Research Article

Auxiliary Teaching System of Higher Mathematics Based on Random Matrix Model

Yabin Xiao, Bin Zhou, Dan He, and Jingzhong Liu

School of Science, Hunan Institute of Technology, Hengyang, Hunan 421003, China

Correspondence should be addressed to Jingzhong Liu; 1994000377@hnit.edu.cn

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With the development of computer technology, computers have become a part of people’s lives and the Internet has connected the world’s networks as a whole. Computer technology is changing people’s study, life, and work. People’s traditional education mode, thinking, content, method, and talent training program have a significant impact. The development from traditional to computer technology-based teaching methods has brought new developments and leaps in educational technology. This paper analyzes the research background, significance, and research status of the advanced mathematics auxiliary teaching system, introduces the related technologies and development modes used in the development of the system, and especially discusses the access database technology by ADO and the mathematical expression based on MathML language. Secondly, starting from the actual teaching, we analyze the functional requirements and performance requirements of the system in detail and make detailed planning and design for the system architecture, database selection, functional modules, etc. The design and implementation process of this teaching system are summarized. The teaching strategy inference engine is the key to the personalization and intelligence of the ICAI system. According to the learning models provided by different students, the system designs a corresponding teaching sequence for the learners by controlling the meta-knowledge of the domain knowledge base. The teaching strategy inference engine cuts the domain knowledge tree, selects the knowledge points suitable for the student, and sorts the selected knowledge points reasonably to generate an optimal teaching sequence. According to the students’ learning situation, combined with the teaching rules in the teaching rule library, the students’ grades are dynamically adjusted, so as to select new learning content for students and provide teaching suggestions in time. The student model is the premise of the ICAI system to achieve individualization and intelligence. The system makes a comprehensive evaluation and diagnosis of students through fuzzy comprehensive evaluation and fuzzy reasoning. On this basis, a cognitive student model is established, which is the teaching strategy that provided the basis for the formulation.

1. Introduction

The use of online teaching has become an irresistible trend, and the most prominent feature of online teaching is that it is globalized, shared, and not limited by time, place, space, etc., providing students with higher and better personalized education. This makes full use of the market mechanism, builds high-quality digital education resources such as online open courses, and promotes the application of online and offline course sharing. This promotes the integration and application of educational information management systems [1–3]. Carrying out the activities of “one teacher, one excellent, one class, one teacher” in the school can improve the professional quality of teachers, make full use of the teaching auxiliary system, and improve the school’s level of education [4]. At present, computer hardware and the Internet have been popularized in my school, and the utilization of rich network resources in teaching is not efficient. To this end, according to the current situation of students and school resources, an auxiliary teaching system is constructed, and an auxiliary teaching system is designed in a targeted manner, so as to combine the traditional teaching with auxiliary teaching, so that the students can gradually move towards the path of love for learning, ability to learn, and autonomous learning [5].

Teaching materials such as courseware, videos, micro-courses, and MOOCs can be added to online teaching to achieve the perfect combination of pictures and texts,
dynamic and static, and sound and video. After a knowledge point is explained, students can conduct their own test evaluations, identify the deficiencies in this part of your knowledge learning [6]. The auxiliary teaching platform is very good for the interaction between teachers and students. Students who encounter difficult problems in the learning process can give feedback to teachers at any time, and teachers can timely answer the problems through the platform. At the same time, other students encountering the same problem can be solved by browsing the platform. Teachers also avoid repeating the same questions. The auxiliary teaching platform realizes the push function. After students ask questions, after background data processing, the same or similar questions are extracted to give answers [7]. The auxiliary teaching platform is not limited by time and space, and students can learn in the auxiliary teaching platform in their spare time. The auxiliary teaching platform not only broadens the thinking of traditional teaching, but also enables students to become the main body of learning. On this platform, students can use the network to effectively preview before class, and teaching resources such as teaching courseware, micro-lectures, and videos can effectively help the students. After class, students use the homework and test links of the online platform to consolidate and improve their knowledge. All aspects of learning before and after class can effectively improve the teaching level [8].

Courseware and other resources in teaching, how to make students preview before class, how to ensure that students can watch teachers’ explanations in time when they encounter problems in the learning process. These are all problems that cannot be solved by the traditional teaching methods. In order to cultivate high-quality applied talents who can adapt to social production and build high-quality applied talents, and cultivate their self-learning and life-long learning ability, students can quickly master the ability of future occupations within the limited time of school. In the normal teaching link, it is very inevitable to develop new learning methods.

System design is the combination of goal and realization, which is an important link of system realization. The fundamental purpose of the system design stage is to transform the system logic scheme reflecting the user’s needs in the analysis stage into an achievable physical scheme based on computers and networks. According to the analysis of the system, fully considering the conditions of economy, technology, operating environment, etc., scientifically and rationally carry out the overall design and specific physical design of the system, and take a solid step for the realization of the system. Starting from the demand analysis, combined with the realization technology, this paper introduces the design process of the system in detail. It summarizes the specific functions to be realized by the system, especially describes the design of the database, lists the overall operation flow chart, and data dictionary of the system; introduces the entity attributes and logical structure design of the database, and focuses on the genetic algorithm of the volume grouping algorithm. The research is discussed, and finally each functional module is described in detail. The system model has been formed, which has laid the foundation for the realization of the auxiliary teaching system.

2. Related Work

A random matrix can be viewed as a matrix whose elements are random numbers [9]. Due to the rapid development of quantum mechanics in the 1940s, the study of random matrix theory has received extensive attention from physicists. It is found that the energy sequence of a quantum system can be approximated by the eigenvalues of the Hermitian matrix, but because the dimension of the data matrix is too large to effectively distinguish each energy sequence, the limiting spectral distribution of all eigenvalues is studied instead [10]. The premise of the classical multivariate statistical analysis method for data analysis is that the dimensionality of the data is assumed to be constant and limited, and the sample size gradually tends to be infinite, which is no longer suitable for the analysis of large-dimensional matrices. In order to solve the above problems, the distribution of eigenvalues of large-dimensional random matrices has been widely discussed and studied, and the famous semicircle law and Marchenko–Pastur law have been discovered, which played a fundamental role in the study of random matrix theory [11–13]. With the development of information and computing science, more and more fields generate and collect a large amount of high-dimensional data, which further promotes the research on random matrix theory, which has also become an important part of modern statistical theory.

The rapid development of network technology promotes the development of modern educational technology and online education is born. Many universities and vocational schools have established their own online teaching platforms [14–16]. Some colleges and universities independently developed by themselves, such as Dalian University of Technology and Wuhan University. Some colleges and universities use the way to establish their own network teaching platform, such as Tsinghua University, the school-enterprise cooperation of China Southern University. It is mainly used to develop the teaching support platform of ASP or JSP technology, database technology, and multimedia technology to realize the teaching activities of students and teachers in the teaching and learning activities of network management [17–19].

In the UK, with support from the government and all sectors of society, the use of the Internet in education has also played a global leadership role [20]. The government issued directives and took various measures to strengthen the computer skills and computer-based teaching in higher education institutions at all levels [21, 22]. The mature online education in the UK is not only used in different schools; the famous University of Liverpool has also promoted online education in other countries. The education system utilizing the network is becoming more and more perfect and efficient, adapting to the way of social education development [23–25]. The UK is promoting a nationwide online learning programme to improve the teachers’ ability to master online information and communication skills [26, 27]. The United Kingdom has established a national Internet access system that enables all primary and secondary school students to use the Internet. The European Education Center in the UK not
only realizes the connectivity of basic education, but also
designs a unified online education plan and rules, and then
develops the most professional online education [28].

3. Methods

3.1. Random Matrix Model. The main idea of the spectral
clustering algorithm evolved from graph theory is to arrange
data as points in space, and there are edges to connect each
point. Short edge weights have a higher proportion.

The graph containing all the data points is cut, so that the
sum of edge weights between different subgraphs is the
smallest, and the sum of edge weights in the subgraphs is the
largest, so as to achieve clustering.

Suppose there is a graph \( G \), the set of points in the graph
is represented by \( V \), and the set of edges is described by \( E \),
which can be written as \( G(V, E) \). Define the weight between
point \( v_i \) and point \( v_j \) as \( w_{ij} \), and let the graph \( G \) be an
undirected graph. For any existing point \( v_i \) in the graph \( G \),
the sum of the weights of all edges connected to it can be
defined as the degree \( d_i \), which is expressed as

\[
d_i = \sum_{j=1}^{n} w_{ij},
\]

From this, an \( n \times n \) dimensional diagonal matrix can be
obtained, called the degree matrix \( D \), the main diagonal of
the degree matrix \( D \) has a value, and corresponds to the sum
of the weights of all the edges connected between the point \( v_i \)
and the point \( v_j \).

This paper uses the full connection method to construct the
adjacency matrix \( W \), and the kernel function for defining
the edge weight can choose polynomial kernel function,
Gaussian kernel function, and Sigmoid kernel function. The
full connection method to construct the adjacency matrix
usually uses the Gauss radial basis kernel function to define
the edge weights, which can be obtained:

\[
w_{ij} = \exp(\sigma^2) - \exp \left\| x_i - x_j \right\|_2^2.
\]

From this, a Laplacian matrix can be constructed. For
any vector \( f \), we have

\[
f^T L f = \sum_{i=0}^{n-1} d_i f_i^2 - \sum_{i=0}^{n-1} \left( 1 - w_{ij} \right) f_i f_j.
\]

Define the tangent weight between \( A \) and \( B \) as

\[
\text{cut} (A, B) = \prod_{i \rightarrow A, j \rightarrow B} \left( 1 - 2w_{ij} \right).
\]

Then, the slicing weight of the set of \( k \) subgraph points
can be expressed as

\[
\text{cut} (A_k) = \prod_{i=0}^{k-1} W \left( A_i, A_i' \right)
\]

Using the RatioCut slicing method can make the weights
and heights of points within the subgraphs and the weights
of points between subgraphs and lows, so as to obtain the
optimal graph cutting method. RatioCut cut graph method:
for each cut graph, it not only needs to minimize cut, but also
needs to maximize the number of points in each subgraph:

\[
\text{Ratio Cut} (A_k) = \prod_{i=0}^{k-1} W \left( A_i, A_i' \right) \frac{1}{2 |A_i - A_i'|}
\]

3.2. Overall System Design. System development is insepara-
bale from the guiding ideology of software engineering. The
development of the system follows the basic idea of modular
programming, dividing the system into several modules,
each of which completes a specific function [29, 30]. Then,
these modules are brought together to form a whole (that is,
a system) to complete the functions of the entire system.
Therefore, the structure of the whole system is clear, the
readability and maintainability are enhanced, and the or-
ganization and management are greatly facilitated.
According to the results of the system functional analysis,
follow a structured system design method.

The question bank management subsystem is a web-
based advanced mathematics question bank management
system. It collects test questions from the “front line,” and
ensures the quality of the test questions in terms of
knowledge points and test difficulty. The autonomous
learning subsystem provides learning methods such as
textbook knowledge, test questions, and courseware. Stu-
dents can make independent arrangements according to
their own learning conditions. The Q&A subsystem provides
functions such as BBS and email Q&A methods. For better
communication, the client only needs to install Mathplayer
to input mathematical symbols, which can facilitate the
communication between students and teachers, which
greatly reflects the interaction in teaching.

The system uses the Internet as the communication
medium and uses the B/S mode architecture to improve the
efficiency and scalability of the system. The B/S mode is a
technical extension based on the C/S mode, and usually has
the following three-tier architecture:

(1) Presentation Layer. It is the user interface part of the
information system, that is, the human–machine
interface, which is the window for the interaction
between the user and the system. The presentation
layer of the “Advanced Mathematics Aided Teaching
System” refers to the web browsers installed on each
browser side. Its main function is to connect users
with the system. For example, administrators can
find and edit functions on it to maintain the system
accordingly. Teachers and students can find the
function options they need from the interface.

(2) Functional Layer. It is the brain of the system,
processing all the instructions of the system. The
functional layer of this system is composed of a Web
server and an application server, so the security is
high.

(3) Data Layer. It consists of a database server, which is
responsible for the access of system data. It occupies
a large amount of space, and requires a high level of server, and the data must be continuously updated.

All databases of the system are stored on the data server, which is more secure and reliable, and also conducive to the improvement of network speed. The user application is installed on the web server, and the user can perform various operations using a browser. The operation interface design should be concise, practical, and the functions should be comprehensive.

“Higher Mathematics Remote Auxiliary Teaching System” makes full use of the characteristics of the network and enriches the teaching methods. At present, all colleges and universities have popularized the network, so the development cost of network equipment is basically zero. In addition, as long as the server side is turned on, client users can use the auxiliary teaching system to learn at anytime and anywhere, so that the learning purpose is clear. The system has powerful information resources, and the teaching content is updated quickly, which promotes the exchange of teaching experience, makes it easier to teach students in accordance with their aptitude, and fully arouses students’ interest in learning advanced mathematics.

3.3. Detailed System Design

3.3.1. User Management Submodule. This module is mainly responsible for the management of user information (including ordinary user registration and password modification) and the management of rights and roles. The realization of the relationship between users and roles is the key to this module.

In order to ensure the integrity and consistency of the system data and the security of the system data, the access rights of users must be controlled, and ordinary registered users have the lowest rights. The system adopts a role-based authority assignment strategy.

In order to facilitate system administrators to manage user permissions and reduce the number of user authorizations, we adopt a role-based permission assignment strategy to grant a set of system-level and object-level permissions to a role [31–33]. Once the user obtains the role, it has all the system-level and object-level permissions that the role has, and the system administrator can also assign permissions to a certain module that the role does not have to specific personnel according to actual needs [34–36]. The access flow of the whole system is shown in Figure 1.

3.3.2. System Maintenance Submodule. The main function of this module is to clean and maintain the test questions and courseware required by the system, as well as the data in BBS, e-mail, and real-time discussion areas, so as to keep the data pertinent and practical. This module has high requirements for managers and is a heavy responsibility work. The administrator should regularly screen, delete, or organize all information materials in the system, including test questions uploaded by teachers, courseware, student emails or BBS posts, and discussion topics. Only by timely sorting can the system be guaranteed. In particular, the valuable information should be loaded into the designated database after being sorted out for user query.

3.3.3. Self-Learning Submodule. The main function of this module is to find the required information timely and accurately according to the students’ requirements, mainly including the advanced mathematics knowledge database and the specialized knowledge database, which is one of the important functions of this teaching system. Starting from the actual teaching needs of the school, conscientiously summarize the knowledge points, carefully screen, from the shallow to the deep; the advanced mathematics knowledge database basically includes all the directions of use of the students in the college stage, special courseware for easier learning and understanding. The knowledge database of the special access book mainly meets the learning needs of students who plan to “access the book.” It mainly includes the real questions of the advanced mathematics examination of the “received book” in the past three years, and each test question has been carefully explained.

3.3.4. Doubt Submodule. The main function of this module improves the interactivity of the teaching system. This system intends to use BBS and email to solve the problem of interaction between teachers, students, or classmates in the network. The learning process of mathematics is boring and requires learning to be calm, but when you encounter a problem that you cannot solve by yourself, it is easy to become overwhelmed and tired of learning. The object of seeking help by email is determined, but it may not be timely. As long as the helper is online in BBS, anyone can leave a message, but the object is not clear, it may be a teacher or a classmate, so the two methods can complement each other’s advantages. In order to facilitate the input and output of special mathematical symbols, the system focuses on the MathML language, which is an XML application used to describe mathematical symbols and express their structure and content. Mathematical content can be transmitted, accessed, and processed through the network. Users can easily display and input mathematical symbols with MathPlayer software, which greatly facilitates the communication of mathematical content.

3.3.5. Group Volume Submodule. The function of this module is that after users log in to the system, they can choose to manually or automatically group volumes according to their personal needs. The range of requirements may include: the difficulty of the test questions, the selected chapters of the test questions, and the type of the test questions. In order to better meet the user’s requirements, the system has set up a temporary test paper library. According to the user’s needs, the temporary test paper is generated manually or automatically.

The test paper users in the test paper library can save some information into their personal accounts according to their personal usage. The test paper can be exported as a
Word document for daily study or work needs, as well as for editing and saving in the future. The data tables created according to the attributes of entity test questions, students, test papers, chapters, etc. are classified into databases. The operation flow design of this module is shown in Figure 2.

3.4. System Database Design. In a specific application environment, construct the optimal database mode, establish the database and its application system, so that it can effectively store data and meet the application requirements of various users.

Database design is the technology of establishing database and its application system, and it is the core technology of information system development and construction.

Database design is the representation and reflection of the most primitive system model. It is well-founded and has clear goals. It is necessary to fully understand the real needs of users and the needs of system implementation.

The biggest function of the database is to store the data on demand, and to carry out various commands accurately and efficiently. Therefore, a reasonable data structure must be designed. The quality of the data structure design directly affects the data query and access efficiency of the system, especially with the operation of the system and the increase of user requirements, the scale of the database will become larger and larger. For a large amount of data, the role of the selected storage structure of the database is particularly obvious.

The database serves the development system, is all the assets of the system, and is a display of the system goal. Starting from demand analysis, screening real and reliable information for storage, and establishing a database has the following goals:

1. The database supports specific and real-time information extraction. The database must store the necessary information, support the information needs determined at design time, and support real-time queries that users may propose.

2. The basic tables of the database should be constructed correctly and efficiently. Each basic table in the database represents a topic and consists of some related fields. The redundancy of the data should be as small as possible, and the corresponding keywords should be used in the entire database to identify.

3. A scientific and reasonable database must be extensible. With the development of the system, the structure design of the database must be flexible.

4. Experimental Results and Analysis

4.1. Experimental Purpose. Whether the NCICAI system can achieve the expected teaching effect must be tested by practice. Evaluation is an intermediary between knowledge and practice [17]. The so-called educational evaluation refers to the process of making a judgment on the extent to which educational activities meet the expected needs by systematically collecting and analyzing information, in order to achieve the value-added process of education. Modern educational evaluation advocates that educational evaluation should take the degree to which educational activities meet the needs of society and individuals as the criterion for value judgment, and its ultimate goal is to increase the value of education. Therefore, the purpose of this experiment is to evaluate the advanced mathematics auxiliary teaching system.

4.2. Experimental Subjects. One of the goals of the assessment lies in the strengths and weaknesses of personalization. For traditional online classrooms, the impact of personalized online teaching should be the criterion for evaluation.

**Figure 1: Access control process of the advanced mathematics auxiliary teaching system.**
Questionnaires were randomly distributed among the students, and the main contents of the survey were the admission scores and the estimated value of their cognitive ability and interest in the subject of “Advanced Mathematics.” Among them, 30 students were selected according to their entrance scores and numbered. Among them, the odd-numbered students were similar to the even-numbered students. The so-called approximate situation means that the students’ entrance scores are similar to their interest in the course, so that the odd-numbered students can be classified. The group was used as the experimental group, and the even group was used as the control group.

Figure 3 shows the students’ preliminary estimates of their cognitive abilities. It is divided into 6 levels: 90–100 is very strong, 80–90 is strong, 70–80 is strong, 60–70 is average, 40–60 is weak, and 0–40 is as weak. It should be pointed out that the cognitive ability here is only a preliminary estimate of the student, which may not be consistent with the actual situation.

Figure 4 shows the preliminary estimation of students’ interest in this course, which is also divided into 6 levels: 90–100 is very interested, 80–90 is interested, 70–80 is relatively interested, 60–70 points for general interest, 40–60 points for not very interested, and 0–40 points for no interest. The interest value is adjusted by the students themselves during the learning process.

4.3. Experimental Results. This system provides self-controlled learning and free learning. Under the control of the system, the system will first evaluate students based on their current cognitive abilities and interests, and then select relevant learning content for students after they have completed all knowledge points (or parts). The method of free study is similar to using traditional online education system.

Divide 30 students into two groups according to odd and even numbers, with 15 students in each group, where the odd group is the experimental group and the even group is the control group. Students use this system to learn, the odd group learns by system control, and the even group learns by free learning. After the study, the two groups were tested in the form of a closed-book written test. The results are shown in Figure 5.

4.4. Analysis of Results. The distribution of the learning efficiency of the two groups is shown in Figure 6. It can be seen from the figure that the learning efficiencies of the two groups are basically randomly distributed. Through learning, both groups basically met the requirements of the syllabus, but the effect of the experimental group using the systematically controlled method of learning was better than that of the control group using the free method of learning.

It can be seen from Figure 7 that for the students in the experimental group, the learning ability value given by the system is basically the same as the final test score, indicating that the student model established by the system is basically reasonable. Students’ over- or underestimation of their own learning ability has been gradually revised in the learning process, which is basically close to the real situation. The comparison between the learning ability value of the control group given by the system and the written test score is shown in Figure 8.

After the experiment, the students had discussions and exchanges. The students put forward some opinions and views on the advanced mathematics auxiliary teaching system, and also pointed out the shortcomings of the system:

(1) The students’ evaluation of the man–machine interface is not too high, and they think that although its operation is simple, the layout is not beautiful enough, which should be improved in the future.

(2) While the students gave positive comments to the system, they also put forward many valuable suggestions. For example, it is best to add a “search”
Figure 3: Students’ preliminary estimates of their cognitive abilities.

Figure 4: Preliminary estimates of students’ interest in this course.

Figure 5: The test results of the two groups in the form of closed-book written test.

function, so that if you encounter difficulties in learning, you can find the corresponding reference resources by entering keywords, which is convenient and quick, avoiding the time-consuming and laborious shortcomings of manual search; the processing examples can be more abundant. The system can add a “message board” to provide an opportunity to communicate and interact with teachers or
The learning ability value of the experimental group

Written test results

Figure 6: The distribution of learning efficiency of the two groups.

The learning ability value of the experimental group

The learning efficiency (%)

Student number

The learning ability value of the control group

Figure 7: The comparison between the learning ability value of the experimental group and the written test score given by the system.

The learning ability value of the control group

Written test results

Figure 8: The comparison between the learning ability value of the control group and the written test score given by the system.
experts, so that when encountering problems that cannot be solved in the system, you can get relevant guidance and help.

5. Conclusion
This paper discusses the design and implementation process of the advanced mathematics auxiliary teaching system in university. The goal of design and realization is to establish a teaching system that uses the network to assist higher mathematics teaching. The content is concise, reasonable, and practical. Using the characteristics of the network, it breaks through the time and space limitations of traditional teaching, which is not only conducive to students’ independent learning, but also has conducive to teachers’ daily teaching work. Focusing on how to realize the expected function of the system, this paper analyzes the software requirements in detail, and builds a system model on this basis. The development technology of advanced numerical auxiliary system is introduced. The B/S architecture is adopted in the system development, and the development language and relational database system are used to realize the system functions. In the detailed design stage, the property diagram and data flow diagram of the system are analyzed in detail, and the system logic structure and database are designed. The paper focuses on the paper-making algorithm. This paper adopts the scientific genetic algorithm to organize the paper, which improves the scientificity and practicability of the test paper. The MathML language is studied, which solves the storage, editing, and display of mathematical expressions, and improves the user’s interactivity on the network. This paper forms a network teaching system with certain intelligence. This paper divides the teaching content into several knowledge points, and uses XML technology to represent the tree-like hierarchical structure formed by the attributes of knowledge points and their interrelations, so as to construct a domain knowledge base. A student model is constructed through fuzzy comprehensive evaluation of students’ cognitive ability and learning situation, and dynamically adjusted according to students’ learning progress. We establish a teaching library based on a production system, and design a teaching strategy reasoning algorithm, so that the system can search for suitable teaching methods and teaching contents in the teaching strategy library and domain knowledge base according to the information provided by the student model. Personalized computer-aided teaching system is a supplement to the traditional classroom teaching mode, and will promote the improvement of education quality.

Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest
The authors declare that they have no known conflicts of interest.

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References


