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Research Article

Numerical and Theoretical Investigation to Estimate Darcy Friction Factor in Water Network Problem Based on Modified Chun-Hui He's Algorithm and Applications

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In this work, we applied the modified Chun-Hui He's algorithm to evaluate for estimation of flow friction factor f for value of friction factor by using Colebrook–White relation. The speedy, precise, and consistent evaluation of flow friction factor f are essential for evaluation of pressure dips and streams in complex network prototypes at distinct values of diameters of pipes. Friction factor estimated outcomes are applied in everyday engineering routine. Numerous computational systems tested for distinguishing of water pipe networks resolution, such as Hardy Cross method (HCM), Newton method (NM), and modified Newton method (MNM), are presented. As a novelty, a modified Newton method tabulated data, graphical results, and comparisons that are presented with different numerical schemes.

1. Introduction

The investigation of a few issues in many fields such as computational physics, computational biology, engineering, environmental sciences, chemistry, and economics in order to resolve real-life nonlinear models with constrained domain. The Newton technique and its variations are effective in solving nonlinear models that occur in real-world problems with reasonable stopping conditions [1, 2]. The flow variation to all loops is immediately removed in this manner, resulting in a high-level merger. This technique nevertheless requires suitable essential assumptions for tributary levels that sustain progression circumstances and are adjacent to the specific stream [3, 4].

The stream systems are energetic, and complex frameworks involve gigantic ventures by private and government sectors. These sectors need sufficient management to control professionally to accomplish goals of their system [5–11]. Unfortunately, now a days, the management of water resource systems is challenging and problematic due to the rapidly increasing customer requirements. This situation is difficult and challenging for conventional methodologies to manage these circumstances [12–14]. Recently, computational methodologies have been tried to tackle these complex circumstances [11, 12]. To explore hydraulic movement in complicated network systems and to satisfy the energy and continuity equations, these quantitative approaches use the Hardy Cross method. Various research laboratories have recently used various techniques to solve these limitations, such as sluggish convergence and recurring dissatisfaction with outcomes, and have failed to meet consumer demand criteria. The failure of others' attempts to eliminate all problems sparked the breakthrough. Furthermore, the Hardy Cross method must be modified due to the nonlinear nature of tube systems [5, 15, 16].

In electrical systems, the relationship between voltage and current with regular resistors is governed by Ohm's law with diodes where resistances depending on current and voltage are nonlinear electrical circuits containing nonlinear components and solving second-order coupled nonlinear Schrödinger equations by using various numerical approaches [3, 4]. HCM is followed by other methods that adopted the Newton–Raphson method, one of the faster effective procedures with higher convergence [17–19]. Toldini and Pilati [20] proposed a global gradient method that is created as a variation to the NM. Such methodologies are usually applied to resolve a system of nonlinear algebraic equations that communicate the behaviors of the hydraulic structures [21–23]. To solve nonlinear problems, numerical approaches are a reasonable alternative. The well-known numerical approaches are visible in [3].

In this study, we believe in a new estimated scheme that is a variation of the standard NM, and we evaluated its efficacy alongside the NM and HCM. At $\mathrm{Re} \geq 4000$, the modified Chun-Hui He's algorithm [24] was used to calculate the friction factor for given pipe diameters in turbulent flow in a confined region, where Re is called Reynolds number and $\varepsilon = 0.05$ is roughness height [25, 26]. At the next level, we used fraction factor values to obtain the water pressure function for various lengths, diameters, and water flow rates in each pipe. All computational data are first incorporated into an Excel sheet, and then, Mathematica code is used to describe the consequences of network systems using Excel data [6].

2. Structure Topology of the Hydraulic System

The first step in characterizing a hydraulic problem is to create a network composition that shows pipe connections in terms of diameter, length, and nodes. Water and utilization levels from suppliers should be allocated to intersection points. Instead of using availability to track pipes, metrics are assigned to each tube and the system's closed loop, as shown in Figure 1. The next step demonstrates the pipes system for preliminary supply of stream is used for utilization in every intersection point and should obey Kirchhoff's law [27]. The overall water entering at an intersection point is approximately the same of that leaves that enter intersection point of the network. The similar preservation law is to satisfy the entire system.

3. Topology of the Hydraulic Model

A scientific explanation of the model can be developed once a complex system configuration is created, along with its loop numbers, pipeline, supply, and resource data. According to the mineralogy theorem of Euler, M nodes (intersection points) and N branches make up the system. In above network problem, we have $N = 13$ and $M = 10$ and $M - 1$ autonomous intersection points, i.e., 9 points and other one point is known as referent point and $N - M + 1 = 13 - 10 + 1 = 4$ autonomous loops. In over problem, point **J** is called referent node.

The Darcy–Weisbach equation and Colebrook–White relation for the Darcy friction factor (f) can be used to analyze this pipe network [8, 28].

$$W_w \approx \Delta w = w_1 - w_2 = \frac{8\mathrm{fL}\mathbf{q}^2}{gD^5\pi^2}, \tag{1}$$

where W_w is the water pressure function. $L = (l_i(m))$, $D = (d_i(m))$ and $\mathbf{q} = (q_i(m^3/s))$ for $i = 1, 2, \ldots, 13$ are the pipes length, diameter, and flow vectors, respectively, and $\mathbf{g}$ is the gravity $(\mathrm{m/s^2})$.

Taking the first derivative of equation (1) and $\mathbf{q}$ assumed as a variable herewith, we have

$$W'_w \approx \frac{\partial W_w(\mathbf{q})}{\partial \mathbf{q}} = \frac{16\mathrm{fL}\mathbf{q}}{gD^5\pi^2}. \tag{2}$$

Darcy friction factor (f) can be described as

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\varepsilon}{3.71\,(\text{diameter})} + \frac{2.51}{\mathrm{Re}.\sqrt{f}}\right]. \tag{3}$$

Colebrook–White (3) is applicable only at turbulent flow in restricted domain at $\mathrm{Re} \geq 4000$, where Re is called Reynolds number and $\varepsilon = 0.05$ is roughness height [25, 26, 28].

3.1. Modified Chun-Hui He's Algorithm for Friction Factor $(\mathbf{f})$. The quick, precise, and dependable evaluation of friction factor (f) in (3) are essential for estimation of pressure falls in complex network models [6, 29]. So, we utilize the modified Chun-Hui He's algorithm [24] to solve the approximate value of friction factor f from equation (3), and we found that the modified Chun-Hui He's algorithm is up-to-date, and an efficient algorithm for solving nonlinear equations exists in real-life applications.

(3) can be redeveloped as follows:

$$F\left(f, \frac{\varepsilon}{D}, \mathrm{Re}\right) = \frac{1}{\sqrt{f}} + 2\log_{10}\left[\frac{\varepsilon}{3.71\,(\text{Diameter})} + \frac{2.51}{\mathrm{Re}.\sqrt{f}}\right] = 0. \tag{4}$$

The friction factor f is used as a variable in this case, and we carefully select preliminary estimates to begin the numerical method in the given restricted domain.

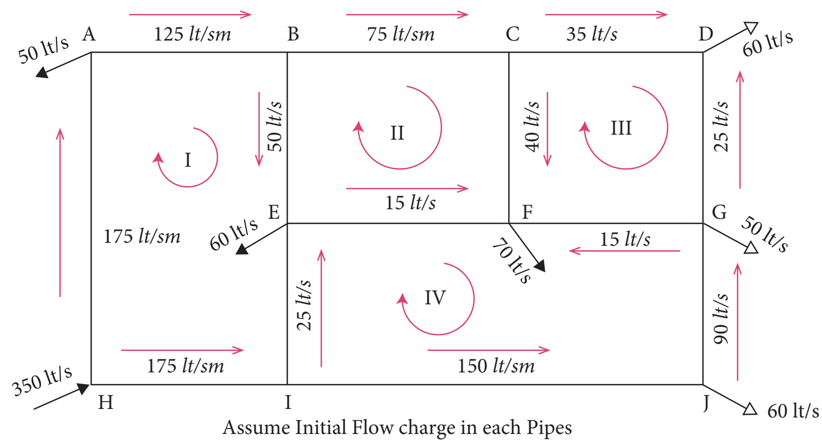

Figure 1: The problem with the water supply network prompted by Kirchhoff's law [3].

3.1.1. Modified Chun-Hui He's Algorithm [24]

Step 1. Necessary condition.

$$F(x_0)F(x_1) < 0, \tag{5}$$

where x_0 and x_1 are the starting assumptions.

Step 2. Ancient Chinese algorithm computes

$$x_2 = x_0 - \frac{F(x_0)}{R(x_0, x_1)}, \tag{6}$$

where $R(x_0, x_1) = (F(x_0) - F(x_1))/(x_0 - x_1)$.

Step 3. Corrector step:

$$x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}. \tag{7}$$

Step 4. Assumptions:

$$\begin{aligned} x_0 &= x_3 - \delta, \\ x_1 &= x_3 + \delta, \end{aligned} \tag{8}$$

$$\delta = -\frac{F(x_3)}{F'(x_3)}. \tag{9}$$

Step 5. If we not get required accuracy, then go back to Step 2.

Table 1 provides the estimated fraction factor f_n flow numbers using the modified Chun-Hui He's Algorithm [24] for various pipe diameters (m).

3.2. Loops Model in Water Network Topology

The system of nonlinear equations along each loop in this section was simulated.

Loop I: W_I,

$$\begin{aligned} &\Delta w_1 + \Delta w_9 - \Delta w_{13} - \Delta w_7 + \Delta w_8 \\ &= \frac{8}{g\pi^2}\left(\frac{f_1 l_1 q_1^2}{d_1^5} + \frac{f_9 l_9 q_9^2}{d_9^5} - \frac{f_{13} l_{13} q_{13}^2}{d_{13}^5} - \frac{f_7 l_7 q_7^2}{d_7^5} + \frac{f_8 l_8 q_8^2}{d_8^5}\right), \\ &= 222.5804287q_1^2 + 536.2707478q_9^2 - 91.8100660q_{13}^2 \\ &\quad - 1708.9539397q_7^2 + 3987.5591927q_8^2. \end{aligned} \tag{10}$$

Loop II: W_{II},

$$\begin{aligned} &\Delta w_2 + \Delta w_{10} - \Delta w_{11} - \Delta w_9 \\ &= \frac{8}{g\pi^2}\left(\frac{f_2 l_2 q_2^2}{d_2^5} + \frac{f_{10} l_{10} q_{10}^2}{d_{10}^5} - \frac{f_{11} l_{11} q_{11}^2}{d_{11}^5} - \frac{f_9 l_9 q_9^2}{d_9^5}\right), \\ &= 278.2255359q_2^2 + 53.6270748q_{10}^2 - 53.6270748q_{11}^2 \\ &\quad - 536.2707478q_9^2. \end{aligned} \tag{11}$$

Loop III: W_{III},

$$\begin{aligned} &\Delta w_3 - \Delta w_4 + \Delta w_{12} - \Delta w_{10} \\ &= \frac{8}{g\pi^2}\left(\frac{f_3 l_3 q_3^2}{d_3^5} - \frac{f_4 l_4 q_4^2}{d_4^5} + \frac{f_{12} l_{12} q_{12}^2}{d_{12}^5} - \frac{f_{10} l_{10} q_{10}^2}{d_{10}^5}\right), \\ &= 278.2255359q_3^2 - 536.2707478q_4^2 + 53.6270748q_{12}^2 \\ &\quad - 53.6270748q_{10}^2. \end{aligned} \tag{12}$$

Table 1: The statistics values of flow, diameter, length, and constants value K displayed along with each value of the fraction factor f of each pipe in the network (Figure 2).

Pipes	No.	f_n	Assume flow (lt/s)	Assume flow (m^3/s)	Diameter (m)	Length (m)	$K = 8\,\text{fL}/\text{gD}^5\Pi^2$
A-B	1	0.4470154801206153	$q_1 = 125$	$q_1 = 0.125$	0.457	120	222.5804287
B-C	2	0.4470154801206153	$q_2 = 75$	$q_2 = 0.075$	0.457	150	278.2255359
C-D	3	0.4470154801206153	$q_3 = 35$	$q_3 = 0.035$	0.457	150	278.2255359
D-G	4	0.4470232660486057	$q_4 = 25$	$q_4 = 0.025$	0.406	160	536.2707478
G-J	5	0.4470154801206153	$q_5 = 90$	$q_5 = 0.090$	0.457	120	222.5804287
J-I	6	0.4470000072957691	$q_6 = 150$	$q_6 = 0.150$	0.609	300	132.4056409
I-H	7	0.4470466694856617	$q_7 = 175$	$q_7 = 0175$	0.304	120	1708.9539397
H-A	8	0.4470466694856617	$q_8 = 175$	$q_8 = 0175$	0.304	280	3987.5591927
B-E	9	0.4470232660486057	$q_9 = 50$	$q_9 = 0.050$	0.406	160	536.2707478
C-F	10	0.4470232660486057	$q_{10} = 40$	$q_{10} = 0.040$	0.406	16	53.6270748
E-F	11	0.4470232660486057	$q_{11} = 15$	$q_{11} = 0.015$	0.406	16	53.6270748
F-G	12	0.4470232660486057	$q_{12} = 15$	$q_{12} = 0.015$	0.406	16	53.6270748
E-I	13	0.44703328769837464	$q_{13} = 25$	$q_{13} = 0.025$	0.355	14	91.8100660

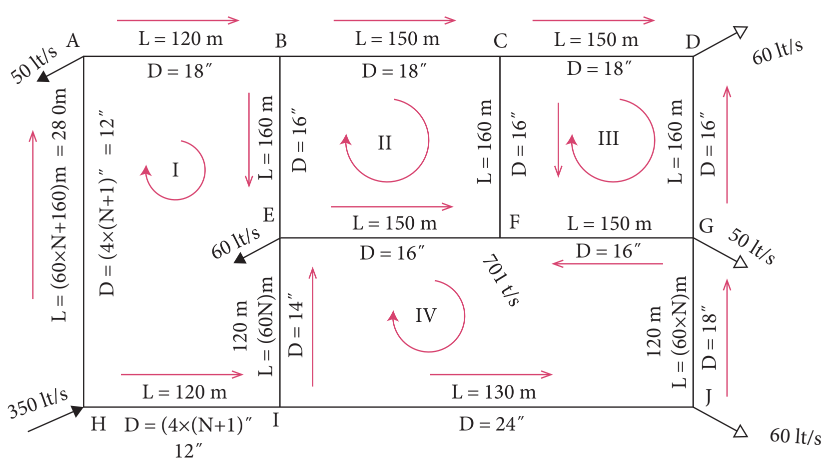

Figure 2: Water supply network problem [3].

Loop IV: W_{IV},

$$
\begin{aligned}
&-\Delta w_5 - \Delta w_6 + \Delta w_{13} + \Delta w_{11} - \Delta w_{12} \\
&= \frac{8}{g\pi^2}\left(-\frac{f_5 l_5 q_5^2}{d_5^5} - \frac{f_6 l_6 q_6^2}{d_6^5} + \frac{f_{13} l_{13} q_{13}^2}{d_{13}^5} + \frac{f_{11} l_{11} q_{11}^2}{d_{11}^5} - \frac{f_{12} l_{12} q_{12}^2}{d_{12}^5}\right). \qquad (13) \\
&= -222.5804287 q_5^2 - 132.4056409 q_6^2 + 91.8100660 q_{13}^2 + 53.6270748 q_{11}^2 - 53.6270748 q_{12}^2.
\end{aligned}
$$

Above loop relations, W_I, W_{II}, W_{III}, and W_{IV} can be written in matrix form:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1
\end{bmatrix}
\times
\begin{bmatrix}
\Delta w_1 \\ \Delta w_2 \\ \Delta w_3 \\ \Delta w_4 \\ \Delta w_5 \\ \vdots \\ \Delta w_{13}
\end{bmatrix}
= 0, \qquad (14)
$$

where $l_1, l_2, \ldots, l_{13}$ are the lengths of each pipe, $d_1, d_2, \ldots, d_{13}$ are the diameters, and water flow in each pipe, as shown in Figure 1, is $q_1, q_2, \ldots, q_{13}$, respectively. The positive sign showing flow direction is clockwise in left matrix of (14) and vice versa [5, 9, 30].

3.3. Nodes Model Topology. In this section, we will simulate the system of linear equation along each node by using Kirchhoff's first law:

(1) Node A: $-q_1 + q_8 - q_{A-\text{output}} = 0$, where $q_{A-\text{output}} = 50\,\text{lt}/s$
(2) Node B: $q_1 - q_2 - q_9 - q_{B-\text{output}} = 0$, where $q_{B-\text{output}} = 0$
(3) Node C: $q_2 - q_3 - q_{10} - q_{C-\text{output}} = 0$, where $q_{C-\text{output}} = 0$
(4) Node D: $q_3 + q_4 - q_{D-\text{output}} = 0$, where $q_{D-\text{output}} = 60\,\text{lt}/s$
(5) Node E: $q_9 + q_{13} - q_{11} - q_{E-\text{output}} = 0$, where $q_{E-\text{output}} = 60\,\text{lt}/s$
(6) Node F: $q_{11} + q_{10} + q_{12} - q_{F-\text{output}} = 0$, where $q_{F-\text{output}} = 70\,\text{lt}/s$
(7) Node G: $q_5 - q_4 - q_{12} - q_{G-\text{output}} = 0$, where $q_{G-\text{output}} = 50\,\text{lt}/s$
(8) Node H: $-q_7 - q_8 + q_{H-\text{input}} - q_{H-\text{output}} = 0$, where $q_{H-\text{output}} = 0$, $q_{H-\text{input}} = 350$
(9) Node I: $q_7 - q_6 - q_{13} - q_{I-\text{output}} = 0$, where $q_{I-\text{output}} = 0$
(10) Node J: $q_6 - q_5 - q_{J-\text{output}} = 0$ where $q_{J-\text{output}} = 60\text{lt}/s$and referent.

Now, rewrite the above linear system of equations in matrix form [31]. The drawback of this matrix form is not a linear independent, that is why referent node would be omitted from the system of linear equation.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}_{(9\times 13)} \times \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ \vdots \\ q_{13} \end{bmatrix}_{(13\times 1)} = \begin{bmatrix} q_A - \text{output} = 50 \\ q_B - \text{output} = 0 \\ q_C - \text{output} = 0 \\ q_D - \text{output} = 60 \\ q_E - \text{output} = 60 \\ q_F - \text{output} = 70 \\ q_G - \text{output} = 50 \\ q_{H-\text{output}} - q_{H-\text{input}} = -350 \\ q_I - \text{output} = 0 \end{bmatrix}_{(9\times 1)}. \quad (15)$$

Above node **J** count up as a referent, the left matrix in system (15) shows the rows corresponding to node **A**, node **B**, . . . node **I**, respectively.

3.4. Hardy Cross Method. The Hardy Cross method is applied to multiloop water network problems. Figures 3 and 4 show the complete results of the HCM. This method converges to the required results after 24$^{\text{th}}$ iteration.

The primary stage in resolution the problem is to make a net map display lengths and diameters and networks nodes. Later, we compose the original stream supply all through a pipe network. The selection of original flows must fulfill Kirchhoff's law used in the system, and conservation law is also applicable for the entire network system [3, 13, 14, 27].

Outcomes of HCM in column 7 indicate that after 24$^{\text{th}}$ iteration $-\mathbf{W_w}/|\mathbf{W'_w}|$ (head loss) is nil by Darcy–Weisbach relation (1). The negative sign indicates anticlockwise water flows shown in Figure 1. Assign a sign with new flow of water. We will add in clockwise if ΔQ is +ve and vice versa. Similarly, we will subtract value if ΔQ is −ve and vice versa.

3.5. Newton Method. In this paragraph, hydrological problem is investigated by utilizing the NM having 2$^{\text{nd}}$-order [32]. NM [23] or Newton-like procedure is not essential to take up an original estimate that fulfills the continuity law [33], as shown in Figure 1.

Table 2 provides the numerical results of correction flow of (1) by using conventional NM and change in two consecutive flows in each loop going to zero after 5$^{\text{th}}$ iteration by selecting the random original flow in network problem. The effectiveness of NM is better than HCM.

3.6. MNM (Modified Newton Method). In this subsection, MNM achieves an optimum resolution of the nonlinear model simulated in Sections 3.2 and 3.3 for scrutinizing the hydrological network considering. Present procedure is cost-effective and takes a smaller amount of time to attain the results than the HCM and conventional NM.

Table 3 provides the numerical results of correction flow of (1) by using MNM and change in two consecutive flows in each loop going to zero after three iterations by selecting the random original flow in network problem.

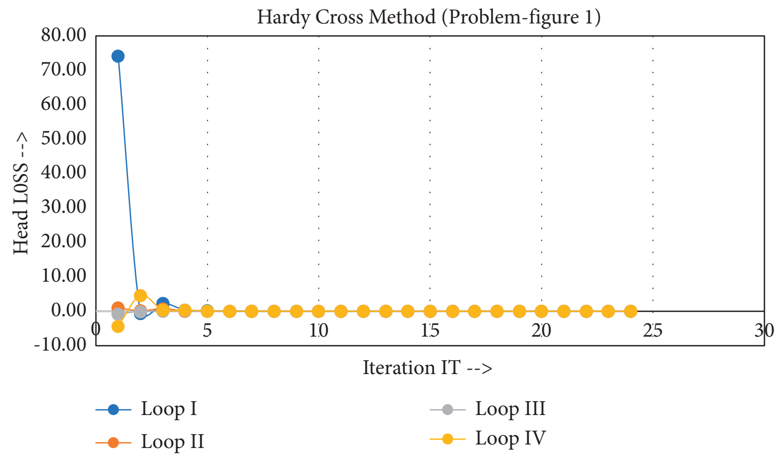

Figure 3: The numerical results of head loss of equation (1) by using HCM and head loss in each loop going to zero after 24^{th} iteration by selecting the random original flow in network problem.

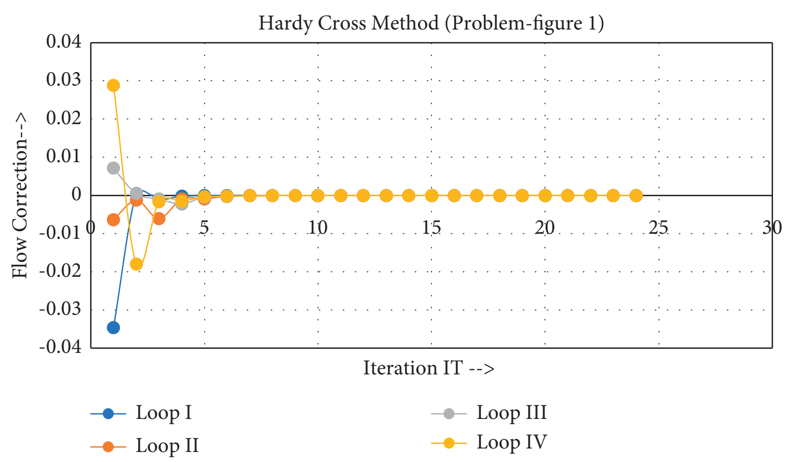

Figure 4: The numerical results of correction flow of equation (1) by using HCM and total corrected flow (m^3/s) in each loop going to nil after 24^{th} iteration with selecting random original flow in network problem, where ΔQ_I, ΔQ_{II}, ΔQ_{III}, and ΔQ_{IV} change in flow of loop I–loop IV, respectively.

Table 2: The water problem (Figure 2) displaying the first iteration of technique NM ([32]). It is not necessary to follow Kirchhoff's law in NM.

Pipes	Loops	Diameter (m)	Length (m)	$\mathbf{q}$ (m^3/s)	Sign (q)	$\mathbf{W}_w$, equation (1)	NM, $\mathbf{q}$ (m^3/s)	$\Delta\mathbf{q}$ (m^3/s)
1		0.457	120	0.125	1	3.48	0.0892094	−0.0357906
9	**I**	0.406	160	0.05	1	1.34	0.0352368	−0.0147632
13		0.355	120	−0.025	−1	−0.49	0.112488	0.087488
7		0.304	120	−0.175	−1	−52.34	0.210791	0.035791
8		0.304	280	0.175	1	122.12	0.139209	−0.035791
	Loop				Σ	**74.11**		**0.0369342**
2		0.457	150	0.075	1	1.57	0.0539726	−0.0210274
10	**II**	0.406	160	0.04	1	0.86	0.017803	−0.022197
11		0.406	150	−0.015	−1	−0.11	0.0877249	0.0727249
9		0.406	160	−0.05	−1	−1.34	0.0352368	−0.0147632
	Loop				Σ	**0.97**		**0.0147373**
3		0.457	150	0.035	1	0.34	0.0361696	0.0011696
4	**III**	0.406	160	−0.025	−1	−0.34	0.0238304	−0.0011696

Table 2: Continued.

Pipes	Loops	Diameter (m)	Length (m)	$\mathbf{q}\,(\mathrm{m}^3/\mathrm{s})$	Sign (q)	$\mathbf{W_w}$, equation (1)	NM, $\mathbf{q}\,(\mathrm{m}^3/\mathrm{s})$	$\Delta\mathbf{q}\,(\mathrm{m}^3/\mathrm{s})$
12		0.406	150	0.015	1	0.11	−0.0355278	−0.0505278
10		0.406	160	−0.04	−1	−0.86	0.017803	−0.022197
	Loop				Σ	**−0.74**		**−0.0727248**
5		0.457	120	−0.09	−1	−1.80	0.0383025	−0.0516975
6	**IV**	0.609	300	−0.15	−1	−2.98	0.0983025	−0.0516975
13		0.355	120	0.025	1	0.49	0.112488	0.087488
11		0.406	150	0.015	1	0.11	0.0877249	0.0727249
12		0.406	150	−0.015	−1	−0.11	−0.0355278	−0.0505278
					Σ	**−4.29**		**0.0062901**

All bold values are based on multidimension.

Table 3: The first iteration of MNM ([34]) for the water problem (Figure 2).

Pipes	Loops	Diameter (m)	Length (m)	$\mathbf{q}\,(\mathbf{m}^3/\mathbf{s})$	Sign (q)	$\mathbf{W_w}$, equation (1)	MNM, $\mathbf{q}\,(\mathbf{m}^3/\mathbf{s})$	$\Delta\mathbf{q}\,(\mathbf{m}^3/\mathbf{s})$
1		0.457	120	0.125	1	3.48	0.0879253	−0.0370747
9	**I**	0.406	160	0.05	1	1.34	0.0315287	−0.0184713
13		0.355	120	−0.025	−1	−0.49	0.115121	0.090121
7		0.304	120	−0.175	−1	−52.34	0.212075	0.037075
8		0.304	280	0.175	1	122.12	0.137925	−0.037075
	Loop				Σ	**74.11**		**0.034575**
2		0.457	150	0.075	1	1.57	0.0563965	−0.0186035
10	**II**	0.406	160	0.04	1	0.86	0.0225996	−0.0174004
11		0.406	150	−0.015	−1	−0.11	0.08665	0.07165
9		0.406	160	−0.05	−1	−1.34	0.0315287	−0.0184713
	Loop				Σ	**0.97**		**0.0171748**
3		0.457	150	0.035	1	0.34	0.0337969	−0.0012031
4	**III**	0.406	160	−0.025	−1	−0.34	0.0262031	0.0012031
12		0.406	150	0.015	1	0.11	−0.0392496	−0.0542496
10		0.406	160	−0.04	−1	−0.86	0.0225996	−0.0174004
	Loop				Σ	**−0.74**		**−0.07165**
5		0.457	120	−0.09	−1	−1.80	0.0369535	−0.0530465
6	**IV**	0.609	300	−0.15	−1	−2.98	0.0969535	−0.0530465
13		0.355	120	0.025	1	0.49	0.115121	−0.0134879
11		0.406	150	0.015	1	0.11	0.08665	0.07165
12		0.406	150	−0.015	−1	−0.11	−0.0392496	−0.0542496
					Σ	**−4.29**		**−0.1021805**

All bold values are based on multidimension.

Table 4: The first iteration of method HCM for the water problem (Figure 2).

Loops	Pipes	Diameter (m)	Length (m)	$\mathbf{q}$ $(\mathrm{m}^3/\mathrm{s})$	Sign (q)	W_w (equation (1))	$\lvert W'_w\rvert$ (equation (2))	Correction	New $\mathbf{q}\,(\mathbf{m}^3/\mathbf{s})$
I	1	0.457	120	$A_1 = 0.125$	1	3.48	55.65		0.09040843
	9	0.406	160	$A_2 = 0.05$	1	1.34	53.63	−0.006320742	0.021729173
	13	0.355	120	$A_3 = -0.025$	−1	−0.49	39.35	0.028735632	−0.088327201
	7	0.304	120	$A_4 = -0.175$	−1	−52.34	598.13		−0.20959157
	8	0.304	280	$A_5 = 0.175$	1	122.12	1395.65		0.14040843
Loop					Σ	**74.11**	**2142.40**		
						$-\mathbf{W_w}/\lvert\mathbf{W'_w}\rvert$	**−0.03459157**		
	2	0.457	150	$B_1 = 0.075$	1	1.57	41.73		0.068679258
II	10	0.406	160	$B_2 = 0.04$	1	0.86	42.90	0.007089585	0.026589673
	11	0.406	150	$B_3 = -0.015$	−1	−0.11	15.08	0.028735632	−0.050056374
	9	0.406	160	$B_4 = -0.05$	−1	−1.34	53.63	−0.03459157	−0.021729173
Loop					Σ	**0.97**	**153.35**		
						$-\mathbf{W_w}/\lvert\mathbf{W'_w}\rvert$	**−0.006320742**		
	3	0.457	150	$C_1 = 0.035$	1	0.34	19.48		0.042089585
III	4	0.406	160	$C_2 = -0.025$	−1	−0.34	26.81		−0.017910415

Table 4: Continued.

Loops	Pipes	Diameter (m)	Length (m)	**q** (m^3/s)	Sign (q)	W_w (equation (1))	$\lvert W_w' \rvert$ (equation (2))	Correction	New **q** (**m^3/s**)
	12	0.406	150	C_3 = 0.015	1	0.11	15.08	0.028735632	−0.006646047
	10	0.406	160	C_4 =−0.04	−1	−0.86	42.90	−0.006320742	−0.026589673
Loop					Σ	**−0.74**	**104.27**		
						$-\mathbf{W_w}/\lvert\mathbf{W_w'}\rvert$	**0.007089585**		
	5	0.457	120	−0.09	−1	−1.80	40.06		−0.061264368
IV	6	0.609	300	−0.15	−1	−2.98	39.72		−0.121264368
	13	0.355	120	0.025	1	0.49	39.35	−0.03459157	0.088327201
	11	0.406	150	0.015	1	0.11	15.08	−0.006320742	0.050056374
	12	0.406	150	−0.015	−1	−0.11	15.08	0.007089585	0.006646047
					Σ	**−4.29**	**149.30**		
						$-\mathbf{W_w}/\lvert\mathbf{W_w'}\rvert$	**0.028735632**		

All bold values are based on multidimension.

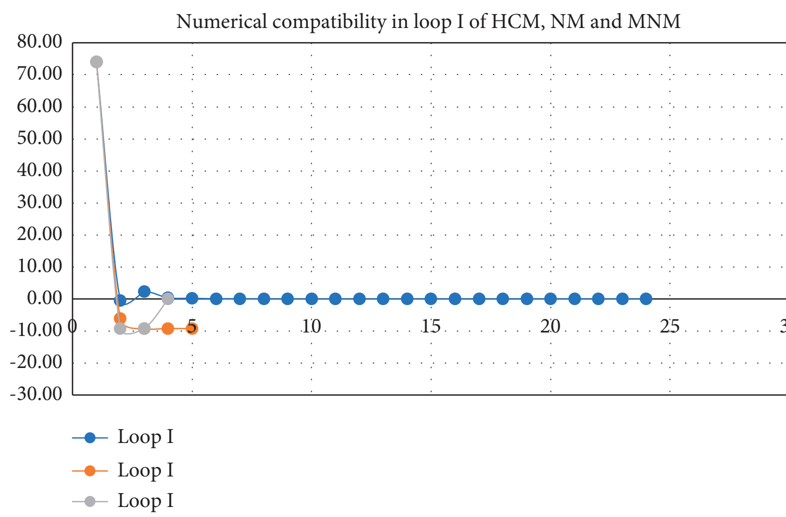

Figure 5: The numerical compatibility of all HCM, NM, and MNM approaches in loop I.

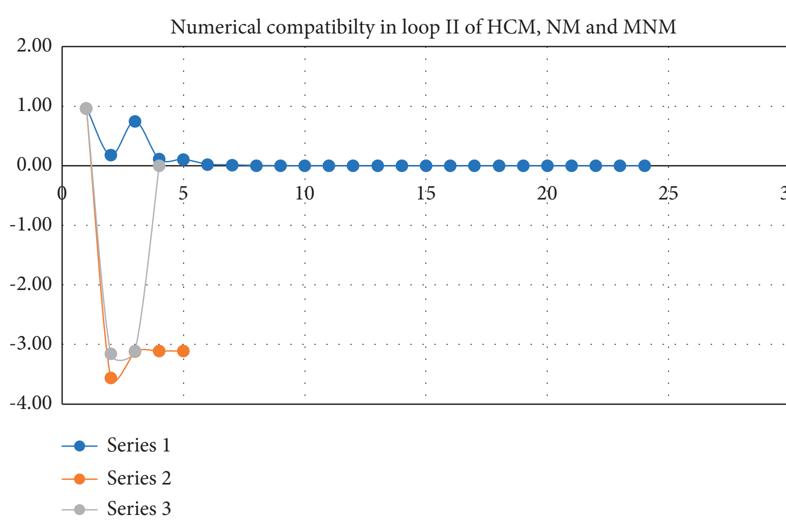

Figure 6: The numerical compatibility of all HCM, NM, and MNM approaches in loop II.

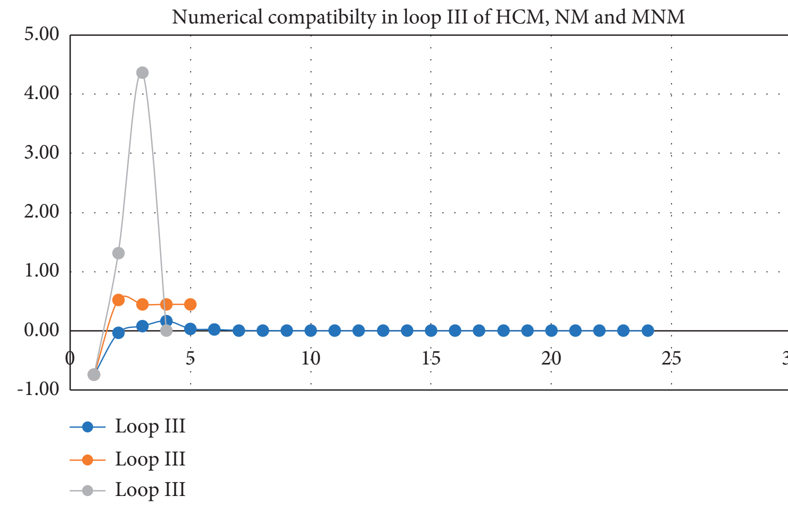

Figure 7: The numerical compatibility of all HCM, NM, and MNM approaches in loop III.

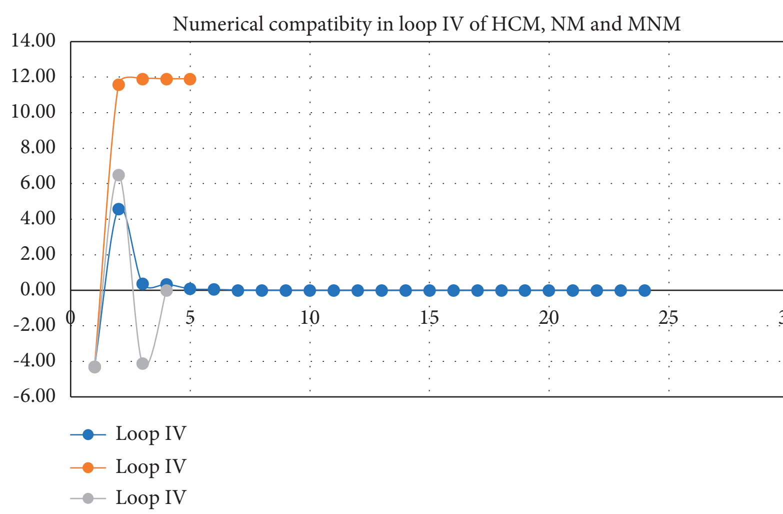

Figure 8: The numerical compatibility of all HCM, NM, and MNM approaches in loop IV.

Table 5

Start node	End node	Pipe index	Hardy Cross method (m^3/s)	Newton method (m^3/s)	Modified NM (m^3/s)
A	B	1	0.0860912	0.0860912	0.0860912
B	C	2	0.057979	0.057979	0.057979
C	D	3	0.0380804	0.0380804	0.0380804
D	G	4	[a] −0.0219196	0.0219196	0.0219196
G	J	5	−0.0854642	0.0854642	0.0854642
J	I	6	−0.145464	0.145464	0.145464
I	H	7	−0.213909	0.213909	0.213909
H	A	8	0.136091	0.136091	0.136091
B	E	9	0.0281122	0.0281122	0.0281122
C	F	10	−0.0198986	0.0198986	0.0198986
E	F	11	0.0365569	Hoboken	0.0365569
F	G	12	−0.0135446	0.0135446	0.0135446
E	I	13	0.0684447	0.0684447	0.0684447

[a]The stream is moving in the opposite direction of the original flow, as indicated by the negative signs. The final water flow of HCM, NM, and MNM is explained in columns 7–9.

4. Analysis and Results Discussion

Several processes must be carried out in order for this to operate. The recommended effort is run on a 10th Generation Intel Core i7 processor, 1 TB SSD, and GTX 1660 Ti (6 GB) graphics card with Windows 10 as the operating system. We used Mathematica 11.2 to do all of the existing simulations. The statistics in Table 4 show only the first iteration of HCM, NM, and MNM, respectively. These results show that HCM reaches its ideal level after 24 rounds, NM after 5 iterations, and MNM after 3 iterations.

Figures 5–8 show the evaluation and assessment of HCM, NM, and MNM in each loop, demonstrating that MNM is less expensive and takes less time to converge than traditional HCM and NM approaches. Table 5 provides the findings and evaluation of head loss in each loop of all approaches.

The optimality status of HCM, NM, and MNM is given in Table 5. We may also check that the original scheme, when compared to HCM and NM, achieves an optimal level with fewer steps and less time. On the other hand, HCM reaches the optimal stage after 24 rounds, NM after 5 iterations, and MNM after 3 iterations. The MNM's key benefit is this statistic. This study's novelty and effectiveness can be seen in these results.

5. Conclusion

The friction factor f is calculated using (3) in the first stage of this study. In a retraction domain Re > 4000, this equation represents the relationship between the tube's innermost diameter, tube roughness, fraction factor, and Reynolds number. This relationship has an implied form that is impossible to solve explicitly. To circumvent this limitation, we numerically solved (3) using the most recent and up-to-date modified Chun-Hui He's Algorithm [24] based on the sizes of each pipe in the system, as shown in Figure 2. The numerical value of the fraction factor was then used to calculate the water pressure function, which was then utilized to estimate the head loss and corrected flow between two consecutive values using the Hardy Cross method, Newton method, and modified Newton method. Tables 2–4 provide that HCM obtains the needed solution after 24 iterations, NM after 5 iterations, and MNM after 3 iterations, all with stopping criteria 1×10^{-10}.

Nomenclature

HCM: Hardy Cross method
π: Ludolph's number (=3.14159)
f: Friction function
l: Length (m/s)
ε: Roughness height
q: Discharge pressure (l/s)
PVC: Polyvinyl chloride
d: Diameter (m)
BVPs: Boundary value problems
MNM: Modified Newton method
g: 9.8 (m/s^2)
f: Friction factor
NM: Newton method
Re: Reynolds number
G_g: : Head loss
IT: Iteration.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

W.A. and M.A. developed methodology and wrote and reviewed the article. W.A. investigated the study and wrote the original draft. W.A., A.J.R., and U.F. collected resources. Z.A. and F.F. collected data. M.A. conceptualized the study. W.A. and J.R. developed software. M.K. performed formal analysis.

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