Research Article

Reselling or Agency Selling? The Optimal Pricing and Return Strategy for the Agricultural Product

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1. Introduction

Since the 18th CPC National Congress, the basic strategy of “targeted poverty alleviation” has promoted the rapid development of poverty alleviation in China. After years of poverty alleviation efforts, most farmers now have owned stable industries, but some are still troubled with the sales of agricultural products. To solve this problem, the Chinese government encouraged farmers to join farmer cooperatives to get rid of the rural economic dilemma of small, scattered, and chaotic. For example, as of 2020, 2.241 million farmer cooperatives were registered in China, attracting nearly half of the farmers in the country to join in [1]. With the support of farmer cooperatives and the significant improvement of rural e-commerce coverage and service level in recent years, more and more farmers abandon the traditional land wholesale mode and turn to cooperative with the e-commerce platform to sell the agricultural products in the name of cooperatives. In 2019, China’s e-commerce sales of agricultural products reached 397.5 billion RMB, nearly eight times that of 2013 [2]. Therefore, distributing through the e-commerce platform can effectively alleviate the sales problem of the agricultural products.

In the cooperation between the farmer cooperative and the e-commerce platform, two types of sales modes are commonly adopted, that is, the reselling mode or the agency selling mode. In the former mode, the cooperative sells the agricultural products to the platform at a wholesale price, and then the platform sets the retail price and the return compensation. In the latter mode, the cooperative enters the platform as a seller, who can set the retail price on his own and needs to share a certain proportion of revenue to the platform. For example, the proportion fee charged by the Tmall ranges from 0.5% to 5%, whereas the proportion fee in JD.com ranges from 1% to 10% [3]. Except for the pricing decisions and profit allocation mechanism, the return policy is also an important factor, especially for the agricultural
products, which is relatively perishable. Generally speaking, the return policy and the return cost are taken by the platform in the reselling mode, whereas they are taken by the cooperative in the agency selling mode. As a result, both the cooperative and the platform have to consider the impact of different sales modes on their own interest.

Motivated by the above observation, we intend to investigate the product sales problem under different sales modes. And we seek to answer the following research questions. First, what are the optimal pricing and return strategy under different sales mode? Secondly, what are the key factors that affect the preference of both firms for different sales mode? Finally, what are the differences of both firms’ decisions and profits under these two sales modes? To answer these questions, we study two types of agricultural product sales modes. One is the reselling mode, wherein the cooperative sells the products to the consumers through the e-commerce platform. These two firms play a Stackelberg game, wherein the leader cooperative first determines the wholesale price per unit sold product and then the follower platform decides the retail price and return compensation. The other is the agency selling mode, in which the cooperative sets the retail price and return compensation and shares part of his revenue to the platform at a given proportion ratio. In our model, we consider a full refund policy for the defective products and a partial refund policy for the nondefective products.

Our contribution is threefold. Firstly, we derive the optimal pricing and return strategy for the supply chain in both modes. An unexpected finding is that, in the reselling mode, both the retail price and the return compensation are independent of the products’ surviving rate. Secondly, we uncover that both firms’ preference for the sales mode hinges on the surviving rate and the revenue sharing proportion. There exists a Pareto improvement region whereby both firms prefer the agency selling mode to the reselling mode. Thirdly, in comparison between these two modes, the retail price is higher while the demand is lower in the reselling mode than in the agency selling mode, owing to the price competition between two firms. Moreover, the high-low relationship of the return compensation and the return quantity in these two modes depends on the surviving rate. The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes our model. Sections 4 and 5 show the equilibrium in the reselling and the agency selling modes, respectively. In Section 6, we compare these two modes and propose some managerial insights. Section 7 concludes the paper and offers some future research directions. All the proofs are relegated to the appendix.

2. Literature Review

Our work belongs to the literature on products return. Davis et al. [4], Mukhopadhyay and Setaputra [5], Griffis et al. [6], and Pei and Paswan [7] point out that return policy has a positive impact on attracting customer. Specifically, when the product quality is high, the excessive refund policy (the refund amount is higher than the product price) can maximize the retailer’s profit (Moorthy and Srinivasan [8], Fruchter and Gerstner [9], Hsiao and Chen [10], etc.). In addition, some relevant literature focuses on the study of return policy. Mukhopadhyay and Setaputra [11] study the full refund and partial refund strategies in the supply chain based on the assumption that the demand and return quantity are linearly related to the selling price and refund amount. Su [12] studies the return strategy of a supply chain composed of one manufacturer and one retailer under the news-vendor framework. Chen and Bell [13] analyze the impact of full refund strategy on pricing, ordering strategy, and supply chain channel members’ profit under a dual channel setting. Li et al. [14] consider the online retailers’ optimal pricing, return, and product quality decision problem; they find that the selling price, refund amount, and product quality have complementary relations; high (low) product quality corresponds to high (low) selling price and high (low) refund amount. Lin et al. [15] examine the optimal pricing and return strategy of retailers faced with two types of customers and fully investigate the impact of freight insurance on customer behavior and retailer decision-making. Our paper differs from the aforementioned literature in two aspects. First, we study the return policy in the reselling mode and the agency selling mode, which enables us to explore the impact of return cost taken by different firms on the pricing and return decisions. Second, we consider different return policy for different types of products, i.e., the full refund policy for the defective products and the partial refund policy for the nondefective products, while they only focus on finding the optimal return policy for all the products.

Our work complements the literature on sales mode selection problems. Jiang et al. [16] study the selection of self-operated products (wholesale mode) and commissioned products (agency selling mode) on the Amazon platform. Hagiu and Wright [17] point out that the marketing activity information of the product will have an important impact on the mode selection of the platform. Abhishek et al. [18] find that when online channel has a positive effect on offline channel, the e-commerce platform prefers the wholesale mode to the agency selling mode, and vice versa. Tan and Carrillo [19] demonstrate that, for the distribution of digital products, the agency mode is always better than the wholesale mode. Geng et al. [20] certify that firms prefer the agency mode when selling core products and add-on products separately and prefer the wholesale mode when selling these two types of products in bundles. Yang et al. [21] consider a two-level supply chain composed of one farmer and one e-commerce platform, facing agency selling and reselling modes. They propose the optimal sales mode selection strategy for both firms in the presence of product decay and live stream marketing. The mix-mode of the reselling and agency selling modes in the e-commerce platform has also attracted the attention of many scholars. Ryan et al. [22] construct a dual channel model and investigate the game of sales modes between the retailer and the e-commerce platform. Tian et al. [23] propose that the selection of agency, reselling, or mix-mode for the suppliers and the intermediate platform depends on the interaction
between order fulfillment cost and upstream competition intensity. Yan et al. [24] consider the introduction of the marketplace channel in a supply chain with one e-tailer and one manufacturer. Moreover, our study is also inspired by some other literature streams, such as channel encroachment (Arya et al. [25], He et al. [26], He et al. [27]) and revenue sharing contract (Cachon and Lariviere [28], He and Zhao [29], Bart et al. [30]). The aforementioned papers, however, only concentrate on the distribution of products, while we introduce the return policy into our model. Specifically, in our model framework, the e-commerce platform takes charge of the products return in the reselling mode, whereas the farmer cooperative is responsible for the products return in the agency selling mode.

3. Model Description

We consider a two-level supply chain composed of a farmer cooperative (he) and an e-commerce platform (she), wherein the former sells the agricultural products to consumers through the latter. We assume that the agricultural product may suffer deterioration loss, due to its perishable feature. We use $\lambda$ to denote the fault rate; then $\lambda = 1 - \lambda$ denotes the surviving rate, similar to He et al. [31] and Yu et al. [32]. We assume that $0 < \lambda < 1/2$. We study two types of sales modes: one is the reselling mode; the other is the agency selling mode, as shown in Figures 1(a) and 1(b), respectively. In reselling mode, the cooperative charges wholesale price $w$ for each unit of sold product. Accordingly, the platform sets the retail price $p$ and the return compensation $r$ and takes on product return and transportation. We assume that the defective products can be returned with money back guarantee, whereas the return compensation for the nondefective products should be lower than the defective products. Similar to Li et al. [14] and Hua et al. [33], the demand function is $D = a - \beta p + \gamma r$, where $a > 0$, $\beta > \gamma > 0$, representing that consumer is more sensitive to retail price than return compensation. The return function for the defective products is $R_d = \lambda D$, called $d$-return quantity. The return function for the nondefective products is $R_n = \lambda \varphi r$, called $n$-return quantity, where $\varphi > \gamma > 0$. The return compensation directly affects the return and does not directly affect the demand, which guarantees that the firm cannot abuse the return policy. For ease of presentation, we define $S = D - R (R = R_d + R_n)$ as the product effective sales volume, which denotes the quantity of product sold that has not been returned.

Figure 1(a) presents the reselling mode, wherein the cooperative acts as a wholesaler while the platform acts as a retailer. In this mode, the cooperative charges the platform a wholesale price $w$, and the platform sets the retail price $p$ and the return compensation $r$. Figure 1(b) presents that the platform assists in selling the agricultural products. In this mode, the cooperative needs to share $\theta$ of his profit with the platform and sets the retail price $p$ and the return compensation $r$ for the nondefective products.

For simplicity, we assume the unit produce cost, return cost of the agricultural product, and savage value of the returned product are zero. All the notations used in this paper are presented in Table 1. Modes $R$ and $A$ represent the reselling mode and the agency selling mode, respectively. For convenience, we define $\bar{\theta} = 1 - \theta$ and $\bar{\omega} = 1 - \lambda - \varphi$.

4. Reselling Mode (Mode $R$)

In this section, we consider the mode in which the cooperative resells the products to the e-commerce platform. In this mode, as illustrated in Figure 1(a), cooperative and the platform play a Stackelberg game, and the decision sequence is shown as follows: (1) as the leader, the cooperative determines the wholesale price $w$; (2) then the follower, the e-commerce platform, decides the retail price $p$ and return compensation $r$; (3) all the demand is realized and the return happens. Based on the demand and return function formulated in Section 3, we formulate the cooperative’s and the platform’s profit function as follows, respectively:

$$n_c^R = wD, \tag{1}$$
$$p_r^R = (p - w)D - \lambda pD - r R_n. \tag{2}$$

Using backward induction, we can derive the platform’s optimal retail price $p$ and return compensation $r$, given the wholesale price $w$, which are shown in the following lemma:

Lemma 1

1. Given $w$, $\pi_p^R$ is jointly concave in $p$ and $r$.
2. Given $w$, the corresponding optimal $p$ and $r$ are

$$p = \frac{2\lambda \varphi + (2\beta \varphi - \gamma^2)w}{\lambda \Delta_1},$$
$$r = \frac{\gamma (\lambda \alpha - \beta w)}{\lambda \Delta_1},$$

where $\Delta_1 = 4\beta \varphi - \gamma^2$.

Lemma 1 demonstrates that the retail price is increasing in $w$, whereas the return compensation is decreasing in $w$. The explanation is as follows. The wholesale price can be treated as the input cost of the platform. Then, she has to set a higher retail price to recover her cost as wholesale price increases. On the contrary, for each unit of the returned product, she has to afford the return cost, as the savage value of the returned products is zero. Then, she has to cut down the return compensation for the nondefective products as wholesale price raises, avoiding taking much too high return cost. Plugging the results obtained in Lemma 1 into the profit function (2), we can derive the equilibrium result of model $R$, demonstrated as the following proposition.

Proof of Proposition 1. In mode $R$:

1. The optimal wholesale price, retail price, and return compensation are $w^* = \frac{\lambda \alpha \varphi}{2\beta}$, $p^* = \alpha (2\beta \varphi + \Delta_1)/ 2\beta \Delta_1$, and $r^* = \alpha \gamma / 2\Delta_1$, respectively.
In this section, we investigate the sales mode that the e-commerce platform assists the farmer cooperative to sell products. The cooperative sets the retail price and return compensation on his own and needs to share some of his profit with the platform; the revenue sharing proportion \( \theta \) satisfies \( 0 < \theta < 1/2 \), in line with [20] and [34]. And the shared profit for the platform is related to the sales revenue, unrelated to the products return. The cooperative’s and platform’s profit functions are characterized as

\[
\pi^A = \theta pD - \lambda pD - \lambda \varphi r^2, \\
\pi^p = \theta pD.
\]

Then we can derive the equilibrium results in the following proposition.

**Proof of Proposition 2.** In mode A:

1. \( \pi^A \) is jointly concave in \( p \) and \( r \).
2. The optimal retail price and return compensation are 
   \[
   p^A = 2\lambda \alpha \varphi / \Delta_2, \quad r^A = \lambda \varphi / \Delta_2. 
   \]
3. The cooperative’s and platform’s optimal profit are
   \[
   \pi^A = \lambda \alpha^2 \varphi / \Delta_2, \quad \pi^p = 4\theta \lambda \alpha^2 \varphi^2 / \Delta_2^2, 
   \]
   respectively,
   where \( \Delta_2 = 4\lambda \beta \varphi - \bar{\omega} \gamma^2 \).

Proposition 2 draws out the optimal pricing and return strategy for the cooperative in the agency selling mode. We can find that both the retail price and the return compensation are slightly increasing in surviving rate and decreasing in sharing proportion (as shown in Figures 3(a) and 4(a)). The explanation is as follows. In a more profitable market, the cooperative can raise the retail price to pursuit a higher marginal profit and meanwhile increases the return compensation to avoid the sharp decline of sales volume, leading to a relatively stable demand, as presented in Figure 3(b). As a result, the d-return quantity and the return quantity decrease. Moreover, although the n-return quantity raises, due to the increase of return compensation, the effective sales volume still increases, owing to the higher surviving rate. Intuitively, both the cooperative’s profit and the total supply chain profit are increasing in surviving rate. Surprisingly, the platform’s profit remains relatively stable as the surviving rate goes up, as can be seen from Figure 3(c). The reason is that the platform’s profit is related to the demand, which is also stable as shown in Figure 3(b), but independent of the products return. Furthermore, we can see from Figure 3(d) that the higher surviving rate leads to the aggravation of the
double marginalization, because the cooperative’s profit increases while the platform’s profit stays relatively stable.

6. Comparison between Two Modes

There exist two sales modes for the cooperative to sell products: the reselling mode and the agency selling mode. These two modes have both advantages and disadvantages for the cooperative, who is the leader in the game. In the reselling mode, the cooperative has to compete with the e-commerce platform but does not need to take the products return. In the agency selling mode, the cooperatives directly control the pricing and products return, wherein the double marginalization can be alleviated. But he has to take the return cost on his own. Therefore, it is necessary to compare the profit under these two sales modes to derive the optimal sales mode selection strategy for the cooperative.

Lemma 2. Comparing the reselling mode and the agency selling mode,

(1) \( p^{R*} > p^{A*}, D^{R*} < D^{A*}, R^{d*} < R^{d*}, S^{R*} < S^{A*} \).

(2) \( r^{R*} < r^{A*}, R^{R*} < R^{A*} \) when \( \theta < \theta_c \),
and \( r^{R*} > r^{A*}, R^{R*} > R^{A*} \) when \( \theta_c < \theta < 1/2 \),
where \( \theta_c = \frac{\bar{\lambda} \Delta}{4 \beta \phi + \Delta} \).
If $\frac{1}{2} < \lambda \leq \lambda_0$, $R^R < R^A$; if $\lambda_0 < \lambda < 1$, $R^R \leq R^A$. when $\theta < \theta_0$ and $R^R > R^A$, when $\theta > \theta_0$.

Lemma 2 demonstrates that the retail price is higher in the reselling mode than that in the agency selling mode (as shown in Figure 4(a)), owing to the price competition between the cooperative and the platform in the former mode. Accordingly, the demand, the $d$-return quantity, and the return quantity are higher in the reselling mode than that in the agency mode (as presented in Figure 4(b)). As can be seen from Figure 4(c), the return compensation and the n-return quantity are lower (higher) in the reselling mode than that in the agency selling mode when the sharing proportion is low (high) enough. The explanation is as follows. Facing low sharing proportion, the cooperative has incentive to set a relatively high return compensation to attract more customers, which can help to earn more profit, even if the n-return quantity raises. In contrast, when he is confronted with high sharing proportion, he has to set a relatively low return compensation to cut down his return cost. We can see from Figure 5(a) that the return quantity in the reselling mode is always lower than that in the agency selling mode when the surviving rate is
Figure 4: Continued.
relatively low, due to the unfavorable market condition and the vertical competition in the supply chain in the former mode. In contrast, if the surviving rate is high enough, return quantity is lower (higher) in the reselling mode than that in the agency selling mode when the sharing proportion is low (high) enough, as shown in Figure 5(b).

**Figure 4:** Comparing equilibrium outcomes in different modes ($\alpha = 1000, \beta = 40, \gamma = 18, \varphi = 20, \bar{\lambda} = 0.8$): (a) variance of prices; (b) variance of sales parameters; (c) variance of return parameters; (d) variance of profits; (e) variance of supply chain profit; (f) variance of profit ratio.

**Figure 5:** Comparing $R$ in different modes ($\alpha = 1000, \beta = 40, \gamma = 18, \varphi = 20$): (a) variance $R$ with $\bar{\lambda} \leq \bar{\lambda}_0 (\bar{\lambda} = 0.8)$; (b) variance of $R$ with $\bar{\lambda} > \bar{\lambda}_0 (\bar{\lambda} = 0.98)$.

The total profit can benefit from the adoption of agency selling mode, and the increase of sharing proportion will alleviate the double marginalization in the agency selling mode, as shown in Figures 4(e) and 4(f). And the comparison of both firms’ profit is characterized in the following proposition.

**Proof of Proposition 3.** Comparing the profits of both firms in the reselling mode and the agency selling mode, we find the following:

1. There exists a platform revenue sharing proportion threshold $\theta_c = \lambda \Delta_1 / 4 \varphi + \Delta_1 < 1/2$; the cooperative prefers the agency selling mode when $0 < \theta \leq \theta_c$ and the reselling mode when $\theta_c < \theta < 1/2$. 

\[ \rho^R = \frac{\pi^R}{\pi^C} \]

\[ \rho^A = \frac{\pi^A}{\pi^C} \]
There exists a platform revenue sharing proportion threshold $\theta_p = \frac{\lambda_{\Delta_1}}{\sqrt{4\beta\phi + \Delta_1}} < 1/2$; the platform prefers the reselling mode when $0 < \theta < \theta_p$ and the agency selling mode when $\theta_p < \theta < 1/2$.

Both firms prefer the agency selling mode when $\theta_p < \theta < \theta_c$.

The total profit in the supply chain performs better in the agency selling mode.

Proposition 3 demonstrates the cooperative’s and the platform’s preferences for the sales mode are in conflict. Intuitively, either firm would like to share more revenue in the agency selling mode. As a result, a higher platform revenue sharing proportion $\theta$ will encourage the platform to adopt the agency mode while motivating the cooperative to take the reselling mode, as shown in Figure 6. Proposition 3 also illustrates another interesting finding; i.e., a Pareto-improving region of the platform revenue sharing proportion $\theta$ always exists (region $AA$ in Figure 6). In other words, when $\theta \in (\theta_p, \theta_c)$, both firms can earn more profit by adopting the agency selling mode instead of the reselling mode, leading to the alleviation of the double marginalization effect and the improvement of the supply chain performance.

Corollary 1. There exists a threshold $\tilde{\theta} = \frac{\lambda_{\Delta_1}}{4\beta\phi + \Delta_1} < 1/2$.

(1) If $0 < \theta < \tilde{\theta}$, the cooperative prefers the reselling mode when $0 < \lambda < \lambda_{\tilde{\theta}}$ and the agency selling mode when $\lambda_{\tilde{\theta}} < \lambda < 1$, where $\lambda_{\tilde{\theta}} = (4\beta\phi + \Delta_1)\theta/\Delta_1$.

(2) If $\tilde{\theta} < \theta < 1/2$, the cooperative always prefers the reselling mode.

In comparison with the cooperative’s decision-making in the reselling mode and the agency selling mode, we can find the advantages for the cooperative in different modes. In the
reselling mode, the cooperative need not afford the products return and enjoys the first-move advantage in the Stackelberg game, but he cannot directly control the retail price and the return compensation. In the agency selling mode, he has to share proportion of the revenue to the platform and bears all the return cost on his own, but he can directly set the retail price and the return compensation. As a consequence, when the revenue sharing proportion is relatively low (0 < θ < 0), he prefers the reselling mode to the agency mode if he owns low quality agricultural products, and vice versa, as illustrated in Figure 7(a). In contrast, when the revenue sharing proportion is too high (θ ≤ 1/2), he cannot accept the agency selling mode, as shown in Figure 7(b). The reason is that the cooperative is only willing to take charge of the return service for the products with high surviving rate; otherwise, the extremely high return cost may harm his interest.

7. Conclusions

This paper analyzes the agricultural products sales problem in a supply chain wherein the farmer cooperative sells its products through the e-commerce platform. Two product sales modes are investigated: the reselling mode and the agency selling mode. During the sales process, the customers who receive defective products can get full refund, while the customers who return the nondefective products can only get partial refund. Firstly, in the reselling mode, as a leader in the game, cooperative needs to decide the wholesale price for each sold product. In response to the wholesale price charged by the cooperative, the platform independently determines the retail price and the return compensation. We find that the retail price and the return compensation are irrespective to the surviving rate, and the double marginalization exists in the supply chain. Secondly, in the agency selling mode, the cooperative needs to share proportion of his profit to the platform. We find that both the retail price and the return compensation are decreasing in revenue sharing proportion. Finally, by comparing these two modes, we find that the cooperative prefers the reselling mode to the agency mode with high revenue sharing proportion, whereas the platform prefers the agency selling mode to the reselling mode with high revenue sharing proportion. Surprisingly, a Pareto improvement region exists, wherein both firms can earn more profit in the agency mode than the reselling mode. Moreover, the cooperative will select the agency selling mode if products’ surviving rate is high enough. On the one hand, compared to the reselling mode, in the agency selling mode, the cooperative will set a lower retail price, leading to a higher demand, due to the mitigating of the price competition. On the other hand, whether the return compensation and the return quantity will increase or decrease hinges on the surviving rate and the revenue sharing proportion.

This paper examines the cooperation between the farmer cooperatives and the e-commerce platforms and gives some preliminary conclusions, but the research still has some limitations. Firstly, this paper only focuses on selling agricultural products in the online channel, without analyzing the impact of traditional retail channel. For example, the study on product sales in dual channel or omnichannel will make the model more practical. Secondly, the impact of the asymmetry information between the e-commerce platform and the farmer cooperative on the decisions of both firms can be incorporated. For instance, when the e-commerce platform owns consumer private return information, the game between these two firms is worth further discussion. Finally, studying the cooperation among the farmers who join in the same farmer cooperative is a meaningful direction for future research.

Appendix

Proof of Lemma 1. Taking partial derivatives of p and r for equation (2), respectively, we can have \( \frac{\partial p}{\partial \beta} = \lambda \alpha + \beta \omega - 2 \lambda \beta \rho + \lambda \gamma r \), \( \frac{\partial p}{\partial \omega} = -w \alpha - \lambda \omega, \frac{\partial^2 p}{\partial \beta^2} = -2 \lambda \beta < 0, \frac{\partial^2 p}{\partial r^2} = 0, \frac{\partial^2 p}{\partial \rho^2} = \lambda \gamma. \) Therefore, it is easy to verify that \( |H| = (\lambda \alpha - \beta \omega)^2 \gamma > 0 \).

Therefore, we can conclude that \( \pi^p \) is jointly concave in \( p \) and \( r \). Thus, we can derive that \( p = 2 \lambda \alpha \beta \omega + (2 \beta \omega - \gamma^2) \omega / \lambda \Delta_1, r = (\lambda \alpha - \beta \omega) / \lambda \Delta_2, \).

Proposition 1. Plugging (3) and (4) into (2), we can obtain \( \frac{d \pi^p}{dp} = 2 \beta \omega (\lambda \alpha - 2 \lambda \omega) / \lambda \Delta_1, \) and \( \frac{d^2 \pi^p}{dp^2} = -4 \beta \lambda \omega / \lambda \Delta_2 < 0. \) Then \( \pi^p \geq \) concave in \( p. \) And we can obtain that \( \omega^R = \lambda \alpha / 2 \beta \omega. \) Plugging it into (3) and (4), it is easy to get the optimal retail price and return compensation.

Proposition 2. Taking partial derivatives of \( p \) and \( r \) for equation (5), respectively, we can have \( \frac{\partial \pi^p}{\partial \alpha} = \beta \omega (\lambda \alpha - \beta \omega) / \lambda \Delta_1, \) \( \frac{\partial \pi^p}{\partial \omega} = -w \alpha - \lambda \omega, \frac{\partial^2 \pi^p}{\partial \alpha^2} = \lambda \alpha \omega - (\lambda \alpha^2 + \beta \omega^2) \Delta_1 < 0. \) Similarly, \( \frac{\partial \pi^p}{\partial \alpha} = \lambda \alpha \omega - (\lambda \alpha^2 + \beta \omega^2) \Delta_1 < 0. \) Thus, we can conclude that \( \pi^p \) is jointly concave in \( p \) and \( r. \) Thus, we can derive that \( p = 2 \lambda \alpha \beta \omega / \lambda \Delta_1, r = \beta \omega / \lambda \Delta_2. \) Similarly, we can derive the other results.

Proof of Lemma 2

(1) Following from Propositions 1 and 2, we have \( \pi^R - \pi^A = \lambda \Delta_1 (2 \beta \omega - \gamma^2) \Delta_1 + \theta (2 \beta p + \Delta_1) / 2 \beta \Delta_1 > 0. \) Besides, \( \pi^R - \pi^A = \alpha \beta \omega / \lambda \Delta_1 < 0, \) \( \pi^R - \pi^A \leq \lambda \omega / \lambda \Delta_1 < 0, \) \( \pi^R - \pi^A = \lambda \alpha / \lambda \Delta_1 < 0. \) In the above, we take partial derivatives of \( \pi^R - \pi^A = \lambda \alpha / \lambda \Delta_1 < 0. \) Similarly, \( \pi^R - \pi^A = \lambda \omega / \lambda \Delta_1 < 0. \) Thus, we can obtain the same result.

(2) \( \pi^R - \pi^A = \alpha \beta \omega / \lambda \Delta_1 < 0. \) In the above, we take partial derivatives of \( \pi^R - \pi^A = \lambda \omega / \lambda \Delta_1 < 0. \) Similarly, \( \pi^R - \pi^A = \lambda \beta \omega / \lambda \Delta_1 < 0. \) Thus, we can obtain the same result.

(3) \( \pi^R - \pi^A = \alpha \beta \omega / \lambda \Delta_1 < 0. \) In the above, we take partial derivatives of \( \pi^R - \pi^A = \lambda \omega / \lambda \Delta_1 < 0. \) Similarly, \( \pi^R - \pi^A = \lambda \beta \omega / \lambda \Delta_1 < 0. \) Thus, we can obtain the same result.
when \(1/2 < \lambda < (2\beta - \gamma)(4\beta\phi + \Delta_1) + \sqrt{(2\beta - \gamma)^2 (4\beta\phi + \Delta_1)^2 - 16\beta^2\phi^2 (2\beta - \gamma)/4\Delta_1} (2\beta - \gamma)\). Then we can easily derive the result. □

**Proposition 3**

(1) Following from Propositions 1 and 2, we can get that 
\[
\pi^R - \pi^A = \lambda\alpha^2\theta[(4\beta\phi + \Delta_1)^2 - 2\lambda\Delta_1 (4\beta\phi + \Delta_1) + \lambda^2\Delta_1^2]/(4\Delta_1\Delta_2^2).
\]
It is easy to verify that the profit gap between these two sales modes is nonpositive when \(0 < \theta \leq \lambda\Delta_1/4\beta\phi + \Delta_1\), and positive when \(\lambda\Delta_1/4\beta\phi + \Delta_1 < \theta < 1/2\).

(2) Similarly, we can get that 
\[
\pi^R - \pi^A = \lambda\alpha^2\theta[(4\beta\phi - \Delta_1)^2 - 2\lambda\Delta_1 (4\beta\phi - \Delta_1) + \lambda^2\Delta_1^2]/(4\Delta_1\Delta_2^2) .
\]
Defining \(f(\theta) = (4\beta\phi - \Delta_1)^2 - 2\lambda\Delta_1 (4\beta\phi - \Delta_1) + \lambda^2\Delta_1^2\), we can find that \(f(0) > 0\) and \(f(1/2) < 0\), which means the sign of \(f(\theta)\) is first positive and then negative in the region \((0, 1/2)\). Then the profit gap is nonnegative when \(0 < \theta < \lambda\Delta_1/(\sqrt{4\beta\phi + \Delta_1})^2\) and negative when \(\theta < \lambda\Delta_1/(\sqrt{4\beta\phi + \Delta_1})^2 < 1/2\).

(3) Since \(\lambda\Delta_1/\sqrt{4\beta\phi + \Delta_1} > \lambda\Delta_1/(\sqrt{4\beta\phi + \Delta_1})^2\), combining the results obtained in parts 1 and 2), we can conclude that \(\pi^R < \pi^A\) and \(\pi^R < \pi^A\) when \(\lambda\Delta_1/(\sqrt{4\beta\phi + \Delta_1})^2 < \theta < \lambda\Delta_1/4\beta\phi + \Delta_1\).

**Proof of Corollary 1.** Following from the proof of Proposition 3, \(\pi^R - \pi^A = \lambda\alpha^2\theta[(4\beta\phi + \Delta_1)^2 - 2\lambda\Delta_1 (4\beta\phi + \Delta_1) + \lambda^2\Delta_1^2]/(4\Delta_1\Delta_2^2)\). Then we can find that (1) if \(\theta \geq \lambda\Delta_1/4\beta\phi + \Delta_1\), \(\pi^R > \pi^A\); (2) if \(\theta < \lambda\Delta_1/4\beta\phi + \Delta_1\), \(\pi^R > \pi^A\) when \(0 < \lambda \leq (4\beta\phi + \Delta_1)/\theta\Delta_1\), and \(\pi^R < \pi^A\) when \((4\beta\phi + \Delta_1)/\theta\Delta_1 < \lambda < 1\). □

**Data Availability**

The data can be obtained from the corresponding author upon request.

**Conflicts of Interest**

All authors declare no conflicts of interest in this paper.

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**References**


