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Research Article

Existence of α_L-Fuzzy Fixed Points of L-Fuzzy Mappings

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In this research article, fixed point theory is beautifully combined with fuzzy set theory. Two fuzzy fixed point theorems of L-fuzzy mappings are established and proved for two different contractive type conditions in the scenario of complete b-metric space. In order to give the strength of these results, nontrivial supportive examples for both results are also provided. The notion of L-fuzzy mappings is a generalized form of fuzzy mappings as well as multivalued mappings. In this approach, our results provide uniqueness, extension, and successive generalizations of many valuable recent and conventional results existing in the literature.

1. Introduction and Preliminaries

Responding to physical problems becomes naiver with the beginning of FS theory which was introduced in 1965 by Zadeh [1], as it benefits in manufacturing the version of fuzziness and flaws stronger and more definite. Now, it is a well-accepted system to grasp confusions originating in different materialistic situations. In 1967, Goguen [2] expanded this idea into the L-FS theory by replacing the interval with a complete distributive lattice L. The concept of a FS is a special case of an L-FS when $L = [0, 1]$. Then, many results were accomplished by various authors for L-FM. Because the notion of distance function plays an energetic part in approximation theory; therefore, FSs and L-FSs have further been practiced in the classical idea of MSs. The Hausdorff distance for α-cut sets of L-FMs was made known by Rashid et al. to study FP theorems for L-FMs. In 1989, Bakhtin [3] introduced the concept of b-MS. In 1993, Czerwik [4] obtained the results of b-MS. By accepting this idea, many researchers gave generalizations of the Banach contractive principle in b-MS. Boriceanu [5], Bota et al. [6] Czerwik [4], Kir and Kiziltunc [7], Kumam et al. [8], and Pacurar [9] obtained the FP theorems in b-MSs. Afterward, many authors derived and calculated the existence of FP of mappings, satisfying a contractive type condition, for example, Abbas et at. [10] obtained fuzzy common FPs for generalized mappings, Ahmad et al. [11] achieved FPs for locally contractive mappings, Azam et al. [12, 13] established FPs and common FPs for FPs, Estruch and Vidal [14]and Frigon and O'Regan [15] constructed FFPs for FPs, Kanwal and Azam [16] obtained common coincidence points for L-FMs and obtained many useful results for FMs and set-valued mappings as the direct consequences of the main result, Lee and Jin Cho [17] did work in proving the FP theorem for fuzzy contractive-type mappings, Phiangsungnoen and Kumam [18, 19] proved FFP theorems for multivalued fuzzy contractions in b-MSs, and Rashid et al. [20, 21] established L-fuzzy fixed points via beta-L admissible pair and coincidence theorem via α-cuts of L-FMs with applications. Gulzar et al. [22, 23] did work on fuzzy algebra and obtained results in this area.

Definition 1 (see [5]). Let Ω be any non-empty-set and $w \geq 1$ be a real-number. A function $d\colon \Omega \times \Omega \longrightarrow R^+$ is called b-metric, if axioms given below are fulfilled for all $\mu, \nu, \xi \in \Omega$:

(1) $d(\mu, \nu) \geq 0$ and $d(\nu, \mu) = 0$ iff $\mu = \nu$
(2) $d(\mu, \nu) = d(\nu, \mu)$
(3) $d(\mu, \xi) \leq w[d(\mu, \nu) + d(\nu, \xi)]$

Then, (Ω, d) is called **b-MS**.

If we take $w = 1$, then b-MS becomes ordinary MS. Hence, set of all MSs is a subset of set of all b-MSs.

Example 1 (see [5]). The set l_p with $0 < p < 1$, where $l_p = \{\{x_n\} \subset \mathbb{R}: \sum_{\iota=1}^{\infty} |\mu_\iota|^p < \infty\}$, together with the function $d: l_p \times l_p \longrightarrow [0, \infty)$,

$$d(\mu, \nu) = \left(\sum_{\iota=1}^{\infty} |\mu_\iota - \nu_\iota|^p\right)^{1/p}, \tag{1}$$

where $\mu = \{\mu_\iota\}$, $\nu = \{\nu_\iota\} \in l_p$ is b metric space with $w = 2^{1/p} > 1$.) Notice that the abovementioned result holds with $0 < p < 1$.

Definition 2 (see [5]). Let (Ω, d) be b-MS and $\{z_n\}$ be a sequence in Ω. Then,

(1) $\{z_n\}$ is called a **convergent sequence** if there exist $z \in \Omega$, such that for all $\varepsilon > 0$ $\exists$ $n_0(\varepsilon) \in \mathbb{N}$ such that for all $n \geq n_0(\varepsilon)$, we have $d(z_n, z) < \varepsilon$. Then, we write $\lim_{n \longrightarrow \infty} z_n = z$.

(2) $\{z_n\}$ is said to be a **Cauchy sequence** if for all $\varepsilon > 0$ $\exists$ $n_0(\varepsilon) \in \mathbb{N}$ such that for each $n \geq n_0(\varepsilon)$, we have $d(z_n, z) < \varepsilon$.

(3) Ω is called **complete** if every Cauchy sequence in Ω is convergent in Ω.

Definition 3 (see [21]). Suppose (Ω, d) be a b-MS, $\mathrm{CB}(\Omega)$ be the set of non-empty closed and bounded subsets of Ω, and $\mathrm{CL}(\Omega)$ be the set of all non-empty closed subsets of Ω. For $z \in \Omega$ and $A, B \in \mathrm{CL}(\Omega)$, we define,

$$\begin{aligned} d(z, A) &= \inf_{a \in A} d(z, a), \\ d(A, B) &= \inf\{d(a, B): a \in A\}. \end{aligned} \tag{2}$$

Let (Ω, d) be a b-MS. Hausdorff b-metric can be defined on $\mathrm{CB}(\Omega)$ induced by d as

$$H(A, B) = \max\left\{\sup_{u \in A} d(u, B), \sup_{v \in B} d(A, v)\right\}, \tag{3}$$

for all $A, B \in \mathrm{CB}(\Omega)$.

Lemma 1 (see [16]). *Let (Ω, d) be a b-MS and A, $B \in CB(\Omega)$,*

(i) If $a \in A$, then $d(a, B) \leq H(A, B)$

(ii) For $A, B \in CB(\Omega)$ and $0 < \delta \in R$. Then, for $a \in A$ there exists $b \in B$ such that

$$d(a, b) \leq H(A, B) + \delta \tag{4}$$

Definition 4 (see [16]). A **partially ordered set** (**poset**) is a set $\mathcal{X}$ with binary relation $\prec$ such that for all $a, b, c \in \mathcal{X}$;

(1) $a \prec a$ (reflexive)

(2) $a \prec b$ and $b \prec a$ implies $a = b$ (antisymmetric)

(3) $a \prec b$ and $b \prec c$ implies $a \prec c$ (transitivity)

Definition 5 (see [16]). A poset $(L, \prec_L)$ is said to be a

(1) **Lattice**; if $r \wedge s \in L$, $r \vee s \in L$ for any $r, s \in L$.

(2) **Complete lattice;** if $\vee B \in L, \wedge B \in L$ for any $B \subseteq L$.

(3) **Distributive lattice;** if $r \vee (s \wedge t) = (r \vee s) \wedge (r \vee t)$, $r \wedge (s \vee t) = (r \wedge s) \vee (r \wedge t)$ for any $r, s, t \in L$.

(4) **Complete distributive lattice;** if $r \vee (\wedge_i s_i) = \wedge_i (r \vee s_i) r \wedge (\vee_i s_i) = \vee_i (r \wedge s_i)$ for any $r_i, s_i \in L$.

(5) **Bounded lattice;** if it is a lattice along with a maximal element 1_L and a minimal element 0_L, which satisfy $0_L \prec_L x \prec_L 1_L$ for every $x \in L$.

Definition 6 (see [1]). A function $W: \Omega \longrightarrow [0, 1]$ is known as FS on a nonempty set Ω

Definition 7 (see [2]). An L-FS W on a nonempty set Ω is a function $W: \Omega \longrightarrow L$, where L is a bounded complete distributive lattice along with 1_L and 0_L.

Remark 1. The class of L-FSs is larger than the class of FSs.

Definition 8 (see [16]). The γ_L-level set of an L-FS W is defined as

$$\begin{aligned} [W]_{\gamma_L} &= \left\{m \in \Omega: \ \gamma_L \prec_L W(m), \text{for } \gamma_L \in \frac{L}{\{0_L\}}\right\}, \\ [W]_{0_L} &= \overline{\{m \in \Omega: 0_L \prec_L W(m)\}}, \end{aligned} \tag{5}$$

where $\overline{D}$ is the closure of the set D (crisp set).

Let $F_L(\Omega)$ be the collection of all L-FSs in Ω.

Definition 9 (see [21]). Let Ω_1 be any set, Ω_2 be a MS. A mapping T is called L-FM, if

$T: \Omega_1 \longrightarrow F_L(\Omega_2)$. An L-FM T is an L-FS on $\Omega_1 \times \Omega_2$ with membership function $T(x)(y)$ The image $T(x)(y)$ is the grade of membership of y in $T(x)$.

Definition 10 (see [16]). Let (Ω, d) be b-MS and $T: \Omega \longrightarrow F_L(\Omega)$ be an L-FM. A point $z \in \Omega$ is the α_L-FFP of T if $z \in [Tz]_{\alpha_L}$ for some $\alpha_L \in L \backslash \{0_L\}$.

Now, for $x \in \Omega, A, B \in F_L(\Omega), \alpha_L \in L \backslash \{0_L\}$ and $[A]_{\alpha_L}, [B]_{\alpha_L} \in \mathrm{CB}(\Omega)$ we define $d(x, S) = \inf \{d(x, a); a \in S\}$; here, S is a subset of Ω:

$$
\begin{aligned}
&p_{\alpha_L}(x, A) = \inf\{d(x,a).; a. \in [A]_{\alpha_L}\},\\
&p_{\alpha_L}(A, B) = \inf\{d(a,b).; a \in [A]_{\alpha_L}, b. \in [B]_{\alpha_L}\},\\
&p(A, B) = \sup_{\alpha_L} p_{\alpha_L}(A, B),\\
&H\left([A]_{\alpha_L}, [B]_{\alpha_L}\right) = \max\left\{\sup_{a\in[A]_{\alpha_L}} d\left(a, [B]_{\alpha_L}\right), \sup_{b\in[B]_{\alpha_L}} d\left(b, [A]_{\alpha_L}\right)\right\}.
\end{aligned} \tag{6}
$$

Remark 2. The function H: $\mathrm{CB}(\Omega)\times\mathrm{CB}(\Omega)\longrightarrow\mathbb{R}$ is a Hausdorff b-metric, where Ω is a b-MS and $\mathrm{CB}(\Omega)$ is the set of all closed and bounded subsets of Ω.

Definition 11 (see [16]). Let Ψ_b be the class of strictly increasing functions, ψ: $[0,\infty)\longrightarrow[0,\infty)$ such that $\sum_{n=0}^{\infty} s^n\psi^n(t) < +\infty$ for each $t>0$, where ψ^n is the nth iterate of ψ. It is known that for each $\psi\in\Psi_b$, we have $\psi(t)<t$ for all $t>0$ and $\psi(0)=0$ for $t=0$.

2. α_L-Fuzzy Fixed Points

In this section, we have obtained two different results to find α_L-fuzzy fixed points (FFP) in complete b-metric spaces (MS) and established significant examples to validate our results.

Theorem 1. *Let (Ω, d) be a complete b-MS with constant $b \geq 1$. Let $T:\Omega\longrightarrow F_L(\Omega)$ be an L-FM and for $x, y\in\Omega, \exists\, \alpha_{L(x)}, \alpha_{L(y)}\in L\backslash\{0_L\}$ such that $[Tx]_{\alpha_{L(x)}}$ and $[Ty]_{\alpha_{L(y)}}$ non – empty and belong to $CB(\Omega)$ satisfying the following condition:*

$$
\begin{aligned}
H\left([Tx]_{\alpha_L(x)}, [Ty]_{\alpha_L(y)}\right) \leq{}& a_1 d\left(x, [Tx]_{\alpha_L(x)}\right) + a_2 d\left(y, [Ty]_{\alpha_{L(y)}}\right) + a_3 d\left(x, [Ty]_{\alpha_{L(y)}}\right)\\
&+ a_4 d\left(y, [Tx]_{\alpha_L(x)}\right) + a_5 d(x,y) + a_6\frac{d\left(x, [Tx]_{\alpha_L(x)}\right)\left(1+d\left(x, [Tx]_{\alpha_L(x)}\right)\right)}{1+d(x,y)}.
\end{aligned} \tag{7}
$$

Also, $a_i\geq 0$, where $i=1,2,3,\ldots,6$ with $a_1+a_2+2ba_3+a_5+a_6<1$, and $\sum_{i=1}^{6} a_i<1$. Then, T has an α_L-FFP.

Proof. Let x_o be an arbitrary point in Ω, since $[Tx_o]_{\alpha_{L(x_0)}}$ is nonempty, so there exists $x_1\in[Tx_o]_{\alpha_{L(x_0)}}$ and $x_2\in[Tx_1]_{\alpha_{L(x_1)}}$ and so on.

Because $[Tx_o]_{\alpha_{L(x_0)}}$ and $[Tx_1]_{\alpha_{L(x_1)}}$ are closed and bounded subsets of Ω.

By Lemma 1,

$$
\begin{aligned}
d(x_1,x_2) &\leq H\left([Tx_o]_{\alpha_{L(x_0)}}, [Tx_1]_{\alpha_{L(x_1)}}\right) + (a_1+ba_3+a_5+a_6),\\
d(x_1,x_2) &\leq a_1 d\left(x_o, [Tx_o]_{\alpha_{L(x_0)}}\right) + a_2 d\left(x_1, [Tx_1]_{\alpha_{L(x_1)}}\right) + a_3 d\left(x_o, [Tx_1]_{\alpha_{L(x_1)}}\right) + a_4 d\left(x_1, [Tx_o]_{\alpha_{L(x_0)}}\right)\\
&\quad + a_5 d(x_o,x_1) + a_6\frac{d\left(x_o, [Tx_o]_{\alpha_{L(x_0)}}\right)\left(1+d\left(x_o, [Tx_o]_{\alpha_{L(x_0)}}\right)\right)}{1+d(x_o,x_1)} + (a_1+ba_3+a_5+a_6),\\
d(x_1,x_2) &\leq a_1 d(x_o,x_1) + a_2 d(x_1,x_2) + a_3 d(x_o,x_2) + a_4 d(x_1,x_1) + a_5 d(x_o,x_1)\\
&\quad + a_6\frac{d(x_o,x_1)(1+d(x_o,x_1))}{1+d(x_o,x_1)} + (a_1+ba_3+a_5+a_6),\\
d(x_1,x_2) &\leq a_1 d(x_o,x_1) + a_2 d(x_1,x_2) + a_3 d(x_o,x_2) + a_5 d(x_o,x_1) + a_6 d(x_o,x_1) + (a_1+ba_3+a_5+a_6),\\
d(x_1,x_2) &\leq a_1 d(x_o,x_1) + a_2 d(x_1,x_2) + ba_3 d(x_o,x_1) + ba_3 d(x_1,x_2)\\
&\quad + a_5 d(x_o,x_1) + a_6 d(x_o,x_1) + (a_1+ba_3+a_5+a_6),
\end{aligned}
$$

$$\begin{aligned} d(x_1, x_2) &\le (a_1 + ba_3 + a_5 + a_6) d(x_o, x_1) + (a_2 + ba_3) d(x_1, x_2) + (a_1 + ba_3 + a_5 + a_6), \\ (1 - (a_2 + ba_3)) d(x_1, x_2) &\le (a_1 + ba_3 + a_5 + a_6) d(x_o, x_1) + (a_1 + ba_3 + a_5 + a_6), \\ d(x_1, x_2) &\le \frac{(a_1 + ba_3 + a_5 + a_6)}{(1 - (a_2 + ba_3))} d(x_o, x_1) + \frac{(a_1 + ba_3 + a_5 + a_6)}{(1 - (a_2 + ba_3))}. \end{aligned} \tag{8}$$

Let $(a_1 + ba_3 + a_5 + a_6)/(1 - (a_2 + ba_3)) = \gamma$ So,

$$d(x_1, x_2) \le \gamma \, d(x_o, x_1) + \gamma. \tag{9}$$

Again, since $x_2 \in [Tx_1]_{\alpha_{L(x_1)}}$ and $x_3 \in [Tx_2]_{\alpha_{L(x_2)}}$ are bounded and closed subsets of Ω. So,

By Lemma 1,

$$\begin{aligned} d(x_2, x_3) &\le H\left([Tx_1]_{\alpha_{L(x_1)}}, [Tx_2]_{\alpha_{L(x_2)}} \right) + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))}, \\ d(x_2, x_3) &\le a_1 d\left(x_1, [Tx_1]_{\alpha_{L(x_1)}}\right) + a_2 d\left(x_2, [Tx_2]_{\alpha_{L(x_2)}}\right) + a_3 d\left(x_1, [Tx_2]_{\alpha_{L(x_2)}}\right) + a_4 d\left(x_2, [Tx_1]_{\alpha_{L(x_1)}}\right) \\ &\quad + a_5 d(x_1, x_2) + a_6 \frac{d\left(x_1, [Tx_1]_{\alpha_{L(x_1)}}\right)\left(1 + d\left(x_1, [Tx_1]_{\alpha_{L(x_1)}}\right)\right)}{1 + d(x_1, x_2)} + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))}, \\ d(x_2, x_3) &\le a_1 d(x_1, x_2) + a_2 d(x_2, x_3) + a_3 d(x_1, x_3) + a_4 d(x_2, x_2) + a_5 d(x_1, x_2) \\ &\quad + a_6 \frac{d(x_1, x_2)(1 + d(x_1, x_2))}{1 + d(x_1, x_2)} + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))}, \\ d(x_2, x_3) &\le a_1 d(x_1, x_2) + a_2 d(x_2, x_3) + a_3 d(x_1, x_3) + a_5 d(x_1, x_2) + a_6 d(x_1, x_2) + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))}, \\ d(x_2, x_3) &\le (a_1 + ba_3 + a_5 + a_6) d(x_1, x_2) + (a_2 + ba_3) d(x_2, x_3) + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))}, \\ (1 - (a_2 + ba_3)) d(x_2, x_3) &\le (a_1 + ba_3 + a_5 + a_6) d(x_1, x_2) + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))}, \\ d(x_2, x_3) &\le \frac{(a_1 + ba_3 + a_5 + a_6)}{(1 - (a_2 + ba_3))} d(x_1, x_2) + \frac{(a_1 + ba_3 + a_5 + a_6)^2}{(1 - (a_2 + ba_3))^2}, \\ d(x_2, x_3) &\le \gamma \, d(x_1, x_2) + \gamma^2. \end{aligned} \tag{10}$$

By using inequality (9), we get

$$\begin{aligned} d(x_2, x_3) &\le \gamma \left[\gamma \, d(x_o, x_1) + \gamma \right] + \gamma^2, \\ d(x_2, x_3) &\le \gamma^2 d(x_o, x_1) + \gamma^2 + \gamma^2, \\ d(x_2, x_3) &\le \gamma^2 d(x_o, x_1) + 2\gamma^2. \end{aligned} \tag{11}$$

Continuing in this way by induction, we obtain a sequence $\{x_n\}$, such that $x_{n-1} \in [Tx_n]_{\alpha_L}$ and $x_n \in [Tx_{n+1}]_{\alpha_L}$, we have

$$id(x_n, x_{n+1}) \le \gamma^n d(x_o, x_1) + n\gamma^n. \tag{12}$$

Now, for positive integers m, n, and $(n > m)$, we have

$$\begin{aligned}
&d(x_m, x_n) \le b[d(x_m, x_{m+1}) + d(x_{m+1}, x_{m+2}) + \ldots + d(x_{n-1}, x_n)],\\
&d(x_m, x_n) \le b\left[\gamma^m d(x_o, x_1) + m\gamma^m + \gamma^{m+1} d(x_o, x_1) + (m+1)\gamma + \ldots + \gamma^{n-1} d(x_o, x_1) + (n-1)\gamma^{n-1}\right],\\
&d(x_m, x_n) \le b\left[\left(\gamma^m + \gamma^{m+1} + \ldots + \gamma^{n-1}\right) d(x_o, x_1) + \sum_{i=m}^{n-1} i\gamma^i\right].
\end{aligned} \tag{13}$$

Because $\gamma^m + \gamma^{m+1} + \ldots + \gamma^{n-1}$ is a geometric series with r (*common ratio*) $= \gamma < 1$, it is hence convergent. So, we can write above inequality as follows:

$$d(x_m, x_n) \le b\left[\frac{\gamma^m}{1-\gamma} d(x_o, x_1) + \sum_{i=m}^{n-1} i\gamma^i\right]. \tag{14}$$

As $\gamma < 1$, and for $m, n \longrightarrow \infty$, the series $\sum_{i=m}^{n-1} i\gamma^i$ converges by the Cauchy root test so,

$$d(x_m, x_n) \longrightarrow 0. \tag{15}$$

Hence, $\{x_n\}$ is the Cauchy sequence in Ω. As Ω is complete. So, $\exists z \in \Omega$ such that $x_n \longrightarrow z$ as $n \longrightarrow \infty$.

Now, we show that z is a L-fuzzy fixed point. For this, consider

$$\begin{aligned}
&d\left(z, [Tz]_{\alpha_L}\right) \le b\left[d(z, x_{n+1}) + d\left(x_{n+1}, [Tz]_{\alpha_L}\right)\right],\\
&d\left(z, [Tz]_{\alpha_L}\right) \le b\left[d(z, x_{n+1}) + H\left([Tx_n]_{\alpha_L}, [Tz]_{\alpha_L}\right)\right],\\
&d\left(z, [Tz]_{\alpha_{L(z)}}\right) \le b\Bigg[d(z, x_{n+1}) + a_1 d\left(x_n, [Tx_n]_{\alpha_{L(x_n)}}\right) + a_2 d\left(z, [Tz]_{\alpha_{L(z)}}\right) + a_3 d\left(x_n, [Tz]_{\alpha_{L(z)}}\right) + a_4 d\left(z, [Tx_n]_{\alpha_{L(x_n)}}\right)\\
&\qquad + a_5 d(x_n, z) + a_6 \frac{d\left(x_n, [Tx_n]_{\alpha_{L(x_n)}}\right)\left(1 + d\left(x_n, [Tx_n]_{\alpha_{L(x_n)}}\right)\right)}{1 + d(x_n, z)}\Bigg],\\
&d\left(z, [Tz]_{\alpha_{L(z)}}\right) \le b\Bigg[d(z, x_{n+1}) + a_1 d(x_n, x_{n+1}) + a_2 d\left(z, [Tz]_{\alpha_{L(z)}}\right) + a_3 d\left(x_n, [Tz]_{\alpha_{L(z)}}\right) + a_4 d(z, x_{n+1})\\
&\qquad + a_5 d(x_n, z) + a_6 \frac{d(x_n, x_{n+1})(1 + d(x_n, x_{n+1}))}{1 + d(x_n, z)}\Bigg],\\
&d\left(z, [Tz]_{\alpha_{L(z)}}\right) \le b\Bigg[d(z, x_{n+1}) + a_1 d(x_n, x_{n+1}) + a_2 d\left(z, [Tz]_{\alpha_{L(z)}}\right) + ba_3 d(x_n, z) + ba_3 d\left(z, [Tz]_{\alpha_{L(z)}}\right)\\
&\qquad + a_4 d(z, x_{n+1}) + a_5 d(x_n, z) + a_6 \frac{d(x_n, x_{n+1})(1 + d(x_n, x_{n+1}))}{1 + d(x_n, z)}\Bigg].
\end{aligned} \tag{16}$$

Also $n \longrightarrow \infty$,

$$\begin{aligned}
&d\left(z, [Tz]_{\alpha_{L(z)}}\right) \le b\left[a_2 d\left(z, [Tz]_{\alpha_{L(z)}}\right) + ba_3 d\left(z, [Tz]_{\alpha_{L(z)}}\right)\right],\\
&d\left(z, [Tz]_{\alpha_{L(z)}}\right) \le b(a_2 + ba_3) d\left(z, [Tz]_{\alpha_{L(z)}}\right), (1 - b(a_2 + ba_3))\\
&d\left(z, [Tz]_{\alpha_{L(z)}}\right) \le 0.
\end{aligned} \tag{17}$$

As $1 - b(a_2 + ba_3) \ne 0$. So, only possibility is $d(z, [Tz]_{\alpha_{L(z)}}) = 0$.

Hence,

$z \in [Tz]_{\alpha_{L(z)}}$.

So, z is α_L-FFP of T. □

Example 2. Let $\Omega = [a, a+1]$ where $a \in R$. Define $d\colon \Omega \times \Omega \longrightarrow R^+$ by $d(\omega, \mu) = |\omega - \mu|$. Define L-fuzzy mapping $T\colon \Omega \longrightarrow F_L(\Omega)$ as

$$T(\omega)(t) = \begin{cases} a+1, & a < t \le \dfrac{\omega}{4},\\[2ex] \dfrac{a+1}{2}, & \dfrac{\omega}{4} < t \le \dfrac{\omega}{3},\\[2ex] \dfrac{a+1}{4}, & \dfrac{\omega}{3} < t \le \dfrac{\omega}{2},\\[2ex] a, & \dfrac{\omega}{2} < t \le a+1. \end{cases} \tag{18}$$

For all $\omega, \mu \in \Omega$. Here, $L = [a, a+1]$ where $a \in R$. Moreover, for all $\omega \in \Omega, \exists \alpha_L(\omega) = \alpha_L(\mu) = a+1$ such that $[T\omega]_{\alpha_L} = [a, \omega/4]$. Hence, for $\mu \in \Omega$, $[T\mu]_{\alpha_L} = [a, \mu/4]$.

As

$$H\left([T\omega]_{\alpha_L}, [T\mu]_{\alpha_L}\right) = \left|\frac{\mu}{4} - \frac{\omega}{4}\right|, \tag{19}$$

$$d\left(\omega, [T\omega]_{\alpha_L}\right) = \left|\omega - \frac{\omega}{4}\right|, \tag{20}$$

$$d\left(\mu, [T\mu]_{\alpha_L}\right) = \left|\mu - \frac{\mu}{4}\right|, \tag{21}$$

$$d(\omega, \mu) = |\omega - \mu|, \tag{22}$$

$$d\left(\omega, [T\mu]_{\alpha_L}\right) = \left|\omega - \frac{\mu}{4}\right|, \tag{23}$$

$$d\left(\mu, [T\omega]_{\alpha_L}\right) = \left|\mu - \frac{\omega}{4}\right|. \tag{24}$$

From (19)–(24), we have

$$\begin{aligned} H\left([T\omega]_{\alpha_L}, [T\mu]_{\alpha_L}\right) \leq{} & \frac{1}{5}\left|\omega - \frac{\omega}{4}\right| + \frac{1}{10}\left|\mu - \frac{\mu}{4}\right| + \frac{1}{15}\left|\omega - \frac{\mu}{4}\right| \\ & + \frac{1}{20}\left|\mu - \frac{\omega}{4}\right| + \frac{1}{25}|\omega - \mu| + \frac{1}{30} \\ & \cdot \left(\frac{|\omega - \omega/4|\,(1 + |\omega - \omega/4|)}{1 + |\omega - \mu|}\right). \end{aligned} \tag{25}$$

Hence, all the conditions of the above theorem are satisfied. There exists ab L-fuzzy fixed point in complete b-metric. So, $a \in \Omega$ is an α_L-FFP of T.

Corollary 1. *Let (Ω, d) be a complete b-MS with constant $b \geq 1$. Let $T\colon \Omega \longrightarrow F(\Omega)$ be a FM and for $x, y \in \Omega, \exists\, \alpha(x), \alpha(y))\varepsilon(0,1]$ such that $[Tx]_{\alpha(x)}$ and $[Ty]_{\alpha(y)}$ benon – empty and belongs to $CB(\Omega)$ satisfying the following condition:*

$$\begin{aligned} H\left([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}\right) \leq{} & a_1 d\left(x, [Tx]_{\alpha(x)}\right) + a_2 d\left(y, [Ty]_{\alpha(y)}\right) + a_3 d\left(x, [Ty]_{\alpha(y)}\right) + a_4 d\left(y, [Tx]_{\alpha(x)}\right) \\ & + a_5 d(x, y) + a_6 \frac{d\left(x, [Tx]_{\alpha(x)}\right)\left(1 + d\left(x, [Tx]_{\alpha(x)}\right)\right)}{1 + d(x, y)}. \end{aligned} \tag{26}$$

Also $a_i \geq 0$, where $i = 1, 2, 3, \ldots, 6$ with $a_1 + a_2 + 2ba_3 + a_5 + a_6 < 1$ and $\sum_{i=1}^{6} a_i < 1$. Then, T has a FFP.

Corollary 2. *Let (Ω, d) be a complete MS. Let $T\colon \Omega \longrightarrow F_L(\Omega)$ be an L-FM and for $x, y \in \Omega, \exists \alpha_{L(x)}, \alpha_{L(y)} \epsilon L/\{0_L\}$ such that $[Tx]_{\alpha_L}$ and $[Ty]_{\alpha_L}$ benon – empty and belong to $CB(\Omega)$ satisfying the following condition:*

$$\begin{aligned} H\left([Tx]_{\alpha_L(x)}, [Ty]_{\alpha_L(y)}\right) \leq{} & a_1 d\left(x, [Tx]_{\alpha_L(x)}\right) + a_2 d\left(y, [Ty]_{\alpha_{L(y)}}\right) + a_3 d\left(x, [Ty]_{\alpha_{L(y)}}\right) + a_4 d\left(y, [Tx]_{\alpha_L(x)}\right) \\ & + a_5 d(x, y) + a_6 \frac{d\left(x, [Tx]_{\alpha_L(x)}\right)\left(1 + d\left(x, [Tx]_{\alpha_L(x)}\right)\right)}{1 + d(x, y)}. \end{aligned} \tag{27}$$

Also $a_i \geq 0$, where $i = 1, 2, 3, \ldots, 6$ with $a_1 + a_2 + 2a_3 + a_5 + a_6 < 1$ and $\sum_{i=1}^{6} a_i < 1$. Then, T has an α_L-FFP.

Corollary 3. *Let (Ω, d) be a complete MS. Let $T\colon \Omega \longrightarrow F(\Omega)$ be a FM and for $x, y \in \Omega, \exists \alpha(x)\alpha(y) \in (0,1]$ such that $[Tx]_{\alpha(x)}$ and $[Ty]_{\alpha(y)}$benon – empty and belongs to $CB(\Omega)$ satisfying the following condition:*

$$\begin{aligned} H\left([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}\right) \leq{} & a_1 d\left(x, [Tx]_{\alpha(x)}\right) + a_2 d\left(y, [Ty]_{\alpha(y)}\right) + a_3 d\left(x, [Ty]_{\alpha(y)}\right) \\ & + a_4 d\left(y, [Tx]_{\alpha(x)}\right) + a_5 d(x, y) + a_6 \frac{d\left(x, [Tx]_{\alpha(x)}\right)\left(1 + d\left(x, [Tx]_{\alpha(x)}\right)\right)}{1 + d(x, y)}. \end{aligned} \tag{28}$$

Also $a_i \geq 0$, where $i = 1, 2, 3, \ldots, 6$ with $a_1 + a_2 + 2a_3 + a_5 + a_6 < 1$ and $\sum_{i=1}^{6} a_i < 1$. Then, T has a FFP.

Theorem 2. *Let (Ω, d) be a complete b-MS with coefficient $s \geq 1$. Let $T\colon \Omega \longrightarrow F_L(\Omega)$ be L-fuzzy mapping and for all*

$x, y \in \Omega$, $\alpha_{L(x)}, \alpha_{L(y)} \epsilon L/\{0_L\}$, $[Tx]_{\alpha_L(x)}$ and $[Ty]_{\alpha(y)}$ be nonempty closed and bounded subsets of Ω. Suppose T satisfies the following multivalued contraction

$$H\Big([Tx]_{\alpha_L(x)}, [Ty]_{\alpha_L(y)}\Big) \leq \psi(d(x, y)) \tag{29}$$

where $\psi \in \Psi_b$. Then, T has an α_L-FFP.

Proof. Let ω_0 be an arbitrary point in Ω. Suppose that there exists $\omega_1 \in [T\omega_0]_{\alpha_L(\omega_0)}$. Because $[T\omega_1]_{\alpha_L(\omega_1)}$ is a nonempty closed and bounded subset of Ω. □

Case 1. If $\omega_0 = \omega_1$, then $\omega_1 = \omega_0 \in [T\omega_0]_{\alpha_L(\omega_0)}$. Hence, ω_0 is the required α_L-FFP of T.

Case 2. If $\omega_1 \in [T\omega_1]_{\alpha_L(\omega_1)}$. Then, ω_1 α_L-FFP of T.

Case 3. Now, we assume that $\omega_0 \neq \omega_1$ and $\omega_1 \in [T\omega_0]_{\alpha_L(\omega_0)}$. so,

$$\begin{aligned}
&0 < d\Big(\omega_1, [T\omega_1]_{\alpha_L(\omega_1)}\Big) \leq H\Big([T\omega_0]_{\alpha_L(\omega_0)}, [T\omega_1]_{\alpha_L(\omega_1)}\Big) \leq \psi(d(\omega_0, \omega_1)),\\
&0 < d\Big(\omega_1, [T\omega_1]_{\alpha_L(\omega_1)}\Big) < \psi(r\ d(\omega_0, \omega_1)), \forall r > 1.
\end{aligned} \tag{30}$$

Because, $[T\omega_1]_{\alpha_L(\omega_1)}$ is a nonempty closed and bounded subset of Ω. Suppose there exists

$\omega_2 \in [T\omega_1]_{\alpha_L(\omega_1)}$ and $\omega_1 \neq \omega_2$ such that

$$0 < d(\omega_1, \omega_2) \leq \psi(d(\omega_0, \omega_1)) \leq \psi(r\ d(\omega_0, \omega_1)). \tag{31}$$

Because $[T\omega_2]_{\alpha_L(\omega_2)}$ is a nonempty closed and bounded subset of Ω. We assume that ω_2 does not belong $[T\omega_2]_{\alpha_L(\omega_2)}$. Then,

$$\begin{aligned}
&0 < d\Big(\omega_2, [T\omega_2]_{\alpha_L(\omega_2)}\Big) \leq H\Big([T\omega_1]_{\alpha_L(\omega_1)}, [T\omega_2]_{\alpha_L(\omega_2)}\Big) \leq \psi(d(\omega_1, \omega_2)),\\
&0 < d\Big(\omega_2, [T\omega_2]_{\alpha_L(\omega_2)}\Big) \leq \psi(d(\omega_1, \omega_2)).
\end{aligned} \tag{32}$$

From (31),

$$\begin{aligned}
&0 < d\Big(\omega_2, [T\omega_2]_{\alpha_L(\omega_2)}\Big) \leq \psi(d(\omega_1, \omega_2)) \leq \psi^2(d(\omega_0, \omega_1)),\\
&0 < d\Big(\omega_2, [T\omega_2]_{\alpha_L(\omega_2)}\Big) \leq \psi(d(\omega_1, \omega_2)) \leq \psi^2(d(\omega_0, \omega_1)) < \psi^2(r\ d(\omega_0, \omega_1)),\\
&0 < d\Big(\omega_2, [T\omega_2]_{\alpha_L(\omega_2)}\Big) < \psi^2(r\ d(\omega_0, \omega_1)).
\end{aligned} \tag{33}$$

Suppose that there exists $\omega_3 \in [T\omega_2]_{\alpha_L(\omega_2)}$ and $\omega_2 \neq \omega_3$ such that

$$0 < d(\omega_2, \omega_3) \leq \psi(d(\omega_1, \omega_2)) < \psi^2(r\ d(\omega_0, \omega_1)). \tag{34}$$

By induction, we can construct a sequence $\{\omega_n\}$ in Ω, such that ω_n does not belongs to $[T\omega_n]_{\alpha_L(\omega_n)}$ and $\omega_{n+1} \in [T\omega_n]_{\alpha_L(\omega_n)}$ and

$$\begin{aligned}
&0 < d\Big(\omega_n, [T\omega_n]_{\alpha_L(\omega_n)}\Big) \leq H\Big([T\omega_{n-1}]_{\alpha_L(\omega_{n-1})}, [T\omega_n]_{\alpha_L(\omega_n)}\Big) \leq \psi(d(\omega_{n-1}, \omega_n)) \leq \psi^n(d(\omega_0, \omega_1)),\\
&0 < d\Big(\omega_n, [T\omega_n]_{\alpha_L(\omega_n)}\Big) \leq \psi^n(d(\omega_0, \omega_1)) < \psi^n(r\ d(\omega_0, \omega_1)), \forall n \in N.
\end{aligned} \tag{35}$$

Now, for $m, n \in N$ with $m > n$, we have

$$\begin{aligned}
&d(\omega_n, \omega_m) \le s[d(\omega_n, \omega_{n+1}) + d(\omega_{n+1}, \omega_m)],\\
&d(\omega_n, \omega_m) \le s\ d(\omega_n, \omega_{n+1}) + s\ d(\omega_{n+1}, \omega_m),\\
&d(\omega_n, \omega_m) \le s\ d(\omega_n, \omega_{n+1}) + s[s[d(\omega_{n+1}, \omega_{n+2}) + d(\omega_{n+2}, \omega_m)]],\\
&d(\omega_n, \omega_m) \le s\ d(\omega_n, \omega_{n+1}) + s^2\ d(\omega_{n+1}, \omega_{n+2}) + s^2 d(\omega_{n+2}, \omega_m).
\end{aligned} \tag{36}$$

Similarly, in this way

$$\begin{aligned}
&d(\omega_n, \omega_m) \le s\ d(\omega_n, \omega_{n+1}) + s^2\ d(\omega_{n+1}, \omega_{n+2}) + s^3 d(\omega_{n+2}, \omega_{n+3}) + \ldots + s^{m-n} d(\omega_{m-1}, \omega_m),\\
&d(\omega_n, \omega_m) \le s\psi^n(r\ d(\omega_0, \omega_1)) + s^2\psi^{n+1}(r\ d(\omega_0, \omega_1)) + s^3\psi^{n+2}(r\ d(\omega_0, \omega_1)) + \ldots + s^{m-n}\psi^{m-1}(r\ d(\omega_0, \omega_1)),\\
&d(\omega_n, \omega_m) \le \frac{1}{s^{n-1}}\left[s^n\psi^n(r\ d(\omega_0, \omega_1)) + s^{n+1}\psi^{n+1}(r\ d(\omega_0, \omega_1)) + s^{n+2}\psi^{n+2}(r\ d(\omega_0, \omega_1)) + \ldots + s^{m-1}\psi^{m-1}(r\ d(\omega_0, \omega_1))\right],\\
&d(\omega_n, \omega_m) \le \frac{1}{s^{n-1}} \sum_{i=0}^{m-1} s^i\psi^i(r\ d(\omega_0, \omega_1)).
\end{aligned} \tag{37}$$

Because i $\psi \in \Psi$, we know that the series $\sum_{i=0}^{m-1} s^i\psi^i(r\ d(\omega_0, \omega_1))$ converges.

For $n \longrightarrow \infty, d(\omega_n, \omega_m) \longrightarrow 0$.

Hence, $\{\omega_n\}$ is a Cauchy sequence in Ω. By the completeness of Ω, there exists $\omega^* \in \Omega$ such that $\lim_{l\longrightarrow\infty}\omega_n = \omega^*$. We claim that $\omega^* \in [T\omega^*]_{\alpha_L(\omega^*)}$.

Because

$$\begin{aligned}
&d\left(\omega^*, [T\omega^*]_{\alpha_L(\omega^*)}\right) \le s\left[d(\omega^*, \omega_{n+1}) + d\left(\omega_{n+1}, [T\omega^*]_{\alpha_L(\omega^*)}\right)\right],\\
&d\left(\omega^*, [T\omega^*]_{\alpha_L(\omega^*)}\right) \le s\Big[d(\omega^*, \omega_{n+1})\\
&\qquad + H\left([T\omega_n]_{\alpha_L(\omega_n)}, [T\omega^*]_{\alpha_L(\omega^*)}\right)\Big],\\
&d\left(\omega^*, [T\omega^*]_{\alpha_L(\omega^*)}\right) \le s[d(\omega^*, \omega_{n+1}) + \psi(d(\omega_n, \omega^*))].
\end{aligned} \tag{38}$$

As $n \longrightarrow \infty$, and $\psi(0) = 0$, which implies that

$$d\left(\omega^*, [T\omega^*]_{\alpha_L(\omega^*)}\right) = 0. \tag{39}$$

As $[T\omega^*]_{\alpha_L(\omega^*)}$ is a closed and bounded subset of Ω. So, $\omega^* \in [T\omega^*]_{\alpha_L(\omega^*)}$.

Hence, ω^* is α_L-FFP of T.

Example 3. Let $\Omega = \{0, 1, 2\}$. Define metric d on Ω as follows:

$$d(\omega, \mu) = \begin{cases} 0, & \text{if } \omega = \mu,\\ \dfrac{1}{6}, & \text{if } \omega \neq \mu \text{ and } \omega, \mu \in \{0, 1\},\\ \dfrac{1}{2}, & \text{if } \omega \neq \mu \text{ and } \omega, \mu \in \{0, 2\},\\ 1, & \text{if } \omega \neq \mu \text{ and } \omega, \mu \in \{1, 2\}. \end{cases} \tag{40}$$

It is a complete b-MS with co-efficient 3/2.

Define an L-FM as

T: $X \longrightarrow F_L(\Omega)$ where $L = [a, b]$.

$$\begin{aligned}
&(T0)(t) =\\
&(T1)(t) = \begin{cases} \dfrac{b}{2}, & t = 0,\\ a, & t = 1, 2, \end{cases}\\
&(T2)(t) = \begin{cases} a, & t = 0,\\ \dfrac{b}{2}, & t = 1, 2. \end{cases}
\end{aligned} \tag{41}$$

Define $\alpha_L\colon \Omega \longrightarrow L/\{a\}$ by $\alpha_L(\omega) = b/2, \forall\, \omega \in \Omega$.
Now, we obtain

$$[T\omega]_{b/2} = \begin{cases} \{0\}, & \omega = 0, 1, \\ \{1\}, & \omega = 2. \end{cases} \tag{42}$$

For $\omega, y \in \Omega$, we get

$$H\left([T0]_{b/2}, [T1]_{b/2}\right) = \max\left\{ \operatorname*{Sup}_{\omega\in[T\omega]_{b/2}} d\left(\omega, [T\mu]_{b/2}\right), \operatorname*{Sup}_{\mu\in[T\mu]b/2} d\left(\mu, [T\omega]_{b/2}\right)\right\}. \tag{43}$$

Define $\psi\colon [0,\infty) \longrightarrow [0,\infty)$ by $\psi(t) = 1/3t\,\forall\, t > 0$. Thus, we have

$$\begin{aligned} H\left([T0]_{b/2}, [T1]_{b/2}\right) &= 0 < \frac{1}{18} \\ &= \frac{1}{3}\frac{1}{6} \\ &= \frac{1}{3} d(0,1), \\ H\left([T0]_{b/2}, [T2]_{b/2}\right) &= \frac{1}{6} \\ &= \frac{1}{3}\frac{1}{2} \\ &= \frac{1}{3} d(0,2), \\ H\left([T1]_{b/2}, [T2]_{b/2}\right) &= \frac{1}{6} < \frac{1}{3}\cdot 1 \\ &= \frac{1}{3} d(1,2). \end{aligned} \tag{44}$$

Therefore, all the conditions of theorem 2 are satisfied. Hence, $\exists$ a point $0 \in \Omega$ such that
$0 \in [T0]_{b/2}$ is an α_L-FFP of T.

Corollary 4. *Let (Ω, d) be a complete b-MS with coefficient $s \geq 1$. Let $T\colon \Omega \longrightarrow F(\Omega)$ be fuzzy mapping and for all $x, y \in \Omega$ and for each $\alpha(x), \alpha(y)\varepsilon(0, 1], [Tx]_{\alpha(x)}$ and $[Ty]_{\alpha(y)}$ be nonempty closed and bounded subsets of Ω. Suppose T satisfies the following multivalued contraction:*

$$H\left([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}\right) \leq \psi(d(x,y)), \tag{45}$$

where $\psi \in \Psi_b$. Then, T has an α-FFP.

Corollary 5. *Let (Ω, d) be a complete MS. Let $T\colon \Omega \longrightarrow F_L$, for all $x, y \in \Omega$ and for each $\alpha_{L(x)}, \alpha_{L(y)}\varepsilon L/\{0_L\}$, $[Tx]_{\alpha_L(x)}$ and $[Ty]_{\alpha_L(y)}$ be a nonempty closed and bounded subset of Ω. Suppose T satisfies the following multivalued contraction:*

$$H\left([Tx]_{\alpha_L(x)}, [Ty]_{\alpha_L(y)}\right) \leq \psi(d(x,y)), \tag{46}$$

where $\psi \in \Psi$. Then, T has an α_L-FFP.

Corollary 6. *Let (Ω, d) be a complete MS. Let $T\colon \Omega \longrightarrow F(\Omega)$ be fuzzy mapping for all $x, y \in \Omega$ and for each $\alpha(x), \alpha(y)\varepsilon(0, 1]$, $[Tx]_{\alpha(x)}$ and $[Ty]_{\alpha(y)}$ be a nonempty closed and bounded subset of Ω. Suppose T satisfies the following multivalued contraction:*

$$H\left([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}\right) \leq \psi(d(x,y)), \tag{47}$$

where $\psi \in \Psi$. Then, T has an α -FFP.

Remark 3. We have extended fixed point theorems having different contractive conditions to L-fuzzy mappings in complete b-metric spaces and obtained some corollaries as direct consequences of our main results. Fixed point theorems are widely used to obtain solutions of some initial value problems (Fredholm integral equations of 1st and 2nd kinds, Volterra integral equations) to find explicit form of implicit functions, etc. Our work will help to solve problems involving situations mentioned above.

3. Conclusion

In the case of the complete b-metric space, two fuzzy fixed point theorems for L-fuzzy mappings are established and proved for two diverse contractive type conditions. Nontrivial supporting examples for both results are also supplied to demonstrate the strength of these findings. Our results give uniqueness, extension, and sequential generalizations of many valuable current and conventional results in the literature using this approach. Some directions for more examinations and work are given in the form of open questions.

(1) Whether these results can be extended to more than one mapping?

(2) In case of complex-valued metric spaces, what type of contractive conditions will be feasible to find fixed points?

Abbreviations

MS: Metric space
FP: Fixed point
FS: Fuzzy set
FM: Fuzzy mapping
FFP: Fuzzy fixed point.

Data Availability

No real data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publications.

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