Hindawi
Mathematical Problems in Engineering
Volume 2023, Article ID 9767024, 1 page
https://doi.org/10.1155/2023/9767024

*Retraction*

# Retracted: Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation

**Mathematical Problems in Engineering**

Received 11 July 2023; Accepted 11 July 2023; Published 12 July 2023



This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

## References

[1] L. Wang, H. Li, J. Zhang, and J. Yang, "Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation," *Mathematical Problems in Engineering*, vol. 2022, Article ID 7419496, 17 pages, 2022.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 7419496, 17 pages
https://doi.org/10.1155/2022/7419496

*Research Article*

# Grey Relational Analysis Method for Probabilistic Double Hierarchy Linguistic Multiple Attribute Group Decision Making and Its Application to College Tennis Classroom Teaching Effect Evaluation

**Lihua Wang, Huiming Li , Jianpeng Zhang , and Jin Yang**

*School of Physical Education, Yunnan Agricultural University, Kunming 650201, Yunnan, China*

Correspondence should be addressed to Huiming Li; 2000021@ynau.edu.cn

Received 25 February 2022; Revised 19 April 2022; Accepted 20 April 2022; Published 10 June 2022

Academic Editor: Darko Božanić



The college tennis classroom teaching effect evaluation is viewed as the multiattribute group decision making (MAGDM). The probabilistic double hierarchy linguistic term set (PDHLTS) not only conforms to people's language expression habit of "adverb + adjective" but also can accurately depict its importance in real MAGDM. Therefore, this paper comes up with the probabilistic double hierarchy linguistic grey relational analysis (PDHL-GRA) method based on the grey relational analysis (GRA) process for MAGDM based on PDHLTS environment and applies it to the college tennis classroom teaching effect evaluation. Finally, a practical case for college tennis classroom teaching effect evaluation is presented to demonstrate the steps of our method, and a comparison analysis illustrates its feasibility and effectiveness.



## 1. Introduction

To better fuse decision information, MAGDM technology came into being [1–5]. After MAGDM theory came into being, it has been widely used in finance, engineering, corporate decision making, and many other aspects [6–10]. In view of the intricateness and fuzzification of decision circumstances [11–15], in many MAGDM issues, expert opinions are often stated as fuzzy data [16–18]. For this reason, Zadeh [19] raised concept of a linguistic variable for approximate reasoning. In many environments, the linguistic variable cannot exactly formulate proficient's perspective. Hence, hesitant fuzzy LTS (HFLTS) was proposed by Rodriguez, Martinez and Herrera [20]. An idea about probabilistic linguistic term sets (PLTSs) was proposed by Pang et al. [21]. Soon afterwards, critical malfunction matters were finished off by the PLAMM and PLWAMM formulas derived by Liu and Teng [22]. The performance estimation system of college teachers was finished off by the PLPA and PLPWG formulas derived by Kobina et al. [23]. Wei et al. [24] built the EDAS method for PL-MAGDM. The extensive similarity measure based on probabilistic language circumstances was derived by Wei et al. [25]. Su et al. [26] defined the PT-TODIM method for PL-MAGDM. Lin et al. [27] defined the probabilistic uncertain linguistic term sets (PULTs). Wang et al. [28] developed the GRP and CRITIC methods for PUL-MAGDM. Wei et al. [29] built the generalized Dice similarity measures for PUL-MAGDM. Zhao et al. [30] built the PUL-TODIM method based on prospect theory. He et al. [31] built the taxonomy-based MAGDM method with probabilistic uncertain linguistic assessment information. He et al. [32] built the bidirectional projection method for PUL-MAGDM. Nevertheless, a few sophisticated proficient estimation perspectives cannot be remarked in existing language terms such as "only a tiny bit poor" or "only a tiny bit good." Hence, Gou et al. [33] made a conceptual layout about double hierarchy linguistic term set (DHLTS) and double hierarchy hesitant fuzzy linguistic term set (DHHFLTS). Many research results have emerged one after another [34–41]. Soon afterwards, Gou et al. [42]

made a project about probabilistic double hierarchy linguistic term set (PDHLTS). Lei et al. [43] built the PDHL-CODAS model to rank online shopping platform. Lei et al. [44] defined a sequence of probabilistic double hierarchy linguistic polymerization formulas. Lei et al. [45] defined the PDHL-EDAS method for MAGDM.

GRA was initially defined by Deng [46] to cope with real MAGDM. Compared with other real MAGDM methods [47–51], the GRA method could consider the shape similarity of every given alternative from PIS as well as NIS. Javanmardi et al. [52] explored grey system theory-based methods and applications in sustainability studies. Javanmardi and Liu [53] explored the human cognitive capacity in understanding systems: a grey system theory perspective. Zhang et al. [54] used the GRA method based on cumulative prospect theory for IF-MAGDM. Javanmardi et al. [55] explored the philosophical paradigm of grey system theory as a postmodern theory. With the purpose of discerning the carbon market, Zhu et al. [56] took advantage of the GRA process as well as EMD. Malek et al. [57] built a revised hybrid GRA for green supply. Kung and Wen [58] used the GRA process to solve grey MADM. Javanmardi and Liu [59] explored grey system theory-based methods and applications in analyzing socioeconomic systems. Javanmardi et al. [60] explored the philosophical foundations of grey system theory. Alptekin et al. [61] solved the low carbon development based on the GRA process. Zhang et al. [62] defined the SF-GRA method based on cumulative prospect theory for MAGDM.

The main contributions of this paper are to utilize the GRA algorithm to build the MAGDM matters on the strength of PDHLTSs. The main research work of this paper is arranged as follows: (1) the GRA is constructed on account of PDHLTSs; (2) the PDHL-GRA method is applied to finish off the MAGDM issue under PDHLTSs; (3) a practical case for college tennis classroom teaching effect evaluation is presented to demonstrate the steps of our method; and (4) a comparison analysis illustrates its feasibility and effectiveness. The framework of this article is as follows. Section 2 reviews some concepts of PDHLTSs. Section 3 designs a PDHL-GRA method for MAGDM with entropy weight. Section 4 provides a practical example to illustrate the method and a comparison analysis illustrates its effectiveness. Finally, Section 5 summarizes this study.

## 2. Preliminaries

First, let us learn some basics about PDHLTS.

*Definition 1.* (see [33]). Let us say $\text{DHL} = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1 \quad , \cdots B;\}$ is a DHLTS, and the definition of the DHLTS is

$$\text{DHLTS} = \{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}, \tag{1}$$

where $\Delta = 1, 2, ..., \Xi\text{DHL}$, the $\Delta-$th double hierarchy linguistic element (DHLE) is narrated as $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}$, the quantity of all DHLEs is $\Xi$DHL, and all DHLEs are sorted in ascending sequence.

*Definition 2* (see [42]). Let us say $\text{DHL} = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ is a DHLTS, and the PDHLTS is created as

$$\text{PDHL}(\lambda) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \text{DHL}, \lambda^{\Delta} \geq 0, \sum_{\Delta=1}^{\Xi P\, DH\, L(\lambda)} \lambda^{\Delta} \leq 1\right\}, \tag{2}$$

where $\Delta = 1, 2, ..., \Xi\text{PDHL}(\lambda)$, the $\Delta-$th probabilistic double hierarchy linguistic element (PDHLE) is narrated as $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta})$, the quantities of all PDHLEs are denoted as $\Xi\text{PDHL}(\lambda)$, and according to $\Upsilon(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}))$, PDHLE is sorted in ascending order; the function is determined by formula (3).

*Definition 3* (see [42]). Let $\text{DHL} = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ be a DHLTS, and $\text{PDHL}(\lambda) = \{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \text{DHL}, \lambda^{\Delta} \geq 0, \sum_{\Delta=1}^{\Xi P\, DH\, L(\lambda)} \lambda^{\Delta} \leq 1\}$ be a PDHLTS. The above conversion function $\Upsilon$ for PDHLE $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta})$ is designed as follows:

$$\begin{aligned}
&\Upsilon: [-A, A] \times [-B, B] \longrightarrow [0, 1], \Upsilon(\vartheta, \Omega),\\
&= \frac{\Omega + (A + \vartheta)B}{2AB} = \varpi,\\
&\Upsilon^{-1}: [0, 1] \longrightarrow [-A, A] \times [-B, B],\\
&\Upsilon^{-1}(\varpi) = [2A\varpi - A]_{\langle I_{B((2A\varpi - A) - [2A\varpi - A])}\rangle or} [2A\varpi - A]\\
&+ 1_{\langle I_{B((2A\varpi - A) - [2A\varpi - A]) - B}\rangle or}.
\end{aligned} \tag{3}$$

Because the probability sum of all PDHLEs in PDHLTS may be less than 1, we had to standardize PDHLTS, and the specific measures are as follows:

$$\text{PDHL}(\lambda) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \text{DHL}, \lambda^{\Delta} \geq 0, \sum_{\Delta=1}^{\Xi\text{PDHL}(\lambda)} \lambda^{\Delta} \leq 1\right\}, \tag{4}$$

where $\tilde{\lambda}^{\Delta} = \lambda^{\Delta} / \sum_{\Delta=1}^{\Xi\text{PDHL}(\lambda)} \lambda^{\Delta}$; $\vartheta \in [-A, A]$; $\Omega \in [-B, B]$; $A, B$ are all integers.

*Definition 4.* (see [43]). Let $\text{DHL} = \{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \quad \Omega = -B, \cdots, -1, 0, 1, \cdots B;\}$ be a DHLTS and $\widetilde{\text{PDHL}}_1(\tilde{\lambda}) = \{\Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}_1^{\Delta}) | \Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle} \in \text{DHL}; \Delta = 1, 2, ..., \Xi\widetilde{\text{PDHL}}_1(\tilde{\lambda})\}$ and $\widetilde{\text{PDHL}}_2(\tilde{\lambda}) = \{\Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}_2^{\Delta}) | \Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle} \in \text{DHL}; \Delta = 1, 2, ..., \Xi\widetilde{\text{PDHL}}_2(\tilde{\lambda})\}$ be two different PDHLTSs, where $\#P\, DH\, \tilde{L}_1(\tilde{\lambda})$, $\#\widetilde{\text{PDHL}}_2(\tilde{\lambda})$ are the lengths of all PDHLEs in $\widetilde{\text{PDHL}}_1(\tilde{\lambda})$ and $\widetilde{\text{PDHL}}_2(\tilde{\lambda})$, respectively. Especially, if $\Xi\widetilde{\text{PDHL}}_1(\tilde{\lambda}) > \Xi\widetilde{\text{PDHL}}_2(\tilde{\lambda})$, then the lengths of $\Xi\widetilde{\text{PDHL}}_1(\tilde{\lambda}) - \Xi\widetilde{\text{PDHL}}_2(\tilde{\lambda})$ DHLEs are raised to $\widetilde{\text{PDHL}}_2(\tilde{\lambda})$. The added PDHLEs should not be greater than any of the elements in the $\text{PDHL}_2(\tilde{\lambda})$, and the probability should be set to 0.

*Definition 5.* (see [42]). Let $\text{PDHL}(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \text{DHL}; \Delta = 1, 2, \ldots, \Xi\text{PDHL}(\tilde{\lambda})\right\}$ be a PDHLTS, and the expected values $\chi(\text{PDHL}(\tilde{\lambda}))$ and deviation degree $\gamma(\text{PDHL}(\tilde{\lambda}))$ of $\text{PDHL}(\tilde{\lambda})$ are built as $\chi(\text{PDHL}_1(\tilde{\lambda})) = \chi(\text{PDHL}_2(\tilde{\lambda}))$:

$$\chi(\text{PDHL}(\tilde{\lambda})) = \frac{\sum_{\Delta=1}^{\Xi\text{PDHL}(\tilde{\lambda})} \Upsilon(\text{PDHL}(\tilde{\lambda}))\tilde{\lambda}^{\Delta}}{\sum_{\Delta=1}^{\text{PDHL}(\tilde{\lambda})} \tilde{\lambda}^{\Delta}},$$
$$\gamma(\text{PDHL}(\tilde{\lambda})) = \frac{\sqrt{\sum_{\Delta=1}^{\Xi\text{PDHL}(\tilde{\lambda})} \left(\Upsilon(\text{PDHL}(\tilde{\lambda}))\tilde{\lambda}^{\Delta} - \chi(\text{PDHL}(\tilde{\lambda}))\right)^2}}{\sum_{\Delta=1}^{\Xi\text{PDHL}(\tilde{\lambda})} \tilde{\lambda}^{\Delta}}. \quad (5)$$

*Definition 6.* (see [43]). Let $\text{DHL} = \left\{\Gamma_{\vartheta\langle I_\Omega\rangle} | \vartheta = -A, \cdots, -1, 0, 1, \cdots A; \quad \Omega = -B, \cdots, -1, 0, 1, \cdots B;\right\}$ be a DHLTS, and $\text{PDHL}_1(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle}(\lambda_1^{\Delta}) | \Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle} \in \text{DHL}; \Delta = 1, 2, \ldots, \Xi\text{PDHL}_1(\tilde{\lambda})\right\}$ and $\text{PDHL}_2(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle}(\lambda_2^{\Delta}) | \Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle} \in \text{DHL}; \Delta = 1, 2, \ldots, \Xi\text{PDHL}_2(\tilde{\lambda})\right\}$ are two PDHLTSs, where $\Xi\text{PDHL}_1(\tilde{\lambda}) = \Xi\text{PDHL}_2(\tilde{\lambda}) = \Xi\text{PDHL}(\tilde{\lambda})$; then, Hamming distance $\text{HD}(\text{PDHL}_1(\tilde{\lambda}), \text{PDHL}_2(\tilde{\lambda}))$ is determined.

$$\text{HD}\left(\text{PDHL}_1(\tilde{\lambda}), \text{PDHL}_2(\tilde{\lambda})\right) = \frac{\sum_{\Delta=1}^{\Xi\text{PDHL}(\tilde{\lambda})} \Upsilon\left(\Gamma^{\Delta}_{1\vartheta\langle I_\Omega\rangle}\right)\lambda_1^{\Delta} - \Gamma^{\Delta}_{2\vartheta\langle I_\Omega\rangle}\lambda_2^{\Delta}}{\Xi\text{PDHL}(\tilde{\lambda})}. \quad (6)$$

## 3. PDHL-GRA Method for MAGDM with Entropy Weight

Now, GRA mean in the context of PDHLTSs is proposed to deal with MAGDM matters. Also, a complete MAGDM issue is narrated as follows. Whole alternatives is shown as $C = \{C_1, C_2, \cdots, C_a\}$, $D = \{D_1, D_2, \cdots, D_b\}$ is denoted a sequence of attributes, and the weight vector is $\mathfrak{J} = (\mathfrak{J}_1, \mathfrak{J}_2, \cdots, \mathfrak{J}_b)$, where $\mathfrak{J}_\sigma \in [0, 1]$, $\sigma = 1, 2, \cdots, b$, $\sum_{\sigma=1}^{b} \mathfrak{J}_b = 1$, and $\text{JK} = \{\text{JK}_1, \text{JK}_2, \cdots, \text{JK}_T\}$ are T experts, and $\mathfrak{R} = (\mathfrak{R}_1, \mathfrak{R}_2, \cdots, \mathfrak{R}_T)$ is weight vector of all experts. Suppose that $q$-th expert $\text{JK}_q$ is evaluated $\tau -$ th alternative $C_\tau$ under $\sigma -$ th attribute $D_\sigma$ as $\text{PDHL}^{(q)}_{\tau\sigma}(\lambda) = \left\{\Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta(q)}_{\tau\sigma}) | \Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle} \in \text{DHL}, \lambda^{\Delta(q)}_{\tau\sigma} \geq 0, \sum_{\Delta=1}^{\Xi\text{PDHL}(\lambda)} \lambda^{\Delta(q)}_{\tau\sigma} \leq 1\right\}$ $(\tau = 1, 2, \cdots, a, \sigma = 1, 2, \cdots, b, q = 1, 2, \cdots, \text{T})$.

Furthermore, PDHL-GRA mean is created to dispose of MAGDM issue with entropy weight.

*Step 1.* Establish all decision makers' decision matrixes PDHLTS $Q^{(q)} = (\text{PDHL}^{(q)}(\lambda))_{a\times b}$.

$$\text{PDHLTS} = \mathop{\oplus}_{q=1}^{\text{T}} w_q \text{PDHL}_q(\tilde{\lambda}), \\ = \left\{\Upsilon^{-1}\left(\cup\left(1 - \prod_{q=1}^{\text{T}}\left(1 - \Upsilon\left(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\right)\right)^{\mathfrak{R}_q}\right)\right)\frac{\sum_{q=1}^{T}\lambda_q^{\Delta}}{q}\right\}. \quad (7)$$

*Step 2.* Convert cost index into benefit index. Let $\text{PDHL}(\tilde{\lambda}) = \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta}) | \Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle} \in \text{DHL}; \Delta = 1, 2, \ldots, \Xi\text{PDHL}(\tilde{\lambda})\right\}$ be a PDHLTS; if $\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}(\tilde{\lambda}^{\Delta})$ is an evaluation on cost, we need to translate it into the benefit evaluation $\Gamma^{\Delta}_{-\vartheta\langle I_\Omega\rangle}(\lambda^{\Delta})$.

*Step 3.* Compute the normalized decision matrix $\tilde{Q}^{(q)} = (\text{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}))_{a\times b}$.

$$\text{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}) = \frac{\sum_{\Delta=1}^{\Xi\text{PDHL}(\tilde{\lambda})} \Upsilon\left(\Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle}\right)\left(\lambda^{\Delta(q)}_{\tau\sigma}\right)}{\sum_{\tau=1}^{a}\sum_{\Delta=1}^{\Xi\text{PDHL}(\tilde{\lambda})} \Upsilon\left(\Gamma^{\Delta(q)}_{\tau\sigma\vartheta\langle I_\Omega\rangle}\right)\left(\lambda^{\Delta(q)}_{\tau\sigma}\right)}, \sigma = 1, 2, \cdots, b. \quad (8)$$

*Step 4.* The proportion of each attribute is calculated depending on the entropy formula.

Entropy [63] is one of the important tools to ascertain the proportion of each attribute.

The first thing to do is ascertaining the normalized decision matrix $\text{NL}_{ij}(p)$:

Secondly, the Shannon entropy $E = (E_1, E_2, \cdots, E_b)$ is obtained by the following formula:

$$E_\sigma = -\frac{1}{\ln a}\sum_{\tau=1}^{a} \text{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda})\ln \text{PDHL}^{(q)}_{\tau\sigma}(\tilde{\lambda}), \quad (9)$$

and $PDHL_{\tau\sigma}^{(q)}(\tilde{\lambda})\ln PDHL_{\tau\sigma}^{(q)}(\tilde{\lambda})$ is defined as 0, if $PDHL_{\tau\sigma}^{(q)}(\tilde{\lambda}) = 0$.

Finally, the attribute weights $\mathfrak{I} = (\mathfrak{I}_1, \mathfrak{I}_2, \cdots, \mathfrak{I}_b)$ are computed:

$$\mathfrak{I}_\sigma = \frac{1 - E_\sigma}{\sum_{\sigma=1}^{b} (1 - E_\sigma)}, \sigma = 1, 2, \cdots, b. \tag{10}$$

*Step 5*. Confirm the probabilistic double hierarchy linguistic positive ideal scheme more than zero (PDHLPIS) and probabilistic double hierarchy linguistic negative ideal scheme less than zero (PDHLNIS):

$$\begin{aligned}
\mathrm{PDHLPIS} &= (\mathrm{PDHLPIS}_1, \mathrm{PDHLPIS}_2, \cdots, \mathrm{PDHLPIS}_b),\\
\mathrm{PDHLPIS} &= (\mathrm{PDHLPIS}_1, \mathrm{PDHLPIS}_2, \cdots, \mathrm{PDHLPIS}_b),\\
\mathrm{PDHLNIS} &= (\mathrm{PDHLNIS}_1, \mathrm{PDHLNIS}_2, \cdots, \mathrm{PDHLNIS}_b),
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\mathrm{PDHLPIS}_\sigma &= \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right) | \Delta = 1, 2, \ldots, \Xi PDHL(\tilde{\lambda})\right\},\\
&= \left\{\max_\tau \Upsilon\left(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right)\right)\right\},\\
\mathrm{PDHLNIS}_\sigma &= \left\{\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right) | \Delta = 1, 2, \ldots, \Xi PDHL(\tilde{\lambda})\right\},\\
&= \left\{\min_\tau \Upsilon\left(\Gamma^{\Delta}_{\vartheta\langle I_\Omega\rangle}\left(\tilde{\lambda}^{\Delta}\right)\right)\right\}.
\end{aligned} \tag{12}$$

*Step 6*. Compute the grey rational coefficients of every given attribute of every given alternative from the PDHLPIS and PDHLNIS.

$$\begin{aligned}
\mathrm{PDHLPIS}(\xi_{\tau\sigma}) &= \frac{\min_{1\le i\le m}\min_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(P\ DH\ LA_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma)}{d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLPIS}_\sigma)},\\
\mathrm{PDHLPIS}(\xi_{\tau\sigma}) &= \frac{\min_{1\le i\le m}\min_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(P\ DH\ LA_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma)}{d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma) + \rho\max_{1\le i\le m}\max_{1\le j\le n} d(\mathrm{PDHLA}_{\tau\sigma}, \mathrm{PDHLNIS}_\sigma)},\\
&\tau = 1, 2, \cdots, a, \sigma = 1, 2, \cdots, b.
\end{aligned} \tag{13}$$

*Step 7*. Figure out the degree of GRC of all given alternatives from PDHLPIS as well as PDHLNIS:

$$\begin{aligned}
\mathrm{PDHLPIS}(\xi_\tau) &= \sum_{\tau=1}^{a} \mathfrak{I}_\tau \mathrm{PDHLPIS}(\xi_{\tau\sigma}), \tau = 1, 2, \cdots, a,\\
\mathrm{PDHLPIS}(\xi_\tau) &= \sum_{\tau=1}^{a} \mathfrak{I}_\tau \mathrm{PDHLNIS}(\xi_{\tau\sigma}), \tau = 1, 2, \cdots, a.
\end{aligned} \tag{14}$$

*Step 8*. Compute each alternative's PDHL relative relational degree (PDHLRRD) of all given alternatives from PDHLPIS:

$$\mathrm{PDHLRRD}_\tau = \frac{\mathrm{PDHLPIS}(\xi_\tau)}{\mathrm{PDHLNIS}(\xi_\tau) + \mathrm{PDHLPIS}(\xi_\tau)}, \tau = 1, 2, \cdots, a. \tag{15}$$

*Step 9*. According to $\mathrm{PDHLRRD}_\tau (\tau = 1, 2, \cdots, a)$. The highest value of $\mathrm{PDHLRRD}_\tau (\tau = 1, 2, \cdots, a)$, the optimal choice is.

## 4. Numerical Example and Comparative Analysis

*4.1. Numerical Example.* Based on the research on the development of tennis teachers in colleges and universities and the evaluation requirements of the new round of basic education curriculum reform, it is of great significance to measure whether tennis teaching meets the expected goals. The core courses in the curriculum reform were implemented, and the fundamental way is to implement curriculum classroom. Curriculum reform embodies an important issue that every school and teacher is thinking about. Classroom evaluation reform to carry out scientific and effective evaluation of classroom teaching and establish an effective evaluation system mechanism should be the core of the curriculum reform. According to the current and future period of teaching reform and development, classroom evaluation should be "developmental classroom evaluation." Classroom evaluation helps to overcome the limitations and deficiencies of current evaluation. Classroom evaluation reflects the latest trend of current teacher evaluation, evaluation of advanced ideas, and evaluation functions. Classroom evaluation conducts reflection and analysis on teachers, evaluates teachers' development potential, teachers' classroom status and the process of value judgment. However, the evaluation of teaching in the field of teaching is a worldwide problem, but also the key to promoting quality education process. There is a clear gap between the current evaluation theories, methods and systems, and quality education. Similar problems exist in teacher teaching evaluation. These serious constraints restricted the promotion of quality education. Therefore, the establishment of the quality

Table 1: The PDHLTS evaluation of all alternatives is provided by $JK_1$.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{-2\langle I_{-1}\rangle}(0.1), \Gamma_{-1\langle I_1\rangle}(0.3), \Gamma_{0\langle I_1\rangle}(0.6)\}$ | $\{\Gamma_{-3\langle I_1\rangle}(0.2), \Gamma_{0\langle I_{-2}\rangle}(0.1), \Gamma_{1\langle I_1\rangle}(0.7)\}$ | $\{\Gamma_{-3\langle I_0\rangle}(1.0)\}$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.5), \Gamma_{-1\langle I_{-2}\rangle}(0.3), \Gamma_{-1\langle I_{-1}\rangle}(0.2)\}$ |
| $C_2$ | $\{\Gamma_{-3\langle I_1\rangle}(1)\}$ | $\{\Gamma_{-3\langle I_0\rangle}(0.3), \Gamma_{0\langle I_2\rangle}(0.7)\}$ | $\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-3\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_{-1}\rangle}(0.2)\}$ | $\{\Gamma_{1\langle I_2\rangle}(0.3), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{-1\langle I_1\rangle}(0.3)\}$ |
| $C_3$ | $\{\Gamma_{0\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.8)\}$ | $\{\Gamma_{3\langle I_0\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.4), \Gamma_{-1\langle I_1\rangle}(0.4)\}$ | $\{\Gamma_{-3\langle I_0\rangle}(0.8), \Gamma_{1\langle I_{-2}\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.1)\}$ | $\{\Gamma_{-3\langle I_1\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.1)\}$ |
| $C_4$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_2\rangle}(0.3), \Gamma_{2\langle I_2\rangle}(0.4)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0.4), \Gamma_{0\langle I_{-3}\rangle}(0.4), \Gamma_{3\langle I_{-2}\rangle}(0.2)\}$ | $\{\Gamma_{-3\langle I_2\rangle}(0.4), \Gamma_{-1\langle I_2\rangle}(0.5), \Gamma_{1\langle I_3\rangle}(0.1)\}$ | $\{\Gamma_{-1\langle I_2\rangle}(0.5), \Gamma_{-1\langle I_3\rangle}(0.5),\}$ |
| $C_5$ | $\{\Gamma_{-2\langle I_0\rangle}(0.5), \Gamma_{-1\langle I_2\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.3)\}$ | $\{\Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.2)\}$ | $\{\Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.6)\}$ | $\{\Gamma_{2\langle I_1\rangle}(1.0)\}$ |

Table 2: The PDHLTS evaluation of all alternatives is provided by $JK_2$.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{-3\langle I_2\rangle}(0.7), \Gamma_{-1\langle I_1\rangle}(0.3)\}$ | $\{\Gamma_{1\langle I_{-1}\rangle}(1.0)\}$ | $\{\Gamma_{1\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_{-2}\rangle}(0.6), \Gamma_{1\langle I_0\rangle}(0.2)\}$ | $\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.4)\}$ |
| $C_2$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.3)\}$ | $\{\Gamma_{-3\langle I_1\rangle}(0.7), \Gamma_{-3\langle I_2\rangle}(0.1), \Gamma_{-3\langle I_3\rangle}(0.2),\}$ | $\{\Gamma_{0\langle I_0\rangle}(1.0)\}$ | $\{\Gamma_{-2\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$ |
| $C_3$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.3), \Gamma_{2\langle I_1\rangle}(0.6)\}$ | $\{\Gamma_{-3\langle I_2\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.2)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0.1), \Gamma_{-2\langle I_2\rangle}(0.9)\}$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0.5), \Gamma_{2\langle I_{-1}\rangle}(0.5)\}$ |
| $C_4$ | $\{\Gamma_{1\langle I_0\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$ | $\{\Gamma_{-2\langle I_0\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.1)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0.8), \Gamma_{-2\langle I_3\rangle}(0.2)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0.3), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{-1\langle I_2\rangle}(0.6)\}$ |
| $C_5$ | $\{\Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_2\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.3)\}$ | $\{\Gamma_{-2\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{0\langle I_2\rangle}(0.1)\}$ | $\{\Gamma_{3\langle I_{-2}\rangle}(1.0)\}$ | $\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_1\rangle}(0.7)\}$ |

Table 3: The PDHLTS evaluation of all alternatives is provided by $JK_3$.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{-1\langle I_1\rangle}(0.4), \Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.4)\}$ | $\{\Gamma_{-3\langle I_{-3}\rangle}(0.3), \Gamma_{-3\langle I_{-2}\rangle}(0.3), \Gamma_{1\langle I_{-2}\rangle}(0.4)\}$ | $\{\Gamma_{1\langle I_0\rangle}(1.0)\}$ | $\{\Gamma_{-2\langle I_{-3}\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.4)\}$ |
| $C_2$ | $\{\Gamma_{0\langle I_3\rangle}(1.0)\}$ | $\{\Gamma_{-1\langle I_0\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.7)\}$ | $\{\Gamma_{2\langle I_{-2}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_0\rangle}(0.2)\}$ | $\{\Gamma_{0\langle I_{-3}\rangle}(0.1), \Gamma_{2\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.7)\}$ |
| $C_3$ | $\{\Gamma_{1\langle I_{-2}\rangle}(0.7), \Gamma_{2\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.1)\}$ | $\{\Gamma_{-1\langle I_3\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{-1\langle I_0\rangle}(0.6)\}$ | $\{\Gamma_{0\langle I_{-3}\rangle}(0.3), \Gamma_{1\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.6)\}$ |
| $C_4$ | $\{\Gamma_{-1\langle I_{-3}\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.1)\}$ | $\{\Gamma_{-2\langle I_{-2}\rangle}(0.4), \Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.5)\}$ | $\{\Gamma_{-3\langle I_{-2}\rangle}(0.4), \Gamma_{-1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-3}\rangle}(0.5)\}$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.4), \Gamma_{2\langle I_2\rangle}(0.3)\}$ |
| $C_5$ | $\{\Gamma_{-3\langle I_{-1}\rangle}(0.5), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.1)\}$ | $\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_3\rangle}(0.6)\}$ | $\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$ | $\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.4)\}$ |

Table 4: The standardized decision matrix is provided by $JK_1$.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{-2\langle I_{-1}\rangle}(0.1), \Gamma_{-1\langle I_1\rangle}(0.3), \Gamma_{0\langle I_1\rangle}(0.6)\}$ | $\{\Gamma_{-3\langle I_1\rangle}(0.2), \Gamma_{0\langle I_{-2}\rangle}(0.1), \Gamma_{1\langle I_1\rangle}(0.7)\}$ | $\{\Gamma_{-3\langle I_0\rangle}(0), \Gamma_{-3\langle I_0\rangle}(0), \Gamma_{-3\langle I_0\rangle}(1.0)\}$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.5), \Gamma_{-1\langle I_{-2}\rangle}(0.3), \Gamma_{-1\langle I_{-1}\rangle}(0.2)\}$ |
| $C_2$ | $\{\Gamma_{-3\langle I_1\rangle}(0), \Gamma_{-3\langle I_1\rangle}(0), \Gamma_{-3\langle I_1\rangle}(1)\}$ | $\{\Gamma_{-3\langle I_0\rangle}(0), \Gamma_{-3\langle I_0\rangle}(0.3), \Gamma_{0\langle I_2\rangle}(0.7)\}$ | $\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-3\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_{-1}\rangle}(0.2)\}$ | $\{\Gamma_{1\langle I_2\rangle}(0.3), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{-1\langle I_1\rangle}(0.3)\}$ |
| $C_3$ | $\{\Gamma_{0\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.8)\}$ | $\{\Gamma_{3\langle I_0\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.4), \Gamma_{-1\langle I_1\rangle}(0.4)\}$ | $\{\Gamma_{-3\langle I_0\rangle}(0.8), \Gamma_{1\langle I_{-2}\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.1)\}$ | $\{\Gamma_{-3\langle I_1\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.1)\}$ |
| $C_4$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_2\rangle}(0.3), \Gamma_{2\langle I_2\rangle}(0.4)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0.4), \Gamma_{0\langle I_{-3}\rangle}(0.4), \Gamma_{3\langle I_{-2}\rangle}(0.2)\}$ | $\{\Gamma_{-3\langle I_2\rangle}(0.4), \Gamma_{-1\langle I_2\rangle}(0.5), \Gamma_{1\langle I_3\rangle}(0.1)\}$ | $\{\Gamma_{-1\langle I_2\rangle}(0), \Gamma_{-1\langle I_2\rangle}(0.5), \Gamma_{-1\langle I_3\rangle}(0.5),\}$ |
| $C_5$ | $\{\Gamma_{-2\langle I_0\rangle}(0.5), \Gamma_{-1\langle I_2\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.3)\}$ | $\{\Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.2)\}$ | $\{\Gamma_{0\langle I_0\rangle}(0), \Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.6)\}$ | $\{\Gamma_{2\langle I_1\rangle}(0), \Gamma_{2\langle I_1\rangle}(0), \Gamma_{2\langle I_1\rangle}(1.0)\}$ |

Table 5: The standardized decision matrix is provided by $JK_2$.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{-3\langle I_2\rangle}(0), \Gamma_{-3\langle I_2\rangle}(0.7), \Gamma_{-1\langle I_1\rangle}(0.3)\}$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0), \Gamma_{1\langle I_{-1}\rangle}(0), \Gamma_{1\langle I_{-1}\rangle}(1.0)\}$ | $\{\Gamma_{1\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_{-2}\rangle}(0.6), \Gamma_{1\langle I_0\rangle}(0.2)\}$ | $\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{2\langle I_2\rangle}(0.4)\}$ |
| $C_2$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.3)\}$ | $\{\Gamma_{-3\langle I_1\rangle}(0.7), \Gamma_{-3\langle I_2\rangle}(0.1), \Gamma_{-3\langle I_3\rangle}(0.2),\}$ | $\{\Gamma_{0\langle I_0\rangle}(0), \Gamma_{0\langle I_0\rangle}(0), \Gamma_{0\langle I_0\rangle}(1.0)\}$ | $\{\Gamma_{-2\langle I_{-3}\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$ |
| $C_3$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0.1), \Gamma_{1\langle I_0\rangle}(0.3), \Gamma_{2\langle I_1\rangle}(0.6)\}$ | $\{\Gamma_{-3\langle I_2\rangle}(0), \Gamma_{-3\langle I_2\rangle}(0.8), \Gamma_{2\langle I_1\rangle}(0.2)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0), \Gamma_{-2\langle I_1\rangle}(0.1), \Gamma_{-2\langle I_2\rangle}(0.9)\}$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0), \Gamma_{1\langle I_{-1}\rangle}(0.5), \Gamma_{2\langle I_{-1}\rangle}(0.5)\}$ |
| $C_4$ | $\{\Gamma_{1\langle I_0\rangle}(0.6), \Gamma_{2\langle I_0\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$ | $\{\Gamma_{-2\langle I_0\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.1)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0), \Gamma_{-2\langle I_1\rangle}(0.8), \Gamma_{-2\langle I_3\rangle}(0.2)\}$ | $\{\Gamma_{-2\langle I_1\rangle}(0.3), \Gamma_{-1\langle I_1\rangle}(0.1), \Gamma_{-1\langle I_2\rangle}(0.6)\}$ |
| $C_5$ | $\{\Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_2\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.3)\}$ | $\{\Gamma_{-2\langle I_{-2}\rangle}(0.5), \Gamma_{-1\langle I_{-1}\rangle}(0.4), \Gamma_{0\langle I_2\rangle}(0.1)\}$ | $\{\Gamma_{3\langle I_{-2}\rangle}(0), \Gamma_{3\langle I_{-2}\rangle}(0), \Gamma_{3\langle I_{-2}\rangle}(1.0)\}$ | $\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_1\rangle}(0.7)\}$ |

Table 6: The standardized decision matrix is provided by $JK_3$.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{-1\langle I_1\rangle}(0.4), \Gamma_{1\langle I_1\rangle}(0.2), \Gamma_{0\langle I_3\rangle}(0.4)\}$ | $\{\Gamma_{-3\langle I_{-3}\rangle}(0.3), \Gamma_{-3\langle I_{-2}\rangle}(0.3), \Gamma_{1\langle I_{-2}\rangle}(0.4)\}$ | $\{\Gamma_{1\langle I_0\rangle}(0), \Gamma_{1\langle I_0\rangle}(0), \Gamma_{1\langle I_0\rangle}(1.0)\}$ | $\{\Gamma_{-2\langle I_{-3}\rangle}(0), \Gamma_{-2\langle I_{-3}\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.4)\}$ |
| $C_2$ | $\{\Gamma_{0\langle I_3\rangle}(0), \Gamma_{0\langle I_3\rangle}(0), \Gamma_{0\langle I_3\rangle}(1.0)\}$ | $\{\Gamma_{-1\langle I_0\rangle}(0), \Gamma_{-1\langle I_0\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.7)\}$ | $\{\Gamma_{2\langle I_{-2}\rangle}(0.4), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{2\langle I_0\rangle}(0.2)\}$ | $\{\Gamma_{0\langle I_{-3}\rangle}(0.1), \Gamma_{2\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.7)\}$ |
| $C_3$ | $\{\Gamma_{1\langle I_{-2}\rangle}(0.7), \Gamma_{2<\langle I_{-1}\rangle}(0.2), \Gamma_{2\langle I_0\rangle}(0.1)\}$ | $\{\Gamma_{-1\langle I_3\rangle}(0.2), \Gamma_{1\langle I_1\rangle}(0.5), \Gamma_{2\langle I_1\rangle}(0.3)\}$ | $\{\Gamma_{2\langle I_{-1}\rangle}(0), \Gamma_{2\langle I_{-1}\rangle}(0.4), \Gamma_{-1\langle I_0\rangle}(0.6)\}$ | $\{\Gamma_{0\langle I_{-3}\rangle}(0.3), \Gamma_{1\langle I_0\rangle}(0.1), \Gamma_{2\langle I_1\rangle}(0.6)\}$ |
| $C_4$ | $\{\Gamma_{-1\langle I_{-3}\rangle}(0.3), \Gamma_{-2\langle I_1\rangle}(0.6), \Gamma_{1\langle I_1\rangle}(0.1)\}$ | $\{\Gamma_{-2\langle I_{-2}\rangle}(0.4), \Gamma_{0\langle I_0\rangle}(0.4), \Gamma_{1\langle I_0\rangle}(0.5)\}$ | $\{\Gamma_{-3\langle I_{-2}\rangle}(0.4), \Gamma_{-1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-3}\rangle}(0.5)\}$ | $\{\Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_1\rangle}(0.4), \Gamma_{2\langle I_2\rangle}(0.3)\}$ |
| $C_5$ | $\{\Gamma_{-3\langle I_{-1}\rangle}(0.5), \Gamma_{-2\langle I_0\rangle}(0.4), \Gamma_{2\langle I_1\rangle}(0.1)\}$ | $\{\Gamma_{1\langle I_{-3}\rangle}(0.1), \Gamma_{1\langle I_{-1}\rangle}(0.3), \Gamma_{1\langle I_3\rangle}(0.6)\}$ | $\{\Gamma_{-3\langle I_{-2}\rangle}(0.6), \Gamma_{-1\langle I_1\rangle}(0.2), \Gamma_{1\langle I_3\rangle}(0.2)\}$ | $\{\Gamma_{-1\langle I_{-2}\rangle}(0.5), \Gamma_{2\langle I_2\rangle}(0.1), \Gamma_{3\langle I_0\rangle}(0.4)\}$ |

Table 7: The overall evaluation matrix.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | $\{\Gamma_{3\langle I_{0.5899}\rangle}(0.1667), \Gamma_{-2\langle I_{0.1756}\rangle}(0.4000), \Gamma_{-2\langle I_{1.2370}\rangle}(0.4333)\}$ | $\{\Gamma_{-3\langle I_{0.7617}\rangle}(0.1667), \Gamma_{-3\langle I_{0.9152}\rangle}(0.1333), \Gamma_{-1\langle I_{2.1598}\rangle}(0.7000)\}$ | $\{\Gamma_{-3\langle I_{0.4904}\rangle}(0.0667), \Gamma_{-3\langle I_{1.5523}\rangle}(0.2000), \Gamma_{-1\langle I_{0.3719}\rangle}(0.7333)\}$ | $\{\Gamma_{-3\langle I_{2.9741}\rangle}(0.3333), \Gamma_{-3\langle I_{1.3877}\rangle}(0.3333), \Gamma_{-2\langle I_{1.5639}\rangle}(0.3333)\}$ |
| $C_2$ | $\{\Gamma_{-3\langle I_{2.0078}\rangle}(0.2000), \Gamma_{-3\langle I_{0.3578}\rangle}(0.0333), \Gamma_{-1\langle I_{2.9446}\rangle}(0.7667)\}$ | $\{\Gamma_{-3\langle I_{0.5894}\rangle}(0.2333), \Gamma_{-3\langle I_{0.6609}\rangle}(0.2333), \Gamma_{-2\langle I_{2.2160}\rangle}(0.5333)\}$ | $\{\Gamma_{-3\langle I_{2.4770}\rangle}(0.3333), \Gamma_{-3\langle I_{1.6502}\rangle}(0.2000), \Gamma_{-1\langle I_{0.1429}\rangle}(0.4667)\}$ | $\{\Gamma_{-3\langle I_{1.8320}\rangle}(0.2000), \Gamma_{-2\langle I_{0.9213}\rangle}(0.3667), \Gamma_{-2\langle I_{2.8878}\rangle}(0.4333)\}$ |
| $C_3$ | $\{\Gamma_{-3\langle I_{2.8891}\rangle}(0.3000), \Gamma_{-3\langle I_{2.4560}\rangle}(0.2000), \Gamma_{-1\langle I_{2.1545}\rangle}(0.5000)\}$ | $\{\Gamma_{-3\langle I_{2.4000}\rangle}(0.1333), \Gamma_{-1\langle I_{0.8095}\rangle}(0.5667), \Gamma_{-2\langle I_{1.2134}\rangle}(0.3000)\}$ | $\{\Gamma_{-3\langle I_{0.3171}\rangle}(0.2667), \Gamma_{-3\langle I_{2.0040}\rangle}(0.2000), \Gamma_{-0\langle I_{0.6000}\rangle}(0.5333)\}$ | $\{\Gamma_{-3\langle I_{0.8605}\rangle}(0.1333), \Gamma_{-2\langle I_{2.4539}\rangle}(0.4667), \Gamma_{-1\langle I_{0.1944}\rangle}(0.4000)\}$ |
| $C_4$ | $\{\Gamma_{-2\langle I_{1.5865}\rangle}(0.4000), \Gamma_{-2\langle I_{1.7293}\rangle}(0.3667), \Gamma_{-2\langle I_{0.6822}\rangle}(0.2333)\}$ | $\{\Gamma_{-3\langle I_{1.0658}\rangle}(0.3667), \Gamma_{-3\langle I_{2.9481}\rangle}(0.4667), \Gamma_{-3\langle I_{2.2129}\rangle}(0.1667)\}$ | $\{\Gamma_{-3\langle I_{0.4637}\rangle}(0.2667), \Gamma_{-3\langle I_{2.5820}\rangle}(0.4667), \Gamma_{-2\langle I_{0.0994}\rangle}(0.2667)\}$ | $\{\Gamma_{-3\langle I_{1.5703}\rangle}(0.2000), \Gamma_{-2\langle I_{0.1473}\rangle}(0.3333), \Gamma_{-2\langle I_{2.9973}\rangle}(0.4667)\}$ |
| $C_5$ | $\{\Gamma_{-3\langle I_{2.7494}\rangle}(0.4000), \Gamma_{-2\langle I_{0.5766}\rangle}(0.3667), \Gamma_{-2\langle I_{0.6239}\rangle}(0.2333)\}$ | $\{\Gamma_{-3\langle I_{1.7130}\rangle}(0.3333), \Gamma_{-2\langle I_{1.0831}\rangle}(0.3667), \Gamma_{-2\langle I_{0.8293}\rangle}(0.3000)\}$ | $\{\Gamma_{-3\langle I_{2.2406}\rangle}(0.2000), \Gamma_{-3\langle I_{2.4432}\rangle}(0.2000), \Gamma_{0\langle I_{0.4612}\rangle}(0.6000)\}$ | $\{\Gamma_{-3\langle I_{2.4888}\rangle}(0.2000), \Gamma_{-3\langle I_{1.5492}\rangle}(0.1000), \Gamma_{1\langle I_{0.6000}\rangle}(0.7000)\}$ |

Table 8: The PDHLPIS.

| $D_1$ | $D_2$ |
|---|---|
| $\left\{\Gamma_{-3\langle I_{2.8891}\rangle}(0.3000), \Gamma_{-3\langle I_{2.4560}\rangle}(0.2000), \Gamma_{-1\langle I_{2.1545}\rangle}(0.5000)\right\}$ | $\left\{\Gamma_{-3\langle I_{2.4000}\rangle}(0.1333), \Gamma_{-1\langle I_{0.8095}\rangle}(0.5667), \Gamma_{-2\langle I_{1.2134}\rangle}(0.3000)\right\}$ |
| $D_3$ | $D_4$ |
| $\left\{\Gamma_{-3\langle I_{2.2406}\rangle}(0.2000), \Gamma_{-3\langle I_{2.4432}\rangle}(0.2000), \Gamma_{0\langle I_{0.4612}\rangle}(0.6000)\right\}$ | $\left\{\Gamma_{-3\langle I_{2.4888}\rangle}(0.2000), \Gamma_{-3\langle I_{1.5492}\rangle}(0.1000), \Gamma_{1\langle I_{0.6000}\rangle}(0.7000)\right\}$ |

Table 9: The PDHLNIS.

| $D_1$ | $D_2$ |
|---|---|
| $\left\{\Gamma_{3\langle I_{0.5899}\rangle}(0.1667), \Gamma_{-2\langle I_{0.1756}\rangle}(0.4000), \Gamma_{-2\langle I_{1.2370}\rangle}(0.4333)\right\}$ | $\left\{\Gamma_{-3\langle I_{1.0658}\rangle}(0.3667), \Gamma_{-3\langle I_{2.9481}\rangle}(0.4667), \Gamma_{-3\langle I_{2.2129}\rangle}(0.1667)\right\}$ |
| $D_3$ | $D_4$ |
| $\left\{\Gamma_{-3\langle I_{0.4637}\rangle}(0.2667), \Gamma_{-3\langle I_{2.5820}\rangle}(0.4667), \Gamma_{-2\langle I_{0.0994}\rangle}(0.2667)\right\}$ | $\left\{\Gamma_{-3\langle I_{2.9741}\rangle}(0.3333), \Gamma_{-3\langle I_{1.3877}\rangle}(0.3333), \Gamma_{-2\langle I_{1.5639}\rangle}(0.3333)\right\}$ |

Table 10: GRC of each alternative from PDHLPIS.

| Alternatives | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | 0.5725 | 1.0000 | 0.2233 | 0.5195 |
| $C_2$ | 0.4343 | 0.5443 | 0.3075 | 0.6043 |
| $C_3$ | 0.5949 | 0.7925 | 1.0000 | 1.0000 |
| $C_4$ | 0.4081 | 0.6043 | 0.3243 | 0.5931 |
| $C_5$ | 1.0000 | 0.5443 | 0.3075 | 0.6281 |

Table 11: GRC of each alternative from PDHLNIS.

| Alternatives | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|
| $C_1$ | 0.4072 | 1.0000 | 0.3067 | 0.5645 |
| $C_2$ | 0.6795 | 1.1100 | 0.8900 | 0.9433 |
| $C_3$ | 0.3312 | 0.4759 | 1.0000 | 0.5047 |
| $C_4$ | 0.8900 | 0.6272 | 0.6043 | 1.0000 |
| $C_5$ | 1.0000 | 0.5645 | 0.3739 | 0.5869 |

Table 12: PDHLPIS$(\xi_\tau)$ and PDHLNIS$(\xi_\tau)$ of every alternative.

| Alternatives | IVIFPIS$(\xi_i)$ | IVIFNIS$(\xi_i)$ |
|---|---|---|
| $C_1$ | 0.6953 | 0.5446 |
| $C_2$ | 0.6089 | 1.0698 |
| $C_3$ | 0.9824 | 0.5046 |
| $C_4$ | 0.6156 | 0.8749 |
| $C_5$ | 0.7575 | 0.6166 |

Table 13: PDHLRRD of each alternative from PDHLPIS.

| Alternatives | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| PDHLRRD$_\tau$ | 0.1748 | 0.4049 | 0.2386 | 0.4233 | 0.5373 |

of classroom education development of the concept of evaluation system is the full implementation of the objective of quality education, and at the same time, it also pushes the design and implementation of teaching activities to a new stage. However, these problems can be attributed to the MAGDM problem. This paper analyzes college tennis classroom teaching effect evaluation problems based on the proposed PDHL-GRA method. There are five given latent college tennis teachers $C = \{C_1, C_2, C_3, C_4, C_5\}$, who may be the best. For the sake of assessing the college tennis classroom teaching effect fairly, three experts $JK = \{JK_1, JK_2, JK_3\}$ (expert's weight $\mathfrak{R} = [0.40, 0.33, 0.27]$) are invited. All experts depict their assessment information through four subsequent attributes: ① $D_1$ is teaching attitude; ② $D_2$ represents the teaching methods; ③ $D_3$ is student feedback; and ④ $D_4$ is teaching quality. Obviously, all attributes are benefit, and $\mathfrak{I} = (\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4)$ is the weight of four attributes where $\mathfrak{I}_\sigma \in [0, 1]$, $\mathfrak{I} = 1, 2, 3, 4$, $\sum_{\sigma=1}^{4} \mathfrak{I}_\sigma = 1$. Suppose that $q-$th expert $JK_q$ evaluated $\tau-$th alternative $C_\tau$ under $\sigma-$th attribute $D_\sigma$ as $PDHL_{\tau\sigma}^{(q)}(\lambda) = \left\{\Gamma_{\tau\sigma\vartheta\langle I_\Omega\rangle}^{\Delta(q)}(\lambda_{\tau\sigma}^{\Delta(q)}) \mid \Gamma_{\tau\sigma\vartheta\langle I_\Omega\rangle}^{\Delta(q)} \in DHL, \lambda_{\tau\sigma}^{\Delta(q)} \geq 0, \sum_{\Delta=1}^{\Xi PDHL(\lambda)} \lambda_{\tau\sigma}^{\Delta(q)} \leq 1\right\}$ $(\tau = 1, 2, \cdots, 5, \sigma = 1, 2, \cdots, 4, q = 1, 2, 3.)$ where the double linguistic hierarchy evaluation information tables are given as follows:

Table 14: The numerical results and rank derived by the PDHL-CODAS.

| | PDHL-TOPSIS | Rank | PDHL-CODAS | Rank |
|---|---|---|---|---|
| $C_1$ | 0.8804 | 5 | −0.5040 | 5 |
| $C_2$ | 0.5782 | 3 | 0.0735 | 3 |
| $C_3$ | 0.6145 | 4 | 0.1672 | 2 |
| $C_4$ | 0.4358 | 2 | 0.0734 | 4 |
| $C_5$ | 0.3846 | 1 | 0.2847 | 1 |
| | The expected values of PDHLWA operator | Rank | The expected values of PDHLWA operator | Rank |
| $C_1$ | 0.4409 | 5 | 0.4433 | 5 |
| $C_2$ | 0.4444 | 4 | 0.6599 | 3 |
| $C_3$ | 0.6488 | 3 | 0.7611 | 2 |
| $C_4$ | 0.7333 | 2 | 0.5609 | 4 |
| $C_5$ | 0.8841 | 1 | 0.8823 | 1 |
| | The expected values of PDHLPWA operator | Rank | The expected values of PDHLPWG operator | Rank |
| $C_1$ | 0.4431 | 5 | 0.4455 | 5 |
| $C_2$ | 0.6466 | 3 | 0.6621 | 3 |
| $C_3$ | 0.5510 | 4 | 0.7633 | 2 |
| $C_4$ | 0.7355 | 2 | 0.5631 | 4 |
| $C_5$ | 0.8863 | 1 | 0.8845 | 1 |

$$\begin{aligned}\Gamma = \{&\Gamma_{-3} = \text{extremely poor}, \Gamma_{-2} = \text{very poor}, \Gamma_{-1} \\ &= \text{poor}, \Gamma_0 = \text{medium}, \\ &\Gamma_1 = \text{good}, \Gamma_2 = \text{very good}, \Gamma_3 = \text{extremely good}\}, \\ \mathrm{I} = \{&\mathrm{I}_{-3} = \text{far form} \mathrm{I}_{-2} = \text{only } a \text{ little}, \mathrm{I}_{-1} \\ &= a \text{ little}, \mathrm{I}_0 = \text{just right}, \\ &\mathrm{I}_1 = \text{much}, \mathrm{I}_2 = \text{very much}, \mathrm{I}_3 = \text{extirely much}\}.\end{aligned} \tag{16}$$

Then, the decision matrixes of each invited expert are expressed in Tables 1–3.

Now, the built PDHL-GRA method is used to select the optimal latent college tennis teacher.

*Step 1*. Standardize the evaluation matrix of the three experts (Tables 4–6).

*Step 2*. According to the weighted average operator, the evaluation of three experts is aggregated into a total decision matrix, which has been converted to the PDHLTSs (see Table 7).

*Step 3*. Calculate the weight of the decision attribute.

$$\mathfrak{I}_1 = 0.1432\mathfrak{I}_2 = 0.3496\mathfrak{I}_3 = 0.3217\mathfrak{I}_4 = 0.1855. \tag{17}$$

*Step 4*. The PDHLPIS and the PDHLNIS are determined according to the global decision matrix, which has been converted to the PDHLTSs (see Tables 8 and 9).

*Step 5*. Figure out the GRC of every alternative from PDHLPIS as well as PDHLNIS (Tables 10 and 11).

*Step 6*. Figure out the degree of GRC of all alternatives from PDHLPIS as well as PDHLNIS (Table 12).

*Step 7*. Calculate the $\text{PDHLRRD}_\tau$ of each given alternative from PDHLPIS (Table 13).

*Step 8*. According to the $\text{PDHLRRD}_\tau$, all given alternatives are ranked, the higher the $\text{PDHLRRD}_\tau$, the better the alternative selected. Evidently, the order is $C_5 > C_4 > C_2 > C_3 > C_1$ and $C_5$ is the best one.

*4.2. Comparative Analysis.* Finally, we compared it with the PDHL-VIKOR method [64], PDHL-CODAS method [43], PDHLWA operator, PDHLWG operator, PDHLPWA operator, and PDHLPWG operator. The results and analysis are as follows (see Table 14). It can be seen from Table 14 that although the six methods are different, the optimal scheme obtained is the same. Only schemes 3 and 4 have slight differences between the PDHLWA operator and other methods. Therefore, the PDHL-GRA method proposed by us can scientifically and effectively solve the investment decision problem.

## 5. Conclusion

Life changes and people's ideas and educational expectation have brought great challenges to contemporary school education, especially to college tennis education. With the gradual development of social needs, schools seem hard to meet the more and more advanced and complex education needs of the society. In order to promote whole-person education to students, family-school cooperation has become one of the effective ways to collect common effort and establish collaboration for education. Family and school cooperation not only provides an opportunity for in-depth development by prioritizing education environment and exploring potentiality of education resources but also is a booster for the development of students' physical and mental health. However, while there are achievements in family-school cooperative management, there are still difficulties and problems. Also, the theoretical basis and teaching practices need further exploration. Affordance theory proposed by Gibson [65] claims that there is an interaction between humans (individuals) and the environment (the nature). There is potentiality of potential act in the affordance environment. Its existence is closely related to actors' capability and understanding of the environment. That is to say, affordance is characterized not only by the environment but also by the individuals and emerges only when the two factors interact. Generally, we may put our focus on the affordance of language, the affordance of social culture, and the affordance of situations. Although focal difference exists between these types of affordances, there are similarities. Classroom management can be considered as an environment created together by the child, the teacher, and the parents, as compared with the traditional classroom management, which put emphasis on the interactive rule of the teacher and the student and the environment managed by the teacher. However, parents' participation in college tennis class management provides a possible route for affordable learning environment. This paper defines an useful method for this kind of issue, since it builds the PDHL-GRA method for college tennis classroom teaching effect evaluation. And then a numerical example is used to evaluate the College tennis classroom teaching effect. Furthermore, to check on the feasibility as well as availability of the new proposed method, useful comparative analysis is also designed. In the near future, we shall pay attention to the consensus reaching process [66–71], influence of DMs' psychological factors [72–77], and how to deal with the situations when criteria weights are incompletely known [78–83].

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## References


[1] J. Ye and S. Du, "Some distances, similarity and entropy measures for interval-valued neutrosophic sets and their relationship," *International Journal of Machine Learning and Cybernetics*, vol. 10, no. 2, pp. 347–355, 2019.

[2] J. Ye, "Multiple attribute group decision-making method with single-valued neutrosophic interval number information," *International Journal of Systems Science*, vol. 50, no. 1, pp. 152–162, 2019.

[3] J. Lu, S. Zhang, J. Wu, and Y. Wei, "COPRAS method for multiple attribute group decision making under picture fuzzy environment and their application to green supplier selection," *Technological and Economic Development of Economy*, vol. 27, no. 2, pp. 369–385, 2021.

[4] F. Lei, G. Wei, W. Shen, and Y. Guo, "PDHL-EDAS method for multiple attribute group decision making and its application to 3D printer selection," *Technological and Economic Development of Economy*, vol. 28, no. 1, pp. 179–200, 2021.

[5] D. Zhang, Y. Su, M. Zhao, and X. Chen, "CPT-TODIM method for interval neutrosophic MAGDM and its application to third-party logistics service providers selection," *Technological and Economic Development of Economy*, vol. 28, no. 1, pp. 201–219, 2021.

[6] H. Garg, M. Munir, K. Ullah, T. Mahmood, and N. Jan, "Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators," *Symmetry*, vol. 10, no. 12, p. 670, 2018.

[7] M. Akram, A. Khan, and A. B. Saeid, "Complex Pythagorean Dombi fuzzy operators using aggregation operators and theirdecision-making," *Expert Systems*, vol. 38, 2021.

[8] H. Zhang, G. Wei, and X. Chen, "Spherical fuzzy Dombi power Heronian mean aggregation operators for multiple attribute group decision-making," *Computational and Applied Mathematics*, vol. 41, no. 3, p. 98, 2022.

[9] H. Zhang, G. Wei, and C. Wei, "TOPSIS method for spherical fuzzy MAGDM based on cumulative prospect theory and combined weights and its application to residential location," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 3, pp. 1367–1380, 2022.

[10] Z. Ali, T. Mahmood, T. Mahmood, K. Ullah, and Q. Khan, "Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management," *Reports in Mechanical Engineering*, vol. 2, no. 1, pp. 105–134, 2021.

[11] W. Yang and Y. Pang, "Hesitant interval-valued Pythagorean fuzzy VIKOR method," *International Journal of Intelligent Systems*, vol. 34, no. 5, pp. 754–789, 2019.

[12] J. Xui, J. Y. Dong, S. P. Wan, and J. Gao, "Multiple attribute decision making with triangular intuitionistic fuzzy numbers based on zero-sum game approach," *Iranian Journal of Fuzzy Systems*, vol. 16, pp. 97–112, 2019.

[13] Z. Xu and S. Zhang, "An overview on the applications of the hesitant fuzzy sets in group decision-making: theory, support and methods," *Frontiers of Engineering Management*, vol. 6, no. 2, pp. 163–182, 2019.

[14] R. Rostamzadeh, A. Esmaeili, A. S. Nia, J. Saparauskas, and M. Keshavarz Ghorabaee, "A fuzzy ARAS method for supply chain management performance measurement in smes under uncertainty," *Transformations in Business and Economics*, vol. 16, pp. 319–348, 2017.

[15] E. K. Zavadskas, J. Antucheviciene, J. Saparauskas, and Z. Turskis, "MCDM methods WASPAS and MULTIMOORA: verification of robustness of methods when assessing alternative solutions," *Economic Computation & Economic Cybernetics Studies & Research*, vol. 47, pp. 5–20, 2013.

[16] M. Yazdani, P. Zarate, A. Coulibaly, and E. K. Zavadskas, "A group decision making support system in logistics and supply chain management," *Expert Systems with Applications*, vol. 88, pp. 376–392, 2017.

[17] H. Hashemi, S. Mousavi, E. Zavadskas, A. Chalekaee, and Z. Turskis, "A new group decision model based on grey-intuitionistic fuzzy-ELECTRE and VIKOR for contractor assessment problem," *Sustainability*, vol. 10, no. 5, p. 1635, 2018.

[18] E. K. Zavadskas, R. Bausys, and I. Mazonaviciute, "Safety evaluation methodology of urban public parks by multi-criteria decision making," *Landscape and Urban Planning*, vol. 189, pp. 372–381, 2019.

[19] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning," *Learning Systems and Intelligent Robots*, vol. 8, pp. 1–10, 1974.

[20] R. M. Rodriguez, L. Martinez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 109–119, 2012.

[21] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Information Sciences*, vol. 369, pp. 128–143, 2016.
[22] P. Liu and F. Teng, "Some Muirhead mean operators for probabilistic linguistic term sets and their applications to multiple attribute decision-making," *Applied Soft Computing*, vol. 68, pp. 396–431, 2018.
[23] A. Kobina, D. Liang, and X. He, "Probabilistic linguistic power aggregation operators for multi-criteria group decision making," *Symmetry*, vol. 9, no. 12, p. 320, 2017.
[24] G. Wei, C. Wei, and Y. Guo, "EDAS method for probabilistic linguistic multiple attribute group decision making and their application to green supplier selection," *Soft Computing*, vol. 25, no. 14, pp. 9045–9053, 2021.
[25] G. Wei, C. Wei, J. Wu, and Y. Guo, "Probabilistic linguistic multiple attribute group decision making for location planning of electric vehicle charging stations based on the generalized Dice similarity measures," *Artificial Intelligence Review*, vol. 54, no. 6, pp. 4137–4167, 2021.
[26] Y. Su, M. Zhao, C. Wei, and X. Chen, "PT-TODIM method for probabilistic linguistic MAGDM and application to industrial control system security supplier selection," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 202–215, 2022.
[27] M. Lin, Z. Xu, Y. Zhai, and Z. Yao, "Multi-attribute group decision-making under probabilistic uncertain linguistic environment," *Journal of the Operational Research Society*, vol. 69, no. 2, pp. 157–170, 2018.
[28] S. Wang, G. Wei, J. Lu, J. Wu, C. Wei, and X. Chen, "GRP and CRITIC method for probabilistic uncertain linguistic MAGDM and its application to site selection of hospital constructions," *Soft Computing*, vol. 26, no. 1, pp. 237–251, 2022.
[29] G. Wei, R. Lin, J. Lu, J. Wu, and C. Wei, "The generalized Dice similarity measures for probabilistic uncertain linguistic MAGDM and its application to location planning of electric vehicle charging stations," *International Journal of Fuzzy Systems*, vol. 24, no. 2, pp. 933–948, 2022.
[30] M. Zhao, H. Gao, G. Wei, C. Wei, and Y. Guo, "Model for network security service provider selection with probabilistic uncertain linguistic TODIM method based on prospect theory," *Technological and Economic Development of Economy*, vol. 28, 2022.
[31] Y. He, G. Wei, and X. Chen, "Taxonomy-based multiple attribute group decision making method with probabilistic uncertain linguistic information and its application in supplier selection," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 2, pp. 3237–3250, 2021.
[32] Y. He, G. Wei, X. Chen, and Y. Wei, "Bidirectional projection method for multi-attribute group decision making under probabilistic uncertain linguistic environment," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 1, pp. 1429–1443, 2021.
[33] X. Gou, H. Liao, Z. Xu, and F. Herrera, "Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method: a case of study to evaluate the implementation status of haze controlling measures," *Information Fusion*, vol. 38, pp. 22–34, 2017.
[34] X. Wang, X. Gou, and Z. Xu, "Assessment of traffic congestion with ORESTE method under double hierarchy hesitant fuzzy linguistic environment," *Applied Soft Computing*, vol. 86, Article ID 105864, 2020.
[35] R. Krishankumar, K. S. Ravichandran, V. Shyam, S. V. Sneha, S. Kar, and H. Garg, "Multi-attribute group decision-making using double hierarchy hesitant fuzzy linguistic preference information," *Neural Computing & Applications*, vol. 32, no. 17, pp. 14031–14045, 2020.
[36] R. Krishankumar, K. S. Ravichandran, S. Kar, P. Gupta, and M. K. Mehlawat, "Double-hierarchy hesitant fuzzy linguistic term set-based decision framework for multi-attribute group decision-making," *Soft Computing*, vol. 25, no. 7, 2020.
[37] R. Krishankumar, K. S. Ravichandran, H. Liao, and S. Kar, "An integrated decision framework for group decision-making with double hierarchy hesitant fuzzy linguistic information and unknown weights," *International Journal of Computational Intelligence Systems*, vol. 13, no. 1, p. 624, 2020.
[38] J. Montserrat-Adell, Z. Xu, X. Gou, and N. Agell, "Free double hierarchy hesitant fuzzy linguistic term sets: an application on ranking alternatives in GDM," *Information Fusion*, vol. 47, pp. 45–59, 2019.
[39] X. Gou, H. Liao, Z. Xu, R. Min, and F. Herrera, "Group decision making with double hierarchy hesitant fuzzy linguistic preference relations: consistency based measures, index and repairing algorithms and decision model," *Information Sciences*, vol. 489, pp. 93–112, 2019.
[40] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Multiple criteria decision making based on distance and similarity measures under double hierarchy hesitant fuzzy linguistic environment," *Computers & Industrial Engineering*, vol. 126, pp. 516–530, 2018.
[41] X. Gou, Z. Xu, and F. Herrera, "Consensus reaching process for large-scale group decision making with double hierarchy hesitant fuzzy linguistic preference relations," *Knowledge-Based Systems*, vol. 157, pp. 20–33, 2018.
[42] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Probabilistic double hierarchy linguistic term set and its use in designing an improved VIKOR method: the application in smart healthcare," *Journal of the Operational Research Society*, vol. 72, no. 12, pp. 2611–2630, 2021.
[43] F. Lei, G. Wei, and X. Chen, "Model-based evaluation for online shopping platform with probabilistic double hierarchy linguistic CODAS method," *International Journal of Intelligent Systems*, vol. 36, no. 9, pp. 5339–5358, 2021.
[44] F. Lei, G. Wei, and X. Chen, "Some self-evaluation models of enterprise's credit based on some probabilistic double hierarchy linguistic aggregation operators," *Journal of Intelligent and Fuzzy Systems*, vol. 40, no. 6, pp. 11809–11828, 2021.
[45] F. Lei, G. Wei, W. Shen, and Y. Guo, "PDHL-EDAS method for multiple attribute group decision making and its application to 3D printer selection," *Technological and Economic Development of Economy*, vol. 28, 2021.
[46] J. L. Deng, "Introduction to grey system," *Journal of Grey System*, vol. 1, pp. 1–24, 1989.
[47] H. Garg, "New exponential operation laws and operators for interval-valued q-rung orthopair fuzzy sets in group decision making process," *Neural Computing & Applications*, vol. 33, no. 20, pp. 13937–13963, 2021.
[48] P. Liu, Y. Li, and F. Teng, "Bidirectional projection method for probabilistic linguistic multi-criteria group decision-making based on power average operator," *International Journal of Fuzzy Systems*, vol. 21, no. 8, pp. 2340–2353, 2019.
[49] W. Xue, Z. Xu, X. Zhang, and X. Tian, "Pythagorean fuzzy LINMAP method based on the entropy theory for railway project investment decision making," *International Journal of Intelligent Systems*, vol. 33, no. 1, pp. 93–125, 2018.
[50] Z. Wu, J. Xu, and Z. Xu, "A multiple attribute group decision making framework for the evaluation of lean practices at

logistics distribution centers," *Annals of Operations Research*, vol. 247, no. 2, pp. 735–757, 2016.

[51] N. Liao, G. Wei, and X. Chen, "TODIM method based on cumulative prospect theory for multiple attributes group decision making under probabilistic hesitant fuzzy setting," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 322–339, 2022.

[52] E. Javanmardi, S. Liu, and N. Xie, "Exploring grey systems theory-based methods and applications in sustainability studies: a systematic review approach," *Sustainability*, vol. 12, no. 11, p. 4437, 2020.

[53] E. Javanmardi and S. Liu, "Exploring the human cognitive capacity in understanding systems: a grey systems theory perspective," *Foundations of Science*, vol. 25, no. 3, pp. 803–825, 2019.

[54] S. Zhang, H. Gao, G. Wei, and X. Chen, "Grey relational analysis method based on cumulative prospect theory for intuitionistic fuzzy multi-attribute group decision making," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 2, pp. 3783–3795, 2021.

[55] E. Javanmardi, S. Liu, and N. Xie, "Exploring the philosophical paradigm of grey systems theory as a postmodern theory," *Foundations of Science*, vol. 25, no. 4, pp. 905–925, 2019.

[56] B. Zhu, L. Yuan, and S. Ye, "Examining the multi-timescales of European carbon market with grey relational analysis and empirical mode decomposition," *Physica A: Statistical Mechanics and Its Applications*, vol. 517, pp. 392–399, 2019.

[57] A. Malek, S. Ebrahimnejad, and R. Tavakkoli-Moghaddam, "An improved hybrid grey relational analysis approach for green resilient supply chain network assessment," *Sustainability*, vol. 9, no. 8, p. 1433, 2017.

[58] C.-Y. Kung and K.-L. Wen, "Applying Grey Relational Analysis and Grey Decision-Making to evaluate the relationship between company attributes and its financial performance-A case study of venture capital enterprises in Taiwan," *Decision Support Systems*, vol. 43, no. 3, pp. 842–852, 2007.

[59] L. Javanmardi and Liu, "Exploring grey systems theory-based methods and applications in analyzing socio-economic systems," *Sustainability*, vol. 11, no. 15, p. 4192, 2019.

[60] E. Javanmardi, S. Liu, and N. Xie, "Exploring the philosophical foundations of grey systems theory: subjective processes, information extraction and knowledge formation," *Foundations of Science*, vol. 26, no. 2, pp. 371–404, 2020.

[61] O. Alptekin, N. Alptekin, and B. Sarac, "Evaluation of low carbon development of European union countries and Turkey using grey relational analysis," *Tehnicki Vjesnik-Technical Gazette*, vol. 25, pp. 1497–1505, 2018.

[62] H. Zhang, G. Wei, and X. Chen, "SF-GRA method based on cumulative prospect theory for multiple attribute group decision making and its application to emergency supplies supplier selection," *Engineering Applications of Artificial Intelligence*, vol. 110, Article ID 104679, 2022.

[63] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.

[64] X. Gou, Z. Xu, H. Liao, and F. Herrera, "Probabilistic double hierarchy linguistic term set and its use in designing an improved VIKOR method: the application in smart healthcare," *Journal of the Operational Research Society*, vol. 72, no. 12, pp. 2611–2630, 2020.

[65] J. J. Gibson, The theory of affordances. In R. E. Shaw & J. Bransford (Eds.), Perceiving, Acting, and Knowing, pp. 67–82, Lawrence Erlbaum, Hillsdale, MI, USA, 1977.

[66] Y. Dong, C.-C. Li, Y. Xu, and X. Gu, "Consensus-based group decision making under multi-granular unbalanced 2-tuple linguistic preference relations," *Group Decision and Negotiation*, vol. 24, no. 2, pp. 217–242, 2015.

[67] Z. Zhang, Y. Gao, and Z. Li, "Consensus reaching for social network group decision making by considering leadership and bounded confidence," *Knowledge-Based Systems*, vol. 204, Article ID 106240, 2020.

[68] Y. Dong, Y. Wu, H. Zhang, and G. Zhang, "Multi-granular unbalanced linguistic distribution assessments with interval symbolic proportions," *Knowledge-Based Systems*, vol. 82, pp. 139–151, 2015.

[69] Z. Zhang, J. Gao, Y. Gao, and W. Yu, "Two-sided matching decision making with multi-granular hesitant fuzzy linguistic term sets and incomplete criteria weight information," *Expert Systems with Applications*, vol. 168, Article ID 114311, 2021.

[70] A. Punkka and A. Salo, "Preference Programming with incomplete ordinal information," *European Journal of Operational Research*, vol. 231, no. 1, pp. 141–150, 2013.

[71] Y. Wu, C.-C. Li, X. Chen, and Y. Dong, "Group decision making based on linguistic distributions and hesitant assessments: maximizing the support degree with an accuracy constraint," *Information Fusion*, vol. 41, pp. 151–160, 2018.

[72] L. F. Autran Monteiro Gomes and L. A. Duncan Rangel, "An application of the TODIM method to the multicriteria rental evaluation of residential properties," *European Journal of Operational Research*, vol. 193, no. 1, pp. 204–211, 2009.

[73] L. F. A. M. Gomes, L. A. D. Rangel, and F. J. C. Maranhão, "Multicriteria analysis of natural gas destination in Brazil: an application of the TODIM method," *Mathematical and Computer Modelling*, vol. 50, no. 1-2, pp. 92–100, 2009.

[74] Z. Jiang, G. Wei, and X. Chen, "EDAS method based on cumulative prospect theory for multiple attribute group decision-making under picture fuzzy environment," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 3, pp. 1723–1735, 2022.

[75] Z. Jiang, G. Wei, and Y. Guo, "Picture fuzzy MABAC method based on prospect theory for multiple attribute group decision making and its application to suppliers selection," *Journal of Intelligent and Fuzzy Systems*, vol. 42, no. 4, pp. 3405–3415, 2022.

[76] Y. Huang, R. Lin, and X. Chen, "An enhancement EDAS method based on prospect theory," *Technological and Economic Development of Economy*, vol. 27, no. 5, pp. 1019–1038, 2021.

[77] N. Liao, H. Gao, G. Wei, and X. Chen, "CPT-MABAC-based multiple attribute group decision making method with probabilistic hesitant fuzzy information," *Journal of Intelligent and Fuzzy Systems*, vol. 41, no. 6, pp. 6999–7014, 2021.

[78] P. Tanadtang, D. Park, and S. Hanaoka, "Incorporating uncertain and incomplete subjective judgments into the evaluation procedure of transportation demand management alternatives," *Transportation*, vol. 32, no. 6, pp. 603–626, 2005.

[79] Z. S. Xu, “Models for multiple attribute decision making with intuitionistic fuzzy information,” *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 15, no. 03, pp. 285–297, 2007.
[80] Z. Chen and W. Yang, “A new multiple attribute group decision making method in intuitionistic fuzzy setting,” *Applied Mathematical Modelling*, vol. 35, no. 9, pp. 4424–4437, 2011.
[81] Z. Zhang and S.-M. Chen, “Group decision making with incomplete q-rung orthopair fuzzy preference relations,” *Information Sciences*, vol. 553, pp. 376–396, 2021.
[82] R. M. Zulqarnain, X. L. Xin, I. Siddique, W. Asghar Khan, and M. A. Yousif, “TOPSIS method based on correlation coefficient under pythagorean fuzzy soft environment and its application towards green supply chain management,” *Sustainability*, vol. 13, no. 4, p. 1642, 2021.
[83] Z.-Y. Chen, F. Xiao, M.-H. Deng, H.-W. Liu, and J.-Q. Wang, “Additive consistency-based decision-making with incomplete probabilistic linguistic preference relations,” *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 405–424, 2022.