Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 8179642, 7 pages
https://doi.org/10.1155/2022/8179642

Research Article

Double-Connected Intuitionistic Space in Double Intuitionistic Topological Spaces

Asmaa Ghassob Raoof,[1] Taha H. Jassim,[1] and Ngiste Amare [2]

[1]*Department of Mathematics, College of Computer Sciences & Mathematics, Tikrit University, Tikrit, Iraq*
[2]*Department of Civil Engineering, Mizan Tepi University, Tepi, Ethiopia*

Correspondence should be addressed to Ngiste Amare; ngiste@mtu.edu.et

Received 11 June 2022; Accepted 28 June 2022; Published 13 December 2022

Academic Editor: Mukesh Soni

The structures of the families of fuzzy sets that arise out of various notions of openness and closeness in a double fuzzy topological space are explored. This research is to present a new portion of space (Double-connected intuitionistic space) in Double intuitionistic topological spaces. Through these concepts, we advance some of their characteristics and relate to themselves.

1. Introduction

Ajmal and Kohli [1] explain connectedness in fuzzy topological spaces. Yasry [2] investigated lectures in advanced topology. El-Hamed et al. [3] presented Double connected spaces. Kandil et al. [4] investigated on flour (intuitionistic) topological spaces. Kendal et al. [5] studied on flour (intuitionistic) compact space. The concept was used to define intuitionistic sets and the intuitionistic gradation of openness by Coker [6–8]. Atanassov and Stoeva [9] describe Intuitionistic fuzzy sets. Atanassov [10] studied more intuitionistic fuzzy sets. Ozcelik and Narli [11] introduced the concept of submaximal intuitionistic topological spaces. Tantawy et al. [12] researched soft connections with double spaces. Selma and Coker [13] examined the concept of connectedness in intuitionistic fuzzy special topological spaces. Levine [14, 15] introduced generalized closed sets in topology. The concept of a fuzzy set was presented by Zadeh in his classic paper from 1965 and 2002 [14, 16]. After that, we introduce a new class of sets in DITS, namely the Double connected set, the separated Double *I* sets, the strongly Double connected, Double CO connected I space, and the Double *I* component. We also presented several examples of each type and concluded that there are relationships between them that were presented through theorems.

2. Preliminaries

We remind the following definitions, which are needed in our effort:

Let $\mathcal{x} \neq \varnothing$, as well as ζ and $\mathcal{L}$ be IS having the form $\zeta = \langle x.\zeta_1.\zeta_2\rangle.\mathcal{L} = \langle x.\mathcal{L}_1.\mathcal{L}_2\rangle$, respectively. Also, $\{\zeta_i\colon i \in I\}$ be an arbitrary family of I S in $\mathfrak{X}$, where $\zeta_i = \langle x.\zeta_i^{(1)}.\zeta_i^{(2)}\rangle$, afterward:

(1) $\tilde{\varnothing} = \langle x.\varnothing.\mathcal{x}\rangle$; $\tilde{\mathcal{x}} = \langle x.\mathcal{x}.\varnothing\rangle$.

(2) $\zeta \subseteq \mathcal{L}$ iff $\zeta_1 \subseteq \mathcal{L}_1$ and$\zeta_2 \supseteq \mathcal{L}_2$.

(3) $\zeta^c = \langle x.\zeta_2.\zeta_1\rangle$.

(4) $\cup\zeta_i = \langle x.\cup\zeta_i^{(1)}.\cap\zeta_i^{(2)}\rangle.\cap\zeta_i = \langle x.\cap\zeta_i^{(1)}.\cup\zeta_i^{(2)}\rangle$ [7]. Let $\mathcal{x}$ be a nonempty set, an intuitionistic set $\mathfrak{B}$ (I S, for short) is an object having g the form $\mathfrak{B} = \langle x.\mathfrak{B}_1.\mathfrak{B}_2\rangle$, where $\mathfrak{B}_1$ and $\mathfrak{B}_2$ are disjoint subset of $\mathcal{x}$. The set $\mathfrak{B}_1$ is called set of members of $\mathfrak{B}$, while $\mathfrak{B}_2$ is called set of nonmembers of B [7]. An intuitionistic topology (IT, for short) on a nonempty set $\mathcal{x}$ is a family T of IS in $\mathfrak{X}$ containing $\tilde{\varnothing}.\tilde{\mathcal{x}}$ and closed under arbitrary unions and finitely intersections. The pair $(\mathcal{x}.T)$ is called ITS [11]. Let $\mathcal{x} \neq \varnothing$.

(1) A Double-set (D-set, for short) $\mathfrak{U}$ is an ordered pair $(\mathfrak{U}_1, \mathfrak{U}_2) \in \wp(\mathcal{x}) \times \wp(\mathcal{x})$, such that $\mathfrak{U}_1 \subseteq \mathfrak{U}_2$.

(2) $D(x) = \{(\mathfrak{U}_1, \mathfrak{U}_2) \in \mathcal{P}(x) \times \mathcal{P}(x), \mathfrak{U}_1 \subseteq \mathfrak{U}_2\}$ is the family of all D-sets on x.
(3) The D-set $\tilde{x} = (x, x)$ is called the universal D-set, and the D-set $\tilde{\emptyset} = (\emptyset, \emptyset)$ is called the empty D-set.
(4) Let $\mathfrak{U} = (\mathfrak{U}_1, \mathfrak{U}_2)$; $\vartheta = (\vartheta_1; \vartheta_2) \in D(x)$:
(1) $(\mathfrak{U}^c) = (\mathfrak{U}_2^c, \mathfrak{U}_1^c)$, where $\mathfrak{U}^c$ is the complement of $\mathfrak{U}$.
(2) $\mathfrak{U} - \vartheta = (U_1 - \vartheta_2, \mathfrak{U}_2 - \vartheta_2)$ [4]. Let x be a nonempty set. The family η of D-sets in x is called a double topology on x if it satisfies the following axioms:
(a) $\emptyset$, $x \in \eta$.
(b) If $\mathfrak{U}$, $\vartheta \in \eta$, then $\mathfrak{U} \cap \vartheta \in \eta$,
(c) If $\{\mathfrak{U}z: z \in Z\} \subseteq \eta$, then $\cup_{z \in Z} \mathfrak{U}z \in \eta$. The pair (x, η) is called a DTS. Each element of η is called an open D-set in x. The complement of open D-set is called closed D-set [4]. Let x be a nonempty set:
(1) IN $(x) = \{\emptyset, x\}$ is a DTS, which is called indiscrete DTS.
(2) dis $(x) = \mathcal{P}(x)) \times \mathcal{P}(x)$ is a DTS, which is called discrete DTS [4]. Let $(\mathfrak{X}, \eta)$ be a DTS and $\partial \in D(\mathfrak{X})$. The double closure of $\mathfrak{U}$, denoted by cl η $(\mathfrak{U})$ defined by cl η $(\mathfrak{U}) = \cap \{\vartheta: \vartheta \in \eta_c$ and $\mathfrak{U} \subseteq \vartheta\}$ [2, 4]. Let $\mathscr{X}$ nonempty set, $w \in \mathscr{X}$ a fixed element in $\mathscr{X}$, and let $\mathscr{M} = \langle x.\mathscr{M}_1.\mathscr{M}_2\rangle$ be an intuitionistic set (IS, for short). The IS $\dot{\mathfrak{w}}$ defined by $\dot{\mathfrak{w}} = \langle x.\{\mathfrak{w}\}.\{\mathfrak{w}\}^c\rangle$ is called an intuitionistic point (I$\mathfrak{w}$ for short) in $\mathscr{X}$. The IS $\ddot{\mathfrak{w}} = \langle x.\emptyset.\{\mathfrak{w}\}^c\rangle$ is called a vanishing I point (VI$\mathfrak{w}$, for short) in $\mathscr{X}$. The IS $\dot{\mathfrak{w}}$ is said to be contained in $\mathscr{M}$ ($\dot{\mathfrak{w}} \in \mathscr{M}$ .for short) only if $\mathfrak{w} \in \mathscr{M}_1$, and similarly IS $\ddot{\mathfrak{w}}$ contained in $\mathscr{M}$ ($\ddot{\mathfrak{w}} \in \mathscr{M}$ for short) only if $w \notin \mathscr{M}_2$. For a given IS w in $\mathscr{X}$, we may write $\mathscr{M} = (\cup\{\mathfrak{w}: \dot{\mathfrak{w}} \in \mathscr{M}\}) \cup (\cup\{\ddot{\mathfrak{w}}: \ddot{\mathfrak{w}} \in \mathscr{M}\})$, and whenever $\mathscr{M}$ is not a proper IS (i.e., if $\mathscr{M}$ is not of the form $\mathscr{M} = \langle x.\mathscr{M}_1.\mathscr{M}_2\rangle$ where $\mathscr{M}_1 \cup \mathscr{M}_2 \neq \mathscr{X}$), then $\mathscr{M} = \cup\{\dot{\mathfrak{w}}: \dot{\mathfrak{w}} \in \mathscr{M}\}$ hold. In general, any IS $\mathscr{M}$ in $\mathscr{X}$ can be written in the form $\mathscr{M} = \dot{\mathscr{M}} \cup \ddot{\mathscr{M}}$ where $\dot{\mathscr{M}} = \cup\{\dot{\mathfrak{w}}: \dot{\mathfrak{w}} \in \mathscr{M}\}$, and $\ddot{\mathscr{M}} = \cup\{\ddot{\mathfrak{w}}: \ddot{\mathfrak{w}} \in \mathscr{M}\}$ [6]. A topological space $\mathscr{X}$ is connected if it cannot be written as $\mathscr{X} = \mathscr{X}_1 \cup \mathscr{X}_2$, where $\mathscr{X}_1$ and $\mathscr{X}_2$ are both open and $\mathscr{X}_1 \cap \mathscr{X}_2 = \emptyset$, otherwise $\mathscr{X}$ is called disconnected [2]. Let $(\mathscr{X}, \eta)$ be a DTS and Υ be a nonempty subset of $\mathscr{X}$. Then, $\eta_\Upsilon = \{q \cap \Upsilon: q \in \mathscr{M}$ and $\Upsilon = (\Upsilon, \Upsilon)\}$ is a double topology on Υ. The DTS $(\Upsilon, \eta_\Upsilon)$ is called a double topological subspace of $(\mathscr{X}, \eta)$ (DT-subspace, for short) [4]. Let $(\mathscr{X}, \eta)$ be a DTS and let $\mathfrak{y}$, $\mathfrak{h} \in D(\mathscr{X})$: $\mathfrak{y}$, $\mathfrak{h}$ are said to be separated double sets (separated D-sets, for short) if cl η $(\mathfrak{y}) \cap \mathfrak{h} = \emptyset$ and cl η $(\mathfrak{h}) \cap \mathfrak{y} = \emptyset$ [3]. Let $(\mathscr{X}, \eta)$ be a DTS, and let be a nonempty subset of $\mathscr{X}$. If there exist two nonempty separated D-sets $\mathfrak{y}$, $\mathfrak{h} \in D(\mathscr{X})$ such that $\mathfrak{y} \cup \mathfrak{h} = \mathsf{N}$, then the D-sets $\mathfrak{y}$ and $\mathfrak{h}$ form a D-separation of N and it is said to be double disconnected set (D-disconnected set, for short). Otherwise, N is said to be double connected set (d-connected set, for short) [3]. The DTS $(\mathscr{X}, \Pi)$ is said to be (1) C_5- disconnected, if $(\mathscr{X}, \Pi)$ has a proper open and closed D-set in Π. (2) C_5-connected, if $(\mathscr{X}, \Pi)$ is not C_5-disconnected [13]. An intuitionistic fuzzy special topological space $(\mathscr{X}, \Pi)$ is said to be strongly connected if there exist nonempty IFSC, SS $\mathfrak{y}$, and $\mathfrak{h}$ in $\mathscr{X}$ such that $\mathfrak{h} \cap \mathfrak{y} = \emptyset$ [13]. Let $(\mathscr{X}, \Pi)$ be a DTS and $\mathsf{y} \subseteq \mathscr{X}$ with $\Pi_h \in (\tilde{\mathsf{y}})$. The double component of y with respect to Π_h is the maximal of all D-connected subsets of (y, Π) containing the D-point Π_h and denoted by $C(\tilde{\mathsf{y}}, \Pi_h)$, i.e., $C(\mathsf{y}, \Pi_h). = \cup\{\tilde{\mathsf{Z}} \subseteq \tilde{\mathsf{y}}: \Pi_h \in \tilde{\mathsf{Z}}, \tilde{\mathsf{Z}}$ is a D-connected set$\}$ [12].

3. Double-Connected Intuitionistic Space in DITS

In this section, we define new kinds of x is called Double connected I space, separated Double I sets, strongly Double connected, Double CO connected I space, and Double I component in Double intuitionistic topological spaces, and joined to other kinds of sets that are defined in this work.

We start this section by the following definitions:

Definition 1. Let x be a nonempty set.

(1) A Double intuitionistic set (Double I set, for short) is an ordered pair $(\mathscr{Q}) = (\langle x.\mathscr{Q}_1.\mathscr{Q}_2\rangle.\langle x.\mathscr{D}_1.\mathscr{D}_2\rangle) \in \mathbf{p}(x) \times \mathbf{p}(x)$ s.t $\boldsymbol{\mathscr{Q}} \subseteq \boldsymbol{\mathscr{D}}$.
(2) DI $(x) = \{(Q, D) \in p(x) \times p(x), Q \subseteq D\}$ is the family of all Double I sets on x.
(3) The Double I set $(\langle x.x.\emptyset\rangle, \langle x.x.\emptyset\rangle = (\tilde{x}, \tilde{x})$ is called the universal Double I set, and the Double I set $(\tilde{\emptyset}, \tilde{\emptyset}) = ((\langle x.\emptyset.x\rangle.\langle x.\emptyset.x\rangle)$ is called the empty Double I set.
(4) Let (Q, D), $(C, G) \in$ Double I (x): 1) $(Q, D)^c = (\mathscr{D}^c, \mathscr{Q}^c) = ((\langle x.\mathscr{D}_1.\mathscr{D}_2\rangle^c.\langle x.\mathscr{Q}_1.\mathscr{Q}_2\rangle^c) = (\langle x.\mathscr{D}_2.\mathscr{D}_1\rangle.\langle x.\mathscr{Q}_2.\mathscr{Q}_1\rangle).2((\mathscr{Q}.\mathscr{D})/(\mathscr{Q}.\mathscr{D}))(\boldsymbol{\mathscr{C}},) = ((\mathscr{Q}/\mathscr{G}), (\mathscr{D}/\mathscr{C}) = ((\langle x.\mathscr{Q}_1.\mathscr{Q}_2\rangle.\ \langle x.\mathscr{D}_1.\mathscr{D}_2\rangle)/\ (\langle x.\mathscr{C}_1.\mathscr{C}_2\rangle.\langle x.\mathscr{G}_1.\mathscr{G}_2\rangle)) = ((\langle x.\mathscr{Q}_1.\mathscr{Q}_2\rangle/\langle x.\mathscr{G}_1.\mathscr{G}_2\rangle), (\langle x.\mathscr{D}_1.\mathscr{D}_2\rangle.\langle x.\mathscr{C}_1.\mathscr{C}_2\rangle)$

Each element of ω is called a DIOS in x. The complement of DIOS is called DICS.

Now, we want to introduce the important theorem to construct the Double intuitionistic topological spaces.

Theorem 1. *Let $x \neq \emptyset$. The family T of all a Double intuitionistic open sets in x is Double intuitionistic topological spaces (DITS).*

Proof
(1) Let $(x.T)$ be intuitionistic topological spaces (ITS), then $\tilde{\emptyset} = \langle x.\emptyset.x\rangle$, $\tilde{x} = \langle x.x.\emptyset\rangle \in \text{IT} \longrightarrow (\tilde{\emptyset}, \tilde{\emptyset}), (\tilde{x}, \tilde{x}) \in$ DITS
(2) Let $(\tau.\sigma), (\phi.\varepsilon) \in \text{DIT} \longrightarrow \tau, \sigma, \varphi, \varepsilon \in$ IT. Since IT is an intuitionistic topology; then, $\tau \cap \in$ IT and $\varphi \cap \varepsilon \in$ IT. Now, let $\mathscr{K} = (\tau.\sigma)$ and $\mathscr{W} = (\phi.\epsilon) \longrightarrow (\mathscr{K}.\mathscr{W}) = ((\tau, \phi), (\sigma.\epsilon)) \in$ DITS.

(3) Let $(\theta_{\mathfrak{r}}.\phi_{\mathfrak{r}})$ be a family of IS and $r \in R$ and $(\theta_{\mathfrak{r}}.\phi_{\mathfrak{r}}) \in$ DIT $\longrightarrow \theta_{\mathfrak{r}}.\phi_{\mathfrak{r}} \in$ ITS, since IT is intuitionistic topology, then $\cup_{\mathfrak{r}\in R}\Theta_{\mathfrak{r}} \in$ IT and $\cup_{\mathfrak{r}\in R}\phi_{\mathfrak{r}} \in$ IT. Thus, $\cup_{\mathfrak{r}\in R}(\Theta_{\mathfrak{r}}.\phi_{\mathfrak{r}}) \in$ DIT. Therefore, DITS is Double intuitionistic topological spaces. □

Definition 2. Let $\mathscr{x}$ nonempty set, $(\tilde{\mathfrak{b}}, \tilde{\mathfrak{b}}) \in \mathscr{x}a$ fixed element in $\mathscr{x}$, and let $(\mathscr{Q}.\mathscr{D}) = (\langle x.\mathscr{Q}_1.\mathscr{Q}_2\rangle.\langle x.\mathscr{D}_1.\mathscr{D}_2\rangle$ be an Double intuitionistic set (DIS).The DIS $(\tilde{\mathfrak{b}}, \tilde{\mathfrak{b}})$ defined by $(\tilde{\mathfrak{b}}, \tilde{\mathfrak{b}}) = (\langle x.\{\mathfrak{b}\}.\{\mathfrak{b}\}^c\rangle.\langle x.\{\mathfrak{b}\}.\{\mathfrak{b}\}^c\rangle)$ is called an Double intuitionistic point (DI$\mathfrak{b}$ for short) in $\mathscr{X}$. The DIS $(\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}})$is said to be contained in $(\mathscr{Q}.\mathscr{D})$, if and only if $(\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}) \in (\mathscr{Q}_1.\mathscr{D}_1)$.

Definition 3. Let $(\mathscr{x}, \omega)$ be a DITS, let $(\mathscr{L}, \text{Ꞙ})$, $(\mathscr{I}, \Omega) \in$ DI $(\mathscr{x})$; then, $(\mathscr{L}, \text{Ꞙ})$, $(\mathscr{I}, \Omega)$ are said to be separated Double intuitionistic sets (separated Double I sets, for short) if cl $(\mathscr{L}, \text{Ꞙ}) \cap (\mathscr{I}, \Omega) = (\tilde{\varnothing}, \tilde{\varnothing})$; and $(\mathscr{L}, \text{Ꞙ}) \cap$ cl$(\mathscr{I}, \Omega) = (\tilde{\varnothing}, \tilde{\varnothing})$. Or (cl $(\mathscr{L}, \text{Ꞙ}) \cap (\mathscr{I}, \Omega)) \cup ((\mathscr{L}, \text{Ꞙ}) \cap$ cl $(\mathscr{I}, \Omega)) = (\tilde{\varnothing}, \tilde{\varnothing})$.

Proposition 1. *Let $(\mathscr{x}, \omega)$ be a DITS and let $(\mathscr{L}, \text{Ꞙ})$, $(\mathscr{I}, \Omega) \in DI$ $(\mathscr{x})$, if $(\mathscr{L}, \text{Ꞙ})$, $(\mathscr{I}, \Omega)$ are separated Double I sets, then $(\mathscr{L}, \text{Ꞙ}) \cap (\mathscr{I}, \Omega) = (\tilde{\varnothing}, \tilde{\varnothing})$.*

Proof. Let $(\mathscr{L}, \text{Ꞙ})$, $(\mathscr{I}, \Omega)$ are separated Double I sets. By definition if cl $(\mathscr{L}, \text{Ꞙ}) \cap (\mathscr{I}, \Omega) = (\tilde{\varnothing}, \tilde{\varnothing})$ and $(\mathscr{L}, \text{Ꞙ}) \cap$ cl $(\mathscr{I}, \Omega) = (\tilde{\varnothing}, \tilde{\varnothing})$. But $(\mathscr{L}, \text{Ꞙ}) \subseteq$ cl $(\mathscr{L}, \text{Ꞙ})$.So, $(\mathscr{L}, \text{Ꞙ}) \cap (\mathscr{I}, \Omega) = (\tilde{\varnothing}, \tilde{\varnothing})$.

The following example shows that the converse is not true: □

Example 1. Let $\mathscr{x} = \{\gamma, \beta, \varepsilon, \Lambda\}$: $\omega = \{(\tilde{\varnothing}, \tilde{\varnothing}), (\tilde{\mathscr{X}}, \tilde{\mathscr{X}}), (\mathscr{x}_1.\mathscr{x}_2), (\mathscr{x}_3.\mathscr{x}_2), (\mathscr{x}_4.\mathscr{x}_2), (\mathscr{x}_4.\mathscr{x}_5), (\mathscr{x}_4.\mathscr{x}_3), (\mathscr{x}_2.\mathscr{x}_2)\}$, where $(\mathscr{x}_1.\mathscr{x}_2) = (\langle x.\{\varepsilon\}.\{\beta.\Lambda\}\rangle, \langle x.\{\gamma.\beta.\varepsilon\}.\{\Lambda\}\rangle)$, $(\mathscr{x}_3.\mathscr{x}_2) = (\langle x.\{\gamma.\beta\}.\{\varepsilon.\Lambda\}\rangle.\langle x.\{\gamma.\beta.\varepsilon\}.\{\Lambda\}\rangle$, $(\mathscr{x}_4.\mathscr{x}_2) = (\langle x.\varnothing.\{\beta.\varepsilon.\Lambda\}\rangle.\langle x.\{\gamma.\beta.\varepsilon\}.\{\Lambda\}\rangle).(\mathscr{x}_4.\mathscr{x}_5) = (\langle x.\varnothing.\{\beta.\varepsilon.\Lambda\}\rangle.\ \langle x.\{\beta\}\{\varepsilon.\Lambda\}\rangle)$, $(\mathscr{x}_4.\mathscr{x}_3) = (\langle x.\varnothing.\{\beta.\varepsilon.\Lambda\}\rangle, \langle x.\{\gamma.\beta\}.\{\varepsilon.\Lambda\}\rangle)$ and $(\mathscr{x}_2.\mathscr{x}_2) = (\langle x.\{\gamma.\beta.\varepsilon\}.\{\Lambda\}\rangle, \langle x.\{\gamma.\beta.\varepsilon\}.\{\Lambda\}\rangle)$. $\omega^c = \{(\tilde{\varnothing}, \tilde{\varnothing}), (\tilde{\mathscr{X}}, \tilde{\mathscr{X}}), (\mathscr{x}_2^c.\mathscr{x}_1^c), (\mathscr{x}_2^c.\mathscr{x}_3^c), (\mathscr{x}_2^c.\mathscr{x}_4^c), (\mathscr{x}_5^c.\mathscr{x}_4^c), (\mathscr{x}_3^c.\mathscr{x}_4^c), (\mathscr{x}_2^c.\mathscr{x}_2^c)\}$, where$(\mathscr{x}_2^c.\mathscr{x}_1^c) = (\langle x.\{\Lambda\}.\{\gamma.\beta.\varepsilon\}\rangle, \langle x.\{\beta.\Lambda\}.\{\varepsilon\}\rangle)$, $(\mathscr{x}_2^c.\mathscr{x}_3^c) = (\langle x.\{\Lambda\}.\{\gamma.\beta.\varepsilon\}\rangle.\langle x.\{\varepsilon.\Lambda\}.\{\gamma.\beta\}\rangle)$. $(\mathscr{x}_2^c.\mathscr{x}_4^c) = (\langle x.\{\Lambda\}.\{\gamma.\beta.\varepsilon\}\rangle.\ \langle x.\{\beta.\varepsilon.\Lambda\}.\varnothing\rangle).(\mathscr{x}_5^c.\mathscr{x}_4^c) = (\langle x.\{\varepsilon.\Lambda\}.\{\beta\}\rangle.\langle x.\{\beta.\varepsilon.\Lambda\}.\varnothing\rangle)$, $(\mathscr{x}_3^c.\mathscr{x}_4^c) = (\langle x.\{\varepsilon.\Lambda\}.\ \{\gamma.\beta\}\rangle, \langle x.\{\beta.\varepsilon.\Lambda\}.\varnothing\rangle)$ and $(\mathscr{x}_2^c.\mathscr{x}_2^c) = (\langle x.\{\Lambda\}.\{\gamma.\beta.\varepsilon\}\rangle, \langle x.\{\Lambda\}.\{\gamma.\beta.\varepsilon\}\rangle)$. Let $(\mathscr{x}_6.\mathscr{x}_5^c) = (\langle x.\{\Lambda\}.\{\gamma.\beta\}\rangle.\langle x.\{\varepsilon.\Lambda\}.\{\beta\}\rangle)$ and $(\mathscr{x}_7.\mathscr{x}_7) = (\langle x.\{\beta\}.\{\gamma.\varepsilon.\Lambda\}\rangle.\langle x.\{\beta\}.\{\gamma.\varepsilon.\Lambda\}\rangle)$*so cl* $(\mathscr{x}_6.\mathscr{x}_5^c) = (\mathscr{x}_3^c.\mathscr{x}_4^c)$. Hence $(\mathscr{x}_3^c.\mathscr{x}_4^c) \cap (\mathscr{x}_7.\mathscr{x}_7) = (\tilde{\varnothing}, \tilde{\varnothing})$, but cl $(\mathscr{x}_6.\mathscr{x}_5^c) \cap (\mathscr{x}_7.\mathscr{x}_7) \neq (\tilde{\varnothing}, \tilde{\varnothing})$. Therefore, $(\mathscr{x}_6.\mathscr{x}_5^c)$, $(\mathscr{x}_7.\mathscr{x}_7)$ is not separated Double I sets.

Proposition 2. *Any two Double I closed (Double I open) subset say (v, u), (s, l) of DITS, $(\mathscr{x}, \omega)$ are separated Double I sets if and only if they are two disjoint.*

Proof. Since separated Double I sets are already two disjoint; then, we only need to show that the Double I sets (v, u), (s, l) are separated Double I sets, they are Double I closed and two disjoint, $(v, u) \cap (s, l) = (\tilde{\varnothing}, \tilde{\varnothing})$. Since cl $(v, u) = (v, u)$ and cl $(s, l) = (s, l) \longrightarrow$ cl $(v, u) \cap (s, l) =$ and $(v, u) \cap$ cl $(s, l) = (\tilde{\varnothing}, \tilde{\varnothing}) \longrightarrow (v, u)$, (s, l) are separated Double I sets.

Now, take (v, u), (s, l) are Double I open and two disjoint $\longrightarrow (v, u) \cap (s, l) = (\tilde{\varnothing}, \tilde{\varnothing})$. Since (v, u) and (s, l) are Double I open $\longrightarrow (v, u)^c$ and $(s, l)^c$ are Double I closed $\longrightarrow$ cl $(v, u)^c = (v, u)^c$, cl $(s, l)^c = (s, l)^c$ $[(v, u) \subseteq (s, l)^c$, $(s, l) \subseteq (v, u)^c] \longrightarrow$ cl $(v, u) \subseteq$ cl $(s, l)^c = (s, l)^c \longrightarrow$ cl $(v, u) \subseteq (s, l)^c$ and cl $(s, l) \subseteq$ cl $(v, u)^c = (v, u)^c \longrightarrow$ cl $(s, l) \subseteq (v, u)^c$. Hence, cl $(v, u) \cap (s, l) = (\tilde{\varnothing}, \tilde{\varnothing})$ and $(v, u) \cap$ cl $(s, l) = (\tilde{\varnothing}, \tilde{\varnothing})$. So, (v, u), (s, l) are separated Double I sets.

The following definition of Double-connected intuitionistic sets in DITS: □

Definition 4. Let $(\mathscr{x}, \omega)$ be a DITS; let (k, Z) be a nonempty subset of $\mathscr{X}$. If there exist two nonempty separated Double I sets $(\mathscr{L}, \text{Ꞙ})$, $(\mathscr{I}, \Omega) \in$ DI $(\mathscr{X})$ such that $(\mathscr{L}, \text{Ꞙ}) \cup (\mathscr{I}, \Omega) = (k, Z)$, then the Double I sets $(\mathscr{L}, \text{Ꞙ})$ and $(\mathscr{I}, \Omega)$ form a Double separation of (k, Z) and it is said to be Double disconnected intuitionistic sets (Double disconnected I sets, for short) Otherwise, (k, Z) is said to be Double-connected intuitionistic sets (Double connected I sets, for short).

The following two examples one of them is Double connected I sets, and the other is not:

Example 2. Let $\mathscr{X} = \{\ell.\mathscr{q}\}$; $\omega = \{(\tilde{\varnothing}, \tilde{\varnothing}), (\tilde{\mathscr{X}}, \tilde{\mathscr{X}}), (\tilde{\varnothing}.\xi_1), (\tilde{\varnothing}.\xi_2), (\tilde{\varnothing}.\xi_3), (\tilde{\varnothing}.\xi_3^c), (\tilde{\varnothing}.\xi_1^c), (\tilde{\varnothing}.\xi_2^c), (\tilde{\varnothing}.\xi_4), (\xi_1.\tilde{\mathscr{X}}), (\xi_3.\tilde{\mathscr{X}}), (\xi_2.\tilde{\mathscr{X}}), (\xi_3^c.\tilde{\mathscr{X}}), (\xi_1^c.\tilde{\mathscr{X}}), (\xi_2^c.\tilde{\mathscr{X}}), (\xi_4.\tilde{\mathscr{X}}), (\xi_3, \xi_1).(\xi_1, \xi_1).(\xi_3, \xi_3).(\xi_2, \xi_2).\ (\xi_3^c.\xi_3^c).(\xi_1^c.\xi_1^c).(\xi_2^c.\xi_2^c).\ (\xi_4, \xi_4).(\xi_3^c, \xi_2).(\xi_1^c, \xi_1).(\xi_1^c, \xi_2).(\xi_1^c.\ \xi_3^c).(\xi_1^c, \xi_4).(\xi_2^c, \xi_1).(\xi_2^c, \xi_3).\ (\xi_2^c, \xi_2).(\xi_2^c, \xi_4).(\xi_4, \xi_1).(\xi_4, \xi_2)\ .(\tilde{\varnothing}.\tilde{\mathscr{X}})\}$ where.$(\tilde{\varnothing}.\xi_1) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{\ell\}.\varnothing\rangle)$. $(\tilde{\varnothing}.\xi_2) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{\mathscr{q}\}.\varnothing\rangle)$. $(\tilde{\varnothing}.\xi_3) = (\langle x.\varnothing.\mathscr{X}\rangle.\ \langle x.\{\ell\}.\{\mathscr{q}\}\rangle)$. $(\tilde{\varnothing}.\xi_3^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\ \langle x.\{\mathscr{q}\}.\{\ell\}\rangle)$. $(\tilde{\varnothing}.\xi_1^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\varnothing.\{\ell\}\rangle)$. $(\tilde{\varnothing}.\xi_2^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\varnothing.\{\mathscr{q}\}\rangle).(\tilde{\varnothing}.\xi_4) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\varnothing.\ \varnothing\rangle).(\xi_1.\tilde{\mathscr{X}}) = (\langle x.\{\ell\}.\varnothing\rangle.\langle x.\mathscr{X}.\varnothing\rangle).(\xi_3.\tilde{\mathscr{X}}) = (\langle x.\{\ell\}.\ \{\mathscr{q}\}\rangle.\langle x.\mathscr{X}.\varnothing\rangle)$, $(\xi_2.\tilde{\mathscr{X}}) = (\langle x.\{\mathscr{q}\}.\varnothing\rangle.\langle x.\mathscr{X}.\varnothing\rangle)$, $(\xi_3^c.\tilde{\mathscr{X}}) = (\langle x.\{\mathscr{q}\}.\{\ell\}\rangle.\langle x.\mathscr{X}.\varnothing\rangle).(\xi_1^c.\tilde{\mathscr{X}}) = (\langle x.\varnothing.\{\ell\}\rangle.\langle x.\ \mathscr{X}.\varnothing\rangle)$, $(\xi_2^c\tilde{\mathscr{X}}) = (\langle x.\varnothing.\{\mathscr{q}\}\rangle.\langle x.\mathscr{X}.\varnothing\rangle)$, $(\xi_4.\tilde{\mathscr{X}}) = (\langle x.\varnothing.\varnothing\rangle.\langle x.\mathscr{X}.\varnothing\rangle).(\xi_3, \xi_1) = (\langle x.\{\ell\}.\{\mathscr{q}\}\rangle.\langle x.\{\ell\}.\varnothing\rangle).(\xi_1, \xi_1) = (\langle x.\{\ell\}.\varnothing\rangle.\langle x.\ \{\ell\}.\varnothing\rangle)$, $(\xi_3, \xi_3) = (\langle x.\{\ell\}.\{\mathscr{q}\}\rangle.\langle x.\{\ell\}.\{\mathscr{q}\}\rangle)$, $(\xi_2, \xi_2) = (\langle x.\{\mathscr{q}\}.\varnothing\rangle.\langle x.\{\mathscr{q}\}.\varnothing\rangle).(\xi_3^c.\xi_3^c) = (\langle x.\{\mathscr{q}\}.\{\ell\}\rangle.\ \langle x.\{\mathscr{q}\}.\{\ell\}\rangle)$, $(\xi_1^c.\xi_1^c) = (\langle x.\varnothing.\{\ell\}\rangle.\langle x.\varnothing.\{\ell\}\rangle)$, $(\xi_2^c, \xi_2^c) = (\langle x.\varnothing.\{\mathscr{q}\}\rangle.\langle x.\varnothing.\{\mathscr{q}\}\rangle)$, $(\xi_4, \xi_4) = (\langle x.\varnothing.\varnothing\rangle.\langle x.\varnothing.\varnothing\rangle).(\xi_3^c, \xi_2) = (\langle x.\{\mathscr{q}\}.\ \{\ell\}\rangle.\langle x.\{\mathscr{q}\}.\varnothing\rangle).(\xi_1^c, \xi_1) = (\langle x.\varnothing.\{\ell\}\rangle.\langle x.\{\ell\}.\varnothing\rangle)$. $(\xi_1^c,\ \xi_2) = (\langle x.\varnothing.\{\ell\}\rangle.\langle x.\{\mathscr{q}\}.\varnothing\rangle)$, $(\xi_1^c.\xi_3^c) = (\langle x.\varnothing.\{\ell\}\rangle.\langle x.\{\mathscr{q}\}.\{\ell\}\rangle)(\xi_1^c, \xi_4) = (\langle x.\varnothing.\{\ell\}\rangle.\langle x.\varnothing.\varnothing\rangle)$, $(\xi_2^c, \xi_1) = (\langle x.\varnothing.\{\mathscr{q}\}\rangle.\langle x.\{\ell\}.\varnothing\rangle).(\xi_2^c, \xi_3) = (\langle x.\varnothing.\{\mathscr{q}\}\rangle.\langle x.\{\ell\}.\ \{\mathscr{q}\}\rangle)$, $(\xi_2^c, \xi_2) = (\langle x.\varnothing.\{\mathscr{q}\}\rangle.\langle x.\{\mathscr{q}\}.\varnothing\rangle)$, $(\xi_2^c, \xi_4) = (\langle x.\varnothing.\{\mathscr{q}\}\rangle.\langle x.\varnothing.\varnothing\rangle)$, $(\xi_4\xi_1) = (\langle x.\varnothing.\varnothing\rangle.\langle x.\{\ell\}.\varnothing\rangle).(\xi_4, \xi_2) = (\langle x.\varnothing.\varnothing\rangle.\langle x.\{\mathscr{q}\}.\varnothing\rangle)$, and$(\tilde{\varnothing}.\tilde{\mathscr{X}}) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\mathscr{X}.\varnothing\rangle)$. Let $(\xi_3, \xi_3) =$ cl $(\xi_3, \xi_3) \cap (\tilde{\varnothing}.\xi_1^c) = (\tilde{\varnothing}, \tilde{\varnothing})$ and $(\xi_3, \xi_3) \cap$ *cl*$(\tilde{\varnothing}.\xi_1^c) = (\tilde{\varnothing}.\xi_1^c) \longrightarrow (\xi_3, \xi_3) \cap (\tilde{\varnothing}.\xi_1^c) = (\tilde{\varnothing}, \tilde{\varnothing})$, such that $(\xi_3, \xi_3) \cup (\tilde{\varnothing}.\xi_1^c) = (\xi_3, \xi_1)$. Therefore, $(\xi_3, \xi_3).(\tilde{\varnothing}.\xi_1^c)$ are not Double connected I sets.

Example 3. Let $\mathscr{X} = \{\text{ɕ}, \text{ȡ}, \text{ɤ}\}$; $\omega = \{(\tilde{\varnothing}, \tilde{\varnothing}), (\tilde{\mathscr{X}}, \tilde{\mathscr{X}}), (\Theta_1.\Theta_2), (\Theta_2.\Theta_3), (\Theta_4.\Theta_5), (\tilde{\varnothing}.\Theta_4)\}$ where $(\Theta_1.\Theta_2) = (\langle x.\{\text{ɤ}\}.\{\text{ɕ}.\text{ȡ}\}\rangle, \langle x.\{\text{ɕ}.\text{ɤ}\}.\{\text{ȡ}\}\rangle)$, $(\Theta_2.\Theta_3) = (\langle x.\{\text{ɕ}.\text{ɤ}\}.\{\text{ȡ}\}\rangle.\ \langle x.\{\text{ɕ}.\text{ɤ}\}.\varnothing\rangle).($

$\Theta_4.\Theta_5) = (\langle x.\{\mathfrak{E}\}.\{\mathfrak{d}.\mathfrak{r}\}\rangle.\langle x.\{\mathfrak{E}\}.\{\mathfrak{r}\}\rangle)$, and $(\widetilde{\varnothing}.\Theta_4) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{\mathfrak{E}\}.\{\mathfrak{d}.\mathfrak{r}\}\rangle)$. $\omega^c = \{(\widetilde{\varnothing},\widetilde{\varnothing}),(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}), (O_2^c.\Theta_1^c).(O_3^c.\Theta_2^c).(O_5^c.\Theta_4^c).(O_4^c.\widetilde{\mathscr{X}})\}$ where $(O_2^c.\Theta_1^c) = (\langle x.\{\mathfrak{d}\}.\{\mathfrak{E}.\mathfrak{r}\}\rangle,\langle x.\{\mathfrak{E}.\mathfrak{d}\}.\{\mathfrak{r}\}\rangle)$, $(O_3^c.\Theta_2^c) = (\langle x.\varnothing.\{\mathfrak{E}.\mathfrak{r}\}\rangle, \langle x.\{\mathfrak{d}\}.\{\mathfrak{E}.\mathfrak{r}\}\rangle)$, $(O_5^c.\Theta_4^c) = (\langle x.\{\mathfrak{r}\}.\{\mathfrak{E}\}\rangle,\langle x.\{\mathfrak{d}.\mathfrak{r}\}.\{\mathfrak{E}\}\rangle)$, and $(O_4^c.\widetilde{\mathscr{X}}) = (\langle x.\{\mathfrak{d}.\mathfrak{r}\}.\{\mathfrak{E}\}\rangle.< x,\ \mathscr{X},\ \varnothing >)$. Let $(\Theta_1.\Theta_2)$ and $(\Theta_2.\Theta_3)$. Then, $(O_4^c.\widetilde{\mathscr{X}}) = \mathrm{cl}\ (\Theta_1.\Theta_2)\cap(\Theta_2.\Theta_3) = (\Theta_1.\Theta_3)$ and $(\Theta_1.\Theta_2)\cap cl(\Theta_2.\Theta_3) = (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})) \longrightarrow (\Theta_1.\Theta_2)\cap(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) = (\Theta_1.\Theta_2)$ and $(\Theta_1.\Theta_2)\cup(\Theta_2.\Theta_3) = (\Theta_2.\Theta_3)$. Hence, $(\Theta_1\Theta_2).(\Theta_2.\Theta_3)$ are Double connected *I* sets.

Definition 5. Let (x, ω) be a DITS; If there exist two nonempty separated Double *I* sets $(\mathscr{L}, \mathfrak{F})$, $(\mathscr{I}, \Omega) \in \mathrm{DI}\ (x)$ such that $(\mathscr{L}, \mathfrak{F}) \cup (\mathscr{I},\Omega)) = (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})$; then, $(\mathscr{L}, \mathfrak{F})$ and $(\mathscr{I}, \Omega)$ are said to be Double *I* division for DITS (x, ω). (x, ω) is said to be Double disconnected intuitionistic space (Double disconnected *I* space, for short), if (x, ω) has a Double *I* division. Otherwise, (x, ω) is said to be Double-connected intuitionistic space (Double connected *I* space, for short).

Example 4. Let $x = \{40, 41, 42\}$; $\omega = \{(\widetilde{\varnothing},\widetilde{\varnothing}), (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}), (\sigma_1.\sigma_1), (\sigma_2.\sigma_3), (\sigma_4.\sigma_5), (\sigma_1^c.\sigma_1^c)\}$ where $(\sigma_1.\sigma_1) = (\langle x.\{40.42\}.\{41\}\rangle, \langle x.\{40.42\}.\{41\}\rangle)$, $(\sigma_2.\sigma_3) = (\langle x.\{41\}.\{42\}\rangle.\langle x.\{40.41\}.\varnothing\rangle).(\sigma_4.\sigma_5) = (\langle x.\varnothing.\{41.42\}\rangle.\langle x.\{40\}.\{41\}\rangle)$ and $(\sigma_1^c.\sigma_1^c) = (\langle \mathrm{x}.\{41\}.\{40.42\}>.\langle \mathrm{x}.\{41\}.\{40.42\}\rangle)$. $\phi^c = \{(\widetilde{\varnothing},\widetilde{\varnothing}), (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}), (\sigma_1^c.\sigma_1^c).(\sigma_3^c.\sigma_2^c).(\sigma_5^c.\sigma_4^c).(\sigma_1.\sigma_1)\}$ where $(\sigma_1^c.\sigma_1^c) =(\langle x\{41\}\{4042\}\rangle,\langle x.\{41\}.\{40.42\}\rangle)$, $(\sigma_3^c.\sigma_2^c) =(\langle x.\varnothing.\{40.41\}\rangle,\langle x.\{42\}.\{41\}\rangle)$, $(\sigma_5^c.\sigma_4^c) = (\langle x.\{41\}.\{40\}\rangle, \langle x.\{41.42\}.\varnothing\rangle)$ and $(\sigma_1.\sigma_1) = (\langle x.\{40.42\}.\{41\}\rangle, \langle x.\{40.42\}.\{41\}\rangle)$. Let $(\sigma_1.\sigma_1) = \mathrm{cl}\ (\sigma_4.\sigma_5)\cap(\sigma_4.\sigma_5) = (\sigma_4.\sigma_5)$ and $(\sigma_4.\sigma_5)\cap \mathrm{cl}(\sigma_4.\sigma_5) = (\sigma_1.\sigma_1)$ and $(\sigma_4.\sigma_5)\cup(\sigma_4.\sigma_5) \neq (\mathscr{X},\mathscr{X})$. So, $(\sigma_4.\sigma_5)$ is Double connected *I* spaces.

Remark 1

(1) (x, IN) is Double connected *I* space always since the only Double *I* open sets are $(\widetilde{\varnothing},\widetilde{\varnothing}), (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})$, and this *I* sets not make the *I* space is Double disconnected.

(2) $(\mathscr{X}, \mathrm{dis})$ is Double disconnected *I* space if x contains more than two element, since there exist $(\mathscr{L},\mathfrak{F})$, $\widetilde{\varnothing} \neq (\mathscr{L},\mathfrak{F}) \subseteq x \longrightarrow (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) = (\mathscr{L},\mathfrak{F}) \cup (\mathscr{L},\mathfrak{F})^c$, $[(\mathscr{L},\mathfrak{F}),(\mathscr{L},\mathfrak{F})^c] \in \mathrm{dis}\ (\mathscr{L},\mathfrak{F}) \cap (\mathscr{L},\mathfrak{F})^c = (\widetilde{\varnothing},\widetilde{\varnothing})$, $(\mathscr{L},\mathfrak{F}) \neq (\widetilde{\varnothing},\widetilde{\varnothing})$, and $(\mathscr{L},\mathfrak{F})^c \neq (\widetilde{\varnothing},\widetilde{\varnothing})$.

Theorem 2. *Let (x,ω) is Double connected I space, if f $(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})$ cannot be written as the union of two disjoint nonempty Double I closed sets.*

Proof. $(\Longrightarrow)$ Suppose that (x,ω) is Double connected to prove that $(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) \neq (\Omega, \Upsilon) \cup (\Upsilon, \Pi)$, (Ω, Υ) and (Υ, Π) are Double *I* closed set, $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$. Let $(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) = (\Omega, \Upsilon) \cup (\Upsilon, \Pi)$, (Ω, Υ) and (Υ, Π) are Double *I* closed set, $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing}) \longrightarrow \mathrm{cl}\ (\Omega, \Upsilon) \subseteq (\Upsilon, \Pi)^c = (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})\backslash(\Upsilon, \Pi) \subseteq (\Upsilon, \Pi)$, but $(\Omega, \Upsilon) \subseteq \mathrm{cl}\ (\Omega, \Upsilon)$, so that $(\Omega, \Upsilon) = (\Upsilon, \Pi)^c \wedge (\Upsilon, \Pi) = (\Omega, \Upsilon)^c \longrightarrow (\Omega, \Upsilon) \in \omega \wedge (\Upsilon, \Pi) \in \omega$ (since $(\Omega, \Upsilon) = (\Upsilon, \Pi)^c \wedge (\Upsilon, \Pi)$ is Double *I* closed set and $(\Upsilon, \Pi) = (\Omega, \Upsilon)^c \wedge (\Omega, \Upsilon)$ is Double *I* closed set) $\longrightarrow (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) = (\Omega, \Upsilon) \cup (\Upsilon, \Pi)$ $[(\Omega, \Upsilon), (\Upsilon, \Pi) \in \omega]$, $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing}) \longrightarrow (x, \omega)$ is Double disconnected, which a contradiction (since (x, ω) is Double connected) $\longrightarrow (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) \neq (\Omega, \Upsilon) \cup (\Upsilon, \Pi)$, (Ω, Υ) and (Υ, Π) are Double *I* closed set, $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$. Hence, $(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})$ cannot be written as the union of two disjoint nonempty Double *I* closed sets $(\Leftarrow)$. Suppose that $(\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) \neq (\Omega, \Upsilon) \cup (\Upsilon, \Pi)$, (Ω, Υ), and (Υ, Π) are Double *I* closed set, $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ to prove that (x,ω) is Double connected *I* space. Let (x,ω) is Double disconnected *I* space $\longrightarrow (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}}) = (\Omega, \Upsilon) \cup (\Upsilon, \Pi)$ $[(\Omega, \Upsilon), (\Upsilon, \Pi) \in \omega]$, $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing}) \longrightarrow (\Omega, \Upsilon) = (\Upsilon, \Pi)^c \wedge (\Upsilon, \Pi) = (\Omega, \Upsilon)^c \longrightarrow (\Omega, \Upsilon)$ and $(\Upsilon, \Pi)$are Double *I* closed set, which is a contradiction. Since the complement of every one of them is Double *I* open set and this contradiction with hypotheses. Therefore, $(x,\ \Psi)$ is Double connected *I* space. □

Theorem 3. *Let (x, ω) be a DITS and let $\yen$ be a nonempty subset of x. Then, if (Ω, Υ) and (Υ, Π) are Double I closed sets in $\yen$; then, (Ω, Υ) and (Υ, Π) are separated Double I sets in $\yen$ if and only if (Ω, Υ) and (Υ, Π) are separated Double I sets in x.*

Proof. $\mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\yen},\widetilde{\yen}) \cap \mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi)$; $(\Upsilon, \Pi) \subseteq (\widetilde{\yen},\widetilde{\yen}) = (\widetilde{\yen},\widetilde{\yen}) \cap (\Upsilon, \Pi) \cap \mathrm{cl}\ (\Omega, \Upsilon) = (\Upsilon, \Pi) \cap (\widetilde{\yen}.\widetilde{\yen}) \cap \mathrm{cl}\ (\Omega, \Upsilon) = (\Upsilon, \Pi) \cap \mathrm{cl}_{\omega\yen}\ (\Omega, \Upsilon) = (\widetilde{\varnothing},\widetilde{\varnothing})$. Similarly, we have $\mathrm{cl}\ (\Upsilon, \Pi) \cap (\Omega, \Upsilon) = (\widetilde{\yen},\widetilde{\yen}) \cap \mathrm{cl}\ (\Upsilon, \Pi) \cap (\Omega, \Upsilon)$; $(\Omega, \Upsilon) \subseteq (\widetilde{\yen},\widetilde{\yen}) = (\widetilde{\yen},\widetilde{\yen}) \cap (\Omega, \Upsilon) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\Omega, \Upsilon) \cap (\widetilde{\yen},\widetilde{\yen}) \cap \mathrm{cl}\ (\Upsilon, \Pi) = (\Omega, \Upsilon) \cap \mathrm{cl}_{\omega\yen}\ (\Upsilon, \Pi) = (\widetilde{\varnothing},\widetilde{\varnothing})$.

Conversely, $\mathrm{cl}_{\omega\yen}\ (\Omega, \Upsilon) \cap (\Upsilon, \Pi) = (\widetilde{\yen},\widetilde{\yen}) \cap \mathrm{cl}\ (\Omega, \Upsilon) \cap (\Upsilon,\Pi) = (\widetilde{\yen},\widetilde{\yen}) \cap [\mathrm{cl}(\Omega,\Upsilon) \cap (\Upsilon, \Pi)] = (\widetilde{\yen},\widetilde{\yen}) \cap (\widetilde{\varnothing},\widetilde{\varnothing}) = (\widetilde{\varnothing},\widetilde{\varnothing})$. Also, $\mathrm{cl}_{\omega\yen}\ (\Upsilon, \Pi) \cap (\Omega, \Upsilon) = (\widetilde{\yen},\widetilde{\yen}) \cap \mathrm{cl}\ (\Upsilon, \Pi) \cap (\Omega,\Upsilon) = (\widetilde{\yen},\widetilde{\yen}) \cap [\mathrm{cl}(\Upsilon, \Pi) \cap (\Omega,\Upsilon)] = (\widetilde{\yen},\widetilde{\yen}) \cap (\widetilde{\varnothing},\widetilde{\varnothing}) = (\widetilde{\varnothing},\widetilde{\varnothing})$. □

Theorem 4. *(x,ω) is Double disconnected I space if f there exist a nonempty proper subset of x, which are both Double I open and Double I closed sets in x.*

Proof. Suppose that $(\mathfrak{l}, \mathfrak{f})$,$(\mathrm{k}, \eta)$ are a nonempty proper subset of x, which are both Double *I* open and Double *I* closed sets to prove x is Double disconnected. Let $(\mathrm{k}, \eta)^c = (\mathfrak{l}, \mathfrak{f})^c$; then, $(\mathrm{k}, \eta) \neq (\widetilde{\varnothing},\widetilde{\varnothing})$ and $(\mathfrak{l}, \mathfrak{f}) \neq (\widetilde{\varnothing},\widetilde{\varnothing})$, Moreover, $(\mathfrak{l}, \mathfrak{f}) \cup (\mathrm{k}, \eta) = (\widetilde{\mathscr{X}},\widetilde{\mathscr{X}})$ and

$$(\mathfrak{l},\mathfrak{f}) \cap (\mathrm{k},\eta) = (\widetilde{\varnothing},\widetilde{\varnothing}). \tag{1}$$

Since $(\mathfrak{l}, \mathfrak{f})$ is Double *I* closed as well as Double *I* open, then (k, η) is also Double *I* open and Double *I* closed subset of x, so $(\mathfrak{l},\mathfrak{f}) = \mathrm{cl}\ (\mathfrak{l},\mathfrak{f})$ and $(\mathrm{k},\eta) = \mathrm{cl}\ (\mathrm{k},\eta) \longrightarrow \mathrm{cl}\ (\mathfrak{l},\mathfrak{f}) \cap (\mathrm{k},\eta) = (\widetilde{\varnothing},\widetilde{\varnothing})$ (from (1)) and $\mathrm{cl}\ (\mathrm{k},\eta) \cap (\mathfrak{l},\mathfrak{f}) = (\widetilde{\varnothing},\widetilde{\varnothing})$. Hence, $\mathscr{X}$ is Double disconnected *I* spaces.

Conversely: suppose that x is Double disconnected, then there exist a nonempty subset $(\mathfrak{l},\mathfrak{f})$, (k,η) of x such that $(\mathfrak{l},\mathfrak{f})$

∪ (ƙ, η) = $(\tilde{\mathcal{X}}, \tilde{\mathcal{X}})$; (ʅ, ŧ) ≠ $(\tilde{\varnothing}, \tilde{\varnothing})$, (ƙ, η) ≠ $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ cl (ʅ, ŧ) ∩ (ƙ, η) = $(\tilde{\varnothing}, \tilde{\varnothing})$ and (ʅ, ŧ) ∩ cl (ƙ, η) = $(\tilde{\varnothing}, \tilde{\varnothing})$.Since (ʅ, ŧ) ⊆ cl (ʅ, ŧ) ⟶ cl (ʅ, ŧ) ∩ (ƙ, η) = $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ (ʅ, ŧ) ∩ (ƙ, η) = $(\tilde{\varnothing}, \tilde{\varnothing})$. Hence, (ʅ, ŧ) =(ƙ, η)^c (since (ʅ, ŧ)⊆ (ƙ, η)^c = $((\tilde{\mathcal{X}}, \tilde{\mathcal{X}})$\(ƙ, η)) = (($\tilde{\mathcal{X}}$\ η). ($\tilde{\mathcal{X}}$\ ƙ)) = (ƙ, η)^c) and (ƙ, η) ∪ (ƙ, η)^c = $(\tilde{\mathcal{X}}, \tilde{\mathcal{X}})$, (ƙ, η) ⊆ (($\tilde{\mathcal{X}}, \tilde{\mathcal{X}}$)\ (ʅ, ŧ)) = (($\tilde{\mathcal{X}}$\ŧ), ($\tilde{\mathcal{X}}$\ ʅ)) = (ʅ, ŧ)^c ⟶ (ƙ, η) = (ʅ, ŧ)^c is a proper subset of x. Now (ʅ, ŧ) ∪ cl (ƙ, η) = $(\tilde{\mathcal{X}}, \tilde{\mathcal{X}})$. Also, (ʅ, ŧ) ∩ cl (ƙ, η) = $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ (ƙ, η) = (cl (ƙ, η))^c. Similarly, cl (ʅ, ŧ) ∩ (ƙ, η) = $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ (ʅ, ŧ) = (cl (ʅ, ŧ))^c. □

Theorem 5. *Let (x, Ѡ) be a DITS and let $\mathcal{H}$ be a nonempty subset of x such that ($\mathcal{H}$, Ѡ$_{\mathcal{H}}$) is Double connected, if (Ω, ϒ) and (ϒ, Π) are separated Double I sets such that $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆ (Ω, ϒ) ∪ (ϒ, Π), then $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆ (Ω, ϒ)or $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆ (ϒ, Π).*

Proof. Since $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆ (Ω, ϒ) ∪ (ϒ, Π), we have cl (Ω, ϒ) ∩ (ϒ, Π) = $(\tilde{\varnothing}, \tilde{\varnothing})$, and (Ω, ϒ) ∩ cl (ϒ, Π) = $(\tilde{\varnothing}, \tilde{\varnothing})$, then $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ = $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ [(Ω, ϒ) ∪ (ϒ, Π)] = [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω,ϒ)] ∪ [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π)] ((by Theorem 4)) $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ) and $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π) are separated Double *I* sets of $\mathcal{H}$. Let $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ) and $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π) are nonempty ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ) ≠ $(\tilde{\varnothing}, \tilde{\varnothing})$ and $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π) ≠ $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ)] ∩ cl[$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π)] = [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ)] ∩ [cl $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ cl (ϒ, Π)] = [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ cl $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$] ∩ [(Ω, ϒ) ∩ cl (ϒ, Π)] = $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ cl $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ $(\tilde{\varnothing}, \tilde{\varnothing})$ = $(\tilde{\varnothing}, \tilde{\varnothing})$. Similarly, cl [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ)] ∩ [$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π)] = $(\tilde{\varnothing}, \tilde{\varnothing})$. *i.* e, $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ) and $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π) are separated Double *I* sets. So ($\mathcal{H}$, Ѡ$_{\mathcal{H}}$) is Double disconnected, which is a contradiction. Let $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω)= $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ = $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π) ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆(ϒ, Π). Let $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π)= $(\tilde{\varnothing}, \tilde{\varnothing})$ ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ = $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (Ω, ϒ) ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆(Ω, ϒ). Therefore, $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆(Ω, ϒ) or $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆(ϒ, Π). □

Theorem 6. *Let (x, Ѡ) be a DITS and let $\mathcal{H}$ be a nonempty subset of x such that ($\mathcal{H}$, Ѡ$_{\mathcal{H}}$) is Double connected and* 𝕽 *be a subset of x, such that $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆($\tilde{𝕽}$, $\tilde{𝕽}$)⊆cl$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$, then (𝕽, Ѡ$_{𝕽}$) is Double connected subspace of (x, Ѡ). In particular, (cl $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$, Ѡ$_{cl}$$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$) is Double connected subspace of (x, Ѡ), ϒ) ƙ, η).*

Proof. Let (𝕽, Ѡ$_{𝕽}$)is Double disconnected subspace of (x, Ѡ), then ($\tilde{𝕽}$, $\tilde{𝕽}$) has a Double separation (Ω, ϒ)and (ϒ, Π) such that cl (Ω, ϒ) ∩ (ϒ, Π) = $(\widetilde{\varnothing,\varnothing})$ and (Ω, ϒ) ∩ cl (ϒ, Π) = $(\widetilde{\varnothing,\varnothing})$ and (Ω, ϒ) ∪ (ϒ, Π) = ($\tilde{𝕽}$, $\tilde{𝕽}$), we have $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆($\tilde{𝕽}$, $\tilde{𝕽}$) ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆(Ω, ϒ) ∪ (ϒ, Π) ⟶ $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆(Ω, ϒ) or $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆ (ϒ, Π) (by Theorem 5). Let $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⊆ (Ω, ϒ) ⟶ cl$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$⊆cl (Ω, ϒ) ⟶ cl$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π)⊆cl (Ω, ϒ) ∩ (ϒ, Π) ⟶ cl$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ, Π) ⊆ $(\widetilde{\varnothing,\varnothing})$, but $(\widetilde{\varnothing,\varnothing})$ ⊆cl$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ∩ (ϒ,Π).i.e.,

$$\text{cl}(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})\bullet \cap (\Upsilon, \Pi)(\widetilde{\varnothing,\varnothing}). \quad (2)$$

Again ($\tilde{𝕽}$, $\tilde{𝕽}$) ⊆ cl $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⟶ (Ω,ϒ) ∪ (ϒ,Π) ⊆ cl $(\tilde{\mathcal{H}}, \tilde{\mathcal{H}})$ ⟶ (ϒ,Π) ⊆ cl.

$$(\tilde{\mathcal{H}}, \tilde{\mathcal{H}}) \longrightarrow (\Upsilon.\Pi) \cap \text{cl}(\tilde{\mathcal{H}}, \tilde{\mathcal{H}}) = (\Upsilon, \Pi). \quad (3)$$

From (2) and (3), we have (ϒ,Π) = $(\widetilde{\varnothing,\varnothing})$, which is a contradiction. So, (𝕽,Ѡ$_{𝕽}$) is Double connected. Similarly, cl ($\tilde{\mathcal{H}}$ <i> </i> $\tilde{\mathcal{H}}$) is Double connected. □

Theorem 7. *Let (x, Ѡ) be a DITS, if $\left\{(\mathcal{Y}_\beta.\mathcal{V}_\beta).Ѡ_{(\mathcal{Y}_\beta.\mathcal{V}_\beta)}: \beta \in J\right\}$ is a family of nonempty Double connected subspace of $\mathcal{X}$, $\cap_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ is nonempty; then, $(\mathbb{Z}_1.\mathbb{Z}_2)$ = $(\cup_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta).Ѡ_{(\cup_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta))}$ is Double connected subspace of (x, Ѡ).*

Proof. Let $\left\{(\mathcal{Y}_\beta.\mathcal{V}_\beta).Ѡ_{(\mathcal{Y}_\beta.\mathcal{V}_\beta)}: \beta \in J\right\}$ is Double connected subspace of (x, Ѡ), $\cap_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ≠ $(\widetilde{\varnothing,\varnothing})$ to show that $(\mathbb{Z}_1.\mathbb{Z}_2) = \cup_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ is Double connected, if possible suppose that $(\mathbb{Z}_1.\mathbb{Z}_2)$ is Double disconnected there exist two nonempty disjoint Double *I* open sets (π, μ), (ζ, ρ) such that (π, μ) ∩ (ζ, ρ) = $(\widetilde{\varnothing,\varnothing})$, (ζ, ρ) ∩ $(\mathbb{Z}_1.\mathbb{Z}_2)$ = $(\widetilde{\varnothing,\varnothing})$ ⟶ [(π, μ) ∩ $(\mathbb{Z}_1.\mathbb{Z}_2)$] ∩ [(ζ, ρ) ∩ $(\mathbb{Z}_1.\mathbb{Z}_2)$] = $(\widetilde{\varnothing,\varnothing})$ and [(π, μ) ∩ $(\mathbb{Z}_1.\mathbb{Z}_2)$] ∪ [(ζ, ρ) ∩ $(\mathbb{Z}_1.\mathbb{Z}_2)$]= (($\mathbb{Z}_1.\mathbb{Z}_2$) ⟶ = [(π, μ) ∪ (ζ, ρ)] ∩ $(\mathbb{Z}_1.\mathbb{Z}_2)$ ⟶ $(\mathbb{Z}_1.\mathbb{Z}_2)$ ⊆ [(π, μ) ∪ (ζ, ρ)], which is a contradiction, now ∩ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ≠ .Choose a Double *I* point ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ $\cap_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ $\cup_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ $(\mathbb{Z}_1.\mathbb{Z}_2)$ ⟶ ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ [(π, μ) ∪ (ζ, ρ)] ⟶ ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ (π, μ) and ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ (ζ, ρ). Let ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ (π, μ) also $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ $\cup_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ $(\mathbb{Z}_1.\mathbb{Z}_2)$ ⊆ [(π, μ) ∪ (ζ, ρ)] ⟶ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ [(π, μ) ∪ (ζ, ρ)] ⟶ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ [(π, μ) ∪ (ζ, ρ)] = $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ [$(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ (π, μ)] ∪ [$(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ (ζ, ρ)] = $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ [$(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ (π, μ)] ∩ [$(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ (ζ, ρ)] = $(\widetilde{\varnothing,\varnothing})$.Also, $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ (π, μ), $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ∩ (ζ, ρ) are nonempty two disjoint Double *I* open set of $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ [(π, μ) ∪ (ζ, ρ)], so $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ is Double connected either $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ (π, μ) or $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ (ζ, ρ) ⟶ ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ (π, μ), ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⟶ $(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ (π, μ) ⟶ $\cup_{\beta\in J}(\mathcal{Y}_\beta.\mathcal{V}_\beta)$ ⊆ (π, μ)) ⟶ $(\mathbb{Z}_1.\mathbb{Z}_2)$ ⊆ (π, μ), (π, μ) ∩(ζ, ρ) = $(\widetilde{\varnothing,\varnothing})$ ⟶ $(\mathbb{Z}_1.\mathbb{Z}_2)$ ∩ (ζ, ρ) = $(\widetilde{\varnothing,\varnothing})$, which is a contradiction to the fact that $(\mathbb{Z}_1.\mathbb{Z}_2)$ ∩ (ζ, ρ) ≠ $(\widetilde{\varnothing,\varnothing})$ and ($\tilde{\mathfrak{b}}.\tilde{\mathfrak{b}}$) ∈ (ζ, ρ), which a contradiction ⟶ $(\mathbb{Z}_1.\mathbb{Z}_2)$ is Double connected subspace of (x,Ѡ). □

Theorem 8. *Let (x, ΨѠ) be a DITS, then, if (x, Ѡ) is Double disconnected I space, then (x, $(\delta_1.\delta_2)$ are disconnected I space.*

Proof. Suppose that (x, Ѡ) is Double disconnected, then there exist [$(q_1.q_2)$, $(\mathcal{J}_1.\mathcal{J}_2)$]∈ Ψ such that $(q_1.q_2)$ ∩ $(\mathcal{J}_1.\mathcal{J}_2)$ = $(\widetilde{\varnothing,\varnothing})$ and $(q_1.q_2)$ ∪ $(\mathcal{J}_1.\mathcal{J}_2)$ = $(\tilde{\mathcal{X}}, \tilde{\mathcal{X}})$. So, q_1 ∩$\mathcal{J}_1$ = $\tilde{\varnothing}$, q_2 ∩ $\mathcal{J}_2$ = $\tilde{\varnothing}$ and q_1 ∪ $\mathcal{J}_1$ = $\tilde{\mathcal{X}}$, q_2 ∪ $\mathcal{J}_2$ = $\tilde{\mathcal{X}}$ .Hence, $(q_1.q_2)$ ∩ $(\mathcal{J}_1.\mathcal{J}_2)$ = $(\widetilde{\varnothing,\varnothing})$ and $(q_1.q_2)$ ∪ $(\mathcal{J}_1.\mathcal{J}_2)$ = $(\tilde{\mathcal{X}}, \tilde{\mathcal{X}})$,[$(q_1.q_2)$, $(\mathcal{J}_1.\mathcal{J}_2)$]∈ $(\delta_1.\delta_2)$.Therefore, (x, δ_1) and (x, δ_2) are disconnected *I* space.

The following example shows that the converse is not true in general: □

Example 5. Let $\mathcal{X} = \{i.j.h\}$; $\delta_1 = \{\tilde{\varnothing}, \tilde{\mathcal{X}}, \mathcal{M}_1.\mathcal{M}_2\}$, where $\mathcal{M}_1 = \langle x.\{i.j\}.\{h\}\rangle$, $\mathcal{M}_2 = \langle x.\{h\}.\{i.j\}\rangle$, and $\delta_2 = \left\{\tilde{\varnothing}.\tilde{\mathcal{X}}.\mathcal{M}_3.\mathcal{M}_4\right\}$, where $\mathcal{M}_3 = \langle x.\{i.h\}.\{j\}\rangle$ and $\mathcal{M}_4 = \langle x.\{j\}.\{i.h\}\rangle$. Then, ($x$, δ_1) and (x, δ_2) are intuitionistic topological spaces and disconnected I space. Since Ѡ = $(\delta_1.\delta_2)$ = {($\tilde{\varnothing}$, $\tilde{\varnothing}$), ($\tilde{\mathcal{X}}$, $\tilde{\mathcal{X}}$), ($\tilde{\varnothing}.\mathcal{M}_3$), ($\tilde{\varnothing}.\mathcal{M}_4$), ($\tilde{\varnothing}.\tilde{\mathcal{X}}$),

$(\mathscr{M}_1.\tilde{\mathscr{X}})$, $(\mathscr{M}_2.\tilde{\mathscr{X}})$, $(\mathscr{M}_2.\mathscr{M}_3)\}$, where $(\tilde{\varnothing}.\mathscr{M}_3) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{i.\hbar\}.\{j\}\rangle)$, $(\tilde{\varnothing}.\mathscr{M}_4) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{j\}.\{i.\hbar\}\rangle)$, $(\tilde{\varnothing}.\tilde{\mathscr{X}}) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, $(\mathscr{M}_1.\tilde{\mathscr{X}}) = (\langle x.\{i.j\}.\{\hbar\}\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, $(\mathscr{M}_2.\tilde{\mathscr{X}}) = (\langle x.\{\hbar\}.\{i.j\}\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, and $(\mathscr{M}_2.\mathscr{M}_3) = (\langle x.\{\hbar\}.\{i.j\}\rangle.\langle x.\{i.\hbar\}.\{j\}\rangle$. $Ѡ^c = (\delta_2^c.\delta_1^c) = \{(\tilde{\varnothing},\tilde{\varnothing}),(\tilde{\mathscr{X}},\tilde{\mathscr{X}}),(\mathscr{M}_3^c.\tilde{\mathscr{X}}).(\mathscr{M}_4^c.\tilde{\mathscr{X}}).(\tilde{\varnothing}.\tilde{\mathscr{X}}).(\tilde{\varnothing}.\mathscr{M}_1^c).(\tilde{\varnothing}.\mathscr{M}_2^c).(\mathscr{M}_3^c.\mathscr{M}_2^c)\}$, where $(\mathscr{M}_3^c.\tilde{\mathscr{X}}) = (\langle x.\{j\}.\{i.\hbar\}\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, $(\mathscr{M}_4^c.\tilde{\mathscr{X}}) = (\langle x.\{i.\hbar\}.\{j\}\rangle,\langle x,\mathscr{X},\varnothing\rangle)$, $(\tilde{\varnothing}.\tilde{\mathscr{X}}) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, $(\tilde{\varnothing}.\mathscr{M}_1^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{\hbar\}.\{i.j\}\rangle)$, $(\tilde{\varnothing}.\mathscr{M}_2^c) = \langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{i.j\}.\{\hbar\}\rangle)$, and $(\mathscr{M}_3^c.\mathscr{M}_2^c) = (\langle x.\{j\}.\{i.\hbar\}\rangle,\langle x.\{i.j\}.\{\hbar\}\rangle)$. Therefore, $(x, Ѡ)$ is not Double disconnected I space.

Definition 6. Let $(x, Ѡ)$ be a DITS is said to be the following:

(1) Double CO disconnected, if $(x, Ѡ)$ has a proper Double I open and Double I closed sets in x.

(2) Double CO connected, if $(x, Ѡ)$ is not Double CO disconnected.

Definition 7. The DITS $(x, Ѡ)$ is said to be the following:

(1) Strongly Double connected I space, if there exist no nonempty Double I closed sets $[(\nu,\mu),(\eta,\epsilon)] \in x$. Such that $(\nu,\mu)\cap(\eta,\epsilon) = (\tilde{\varnothing}\tilde{\varnothing})$.

(2) Strongly Double disconnected I space, if $(x, Ѡ)$ is not strongly Double connected I space.

Proposition 3. *$(x, Ѡ)$ is strongly Double connected, if and only, if there exist no Double I open sets $(\nu,\mu),(\eta,\epsilon)$ in x such that $(\nu,\mu) \neq (\tilde{\mathscr{X}},\tilde{\mathscr{X}}) \neq (\eta,\epsilon)$and $(\nu,\mu)\cup(\eta,\epsilon) = (\tilde{\mathscr{X}},\tilde{\mathscr{X}})$.*

Proof. Let (ν,μ), (η,ϵ) be Double I open sets in x such that $(\nu,\mu) \neq (\tilde{\mathscr{X}},\tilde{\mathscr{X}}) \neq (\eta,\epsilon)$. If we take $(k, t) = (\nu,\mu)^c$ and $(n, m) = (\eta,\epsilon)^c$, then (k, t) and (n, m) become Double I closed sets in x and $(k, t) \neq (\tilde{\varnothing},\tilde{\varnothing}) \neq (n, m)$ and $(k, t)\cap(n, m) = (\tilde{\varnothing},\tilde{\varnothing})$, which is a contradiction.

Conversely, it is obvious. □

Remark 2. Strongly Double connectedness does not imply Double CO connectedness, and Double CO connectedness does not imply Strongly Double connectedness.

Example 6. Let $x = \{\Omega.\tau.\mu\}$; $Ѡ = \{(\tilde{\varnothing},\tilde{\varnothing}),(\tilde{\mathscr{X}},\tilde{\mathscr{X}}),(\phi_1.\phi_2),(\phi_3.\phi_3),(\tilde{\varnothing}.\phi_4),(\phi_1.\phi_1)\}$where $(\phi_1.\phi_2) = (\langle x.\{\Omega\}.\{\tau.\mu\}\rangle,\langle x.\{\Omega.\tau\}.\varnothing\rangle),(\phi_3.\phi_3) = (\langle x.\{\tau.\mu\}.\{\Omega\}\rangle.\langle x.\{\tau.\mu\}.\{\Omega\}\rangle)$. $(\tilde{\varnothing}.\phi_4) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{\tau\}.\{\Omega\}\rangle)$ and $(\phi_1.\phi_1) = (\langle x.\{\Omega\}.\{\tau.\mu\}\rangle.\langle x.\{\Omega\}.\{\tau.\mu\}\rangle)$. $Ѡ^c = \{(\tilde{\varnothing},\tilde{\varnothing}),(\tilde{\mathscr{X}},\tilde{\mathscr{X}}),(\phi_2^c.\phi_1^c).(\phi_1.\phi_1)(\phi_4^c.\tilde{\mathscr{X}}),(\phi_3.\phi_3)\}$,where $(\phi_2^c.\phi_1^c) = (\langle x.\varnothing.\{\Omega.\tau\}\rangle,\langle x.\{\tau.\mu\}.\{\Omega\}\rangle)$, $(\phi_1.\phi_1) = (\langle x.\{\Omega\}.\{\tau.\mu\}\rangle.\langle x.\{\Omega\}.\{\tau.\mu\}\rangle)$. $(\phi_4^c.\tilde{\mathscr{X}}) = (\langle x.\{\Omega\}.\{\tau\}\rangle,\langle x,\mathscr{X},\varnothing\rangle)$, and $(\phi_3.\phi_3) = (\langle x.\{\tau.\mu\}.\{\Omega\}\rangle.\langle x.\{\tau.\mu\}.\{\Omega\}\rangle)$. Therefore, $(x, Ѡ)$ is strongly Double connected, but is not Double CO connected, for there exist $(\phi_1.\phi_1)$, $(\phi_3.\phi_3)$ are both Double I open and Double I closed sets.

Example 7. Let $x = \{a.ƀ.C.Đ\}$; $Ѡ = \{(\tilde{\varnothing},\tilde{\varnothing}),(\tilde{\mathscr{X}},\tilde{\mathscr{X}}),(Ɣ_1.Ɣ_4),(Ɣ_2.Ɣ_3),(Ɣ_3.\tilde{\mathscr{X}}).(Ɣ_2.Ɣ_4),(Ɣ_5.Ɣ_1),(Ɣ_4.\tilde{\mathscr{X}}),(Ɣ_5.Ɣ_2),(Ɣ_4.Ɣ_4),(Ɣ_5.Ɣ_4),(Ɣ_2.\tilde{\mathscr{X}}),(Ɣ_2.Ɣ_2),(Ɣ_5.Ɣ_5),(Ɣ_3.Ɣ_3)\}$, where $(Ɣ_1.Ɣ_4) = (\langle x.\{ƀ.C\}.\{Đ\}\rangle.\langle x.\{a.ƀ.C\}.\varnothing\rangle)$, $(Ɣ_2.Ɣ_3) = (\langle x.\{a\}.\{C\}\rangle.\langle x.\{a.Đ\}.\{C\}\rangle)$, $(Ɣ_3.\tilde{\mathscr{X}}) = (\langle x.\{a.Đ\}.\{C\}\rangle,\langle x,\mathscr{X},\varnothing\rangle)$, $(Ɣ_2.Ɣ_4) = (\langle x.\{a\}.\{C\}\rangle.\langle x.\{a.ƀ.C\}.\varnothing\rangle)$, $(Ɣ_5.Ɣ_1) = (\langle x.\varnothing.\{C.Đ\}\rangle.\langle x.\{ƀ.C\}.\{Đ\}\rangle)$, $(Ɣ_4.\tilde{\mathscr{X}}) = (\langle x.\{a.ƀ.C\}.\varnothing\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, $(Ɣ_5.Ɣ_2) = (\langle x.\varnothing.\{C.Đ\}\rangle.\langle x.\{a\}.\{C\}\rangle)$, $(Ɣ_4.Ɣ_4) = (\langle x.\{a.ƀ.C\}.\varnothing\rangle.\langle x.\{a.ƀ.C\}.\varnothing\rangle)$, $(Ɣ_5.Ɣ_4) = (\langle x.\varnothing.\{C.Đ\}\rangle.\langle x.\{a.ƀ.C\}.\varnothing\rangle),(Ɣ_2.\tilde{\mathscr{X}}) = (\langle x.\{a\}.\{C\}\rangle.\langle x,\mathscr{X},\varnothing\rangle)$, $(Ɣ_2.Ɣ_2) = (\langle x.\{a\}.\{C\}\rangle.\langle x.\{a\}.\{C\}\rangle)$, $(Ɣ_5.Ɣ_5) = (\langle x.\varnothing.\{C.Đ\}\rangle.\langle x.\varnothing.\{C.Đ\}\rangle)$, and $(Ɣ_3.Ɣ_3) = (\langle x.\{a.Đ\}.\{C\}\rangle.\langle x.\{a.Đ\}.\{C\}\rangle))$. $Ѡ^c = \{(\tilde{\varnothing},\tilde{\varnothing}),(\tilde{\mathscr{X}},\tilde{\mathscr{X}}),(Ɣ_4^c.Ɣ_1^c).(Ɣ_3^c.Ɣ_2^c).(Ɣ_4^c.Ɣ_2^c).(Ɣ_1^c.Ɣ_5^c).(\tilde{\varnothing}.Ɣ_4^c).(Ɣ_2^c.Ɣ_5^c).(Ɣ_4^c.Ɣ_4^c).(Ɣ_4^c.Ɣ_5^c).(\tilde{\varnothing}.Ɣ_2^c),(Ɣ_2^c.Ɣ_2^c).(Ɣ_5^c.Ɣ_5^c).(Ɣ_3^c.Ɣ_3^c).(\tilde{\varnothing}.Ɣ_3^c)\}$, where $(Ɣ_4^c.Ɣ_1^c) = (\langle x.\varnothing.\{a.ƀ.C\}\rangle.\langle x.\{Đ\}.\{ƀ.C\}\rangle)$, $(Ɣ_3^c.Ɣ_2^c) = (\langle x.\{C\}.\{a.Đ\}\rangle.\langle x.\{C\}.\{a\}\rangle)$, $(Ɣ_4^c.Ɣ_2^c) = (\langle x.\varnothing.\{a.ƀ.C\}\rangle.\langle x.\{C\}.\{a\}\rangle)$, $(Ɣ_1^c.Ɣ_5^c) = \langle x.\{Đ\}.\{ƀ.C\}\rangle.\langle x.\{C.Đ\}.\varnothing\rangle)$, $(\tilde{\varnothing}.Ɣ_4^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\varnothing.\{a.ƀ.C\}\rangle)$, $(Ɣ_2^c.Ɣ_5^c) = (\langle x.\{C\}.\{a\}\rangle.\langle x.\{C.Đ\}.\varnothing\rangle)$, $(Ɣ_4^c.Ɣ_4^c) = (\langle x.\varnothing.\{a.ƀ.C\}\rangle.\langle x.\varnothing.\{a.ƀ.C\}\rangle$, $(Ɣ_4^c.Ɣ_5^c) = (\langle x.\varnothing.\{a.ƀ.C\}\rangle.(\langle x.\varnothing.\{C.Đ\}\rangle$, $(\tilde{\varnothing}.Ɣ_2^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{C\}.\{a\}\rangle)$, $(Ɣ_2^c.Ɣ_2^c) = (\langle x.\{C\}.\{a\}\rangle.\langle x.\{C\}.\{a\}\rangle)$, $(Ɣ_5^c.Ɣ_5^c) = (\langle x.\{C.Đ\}.\varnothing\rangle.\langle x.\{C.Đ\}.\varnothing\rangle)$, $(Ɣ_3^c.Ɣ_3^c) = (\langle x.\{C\}.\{a.Đ\}\rangle.\langle x.\{C\}.\{a.Đ\}\rangle)$, and $(\tilde{\varnothing}.Ɣ_3^c) = (\langle x.\varnothing.\mathscr{X}\rangle.\langle x.\{C\}.\{a.Đ\}\rangle)$. $(x, Ѡ)$ is Double CO connected, but it is not strongly Double connected, for there exist $[(Ɣ_1.Ɣ_4),(Ɣ_3.Ɣ_3)] \in \Psi$, and $(Ɣ_1.Ɣ_4)\cup(Ɣ_3.Ɣ_3) = (\tilde{\mathscr{X}},\tilde{\mathscr{X}})$.

Definition 8. Let $(x, Ѡ)$ be a DITS and $(\tilde{y},\tilde{y}) \subseteq (\tilde{\mathscr{X}},\tilde{\mathscr{X}})$ with $(\tilde{ƀ}.\tilde{ƀ}) = (\langle x.\{ƀ\}.\{ƀ\}^c\rangle.\langle x.\{ƀ\}.\{ƀ\}^c\rangle) \in$ DI $ƀ$ (y). The union of all Double connected subsets of $(\tilde{y},\tilde{y})$ containing the Double I point $(\tilde{ƀ}.\tilde{ƀ})$ is called Double I component of y with respect to $(\tilde{ƀ}.\tilde{ƀ})$, denoted by $(c_1.c_2)$ $[(\tilde{y},\tilde{y}),(\tilde{ƀ}.\tilde{ƀ})]$. *i.e.*, $(c_1.c_2)$ $[(\tilde{y},\tilde{y}),(\tilde{ƀ}.\tilde{ƀ})] = \cup\{(\nu,\mu)\subseteq(\tilde{y},\tilde{y})$: $(\tilde{ƀ}.\tilde{ƀ}) \in (\nu,\mu)$, and (ν,μ) is Double connected I set $\}$.

Example 8. Let $x = \{Ɛ.p.Ʊ\}$; $Ѡ = \{(\tilde{\varnothing},\tilde{\varnothing}),(\tilde{\mathscr{X}},\tilde{\mathscr{X}}),(Ʈ_1.Ʈ_1),(Ʈ_1^c.Ʈ_1^c)\}$, where $(Ʈ_1.Ʈ_1)(\langle x.\{Ɛ\}.\{p.Ʊ\}\rangle.\langle x.\{Ɛ\}.\{p.Ʊ\}\rangle)$, $(Ʈ_1^c.Ʈ_1^c) = (\langle x.\{p.Ʊ\}.\{Ɛ\}\rangle.\langle x.\{p.Ʊ\}.\{Ɛ\}\rangle)$. Let $(Ʈ_1.Ʈ_1) = (\tilde{Ɛ}.\tilde{Ɛ}) = (\langle x.\{Ɛ\}.\{p.Ʊ\}\rangle.\langle x.\{Ɛ\}.\{p.Ʊ\}\rangle) \subseteq x \longrightarrow (Ʈ_1.Ʈ_1)$ is Double connected. $(Ʈ_2.Ʈ_2) = (\tilde{p}.\tilde{p}) = (\langle x.\{p\}.\{Ɛ.Ʊ\}\rangle.\langle x.\{p\}.\{Ɛ.Ʊ\}\rangle) \subseteq x \longrightarrow (Ʈ_2.Ʈ_2)$ is do double connected. $(Ʈ_3.Ʈ_3) = (\tilde{Ʊ}.\tilde{Ʊ}) = ((\langle x.\{Ʊ\}.\{Ɛ.p\}\rangle.\langle x.\{Ʊ\}.\{Ɛ.p\}\rangle) \subseteq x \longrightarrow (Ʈ_3.Ʈ_3)$ is Double connected. $(Ʈ_4.Ʈ_4) = (\{p.Ʊ\}.\widetilde{\{p.Ʊ\}}) = (\langle x.\{p.Ʊ\}.\{Ɛ\}\rangle.\langle x.\{p.Ʊ\}.\{Ɛ\}\rangle)$, and subE; $x \longrightarrow (Ʈ_4.Ʈ_4)$ is Double connected. $(Ʈ_5.Ʈ_5) = (\{Ɛ.p\}.\widetilde{\{Ɛ.p\}}) = (\langle x.\{Ɛ.p\}.\{Ʊ\}\rangle.\langle x.\{Ɛ.p\}.\{Ʊ\}\rangle) \subseteq x \longrightarrow (Ʈ_5.Ʈ_5)$ is not Double connected. $(Ʈ_6.Ʈ_6) = (\{Ɛ.Ʊ\}.\widetilde{\{Ɛ.Ʊ\}}) = (\langle x.\{Ɛ.Ʊ\}.\{p\}\rangle.\langle x.\{Ɛ.Ʊ\}.\{p\}\rangle) \subseteq x \longrightarrow (Ʈ_6.Ʈ_6)$ is not Double connected.

Note that, $(Ʈ_1.Ʈ_1)$, $(Ʈ_2.Ʈ_2)$, $(Ʈ_3.Ʈ_3)$, and $(Ʈ_4.Ʈ_4)$ are Double connected to find the Double I component of any Double I point in x, so $(Ʈ_1.Ʈ_1) \subseteq (Ʈ_1.Ʈ_1)$ only. Hence, $(Ʈ_1.Ʈ_1)$ is Double I component.

$(Ʈ_2.Ʈ_2) \subseteq (Ʈ_2.Ʈ_2)$ and $(Ʈ_4.Ʈ_4)$, so $(Ʈ_2.Ʈ_2)$ is not Double I component $(Ʈ_3.Ʈ_3) \subseteq (Ʈ_3.Ʈ_3)$ and $(Ʈ_4.Ʈ_4)$, so $(Ʈ_3.Ʈ_3)$ is not Double I component, $(\mathsf{T}_4.\mathsf{T}_4) \subseteq (\mathsf{T}_4.\mathsf{T}_4)$ only. Hence, $(\mathsf{T}_4.\mathsf{T}_4)$ is Double I component. Therefore, $(\mathsf{T}_1.\mathsf{T}_1)$, $(\mathsf{T}_4.\mathsf{T}_4)$ are Double I component $\longrightarrow$ $(\mathsf{T}_1.\mathsf{T}_1) \cup (\mathsf{T}_4.\mathsf{T}_4) = (\tilde{\mathscr{X}}, \tilde{\mathscr{X}})$.

Theorem 9. *Every Double I component of a DITS is Double I closed set.*

Proof. Let $(\mathscr{x}, \omega)$be a DITS and let $(\mathbb{c}_1.\mathbb{c}_2)$ $[(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]$ be a Double I component of the DITS $(\mathscr{x}, \omega)$ with respect to an arbitrary Double I point $(\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}}) \in \text{DI}\ \mathfrak{b}\ (\mathscr{x})$.Then, $(\mathbb{c}_1.\mathbb{c}_2)$ $[(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]$ is Double connected subset of $\mathscr{x}$ (by Theorem 6 and Theorem 7), cl $[(\mathbb{c}_1.\mathbb{c}_2)\ [(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]]$ is Double connected subset of $(\tilde{\mathscr{X}}, \tilde{\mathscr{X}})$ containing $(\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})$, then cl $[(\mathbb{c}_1.\mathbb{c}_2)\ [(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]] \subseteq (\mathbb{c}_1.\mathbb{c}_2)\ [(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]$.But $(\mathbb{c}_1.\mathbb{c}_2)$ $[(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})] \subseteq$ cl $[(\mathbb{c}_1.\mathbb{c}_2)\ [(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]]$.Therefore, $(\mathbb{c}_1.\mathbb{c}_2)$ $[(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})] =$ cl $[(\mathbb{c}_1.\mathbb{c}_2)\ [(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]]$. This indicates the Double I component $(\mathbb{c}_1.\mathbb{c}_2)$ $[(\tilde{y}, \tilde{y}), (\tilde{\mathfrak{b}}. \tilde{\mathfrak{b}})]$ is Double I closed set. □

4. Conclusions

In this paper, we got the following next results:

(1) We have presented a new set of the following concepts: Double intuitionistic set (DIS) (resp., Double intuitionistic topological spaces (DITS)), Double I point, Double connected $_\Psi I$ set, separated Double I sets, strongly Double connected, Double CO connected I space, and Double I component $_\Psi$ in DITS.

(2) Study the basic characteristics and qualities related to these types and the relationships between them; giving examples is incorrect.

Data Availability

The data underlying the results presented in the study are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] N. Ajmal and J. K. Kohli, "Connectedness in fuzzy topological spaces," *Fuzzy Sets and System*, vol. 31, pp. 369–388, 1989.

[2] A. Z. A. Yasry, *Lectures in Advanced Topology*, College of Science for Women, Baghdad, Iraq, 2013.

[3] O. A. El-Hamed, A. S. Kandil, S. A. Ali, and S. H. Shaliel, "Double connected spaces," *Journal of New Theory*, vol. 17, pp. 1–17, 2017.

[4] A. Kandil, O. A. E. Tantawy, and M. Wafaie, "On flou (intuitionistic) topological spaces," *Journal of Fuzzy Mathematics*, vol. 15, no. 2, pp. 1–23, 2007.

[5] A. Kendal, O. A. E. Tantawy, and M. Wafaie, "On flou (intuitionistic) compact space," *Journal of Fuzzy Mathematics*, vol. 17, no. 2, pp. 275–294, 2009.

[6] D. Coker, "An introduction to intuitionistic topological spaces," *Busefal*, vol. 81, pp. 51–56, 2000.

[7] D. Coker, "A note on Intuitionistic sets and Intuitionistic points," *Turkish Journal of Mathematics*, vol. 20, pp. 343–351, 1996.

[8] D. Coker, "An introduction to intuitionistic fuzzy topological spaces," *Fuzzy Sets and Systems*, vol. 88, no. 1, pp. 81–89, 1997.

[9] K. T. Atanassov and S. Stoeva, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[10] K. T. Atanassov, "More on intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 33, no. 1, pp. 37–45, 1989.

[11] A. Z. Ozcelik and S. Narli, "On submaximality intuitionistic topological spaces," *International Journal of Mathematical and Computational Sciences*, vol. 1, no. 1, pp. 139–141, 2007.

[12] O. A. E. Tantawy, S. A. EI-Sheikh, and S. Hussien, "Soft connected of double spaces," *South Asian Journal of Mathametics*, vol. 6, no. 5, pp. 249–262, 2016.

[13] O. Selma and D. Coker, "On connectedness in intuitionistic fuzzy special topolgical spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. 21, no. 1, pp. 33–40, 1998.

[14] T. K. Mondal and S. Samanta, "On intuitionistic gradation of openness," *Fuzzy Sets and Systems*, vol. 131, no. 3, pp. 323–336, 2002.

[15] N. Levine, "Generalized closed sets in topology," *Rendiconti del Circolo Matematico di Palermo*, vol. 19, no. 1, pp. 89–96, 1970.

[16] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.