$$\begin{cases} \bar{\varpi}_0 = \left[i_kh + \tau^{sc}_{i_k}, i_kh + h + \tau^{sc}_M\right), \\ \bar{\varpi}_\delta = \left[i_kh + \delta h + \tau^{sc}_M, i_kh + (\delta + 1)h + \tau^{sc}_M\right), \\ \bar{\varpi}_n = \left[i_kh + \varepsilon h + \tau^{sc}_M, i_{k+1}h + \tau^{sc}_{i_{k+1}}\right). \end{cases} \tag{9}$$

Then the interval $[i_kh + \tau^{sc}_{i_k}, i_{k+1}h + \tau^{sc}_{i_{k+1}})$ can be divided into the following $\varepsilon + 1$ subintervals

$$\left[i_kh + \tau^{sc}_{i_k}, i_{k+1}h + \tau^{sc}_{i_{k+1}}\right) = \bar{\varpi}_0 \cup \left\{\cup_{\delta=1}^{\varepsilon-1} \bar{\varpi}_\delta\right\} \cup \bar{\varpi}_n. \tag{10}$$

Define the segmentation function as

$$\rho(t) = \begin{cases} t - i_kh, & t \in \bar{\varpi}_0, \\ t - i_kh - \delta h, & t \in \bar{\varpi}_\delta, \quad \delta = 1, 2, \ldots, \varepsilon - 1, \\ t - i_kh - \varepsilon h, & t \in \bar{\varpi}_n. \end{cases} \tag{11}$$

From (9) and (11), we can obtain

$$\tau^{sc}_m \leq \rho(t) \leq h + \tau^{sc}_M. \tag{12}$$

At this point, define the error vector

$$\vartheta_k(t) = \begin{cases} 0, & t \in \bar{\varpi}_0, \\ y(i_kh) - y(i_kh + \delta h), & t \in \bar{\varpi}_\delta, \\ y(i_kh) - y(i_kh + \varepsilon h), & t \in \bar{\varpi}_n. \end{cases} \tag{13}$$

From the definition of $\vartheta_k(t)$, the following equation holds for $t \in [i_kh + \tau^{sc}_{i_k}, i_{k+1}h + \tau^{sc}_{i_{k+1}})$

$$y(i_k h) = \vartheta_k(t) + y(t-\rho(t)), \quad (14)$$

$$\vartheta_k^T(t)V\vartheta_k(t) \le \sigma y^T(t-\rho(t))Vy(t-\rho(t)). \quad (15)$$

Construct an observer of the following form on the controller side as

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + L[y(t-\rho(t)) + \vartheta_k(t) - \hat{y}(t-\rho(t))], \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \quad (16)$$

where $\hat{x}(t)$ represents the state of the observer, $\hat{y}(t)$ represents the output of the observer, and L is the gain matrix of the observer to be designed.

The following observer-based feedback control law is used as

$$\hat{u}(t) = K\hat{x}(t), \quad (17)$$

where K is the controller gain matrix.

Remark 3. Due to the time-delay $\tau_{i_k}^{ca}$ in the controller-to-actuator network, the input of the control object is different from the input of the observer, and the following equation holds true:

$$u(t) = \hat{u}\left(t-\tau_{i_k}^{ca}\right) = K\hat{x}\left(t-\tau_{i_k}^{ca}\right). \quad (18)$$

Define $e(t) = x(t) - \hat{x}(t)$, the augmentation vector $z(t) = [\,x^T(t)\ \ e^T(t)\,]^T$, $h_1 = \tau_m^{sc}, h_2 = \tau_M^{sc} + h$, and $h_3 = \tau_m^{ca}$, $h_4 = \tau_M^{ca}$. Based on the above analysis, the closed-loop network control system can be written in the following form:

$$\begin{cases} \dot{z}(t) = (A_1 + \overline{U}\overline{\Xi}(t)\overline{H}_1 + B_1K_1)z(t) - (B_2 + \overline{U}\overline{\Xi}(t)\overline{H}_2)\mathrm{F}K_1 z\left(t-\tau_{i_k}^{ca}\right) + L_1C_1z(t-\rho(t)) + (D_1 + \overline{U}\overline{\Xi}(t)\overline{H}_3)\omega(t) + L_1\vartheta_k(t), \\ z(t) = \phi(t), \quad \forall t \in [t_0 - \max(h_4, h_3, h_2, h_1), t_0 - h_1). \end{cases} \quad (19)$$

where

$$A_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ B \end{bmatrix}, B_2 = \begin{bmatrix} B \\ B \end{bmatrix}, C_1 = [\,0\ \ C\,], D_1 = \begin{bmatrix} D \\ D \end{bmatrix}, K_1 = [\,-K\ \ K\,], L_1 = \begin{bmatrix} 0 \\ -L \end{bmatrix}, \overline{U} = \begin{bmatrix} U & 0 \\ 0 & U \end{bmatrix},$$
$$\overline{\Xi}(t) = \begin{bmatrix} \Xi(t) & 0 \\ 0 & \Xi(t) \end{bmatrix}, \overline{H}_1 = \begin{bmatrix} H_1 & 0 \\ H_1 & 0 \end{bmatrix}, \overline{H}_2 = \begin{bmatrix} H_2 \\ H_2 \end{bmatrix}, \overline{H}_3 = \begin{bmatrix} H_3 \\ H_3 \end{bmatrix}. \quad (20)$$

At this point, condition (2) becomes

$$\omega^T(t)\omega(t) + \omega^T(t)\overline{\Theta}z(t) \le 0, \quad (21)$$

where $\overline{\Theta} = \Theta C[\,I\ \ 0\,]$.

Remark 4. In a real industrial system, the actuator failure indication matrix $F = \mathrm{diag}\{0, 0\}$ means the actuator is completely failed and the controller will not be able to realize the control of the controlled object. Therefore, this paper only considers the case of partial actuator failure, and the actuator can still work normally within a certain range even though it has failed.

3. Main Results

3.1. System Stability Analysis. The following lemmas will be used in the derivation of the expected result.

Lemma 1. *(Jensen's inequality) [24] For any constant matrix $Q \in R^{n\times n}$, $Q = Q^T \ge 0$, scalar $0 \le d_1 \le d_2$, the vector-valued function $\dot{x}$: $[-d_2 - d_1] \longrightarrow R^n$, the following inequality holds true:*

$$-(d_2 - d_1)\int_{t-d_2}^{t-d_1} \dot{x}^T(\alpha)Q\dot{x}(\alpha)d\alpha \le -\begin{bmatrix} x(t-d_1) \\ x(t-d_2) \end{bmatrix}^T \begin{bmatrix} Q & -Q \\ * & Q \end{bmatrix} \begin{bmatrix} x(t-d_1) \\ x(t-d_2) \end{bmatrix}. \quad (22)$$

Lemma 2. *[25] Suppose that $l_1, l_2, \ldots l_N$: $R^m \longrightarrow R$ has positive values in a subset of the open set D, $D \in R^m$, then the mutually inverse convex combination of l_i in the set D satisfies*

$$\min_{\left\{\gamma_i|\gamma_i>0,\sum_i \gamma_i=1\right\}} \sum_i \frac{1}{\gamma_i} l_i(t) = \sum_i l_i(t) + \max_{f_{i,j}(t)} \sum_{i\neq j} f_{i,j}(t), \tag{23}$$

where

$$\left\{ f_{i,j}: R^m \longrightarrow R, f_{j,i}(t) = f_{i,j}(t), \begin{bmatrix} l_i(t) & f_{i,j}(t) \\ f_{j,i}(t) & l_i(t) \end{bmatrix} \geq 0 \right\}. \tag{24}$$

Lemma 3. *[26] Given a symmetric matrix Λ_1 and real matrices of any proper dimensions Λ_2 and Λ_3, for all $\Delta \in \Omega$, there is $\Lambda_1 + \Lambda_2\Delta\Lambda_3 + \Lambda_3^T\Delta\Lambda_2^T < 0$, where $\Omega = \{\Delta = diag\{\Delta_1, \ldots, \Delta_k, p_1 I, \ldots, p_l I\}: \|\Delta\| \leq 1, \quad \Delta_i \in R^{n_i\times n_i}, \quad i = 1, \ldots, k, \ p_j \in R, \ j = 1, \ldots, g, k, g \in Z^+\}$, when and only when there exists $\Gamma \in \chi$ satisfying*

$$\begin{bmatrix} \Lambda_1 + \Lambda_3^T\Gamma\Lambda_3 & \Lambda_2 \\ * & -\Gamma \end{bmatrix} < 0, \tag{25}$$

where $\chi = \left\{diag\left\{s_1 I, \ldots, s_k I, S_1, \ldots S_g\right\}: \quad 0 < s_i \in R, 0 < S_j \in R^{n_j\times n_j}, k, g \in Z^+\right\}$, in particular, when $k = 1, g = 0$, and this inequality $\Lambda_1 + \Lambda_2\Delta_1\Lambda_3 + \Lambda_3^T\Delta_1^T\Lambda_2^T < 0$ is equivalent to $\Lambda_1 + s_1\Lambda_3^T\Lambda_3 + s_1^{-1}\Lambda_2\Lambda_2^T < 0$.

Definition 1. [27] If it is globally asymptotically stable for all $\psi(\cdot) \in \ell[0,\Theta]$, then the Lurie system (18) is absolutely stable within $\ell[0,\Theta]$.

Theorem 1. *Consider the system shown in Figure 1, for given scalars $h_1, h_2, h_3, h_4, \sigma$, actuator failure indication matrix F, and gain matrices K and L, if there exists matrices of proper dimensions M, N and positive definite matrices $W > 0, V > 0, S_i > 0, P_i > 0$, where $i = 1, 2, 3, 4$, then we have*

$$\begin{bmatrix} \aleph_{11} & \aleph_{12} \\ * & \aleph_{22} \end{bmatrix} < 0, \quad \begin{bmatrix} P_2 & M \\ * & P_2 \end{bmatrix} > 0, \quad \begin{bmatrix} P_4 & N \\ * & P_4 \end{bmatrix} > 0, \tag{26}$$

where

$$\aleph_{11} = \begin{bmatrix} \phi_{11} & P_1 & \mathrm{WL}_1C_1 & 0 & \phi_{15} & P_3 & 0 & \mathrm{WL}_1 & \phi_{19} \\ * & \phi_{22} & \phi_{23} & M & 0 & 0 & 0 & 0 & 0 \\ * & * & \phi_{33} & \phi_{34} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & 0 & 0 \\ * & * & * & * & * & \phi_{66} & N & 0 & 0 \\ * & * & * & * & * & * & \phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -V & 0 \\ * & * & * & * & * & * & * & * & -2I \end{bmatrix}, \tag{27}$$

$$\aleph_{12} = \left[h_1\zeta_1^T P_1 \, (h_2 - h_1)\zeta_1^T P_2 h_3\zeta_1^T P_3 \, (h_4 - h_3)\zeta_1^T P_4\right], \aleph_{22} = \mathrm{diag}\{-P_1, -P_2, -P_3, -P_4\},$$

$$\zeta_1 = \left[\,(A_1 + \overline{U\Xi}(t)\overline{H}_1 + B_1K_1)\,0 \;\; L_1C_1 \;\; 0 - (B_2 + \overline{U\Xi}(t)\overline{H}_2)\mathrm{FK}_1\,0 \;\; 0 \;\; L_1\;(D_1 + \overline{U\Xi}(t)\overline{H}_3)\,\right],$$

$$\phi_{11} = W(A_1 + \overline{U\Xi}(t)\overline{H}_1 + B_1K_1) + (A_1 + \overline{U\Xi}(t)\overline{H}_1 + B_1K_1)^T W + \mathrm{WB}_1K_1 + K_1^T B_1^T W + S_1 + S_2 + S_3 + S_4 - P_1 - P_3,$$

$$\phi_{15} = -W(B_2 + \overline{U\Xi}(t)\overline{H}_2)\mathrm{FK}_1, \phi_{19} = W(D_1 + \overline{U\Xi}(t)\overline{H}_3) - \overline{\Theta}^T, \phi_{22} = -S_1 - P_1 - P_2, \phi_{23} = -M + P_2, \overline{C} = [\,C \;\; 0\,],$$

$$\phi_{33} = -2P_2 + M + M^T + \sigma\overline{C}^T V\overline{C}, \phi_{34} = -M + P_2, \phi_{44} = -S_2 - P_2, \phi_{55} = -2P_4 + N + N^T, \phi_{56} = P_4^T - N^T,$$

$$\phi_{57} = P_4 - N, \phi_{66} = -S_3 - P_3 - P_4, \phi_{77} = -S_4 - P_4.$$

Then the Lurie system (19) is robust and absolutely stable within $\ell[0,\Theta]$.

Proof. Construct a Lyapunov–Krasovskii function

$$V(t,z(t)) = V_1(t,z(t)) + V_2(t,z(t)) + V_3(t,z(t)), \tag{28}$$

where

$$\begin{aligned}
V_1(t,z(t)) &= z^T(t)Wz(t),\\
V_2(t,z(t)) &= \int_{t-h_1}^{t} z^T(\alpha)S_1z(\alpha)\mathrm{d}\alpha + \int_{t-h_2}^{t} z^T(\alpha)S_2z(\alpha)\mathrm{d}\alpha + \int_{t-h_3}^{t} z^T(\alpha)S_3z(\alpha)\mathrm{d}\alpha + \int_{t-h_4}^{t} z^T(\alpha)S_4z(\alpha)\mathrm{d}\alpha,\\
V_3(t,z(t)) &= h_1\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot z^T(\alpha)P_1\dot z(\alpha)\mathrm{d}\alpha\mathrm{d}\theta + (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot z^T(\alpha)P_2\dot z(\alpha)\mathrm{d}\alpha\mathrm{d}\theta + h_3\int_{-h_3}^{0}\int_{t+\theta}^{t}\dot z^T(\alpha)P_3\dot z(\alpha)\mathrm{d}\alpha\mathrm{d}\theta\\
&\quad + (h_4-h_3)\int_{-h_4}^{-h_3}\int_{t+\theta}^{t}\dot z^T(\alpha)P_4\dot z(\alpha)\mathrm{d}\alpha\mathrm{d}\theta.
\end{aligned}\tag{29}$$

By taking derivation of $V(t,z(t))$ for t along the system (19), it can be obtained that

$$\dot V(t,z(t)) = \dot V_1(t,z(t)) + \dot V_2(t,z(t)) + \dot V_3(t,z(t)), \tag{30}$$

where

$$\begin{aligned}
\dot V_1(t,z(t)) &= \dot z^T(t)Wz(t) + z^T(t)W\dot z(t) = 2z^T(t)W\dot z(t),\\
\dot V_2(t,z(t)) &= z^T(t)S_1z(t) - z^T(t-h_1)S_1z(t-h_1) + z^T(t)S_2z(t) - z^T(t-h_2)S_2z(t-h_2) + z^T(t)S_3z(t)\\
&\quad - z^T(t-h_3)S_3z(t-h_3) + z^T(t)S_4z(t) - z^T(t-h_4)S_4z(t-h_4),\\
\dot V_3(t,z(t)) &= h_1^2\dot z^T(t)P_1\dot z(t) - h_1\int_{t-h_1}^{t}\dot z^T(\alpha)P_1\dot z(\alpha)\mathrm{d}\alpha + (h_2-h_1)^2\dot z^T(t)P_2\dot z(t) - (h_2-h_1)\int_{t-h_2}^{t-h_1}\dot z^T(\alpha)P_2\dot z(\alpha)\mathrm{d}\alpha\\
&\quad + h_3^2\dot z^T(t)P_3\dot z(t) - h_3\int_{t-h_3}^{t}\dot z^T(\alpha)P_3\dot z(\alpha)\mathrm{d}\alpha + (h_4-h_3)^2\dot z^T(t)P_4\dot z(t) - (h_4-h_3)\int_{t-h_4}^{t-h_3}\dot z^T(\alpha)P_4\dot z(\alpha)\mathrm{d}\alpha.
\end{aligned}\tag{31}$$

Applying Lemma 1 and Lemma 2 to the integral term in (29), the following equations hold true:

$$-h_1\int_{t-h_1}^{t}\dot z^T(\alpha)P_1\dot z(\alpha)\mathrm{d}\alpha \le -\varphi_1^T(t)\Pi_1\varphi_1(t), \tag{32}$$

$$\begin{aligned}
-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot z^T(\alpha)P_2\dot z(\alpha)\mathrm{d}\alpha &= -(h_2-h_1)\int_{t-\tau(t)}^{t-h_1}\dot z^T(\alpha)P_2\dot z(\alpha)\mathrm{d}\alpha - (h_2-h_1)\int_{t-h_2}^{t-\tau(t)}\dot z^T(\alpha)P_2\dot z(\alpha)\mathrm{d}\alpha\\
&\le -\varphi_2^T(t)\Pi_2\varphi_2(t),
\end{aligned}\tag{33}$$

$$-h_3\int_{t-h_3}^{t}\dot z^T(\alpha)P_3\dot z(\alpha)\mathrm{d}\alpha \le -\varphi_3^T(t)\Pi_3\varphi_3(t), \tag{34}$$

$$\begin{aligned}
-(h_4-h_3)\int_{t-h_4}^{t-h_3}\dot z^T(\alpha)P_4\dot z(\alpha)\mathrm{d}\alpha &= -(h_4-h_3)\int_{t-\tau(t)}^{t-h_3}\dot z^T(\alpha)P_4\dot z(\alpha)\mathrm{d}\alpha - (h_4-h_3)\int_{t-h_4}^{t-\tau(t)}\dot z^T(\alpha)P_4\dot z(\alpha)\mathrm{d}\alpha\\
&\le -\varphi_4^T(t)\Pi_4\varphi_4(t),
\end{aligned}\tag{35}$$

where

$$\varphi_1^T(t) = \begin{bmatrix} z(t) \\ z(t-h_1) \end{bmatrix}^T, \varphi_2^T = \begin{bmatrix} z(t-h_1) \\ z(t-\rho(t)) \\ z(t-h_2) \end{bmatrix}^T, \varphi_3^T(t) = \begin{bmatrix} z(t) \\ z(t-h_3) \end{bmatrix}^T, \varphi_4^T = \begin{bmatrix} z(t-h_3) \\ z\left(t-\tau_{i_k}^{ca}\right) \\ z(t-h_4) \end{bmatrix}^T, \Pi_1 = \begin{bmatrix} P_1 & -P_1 \\ * & P_1 \end{bmatrix},$$
$$\Pi_2 = \begin{bmatrix} P_2 & M-P_2 & -M \\ * & 2P_2-M-M^T & M-P_2 \\ * & * & P_2 \end{bmatrix}, \Pi_3 = \begin{bmatrix} P_3 & -P_3 \\ * & P_3 \end{bmatrix}, \Pi_4 = \begin{bmatrix} P_4 & N-P_4 & -N \\ * & 2P_4-N-N^T & N-P_4 \\ * & * & P_4 \end{bmatrix}. \tag{36}$$

Substituting (32), (33), (34), (35) into (29) and combining (15) and (21), we can obtain

$$\begin{aligned} \dot{V}(t,z(t)) \le{}& 2z^T(t)W\dot{z}(t) + z^T(t)S_1z(t) - z^T(t-h_1)S_1z(t-h_1) + z^T(t)S_2z(t) - z^T(t-h_2)S_2z(t-h_2) \\ &+ z^T(t)S_3z(t) - z^T(t-h_3)S_3z(t-h_3) + z^T(t)S_4z(t) - z^T(t-h_4)S_4z(t-h_4) \\ &+ h_1^2\dot{z}^T(t)P_1\dot{z}(t) + (h_2-h_1)^2\dot{z}^T(t)P_2\dot{z}(t) + h_3^2\dot{z}^T(t)P_3\dot{z}(t) + (h_4-h_3)^2\dot{z}^T(t)P_4\dot{z}(t) \\ &+ \sigma y^T(t-\rho(t))Vy(t-\rho(t)) - \vartheta_k^T(t)V\vartheta_k(t) - 2\omega^T(t)\omega(t) - 2\omega^T(t)\overline{\Theta}z(t) \\ \le{}& \eta^T(t)\left(\aleph_{11} - \aleph_{12}\aleph_{22}^{-1}\aleph_{12}^T\right)\eta(t), \end{aligned} \tag{37}$$

where $\eta^T(t) = \left[z^T(t)z^T(t-h_1)z^T(t-\rho(t))z^T(t-h_2)z^T(t-\tau_{i_k}^{ca})z^T(t-h_3)z^T(t-h_4)\vartheta_k^T(t)\omega^T(t)\right]$.

From Schur's complement lemma and Definition 1, if (36) is less than 0, it is known that the Lurie system (19) is globally asymptotically stable for any $\ell[0,\Theta]$ and therefore is absolutely stable. □

3.2. Joint Design of the Observer and the Controller under the Event-Triggering Mechanism

Theorem 2. *Consider the system shown in Figure 1, for given scalars $h_1, h_2, h_3, h_4, \sigma$, actuator fault indication matrix F, and gain matrices K and L, if there exist matrices of suitable dimensions $\overline{M}, \overline{N}, Y_1, Y_2$ and positive definite matrices $X>0, V>0, Y>0, \overline{S}_i>0, \overline{P}_i>0,\quad i=1,2,3,4,$ then we have*

$$\begin{bmatrix} \widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} \\ * & \widetilde{\Lambda}_{22} \end{bmatrix} < 0, \tag{38}$$

$$\begin{bmatrix} P_2 & M \\ * & P_2 \end{bmatrix} > 0, \begin{bmatrix} P_4 & N \\ * & P_4 \end{bmatrix} > 0, \tag{39}$$

where

$$\tilde{\Lambda}_{11} = \begin{bmatrix} \tilde{\phi}_{11} & \overline{P}_1 & -I_2\overline{Y}_2I_2 & 0 & \tilde{\phi}_{15} & \overline{P}_3 & 0 & -I_2\overline{Y}_2C & \tilde{\phi}_{19} \\ * & \tilde{\phi}_{22} & \tilde{\phi}_{23} & \overline{M} & 0 & 0 & 0 & 0 & 0 \\ * & * & \tilde{\phi}_{33} & \tilde{\phi}_{34} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\phi}_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\phi}_{55} & \tilde{\phi}_{56} & \tilde{\phi}_{57} & 0 & 0 \\ * & * & * & * & * & \tilde{\phi}_{66} & \overline{N} & 0 & 0 \\ * & * & * & * & * & * & \tilde{\phi}_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\mathrm{CYC}^T & 0 \\ * & * & * & * & * & * & * & * & -2I \end{bmatrix},$$

$$\begin{aligned} &\tilde{\Lambda}_{12} = \left[h_1\tilde{\zeta}^T \ (h_2-h_1)\tilde{\zeta}^T \ h_3\tilde{\zeta}^T \ (h_4-h_3)\tilde{\zeta}^T\tilde{\xi}_1^T\tilde{\xi}_2^T\right], \tilde{\Lambda}_{22} = \operatorname{diag}\left\{\overline{P}_1 - 2\overline{X}, \overline{P}_2 - 2\overline{X}, \overline{P}_3 - 2\overline{X}, \overline{P}_4 - 2\overline{X}, -sI, -s^{-1}I\right\}, \\ &\tilde{\zeta} = \left[\, A_1\overline{X} + B_1\overline{Y}_1I_1 \ \ 0 \ \ -I_2^T\overline{Y}_2I_2 \ \ 0 \ \ -B_2F\overline{Y}_1I_1 \ \ 0 \ \ 0 \ \ -I_2^T\overline{Y}_2C^T \ \ D_1 \,\right], \tilde{\xi}_1 = \left[\, \overline{U}^T \ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \ \ \overline{U}^T \ \ \overline{U}^T \ \ \overline{U}^T \ \ \overline{U}^T \,\right], \\ &\tilde{\xi}_2 = \left[\, \overline{XH}_1 \ \ 0\ 0\ 0 \ \ -\overline{H}_2F\overline{Y}_1I_1 \ \ 0\ 0\ 0 \ \ \overline{H}_3 \ \ 0\ 0\ 0\ 0 \,\right], \tilde{\phi}_{11} = A_1\overline{X} + B_1\overline{Y}_1I_1 + \overline{X}A_1^T + I_1^T\overline{Y}_1^TB_1^T + \overline{S}_1 + \overline{S}_2 + \overline{S}_3 + \overline{S}_4 \\ &\qquad - \overline{P}_1 - \overline{P}_3, \\ &\tilde{\phi}_{15} = -B_2F\overline{Y}_1I_1, \tilde{\phi}_{19} = D_1 - \overline{X\Theta}^T, \tilde{\phi}_{22} = -\overline{S}_1 - \overline{P}_1 - \overline{P}_2, \tilde{\phi}_{23} = -\overline{M} + \overline{P}_2, \tilde{\phi}_{33} = -2\overline{P}_2 + \overline{M} + \overline{M}^T + \sigma I_3^TYI_3, \tilde{\phi}_{34} = -\overline{M} + \overline{P}_2, \\ &\tilde{\phi}_{44} = -\overline{S}_2 - \overline{P}_2, \tilde{\phi}_{55} = -2\overline{P}_4 + \overline{N} + \overline{N}^T, \tilde{\phi}_{56} = \overline{P}_4^T - \overline{N}^T, \tilde{\phi}_{57} = \overline{P}_4 - \overline{N}, \tilde{\phi}_{66} = -\overline{S}_3 - \overline{P}_3 - \overline{P}_4, \tilde{\phi}_{77} = -\overline{S}_4 - \overline{P}_4, I_1 = [\, -I \ \ I \,], \\ &I_2 = [\, 0 \ \ I \,], \end{aligned} \tag{40}$$

then the Lurie system (19) is robust and absolutely stable within $\ell[0,\Theta]$, controller gain matrix K, observer gain matrix L, then the event-triggered weight matrix V can be obtained from the following equation

$$K = Y_1X^{-1}, L = Y_2X^{-1}C^{+}, V = C^{+T}X^{-1}\mathrm{Y}\mathrm{X}^{-1}C^{+}. \tag{41}$$

Proof. : From Schur's complement lemma and Lemma 3, $\aleph_{11} - \aleph_{12}\aleph_{22}^{-1}\aleph_{12}^T$ can be transformed into

$$\aleph_{11} - \aleph_{12}\aleph_{22}^{-1}\aleph_{12}^T \le \Lambda_{11} - \Lambda_{12}\Lambda_{22}^{-1}\Lambda_{12}^T, \tag{42}$$

where

Table 1: τ_M^{sc} or τ_M^{ca} under different event-triggering thresholds σ.

σ	0.4	0.3	0.2	0.1	0.05
Given $\tau_M^{sc} = 0.1$, the upper bound of τ_M^{ca}	1.532	1.541	1.557	1.586	1.611
Given $\tau_M^{ca} = 0.1$, the upper bound of τ_M^{sc}	1.546	1.548	1.552	1.561	1.569

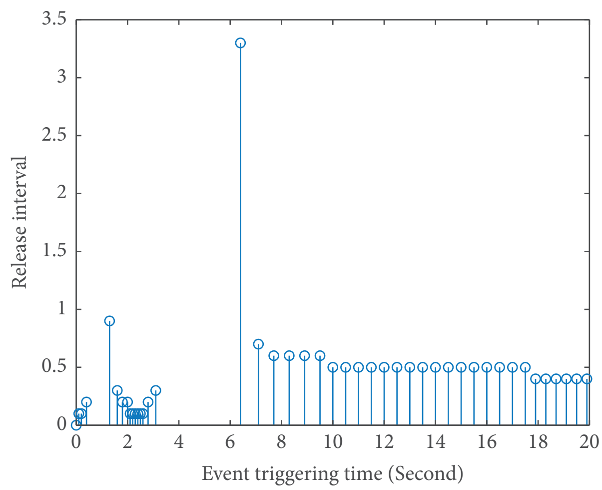

Figure 2: Event-triggering time and intervals when the actuator is normal.

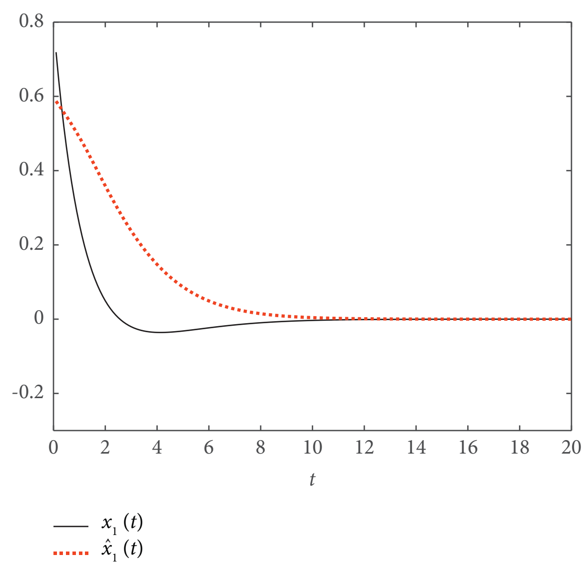

Figure 3: System state when the actuator is normal $x_1(t)$ and its estimated value $\hat{x}_1(t)$.

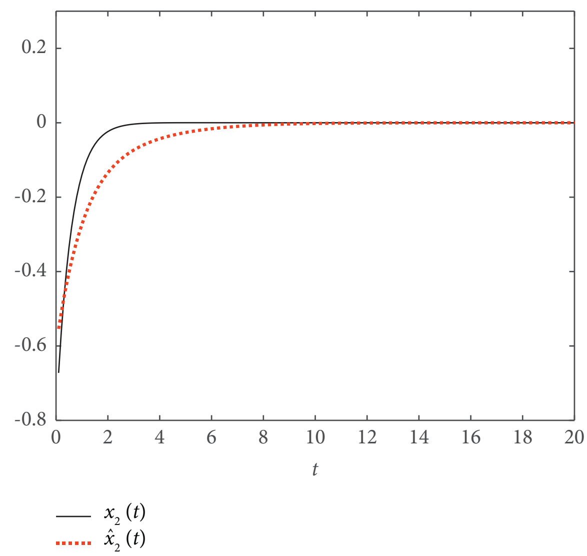

Figure 4: System state when the actuator is normal $x_2(t)$ and its estimated value $\hat{x}_2(t)$.

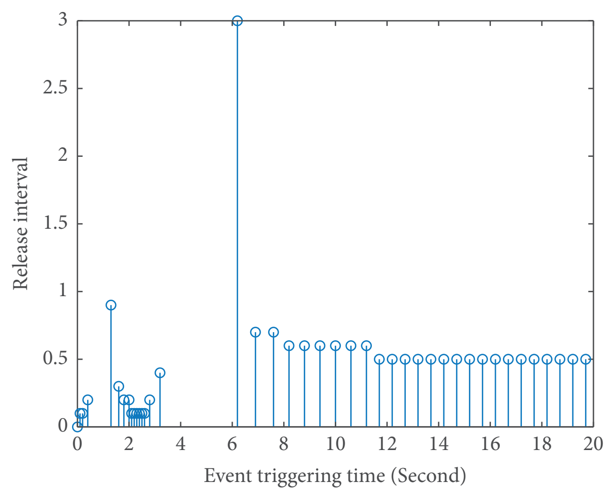

Figure 5: Event-triggering time and intervals in case of $F = \text{diag}\{1, 0\}$.

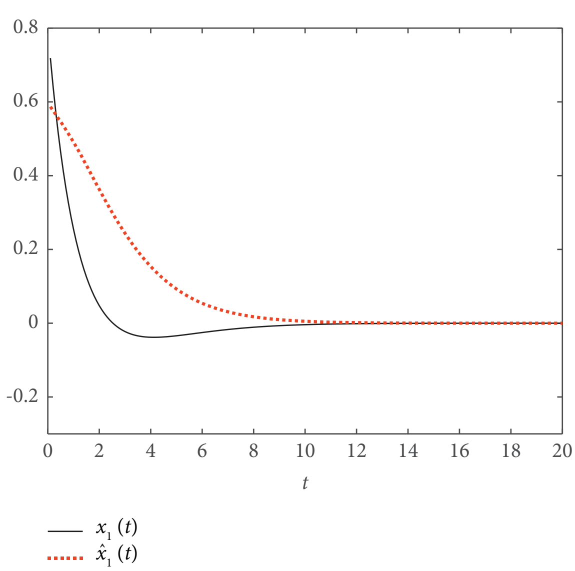

Figure 6: System state $x_1(t)$ and its estimated value $\hat{x}_1(t)$ in case of $F = \text{diag}\{1, 0\}$.

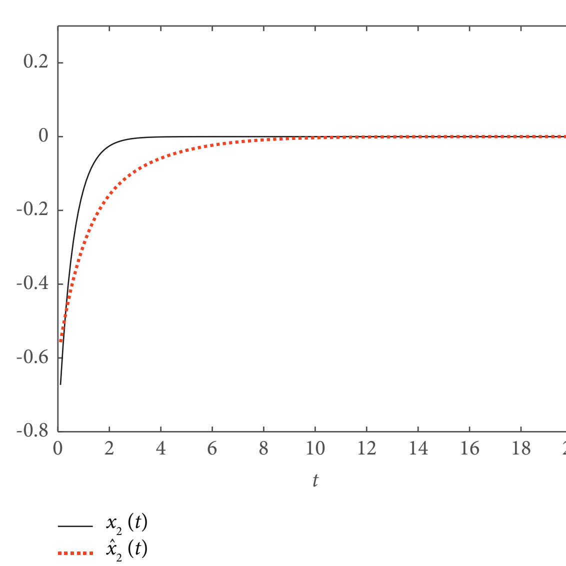

Figure 7: System state $x_2(t)$ and its estimated value $\hat{x}_2(t)$ in case of $F = \text{diag}\{1, 0\}$.

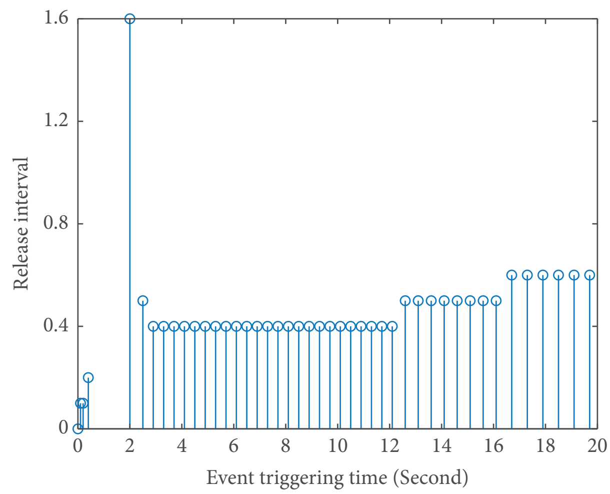

FIGURE 8: Event-triggering time and intervals in case of $F = \text{diag}\{0, 1\}$.

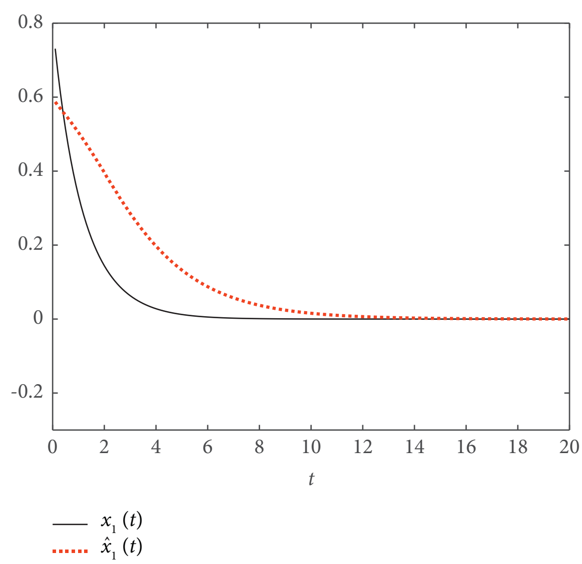

FIGURE 9: System state $x_1(t)$ and its estimated value $\hat{x}_1(t)$ in case of $F = \text{diag}\{0, 1\}$.

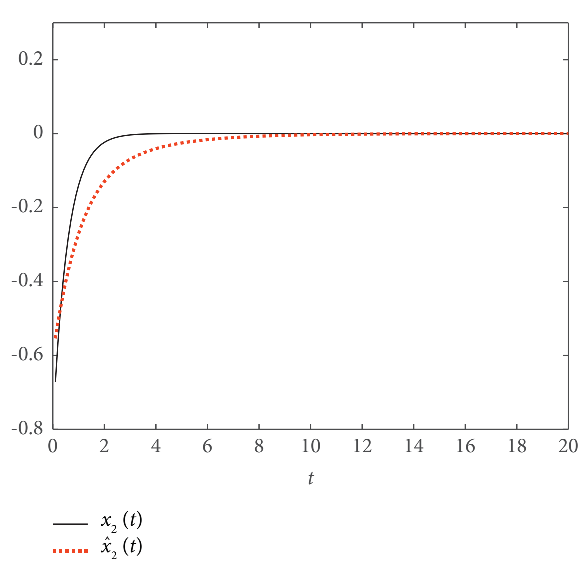

FIGURE 10: System state $x_2(t)$ and its estimated value $\hat{x}_2(t)$ in case of $F = \text{diag}\{0, 1\}$.

$$\Lambda_{11}=\begin{bmatrix}\overline{\phi}_{11} & P_1 & WL_1C_1 & 0 & \overline{\phi}_{15} & P_3 & 0 & WL_1 & \overline{\phi}_{19}\\ * & \phi_{22} & \phi_{23} & M & 0 & 0 & 0 & 0 & 0\\ * & * & \phi_{33} & \phi_{34} & 0 & 0 & 0 & 0 & 0\\ * & * & * & \phi_{44} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & 0 & 0\\ * & * & * & * & * & \phi_{66} & n & 0 & 0\\ * & * & * & * & * & * & \phi_{77} & 0 & 0\\ * & * & * & * & * & * & * & -V & 0\\ * & * & * & * & * & * & * & * & -2I\end{bmatrix}$$

$$\begin{aligned}
\Lambda_{12} &= \left[h_1\overline{\zeta}^T P_1\ (h_2-h_1)\overline{\zeta}^T P_2\ h_3\overline{\zeta}^T P_3\ (h_4-h_3)\overline{\zeta}^T P_4 \xi_1^T \xi_2^T\right], \Lambda_{22}=\mathrm{diag}\left\{-P_1,-P_2,-P_3,-P_4,-sI,-s^{-1}I\right\},\\
\xi_1^T &= \left[\ (W\overline{U})^T\ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \ (P_1\overline{U})^T\ \ (P_2\overline{U})^T\ \ (R_3\overline{U})^T\ \ (P_4\overline{U})^T\ \right]^T,\\
\xi_2^T &= \left[\ \overline{H}_1\ \ 0\ 0\ 0\ -\overline{H}_2FK_1\ \ 0\ 0\ 0\ \ \overline{H}_3\ \ 0\ 0\ 0\ 0\right]^T,\\
\overline{\zeta} &= \left[\ A_1+B_1K_1\ \ 0\ \ L_1C_1\ \ 0\ -B_2FK_1\ \ 0\ \ 0\ \ L_1\ \ D_1\ \right], \overline{\phi}_{11}=WA_1+WB_1K_1+A_1^TW+K_1^TB_1^TW+S_1+S_2+S_3\\
&\quad +S_4-P_1-P_3,\\
\overline{\phi}_{15} &= -WB_2FK_1, \overline{\phi}_{19}=WD_1-\overline{\Theta}^T, \phi_{22}=-S_1-P_1-P_2, \phi_{23}=-M+P_2, \phi_{33}=-2P_2+M+M^T+\sigma\overline{C}^TV\overline{C}, \phi_{34}=-M+P_2,\\
\phi_{44} &= -S_2-P_2, \phi_{55}=-2P_4+N+N^T, \phi_{56}=P_4^T-N^T, \phi_{57}=P_4-N, \phi_{66}=-S_3-P_3-P_4, \phi_{77}=-S_4-P_4, \overline{C}=\left[\ C\ \ 0\ \right].
\end{aligned}\tag{43}$$

Due to the presence of nonlinear terms in $\Lambda_{11}-\Lambda_{12}\Lambda_{22}^{-1}\Lambda_{12}^T$, it is inconvenient to solve the controller, so take $W=\mathrm{diag}\{\overline{W},\overline{W}\}$, and define $X=\overline{W}^{-1}$, $\overline{X}=W^{-1}=\mathrm{diag}\{X, X\}$, $Y_1=KX$, $Y_2=LCX$, $Y=XC^TVCX$, $\overline{Y}_1=\left[\ Y_1\ \ -Y_1\ \right]$, $\overline{Y}_2=\begin{bmatrix}0 & 0\\ 0 & -Y_2\end{bmatrix}$, $\overline{M}=\overline{X}M\overline{X}$, $\overline{N}=\overline{X}N\overline{X}$, $\overline{P}_i=\overline{X}P_i\overline{X}$, $\overline{S}_i=\overline{X}S_i\overline{X}$, $i=1,2,3,4$. Pre and postmultiply $\mathrm{diag}\{\overline{X},\overline{X},\overline{X},\overline{X},\overline{X},\overline{X},\overline{X},CXC^T,I,P_1^{-1},P_2^{-1},P_3^{-1},P_4^{-1},I,I\}$ on both sides of the matrix $\begin{bmatrix}\Lambda_{11} & \Lambda_{12}\\ * & \Lambda_{22}\end{bmatrix}$, noting that since $\overline{P}_1>0,\overline{P}_2>0$, $\overline{P}_3>0,\overline{P}_4>0$, there must be

$$\begin{cases}(\overline{P}_1-\overline{X})\overline{P}_1^{-1}(\overline{P}_1-\overline{X})\geq 0,\\ (\overline{P}_2-\overline{X})\overline{P}_2^{-1}(\overline{P}_2-\overline{X})\geq 0,\\ (\overline{P}_3-\overline{X})\overline{P}_3^{-1}(\overline{P}_3-\overline{X})\geq 0,\\ (\overline{P}_4-\overline{X})\overline{P}_4^{-1}(\overline{P}_4-\overline{X})\geq 0.\end{cases}\tag{44}$$

Equation (43) is equivalent to

$$\begin{cases}-\overline{X}\overline{P}_1^{-1}\overline{X}\leq\overline{P}_1-2\overline{X},\\ -\overline{X}\overline{P}_2^{-1}\overline{X}\leq\overline{P}_2-2\overline{X},\\ -\overline{X}\overline{P}_3^{-1}\overline{X}\leq\overline{P}_3-2\overline{X},\\ -\overline{X}\overline{P}_4^{-1}\overline{X}\leq\overline{P}_4-2\overline{X}.\end{cases}\tag{45}$$

That is, we can get the linear matrix inequality (38), where $K=Y_1X^{-1}$, $L=Y_2X^{-1}C^+$ and $V=C^{+T}X^{-1}YX^{-1}C^+$. □

Remark 5. Since the matrix C is a row full rank matrix of $m\times n$, the matrix C^+ of $n\times m$ exists such that $CC^+=I$, C^+ is called the right inverse of C, and the observer gain matrix $L=Y_2X^{-1}C^+$ is obtained from $Y_2=LCX$.

Remark 6. Since $X>0, V>0, CX\neq 0, C$ is a row full rank matrix, there must exist $(CX)^TV(CX)>0$; $Y=XC^TVCX$ is a nonsingular matrix, which yields the event-triggered weight matrix $V=C^{+T}X^{-1}YX^{-1}C^+$.

Remark 7. The upper and lower bounds of time-delay are introduced in the proof, which makes the conclusion less conservative. In addition to the decision variables required for constructing Lyapunov function, no other free-weighting-matrix is introduced, which avoids the computational burden caused by too many decision variables and the possible conservatism caused by the optimization of too many decision variables. A modified Jensen's inequality is used in the paper. This inequality has a tighter integration bound, which reduces the computational complexity while reducing the conservatism of the conclusion.

4. Simulation Example

To verify the effectiveness and usability of the method, consider the following Lurie system, where

$$A=\begin{bmatrix}0.1 & 0.1\\ 0 & -0.9\end{bmatrix}, B=\begin{bmatrix}1.2 & 0\\ 0 & 0.1\end{bmatrix}, C=\begin{bmatrix}0.1 & 0.1\end{bmatrix}, D=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}, U=\begin{bmatrix}0.2 & 0\\ 0 & 0\end{bmatrix}, H_1=\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix}, H_2=\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix},$$
$$H_3=\begin{bmatrix}0\\ 0.1\end{bmatrix}, \Xi(t)=\begin{bmatrix}\sin t & 0\\ 0 & \cos t\end{bmatrix}, \psi(\cdot)\in \ell[0,1]. \quad (46)$$

Obviously this system is not stable.

According to Theorem 2, when the other parameters are chosen constant, the upper bound of the allowed time-delay of the system varies with the event-triggered threshold σ as shown in Table 1.

As can be seen from Table 1, the smaller the event-triggered threshold σ is, the larger the upper bound of the allowed time-delay is. Therefore, in order to mitigate the impact of time-delay of the system performance, the event threshold σ can be appropriately decreased.

Assume that an event-triggering threshold is set as $\sigma = 0.1$, a system sampling period is given as $h = 0.01s$, and the upper and lower bounds of the time-delay are $h_1 = 0.01, h_2 = 0.11, h_3 = 0.01, h_4 = 0.11$. The nonlinear function $\psi(\cdot)$ is taken as the saturation function $\mathrm{sat}(\cdot)$, and the initial state of the system $x(0) = \begin{bmatrix}-0.8 & 0.8\end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix}-0.6 & 0.6\end{bmatrix}^T$.

According to Theorem 2, given the fault indication matrix $F=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, the controller gain matrix, observer gain matrix, and event-triggered weight matrix for normal operation of the system are as follows:

$$K=\begin{bmatrix}-0.1945 & -0.0245\\ -0.0431 & -0.1645\end{bmatrix}, L=\begin{bmatrix}4.1849\\ 0.8596\end{bmatrix}, V=4.6857. \quad (47)$$

The triggering instants are shown in Figure 2, the state and its estimated value curves of the system are shown in Figures 3–4.

Given the fault indication matrix $F=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$, the controller gain matrix, observer gain matrix, and event-triggered weight matrix for a system actuator failure are as follows:

$$K=\begin{bmatrix}-0.1973 & -0.0185\\ 0.1882 & -0.7919\end{bmatrix}, L=\begin{bmatrix}4.1656\\ 0.8093\end{bmatrix}, V=4.6690. \quad (48)$$

The triggering instants are shown in Figure 5, the state and its estimated value curves of the system are shown in Figures 6–7.

Given the fault indication matrix $F=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$, the controller gain matrix, observer gain matrix, and event-triggered weight matrix for a system actuator failure are as follows:

$$K=\begin{bmatrix}-0.1942 & -0.0246\\ -0.0426 & -0.1639\end{bmatrix}, L=\begin{bmatrix}4.1673\\ 0.8359\end{bmatrix}, V=4.5781. \quad (49)$$

The triggering instants are shown in Figure 8, and the state and its estimated value curves of the system are shown in Figures 9–10.

As can be seen from Figures 2, 5, and 8, the system requires much less data to be transmitted near the equilibrium point than during the transient process, which is exactly in line with the expectation of transmitting data according to the control demand and can effectively save network communication resources, reduce waste, and save resources compared to the time-triggered mechanism. As can be seen from Figures 6, 7, 9, and 10, the system remains stable with effective fault-tolerant control in the event of an actuator failure.

5. Conclusion

In this paper, a robust fault-tolerant controller is designed for the Lurie NCS with time-delay and uncertainty from the sensor to the controller and from the controller to the actuator, which makes the system fault-tolerant effective and keeps the system stable in case of an actuator failure. By constructing an observer on the controller node, a mathematical model of the observer-based event-triggered mechanism is established to ensure the stability of the closed-loop system while also reducing the amount of data transmission in the network, which in turn saves network bandwidth resources. According to Lyapunov stability theory, sufficient conditions for the stability of the closed-loop system are derived, and the collaborative design of the event-triggered mechanism, fault-tolerant controller, and the observer is realized. In future research, the focus will be on more complex systems based on event-triggered mechanisms, such as T-S fuzzy systems that simultaneously consider the effects of packet loss and external disturbances.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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