Research Article

DOA Estimation for Noncircular Signals under Strong Impulsive Noise

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Direction of arrival (DOA) estimation under impulsive noise has been an important research area. Most present methods are based on the fractional lower order statistics, while the computation load is heavy, and the estimation property degrades when the noise impact is strong. To get around this conundrum, a novel solution is presented, where the noncircular signals are introduced for modelling, a filtering preprocessing method is introduced to eliminate the impulsive noise, and a matrix reconstruction method is presented to smooth the residual noise. Firstly, the filtering preprocessing method is implemented to cut out the impulsive noise. Secondly, the characteristic of the noncircular signal is utilized to extend the array aperture. Thirdly, a new matrix reconstruction method is proposed to smooth the residual noise. Finally, the classical ESPRIT algorithm is adopted to estimate the DOAs. Simulations under different comparison of dimensions are conducted, and the mainstream methods are selected as comparison. The simulation results illustrate the outstanding performance of the proposed method in a strong impulsive noise environment.

1. Introduction

The direction of arrival (DOA) estimation technique is widely applied in radar [1], electronic warfare [2], communication [3], and other military and civilian fields. The classical direction finding algorithms are mostly modelled based on the Gaussian noise [4, 5]. While in most practical engineering applications, the noise does not satisfy the ideal Gaussian distribution, and it shows impact characteristics, such as the cosmic electromagnetic wave, low frequency atmospheric noise, and all kinds of human interference noise, which can be called as the impulsive noise [6]. The majority of traditional DOA estimation algorithms fail to work under the impulsive noise environment. Meanwhile, most signals in practical engineering application, such as the amplitude modulation (AM) signal, multiple amplitude shift keying (MASK) signal, and the binary phase shift keying

(BPSK) signal, have noncircular (NC) characteristics [7, 8], which means their elliptic covariance is not zero, and they are referred to as noncircular signals. Consequently, it is of great theoretical sense and practical significance to study on the noncircular direction-finding algorithm under the impulsive noise.

Studies have shown that the alpha-stable distribution can be exploited to model the impulsive noise, and the characteristic exponent $\alpha\,(0 < \alpha \le 2)$ is employed to describe the intensity of the impulsive noise [9]. A series of new ideas and methods for DOA estimation in impulsive noise have been put forward by scholars at home and abroad, and some remarkable achievements have been reached. The present DOA estimation algorithms are mainly based on the fractional low-order statistics [10, 11], multidimensional optimization [12], amplitude threshold processing [13], etc. The fractional low-order moment (FLOM) and the

fractional low-order covariance (FLOC) algorithms were proposed in the early stage for direction finding of impulsive noise, which fail to estimate or degrade under strong impulsive noise background. By reconstructing the FLOC matrix, the reconstructed FLOC (RFLOC) method was proposed [11], which improved the estimation performance effectively. However, these methods need to calculate the fractional low-order moments, which has high computational complexity and require a proper low-order prior parameter p, which influences the accuracy of estimation greatly. Based on the sparse theory and the infinite norm, the intelligent optimization direction finding method was presented, which can improve the estimation performance to a certain degree. While this kind of algorithm has a huge amount of calculation, the adaptability to the strong impulsive noise environment is poor. The amplitude threshold processing method can weaken the impulse effectively [13], but a proper prior parameter is required. In conclusion, the present methods cannot meet the requirements of engineering application, the DOA estimation method with outstanding accuracy and small computational complexity for the strong impulsive noise is needed urgently.

By studying the property of the NC signals, based on the multiple signal classification (MUSIC) [14] algorithm and the estimation of signal parameters via rotational invariance techniques (ESPRIT) [15], the NC-MUSIC [16], and the NC-ESPRIT [17] were proposed, which can extend the array aperture and improve the estimation performance. The NC-MUSIC algorithm has a high resolution, and an improved method was presented to cut down the computational complexity [18], but they all require mutidimensional spectral peak search, and the computational load is heavy, which is not applicable in engineering implementation. Compared with the NC-MUSIC algorithm, the NC-ESPRIT algorithm has a much small computational load, but it has a loss of array aperture when constructing the rotation invariant matrix. Therefore, it is of great significance to study on the NC-ESPRIT algorithm with high resolution.

In order to estimate the DOAs of the NC signals in impulsive noise, a noncircular generalized covariance ESPRIT method was proposed [19], which has good robustness, but it fails to estimate in strong impulsive noise ($\alpha < 1$). By researching and analysing the distribution property of the impulsive noise, a novel method based on the median value filtering was proposed utilizing the low probability and randomness of the impulsive noise [20], which is effective in strong impulsive noise as well. Inspired by the median value filtering, a filtering preprocessing (FP) method is introduced to eliminate the impulsive noise, and the noncircular characteristic is utilized to extend the array aperture. Moreover, a new matrix reconstruction (MR) method is proposed to smooth the residual noise, so that the ESPRIT algorithm can be exploited. The presented method, termed as NC-FP-MR-ESPRIT, is verified to be effective in strong impulsive noise, and the presented method is suitable for an engineering application as well.

2. Data Model

The array received data model is established as follows: K uncorrelated far-field narrowband noncircular signal with the wavelength λ impinging on the uniform linear array (ULA), the interspacing between adjacent sensors is d, and the number of array sensors is M. The array noise is impulsive noise, which can be modelled by the α stable distribution. And, the array of received data at the time snapshot t can be expressed as follows:

$$X(t) = AS(t) + N(t), \tag{1}$$

where $X(t) = [x_1(t), \ldots, x_M(t)]^T$, $S(t) = [s_1(t), \ldots, s_K(t)]^T$ and $N(t) = [n_1(t), \ldots, n_M(t)]^T$ are the vectors of the array received data, signals and the impulsive noise, respectively. $A = [a(\theta_1), \ldots, a(\theta_K)]$, where $a(\theta_i) = [1, u_k, \ldots, u_k^{M-1}]^T$, is the array manifold matrix, and $u_k = \exp(-j2\pi d \cos\theta_k/\lambda)$, where θ_k is the incident angle of the k th noncircular signal.

Usually, the α stable distribution is utilized to model the impulsive noise. Since there is no fixed probability density function (PDF) for α stable distribution, the characteristic function is employed.

$$\varphi(w) = \exp(j\varepsilon w - \gamma|w|^\alpha), \tag{2}$$

where $\alpha \in (0, 2]$, is the characteristic exponent of the α stable distribution, which determines the strength of the impulse. The smaller the value of α, the stronger of the impulse. ε is the position parameter, representing the symmetric point of the PDF of the α stable distribution, which is equivalent to the mean value of the second-order process. γ is the dispersion coefficient, indicating the dispersion degree of the sample relative to the mean, which is similar to the variance of the second-order process. The impulsive noise discussed in this paper satisfies the standard α stable distribution, i.e., $\varepsilon = 0$, $\gamma = 1$.

According to the property of the noncircular signal, we have $E[s^2(t)] \neq 0$. When the noncircular rate of the noncircular signal $s_i(t)$ is 1, we have $s_i(t) = s_{iR}(t)e^{j\phi_i}$, where $s_{iR}(t)$ is the real signal, and ϕ_i is the noncircular phase. Then, equation (1) is equivalent to the following equation:

$$X(t) = A\Psi S_R(t) + N(t), \tag{3}$$

where $S_R(t)$ is the vector of the real signals, and Ψ is a diagonal matrix with $e^{j\phi_i}$ $(i = 1, \ldots, K)$ on its diagonal position.

3. The NC-FP-MR-ESPRIT Method under Strong Impulsive Noise

3.1. The Filtering Preprocessing Method under Strong Impulsive Noise. The characteristic exponent of the α stable distribution determines the intensity of the impulse noise. The smaller the value of α is, the stronger the impulse is. According to the distribution property of the impulsive noise, the probability of a sampling point with a very large value is small. Most of the time, the value is smaller than the mean or median value of the α stable distribution. The

impact of the impulsive noise is random. It only produces large impact values in a few locations, and the amplitude is relatively small in most locations without impact. But we cannot predict when it will produce a large impact.

For a long time, most of the direction-finding algorithms for impulsive noise are trying to adapt to the noise, such as the methods based on the fractional lower order statistics. Actually, some methods of noise removal in imagine processing are worth using for reference. The median filter is a nonlinear digital filter technology, which is widely exploited to remove noise in imagines or other signals. It has been worked well on eliminating isolated noise points in image processing. Considering the distribution characteristics of the impulsive noise, the impact of the noise is generated randomly and isolated. Therefore, a filtering preprocessing method is presented based on the median filter to eliminate the impulsive noise.

Suppose the snapshot number is L, we have $X = [X(1), \ldots, X(L)]$. Perform the median filtering on the array received data, and the window length is $2P + 1$, where P is an integer.

$$X_m(i, t) = A_m(i, t) \frac{X(i, t)}{\overline{X}(i, t)}, \tag{4}$$

where $\overline{X} = abs(X)$, $A_m(i, t) = \text{median}(\overline{X}(i, t - P: t + P))$, and the function $abs(*)$ represents computing the absolute value, and $\text{median}(*)$ represents finding the median of the vector. While in the case of some strong impulsive noise, the smoothing effect of the median filtering degrades. There are still some local outliers, which cannot support the DOA estimation. To solve this problem, the global median is introduced to adjust the filtering results. After the filtering preprocessing, the array received data at place (i, t) can be expressed as follows:

$$X_{FP}(i, t) = \begin{cases} T_m(i) \dfrac{X(i, t)}{\overline{X}(i, t)}, & A_m(i, t) > T_m(i) \\ \\ A_m(i, t) \dfrac{X(i, t)}{\overline{X}(i, t)}, & A_m(i, t) \leq T_m(i) \end{cases}, \tag{5}$$

where $T_m(i) = \text{median}(\overline{X}(i, :))$.

The array received data after the filtering preprocessing can be formulated as follows:

$$X_{FP} = A\Psi S_R + N_{FP}, \tag{6}$$

where N_{FP} is the residual noise after the filtering preprocessing. Since the impulsive noise has been eliminated, the residual noise N_{FP} is not impulsive any more. The traditional DOA estimation algorithm based on the second-order moment can be employed. The proposed filtering preprocessing method does not require prior parameters, and the procedure of processing is simple. Besides, the computational complexity of the FP method is low, and it is easy for engineering to implement. Figure 1 shows the effect

of the presented method to eliminate the impact noise at different characteristic exponent α, when the generalized signal to noise (GSNR) is set to 5 dB.

Figure 1 shows that the proposed filtering preprocessing method can cut out the impact noise effectively. When $\alpha = 1.2$, the impulse noise is not very strong, and the eliminating effect is excellent, as shown in Figure 1(b). As α reduced to 0.4 in Figure 1(a), the impact of noise is very strong, the proposed FP method can still work well. In conclusion, it has been verified the validity of the FP method in eliminating the impulsive noise, whenever it is strong or not. It is of great significance to take the FP method as a preprocessing method for the DOA estimation in impulsive noise background, which can simplify the estimation process and improve the estimation performance greatly. Moreover, the FP method is easy to implement in engineering.

3.2. The NC Matrix Reconstruction Method.

According to the noncircular property of the NC signal, the array can be extended virtually. The extended array received data after the filtering preprocessing can be obtained according to (6).

$$Y = \begin{bmatrix} X_{FP} \\ X_{FP}^* \end{bmatrix} = \begin{bmatrix} A\Psi \\ A^*\Psi^* \end{bmatrix} S_R + \begin{bmatrix} N_{FP} \\ N_{FP}^* \end{bmatrix} = BS_R + N_E, \tag{7}$$

where $B = [A\Psi; A^*\Psi^*] = [b(\theta_1, \phi_1), \ldots, b(\theta_K, \phi_K)]$, $b(\theta_i, \phi_i) = [a(\theta_i)e^{j\phi_i}; a^*(\theta_i)e^{-j\phi_i}]$, and $N_E = [N_{FP}; N_{FP}^*]$. Then, the extended covariance matrix can be formulated as follows:

$$R = E[YY^H] = BR_{SR}B^H + R_N = \begin{bmatrix} R_1 & R_2 \\ R_2^* & R_1^* \end{bmatrix}, \tag{8}$$

where $R_{SR} = E[S_R S_R^H]$, $R_N = E[N_E N_E^H]$, $R_1 = E[X_{FP} X_{FP}^H]$, and $R_2 = E[X_{FP} X_{FP}^T]$. Obviously, the calculational complexity of R can be reduced by reconstructing the extended covariance matrix R with R_1 and R_2.

Furthermore, in order to smooth the residual noise after the filtering preprocessing, the matrix reconstruction method is presented. As for R_1 and R_2, the value at position (i, j) can be expressed as follows:

$$R_1(i, j) = \sum_{k=1}^{K} u_k^{i-j} \sigma_{S1k}^2 + \sigma_{N1ij}^2, \tag{9}$$

$$R_2(i, j) = \sum_{k=1}^{K} u_k^{i+j-2} \sigma_{S2k}^2 + \sigma_{N2ij}^2, \tag{10}$$

where σ_{S1k}^2, σ_{S2k}^2 and σ_{N1ij}^2, σ_{N2ij}^2 are the power of signal and noise, respectively. We can learn from (9) that the values on the main and auxiliary diagonal position of R_1 are equal, respectively. And, (10) illustrates that the values on the main and auxiliary antidiagonal position of R_2 are equal, respectively. Then, the smoothed covariance matrix $\widetilde{R}_1$ and $\widetilde{R}_2$ are constructed as follows:
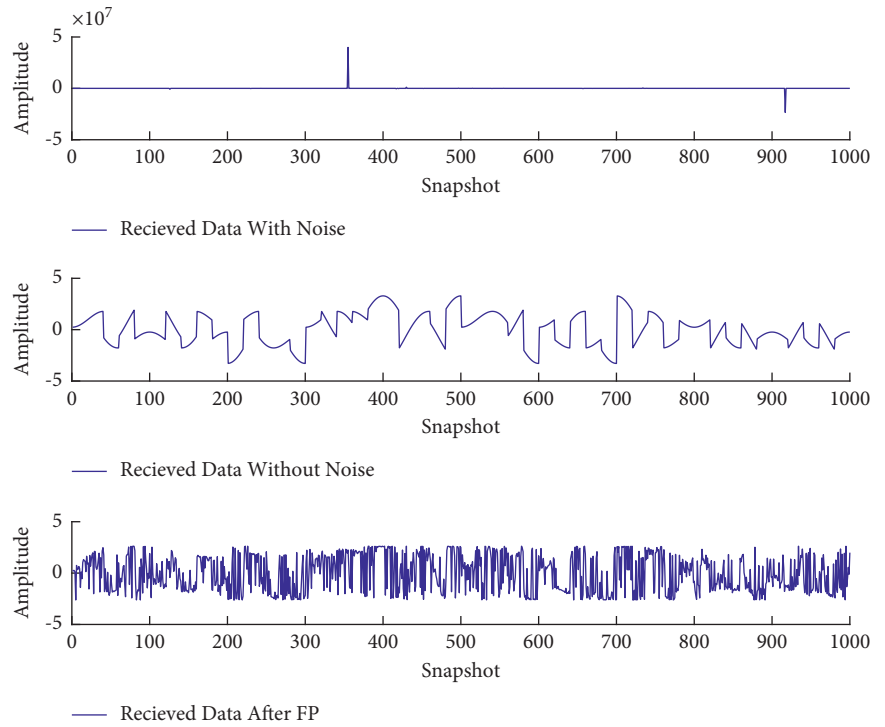
(a)
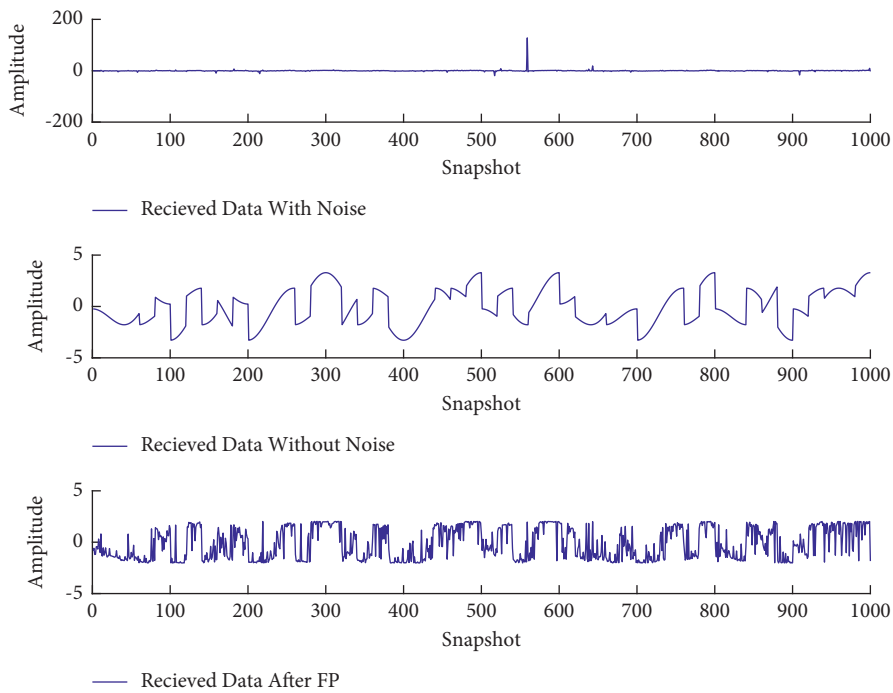
(b)

FIGURE 1: The smoothing effect at different α (a) $\alpha = 0.4$, (b) $\alpha = 1.2$.

$$\widetilde{R}_1 = \begin{bmatrix} r(0) & r(-1) & \cdots & r(1-M) \\ r(1) & r(0) & \cdots & r(2-M) \\ \vdots & \vdots & \ddots & \vdots \\ r(M-1) & r(M-2) & \cdots & r(0) \end{bmatrix}, \quad (11)$$

$$\widetilde{R}_2 = \begin{bmatrix} g(1-M) & g(2-M) & \cdots & g(0) \\ g(2-M) & g(3-M) & \cdots & g(1) \\ \vdots & \vdots & \ddots & \vdots \\ g(0) & g(1) & \cdots & g(M-1) \end{bmatrix}, \quad (12)$$

where

$$r(n) = \frac{1}{M-|n|} \sum R_1(i,j), \forall (i-j=n), \quad (13)$$

$$g(n) = \frac{1}{M-|n|} \sum R_2(i,j), \forall (i+j=M+1+n). \quad (14)$$

So far, the proposed preprocessing method for the impulsive noise has been carried out, and the classical DOA estimation algorithm based on the second-order moment could be utilized directly.

3.3. DOA Estimation for NC Signals with the ESPRIT Algorithm. The reconstructed extended covariance matrix can be expressed as follows:

$$\widetilde{R} = \begin{bmatrix} \widetilde{R}_1 & \widetilde{R}_2 \\ \widetilde{R}_2^* & \widetilde{R}_1^* \end{bmatrix}. \quad (15)$$

Eigenvalue decomposition is applied to $\widetilde{R}$, then the signal subspace U_S could be obtained, which is composed of the K eigenvectors corresponding to the largest K eigenvalue. Since the subspace spanned by the signal subspace and the array manifold matrix are the same, there is a unique nonsingular matrix T, which satisfies the following equation:

$$U_S = BT. \quad (16)$$

In order to implement the ESPRIT algorithm, construct the submatrices with the rotational invariant property.

$$\begin{cases} U_{S1} = [U_S(1:M-1); U_{S1}(M+2:2M)] \\ U_{S2} = [U_S(2:M); U_{S1}(M+1:2M-1)] \end{cases}, \quad (17)$$

where $U_S(m:n,:)$ represents taking the m th to n th row of U_S to form a new matrix. Then, we have $U_{S1} = B_1 T$, and $U_{S2} = B_1 \Phi T$, where $B_1 = [B(1:M-1,:); B(M+2:2M,:)]$, and $\Phi = \text{diag}\{u_1, \ldots, u_K\}$.

$$U_{S2} = U_{S1} T^{-1} \Phi T = U_{S1} \mathbf{J}, \quad (18)$$

where $J = T^{-1}\Phi T = (U_{S1}^H U_{S1})^{-1} U_{S1}^H U_{S2}$, and the DOAs could be estimated according to the eigenvalues of J.

3.4. The Process of the NC-FP-MR-ESPRIT Method. According to the previous statement, the scheme of the proposed NC-FP-MR-ESPRIT method is summarized as follows:

Step 1. Establish the array data model according to equation (1), the noise is modelled by the α stable distribution

Step 2. Apply filtering preprocessing method to the array data using equation (5), and the data matrix without impact X_{FP} can be obtained

Step 3. Construct the extended covariance matrix R by equation (8), and reconstruct R by equations (11)–(14), so that the reconstructed extended covariance matrix $\widetilde{R}$ can be obtained

Step 4. Apply eigenvalue decomposition to $\widetilde{R}$, then the signal subspace U_S could be obtained, and the ESPRIT algorithm is implemented for DOA estimation

4. Simulation

In this section, several simulations were carried out to verify the outstanding estimation property of the proposed NC-FP-MR-ESPRIT method. The RFLOC in document [11], is expanded for NC signals DOA estimation, termed as NC-RFLOC. The two solutions in document [20], termed as MED-ESPRIT and MEDC-ESPRIT, are selected for comparison because of their excellent performance in strong impulsive noise. During the simulations, the NC signals are modelled as the BPSK signals, the spacing of the adjacent array sensors is 0.5λ, and the incident angle is range from $0°$ to $180°$. If not specified, the number of snapshots is 1000, and each simulation condition is carried out 500 times. The generalized signal to noise ratio ρ_{GSNR} is defined as follows:

$$\rho_{GSNR} = 10\lg \left\{ \frac{E\left[|s(t)|^2\right]}{\gamma} \right\}, \quad (19)$$

where $E[|s(t)|^2]$ represent the power of the signal. The estimation is considered successful when the difference between the estimated value and the real value is less than or equal to $1°$. And, the estimated root mean square error (RMSE) is calculated as follows:

$$\theta_{RMSE} = \sqrt{\frac{1}{KQ} \sum_{i=1}^{K} \sum_{j=1}^{Q} \left(\theta_i - \theta_{ij}\right)^2}, \quad (20)$$

where Q is the times of the independent experiment, θ_{ij} is the j th estimated value of θ_i.

4.1. Simulation A. Assume four NC signals incident from $40°$, $70°$, $100°$ and $135°$, Figure 2 illustrates the estimation results of the proposed NC-FP-MR-ESPRIT method with $\alpha = 0.5$ and GSNR = 20 dB. The number of the array sensors is 4.50 times independent experiments are carried out.

Figure 2 illustrates that the proposed NC-FP-MR-ESPRIT method can estimate 4 NC signals correctly when the number of array sensors is 4, while the common ESPRIT method can only estimate 3 signals at most. Taking advantage of the noncircularity property of the NC signals, the NC-FP-MR-ESPRIT method can extend the array aperture virtually, and the estimation performance can be improved
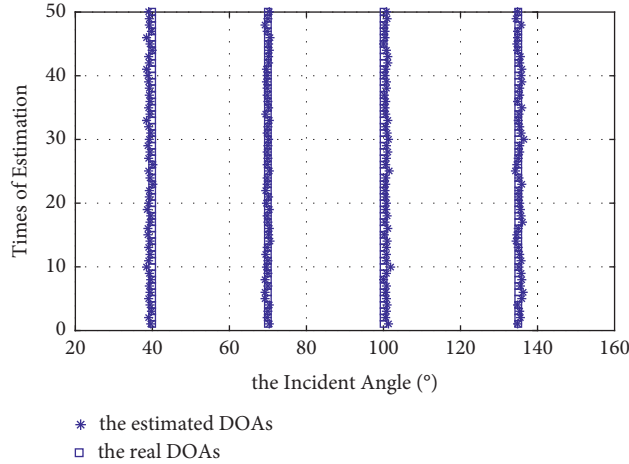
* the estimated DOAs
□ the real DOAs

FIGURE 2: The estimation results of the NC-FP-MR-ESPRIT method.
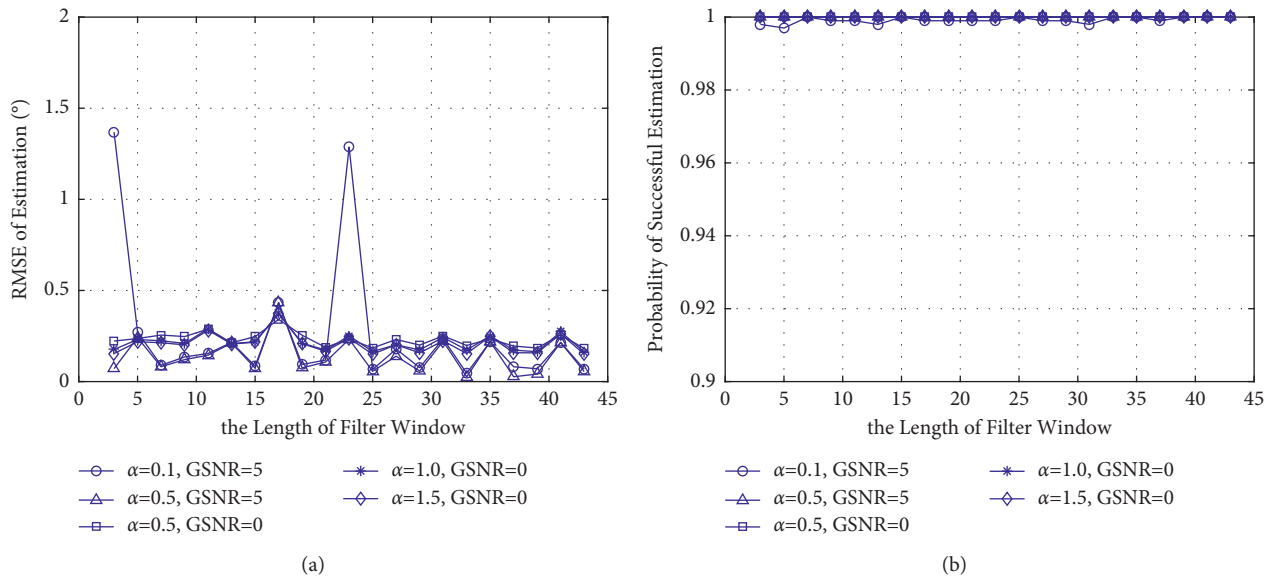

(a)

(b)

FIGURE 3: The estimation property via the length of the filtering window, (a). root mean square error of estimation, (b). probability of successful estimation.

as well. Since the characteristic exponent α is 0.5 in this simulation, which verifies the effectiveness of the NC-FP-MR-ESPRIT method under a strong impulsive noise environment.

4.2. Simulation B.

Consider two NC signals incident from $60°$ and $80°$, Figure 3 illustrates the estimation performance of the proposed NC-FP-MR-ESPRIT method via the length of filtering window with different α and GSNR.

Figure 3 illustrates that the estimation property of the proposed NC-FP-MR-ESPRIT method improves with the increase of GSNR, and there is no obvious difference as the characteristic exponent α changing. Besides, the length of the filtering window does not influence the results of estimation very much, while the computational load increases as the window length growing. Figure 3(a) shows that the proposed method fails to work occasionally when the impact

of the noise is extremely strong, as $\alpha = 0.1$, GSNR = 0 dB. But this is rare and acceptable, as shown in Figure 3(b). Considering the computational complexity and the robust performance of the DOA estimation, the length of the filtering window is advised to choose from 10 to 25, and the length of the filtering window is set to 21 in the following simulations.

4.3. Simulation C.

Consider two NC signals incident from $50°$ and $70°$, Figures 4 and 5 illustrate the estimation performance of the 5 methods via GSNR with $\alpha = 0.4$ and $\alpha = 1.4$, respectively. The parameter of the RFLOC is 0.1 and 0.4 when $\alpha = 0.4$ and $\alpha = 1.4$, respectively.

Figures 4 and 5 show that the estimation property of the 5 solutions become better and better with the increase of the GSNR, and the proposed NC-FP-MR-ESPRIT method has the best performance at both weak and strong impulsive noise. Figure 5(b) shows that the 5 solutions work well when
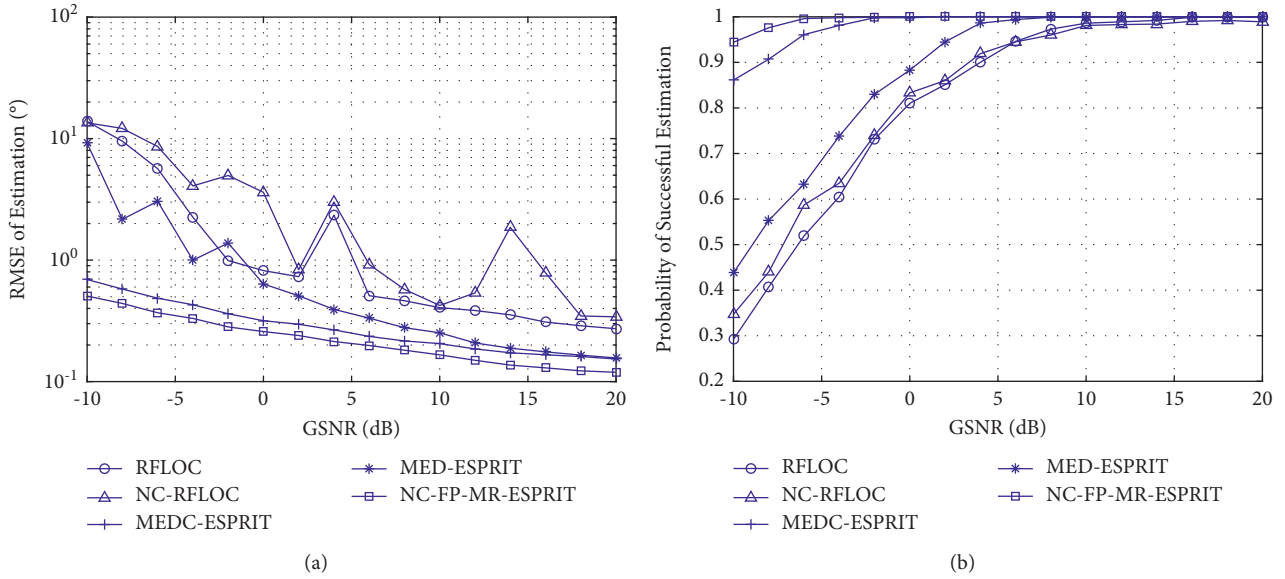
(a)

(b)

FIGURE 4: The estimation performance via GSNR with $\alpha = 0.4$, (a). root mean square error of estimation, (b). probability of successful estimation.
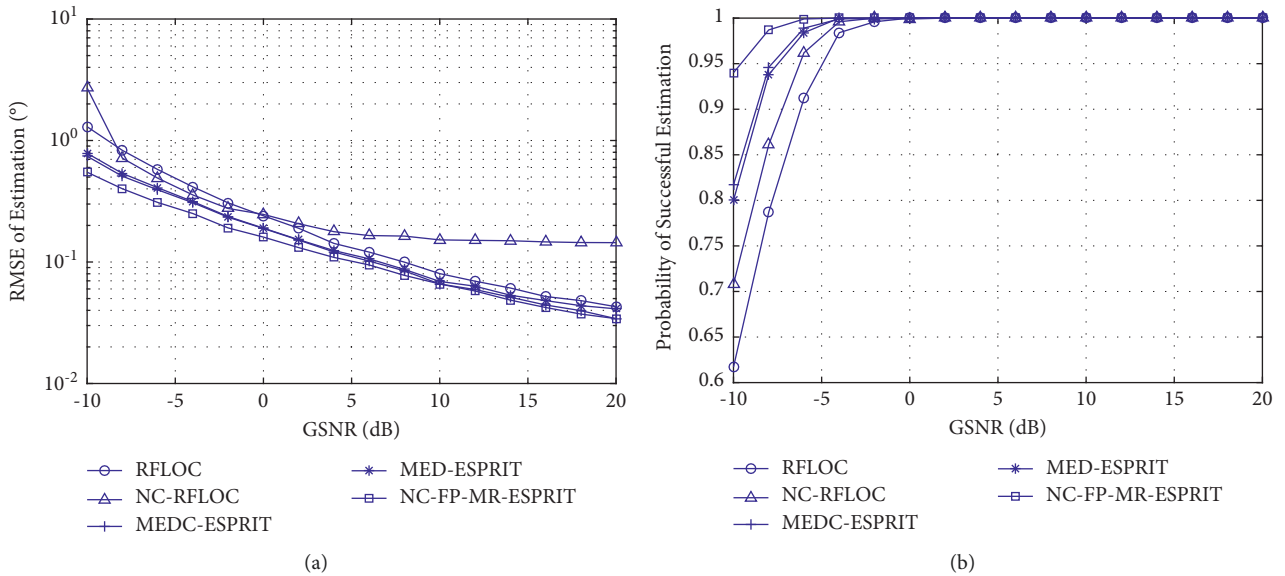

(a)

(b)

FIGURE 5: The estimation performance via GSNR with $\alpha = 1.4$, (a). root mean square error of estimation, (b). probability of successful estimation.

the impulse is weak, while the performance of the FLOC, the NC-FLOC, and the MED-ESPRIT degrade greatly in strong impulsive noise background, as shown in Figures 4(a) and 4(b). Moreover, we can learn from Figures 4 and 5 that the proposed NC-FP-MR-ESPRIT method has excellent estimation property at low GSNR compared with the other 4 solutions, which indicate that the NC-FP-MR-ESPRIT method has an excellent capacity to adaptive the low GSNR environment. This simulation illustrates that the FP method and the MR method proposed in this paper can improve the GSNR condition in both strong and weak impulsive noise background effectively.

4.4. Simulation D. Assume two NC signals incident from 45° and 65°, Figure 6 illustrates the estimation performance of the 5 solutions via characteristic exponent α with GSNR = 20 dB. The parameter of the RFLOC is $\alpha/3$.

Figure 6 shows that the performance of the 5 solutions improve with the increase of the characteristic exponent α, and the proposed NC-FP-MR-ESPRIT method has the best performance. We can learn from Figure 6(b) that the NC-FP-MR-ESPRIT method is effective at extremely strong impulsive noise, when $\alpha = 0.1$. Moreover, the proposed NC-FP-MR-ESPRIT method has similar excellent estimation property at both weak and strong impulsive noise, which
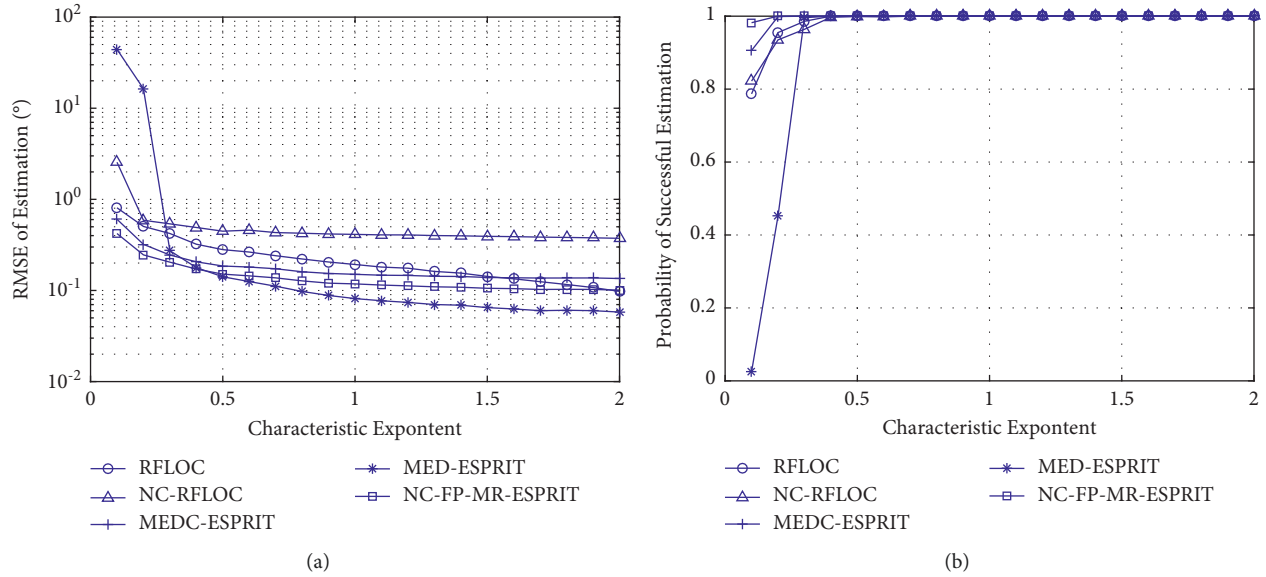
(a)

(b)

FIGURE 6: The estimation performance via α, (a). root mean square error of estimation, (b). probability of successful estimation.

indicates that the NC-FP-MR-ESPRIT method has the excellent robust performance to impulsive noise. Figure 6(a) illustrates the resolution of the proposed method is lower than the MED-ESPRIT when $\alpha > 0.5$, due to the global limit processing in the FP method for extremely strong impulsive noise. There is a loss of useful signal in the FP method, especially when the impact is not strong. Figure 6(b) indicates the MED-ESPRIT fails to work when $\alpha < 0.4$, that is due to the outliers after the median filter in strong impulsive noise. The FP method proposed in this paper has fixed this problem. Besides, the MR method is presented to optimize the estimation performance by smoothing the residual noise. To sum up, the impulse noise is eliminated effectively by the FP method and the MR method, so that the proposed NC-FP-MR-ESPRIT method can work well under both strong and weak impulsive noise background.

5. Conclusions

In this paper, the NC-FP-MR-ESPRIT method is presented for the DOA estimation of noncircular signals in the present of strong impulsive noise. Two methods are presented to process the array received data under impulsive noise, which can eliminate and smooth the impact composition of the impulsive noise effectively so that the computation could be simplified and the classical ESPRIT could be exploited for DOA estimation.

By analysing the distribution characteristics of the impulsive noise, the filtering preprocessing method, based on the median filtering, is proposed to eliminating the impact composition of the noise. Moreover, the characteristic of the noncircular signal is utilized to extend the array aperture, and the matrix reconstruction method is put forward to smooth the residual noise after the filtering preprocessing. Theoretical analysis and simulations have been conducted sufficiently, which illustrate the outstanding performance of

the proposed NC-FP-MR-ESPRIT method at low GSNR and a strong impulsive noise environment. Most of all, the low computational complexity and good adaptive property to strong impulsive noise of the proposed method is valuable for engineering implementation.

Data Availability

The data and figures used to support this article have been included in this paper. Further details can be provided upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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