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Research Article

Interval-Valued *q*-Rung Orthopair Fuzzy Choquet Integral Operators and Their Application in Group Decision-Making

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In this paper, we develop a multiattribute group decision-making (MAGDM) method to solve problems with interactive attributes under interval-valued q-rung orthopair fuzzy set (IVq-ROFS) environment. Firstly, the interval-valued q-rung orthopair fuzzy Choquet integral average (IVq-ROFCA) operator is proposed to aggregate interval-valued q-rung orthopair fuzzy information. Then, we investigate the interval-valued q-rung orthopair fuzzy Choquet integral geometric (IVq-ROFCG) operator and offer several related properties. More importantly, for handling problems with interdependence between attributes for IVq-ROFS, a MAGDM method is developed based on the IVq-ROFCA operator. Finally, an example of the warning management system for hypertension is given to illustrate the proposed method, and parameter analysis and comparison analysis further verify the feasibility and validity of the proposed method.

1. Introduction

As a powerful and practical approach, fuzzy set (FS) [1] has been deeply used in various areas, such as medical treatment, manufacturing, and education . With the increase in people's awareness of complex and uncertain issues, fuzzy theories and methods have received great attention [2–5]. Since decision-makers need to deal with the possibilities of support, opposition, and neutrality in real life, Atanassov [6] proposed the intuitionistic fuzzy set (IFS), that is, the sum of the membership degree (u) and the nonmembership degree (v) satisfies $u + v \leq 1$. In response to the condition $u + v > 1$, Yager [7] investigated the Pythagorean fuzzy set (PFS) ($u^2 + v^2 \leq 1$). As a wider range compared with intuitionistic fuzzy, Verma and Merigo [8] defined a generalized hybrid trigonometric Pythagorean fuzzy similarity measure and developed the approach for Pythagorean fuzzy decision-making; Bakioglu and Atahan [9] addressed the prioritization of risks involved with self-driving vehicles by the proposed new hybrid MCDM method based on AHP and TOPSIS VIKOR under Pythagorean fuzzy environment; Peng and Yang [10] defined interval-valued Pythagorean fuzzy sets (IVPFSs) and developed two interval-valued Pythagorean fuzzy aggregation operators to solve MAGDM problems. Gradually, for dealing with much more complicated problems, Yager [11] proposed the q-rung orthopair fuzzy set (q-ROFS) ($u^q + v^q \leq 1, q \geq 1$) in 2016 and extended the fuzzy set to a wider range of applications based on the different values of q. Researchers have put forward numerous excellent results in recent years. Yager et al. [12, 13] further deduced the related properties and mathematical principles of q-ROFS. Liu and Wang proposed several weighted average operators of q-rung orthopair fuzzy numbers (q-ROFNs) [14]. Liu and Liu developed the q-rung orthopair fuzzy Bonferroni mean operator to deal with problems [15]. Xing et al. [16] developed the q-rung orthopair fuzzy point weighted aggregation operator and applied it to q-rung orthopair fuzzy decision-making. Garg investigated the trigonometric operation-based q-rung orthopair fuzzy aggregation operator for processing fuzzy information [17] and introduced a novel concept of the connection number-based q-rung orthopair fuzzy set (CN-q-ROFS) [18]. Hussain et al. [19] proposed q-rung orthopair fuzzy soft weighted averaging, q-rung orthopair fuzzy soft ordered weighted averaging, and q-rung orthopair fuzzy soft hybrid averaging operators. Verma [20] defined two order-α

divergence measures for q-ROFS to quantify the information of discrimination that determine completely unknown or partially known attribute weights. Aydemir and Gunduz [21] presented neutrality average and neutrality geometric aggregation operators and further designed a general score function for q-ROFS based on power aggregation operators. Liu et al. [22] constructed the normalized bidirectional projection model and generalized knowledge-based entropy measure under q-rung orthopair fuzzy environment. In particular, it is better to address strongly uncertain decision-making problems via IVq-ROFS, such that Garg [23] defined q-connection numbers (q-CNs) and gave some new q-exponential operation laws (q-EOLs) and operators over q-CNs for IVq-ROFS and presented the possibility of comparison between IVq-ROFSs [24]. Garg [25] introduced a novel concept of interval-valued q-rung orthopair fuzzy preference relations (IVq-ROFPRs) and then proposed additive consistent of IVq-ROFPR and programming model to derive the weights of alternatives.

However, attributes are not independent in decision-making, and there are more mutual influences and correlations that can be appropriately solved by the Choquet integral [26]. The Choquet integral comes with decision-making problems with independent attributes in handy [27–29], which had been conducted in an in-depth research study Tan and Chen defined the intuitionistic of the fuzzy measure based on the Choquet integral [30] and investigated the induced Choquet ordered averaging operator for aggregating expert evaluation [31]. Tan developed the generalized interval-valued intuitionistic fuzzy geometric aggregation (GIIFGA) operator and combined TOPSIS to deal with MAGDM problems [32]. Xu [33] investigated the intuitionistic fuzzy Choquet integral average operator and geometric operator, which are applied to address MAGDM problems with IFS. It is significant to mention that Grabisch [34] presented a synthesis on the application of fuzzy integrals and extended the application of fuzzy integrals in various fields. Considering the advantages of fuzzy integrals that simulate the interaction between standards in a flexible way, scholars have conducted a lot of studies on fuzzy integrals (including Choquet integral) group decision-making (GDM) methods to effectively solve decision problems. Wu et al. [35] extended some operational properties of intuitionistic fuzzy values (IFVs) and then studied the aggregation properties of the intuitionistic fuzzy-valued Choquet integral (IFCI) and the intuitionistic fuzzy-valued conjugate Choquet integral (IFCCI). Xing et al. [36] developed the Choquet integral based on q-rung orthopair fuzzy environment and proposed q-rung orthopair fuzzy decision-making methods, Keikha et al. [37] combined the Choquet integral and the TOPSIS method to process fuzzy information, and Teng and Liu [38] developed the generalized Shapley probabilistic linguistic Choquet average (GS-PLCA) operator and investigated a method to solve large group decision-making (LGDM) issues. The in-depth study of the fusion application of the Choquet integral and GDM methods is of practical significance for solving complex and uncertain problems.

Although the aforementioned studies brilliantly handle complex decision-making problems, they cannot be applied to the decision-making problem where attributes are dependent under the interval-valued q-rung orthopair fuzzy environment. Accordingly, this paper develops two interval-valued q-rung orthopair fuzzy Choquet integral operators for aggregating fuzzy information. Subsequently, a MAGDM method is constructed by employing the proposed IVq-ROFCA operator, where interaction attributes of alternatives among the MAGDM problem are taken into account. Finally, compared with existing methods, the practicability and superiority of the proposed method are demonstrated.

The remainder of this paper is constructed as follows: Section 2 reviews the concept of the IVq-ROFS and Choquet integral operator, Section 3 proposes the IVq-ROFCA operator and the IVq-ROFCG operator and extends several weighted and ordered operators, Section 4 introduces a MAGDM method based on the IVq-ROFCA operator, Section 5 gives an illustrated case and then provides parameter analysis and comparison analysis, and Section 6 concludes this paper.

2. Preliminaries

In this section, we make a brief review of the IVq-ROFS and Choquet integral.

Definition 1 (see [11]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, $\widetilde{a} = \{x_i, t_{\widetilde{a}}(x_i), f_{\widetilde{a}}(x_i) | x_i \in X\}$ is a q-ROFS, where $t_{\widetilde{a}}\colon X \longrightarrow [0,1]$, $f_a\colon X \longrightarrow [0,1]$ and the following equation holds:

$$0 \leq (t_{\widetilde{a}}(x_i))^q + (f_{\widetilde{a}}(x_i))^q \leq 1, \tag{1}$$

where $q \geq 1$. For all $x_i \in X$, $t_{\widetilde{a}}(x_i)$ is the degree of membership, $f_{\widetilde{a}}(x_i)$ is the degree of nonmembership, and the degree of indeterminacy $\pi_{\widetilde{a}}(x_i)$ is shown in as follows:

$$\pi_{\widetilde{a}}(x_i) = \sqrt[q]{1 - t_{\widetilde{a}}(x_i)^q - f_{\widetilde{a}}(x_i)^q}\,(q \geq 1). \tag{2}$$

Definition 2 (see [39]). Given a fixed set $X = \{x_1, x_2, \ldots, x_n\}$, IVq-ROFS a on X is defined as

$$a = \{ < x_i, t_a(x_i), f_a(x_i) > | x_i \in X\}. \tag{3}$$

The membership is represented by interval values $t_a(x_i) = [t_a^-(x_i), t_a^+(x_i)] \subseteq [0,1]$ and the nonmembership is $f_a(x_i) = [f_a^-(x_i), f_a^+(x_i)] \subseteq [0,1]$, $0 \leq (t_a^+(x_i))^q + (f_a^+(x_i))^q \leq 1$, $(q \geq 1)$. The indeterminacy degree of a is shown as follows:

$$\begin{aligned}\pi_a(x_i) &= [\pi_a^-(x_i), \pi_a^+(x_i)] \\ &= \left[\sqrt[q]{1 - (t_a^+(x_i))^q - (f_a^+(x_i))^q}, \sqrt[q]{1 - (t_a^-(x_i))^q - (f_a^-(x_i))^q}\right].\end{aligned} \tag{4}$$

In particular, IVq-ROFS extends the application of interval-valued fuzzy sets in decision-making problems. When $q = 1$, IVq-ROFS would reduce to the interval-valued intuitionistic fuzzy set (IVIFS); when $q = 2$, IVq-ROFS would transform to IVPFS.

Definition 3 (see [39]). Let $a_1 = <[t^-_{a_1}, t^+_{a_1}], [f^-_{a_1}, f^+_{a_1}]>$ and $a_2 = <[t^-_{a_2}, t^+_{a_2}], [f^-_{a_2}, f^+_{a_2}]>$ be two interval-valued q-rung orthopair fuzzy numbers (IVq-ROFNs), $q \geq 1$. Equations (5)–(8) hold

$$a_1 \oplus a_2 = < \begin{bmatrix} \sqrt[q]{\left(t^-_{a_1}\right)^q + \left(t^-_{a_2}\right)^q - \left(t^-_{a_1}\right)^q\left(t^-_{a_2}\right)^q}, \\ \sqrt[q]{\left(t^+_{a_1}\right)^q + \left(t^+_{a_2}\right)^q - \left(t^+_{a_1}\right)^q\left(t^+_{a_2}\right)^q} \end{bmatrix}, \left[f^-_{a_1} f^-_{a_2}, f^+_{a_1} f^+_{a_2}\right] >, \tag{5}$$

$$a_1 \otimes a_2 = < \left[t^-_{a_1} t^-_{a_2}, t^+_{a_1} t^+_{a_2}\right], \begin{bmatrix} \sqrt[q]{\left(f^-_{a_1}\right)^q + \left(f^-_{a_2}\right)^q - \left(f^-_{a_1}\right)^q\left(f^-_{a_2}\right)^q}, \\ \sqrt[q]{\left(f^+_{a_1}\right)^q + \left(f^+_{a_2}\right)^q - \left(f^+_{a_1}\right)^q\left(f^+_{a_2}\right)^q} \end{bmatrix} >, \tag{6}$$

$$\lambda a_1 = < \left[\sqrt[q]{1 - \left(1 - \left(t^-_{a_1}\right)^q\right)^\lambda}, \sqrt[q]{1 - \left(1 - \left(t^+_{a_1}\right)^q\right)^\lambda} \right], \left[\left(f^-_{a_1}\right)^\lambda, \left(f^+_{a_1}\right)^\lambda \right] >, \tag{7}$$

$$a_1^\lambda = < \left[\left(t^-_{a_1}\right)^\lambda, \left(t^+_{a_1}\right)^\lambda \right], \left[\sqrt[q]{1 - \left(1 - \left(f^-_{a_1}\right)^q\right)^\lambda}, \sqrt[q]{1 - \left(1 - \left(f^+_{a_1}\right)^q\right)^\lambda} \right] >. \tag{8}$$

Definition 4 (see [39]). For the IVq-ROFN $a = <[t^-_a, t^+_a], [f^-_a, f^+_a]>$, the score function is defined as

$$S(a) = \frac{1}{2}\left[(t^-_a)^q + (t^+_a)^q - (f^-_a)^q - (f^+_a)^q \right], (q \geq 1). \tag{9}$$

Definition 5 (see [39]). For the IVq-ROFN $a = <[t^-_a, t^+_a], [f^-_a, f^+_a]>$, the accuracy function is defined as

$$H(a) = \frac{1}{2}\left[(t^-_a)^q + (t^+_a)^q + (f^-_a)^q + (f^+_a)^q \right], (q \geq 1). \tag{10}$$

Definition 6 (see [39]). For two IVq-ROFNs a_1 and a_2, the comparison method is defined as follows:

(1) If $S(a_1) > S(a_2)$, then $a_1 > a_2$

(2) If $S(a_1) < S(a_2)$, then $a_1 < a_2$

(3) If $S(a_1) = S(a_2)$, and if $H(a_1) > H(a_2)$, then $a_1 > a_2$; if $H(a_1) < H(a_2)$, then $a_1 < a_2$; if $H(a_1) = H(a_2)$, then $a_1 = a_2$

Definition 7 (see [26]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse, f be a positive real-valued function, and μ be the fuzzy measure on X. Then, the discrete Choquet integral of f on fuzzy measure μ is defined as

$$\int f \mathrm{d}\mu = \sum_{i=1}^{n} f\left(x_{\sigma(i)}\right)\left[\mu\left(B_{\sigma(i)}\right) - \mu\left(B_{\sigma(i-1)}\right)\right], \tag{11}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ that satisfies $f(x_{\sigma(1)}) \geq f(x_{\sigma(2)}) \geq \cdots \geq f(x_{\sigma(n)})$, $B_{\sigma(I)} = \{x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(i)}\}$ $(i = 1, 2, \ldots, n, B_{\sigma(0)} = \varnothing)$.

3. Several Interval-Valued q-Rung Orthopair Fuzzy Choquet Integral Operators

In this section, we investigate IVq-ROFCA and IVq-ROFCG operators, discuss their properties, and extend their weighted operators.

3.1. IVq-ROFCA

Definition 8. Let μ be the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$ $(\mu(\varnothing) = 0)$ and $a(x_i) = <[t^-_a(x_i), t^+_a(x_i)], [f^-_a(x_i), f^+_a(x_i)]> (i = 1, 2, \ldots, n)$ are IVq-ROFNs. The IVq-ROFCA operator is defined as

$$\begin{aligned} (C_1)\int a \mathrm{d}\mu &= IVq-ROFCA\left(a(x_1), a(x_2), \ldots, a(x_n)\right) \\ &= \sum_{i=1}^{n}\left[\mu\left(B_{\sigma(i)}\right) - \mu\left(B_{\sigma(i-1)}\right)\right] a\left(x_{\sigma(i)}\right), \end{aligned} \tag{12}$$

where $(C_1)\int \alpha \mathrm{d}\mu$ indicates the Choquet integral, and $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ denotes a permutation of $(1, 2, \ldots, n)$ that satisfies $a(x_{\sigma(1)}) \geq a(x_{\sigma(2)}) \geq \cdots \geq a(x_{\sigma(n)})$, $B_{\sigma(I)} = \{x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(i)}\}$ $(i = 1, 2, \ldots, n, B_{\sigma(0)} = \varnothing)$. The IVq-

ROFCA operator aggregates information according to the fuzzy measures between attributes, and it is easy to find that the aggregation results obtained are still IVq-ROFNs.

Theorem 1. *For IVq-ROFNs* $a(x_i) = <[t_a^-(x_i), t_a^+(x_i)], [f_a^-(x_i), f_a^+(x_i)]> (i = 1, 2, \ldots, n)$, $\mu(\mu(\emptyset) = 0)$ *is the fuzzy measure on the nonempty finite set* $X = \{x_1, x_2, \ldots, x_n\}$, *and the IVq-ROFCA operator can be expressed as*

$$IVq-ROFCA(a(x_1), a(x_2), \ldots, a(x_n)) = <\left[\begin{array}{l}\sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}\end{array}\right],\left[\begin{array}{l}\prod_{i=1}^{n}\left(f_a^-\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}, \\ \prod_{i=1}^{n}\left(f_a^+\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}\end{array}\right]>. \tag{13}$$

Proof. Based on Definition 3, we prove equation (13) by mathematical induction.

If $n = 2$,

$$\begin{aligned} IVq-ROFCA(a(x_1), a(x_2)) &= \prod_{i=1}^{2}\left[\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)\right]a\left(x_{\sigma(i)}\right) \\ &= <\left[\begin{array}{l}\sqrt[q]{1-\prod_{i=1}^{2}\left(1-t_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}, \\ \sqrt[q]{1-\prod_{i=1}^{2}\left(1-t_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}\end{array}\right],\left[\begin{array}{l}\prod_{i=1}^{2}\left(f_a^-\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}, \\ \prod_{i=1}^{2}\left(f_a^+\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}\end{array}\right]>. \end{aligned} \tag{14}$$

If $n = k$,

$$IVq-ROFCA(a(x_1), a(x_2), \ldots, a(x_k)) = <\left[\begin{array}{l}\sqrt[q]{1-\prod_{i=1}^{k}\left(1-t_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}, \\ \sqrt[q]{1-\prod_{i=1}^{k}\left(1-t_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}\end{array}\right],\left[\begin{array}{l}\prod_{i=1}^{k}\left(f_a^-\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}, \\ \prod_{i=1}^{k}\left(f_a^+\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}\end{array}\right]>. \tag{15}$$

If $n = k + 1$, the results of IVq-ROFCA are as follows:

$$
\begin{aligned}
&IVq-ROFCA\left(a\left(x_1\right),a\left(x_2\right),\ldots,a\left(x_{k+1}\right)\right)=\prod_{i=1}^{k+1}\left[\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)\right]a\left(x_{\sigma(i)}\right)\\
&=\prod_{i=1}^{k}\left[\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)\right]a\left(x_{\sigma(i)}\right)\oplus\left[\mu\left(B_{\sigma(k+1)}\right)-\mu\left(B_{\sigma(k)}\right)\right]a\left(x_{\sigma(k+1)}\right)\\
&=<\begin{bmatrix}\sqrt[q]{1-\prod_{i=1}^{k}\left(1-t_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}},\\ \sqrt[q]{1-\prod_{i=1}^{k}\left(1-t_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}\end{bmatrix},\begin{bmatrix}\prod_{i=1}^{k}\left(f_a^-\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)},\\ \prod_{i=1}^{k}\left(f_a^+\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}\end{bmatrix}>\\
&\oplus<\begin{bmatrix}\sqrt[q]{1-\left(1-t_a^-\left(x_{\sigma(k+1)}\right)^q\right)^{\mu\left(B_{\sigma(k+1)}\right)-\mu\left(B_{\sigma(k)}\right)}},\\ \sqrt[q]{1-\left(1-t_a^+\left(x_{\sigma(k+1)}\right)^q\right)^{\mu\left(B_{\sigma(k+1)}\right)-\mu\left(B_{\sigma(k)}\right)}}\end{bmatrix},\begin{bmatrix}\left(f_a^-\left(x_{\sigma(k+1)}\right)\right)^{\mu\left(B_{\sigma(k+1)}\right)-\mu\left(B_{\sigma(k)}\right)},\\ \left(f_a^+\left(x_{\sigma(k+1)}\right)\right)^{\mu\left(B_{\sigma(k+1)}\right)-\mu\left(B_{\sigma(k)}\right)}\end{bmatrix}>\\
&=<\begin{bmatrix}\sqrt[q]{1-\prod_{i=1}^{k+1}\left(1-t_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}},\\ \sqrt[q]{1-\prod_{i=1}^{k+1}\left(1-t_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}\end{bmatrix},\begin{bmatrix}\prod_{i=1}^{k+1}\left(f_a^-\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)},\\ \prod_{i=1}^{k+1}\left(f_a^+\left(x_{\sigma(i)}\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}\end{bmatrix}>.
\end{aligned}
\tag{16}
$$

Equation (13) holds, and Theorem 1 holds. □

Example 1. Suppose there are three suppliers $\{x_1, x_2, x_3\}$ to be chosen. The core competitiveness of suppliers can be evaluated by three criteria $\{C_1, C_2, C_3\}$: C_1 denotes the level of technological innovation, C_2 denotes the ability of circulation control, and C_3 denotes the capability of management. The decision matrix $A = (a_{ij})_{3\times3}$ is generated by evaluation of experts, in which the IVq-ROFN evaluation values $a_{ij} = <[t^-_{a_{ij}}, t^+_{a_{ij}}], [f^-_{a_{ij}}, f^+_{a_{ij}}]> (i, j = 1, 2, 3)$ are represented in Table 1. Now, it needs to evaluate the core competitiveness of $\{x_1, x_2, x_3\}$ according to the decision matrix $A = (a_{ij})_{3\times3}$, which provides the reference for the manufacturing enterprise to select the best solution. The fuzzy measure μ between attributes are set as follows:

$$
\begin{aligned}
\mu(\varnothing) &= 0,\\
\mu(\{C_1, C_2, C_3\}) &= 1,\\
\mu(\{C_1\}) &= \mu(\{C_2\})\\
&= 0.4,\\
\mu(\{C_3\}) &= 0.3,\\
\mu(\{C_1, C_2\}) &= 0.5,\\
\mu(\{C_1, C_3\}) &= \mu(\{C_2, C_3\})\\
&= 0.8.
\end{aligned}
\tag{17}
$$

Let $q = 3$, all the IVq-ROFNs in A satisfy $t^+_{a_{ij}} + f^+_{a_{ij}} \le 1$, and the results are obtained as follows:

$$
\begin{aligned}
r_1 &= IVq-ROFCA\left(x_1(C_1), x_1(C_2), x_1(C_3)\right)\\
&= <[0.57, 0.77], [0.39, 0.54]>,\\
r_2 &= IVq-ROFCA\left(x_2(C_1), x_2(C_2), x_2(C_3)\right)\\
&= <[0.49, 0.70], [0.41, 0.57]>,\\
r_3 &= IVq-ROFCA\left(x_3(C_1), x_3(C_2), x_3(C_3)\right)\\
&= <[0.52, 0.72], [0.34, 0.46]>.
\end{aligned}
\tag{18}
$$

It is easy to find r_1, r_2, and r_3 are IVq-ROFNs. According to equation (9), $S(r_1) > S(r_3) > S(r_2)$ can be derived, which indicates that the supplier x_1 is better than x_2 and x_3.

Theorem 2. *Suppose $a_i = <[t^-_{a_i}, t^+_{a_i}], [f^-_{a_i}, f^+_{a_i}]> (i = 1, 2, \ldots, n)$ are IVq-ROFNs, μ is the fuzzy measure on a nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$, and then, the following four properties of the IVq-ROFCA operator hold*

(1) *Idempotency: for an IVq-ROFN $a = <[t^-_a, t^+_a], [f^-_a, f^+_a]>$, if $a_i = a (i = 1, 2, \ldots, n)$, then $IVq-ROFCA(a_1, a_2, \ldots, a_n) = a$*

(2) *Boundedness: if $a_{min} = <[min(t^-_{a_i}), min(t^+_{a_i})], [max(f^-_{a_i}), max(f^+_{a_i})]>$ and $a_{max} = <[max(t^-_{a_i}), max(t^+_{a_i})], [min(f^-_{a_i}), min(f^+_{a_i})]>$, then $a_{\min} \le IVq-ROFCA(a_1, a_2, \ldots, a_n) \le a_{max}$*

(3) *Commutativity: suppose $(a'_1, a'_2, \ldots, a'_n)$ is a permutation of $(a_1, a_2, \ldots, a_n)$, then $IVq-ROFCA(a_1, a_2, \ldots, a_n) = IVq-ROFCA(a'_1, a'_2, \ldots, a'_n)$*

(4) *Monotonicity: if $\beta_i = <[t^-_{\beta_i}, t^+_{\beta_i}], [f^-_{\beta_i}, f^+_{\beta_i}]> (i = 1, 2, \ldots, n)$ are IVq-ROFNs and $t^-_{a_i} \le t^-_{\beta_i}, t^+_{a_i} \le t^+_{\beta_i}, f^-_{a_i} \ge$*

Table 1: Decision matrix $A = (a_{ij})_{3\times 3}$.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$
$\mathbf{x}_1$	<[0.7, 0.9], [0.3, 0.5]>	<[0.3, 0.4], [0.5, 0.6]>	<[0.5, 0.6], [0.4, 0.5]>
$\mathbf{x}_2$	<[0.6, 0.8], [0.4, 0.5]>	<[0.3, 0.5], [0.5, 0.7]>	<[0.4, 0.7], [0.3, 0.5]>
$\mathbf{x}_3$	<[0.6, 0.8], [0.3, 0.4]>	<[0.4, 0.6], [0.4, 0.5]>	<[0.5, 0.7], [0.3, 0.5]>

$f^-_{\beta_i}, f^+_{a_i} \geq f^+_{\beta_i}$, then $IVq-ROFCA(a_1, a_2, \ldots, a_n) \leq IVq-ROFCA(\beta_1, \beta_2, \ldots, \beta_n)$

The proof of Theorem 2 is given as follows.

Proof. (1) Because all elements in the a_i are equal and $a_i = a$,

$$\begin{aligned}
\mu(x_i) &= \frac{1}{n} \text{ and } \mu\left(B_{\sigma(i)}\right) - \mu\left(B_{\sigma(i-1)}\right) \\
&= \frac{1}{n}, \\
&IVq-ROFCA(a_1, a_2, \ldots, a_n) \\
&= <\left[\begin{array}{c} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t^{-q}_{a_{\sigma(i)}}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t^{+q}_{a_{\sigma(i)}}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}} \end{array}\right], \left[\begin{array}{c} \prod_{i=1}^{n}\left(f^-_{a_{\sigma(i)}}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}, \\ \prod_{i=1}^{n}\left(f^+_{a_{\sigma(i)}}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \end{array}\right]> \\
&= <\left[\begin{array}{c} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t^{-q}_{a}\right)^{(1/n)}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t^{+q}_{a}\right)^{(1/n)}} \end{array}\right], \left[\begin{array}{c} \prod_{i=1}^{n}\left(f^-_{a}\right)^{(1/n)}, \\ \prod_{i=1}^{n}\left(f^+_{a}\right)^{(1/n)} \end{array}\right]> \\
&= <\left[\begin{array}{c} \sqrt[q]{1-\left(1-t^{-q}_{a}\right)}, \\ \sqrt[q]{1-\left(1-t^{+q}_{a}\right)} \end{array}\right], \left[\begin{array}{c} f^-_{a}, \\ f^+_{a} \end{array}\right]> \\
&= <\left[\begin{array}{c} \sqrt[q]{t^{-q}_{a}}, \\ \sqrt[q]{t^{+q}_{a}} \end{array}\right], \left[\begin{array}{c} f^-_{a}, \\ f^+_{a} \end{array}\right]> \\
&= a.
\end{aligned} \tag{19}$$

(2) According to equation (9), it can be seen that

$$\begin{aligned}
S\left(a_{\min}\right) &= \frac{1}{2}\left[\left(\min\left(t^-_{a_i}\right)\right)^q + \left(\min\left(t^+_{a_i}\right)\right)^q - \left(\max\left(f^-_{a_i}\right)\right)^q - \left(\max\left(f^+_{a_i}\right)\right)^q\right], \\
S\left(a_{\max}\right) &= \frac{1}{2}\left[\left(\max\left(t^-_{a_i}\right)\right)^q + \left(\max\left(t^+_{a_i}\right)\right)^q - \left(\min\left(f^-_{a_i}\right)\right)^q - \left(\min\left(f^+_{a_i}\right)\right)^q\right].
\end{aligned} \tag{20}$$

For any $t^-_{a_i}$, it satisfies

$$
\begin{aligned}
&\left(\min\left(t_{a_i}^-\right)\right)^q \le \left(t_{a_i}^-\right)^q \\
&\le \left(\max\left(t_{a_i}^-\right)\right)^q, 1-\left(\min\left(t_{a_i}^-\right)\right)^q \\
&\ge 1-\left(t_{a_i}^-\right)^q \\
&\ge 1-\left(\max\left(t_{a_i}^-\right)\right)^q; \\
&\prod_{i=1}^{n}\left(1-\left(\min\left(t_{a_i}^-\right)\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \\
&\ge \prod_{i=1}^{n}\left(1-\left(t_{a_i}^-\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \\
&\ge \prod_{i=1}^{n}\left(1-\left(\max\left(t_{a_i}^-\right)\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \\
&1-\prod_{i=1}^{n}\left(1-\left(\min\left(t_{a_i}^-\right)\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \\
&\le 1-\prod_{i=1}^{n}\left(1-\left(t_{a_i}^-\right)\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \\
&\le 1-\prod_{i=1}^{n}\left(1-\left(\max\left(t_{a_i}^-\right)\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}.
\end{aligned}
\tag{21}
$$

Namely,

$$
\left(\min\left(t_{a_i}^-\right)\right)^q \le 1-\prod_{i=1}^{n}\left(1-\left(t_{a_i}^-\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \le \left(\max\left(t_{a_i}^-\right)\right)^q.
\tag{22}
$$

Similarly,

$$
\begin{aligned}
&\left(\min\left(t_{a_i}^+\right)^q\right) \le -\prod_{i=1}^{n}\left(1-\left(t_{a_i}^+\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \le \left(\max\left(t_{a_i}^+\right)\right)^q, \\
&\left(\min\left(f_{a_i}^-\right)\right)^q \le \prod_{i=1}^{n}\left(\left(\left(f_{a_i}^-\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \le \left(\max\left(f_{a_i}^-\right)\right)^q, \\
&\left(\min\left(f_{a_i}^+\right)\right)^q \le \prod_{i=1}^{n}\left(\left(\left(f_{a_i}^-\right)^q\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \le \left(\max\left(f_{a_i}^+\right)\right)^q,
\end{aligned}
\tag{23}
$$

sorted out

$$
\begin{aligned}
S\left(a_{\min}\right) &\le S\left(IVq-ROFCA\left(a_1,a_2,\ldots,a_n\right)\right) \\
&\le S\left(a_{\max}\right), \\
a_{\min} &\le IVq-ROFCA\left(a_1,a_2,\ldots,a_n\right) \le a_{\max}.
\end{aligned}
\tag{24}
$$

(3) According to Definition 8, it can be easily derived that

$$
\begin{aligned}
&IVq-ROFCA\left(a_1,a_2,\ldots,a_n\right) = \\
&<\left[\begin{array}{l}\sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_{a_{\sigma(i)}}^{-q}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_{a_{\sigma(i)}}^{+q}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}\end{array}\right],\left[\begin{array}{l}\prod_{i=1}^{n}\left(f_{a_{\sigma(i)}}^-\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}, \\ \prod_{i=1}^{n}\left(f_{a_{\sigma(i)}}^+\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}\end{array}\right]> \\
&= IVq-ROFCA\left(a_1',a_2',\ldots,a_n'\right)\square.
\end{aligned}
\tag{25}
$$

(4) Based on $t_{a_i}^- \le t_{\beta_i}^-$ and Definition 8, it can be found that

$$
\begin{aligned}
t_{a_{\sigma(i)}}^{-q} &\le t_{\beta_{\sigma(i)}}^{-q}, \\
1-t_{a_{\sigma(i)}}^{-q} &\ge 1-t_{\beta_{\sigma(i)}}^{-q}.
\end{aligned}
\tag{26}
$$

Thus,

$$
\begin{aligned}
&\prod_{i=1}^{n}\left(1-t_{a_{\sigma(i)}}^{-q}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)} \ge \prod_{i=1}^{n}\left(1-t_{\beta_{\sigma(i)}}^{-q}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}, \\
&\sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_{a_{\sigma(i)}}^{-q}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}} \le \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_{\beta_{\sigma(i)}}^{-q}\right)^{\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i-1)}\right)}}.
\end{aligned}
\tag{27}
$$

Similarly,

$$\sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_{a_{\sigma(i)}}^{+q}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}} \leq \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_{\beta_{\sigma(i)}}^{+q}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}},$$
$$\prod_{i=1}^{n}\left(f_{a_{\sigma(i)}}^{-}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})} \geq \prod_{i=1}^{n}\left(f_{\beta_{\sigma(i)}}^{-}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}, \tag{28}$$
$$\prod_{i=1}^{n}\left(f_{a_{\sigma(i)}}^{+}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})} \geq \prod_{i=1}^{n}\left(f_{\beta_{\sigma(i)}}^{+}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}.$$

According to Definition 6, it is obviously discovered that $IVq-ROFCA(a_1, a_2, \ldots, a_n) \leq IVq-ROFCA(\beta_1, \beta_2, \ldots, \beta_n)$ □.

For some special relationships that exist between evaluation values in decision-making, we then develop the following operators to facilitate calculation. □

Definition 9. Let μ be the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$ $(\mu(\varnothing) = 0)$ and $a(x_i) = \langle [t_a^-(x_i), t_a^+(x_i)], [f_a^-(x_i), f_a^+(x_i)] \rangle$ $(i = 1, 2, \ldots, n)$ be IVq-ROFNs.

(1) If $\mu(B \cup C) = \mu(B) + \mu(C)$ for all $B, C \subseteq X$, $B \cap C = \varnothing$, and it is independent for any elements in X that means $\mu(B) = \sum_{x_i \in B}\mu(\{x_i\})$. The IVq-ROFCA operator transforms to the interval-valued q-rung orthopair fuzzy weighted Choquet average (IVq-ROFWCA) operator, which is expressed as

$$IVq-ROFWCA(a(x_1), a(x_2), \ldots, a(x_n)) = \left\langle \begin{bmatrix} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^-(x_{\sigma(i)})^q\right)^{\mu(\{x_i\})}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^+(x_{\sigma(i)})^q\right)^{\mu(\{x_i\})}} \end{bmatrix}, \begin{bmatrix} \prod_{i=1}^{n}\left(f_a^-(x_{\sigma(i)})\right)^{\mu(\{x_i\})}, \\ \prod_{i=1}^{n}\left(f_a^+(x_{\sigma(i)})\right)^{\mu(\{x_i\})} \end{bmatrix} \right\rangle. \tag{29}$$

(2) If $\mu(B) = \sum_{i=1}^{|B|}\lambda_i$ for all $B \subseteq X$, we have $\lambda_i = \mu(B_{\sigma(i)}) - \mu(B_{\sigma(i-1)})$ $(i = 1, 2, \ldots, n)$, where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ and $\sum_{i=1}^{n}\lambda_i = 1\,(\lambda_i \geq 0)$. Furthermore, the IVq-ROFCA operator reduces to the interval-valued q-rung orthopair fuzzy order Choquet average (IVq-ROFOCA) operator that is represented as

$$IVq-ROFOCA(a(x_1), a(x_2), \ldots, a(x_n)) = \left\langle \begin{bmatrix} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^-(x_{\sigma(i)})^q\right)^{\lambda_i}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^+(x_{\sigma(i)})^q\right)^{\lambda_i}} \end{bmatrix}, \begin{bmatrix} \prod_{i=1}^{n}\left(f_a^-(x_{\sigma(i)})\right)^{\lambda_i}, \\ \prod_{i=1}^{n}\left(f_a^+(x_{\sigma(i)})\right)^{\lambda_i} \end{bmatrix} \right\rangle. \tag{30}$$

(3) If $\mu(B) = Q(\sum_{x_i \in B}(\mu\{x_i\})$ for all $B \subseteq X$, where $Q$is a basic unit-interval monotonic function, satisfies monotonicity in [0,1] and follows properties: $(i) Q(0) = 0$; $(ii) Q(1) = 1$; $(iii)$for $x > y, Q(x) \geq Q(y)$. For $w_i = \mu(B_\sigma(i)) - \mu(B_{\sigma(i-1)}) = Q(\sum_{j \leq i}\mu(\{x_{\sigma(i)}\})) - Q(\sum_{j<i}\mu(\{x_{\sigma(i)}\}))$ $(i = 1, 2, \ldots, n)$, where $w = (w_1, w_2, \ldots, w_n)^T$ and $\sum_{i=1}^{n} w_i = 1\,(w_i \geq 0)$, the IVq-ROFCA operator changes to the interval-valued q-rung orthopair fuzzy order weighted Choquet average (IVq-ROFOWCA) operator that is proposed as

$$IVq-ROFOWCA(a(x_1), a(x_2), \ldots, a(x_n)) = \left\langle \begin{bmatrix} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^-(x_{\sigma(i)})^q\right)^{w_i}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-t_a^+(x_{\sigma(i)})^q\right)^{w_i}} \end{bmatrix}, \left[\prod_{i=1}^{n}\left(f_a^-(x_{\sigma(i)})\right)^{w_i}, \prod_{i=1}^{n}\left(f_a^+(x_{\sigma(i)})\right)^{w_i}\right] \right\rangle. \tag{31}$$

Especially, if $\mu(\{x_i\}) = (1/n)$ $(i = 1, 2, \ldots, n)$, then the IVq-ROFOWCA operator reduces to the IVq-ROFOCA operator. In addition, we develop the IVq-ROFCG operator in Section 3.2.

3.2. IVq-ROFCG

Definition 10. Let μ be the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$ $(\mu(\varnothing) = 0)$ and $a(x_i) = \langle [t_a^-(x_i), t_a^+(x_i)], [f_a^-(x_i), f_a^+(x_i)] \rangle$ $(i = 1, 2, \ldots, n)$ be IVq-ROFNs. Then, the IVq-ROFCG operator is defined as

$$(C_2)\int a\mathrm{d}\mu = IVq-ROFCG(a(x_1), a(x_2), \ldots, a(x_n)) = \prod_{i=1}^{n}\left[\mu(B_{\sigma(i)}) - \mu(B_{\sigma(i-1)})\right]a(x_{\sigma(i)}), \tag{32}$$

where $(C_2)\int a d\mu$ indicates the Choquet integral, $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots n)$, which satisfies $a(x_{\sigma(1)}) \geq a(x_{\sigma(2)}) \geq \cdots \geq a(x_{\sigma(n)})$, $B_{\sigma(i)} = \{x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(i)}\}$ $(i = 1, 2, \ldots, n, B_{\sigma(0)} = \varnothing)$.

Theorem 3. *If μ is the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$ and $a(x_i) = < [t_a^-(x_i), t_a^+(x_i)], [f_a^-(x_i), f_a^+(x_i)] > (i = 1, 2, \ldots n)$ are IVq-ROFNs, $\mu(\varnothing) = 0$. Then, the IVq-ROFCG operator is represented by*

$$IVq-ROFCG(a(x_1), a(x_2), \ldots, a(x_n)) = < \left[\begin{array}{l} \prod_{i=1}^{n}\left(t_a^-\left(x_{\sigma(i)}\right)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}, \\ \prod_{i=1}^{n}\left(t_a^+\left(x_{\sigma(i)}\right)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})} \end{array}\right], \left[\begin{array}{l} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-f_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-f_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})}} \end{array}\right] >. \quad (33)$$

The proof of Theorem 3 is similar to Theorem 1, and the proof is omitted.

Theorem 4. *If $a_i = < [t_{a_i}^-, t_{a_i}^+], [f_{a_i}^-, f_{a_i}^+] > (i = 1, 2, \ldots, n)$ is a set of IVq-ROFNs, μ is the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$, then the following properties of the IVq-ROFC G operator hold:*

(1) Idempotency: for an IVq-ROFN $a = < [t_a^-, t_a^+], [f_a^-, f_a^+] >$, if $a_i = a (i = 1, 2, \ldots, n)$, then $IVq-ROFCG(a_1, a_2, \ldots, a_n) = a$

(2) Boundedness: if $a_{min} = < [min(t_{a_i}^-), min(t_{a_i}^+)], [max(f_{a_i}^-), max(f_{a_i}^+)] >$ and $a_{max} = < [max(t_{a_i}^-), max(t_{a_i}^+)], [min(f_{a_i}^-), min(f_{a_i}^+)] >$, then $a_{\min} \leq IVq-ROFCG(a_1, a_2, \ldots, a_n) \leq a_{max}$

(3) Commutativity: suppose that $(a_1', a_2', \ldots, a_n')$ is a permutation of $(a_1, a_2, \ldots, a_n)$, then $IVq-ROFCG(a_1, a_2, \ldots, a_n) = IVq-ROFCG(a_1', a_2', \ldots, a_n')$

(4) Monotonicity: if $\beta_i = < [t_{\beta_i}^-, t_{\beta_i}^+], [f_{\beta_i}^-, f_{\beta_i}^+] > (i = 1, 2, \ldots, n)$ is a set of IVq-ROFNs and $t_{a_i}^- \leq t_{\beta_i}^-, t_{a_i}^+ \leq t_{\beta_i}^+, f_{a_i}^- \geq f_{\beta_i}^-, f_{a_i}^+ \geq f_{\beta_i}^+$, then $IVq-ROFCG(a_1, a_2, \ldots, a_n) \leq IVq-ROFCG(\beta_1, \beta_2, \ldots, \beta_n)$

The proof of Theorem 4 is similar to Theorem 2, and the proof is omitted.

Definition 11. Let μ be the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$ $(\mu(\varnothing) = 0)$ and $a(x_i) = < [t_a^-(x_i), t_a^+(x_i)], [f_a^-(x_i), f_a^+(x_i)] > (i = 1, 2, \ldots, n)$ be n IVq-ROFNs. Then,

(1) If $\mu(B \cup C) = \mu(B) + \mu(C)$ for all $B, C \subseteq X$, $B \cap C = \varnothing$, and it is independent for any elements in X that means $\mu(B) = \sum_{x_i \in B} \mu(\{x_i\})$. Then, the IVq-ROFCG operator transforms to the interval-valued q-rung orthopair fuzzy weighted Choquet geometric (IVq-ROFWCG) operator that is represented by

$$IVq-ROFWCG(a(x_1), a(x_2), \ldots, a(x_n)) = < \left[\prod_{i=1}^{n}\left(t_a^-\left(x_{\sigma(i)}\right)\right)^{\mu(\{x_i\})}, \prod_{i=1}^{n}\left(t_a^+\left(x_{\sigma(i)}\right)\right)^{\mu(\{x_i\})}\right], \left[\begin{array}{l} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-f_a^-\left(x_{\sigma(i)}\right)^q\right)^{\mu(\{x_i\})}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-f_a^+\left(x_{\sigma(i)}\right)^q\right)^{\mu(\{x_i\})}} \end{array}\right] >. \quad (34)$$

(2) If $\mu(B) = \sum_{i=1}^{|B|} \lambda_i$ for all $B \subseteq X$, we have $\lambda_i = \mu(B_{\sigma(i)}) - \mu(B_{\sigma(i-1)}) (i = 1, 2, \ldots, n)$, where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ and $\sum_{i=1}^{n} \lambda_i = 1 (\lambda_i \geq 0)$, the IVq-ROFCG operator reduces to the interval-valued q-rung orthopair fuzzy order Choquet geometric (IVq-ROFOCG) operator expressed by

$$IVq-ROFOCG(a(x_1), a(x_2), \ldots, a(x_n)) = < \left[\begin{array}{l} \prod_{i=1}^{n}\left(t_a^-\left(x_{\sigma(i)}\right)\right)^{\lambda_i}, \\ \prod_{i=1}^{n}\left(t_a^+\left(x_{\sigma(i)}\right)\right)^{\lambda_i} \end{array}\right], \left[\begin{array}{l} \sqrt[q]{1-\prod_{i=1}^{n}\left(1-f_a^-\left(x_{\sigma(i)}\right)^q\right)^{\lambda_i}}, \\ \sqrt[q]{1-\prod_{i=1}^{n}\left(1-f_a^+\left(x_{\sigma(i)}\right)^q\right)^{\lambda_i}} \end{array}\right] >. \quad (35)$$

(3) If $\mu(B)=Q(\sum_{x_i\in B}\mu(\{x_i\})$ for all $B\subseteq X$, where Q is a basic unit-interval monotonic function, satisfies monotonicity in [0,1] and follows properties: $(i)Q(0)=0$; $(ii)Q(1)=1$; $(iii)Q(x)\geq Q(y)$ for $x>y$. Then, we let $w_i=\mu(B_{\sigma(i)})-\mu(B_{\sigma(i-1)})=Q(\sum_{j\leq i}\mu(\{x_{\sigma(i)}\}))-Q(\sum_{j<i}\mu(\{x_{\sigma(i)}\}))$ $(i=1,2,\ldots,n)$, where $w=(w_1,w_2,\ldots,w_n)^T$ and $\sum_{i=1}^n w_i=1(w_i\geq 0)$. Then, the IVq-ROFCG operator changes to the interval-valued q-rung orthopair fuzzy order weighted Choquet geometric (IVq-ROFOWCG) operator that is shown as

$$IVq-ROFOWCG(a(x_1),a(x_2),\ldots,a(x_n))=$$
$$<\left[\begin{array}{c}\prod_{i=1}^n\left(t_a^-\left(x_{\sigma(i)}\right)\right)^{w_i},\\ \prod_{i=1}^n\left(t_a^+\left(x_{\sigma(i)}\right)\right)^{w_i}\end{array}\right],\left[\begin{array}{c}\sqrt[q]{1-\prod_{i=1}^n\left(1-f_a^-\left(x_{\sigma(i)}\right)^q\right)^{w_i}},\\ \sqrt[q]{1-\prod_{i=1}^n\left(1-f_a^+\left(x_{\sigma(i)}\right)^q\right)^{w_i}}\end{array}\right]>. \quad (36)$$

Especially, if $\mu(\{x_i\})=(1/n)(i=1,2,\ldots,n)$, it is obvious to find that the IVq-ROFOWCG operator reduces to the IVq-ROFOCG operator.

4. A MAGDM Method Based on the IVq-ROFCA Operator

In this section, we develop a MAGDM method based on the IVq-ROFCA operator. For a MAGDM problem, $X=\{x_1,x_2,\ldots,x_m\}$ is the set of alternatives, $C=\{C_1,C_2,\ldots,C_n\}$ is the set of interaction between attributes, and $\mu(C_j)(j=1,2,\ldots,n)$ is the fuzzy measure of attributes, $e=\{e_1,e_2,\ldots,e_t\}$ indicates the set of experts, and $\mu(e_k)(k=1,2,\ldots,t)$ is the fuzzy measure of experts. For each expert, a decision matrix of different alternatives is $A=(a_{ij})_{m\times n}=<[t_{a_{ij}}^-,t_{a_{ij}}^+],[f_{a_{ij}}^-,f_{a_{ij}}^+]>$, where a_{ij} are IVq-ROFNs that satisfy $(t_{a_{ij}}^+)^q+(f_{a_{ij}}^+)^q\leq 1(q\geq 1)$. For the k th expert e_k, the decision matrix is expressed as $A^{(k)}=(a_{ij}^{(k)})_{m\times n}$, $a_{ij}^{(k)}$ denotes the kth expert's evaluation of alternative x_i with the attribute C_j. Because of the differences between physical dimensions of attributes, the decision matrix needs to be standardized. Correspondingly, Ω_1 indicates the benefit type and Ω_2 indicates the cost type, and the standardization processing is shown in equation (37) and the complement operation of fuzzy numbers is shown in equation (38):

$$r_{ij}^{(k)}=\begin{cases}a_{ij}^{(k)}, & a_{ij}^{(k)}\in\Omega_1,\\ \left(a_{ij}^{(k)}\right)^c, & a_{ij}^{(k)}\in\Omega_2,\end{cases}(i=1,2,\ldots,m,j=1,2,\ldots,n), \quad (37)$$

$$\left(a_{ij}^{(k)}\right)^c=<\left[f_{a_{ij}^{(k)}}^-,f_{a_{ij}^{(k)}}^+\right],\left[t_{a_{ij}^{(k)}}^-,t_{a_{ij}^{(k)}}^+\right]>(i=1,2,\ldots,m,j=1,2,\ldots,n). \quad (38)$$

The steps of the MAGDM method based on the IVq-ROFCA operator are as follows:

(1) According to the evaluation matrix given by experts, the appropriate value of q is chosen. When the amount of data is small, the value of q can be determined by observation. If the amount of data is large, q can be determined by the traversal method utilizing $(t_{a_{ij}^{(k)}}^+)^q+(f_{a_{ij}^{(k)}}^+)^q\leq 1(q\geq 1)$.

(2) Using equations (37) and (38), the decision matrix $A^{(k)}$ is converted into the standardize matrix $A'^{(k)}$.

(3) $A'^{(k)}=(a_{ij}'^{(k)})_{m\times n}$ is aggregated into $R=(r_{ij})_{m\times n}$ employing the IVq-ROFCA operator, and the collection matrix is derived by

$$r_{ij}=IVq-ROFCA\left(a_{ij}'^{(1)},a_{ij}'^{(2)},\ldots,a_{ij}'^{(t)}\right)(i=1,2,\ldots,m,j=1,2,\ldots,n), \quad (39)$$

where $a_{ij}'^{(t)}$ represents the IVq-ROFNs in the decision matrix given by the t-th expert.

(4) Then, the collective value r_i of x_i is derived by equation (40), which aggregates the fuzzy information corresponding to attributes:

$$r_i=IVq-ROFCA(r_{i1},r_{i2},\ldots,r_{in})(i=1,2,\ldots,m). \quad (40)$$

(5) The score value and the accuracy value of r_i are obtained according to equations (9) and (10), which determine the ranking of alternatives.

5. Case of the Warning Management System for Hypertension

5.1. Decision Result by the Proposed MAGDM Method. In order to improve the management efficiency of doctors, we

Table 2: The fuzzy measures of attributes.

Fuzzy measures of $\{\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3, \mathbf{C}_4\}$	
$\mu(\{\varnothing\}) = 0$	$\mu(\{C_2, C_3\}) = 0.45$
$\mu(\{C_1\}) = 0.2$	$\mu(\{C_2, C_4\}) = 0.5$
$\mu(\{C_2\}) = 0.2$	$\mu(\{C_3, C_4\}) = 0.57$
$\mu(\{C_3\}) = 0.2\ 5$	$\mu(\{C_1, C_2, C_3\}) = 0.65$
$\mu(\{C_4\}) = 0.35$	$\mu(\{C_1, C_2, C_4\}) = 0.77$
$\mu(\{C_1, C_2\}) = 0.42$	$\mu(\{C_1, C_3, C_4\}) = 0.8$
$\mu(\{C_1, C_3\}) = 0.45$	$\mu(\{C_2, C_3, C_4\}) = 0.8$
$\mu(\{C_1, C_4\}) = 0.5$	$\mu(\{C_1, C_2, C_3, C_4\}) = 1.0$

plan to develop a daily follow-up warning management system for hypertension. The warning degrees are divided into five levels that are represented in five colors denoted by $X = \{x_1, x_2, x_3, x_4, x_5\}$. x_1 indicates that the resident shall seek treatment immediately, showing red; x_2 indicates the resident needs treatment in time, showing orange; x_3 indicates the resident should be treated for the timely follow-up to promote management, showing yellow; x_4 indicates that blood pressure management may be required in the future, showing blue; and x_5 indicates that the blood pressure of the resident is normal, and the color is green. The warning levels are related to numerous factors of hypertension patients, among which the blood pressure measurements, related diseases, related risk factors, and follow-up time intervals are represented as the attributes denoted by $C = \{C_1, C_2, C_3, C_4\}$. From a managerial perspective, C_1, C_2, and C_3 are benefit types, and C_4 is the cost type. With different importance, the fuzzy measures of attributes are listed in Table 2.

Without taking medicine, a 60-year-old resident has a measured value and related factors as follows: systolic blood pressure is 153 mmHg, diastolic blood pressure is 98 mmHg, there is no relevant disease record, obesity and family genetic history, and the doctor had no follow-up record of him. Three experts $e = \{e_1, e_2, e_3\}$ are invited to evaluate the warning degree of this resident using IVq-ROFNs, and the fuzzy measures of them are $\mu\{e_1\} = \mu\{e_2\} = \mu\{e_3\} = 0.4$, $\mu\{e_1, e_2\} = \mu\{e_2, e_3\} = \mu\{e_1, e_3\} = 0.7$, $\mu\{e_1, e_2, e_3\} = 1$. Tables 3–5 are the evaluation values of the three experts after standardization. Subsequently, the decision result is determined utilizing the proposed MAGDM method.

We set $q = 3$ via observation, and then, the collective matrix R is obtained by equation (39) which is presented in Table 6.

The comprehensive attribute values of each alternative are obtained through equation (40), and the results are obtained as follows:

$$\begin{aligned}
r_1 &= <[0.63, 0.74], [0.32, 0.46]>,\\
r_2 &= <[0.67, 0.75], [0.27, 0.38]>,\\
r_3 &= <[0.71, 0.78], [0.32, 0.42]>,\\
r_4 &= <[0.56, 0.64], [0.40, 0.50]>,\\
r_5 &= <[0.47, 0.59], [0.57, 0.68]>.
\end{aligned} \tag{41}$$

The score of each alternative is determined by equation (9):

$$\begin{aligned}
S(r_1) &= 0.53,\\
S(r_2) &= 0.65,\\
S(r_3) &= 0.73,\\
S(r_4) &= 0.25,\\
S(r_5) &= -0.19.
\end{aligned} \tag{42}$$

The ranking result of alternatives' scores is derived as follows:

$$S(r_3) > S(r_2) > S(r_1) > S(r_4) > S(r_5). \tag{43}$$

Therefore, the ranking of alternatives is $x_3 > x_2 > x_1 > x_4 > x_5$, and x_3 is the optimal selection, which means the doctor is required for timely follow-up to promote management of blood pressure of this patient who receives yellow warning.

5.2. Parameter Analysis. For further studying the influence of q, we analyze different values of q in this subsection, which are 2, 3, 4, and 5 in this case, and the results are listed in Table 7 and Figure 1.

It can be seen from Table 7 that the score rankings are always $S(r_3) > S(r_2) > S(r_1) > S(r_4) > S(r_5)$, and the early-warning results are all yellow warnings, which are consistent

Table 3: Decision matrix $A'^{(1)}$ by e_1.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{x}_1$	< [0.7,0.8], [0.2,0.3] >	< [0.5,0.6], [0.6,0.7] >	< [0.8,0.9], [0.1,0.2] >	<[0.1,0.2], [0.8,0.95] >
$\mathbf{x}_2$	< [0.8,0.9], [0.1,0.2] >	<[0.5,0.55], [0.4,0.5] >	<[0.8,0.9], [0.1,0.15] >	<[0.1,0.2], [0.8,0.95] >
$\mathbf{x}_3$	<[0.9,0.95], [0.1,0.2] >	< [0.5,0.6], [0.3,0.4] >	<[0.8,0.85], [0.2,0.3] >	< [0.1,0.2], [0.8,0.9] >
$\mathbf{x}_4$	< [0.6,0.7], [0.4,0.5] >	< [0.4,0.5], [0.3,0.4] >	<[0.75,0.8], [0.2,0.3] >	< [0.2,0.3], [0.7,0.8] >
$\mathbf{x}_5$	< [0.1,0.2], [0.8,0.9] >	< [0.2,0.3], [0.5,0.6] >	<[0.7,0.85], [0.3,0.4] >	< [0.2,0.3], [0.6,0.7] >

Table 4: Decision matrix $A'^{(2)}$ by e_2.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{x}_1$	<[0.6,0.7],[0.2,0.3]>	<[0.6,0.65],[0.4,0.5]>	<[0.8,0.9], [0.1,0.2]>	<[0.1,0.2],[0.8,0.95]>
$\mathbf{x}_2$	<[0.8,0.85],[0.1,0.15]>	<[0.5,0.55],[0.4,0.5]>	<[0.8,0.85],[0.1,0.15]>	<[0.1,0.2],[0.8,0.95]>
$\mathbf{x}_3$	<[0.85,0.9],[0.1,0.15]>	<[0.5,0.6], [0.3,0.4]>	<[0.8,0.85], [0.2,0.3]>	<[0.1,0.2],[0.85,0.9]>
$\mathbf{x}_4$	<[0.4,0.5],[0.3,0.4]>	<[0.4,0.5], [0.3,0.4]>	<[0.75,0.8], [0.2,0.3]>	<[0.2,0.3],[0.85,0.9]>
$\mathbf{x}_5$	<[0.1,0.2],[0.8,0.95]>	<[0.2,0.3], [0.5,0.6]>	<[0.7,0.85], [0.3,0.4]>	<[0.2,0.3],[0.7,0.85]>

Table 5: Decision matrix $A'^{(3)}$ by e_3.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{x}_1$	< [0.7,0.8],[0.2,0.3] >	<[0.5,0.6],[0.4,0.5] >	<[0.8,0.9], [0.1,0.2]>	< [0.1,0.2],[0.8,0.95] >
$\mathbf{x}_2$	< [0.85,0.9],[0.1,0.2] >	<[0.5,0.55],[0.4,0.5] >	<[0.8,0.85],[0.1,0.15]>	< [0.1,0.2],[0.8,0.95] >
$\mathbf{x}_3$	<[0.9,0.95],[0.15,0.2] >	< [0.6,0.7], [0.3,0.4] >	<[0.8,0.85],[0.2,0.3]>	< [0.1,0.2],[0.85,0.9] >
$\mathbf{x}_4$	< [0.6,0.7],[0.4,0.5] >	< [0.5,0.6], [0.3,0.4] >	<[0.75,0.85],[0.2,0.3] >	< [0.2,0.3],[0.85,0.9] >
$\mathbf{x}_5$	< [0.1,0.2],[0.8,0.9] >	< [0.4,0.5], [0.7,0.8] >	< [0.75,0.8], [0.3,0.4] >	<[0.2,0.3],[0.75,0.85] >

Table 6: Collective matrix R.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{r}_1$	<[0.67,0.78], [0.20,0.30]>	<[0.54,0.62], [0.47,0.57]>	<[0.80,0.90], [0.10,0.20]>	<[0.10,0.20], [0.80,0.95]>
$\mathbf{r}_2$	<[0.82,0.89], [0.10,0.18]>	<[0.50,0.55], [0.40,0.50]>	<[0.80,0.87], [0.10,0.15]>	<[0.10,0.20], [0.80,0.95]>
$\mathbf{r}_3$	<[0.89,0.94], [0.11,0.18]>	<[0.54,0.64], [0.30,0.40]>	<[0.80,0.85], [0.20,0.30]>	<[0.10,0.20], [0.83,0.90] >
$\mathbf{r}_4$	<[0.56,0.66], [0.37,0.47]>	<[0.44,0.54], [0.30,0.40]>	<[0.75,0.82], [0.20,0.30]>	<[0.20,0.30], [0.79,0.86]>
$\mathbf{r}_5$	<[0.10,0.20], [0.80,0.91]>	<[0.29,0.39], [0.55,0.65]>	<[0.72,0.84], [0.30,0.40]>	<[0.20,0.30], [0.67,0.79]>

with the results obtained in Table 7. Hence, the decision result does not change due to the variation in the value of q. However, with the increase in the q value, it can be discovered that scores of the alternatives are declining completely but show different trends of downward. Considering the practical application, there is not much practical significance for IVq-ROFNs with the large value of q, which often does not exceed 5. Therefore, the value of q will not have an impact on the final decision result.

5.3. Comparison Analysis. In this subsection, the proposed method is used to solve the case in [32], which is compared with the method given in [32, 40]. There are 5 alternatives with 4 attributes C_1, C_2, C_3, and C_4 for 3 experts e_1, e_2, and e_3 to make a decision, and the evaluation matrices $A^{(1)}$, $A^{(2)}$, and $A^{(3)}$ of interval-valued intuitionistic fuzzy numbers (IVIFNs) given are listed in Tables 8–10. Besides, the fuzzy measures of experts are $\mu\{e_1\} = \mu\{e_2\} = \mu\{e_3\} = 0.4$, $\mu\{e_1, e_2\} = \mu\{e_2, e_3\} = \mu\{e_1, e_3\} = 0.73$, and $\mu\{e_1, e_2, e_3\} = 1$, and the fuzzy measures of attributes are shown in Table 11.

If μ is the fuzzy measure on the nonempty finite set $X = \{x_1, x_2, \ldots, x_n\}$ $(\mu(\varnothing) = 0)$, for an IVIFS $\hat{a}(x_i) = <[t_{\hat{a}}^{-}(x_i), t_{\hat{a}}^{+}(x_i)], \ [f_{\hat{a}}^{-}(x_i), f_{\hat{a}}^{+}(x_i)]> (i = 1, 2, \ldots, n)$, the GIIFGA operator and the interval-valued

Table 7: Decision result with $q = 2,3,4$, and5.

q	Results of IVq-ROFCA	Score of alternatives	Ranking of alternatives
2	$r_1 = <[0.60, 0.71], [0.32, 0.46]> r_2 = <[0.64, 0.73], [0.27, 0.38]> r_3 = <[0.69, 0.76], [0.32, 0.42]> r_4 = <[0.53, 0.62], [0.40, 0.50]> r_5 = <[0.42, 0.54], [0.57, 0.68]>$	$S(r_1) = 0.55 S(r_2) = 0.73 S(r_3) = 0.77 S(r_4) = 0.26 S(r_5) = -0.32$	$x_3 > x_2 > x_1 > x_4 > x_5$
3	$r_1 = <[0.63, 0.74], [0.32, 0.46]> r_2 = <[0.67, 0.75], [0.27, 0.38]> r_3 = <[0.71, 0.78], [0.32, 0.42]> r_4 = <[0.56, 0.64], [0.40, 0.50]> r_5 = <[0.47, 0.59], [0.57, 0.68]>$	$S(r_1) = 0.53 S(r_2) = 0.65 S(r_3) = 0.73 S(r_4) = 0.25 S(r_5) = -0.19$	$x_3 > x_2 > x_1 > x_4 > x_5$
4	$r_1 = <[0.65, 0.75], [0.32, 0.46]> r_2 = <[0.69, 0.77], [0.27, 0.38]> r_3 = <[0.73, 0.80], [0.32, 0.42]> r_4 = <[0.58, 0.66], [0.40, 0.50]> r_5 = <[0.51, 0.62], [0.57, 0.68]>$	$S(r_1) = 0.44 S(r_2) = 0.55 S(r_3) = 0.65 S(r_4) = 0.21 S(r_5) = -0.10$	$x_3 > x_2 > x_1 > x_4 > x_5$
5	$r_1 = <[0.67, 0.77], [0.32, 0.46]> r_2 = <[0.71, 0.78], [0.27, 0.38]> r_3 = <[0.75, 0.81], [0.32, 0.42]> r_4 = <[0.60, 0.68], [0.40, 0.50]> r_5 = <[0.54, 0.65], [0.57, 0.68]>$	$S(r_1) = 0.37 S(r_2) = 0.46 S(r_3) = 0.57 S(r_4) = 0.18 S(r_5) = -0.04$	$x_3 > x_2 > x_1 > x_4 > x_5$

Table 8: Decision matrix $A^{(1)}$.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{x}_1$	$<[0.4,0.5],[0.3,0.4]>$	$<[0.4,0.6],[0.2,0.4]>$	$<[0.1,0.3],[0.5,0.6]>$	$<[0.3,0.4],[0.3,0.5]>$
$\mathbf{x}_2$	$<[0.6,0.7],[0.2,0.3]>$	$<[0.6,0.7],[0.2,0.3]>$	$<[0.4,0.7],[0.1,0.2]>$	$<[0.5,0.6],[0.1,0.3]>$
$\mathbf{x}_3$	$<[0.6,0.7],[0.1,0.2]>$	$<[0.5,0.6],[0.3,0.4]>$	$<[0.5,0.6],[0.1,0.3]>$	$<[0.4,0.5],[0.2,0.4]>$
$\mathbf{x}_4$	$<[0.3,0.4],[0.2,0.3]>$	$<[0.6,0.7],[0.1,0.3]>$	$<[0.3,0.4],[0.1,0.2]>$	$<[0.3,0.7],[0.1,0.2]>$
$\mathbf{x}_5$	$<[0.7,0.8],[0.1,0.2]>$	$<[0.3,0.5],[0.1,0.3]>$	$<[0.5,0.6],[0.2,0.3]>$	$<[0.3,0.4],[0.5,0.6]>$

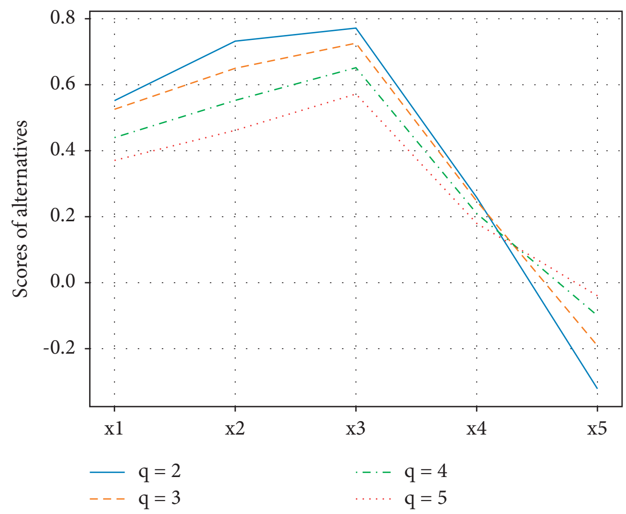

Figure 1: Decision result with $q = 2,3,4$, and 5.

Table 9: Decision matrix $A^{(2)}$.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{x}_1$	$<[0.3,0.4],[0.4,0.5]>$	$<[0.5,0.6],[0.1,0.3]>$	$<[0.4,0.5],[0.3,0.4]>$	$<[0.4,0.6],[0.2,0.4]>$
$\mathbf{x}_2$	$<[0.3,0.6],[0.3,0.4]>$	$<[0.4,0.7],[0.1,0.2]>$	$<[0.5,0.6],[0.2,0.3]>$	$<[0.6,0.7],[0.2,0.3]>$
$\mathbf{x}_3$	$<[0.6,0.8],[0.1,0.2]>$	$<[0.5,0.6],[0.1,0.2]>$	$<[0.5,0.7],[0.2,0.3]>$	$<[0.1,0.3],[0.5,0.6]>$
$\mathbf{x}_4$	$<[0.4,0.5],[0.3,0.5]>$	$<[0.5,0.8],[0.1,0.2]>$	$<[0.2,0.5],[0.3,0.4]>$	$<[0.4,0.7],[0.1,0.2]>$
$\mathbf{x}_5$	$<[0.6,0.7],[0.2,0.3]>$	$<[0.6,0.7],[0.1,0.2]>$	$<[0.5,0.7],[0.2,0.3]>$	$<[0.6,0.7],[0.1,0.3]>$

Table 10: Decision matrix $A^{(3)}$.

	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{C}_4$
$\mathbf{x}_1$	$<[0.2,0.5],[0.3,0.4]>$	$<[0.4,0.5],[0.1,0.2]>$	$<[0.3,0.6],[0.2,0.3]>$	$<[0.3,0.7],[0.1,0.3]>$
$\mathbf{x}_2$	$<[0.2,0.7],[0.2,0.3]>$	$<[0.3,0.6],[0.2,0.4]>$	$<[0.4,0.7],[0.1,0.2]>$	$<[0.5,0.8],[0.1,0.2]>$
$\mathbf{x}_3$	$<[0.5,0.6],[0.3,0.4]>$	$<[0.7,0.8],[0.1,0.2]>$	$<[0.5,0.6],[0.2,0.3]>$	$<[0.4,0.5],[0.3,0.4]>$
$\mathbf{x}_4$	$<[0.3,0.6],[0.2,0.4]>$	$<[0.4,0.6],[0.2,0.3]>$	$<[0.1,0.4],[0.3,0.6]>$	$<[0.3,0.7],[0.1,0.2]>$
$\mathbf{x}_5$	$<[0.6,0.7],[0.1,0.3]>$	$<[0.5,0.6],[0.3,0.4]>$	$<[0.5,0.6],[0.2,0.3]>$	$<[0.5,0.6],[0.2,0.4]>$

Table 11: Fuzzy measures of attributes.

Fuzzy measures of attributes	
$\mu\{C_1\} = 0.4$	$\mu\{C_2, C_4\} = 0.43$
$\mu\{C_2\} = 0.25$	$\mu\{C_3, C_4\} = 0.54$
$\mu\{C_3\} = 0.37$	$\mu\{C_1, C_2, C_3\} = 0.88$
$\mu\{C_4\} = 0.2$	$\mu\{C_1, C_2, C_4\} = 0.75$
$\mu\{C_1, C_2\} = 0.6$	$\mu\{C_1, C_3, C_4\} = 0.84$
$\mu\{C_1, C_3\} = 0.7$	$\mu\{C_2, C_3, C_4\} = 0.73$
$\mu\{C_1, C_4\} = 0.56$	$\mu\{C_1, C_2, C_3, C_4\} = 1$
$\mu\{C_2, C_3\} = 0.68$	

intuitionistic fuzzy Einstein geometric Choquet integral (IVIFEGC) operator are defined as follows:

$$
\begin{aligned}
& GIIFGA(\hat{a}(x_1),\hat{a}(x_2),\ldots,\hat{a}(x_n)) = \\
& <\left[\begin{array}{c}\prod_{i=1}^{n}\left(t_{\hat{a}}^{-}(x_{\sigma(i)})\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})},\\ \prod_{i=1}^{n}\left(t_{\hat{a}}^{+}(x_{\sigma(i)})\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}\end{array}\right],\left[\begin{array}{c}1-\prod_{i=1}^{n}\left(1-f_{\hat{a}}^{-}(x_{\sigma(i)})\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})},\\ 1-\prod_{i=1}^{n}\left(1-f_{\hat{a}}^{+}(x_{\sigma(i)})\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}\end{array}\right]>, \\
& IVIFEGC(\hat{a}(x_1),\hat{a}(x_2),\ldots,\hat{a}(x_n)) = \\
& <\left[\begin{array}{c}\dfrac{2\prod_{j=1}^{n} t_{\hat{a}}^{-}\left(x_{\sigma(i)}\right)_{\sigma(j)}^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}}{\prod_{j=1}^{n}\left(2-t_{\hat{a}}^{-}\left(x_{\sigma(i)}\right)_{\sigma(j)}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}+\prod_{j=1}^{n} t_{\hat{a}}^{-}\left(x_{\sigma(i)}\right)_{\sigma(j)}^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}},\\ \dfrac{2\prod_{j=1}^{n} t_{\hat{a}}^{+}(x_i)_{\sigma(j)}^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}}{\prod_{j=1}^{n}\left(2-t_{\hat{a}}^{+}(x_i)_{\sigma(j)}\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}+\prod_{j=1}^{n} t_{\hat{a}}^{+}(x_i)_{\sigma(j)}^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}}\end{array}\right], \\
& \left[\begin{array}{c}\dfrac{\prod_{j=1}^{n}\left(1+f_{\hat{a}}^{-}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}-\prod_{j=1}^{n}\left(1-f_{\hat{a}}^{-}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}}{\prod_{j=1}^{n}\left(1+f_{\hat{a}}^{-}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}+\prod_{j=1}^{n}\left(1-f_{\hat{a}}^{-}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}},\\ \dfrac{\prod_{j=1}^{n}\left(1+f_{\hat{a}}^{+}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}-\prod_{j=1}^{n}\left(1-f_{\hat{a}}^{+}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}}{\prod_{j=1}^{n}\left(1+f_{\hat{a}}^{+}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}+\prod_{j=1}^{n}\left(1-f_{\hat{a}}^{+}(x_i)\right)^{\mu(B_{\sigma(i)})-\mu(B_{\sigma(i+1)})}}\end{array}\right]>,
\end{aligned}
\tag{44}
$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots n)$ that satisfies $\hat{a}(x_{\sigma(1)})\le\hat{a}(x_{\sigma(2)})\le\cdots\le\hat{a}(x_{\sigma(n)}), B_{\sigma(i)} = \{x_{\sigma(i)}, x_{\sigma(i+1)},\ldots,x_{\sigma(n)}\}\ (i = 1,2,\ldots,n, B_{\sigma(n+1)} = \varnothing)$.

In this case, we set $q=1$, IVq-ROFNs are equal to IVIFNs, which are applied to the comparison with existing methods. The obtained results of different methods are shown in Figure 2 and Table 12.

In this case, compared with the method proposed in [40], the same best and worst solutions are obtained by the method proposed in this paper, and obvious differences in the scores of alternatives are discovered. However, there are differences between x_2 and x_3, and the small deviation is derived between them. From the method proposed in [32], the decision result is completely different from the other two methods, and the closeness of each alternative has a few differences. We analyze the reason as follows.

In this case, the membership degrees of IVIFNs are generally greater than nonmembership degrees. From the method proposed in [32], multiple decision matrices are aggregated by CITOPSIS, which reduces the difference in the influence of the membership and nonmembership of each alternative on the results. Thus, the deviations of alternatives are not obvious. Contrarily, the method proposed in this paper employs IVq-ROFCA to process fuzzy information, which further highlights the degree of expert support for the alternatives. The collective matrices of GIIFGA and IVq-ROFCA of Tables 8–10 are listed in Tables 13 and 14, respectively. According to equation (9), the scores of these two matrices are presented in Figure 3.

From Figure 3, the abscissa in the figure represents the alternatives with different attributes and the ordinate indicates their scores. It is easy to find there are only few differences between membership and nonmembership, except for the evaluation of x_2 with C_1. Correspondingly, the span of the interval in membership with a wide range can be found in the experts' evaluation of x_2 with C_1 from Tables 8–10, such as [0.2, 0.7]. Considering the above, there are differences in the selection of x_2 between the method proposed in this paper and the proposed method in [32]. On the other hand, it reflects that more advantages of the method proposed in this paper will be discovered, while the

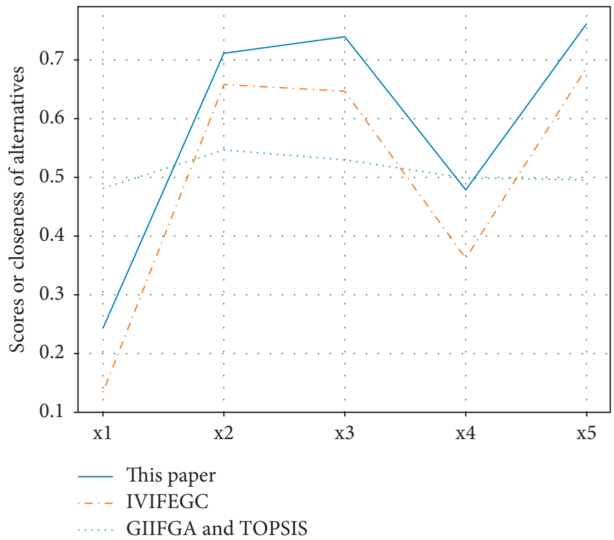

FIGURE 2: Ranking of different methods.

TABLE 12: Results by different methods.

Operators	Score or closeness	Alternative ranking
Proposed method	Score: $S(x_1) = 0.16S(x_2) = 0.71S(x_3) = 0.74S(x_4) = 0.38S(x_5) = 0.77$	$x_5 > x_3 > x_2 > x_4 > x_1$
IVIFEGC [40]	Score: $S(x_1) = 0.05S(x_2) = 0.66S(x_3) = 0.65S(x_4) = 0.26S(x_5) = 0.70$	$x_5 > x_2 > x_3 > x_4 > x_1$
Choquet integral-based TOPSIS (CITOPSIS) [32]	Closeness: $r(x_1) = 0.4817r(x_2) = 0.5465r(x_3) = 0.5297r(x_4) = 0.4990r(x_5) = 0.4958$	$x_2 > x_3 > x_4 > x_5 > x_1$

TABLE 13: Collective matrix by GIIFGA.

	C_1	C_2	C_3	C_4
x_1	<[0.3017,0.4645], [0.2685,0.3687]>	<[0.4373,0.5650], [0.1282,0.2983]>	<[0.2452,0.4685], [0.3257,0.4280]>	<[0.3299,0.5720], [0.1911,0.3925]>
x_2	<[0.3463,0.5386], [0.2917,0.3925]>	<[0.4353,0.6715], [0.1683,0.2983]>	<[0.4248,0.6715], [0.1282,0.2283]>	<[0.5310,0.7083], [0.1343,0.2616]>
x_3	<[0.5712,0.7083], [0.1590,0.2598]>	<[0.5720,0.6732], [0.1590,0.2598]>	<[0.5000,0.6382], [0.1683,0.3000]>	<[0.2751,0.4356], [0.3257,0.4622] >
x_4	<[0.3242,0.4996], [0.2283,0.3990]>	<[0.5000,0.7083], [0.1282,0.2616]>	<[0.1951,0.4306], [0.2260,0.3966]>	<[0.3366,0.7000], [0.1000,0.2000]>
x_5	<[0.6382,0.7384], [0.1282,0.2616]>	<[0.4685,0.6075], [0.1716,0.2982]>	<[0.5000,0.6382], [0.2000,0.3000]>	<[0.4685,0.5720], [0.2614,0.4280]>

TABLE 14: Collective matrix by IVq-ROFCA.

	C_1	C_2	C_3	C_4
$\mathbf{x}_1$	<[0.3122,0.4748], [0.3242,0.4248]>	<[0.4288,0.5694], [0.1320,0.2944]>	<[0.2575,0.4686], [0.3219,0.4278]>	<[0.3285,0.5722], [0.1871,0.3977]>
x_2	<[0.4152,0.6758], [0.2231,0.3242]>	<[0.4632,0.6701], [0.1659,0.2957]>	<[0.4288,0.6758], [0.1206,0.2231]>	<[0.5292,0.7056], [0.1206,0.2624]>
x_3	<[0.5694,0.7043], [0.1437,0.2514]>	<[0.5776,0.6818], [0.1552,0.2639]>	<[0.5000,0.6299], [0.1516,0.3000]>	<[0.3306,0.4524], [0.2928,0.4563]>
x_4	<[0.3285,0.5004], [0.2231,0.3787]>	<[0.5143,0.7043], [0.1257,0.2689]>	<[0.2116,0.4288], [0.1933,0.3456]>	<[0.3285,0.7000], [0.1000,0.2000]>
x_5	<[0.6435,0.7449], [0.1206,0.2551]>	<[0.4614,0.5953], [0.1437,0.2957]>	<[0.5000,0.6299], [0.2000,0.3000]>	<[0.4614,0.5647], [0.2393,0.4353]>

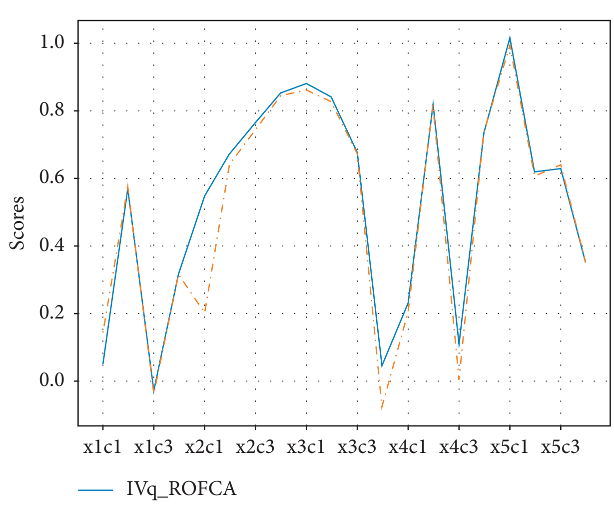

FIGURE 3: The scores of different matrices obtained by GIFFGA and IVq-ROFCA.

degree of support of experts for the alternatives is significantly higher than that of the opposition.

Above all, the proposed method employs the Choquet integral to aggregate interval-valued q-rung orthopair fuzzy information in this paper, which is a powerful tool to deal with MAGDM problems with dependent attributes. Moreover, the proposed method will highlight the support of experts for the alternatives and better reflect the superiority of the optimal alternative. Nevertheless, the proposed operators and the MAGDM method are used to handle decision problems for IVq-ROFNs, which are not able to be applied to different fuzzy environments. Besides, as a great impact on decision results remains to be studied, the degree of consensus among experts needs to be further studied in the proposed method.

6. Conclusion

The Choquet integral is an efficient tool to solve decision problems with interaction between attributes. For addressing complex MAGDM problems under interval-valued q-rung orthopair fuzzy information, we develop the IVq-ROFCA operator and the IVq-ROFCG operator and discuss some properties of them, including idempotency, commutativity, monotonicity, and boundedness. Particularly, we further design Choquet integral weighted and ordered operators for IVq-ROFS. Subsequently, a novel method is devised to process fuzzy information employing the IVq-ROFCA operator. Finally, a case of early-warning in the daily management of hypertension is given to illustrate the proposed method, and the results obtained are consistent with the actual situation provided by medical experts. The feasibility and effectiveness of the proposed method are proved by sensitivity analysis and comparative analysis additionally. Ulteriorly, our future research will focus on consensus models of IVq-ROFS, for instance, the MAGDM method with the Choquet integral based on consistency and consensus of experts. Moreover, considering the broad development prospect of fuzzy theory in the era of big data [41], the integration of the decision-making method, machine learning and big data are also one of our research directions in the future.

Data Availability

All data used to support the findings of the study are included within the article.

Disclosure

The innovation of this paper has been preprinted in arXiv: "Interval-Valued q-Rung Orthopair Fuzzy Choquet Integral Operators and Its Application in Group Decision Making" [42] by the authors of this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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