Research Article
The Extended MOORA Method Based on Fermatean Fuzzy Information

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The intelligent manufacturing system (IMS) is a framework that improves productivity by organizing the logical features involved in manufacturing. The procedure of intelligent manufacturing owns the capability to self-control the manufacturing of the products according to the specifications of design. Different IMSs are designed to deal with continuous changes in market which can adjust to make the modified environment easier. The central idea of this research article is to select an IMS that can adapt the updated situations faster than the existing competing systems and provide higher benefits in utilizing new possibilities. To select such IMS, the applicability of multiobjective optimization on the basis of the ratio analysis (MOORA) method has been explored using Fermatean fuzzy sets. The Fermatean fuzzy aggregated weighted operators are used to construct the decision matrices. Then, the ratio analysis-based MOORA method is developed to accomplish the ranking of under consideration IMSs. Furthermore, the conversion of qualitative attributes into quantitative attributes has been performed using Fermatean fuzzy numbers (FFNs). Finally, a brief comparative analysis of the developed technique with existing models is narrated to reveal the flexibility of the Fermatean fuzzy MOORA method.

1. Introduction

Intelligent manufacturing systems (IMoS) are capable of furnishing the adding performance and stoutly changing systems to manage with the request conditions. They can grease largely complex manufacturing systems as well as colorful degrees of functionality of products. As they can handle design changes as snappily as possible, they can fluently acclimatize themselves to the changes in the request and satisfy client conditions, which are utmost of the time too versatile. There are different manufacturing systems, including the lean MS, agile MS, leagile MS, computer-integrated MS, flexible MS, bionic MS, holonic MS, and fractal MS. Lean manufacturing consults to the demand of lean practices, ideas, and tools for the progress and production of physical products. Several inventors are applying lean manufacturing principles to completely remove the waste, optimize processes for increase of creativity, and reduce time to market in a fast moving and constantly changing worldwide market place. Agile manufacturing is mainly a technique chosen by manufacturing organizations to make things. It keeps in mind those techniques, means, and production which the firm utilizes to respond according to the change in the demands of consumers within the industry. In the running situation, leagile manufacturing is the most important research field in the operational management area, but there is a lack of research concentrating on leagile manufacturing industries. The advantages of lean and agile systems are combined in the leagile system. Flexible manufacturing has the capacity to adjust with new conditions according to the requirements of the products without any change in the quality of the products. A flexible MS is the
type of production system that can reduce both production instances and requirement of supply amount. Computer incorporated manufacturing (CIM) is the producing approach of the use of computer systems to govern the complete manufacturing process. It is the use of integrated systems and data communications coupled with new managerial philosophers. The major benefit of CIM is that it has the capability to design automated manufacturing processes. The holonic manufacturing system (HMS) has the capacity to give moldable planning/construction to the changes and ambiguities of the manufacturing terms. The HMS has elaborated to cover the demands of consumers who are adapted to the capability of the company.

1.1. Objectives. Motivated by the distinctive features of Fermatean fuzzy sets (FFSs) and MOORA technique, we have designed a novel FF-MOORA method. The objectives of this research are as follows:

1. The main objective of this study is the development of decision-making technique, namely, the FF-MOORA method to answer the MAGDM (multi-attribute group decision-making) issues under the framework of FFSs

2. To aggregate the decision data provided by experts in the form of FFFNs (Fermatean fuzzy numbers), FFWA (Fermatean fuzzy weighted averaging) operator and FFWG (Fermatean fuzzy weighted geometric) operators are designed. These operators are useful to obtain the subordinate ranking of alternatives.

3. To demonstrate the applicability of our developed technique, we implement the proposed FF-MOORA method to some real case examples. We have implemented the designed approach to measure the quality of IMS and AMS.

4. To demonstrate the robustness and authenticity of our developed work, we compare our developed method to check its authenticity with the previous knowledge.

Hence, the proposed model would be able to own the following benefits and advantages. The FF membership grades generalize the intuitionistic fuzzy and Pythagorean fuzzy grades as they develop the suitable area of unsure facts and figures. The generalization of intuitionistic fuzzy numbers and Pythagorean fuzzy numbers to FFFNs increases the flexibility of the furnished uncertain records and increases the applicability of the MOORA approach to the machine where the membership capabilities are complex or not viable to identify completely. The proposed method allows the assessment of all options and their corresponding ratings in form of linguistic variables. These variables are expressed through FFFNs, which enhances the ability of the system and can increase the applicability of the MOORA technique. The FF-MOORA method represents a scientific way and a computational ratio to pick the best opportunity. The analysis to determine the best alternative can be carried out in low setup time while using the FF-MOORA method.

1.2. Literature Review. Recently, there has been a rapid progress in the area of multiple criteria decision-making (MCDM). The basic motive of MCDM is to provide the strategies that rank alternatives or pick out the maximum suitable alternative number of the series of possible options corresponding to special criteria [1]. The common existence of MCDM problems in modern life ensures the applicability of new theories of MCDM in various fields, including macroeconomic domain, military affair, management, and industrial engineering [2]. Similarly, various multicriteria methodologies have been utilized to deliver assistance in the complicated task of establishing the decisions [3]. For this purpose, elimination and choice translation reality (ELECTRE) [4], decision support system (DSS) [5], data envelopment analysis (DEA) [6], analytic hierarchy process (AHP) [7], technique for order of preference by similarity to ideal solution (TOPSIS) [8], dimensional analysis (DA) [9], multicriteria optimization and compromise solution (vsekriterijumska optimizacija i kompromisno resenje, VIKOR) [10], analytic network process (ANP) [11], multiobjective optimization on the basis of ratio analysis (MOORA) [12], and preference selection index (PSI) [13] are the most common methodologies in the literature. Furthermore, there are various classical MCDM problems in which only the crisp data are utilized to appraise the alternatives corresponding to every criterion, and preferences of criteria are considered in nonfuzzy logic. Note that the traditional MOORA method is sufficient to establish the evaluations and rankings of the alternatives in a crisp environment without any complexity. However, there are numerous MCDM problems in real life in which the preferences of DMs to appraise the attributes and criteria values are represented using linguistic terms containing uncertainty and hesitation [1, 4, 14]. In such cases, the crisp MOORA method expresses few drawbacks to manipulate the fuzzy and qualitative information involved in a problem of MCDM [15, 16].

The fuzzy set (FS) is an interesting analytical tool to deal with vagueness and uncertainty [17, 18]. FSs were drastically applied in MCDM so that people can recover from the drawbacks and limitations of classical techniques. The MOORA method was applied for the selection of the best IMS beneath fuzzy surroundings by Mandal and Sarkar [19]. The MAIRCA (multiatribute ideal real comparative analysis) technique under fuzzy sets was developed by Boral et al. [20]. An integrated fuzzy AHP and fuzzy MOORA technique was developed to study the hassle of the economic engineering area [21]. In the proposed technique, the weight values of criteria had been obtained using the fuzzy AHP, and fuzzy MOORA was carried out for the ranking of alternatives. Emovon et al. [22] proposed the utility of the fuzzy MOORA technique inside the layout and fabrication of an automated hammering machine. The authors applied a case of the shaft to illustrate the suitability and applicability of the developed technique. The ranking and evaluation of renewable energy sources in Turkey were proposed in [23]. The important weights of the assessment criteria were calculated by the fuzzy entropy method.
Furthermore, contemporary analysis reveals that MCDM techniques are being developed combining with intuitionistic fuzzy sets (IFSs). Mainly, the IFSs, proposed by Atanassov [24], are a generalization of classical FSs. According to literature over the last decade, great attention has been paid by the academics to the benefits of IFSs in MCDM [3, 25]. The combination of MOORA and IFSs for the choice of suppliers was proposed by Perez-Dominguez et al. [26]. Many MCDM problems under IFS are discussed in [27]. Moreover, the ranking for the selection of materials for mushroom cultivation was obtained through the MOORA method that was based on new score functions of interval-valued IFSs [28]. Some other aggregates of IFSs and rough numbers developed to cope with internal and external vagueness in evaluation of risks by Huang et al. [29]. The proposed model utilized the rough numbers to handle external uncertainty, while IFSs have been used to deal with inner vagueness. Interval-valued IFSs were utilized to deal with the uncertainty and hesitation in FMEA by Huang and Xiao [30]. The authors additionally advanced an Excel system to limit the computational burden.

Pythagorean fuzzy sets (PFSs) [31] are defined to relax the limitation of orthopair belonging degrees to the weaker supposition that \( \mu^2(x) + \nu^2(x) \leq 1 \) has to keep all through. There are number of aggregation operators which are defined under PFSs [32]. The MOORA method was combined with PFSs by Prez-Domínguez et al. [33]. An ELECTRE-I technique takes gain of the superiority of PFSs within the expression of human evaluations [34]. Furthermore, PFH-TOPSIS and PFH-ELECTRE-I, two modified techniques, were proposed by Akram et al. [35]. A novel method for hazard evaluation in FMEA under PF statistics was proposed by Luqman et al. [36]. In this method, risk precedence indexes are calculated through triangular PF numbers, and the applicability of the proposed version has been proved through a case of the steam valve gadget. A novel technique to gain risk ratings in FMEA developed by means of combining the interval-valued PFSSs and the prolonged MULTIMOORA method [37]. Recently, Akram et al. [38] proposed the ELECTRE-I approach under hesitant PF surroundings to measure and rank risks in FMEA. The authors used an outranking decision graph to achieve the most important failure mode. The version is tested via the look of actual-existence applications, which include the prevention of toddler abduction and chance evaluation of healthcare failure modes in blood transfusion. Huang et al. [39] introduced the MULTIMOORA method under PF information that was based on two novel distance measure of PFSSs, i.e., Dice distance and Jaccard distance. Moreover, a novel score function which is based on determinacy degree and indeterminacy degree is proposed to represent approximate PFSSs. Li et al. [40] proposed the PF-MULTIMOORA method to determine the passenger satisfaction level of the public transport system under large groups. The effectiveness of the proposed passenger satisfaction evaluation method is demonstrated by studying the rail transit network in Shanghai. A new area of fuzzy graphs, namely, linguistic \( q \)-rung orthopair fuzzy graphs (Lq-ROFGs) was introduced by Akram et al. [41]. The authors proposed certain new ideas, including generalized product-connectivity energy, product-connectivity energy, and Laplacian energy. Furthermore, the \( q \)-rung orthopair fuzzy linguistic family of point aggregation operators under linguistic \( q \)-rung orthopair fuzzy sets was proposed in [42]. The point-weighted aggregation operators were introduced with the help of arithmetic and geometric operators.

Fermatean fuzzy sets (FFSSs), first developed by Senapati and Yager [43, 44], can handle uncertain information more effectively within the procedure of choice making. The authors have proposed an example to show the reasonability of the FFS, i.e., whenever a person wants to assign the priority for the degree of an attribute corresponding to a criterion, he/she may assign the degree to which the attribute fulfills the criterion as 0.9, and similarly, when the attribute does not fulfill the criterion as 0.6, he/she can definitely get 0.9 + 0.6 > 1; therefore, it does not satisfy the constraint of IFSs. Also, we have 0.9^2 + 0.6^2 = 1.17 > 1, which does not follow the condition of PFS. However, we can get 0.9^2 + 0.6^2 = 0.945 < 1, which is suitable to engage the FFS to capture it, that is, the FFSs have more uncertainties than IFSs and PFSSs and are capable of dealing with higher levels of uncertainties. A new hybrid model, namely, complex Fermatean fuzzy N-soft set was introduced and applied by Akram et al. [45]. A combination of FFSSs and soft expert sets was proposed by Akram et al. [46]. The authors described the proposed model with the help of numerical examples. Shahzadi et al. [47] presented a novel multiple-attribute decision-making (MADM) technique based on FFS and Hamacher operator. Some valuable contributions using FFSSs and hybrid models have been done in [48–50]. A novel technique that is based on the MULTIMOORA approach, the maximizing deviation method, and Einstein aggregation operators under FF environment was proposed by Rani and Mishra [51]. Akram et al. [52] introduced a novel technique by merging the FFSSs with linguistic term sets. Furthermore, the authors presented various linguistic FF Hamy mean operators, namely, the linguistic FF Hamy mean operator, the linguistic FF dual Hamy mean operator, the linguistic FF weighted Hamy mean operator, and the linguistic FF weighted dual Hamy mean operator.

Mahmood et al. [53] introduced the generalized MULTIMOORA technique under \( T \)-spherical fuzzy environment. The authors proposed the concept of \( T \)-spherical fuzzy Dombi prioritized weighted arithmetic aggregation operators, \( T \)-spherical fuzzy Dombi prioritized arithmetic, \( T \)-spherical fuzzy Dombi prioritized geometric, and \( T \)-spherical fuzzy Dombi prioritized weighted geometric aggregation operators to represent the interrelations between any number of \( T \)-spherical fuzzy sets. The dual hesitant \( q \)-rung orthopair fuzzy 2-tuple linguistic sets were introduced by Naz et al. [54]. An extended MULTIMOORA method based on interval 2-tuple Pythagorean fuzzy linguistic variables was introduced in [55]. The score, accuracy, and Hamming distance of the proposed theory were also introduced in the proposed work. A novel MAGDM technique, namely, the ELECTRE-II method under PF information was proposed by Akram et al. [56]. The proposed technique is based on three kinds of PF outranking sets, such as concordance, indifferent,
and discordance sets. The generalized EDAS method based on MSM operators under 2-tuple linguistic-spherical fuzzy information was proposed in [57]. A novel technique to find out the neutrosophic shortest path was proposed in [58] by considering the Gaussian valued neutrosophic numbers. Akram et al. [59] proposed an interval-valued FF fractional transportation problem and discussed the numerical examples of the proposed study. Ejegwa and Zuakwagh [60] established FF composite relation, which is based on max-average rule to maximize the applicability of FFSs in machine learning, through soft computing approach. They also provided some numerical examples to prove the superiority of the proposed approach over the existing procedures. A combination of the matrix approach to robustness analysis (MARA) and fuzzy DEMATEL-based ANP (FDANP) was proposed in [61]. The results showed that by concerning the environmental situations and the possible future of Iran, education service is the most robust business to start. Recently, Akram et al. [62] proposed an integrated MULTI-MOORA method with 2-tuple linguistic FFSs and studied its application in urban quality of life selection problem. Further recent studies on MCDM methods under fuzzy information can be handled through the proposed technique. Being motivated by the benefits of FFSs and the MOORA method, we propose a novel technique of MCDM by generalizing the MOORA under FF environment. In this regard, the contribution and originality of the proposed work can be summarized as follows: First, the MOORA method is proposed under FF information to deal with any other kind of arguments rather than the crisp knowledge and to generalize its applicability to more extensive areas. Second, two types (qualitative as well as quantitative) of information can be handled through the proposed technique.

The remainder of the article is arranged as follows: Some fundamental concepts of FFSs are discussed in Section 2. We talk about certain operations and distance measures for FFSs. In Section 3, the FF-MOORA technique is developed. Section 4 narrates the capability and effectiveness of the proposed method with the aid of way of solving some numerical examples. We apply the FF-MOORA technique for the selection of the best intelligent manufacturing system. In Section 5, we present a comprehensive comparison of the FF-MOORA approach with some existing techniques. Section 6 presents some advantages and disadvantages of the proposed scheme. The last section deals with conclusions and future research directions.

2. Preliminaries

**Definition 1** (See [43]). A Fermatean fuzzy set (FFS) \( \varphi \) on a nonempty set \( V \) is given by \( \varphi = \{ (x, \mu_\varphi(x), \nu_\varphi(x)) \} \), where \( \mu_\varphi: V \rightarrow [0,1] \), \( \nu_\varphi: V \rightarrow [0,1] \), and \( \omega_\varphi(x) = \sqrt{1 - (\mu_\varphi(x))^2} - (\nu_\varphi(x))^3 \) specify membership degree (MD), nonmembership degree (NMD), and indeterminacy degree (ID), respectively. Note that, FFSs are components of the FFS.

**Definition 2** (See [43]). The score-valued function and accuracy-valued function for FFN \( \varphi = (\mu_\varphi, \nu_\varphi) \) are given by

\[
S(\varphi) = \mu_\varphi^3 - \nu_\varphi^3, \text{ where } S(\varphi) \in [-1,1],
\]

\[
A(\varphi) = \mu_\varphi^3 + \nu_\varphi^3, \text{ where } A(\varphi) \in [0,1].
\]

**Definition 3** (See [43]). Consider two FFSs \( \varphi_1 = (\mu_{\varphi_1}, \nu_{\varphi_1}) \) and \( \varphi_2 = (\mu_{\varphi_2}, \nu_{\varphi_2}) \).

1. If \( S(\varphi_1) < S(\varphi_2) \), then \( \varphi_1 \prec \varphi_2 \).
2. If \( S(\varphi_1) > S(\varphi_2) \), then \( \varphi_1 \succ \varphi_2 \).
3. If \( S(\varphi_1) = S(\varphi_2) \), then
   - (a) If \( A(\varphi_1) < A(\varphi_2) \), then \( \varphi_1 \prec \varphi_2 \).
   - (b) If \( A(\varphi_1) > A(\varphi_2) \), then \( \varphi_1 \succ \varphi_2 \).
   - (c) If \( A(\varphi_1) = A(\varphi_2) \), then \( \varphi_1 \sim \varphi_2 \).

**Definition 4** (See [43]). Let \( \varphi_1 = (\mu_{\varphi_1}, \nu_{\varphi_1}) \) and \( \varphi_2 = (\mu_{\varphi_2}, \nu_{\varphi_2}) \) be two FFSs. The operational laws between them are as follows:

1. (i) \( \varphi_1^\circ = (\nu, \mu) \)
2. (ii) \( \varphi_1 \leq \varphi_2 \) if \( \mu_{\varphi_1} \leq \mu_{\varphi_2} \) and \( \nu_{\varphi_1} \leq \nu_{\varphi_2} \)
3. (iii) \( \varphi_1 \cdot \varphi_2 = (\mu_{\varphi_1} + \mu_{\varphi_2} - \mu_{\varphi_1} \mu_{\varphi_2}, \nu_{\varphi_1} \nu_{\varphi_2}) \)
4. (iv) \( \varphi_1 \odot \varphi_2 = (\mu_{\varphi_1} \mu_{\varphi_2}, \sqrt{\nu_{\varphi_1} + \nu_{\varphi_2} - \nu_{\varphi_1} \nu_{\varphi_2}}) \)
5. (v) \( \lambda \varphi_1 = (\sqrt{1 - (1 - \mu_{\varphi_1})^2}, \nu_{\varphi_1}) \)
6. (vi) \( \varphi_1^\circ = (\mu_{\varphi_1}^3, \sqrt{1 - (1 - \nu_{\varphi_1})^3}) \)

3. The Fermatean Fuzzy-MOORA Method

The FF-MOORA method consists of the following steps as shown in Figure 1.

Let \( \tilde{\varrho} = [\tilde{\varphi}_1, \tilde{\varphi}_2, \ldots, \tilde{\varphi}_n] \) be alternatives and \( h = [h_1, h_2, \ldots, h_m] \) be parameters for the evaluation of objects. The Fermatean fuzzy-MOORA (FF-MOORA) method is given as follows:

Step 1. First of all, take a team of decision makers (DMs) and establish the significance of every decision maker. Let \( \text{DMs} = \{ \text{DM}_1, \text{DM}_2, \ldots, \text{DM}_r \} \) be the group of DMs. The linguistic term defined by Fermatean fuzzy numbers will judge the importance of each DM. Table 1 shows the linguistic values (LVs) given by Fermatean fuzzy numbers for ranking the value of decisions of each decision maker.

Suppose \( \text{DM}_k = \{ \mu_k, \nu_k, \pi_k \} \) is the Fermatean fuzzy number for ranking the \( k \)th decision maker. To find the weight of \( k \)th decision maker, the expression given in the following equation will be used.

\[
\lambda_k = \frac{(\mu_k^3 + \pi_k^3)(\mu_k^3 + \nu_k^3)}{\sum_{k=1}^{l} (\mu_k^3 + \pi_k^3)(\mu_k^3 + \nu_k^3)},
\]

where \( \sum_{k=1}^{l} \lambda_k = 1 \).

Step 2. Calculate the values of parameters. Usually, all parameters may not have the equal value, and DMs may assign various opinions about the same parameter. Therefore, every suggestion has to be taken and joined...
Set up a group of policy makers and grab the preferences

Determine the set of attributes

Determine the set of alternatives

Construc the FF decision matrix

Mention the priorities of parameters

Construct the weighted FF decision matrix

Aggregate the FF decision matrix according to linguistic variables

Decision matrix is aggregated through FFWA operator

Sum up the values of cost and benefit criteria

Defuzzify the obtained data using score and accuracy values of FFSs

Determine the participation value of each alternative

Choose the best alternative with maximum participation value

End

Figure 1: Flowchart for the FF-MOORA method.
into single one. In Table 1, LVs are given that are used to rank the values of parameter by each decision maker. Suppose that $\omega^k_j = \{\mu^k_j, v^k_j, \pi^k_j\}$ is the Fermatean fuzzy number consigned to the parameter $h_j$ by the $k$th DM. The parameters’s weights are calculated by the Fermatean fuzzy weighted averaging (FFWA) operator given [44].

$$w_j = FFWA(w_j^{(1)}, w_j^{(2)}, \ldots, w_j^{(j)}) = \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \ldots \lambda_j w_j^{(j)} = \left( \sum_{k=1}^{j} \lambda_k \mu^k_j \right) \left( \sum_{k=1}^{j} \lambda_k v^k_j \right).$$

Table 1: LVs for ranking the value of DMs and parameters.

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>FFNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely important (EI)</td>
<td>(0.82, 0.70)</td>
</tr>
<tr>
<td>Inconsiderable (IN)</td>
<td>(0.36, 0.41)</td>
</tr>
<tr>
<td>Quite worthless (QW)</td>
<td>(0.21, 0.70)</td>
</tr>
<tr>
<td>Noteworthy (NW)</td>
<td>(0.73, 0.1)</td>
</tr>
<tr>
<td>Normal (N)</td>
<td>(0.42, 0.52)</td>
</tr>
</tbody>
</table>

Step 3. For ranking of alternatives $\partial_i$, form the aggregated Fermatean fuzzy decision matrix (FFDM) on the basis of opinions of DMs. Let $R^k = (x^k_{ij})_{n \times m}$ be a FFD, LVs are given in Table 2 for ranking the alternatives according to parameters. The views of all DMs are combined into an aggregated FFD by applying the FFWA operator. Now, $R = (x_{ij})_{n \times m}$ is

$$x_{ij} = FFWA(x_{ij}^{(1)}, x_{ij}^{(2)}, \ldots, x_{ij}^{(k)}) = \lambda_1 x_{ij}^{(1)} \oplus \lambda_2 x_{ij}^{(2)} \oplus \ldots \lambda_k x_{ij}^{(k)}.$$ (4)

The FFD is characterized as follows:

$$FFDM = R = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}.$$ (5)

particularly,

$$R = \begin{bmatrix} \{\mu_{h_1}(h_1), v_{h_1}(h_1), \pi_{h_1}(h_1)\} & \cdots & \{\mu_{h_m}(h_m), v_{h_m}(h_m), \pi_{h_m}(h_m)\} \\ \vdots & \ddots & \vdots \\ \{\mu_{h_1}(h_1), v_{h_1}(h_1), \pi_{h_1}(h_1)\} & \cdots & \{\mu_{h_m}(h_m), v_{h_m}(h_m), \pi_{h_m}(h_m)\} \end{bmatrix}.$$ (6)

Step 4. Determine the aggregated weighted FFD (AWFFDM). The AWFFDM is determined by using the FFD, attained from Step 3, and weights of criteria, attained from Step 2. The components of AWFFDM are computed using the following equation:

$$AWFFDM = \left\{ x_{i1}, \sqrt{\frac{3}{2}} \mu_{h_i}(x), \mu_{h_i}(x) \right\} + y_w(x) - v_{h_i}(x), v_w(x) \right\},$$

$$AWFFDM = R = \left[ \begin{bmatrix} \{\mu_{h_1}(h_1), v_{h_1}(h_1), \pi_{h_1}(h_1)\} & \cdots & \{\mu_{h_m}(h_m), v_{h_m}(h_m), \pi_{h_m}(h_m)\} \\ \vdots & \ddots & \vdots \\ \{\mu_{h_1}(h_1), v_{h_1}(h_1), \pi_{h_1}(h_1)\} & \cdots & \{\mu_{h_m}(h_m), v_{h_m}(h_m), \pi_{h_m}(h_m)\} \end{bmatrix} \right].$$ (7)

Step 5. At this step, we identify the benefit and cost parameters to find the sum of benefit and cost parameters. The benefit parameters are those where highest values are required, and on the other side, the cost parameters are those where the lowest values are required.

The sum of benefit parameters is calculated as

$$BN_{h_i} = \sum_{i=1}^{n} (\mu_{h_i}(h_i), v_{h_i}(h_i)).$$ (8)

$BN_{h_i}$ represents the benefit parameters for alternatives $i = 1, 2, \ldots, n$. $h_i = 1, 2, \ldots, g$ represents the maximal parameter.

The sum of cost parameters is calculated as
Table 2: LVs for ranking the alternatives.

<table>
<thead>
<tr>
<th>LVs</th>
<th>FFNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptionally low (ExL)</td>
<td>(0.3, 0.5)</td>
</tr>
<tr>
<td>Extremely low (EL)</td>
<td>(0.35, 0.43)</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>(0.36, 0.56)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.4, 0.73)</td>
</tr>
<tr>
<td>Below average (BA)</td>
<td>(0.42, 0.3)</td>
</tr>
<tr>
<td>Average (A)</td>
<td>(0.47, 0.21)</td>
</tr>
<tr>
<td>Above average (AA)</td>
<td>(0.50, 0.62)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(0.55, 0.38)</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>(0.60, 0.18)</td>
</tr>
<tr>
<td>Extremely high (EH)</td>
<td>(0.72, 0.50)</td>
</tr>
<tr>
<td>Exceptionally high (ExH)</td>
<td>(0.83, 0.42)</td>
</tr>
</tbody>
</table>

\[
\text{C Nh}_j = \sum_{j=1}^{m} \left( \mu_3(h_j), \nu_3(h_j) \right). \tag{9}
\]

\[
\text{C Nh}_j \text{ represents the cost parameters for alternatives } i = 1, 2, \ldots, n. \quad h_j = g + 1, g + 2, \ldots, m \text{ shows the minimal parameter.}
\]

Step 6. At this step, defuzzification of the sum of benefits and cost parameters is performed. For the defuzzification of highest \( h_i \) and lowest \( h_j \) parameters, we apply the following equations:

\[
\text{Nh}_i = \mu_3^{3}(h_i) - \nu_3^{3}(h_i); \\
\text{Nh}_j = \mu_3^{3}(h_j) - \nu_3^{3}(h_j). \tag{10}
\]

Step 7. To know the participation of each alternative, utilize the following expression:

\[
N \tilde{d}_i = \text{Nh}_i - \text{Nh}_j. \tag{11}
\]

The value of \( N \tilde{d}_i \) may be positive and negative; it depends upon the total value of profitable and loss criteria in the decision matrix. The superior option has the largest value of \( N \tilde{d}_i \), and the inferior option has the lowest value of \( N \tilde{d}_i \).

Step 8. Set up the alternatives in the decreasing order of \( N \tilde{d}_i \) and choose the best alternative with the highest value of \( N \tilde{d}_i \).

4. Numerical Interpretation

In order to explain the proposed method, two numerical cases are represented as follows:

\[
\lambda_{1,2} = \frac{(0.73^3 + 0.83^3(0.73^3/0.73^3 + 0.1^3))}{(0.73^3 + 0.83^3(0.73^3/0.73^3 + 0.1^3)) + (0.73^3 + 0.83^3(0.73^3/0.73^3 + 0.1^3))} \tag{12}
\]

\[
= 0.5.
\]

4.1. The Intelligent Manufacturing System. The IMS is an advanced approach of production which combines the compatibilities of human beings, machinery, and processes to obtain the better feasible production outcome. The manufacturing system applies the entire process of gathering inputs, arranging them, and transforming them into the required output. The IMS seeks to achieve optimal utilization of manufacturing resources, minimize wastage, and add value to the business. The main objective of this application is the selection of the best IMS. Two decision makers and six intelligent manufacturing systems are under taken for the selection of the best one to compete with the modern requirements of the world. Five parameters are under consideration for describing the important characteristics of the intelligent manufacturing systems. The following criteria are under consideration:

(i) Cost: The information is not desirable if the solution is more costly than the problem. The cost of gathering data and processing it into information must be weighed against the benefits derived from using such information.

(ii) Lead time: That is, information must be delivered at the right time and the right place to the right person.

(iii) Quality: Information is good only if it is relevant. This means that it should be pertinent and meaningful to the decision maker and should be in his area of responsibility.

(iv) Service level: Information should be to the point and just enough, no more, no less.

(v) Product variety: It should be accurate, consistent with facts, and verifiable. Inadequate or incorrect information generally leads to decisions of poor quality.

(vi) Robustness: Since information is already in a summarized form, it must be understood by the receiver so that he will interpret it correctly.

The set of IMSs is given by \( \{\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_6\} \).

Step 1. Form a team of DMrs and find the significance of everyone. Two DMrs form the team and their values are given in Table 3. Table 1 shows the ranking in terms of linguistic values. To get the weight of every decision maker, equation (2) is used, and every decision maker has equal worth.
Step 2. Identify the significance of each parameter. Tables 3 and 4 show the analysis of every DM about the significance of parameters in terms of LVs.

The viewpoints of DMs are unified by (3) which are given by

\[
W = \begin{bmatrix}
0.39, 0.47 \\
0.39, 0.47 \\
0.82, 0.5 \\
0.58, 0.31 \\
0.58, 0.31
\end{bmatrix}^T
\]  \tag{13}

Step 3. Establish the AFFDM showing the ratings of objects \( \tilde{O}_i \) based on the views of DMs. Table 5 shows the ratings assigned by every DM. The AFFDM is given as follows:

\[
R = \begin{bmatrix}
(0.5, 0.46) & (0.46, 0.47) & (0.72, 0.5) & (0.61, 0.56) & (0.54, 0.20) & (0.51, 0.30) \\
(0.60, 0.49) & (0.49, 0.42) & (0.45, 0.62) & (0.54, 0.47) & (0.60, 0.36) & (0.69, 0.4) \\
(0.47, 0.21) & (0.61, 0.56) & (0.45, 0.68) & (0.39, 0.43) & (0.53, 0.5) & (0.47, 0.21) \\
(0.38, 0.65) & (0.39, 0.37) & (0.49, 0.42) & (0.45, 0.26) & (0.55, 0.38) & (0.72, 0.50) \\
(0.65, 0.32) & (0.56, 0.62) & (0.44, 0.47) & (0.55, 0.4) & (0.63, 0.36) & (0.58, 0.28) \\
(0.38, 0.65) & (0.38, 0.65) & (0.58, 0.28) & (0.61, 0.56) & (0.47, 0.21) & (0.61, 0.56)
\end{bmatrix}
\]  \tag{14}

Step 4. Construct the AWFFDM, which is given as follows:

\[
R' = \begin{bmatrix}
(0.20, 0.71) & (0.18, 0.72) & (0.59, 0.75) & (0.35, 0.70) & (0.39, 0.28) & (0.30, 0.52) \\
(0.23, 0.73) & (0.19, 0.69) & (0.37, 0.81) & (0.31, 0.63) & (0.44, 0.42) & (0.40, 0.59) \\
(0.18, 0.58) & (0.24, 0.77) & (0.37, 0.84) & (0.23, 0.61) & (0.39, 0.55) & (0.27, 0.45) \\
(0.15, 0.81) & (0.15, 0.67) & (0.40, 0.71) & (0.26, 0.49) & (0.40, 0.44) & (0.42, 0.66) \\
(0.25, 0.64) & (0.22, 0.80) & (0.36, 0.74) & (0.32, 0.59) & (0.46, 0.42) & (0.34, 0.50) \\
(0.15, 0.81) & (0.15, 0.81) & (0.48, 0.64) & (0.35, 0.70) & (0.34, 0.29) & (0.35, 0.70)
\end{bmatrix}
\]  \tag{15}

Step 5. Find the sum of maximal and minimal parameters. Quality, service level, product quality, and robustness are the benefit parameters, while cost and lead time are the cost parameters. The sum of benefit and cost parameters for each object is given in Tables 6 and 7.

Step 6. Here, the defuzzification of benefit parameters's sum and cost parameters's sum is done. The results are given in Tables 8 and 9.

Step 7. Find out the participation of each object. The participation and ranking of each alternative are given in Table 10.

Step 8. Here, the ranking of alternatives is done. The alternatives are ranked as \( \tilde{O}_1 \succ \tilde{O}_2 \succ \tilde{O}_3 \succ \tilde{O}_4 \succ \tilde{O}_5 \succ \tilde{O}_6 \).

4.2 Assembly Manufacturing Company. The proposed case is adopted from an organization of Maquiladoras of Juárez,
México. Here, an assembly manufacturing company is considered, in which various components are manufactured in its production line. A project was implemented for cost reduction and packing items, which is considered as an opportunity area. Five packing suppliers which are proposed by the company to pack the electronic components. To evaluate the best supplier, two decision makers are invited and four criteria are considered to depict the substantial features of the suppliers. The considered criteria are given as follows:

(i) Cost: Probably, the most obvious but equally important factor to take into consideration when looking for new suppliers is cost.

(ii) Accountability: It is easy to work with a supplier who is accountable for his/her mistakes as it makes the supply process faster. Sometimes, one may encounter quality problems, and the last thing one will expect is the supplier to deny responsibility.

(iii) Reliability: A reliable supplier is one who can meet one’s supply needs on time. One must ensure the supplier has the right workforce and equipment to meet one’s requirements.

(iv) Quality: There is often a correlation between cost and quality; the more expensive the product, the better the quality. Regardless of price, there is still a predetermined and agreed level of quality, and one wants to be sure that one’s expectations are met.

The set of alternatives is given by \( \{z_1, z_2, z_3, z_4, z_5\} \). The process to obtain the best alternative is described as follows:

Step 1. Form a team of DMs and find the importance of everyone. Two DMs form the team and their values are given in Table 11. Table 12 shows the ranking in terms of LVs. The LVs to rank the alternatives is given in Table 13. To get the weight of every decision maker, equation (2) is used, and every decision maker has equal worth.

\[
\lambda_{1,2} = \frac{(0.47^3 + 0.96^3)(0.47^3/0.47^3 + 0.25^3)}{(0.47^3 + 0.96^3)(0.47^3/0.47^3 + 0.25^3) + (0.47^3 + 0.96^3)(0.47^3/0.47^3 + 0.25^3)}
\]

\[
= 0.5.
\]
Table 10: Participation and ranking of alternatives.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>FF-MOORA method</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$</td>
<td>4.19</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>8.4</td>
<td>5</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>-10.32</td>
<td>6</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>-5.72</td>
<td>4</td>
</tr>
<tr>
<td>$\delta_5$</td>
<td>-5.26</td>
<td>3</td>
</tr>
<tr>
<td>$\delta_6$</td>
<td>-4.92</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 11: The eminence of DMs.

<table>
<thead>
<tr>
<th>LVs</th>
<th>DM 1</th>
<th>DM 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>FFNs</td>
<td>(0.47, 0.25, 0.96)</td>
<td>(0.47, 0.25, 0.96)</td>
</tr>
<tr>
<td>Weight</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 12: LVs for ranking the value of decision makers and parameters.

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>FFNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Totally worthless (TW)</td>
<td>(0.31, 0.50)</td>
</tr>
<tr>
<td>Pointless (P)</td>
<td>(0.45, 0.35)</td>
</tr>
<tr>
<td>Ordinary (O)</td>
<td>(0.51, 0.27)</td>
</tr>
<tr>
<td>Remarkable (R)</td>
<td>(0.47, 0.25)</td>
</tr>
<tr>
<td>Predominant (Pr)</td>
<td>(0.71, 0.32)</td>
</tr>
</tbody>
</table>

Step 2. Identify the significance of each parameter. Tables 11 and 14 show the analysis of every DM about the significance of the parameters in terms of LVs.

Step 4. Construct the AWFFDM which is given as follows:

$$R' = \begin{bmatrix}
(0.45, 0.79) & (0.45, 0.54) & (0.51, 0.63) & (0.58, 0.51) \\
(0.27, 0.84) & (0.44, 0.51) & (0.41, 0.67) & (0.48, 0.62) \\
(0.59, 0.72) & (0.45, 0.54) & (0.37, 0.69) & (0.33, 0.69) \\
(0.59, 0.84) & (0.37, 0.64) & (0.21, 0.79) & (0.69, 0.42) \\
(0.66, 0.74) & (0.33, 0.63) & (0.51, 0.63) & (0.52, 0.53)
\end{bmatrix}$$

Step 5. Find the sum of maximal and minimal parameters. Quality and service are the benefit parameters, while cost and lead time are the cost parameters.

The sum of benefit and cost parameters for each object is given in Tables 16 and 17.

Step 6. Here, the defuzzification of benefit parameters’s sum and cost parameters’s sum is done. The results are given in ‘Tables 18 and 19.

Step 7. Find out the participation of each object. Table 20 shows the participation and ranking of each alternative.

Step 8. Here, the ranking of alternatives is done. The alternatives are ranked as $\delta_1 \succ \delta_6 \succ \delta_3 \succ \delta_4 \succ \delta_5 \succ \delta_2$.

5. Comparative Analysis

For the validity and advantages of presented work, we examine the comparison analysis of purposed theory for given
application with present operators such as Fermatean fuzzy Einstein weighted averaging (FFEWA) [49], Fermatean fuzzy Yager weighted averaging (FFYWA) [50], and Fermatean fuzzy Yager weighted geometric (FFYWG) [50] operators. The general procedure to get the better alternative is given as follows:

**Step 1.** To calculate the preference values of alternative, utilize the FFEWA, FFYWA, and FFYWG operators which are given as follows:

- **FFEWA**
  \[
  \text{FFEWA}(\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_m) = \left( \prod_{i=1}^{m} \left( 1 + \mu_i \right)^{w_i} - \prod_{i=1}^{m} \left( 1 - \mu_i \right)^{w_i} \right)^{1/\eta} \left( \prod_{i=1}^{m} \left( 1 - \nu_i \right)^{w_i} \right)^{1/\eta},
  \]

- **FFYWA**
  \[
  \text{FFYWA}(\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_m) = \left( \min \left( 1, \left( \sum_{i=1}^{m} w_i \left( 1 - \mu_i \right)^{\eta} \right)^{1/\eta} \right), \min \left( 1, \left( \sum_{i=1}^{m} w_i \left( 1 - \nu_i \right)^{\eta} \right)^{1/\eta} \right) \right),
  \]

- **FFYWG**
  \[
  \text{FFYWG}(\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_m) = \left( \min \left( 1, \left( \sum_{i=1}^{m} w_i \left( 1 - \mu_i \right)^{\eta} \right)^{1/\eta} \right), \min \left( 1, \left( \sum_{i=1}^{m} w_i \left( 1 - \nu_i \right)^{\eta} \right)^{1/\eta} \right), \right),
  \]

**Step 2.** Apply the score function to find the score values (SVs) of alternatives

**Step 3.** Arrange the alternatives in the decreasing order of SVs and alternative with highest SV is the best option

### 5.1. Results and Discussion

It is far clear from Table 21 and Figure 2 that the effects acquired from the proposed work and existing theories are varying slightly, but the first alternative is identical. Identically, the comparison reveals an advantage of the FF-MOORA method as compared to alternative theories which is associated with the contribution value regarding the assessment of the best alternative. It is clear that the FF-MOORA method is more effective due to the Fermatean fuzzy consideration into counting the membership and nonmembership grades to be functioning in MCDM methods. In this manner, the
proposed approach represents a systematic way and a computational ratio to pick out the best choice. At the same time, the analysis to determine the best alternative can be carried out in low setup time while using the FF-MOORA method. Hence, the presented work is authentic and can be carried out in decision-making problems. There are some obstacles of the existing theories which may be handled by using the way of the proposed work.

6. Advantages and Disadvantages of the Proposed Method

With the aid of comparing the proposed technique based on FF-MOORA with current strategies and analyzing the flexibility and effectiveness of the proposed version, we conclude that the proposed scheme owns the subsequent benefits and advantages:

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>FF MOORA method</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>0.15</td>
<td>3</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.24</td>
<td>2</td>
</tr>
<tr>
<td>$d_3$</td>
<td>-0.06</td>
<td>4</td>
</tr>
<tr>
<td>$d_4$</td>
<td>0.41</td>
<td>1</td>
</tr>
<tr>
<td>$d_5$</td>
<td>-0.15</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 20: Participation and ranking of alternatives.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Ranking results</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF-MOORA method (proposed)</td>
<td>$d_1 &gt; d_2 &gt; d_3 &gt; d_4 &gt; d_5$</td>
</tr>
<tr>
<td>FFEWA [49]</td>
<td>$d_1 &gt; d_2 &gt; d_3 &gt; d_4 &gt; d_5$</td>
</tr>
<tr>
<td>FFYWA [50]</td>
<td>$d_1 &gt; d_2 &gt; d_3 &gt; d_4 &gt; d_5$</td>
</tr>
<tr>
<td>FFYWG [50]</td>
<td>$d_1 &gt; d_2 &gt; d_3 &gt; d_4 &gt; d_5$</td>
</tr>
</tbody>
</table>

Table 21: Comparison analysis.

Figure 2: Comparison of the FF-MOORA method with FFEWA [49], FFYWA [50], and FFYWG [50] operators.

(1) The FF membership grades generalize the IF and PF grades as they develop the suitable area of unsure facts and figures.

(2) The generalization of IFNs and PFNs to FFNs increases the flexibility of the furnished uncertain records and increases the applicability of MOORA approach to the machine where the membership capabilities are complex or not viable to identify completely.

(3) The proposed method allows the assessment of all options and their corresponding ratings in form of linguistic variables. These variables are expressed through FFNs, which enhances the ability of the system and can increase the applicability of the MOORA technique.

(4) The FF-MOORA method represents a scientific way and a computational ratio to pick the best opportunity.
7. Conclusions

The MOORA method under FFSs can be applied very easily by decision makers to evaluate the alternatives, and the most appropriate manufacturing system can be selected, while being fully unaware of the physical interpretation of the decision-making procedure. Furthermore, a minimized performance criterion can be formulated using the FF-MOORA method which is directly proportional to the corresponding effect of the compared criteria values. On the other hand, the MOORA method implements separate mathematical frameworks to benefit the qualitative and nonbenefit criteria of the decision matrix. In this study, we have developed the MOORA method under Fermatean fuzzy environment. Then, the developed model has been applied for the selection of the best manufacturing system. The decision matrices have been constructed with the help of Fermatean fuzzy aggregated weighted operators. The conversion of qualitative attributes into quantitative attributes has been performed using Fermatean fuzzy numbers. After presenting a comprehensive comparison of the proposed technique with other existing techniques, we conclude that the chance of losing data and information in the MOORA method is very small. Moreover, the developed model isolates the subjective part of the evaluation procedure through a combined multiattribute decision-making technique that intimates the flexibility of the FF-MOORA method. In future studies, the approach proposed in this text may be taken into consideration to solve packages to other selection making instances, such as, undertaking choice, supplier selection, and numerous regions of control.

7.1. Limitations and Future Work. To overcome the limitations of the proposed work, it can be extended in the following directions:

(i) The MOORA method can be generalized under Fermatean hesitant fuzzy sets. The model would be useful to rank the alternatives in such decision problems where a decision expert can assign his/her preferences in the form of discrete set in which he/she partially knows the membership and non-membership of an element belonging to FFS.

(ii) The MOORA method can be combined with the multiplicative approach to form the MULTI-MOORA technique, and then, certain hybrid theories, including Fermatean hesitant fuzzy sets, interval-valued FFSs, and hesitant Fermatean fuzzy soft sets, can be applied to deal with uncertainties appearing in decision-making problems.

(iii) Certain aggregation operators such as Dombi prioritized aggregations can be developed on FFSs, and the resulted FF Dombi prioritized aggregation operators can be utilized in the MOORA method to aggregate the imprecise data.

(iv) The $q$–rung orthopair fuzzy sets ($q$–ROFSs) can be applied on the MOORA method to expand the spaces of considered data. These sets include many fuzzy sets with dynamically changing $q$ parameters, i.e., IFSs, PFSs, and FFSs according to the value of $q$ parameter.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Gulfam Shahzadi conceptualized and designed the study, analysed the data, and wrote the manuscript. Anam Luqman conceptualized and designed the study and revised the manuscript. Mohammed M. Ali Al-Shamiri designed the study, analysed the data, and revised the manuscript.

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