Research Article

Entropy and Heat Transfer Analysis for MHD Flow of Cu/Ag -Water-Based Nanofluid on a Heated 3D Plate with Nonlinear Radiation

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This paper scrutinizes the consequences of radiation and heat consumption of MHD convective flow of nanofluid on a heated stretchy plate with injection/suction and convective heating/cooling conditions. The nanofluid encompasses with Cu and Ag nanoparticles. We enforce the suited transformation to remodel the governing mathematical models to ODE models. The HAM (homotopy analysis method) idea is applied to derive the series solutions. The divergence of fluid velocity, temperature, skin friction coefficient, local Nusselt number, entropy generation, and Bejan number on disparate governing parameters is exhibited via graphs and tables. It is seen that the fluid velocity in both directions is subsided when elevating the magnetic field and Forchheimer number. Also, the Cu nanoparticles possess hefty speed compared to Ag nanoparticles because the density of Ag nanoparticles is high compared to that of Cu nanoparticles. The fluid temperature upturns when enlarging the heat generation and radiation parameters. The skin friction coefficients and local Nusselt number are high in Ag nanoparticles than in Cu nanoparticles.

1. Introduction

The fluid thermal conductivity plays a vital role in many industrial and engineering processes, such as heat exchanges, fabrication of malleable, cooling of vehicles, domestic refrigerators, and cancer treatment. However, the standard base fluids such as water, ethylene, and engine oil have low thermal conductivity and do not fulfill the necessary heating and cooling rates. So, strengthening the thermal conductivity is one of the challenging tasks for researchers, which leads to upgrading the heat transfer characteristics. One of the facile methods is for increasing the thermal conductivity, nanosized metals such as copper, graphite, alumina, titanium, carbides, metal oxides, and nitrides are added to common base fluids. These fluids

enhance the heat transfer attributes. The 3D flow of water-based nanofluid past a stretching sheet with three different types of nanoparticles was deliberated by Nadeem et al. [1]. They noticed that the fluid hotness becomes high for the more quantity of nanoparticle volume fraction values. Hosseinzadeh et al. [2] inspected the 3D flow of ethylene glycol-titanium dioxide nanofluid on a porous stretching surface with the convective heating condition. The skin friction coefficient exalts when aggravating the nanoparticle volume fraction parameter. The axisymmetric flow of silver and copper water-based nanofluids between two rotating disks was studied by Rout et al. [3]. They detected that the skin friction coefficient elevates when raising the solid volume fraction. Mishra et al. [4] described the MHD flow of ethylene-glycol-based copper and aluminium oxide

nanofluid on an exponentially stretching sheet in a porous medium. They identified that the more extensive momentum boundary layer occurs in aluminium oxide nanofluid than the copper nanofluid. Various analyses for this direction are found in [5–9].

In recent decades, the study of MHD (magnetohydrodynamics) has obtained much interest for many researchers because of its variety of applications in many industries. Such applications are polymer processes, extrusion of plastic sheets, X-ray radiation, vehicle cooling, fiber filters, electrolytes, magnetic cells, manufacture of loudspeakers, etc. The impact of zero mass flux on a second-grade nanofluid flow on a Riga plate was presented by Rasool and Wakif [10]. They have seen that the momentum boundary layer thickness escalates when enhancing the modified Hartmann number. Akbar and Khan [11] discussed the impact of magnetic field and bioconvective flow of a nanofluid over a stretching sheet. They noticed that the fluid velocity decimates when the magnetic field parameter increases. Chemically reactive MHD flow of Maxwell fluid between stretching disks with Joule heating and viscous dissipation was demonstrated by Khan et al. [12]. Reddy and Chamkha [13] inspected the effects of thermal radiation and magnetic field of Al_2O_3 and TiO_2 -water-based nanofluid on a porous stretching sheet. They acknowledged that the nanoparticle concentration is intensified when raising the magnetic field parameter. MHD mixed convective flow of Oldroyd-B nanofluid between two isothermal stretching disks was illustrated by Hashmi et al. [14]. They recognized that the heat transfer gradient becomes high in the upper disk than the lower disk. Few momentous analyses for this area are collected in [15–23].

The study of fluid flow via porous material has plentiful applications in various fields, such as geothermal operations, catalytic reactors, crude oil production, nuclear waste disposal, solar receivers, and beds of fossil fuels, but this principle was not suitable for higher velocity and uneven porosity problems; see Umavathi et al. [24]. In general, many practical issues have enormous flow speed and nonuniform porosity. In this situation, Forchheimer [25] remodeled the Darcy principle to insert the second-order polynomial into the velocity equation, which altered the consequences of inertia on relative permeability. The 2D radiative flow of water-based viscous nanofluid on a shrinking/stretching sheet embedded in a porous medium with thermal stratification was inspected by Vishnu Ganesh et al. [26]. They revealed that the heat transfer gradient downturns when increasing the porosity parameter. Hayat et al. [27] disclosed the consequence of Darcy–Forchheimer flow of Maxwell fluid with Cattaneo–Christov theory. They uncovered that the fluid speed slumps when enriching the Forchheimer number. The 3D Darcy–Forchheimer porous flow of water-based carbon nanotubes on a bidirectional stretching sheet was examined by Alzahrani [28]. Muhammad et al. [29] analyzed the consequence of 3D Darcy–Forchheimer flow of water-based carbon nanotubes past a heated stretching sheet. They proved that the fluid velocity deteriorates when aggravating the porosity parameter.

Thermally radiative flow is usually confronted when the difference between surface and free stream temperatures is large. In many industrial operations, thermal radiation affects the thermal boundary layer thickness. A few examples are missile technology, nuclear reactors, satellites, and power plants. The abundant analysis concentrates only on linear radiation using the linearized Rosseland approximation theory, but this theory is applicable when the temperature difference between the plate and ambient fluid is less. In many practical problems has this difference is high. In this situation, a nonlinearized Rosseland approximation is applicable. Uddin et al. [30] deliberated the significance of nonlinear thermal radiation of a Sisko nanofluid. They noticed that the fluid temperature mounted when enlarging the temperature ratio parameter. Okuyade et al. [31] deliberated the impact of Soret and Dufour effects of a chemically radiative MHD fluid flow on a vertical plate. The consequence of the MHD flow of carbon nanotubes in a Maxwell nanofluid with thermal radiation was theoretically investigated by Subbarayudu et al. [32]. They detected that the thermal boundary layer thickness was thicker when increasing the values of the radiation parameter. Gireesha et al. [33] analyzed the 3D flow of thermally radiative Jeffrey nanofluid on an uneven stretching sheet. The 2D stagnation point flow of Walter-B nanofluid with nonlinear thermal radiation was presented by Ijaz Khan and Alzahrani [34]. They proved that the temperature ratio parameter leads to decay in the heat transfer rate.

From the analysis mentioned above, the consequence of entropy analysis of a nonlinear thermally radiative flow encompassed with Cu/Ag nanoparticles with the presence of the magnetic field, heat absorption/generation, suction/injection, and velocity slip on a convectively heated stretchy plate on a Darcy–Forchheimer porous medium is not inspected. The entropy generation is closely related to thermodynamic irreversibility and randomness, which occurs in all types of heat transfer equipment. The higher entropy generation rate suppresses system efficiency. Bejan [35] initiated the EGM (entropy generation minimization) model, which is helpful to minimize the energy losses while heat transfer processes and enrich the thermal system efficiency. Also, the Ag and Cu nanoparticles have enormous usage in the market because of their outstanding electrical and thermal properties. The impact of the first-order velocity slip condition is essential for fluids that exhibit wall slip seen in foams, suspensions, emulsions, etc. These types of analysis are the gateways for many researchers to model new thermal equipment in the industry.

2. Mathematical Formulation

The present study considers the 3D Darcy–Forchheimer flow of electrically conducting nanofluid past a stretching sheet in a porous medium. The uniform strength (B_0) of the magnetic field applied in the normal direction and the induced magnetic field were not taken because of the small quantity of the magnetic Reynolds number. The $\hat{x}$ and $\hat{y}$ coordinates can be taken along the surface, and $\hat{z}$ can be assumed perpendicular to the surface. Let us take $\tilde{\Pi}_1 = A\hat{x}$

and $\tilde{\Pi}_2 = A\tilde{y}$ as the velocity components in the $\tilde{x}\tilde{y}$ -plane. The nonlinear thermal radiation effect is included in the energy equation. This effect changes the heat transfer rate in the industrial process, and entropy generation is to minimize the energy losses during these heat transfer processes. Therefore, these effects are considered for designing thermal energy systems. Two different types of nanomaterials, such as copper and silver, are taken, and water is taken as a base fluid. In addition, the first-order velocity slip condition is taken into account; see Figure 1. Based on the above assumption, the governing mathematical model can be defined as follows; see Hayat et al. [36], Nayak et al. [37], and Tarakaramu et al. [38].

FIGURE 1: Physical model of flow.

$$\tilde{\Pi}_{1\tilde{x}} + \tilde{\Pi}_{2\tilde{y}} + \tilde{\Pi}_{3\tilde{z}} = 0, \quad (1)$$

$$\tilde{\Pi}_1\tilde{\Pi}_{1\tilde{x}} + \tilde{\Pi}_2\tilde{\Pi}_{2\tilde{y}} + \tilde{\Pi}_3\tilde{\Pi}_{3\tilde{z}} = \nu_{nf}\tilde{\Pi}_{1\tilde{z}\tilde{z}} - \frac{\sigma_{nf}}{\rho_{nf}}B_0^2\tilde{\Pi}_1 - \frac{\gamma_{nf}}{k_1}\tilde{\Pi}_1 - F\tilde{\Pi}_1^2, \quad (2)$$

$$\tilde{\Pi}_1\tilde{\Pi}_{2\tilde{x}} + \tilde{\Pi}_2\tilde{\Pi}_{2\tilde{y}} + \tilde{\Pi}_3\tilde{\Pi}_{2\tilde{z}} = \nu_{nf}\tilde{\Pi}_{2\tilde{z}\tilde{z}} - \frac{\sigma_{nf}}{\rho_{nf}}B_0^2\tilde{\Pi}_2 - \frac{\gamma_{nf}}{k_1}\tilde{\Pi}_2 - F\tilde{\Pi}_2^2, \quad (3)$$

$$\tilde{\Pi}_1\tilde{T}_{\tilde{x}} + \tilde{\Pi}_2\tilde{T}_{\tilde{y}} + \tilde{\Pi}_3\tilde{T}_{\tilde{z}} = \alpha_{nf}\tilde{T}_{\tilde{z}\tilde{z}} + \frac{16\sigma^*}{3k^*(\rho C p)_{nf}} \left[\tilde{T}^3\tilde{T}_{\tilde{z}\tilde{z}} + 3\tilde{T}^2\tilde{T}_{\tilde{z}}^2 \right] + \frac{Q}{(\rho C p)_{nf}} [\tilde{T} - \tilde{T}_\infty]. \quad (4)$$

where $\tilde{\Pi}_1$, $\tilde{\Pi}_2$, and $\tilde{\Pi}_3$ are the velocity components in $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ directions, ν_{nf} is the kinematic viscosity, ρ_{nf} is the density, σ_{nf} is the electrical conductivity, k_1 is the permeability of the porous medium, F is the nonuniform inertia coefficient, α_{nf} is the thermal diffusivity, σ^* is the Stefan-Boltzmann constant, $C p_{nf}$ is the specific heat capacity, and Q is the heat generation/absorption coefficient.

The corresponding boundary conditions are as follows (see Usman et al. [39]):

$$\tilde{\Pi}_1 = A\tilde{x} + \frac{2 - \sigma_v}{\sigma_v}\lambda_0\tilde{\Pi}_{1\tilde{z}}; \tilde{\Pi}_2 = B\tilde{y} + \frac{2 - \sigma_v}{\sigma_v}\lambda_0\tilde{\Pi}_{2\tilde{z}};$$

$$\tilde{\Pi}_3 = -W; -k_{nf}\tilde{T}_{\tilde{z}} = h_c(\tilde{T}_f - \tilde{T}) \text{ at } \tilde{z} = 0, \quad (5)$$

$$\tilde{\Pi}_1 \longrightarrow 0; \tilde{\Pi}_2 \longrightarrow 0; \tilde{T} \longrightarrow \tilde{T}_\infty \text{ as } \tilde{z} \longrightarrow \infty$$

where A and B are positive constants, W is the suction/injection velocity, σ_v is the coefficient of tangential

momentum accommodation, and λ_0 is the molecular mean free path; see Hayat et al. [36].

Define

$$\psi = \sqrt{\frac{A}{\nu_f}}\tilde{\Pi}_3;$$

$$\tilde{\Pi}_1 = A\tilde{x}f'(\psi);$$

$$\tilde{\Pi}_2 = A\tilde{y}g'(\psi); \quad (6)$$

$$\tilde{\Pi}_3 = -\sqrt{A\nu_f}[f(\psi) + g(\psi)];$$

$$\theta = \frac{\tilde{T} - \tilde{T}_\infty}{\tilde{T}_f - \tilde{T}_\infty}.$$

Substituting equation (6) into equations (2)–(4), we have

$$A_1A_2f'''(\psi) - f'(\psi)f'(\psi) + [f(\psi) + g(\psi)]f''(\psi) - A_2Mf'(\psi) - A_1A_2\lambda f'(\psi) - Frf'(\psi)f'(\psi) = 0, \quad (7)$$

$$A_1A_2g'''(\psi) - g'(\psi)g'(\psi) + [f(\psi) + g(\psi)]g''(\psi) - A_2Mg'(\psi) - A_1A_2\lambda g'(\psi) - Frg'(\psi)g'(\psi) = 0, \quad (8)$$

$$\begin{aligned} & \frac{A_4}{A_5} \frac{1}{\text{Pr}} \theta''(\psi) + [f(\psi) + g(\psi)] \theta'(\psi) + A_4 \frac{1}{\text{Pr}} R d \left[(\Gamma - 1)^3 \left\{ \theta^3(\psi) \theta''(\psi) + 3\theta^2(\psi) \theta'^2(\psi) \right\} \right. \\ & \left. + (\Gamma - 1)^2 \left\{ 3\theta^2(\psi) \theta''(\psi) + 6\theta(\psi) \theta'^2(\psi) \right\} + (\Gamma - 1) \left\{ 3\theta(\psi) \theta''(\psi) + 3\theta'^2(\psi) \right\} + \theta''(\psi) \right] + A_4 H g \theta = 0. \end{aligned} \quad (9)$$

The reduced boundary conditions are

$$\begin{aligned} f'(0) &= 1 + \Lambda f''(0); \\ g'(0) &= c + \Lambda g''(0); \\ f(0) + g(0) &= fw; \\ \theta'(0) &= -\frac{1}{A_5} Bi[1 - \theta(0)]; f'(\infty) \longrightarrow 0; g'(\infty) \longrightarrow 0; \theta(\infty) \longrightarrow 0, \end{aligned} \quad (10)$$

where M is the magnetic field parameter, λ is the porosity parameter, Fr is the Forchheimer number, Pr is the Prandtl number, Rd is the radiation parameter, Γ is the temperature ratio parameter, Hg is the heat generation/absorption parameter, Λ is the slip parameter, c is the ratio parameter, fw is the suction/injection parameter, and Bi is the Biot number.

Also,

$$A_1 = \frac{1}{(1 - \phi)^{2.5}},$$

$$A_2 = \frac{1}{(1 - \phi + \phi \rho_s / \rho_f)},$$

$$\begin{aligned} A_3 &= \left[1 + \frac{3(\sigma_s / \sigma_f - 1)\phi}{\sigma_s / \sigma_f + 2 - (\sigma_s / \sigma_f - 1)\phi} \right], \\ A_4 &= \frac{1}{(1 - \phi + \phi(\rho C p)_s / (\rho C p)_f)}, \\ A_5 &= \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}. \end{aligned} \quad (11)$$

The skin friction coefficients and local Nusselt number are expressed as follows:

$$\begin{aligned} C_{f_x} \sqrt{Re} &= A_1 f''(0); C_{f_y} \sqrt{Re} = \frac{A_1}{c^{3/2}} g''(0); \\ \frac{Nu}{\sqrt{Re}} &= -\left[A_5 \theta'(0) + Rd \left\{ \theta'(0) + (\Gamma - 1)^3 \theta^3(0) \theta'(0) + 3(\Gamma - 1)^2 \theta^2(0) \theta'(0) + 3(\Gamma - 1) \theta(0) \theta'(0) \right\} \right]. \end{aligned} \quad (12)$$

3. Entropy Analysis

The entropy generation (EG) can be defined as

$$S_{\text{gen}} = \frac{k_f}{\tilde{T}_{\infty}^2} \left[\frac{k_{nf}}{k_f} + \frac{16\sigma^* \tilde{T}^3}{3k_f k^*} \right] \tilde{T}_{\tilde{y}}^2 + \frac{1}{\tilde{T}_{\infty}} (\mu_{nf} + \sigma_{nf} B_0^2) (\tilde{\Pi}_1^2 + \tilde{\Pi}_2^2) + \frac{\mu_{nf}}{\tilde{T}_{\infty}} [\tilde{\Pi}_{1z}^2 + \tilde{\Pi}_{2z}^2]. \quad (13)$$

The transformed EG expression is

$$EG = A_3 \text{Re} \theta'^2(\psi) + \text{RdRe} \left[(\Lambda - 1)^3 \theta^3(\psi) \theta'^2(\psi) + 3(\Lambda - 1)^2 \theta^2(\psi) \theta'^2(\psi) + 3(\Lambda - 1) \theta(\psi) \theta'^2(\psi) + \theta'^2(\psi) \right] + (A_3 M + A_1 \lambda) \text{ReBr} \frac{1}{\alpha_1} \left[f'^2(\psi) + g'^2(\psi) \right] + A_1 \text{ReBr} \frac{1}{\alpha_1} \left[f''^2(\psi) + g''^2(\psi) \right], \quad (14)$$

where $\text{Re} = a \tilde{x}^2 / \nu_f$ is the local Reynolds number, $\text{Br} = \mu_f a^2 \tilde{x}^2 / k_f (\tilde{T}_f - \tilde{T}_{\infty})$ is the Brinkman number, and $\alpha_1 = \tilde{T}_f - \tilde{T}_{\infty} / \tilde{T}_{\infty}$ is the temperature difference parameter.

The Bejan number is

$$BE = \frac{\text{entropy generation due to heat and mass transfer}}{\text{total entropy generation}}, \quad (15)$$

$$BE = \frac{Z_1}{Z_2},$$

where

$$Z_1 = A_3 \text{Re} \theta'^2(\psi) + R \, dR \, e \cdot \left[(\Lambda - 1)^3 \theta^3(\psi) \theta'^2(\psi) + 3(\Lambda - 1)^2 \theta^2(\psi) \theta'^2(\psi) + 3(\Lambda - 1) \theta(\psi) \theta'^2(\psi) + \theta'^2(\psi) \right] \\ Z_2 = A_3 \text{Re} \theta'^2(\psi) + R \, dR \, e \left[(\Lambda - 1)^3 \theta^3(\psi) \theta'^2(\psi) + 3(\Lambda - 1)^2 \theta^2(\psi) \theta'^2(\psi) + 3(\Lambda - 1) \theta(\psi) \theta'^2(\psi) + \theta'^2(\psi) \right] \\ + (A_3 M + A_1 \lambda) \text{ReBr} \frac{1}{\alpha_1} \left[f'^2(\psi) + g'^2(\psi) \right] \\ + A_1 \text{ReBr} \frac{1}{\alpha_1} \left[f''^2(\psi) + g''^2(\psi) \right]. \quad (16)$$

4. HAM Solutions

Reduced models (7)–(9) with associated conditions (10) are solved by applying the HAM procedure (see Eswaramoorthi et al. [40] and Loganathan et al. [41]) because this method helps to solve the nonlinear equations and it does not depend on large/small physical parameters. Also, this method provides the great freedom to fix the auxiliary linear operator and the initial guess of unknowns. In addition, this method is used to solve many strongly nonlinear problems in various fields in science and engineering; see Rana and Liao [42].

Initially, we fix the initial approximation as $f_0(\psi) = f w + (1/(1 + \Lambda))(1 - 1/e^\psi)$, $g_0(\psi) = (c/(1 + \Lambda))(1 - 1/e^\psi)$, and $\theta_0(\psi) = Bi/(Bi + A_5)e^\psi$, the linear operator is $L_f = f'''' - f$, $L_g = g'''' - g$, and $L_\theta = \theta'' - \theta$, and the property

$L_f [J_1 + J_2 e^\psi + J_3 1/e^\psi] = 0$, $L_g [J_4 + J_5 e^\psi + J_6 1/e^\psi] = 0$, and $L_\theta [J_7 e^\psi + J_8 1/e^\psi] = 0$ where $J_l (l = 1 - 8)$ are constants.

After substituting the i^{th} -order HAM, we get

$$f_i(\eta) = f_i^\circ(\psi) + J_1 + J_2 e^\psi + J_3 \frac{1}{e^\psi}, \\ g_i(\eta) = g_i^\circ(\psi) + J_4 + J_5 e^\psi + J_6 \frac{1}{e^\psi}, \quad (17) \\ \theta_i(\eta) = \theta_i^\circ(\psi) + J_7 e^\psi + J_8 \frac{1}{e^\psi}.$$

Here, $f_i^\circ(\psi)$, $g_i^\circ(\psi)$, and $\theta_i^\circ(\psi)$ are the particular solutions.

The solutions have the parameters h_f , h_g , and h_θ , and these parameters are responsible for the convergence of HAM solutions; see Eswaramoorthi et al. [43]. In copper

FIGURE 2: h -graph for $f''(0)$, $g''(0)$, and $\theta'(0)$ with $\Lambda = 1.0, M = 0.1, \lambda = 0.3, Fr = 0.5, fw = 0.3, Bi = 0.2, Hg = -0.5, Rd = 0.2, c = 0.5, \phi = 0.1, \Gamma = 1.1$, and $Pr = 6.2$.

nanofluid, the range values are $-1.1 \leq h_f \leq -0.5$, $-1.2 \leq h_g \leq -0.3$, and $-1.7 \leq h_\theta \leq -0.7$, and in silver nanofluid, they are $-1.2 \leq h_f \leq -0.3$, $-1.3 \leq h_g \leq -0.4$, and $-1.6 \leq h_\theta \leq -0.6$; see Figures 2(a)–2(c). We set $h_f = h_g = -0.8$ and $h_\theta = -1.2$ for getting higher accuracy.

5. Results and Discussion

In this section, we present the consequences of physical parameters on $\tilde{x}$ -direction velocity ($f'(\psi)$), $\tilde{y}$ -direction velocity ($g'(\psi)$), temperature profile ($\theta(\psi)$), skin friction coefficients $C_{f_{\tilde{x}}} \sqrt{Re}$ and $C_{f_{\tilde{y}}} \sqrt{Re}$, local Nusselt number ($Nu / \sqrt{Re}$), entropy generation ($N_s(\psi)$), and Bejan number (BE). Table 1 provides the thermal conductivity, density, and specific heat capacity of copper, silver, and water. The thermophysical properties of the nanofluid are displayed in Table 2. The HAM order of approximation is illustrated in Table 3. It is noted that 12th order is enough for all calculations. Table 4 portrays the variations of skin friction coefficients and local Nusselt number for various combinations of Fr , λ , Λ , and fw . It is noticed that the surface shear stress in both directions declines when the values of fw increase. The heat transfer gradient enhances when enriching the magnitude of fw . The skin friction coefficient and local

TABLE 1: Physical properties; see Rout et al. [3].

Physical characteristics	Cu	Ag	Water
k (W/mK)	401	429	0.613
ρ (kg/m ³)	8933	10490	997.1
C_p (J/kgK)	385	235	4179

Nusselt number are high in *Cu* nanoparticles compared to those of *Ag* nanoparticles. The non-Darcy–Forchheimer flow has higher skin friction coefficient and local Nusselt number than Darcy–Forchheimer flow. Also, slip parameters control the surface shear stress and reduce the heat transfer gradient.

The consequences of M , Fr , γ , and Λ on $\tilde{x}$ - and $\tilde{y}$ -direction velocity profiles are displayed in Figures 3(a)–3(d) and Figures 4(a)–4(d). It is proved that both direction velocities and their associated boundary layer thicknesses diminish for the more presence of M , Fr , γ , and Λ . Physically, the presence of a magnetic field generates the Lorentz force, and this force affects the fluid motion and reduces the momentum boundary layer thickness. The magnitude of the porosity parameter tends to enhance the fluid resistance during the flow, and this causes to decimate the fluid velocity and diminish the momentum boundary

TABLE 2: Thermophysical properties; see Mahanthesh et al. [44].

Properties	Nanoliquid
Dynamic viscosity	$\mu_{nf} = \mu_f / (1 - \phi)^{2.5}$
Density	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$
Heat capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$
Thermal conductivity	$k_{nf}/k_f = ((k_s + 2k_f) - 2\phi(k_f - k_s)/(k_s + 2k_f) + \phi(k_f - k_s))$
Electrical conductivity	$\sigma_{nf}/\sigma_f = (1 + 3(\sigma_s/\sigma_f - 1)\phi/(\sigma_s/\sigma_f - 2) - (\sigma_s/\sigma_f - 1)\phi)$

TABLE 3: Order of approximations.

Order	Cu			Ag		
	$-f''(0)$	$-g''(0)$	$-\theta'(0)$	$-f''(0)$	$-g''(0)$	$-\theta'(0)$
1	0.56170	0.26835	0.13882	0.56516	0.27701	0.13888
5	0.57619	0.27685	0.14169	0.58415	0.28083	0.14155
10	0.57642	0.27700	0.14175	0.58458	0.28112	0.14161
12	0.57642	0.27700	0.14175	0.58459	0.28112	0.14162
15	0.57642	0.2770	0.14175	0.58459	0.28112	0.14162
20	0.57642	0.27700	0.14175	0.58459	0.28112	0.14162
25	0.57642	0.27700	0.14175	0.58459	0.28112	0.14162
30	0.57642	0.27700	0.14175	0.58459	0.28112	0.14162
35	0.57642	0.27700	0.14175	0.58459	0.28112	0.14162
40	0.57642	0.27700	0.14175	0.58459	0.28112	0.14162

TABLE 4: Skin friction coefficients and local Nusselt number for various combinations of λ , Fr , Λ , and fw .

λ	Fr	Λ	fw	Cu			Ag		
				$C_{f_x} \sqrt{Re}$	$C_{f_y} \sqrt{Re}$	Nu	$C_{f_x} \sqrt{Re}$	$C_{f_y} \sqrt{Re}$	Nu
0.3	0.5	0	-0.4	-1.72296	-1.97777	0.20763	-1.76959	-2.01957	0.18195
			-0.2	-1.87913	-2.20929	0.21214	-1.93811	-2.26941	0.18558
			0.0	-2.05072	-2.46583	0.21558	-2.12419	-2.54782	0.18834
			0.2	-2.23793	-2.74704	0.21818	-2.32808	-2.85441	0.19044
			0.4	-2.44060	-3.05198	0.22020	-2.54959	-3.18801	0.19207
0.3	0.5	1	-0.4	-0.63968	-0.82752	0.20305	-0.64466	-0.83180	0.17832
			-0.2	-0.67002	-0.88095	0.20856	-0.67648	-0.88817	0.18272
			0.0	-0.70154	-0.93608	0.21292	-0.70962	-0.94646	0.18622
			0.2	-0.73383	-0.99183	0.21625	-0.74362	-1.00543	0.18888
			0.4	-0.76641	-1.04707	0.21876	-0.77788	-1.06369	0.19090
0	0	0	-0.4	-1.38430	-1.59075	0.20804	-1.42585	-1.63195	0.18228
			-0.2	-1.54373	-1.82799	0.21241	-1.59788	-1.88787	0.18579
			0.0	-1.72093	-2.09332	0.21574	-1.79008	-2.17572	0.18847
			0.2	-1.91593	-2.38593	0.21827	-2.00247	-2.49454	0.19052
			0.4	-2.12825	-2.70430	0.22035	-2.23450	-2.84251	0.19225
0	0	1	-0.4	-0.56625	-0.71550	0.20410	-0.57259	-0.72225	0.17913
			-0.2	-0.60298	-0.78024	0.20929	-0.61105	-0.79033	0.18328
			0.0	-0.64139	-0.84740	0.21338	-0.65137	-0.86108	0.18657
			0.2	-0.68090	-0.91540	0.21653	-0.69285	-0.93267	0.18909
			0.4	-0.72074	-0.98251	0.21892	-0.73461	-1.00308	0.19102

layer thickness. In addition, it is observed that the momentum boundary layer thickness is high in *Cu* nanoparticles than in *Ag* nanoparticles. Physically, the *Cu* nanoparticles have a small density compared to *Ag* nanoparticles. Figures 5(a)–5(d) delineate the changes of the temperature profile for disparate values of Bi , Hg , and Rd . It is noticed that the fluid hotness and thermal boundary layer thickness increase when enhancing the values of convective heating, heat generation/absorption, and radiation parameters. However, the convective cooling parameter leads to

weakening the fluid temperature. Physically, the higher magnitude of the convective heating parameter enriches the heat transfer coefficient, and this tends to enhance the fluid hotness and raise the thermal boundary layer thickness. Additionally, it is noted that the larger thermal boundary layer thickness occurs in *Ag* nanoparticles than in *Cu* nanoparticles. Physically, the *Ag* nanoparticles have larger thermal conductivity compared to the *Cu* nanoparticles. The impact of ϕ on velocity and temperature profiles is presented in Figures 6(a) and 6(b). It is seen that both direction

FIGURE 3: The $\tilde{x}$ -direction velocity profile for dissimilar values of M (a), Fr (b), γ (c), and Λ (d).

FIGURE 4: Continued.

FIGURE 4: The $\tilde{y}$ -direction velocity profile for dissimilar values of M (a), Fr (b), γ (c), and Λ (d).

FIGURE 5: The temperature profile for dissimilar values of $Bi \geq 0$ (a), $Bi \leq 0$ (b), Hg (c), and Rd (d).

velocities decrease and temperature enhances for raising the values of ϕ . Physically, the higher value of ϕ gives more thermal conductivity of the nanofluid, and this causes to enrich the fluid temperature.

Figures 7(a) and 7(b) and Figures 8(a) and 8(b) indicate the outcomes of fw , M , and γ on skin friction coefficient in both directions. It is noticed that the surface shear stress slashes when heightening the quantity of fw and M , and it

FIGURE 6: The $\tilde{x}$ - and $\tilde{y}$ -direction velocity (a) and temperature (b) profiles for dissimilar values of ϕ .

FIGURE 7: The skin friction coefficient ($\tilde{x}$ -direction) for different combinations of fw , M , and γ .

FIGURE 8: The skin friction coefficient ($\tilde{y}$ -direction) for different combinations of fw , M , and γ .

FIGURE 9: The local Nusselt number for different combinations of fw , γ , Hg , and Rd . FIGURE 10: The entropy generation profile for dissimilar values of Rd (a), Re (b), Br (c), and fw (d).

exalts when rising the values of γ on both directions. Physically, the velocity slip parameter affects the fluid velocity, and as a result, it enhances the surface shear stress. The higher quantity of M strengthens the Lorentz force, which affects the fluid motion and thus decreases the surface shear stress. Permeability is the calculation of the capacity of porous material to permit the fluid to pass through it. The rise in the parameter enhances the shear stress, which is the

force of friction. The larger values of fw mean a larger amount of fluid sucked away from the plate. This causes to suppress the fluid speed and reduce the shear stress. The larger skin friction coefficient occurs in Cu nanoparticles than in Ag nanoparticles since Ag nanoparticles exhibit less surface drag compared to Cu nanoparticles. The variations of the local Nusselt number for dissimilar values of fw , γ , Hg , and Rd on both nanoparticles are illustrated in

FIGURE 11: The Bejan number for dissimilar values of Rd (a), Hg (b), fw (c), and Br (d).

Figures 9(a) and 9(b). From these figures, it is seen that the heat transfer gradient intensifies for larger values of fw and Rd and decays when enriching the values of γ and Hg . Physically, the velocity slip parameter leads to enhancing the fluid temperature inside the boundary, which creates the reduction of the temperature gradient. The suction means the heated fluid is taken away from the surface, resulting in the enhancement of the heat transfer gradient. The higher values of Rd with $Hg < 0.3$ reflect more heat transfer from a hot place to a cold place which results in the enhancement of the heat transfer gradient. However, downfall occurs when $Hg > 0.3$. The porosity parameter decreases the fluid motion, which results in the reduction of the heat transfer rate. In addition, the higher heat transfer gradient accounted for Cu nanoparticles than Ag nanoparticles. Figures 10(a)–10(d) portray the reactions of entropy generation for the diverse quantity of Rd , Re , Br , and fw . It is seen that the entropy generation upsurges when increasing the quantity of Rd , Re , and Br , and quite the opposite behavior is attained for larger values of fw . The higher magnitude of Rd leads to enriching the heat transfer rate, so more heat is produced due to which the entropy generation rises. The Bejan number for different values of Rd , Hg , fw , and Br is plotted in

Figures 11(a)–11(d). It is concluded that the Bejan number enriches when rising Rd , Hg , fw , and Br values.

6. Conclusions

The consequences of heat consumption and radiation of an MHD Darcy–Forchheimer flow of water-based nanofluid past a 3D plate are analytically investigated. There are two types of nanoparticles such as Cu and Ag which are taken into account. The suitable variables are implemented to remodel the governing PDEs into ODEs. The resultant models are analytically solved by applying the HAM procedure. In addition, the impact of radiation, heat consumption/generation, and suction/injection on entropy generation and the Bejan number is analyzed. The key outcomes of our investigation are as follows:

- (i) The $\check{x}$ - and $\check{y}$ -direction velocities are decreasing functions of magnetic field and slip parameters
- (ii) The fluid temperature enhances when rising the convective heating, heat generation/absorption, and radiation parameters, and it reduces when escalating the convective heating parameter

- (iii) The surface shear stress in both directions is suppressed when enhancing the magnetic field and porosity parameters
- (iv) The heat transfer gradient accelerates when raising the values of the radiation parameter, and it reduces when heightening the heat generation/absorption parameter
- (v) The entropy generation increases when escalating the radiation parameter, Reynolds number, Brinkman number, and suction/injection parameter
- (vi) The Bejan number upsurges when enriching the quantity of radiation, heat generation/absorption parameters, Brinkman number, and suction/injection parameter

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to this work and read and approved the final version of the manuscript.

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