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Research Article

Aggregation Operators for Interval-Valued Intuitionistic Fuzzy Hypersoft Set with Their Application in Material Selection

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The intuitionistic fuzzy hypersoft set (IFHSS) is the most generalized form of the intuitionistic fuzzy soft set used to resolve uncertain and vague data in the decision-making process, considering the parameters' multi-sub-attributes. Aggregation operators execute a dynamic role in assessing the two prospect sequences and eliminating anxieties from this perception. This paper prolongs the IFHSS to interval-valued IFHSS (IVIFHSS), which proficiently contracts with hesitant and unclear data. It is the most potent technique for incorporating insecure data into decision-making (DM). The main objective of this research is to develop the algebraic operational laws for IVIFHSS. Furthermore, using the algebraic operational law, some aggregation operators (AOs) for IVIFHSS have been presented, such as interval-valued intuitionistic fuzzy hypersoft weighted average (IVIFHSWA) and interval-valued intuitionistic fuzzy hypersoft weighted geometric (IVIFHSWG) operators with their essential properties. Multi-criteria group decision-making (MCGDM) technique is vigorous for material selection. However, conventional methods of MCGDM regularly provide inconsistent results. Based on the expected AOs, industrial enterprises propose a robust MCGDM material selection method to meet this shortfall. The real-world application of the planned MCGDM method for cryogenic storing vessel material selection (MS) is presented. The implication is that the designed model is more efficient and consistent in handling information based on IVIFHSS.

1. Introduction

MCGDM is deliberated as the most suitable method for verdict the adequate alternative from all probable choices, following conditions or features. Maximum judgments are taken when the intentions and confines are usually unspecified or unclear in real-life circumstances. Zadeh presented the notion of the fuzzy set (FS) [1] to overcome such vagueness and doubts in decision-making (DM). Turksen [2] presented the interval-valued FS (IVFS) with fundamental operations. If the experts consider a membership degree (MD) and a non-membership degree (NMD) in the DM procedure, the FS theories cannot handle the situation. Atanassov [3] resolved the abovementioned limitations and developed the intuitionistic fuzzy set (IFS). Garg and Rani [4] projected some distance measures under IFS setting to resolve DM obstacles. Wang and Liu [5] introduced several operations such as Einstein product, Einstein sum, etc., and AOs for IFS. Garg [6] developed the cosine similarity measures (SM) for IFS considering the interaction between the couples of MD and NMD. Atanassov [7] introduced the topological operators and discussed some essential properties. Garg and Kumar [8] projected the SM to extend the power of distinct IFS. Ejegwa and Agbetayo [9] developed several SM and distance measures under the IFS environment and used their presented measures to resolve DM complications. To measure their relation, Garg and Rani [10] established the correlation coefficient (CC) for complex IFS. Khan et al. [11] offered a MADM technique using complex T-spherical fuzzy power AOs. Atanassov [12] introduced the interval-valued intuitionistic fuzzy set (IVIFS) with some basic operations. Wang et al. [13] proposed the weighted average AOs for IVIFS and established a multi-criteria decision-making (MCDM) technique to resolve DM obstacles. Arora and Garg [14] prolonged the linguistic IFS with prioritized AOs. Garg and Rani [15] settled the MULTIMOORA technique under IFS information using their presented AOs. Xu and Chen [16] developed the weighted geometric and hybrid weighted geometric AOs for IVIFS. They also constructed the multi-attribute decision-making (MADM) technique using their established AOs to resolve DM issues.

Jia and Zhang [17] prolonged the weighted arithmetic AOs for IVIFS and presented the multi-attribute group decision-making (MAGDM) model. Xu and Gou [18] developed several DM methodologies under the IVIFS setting and utilized their methodologies in various real-life problems. Ze-Shui [19] proposed the weighted arithmetic and geometric AOs for IVIFS. Mu et al. [20] protracted the Zhenyuan average and geometric AOs for IVIFS. They also established some DM approaches to resolve MADM obstacles using Zhenyuan AOs. Zhang [21] developed the Bonferroni mean geometric AOs under the IVIFS setting and presented the MAGDM approach. Park et al. [22] proposed the hybrid geometric aggregation operator for IVIFS and utilized it for MAGDM problems. Gupta et al. [23] developed a corrective model for determining the weight of experts. The weight information of experts is conveyed by interval-valued intuitionistic fuzzy numbers (IVIFNs). Garg and Kumar [24] extended the AOs with their fundamental properties under the linguistic IVIFS environment to solve group decision-making problems. Peng and Yang [25] presented the idea of interval-valued PFS (IVPFS) and prolonged the AOs under-considered environment. Rahman et al. [26] offered geometric and ordered AOs for IVPFS and used their established operators to resolve DM issues.

The above-stated FS, IVFS, IFS, IVIFS, PFS, and IVPFS cannot deal with the parametrized values of the alternatives. Molodtsov [27] introduced soft sets (SS) theory and explained some basic operations with their features to handle confusion and uncertainties. Fatimah et al. [28] extended the concept of SS to N-soft set with some basic operations and their properties. Maji et al. [29] extended the SS theory and developed some fundamental operations. Yuksel et al. [30] extended the SS theory to soft expert sets and utilized their theory to calculate the patient's prostate cancer risk. Maji et al. [31] introduced the fuzzy soft set theory by merging SS and FS. Fatimah and Alcantud [32] proposed the multi fuzzy–soft set theory with fundamental operations and their properties. They also established a DM methodology employing their progressive approach to resolve DM obstacles. Garg et al. [33] presented the spherical fuzzy soft topology with some fundamental operations and discussed their properties. Maji et al. [34] developed basic operations for their properties for the intuitionistic fuzzy soft set (IFSS). Arora and Garg [35] proposed the AOs for IFSS and utilized their developed AOs to solve MCDM obstacles. Garg and Arora [36] extended the TOPSIS technique by employing the CC under the IFSS environment. They also developed the Maclaurin symmetric mean AOs for the IFSS setting [37]. Garg and Arora [38] proposed the idea of generalized IFSS with some fundamental operations and essential properties. Jiang et al. [39] introduced the interval-valued IFSS (IVIFSS) with some basic operations and their properties. Zulqarnain et al. [40] planned the TOPSIS technique for IVIFSS based on correlation measures to solve MADM problems. Smarandache [41] projected the idea of the hypersoft set (HSS), which penetrates multiple sub-attributes in the parameter function f, which is a characteristic of the Cartesian product with the n attribute. Associated with SS and other prevailing ideas, Smarandache HSS is the most appropriate model that grips the deliberated constraints' multiple sub-attributes. Zulqarnain et al. [42] extended the TOPSIS approach using the correlation coefficient for IFHSS to solve MADM complications. Zulqarnain et al. [43] prolonged the AOs for the IFHSS setting and established a DM technique based on their developed AOs. Jafar et al. [44] developed the intuitionistic fuzzy hypersoft matrices with fundamental operations. Debnath [45] introduced the IVIFHSS with several fundamental operations and their properties. Sunthrayuth et al. [46] established a novel MCDM technique based on Einstein's weighted average operator for Pythagorean fuzzy hypersoft sets. IFHSS plays a vital role in decision-making by combining multiple sources into a single value. IFHSS is a hybrid intellectual structure of IFSS. A boosted sorting development captivates the investigators to crash unsolved and insufficient facts. Interpreting the exploration consequences, it is concluded that the IFHSS performs an energetic part in DM by assembling several causes into a solitary value. Therefore, to inspire the current research on IVIFHSS, we will describe AOs built on irregular information. The core objectives of the present study are as follows:

(i) IVIFHSS deals competently with multidimensional concerns by looking at the multi-sub-attributes of the considered parameters in the DM procedure. To preserve this benefit in concentration, we extend IFHSS to IVIFHSS and set up AOs for IVIFHSS.

(ii) AOs for IVIFHSS are well-known attractive estimate AOs. It has been observed that the prevailing AOs aspect is irresponsible for scratching the correct detection of the DM process. To overcome these specific complications, these existing AOs need to be reviewed. We introduce the advanced operational laws for interval-valued intuitionistic fuzzy hypersoft numbers (IVIFHSNs).

(iii) IVIFHSWA and IVIFHSWG operators have been introduced with their essential features using developed operational laws.

(iv) A new algorithm based on planned operators has been established to solve the problems of MCGDM under the IVIFHSS scenario.

(v) Material selection is an essential feature of manufacturing as it understands the stable conditions for all components. MS is a complex but essential step in professional development. Lack of material selection will damage the manufacturer's efficiency, productivity, and eccentricity.

(vi) A comparative analysis of the latest MCGDM technique and existing methods is presented to consider the utility and superiority.

The organization of this research is estimated to be as follows: The Section 2 of this study contains some basic concepts that help us develop the structure of the later research. Section 3 introduces some new operational laws for IVIFHSN. Also, in the same section, IVIFHSWA and IVIFHSWG operators are presented based on the basic features of our developed operators. In Section 4, an MCGDM approach is developed based on the proposed AOs. A numerical example for material selection in the manufacturing industry is discussed in the same section to confirm the practicality of the established technique. In addition, Section 5 provides a brief comparative analysis to confirm the validity of the advanced approach.

2. Preliminaries

This section contains some basic definitions that will structure the following work.

Definition 1 (see [27]). Let U and $\mathbb{N}$ be the universe of discourse and set of attributes, respectively. Let $\mathscr{P}(U)$ be the power set of U and $A\subseteq\mathbb{N}$. A pair (Ω, A) is called a SS over U, and its mapping is expressed as follows:

$$\Omega\colon A \longrightarrow \mathscr{P}(U). \tag{1}$$

Also, it can be defined as follows:

$$(\Omega, \mathbf{A}) = \{\Omega(\mathbf{t}) \in \mathscr{P}(\mathbf{U})\colon \mathbf{t} \in \mathbb{N}, \Omega(\mathbf{t}) = \varnothing \text{ if } t \notin A\}. \tag{2}$$

Definition 2 (see [41]). Let U be a universe of discourse and $\mathscr{P}(U)$ be a power set of U and $t = \{t_1; t_2; t_3, \ldots, t_n\}$, $n \geq 1$ and T_i represented the set of attributes and their corresponding sub-attributes, such as $T_i \cap T_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1, 2, 3 \ldots, n\}$. Assume $T_1 \times T_2 \times T_3 \times \ldots \times = A = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha; 1 \leq k \leq \beta;$ and $1 \leq l \leq \gamma$, and $\alpha; \beta; \gamma \in N$, Then the pair $(\mathscr{F}, T_1 \times T_2 \times T_3 \times \ldots \times T_n) = (\Omega, A)$ is known as HSS and defined as follows:

$$\Omega\colon T_1 \times T_2 \times T_3 \times \cdots \times T_n = A \longrightarrow \mathscr{P}(U). \tag{3}$$

It is also defined as

$$(\Omega, \dddot{A}) = \left\{\check{d}, \Omega_{\dddot{A}}(\check{d})\colon\ \check{d} \in \dddot{A}, \Omega_{\dddot{A}}(\check{d}) \in \mathscr{P}(U)\right\}. \tag{4}$$

Definition 3 (see [12]). U be a universe of discourse, and A be any subset of U. Then, the IVIFS A over U is defined as:

$$A = \left\{\left(x, \left(\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]\right)\right) \middle| t \in U\right\}, \tag{5}$$

where, $[\kappa_A^l(t), \kappa_A^u(t)]$ and $[\delta_A^l(t), \delta_A^u(t)]$ represents the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A^u(t)$, $\delta_A^l(t), \delta_A^u(t) \in [0, 1]$ And satisfied the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$.

Definition 4 (see [39]). Let U be a universe of discourse and $\mathbb{N}$ be a set of attributes. Then a pair $(\Omega, \mathbb{N})$ is called an IVIFSS over U. Its mapping can be expressed as

$$\Omega\colon \mathbb{N} \longrightarrow IK^U, \tag{6}$$

where IK^U represents the collection of interval-valued intuitionistic fuzzy subsets of the universe of discourse U.

$$(\Omega, \mathbb{N}) = \left\{x, \left(\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]\right) \middle| t \in A\right\}, \tag{7}$$

where, $[\kappa_A^l(t), \kappa_A^u(t)]$, $[\delta_A^l(t), \delta_A^u(t)]$ represents the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A^u(t)$, $\delta_A^l(t), \delta_A^u(t) \in [0, 1]$ And satisfied the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$ and. $A \subset \mathbb{N}$.

Definition 5 (see [41]). Let U be a universe of discourse and $\mathscr{P}(U)$ be a power set of U and $t = \{t_1; t_2; t_3, \ldots, t_n\}$, $n \geq 1$ and T_i represented the set of attributes and their corresponding sub-attributes, such as $T_i \cap T_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1, 2, 3 \ldots, n\}$. Assume $T_1 \times T_2 \times T_3 \times \ldots \times = A = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha; 1 \leq k \leq \beta;$, and $1 \leq l \leq \gamma$, and $\alpha; \beta; \gamma \in N$. Then the pair $(\mathscr{F}, T_1 \times T_2 \times T_3 \times \ldots \times T_n) = (\Omega, A)$ is known as IFHSS and defined as follows:

$$\Omega\colon T_1 \times T_2 \times T_3 \times \ldots \times T_n = A \longrightarrow IFS^U. \tag{8}$$

It is also defined as $(\Omega, \mathrm{A}) = \Big\{(\check{d}, \Omega_{\dddot{A}}(\check{d}))\colon\ \check{d} \in \dddot{A}$, $\Omega_{\dddot{A}}(\check{d}) \in IFS^U \in [0, 1]\}$, where $\Omega_{\dddot{A}}(\check{d}) = \{\zeta, \kappa_{\Omega(\check{d})}(\zeta)$, $\delta_{\Omega(\check{d})}(\zeta)\colon\ \zeta \in U\}$, where $\kappa_{\Omega(\check{d})}(\zeta)$ and $\delta_{\Omega(\check{d})}(\zeta)$ represents the MD and NMD, respectively, such as $\kappa_{\Omega(\check{d})}(\zeta)$, $\delta_{\Omega(\check{d})}(\zeta) \in [0, 1]$, and $0 \leq \kappa_{\Omega(\check{d})}(\zeta) + \delta_{\Omega(\check{d})}(\zeta) \leq 1$.

Definition 6 (see [45]). Let U be a universe of discourse and $\mathscr{P}(U)$ be a power set of U and $t = \{t_1; t_2; t_3, \ldots, t_n\}$, $n \geq 1$ and T_i represented the set of attributes and their corresponding sub-attributes, such as $T_i \cap T_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1, 2, 3, \ldots, n\}$. Assume $T_1 \times T_2 \times T_3 \times \cdots \times = A = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha; 1 \leq k \leq \beta;$ and $1 \leq l \leq \gamma$, and $\alpha; \beta; \gamma \in N$. Then the pair $(\mathscr{F}, T_1 \times T_2 \times T_3 \times \cdots \times T_n) = (\Omega, A)$ is known as IVIFHSS and defined as follows:

$$\Omega: T_1 \times T_2 \times T_3 \times \ldots \times T_n = A \longrightarrow IVIFHS^U. \quad (9)$$

It is also defined as $(\Omega, A)\left\{(\check{d}, \Omega_{\overset{i}{A}}(\check{d})): \check{d} \in \overset{i}{A}, \Omega_{\overset{i}{A}}(\check{d}) \in IVPFS^U \in [0,1]\right\}$, where $\Omega_{\overset{i}{A}}(\check{d}) = \{\zeta, \kappa_{\Omega(\check{d})}(\zeta), \delta_{\Omega(\check{d})}(\zeta): \zeta \in U\}$, and $\kappa_{\Omega(\check{d})}(\zeta) = [\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta)]$, $\delta_{\Omega(\check{d})}(\zeta) = [\delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta)] - b \pm \sqrt{b^2 - 4ac}/2a$, where $\kappa_{\Omega(\check{d})}(\zeta)$ and $\delta_{\Omega(\check{d})}(\zeta)$ represents the MD and NMD intervals, respectively, such as, $\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta), \delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta) \in [0,1]$, and $0 \le \kappa^u_{\Omega(\check{d})}(\zeta) + \delta^u_{\Omega(\check{d})}(\zeta) \le 1$ The IVIFHSN can be stated as $\mathscr{F} = ([\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta)], [\delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta)])$.

To compute the alternative ranking, the score function and accuracy function for IVIFHSS can be stated as, if $\mathscr{F} = ([\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta)], [\delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta)])$ be an IVIFHSN. Then,

$$S(\mathscr{F}) = \frac{\kappa^l_{\Omega(\check{d})}(\zeta) + \kappa^u_{\Omega(\check{d})}(\zeta) + \delta^l_{\Omega(\check{d})}(\zeta) + \delta^u_{\Omega(\check{d})}(\zeta)}{4}. \quad (10)$$

And

$$A(\mathscr{F}) = \frac{\left(\kappa^l_{\Omega(\check{d})}(\zeta)\right)^2 + \left(\kappa^u_{\Omega(\check{d})}(\zeta)\right)^2 + \left(\delta^l_{\Omega(\check{d})}(\zeta)\right)^2 + \left(\delta^u_{\Omega(\check{d})}(\zeta)\right)^2}{2}. \quad (11)$$

3. Aggregation Operators for Interval Valued Intuitionistic Fuzzy Hypersoft Sets

We will extend the IVIFHSS with some fundamental concepts and present the operational laws for IVIFHSNs in the following section. Moreover, we prolong the IVIFHSWA and IVIFHSWG operators by utilizing the developed operational laws.

Definition 7. Let $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$, $\mathscr{F}_{\check{d}_{11}} = ([\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}], [\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}])$, and $\mathscr{F}_{\check{d}_{12}} = ([\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}], [\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}])$ be three IVIFHSNs and β be a positive real number, and by algebraic norms, we have

$$\begin{aligned}
&1.\ \mathscr{F}_{\check{d}_{11}} \oplus \mathscr{F}_{\check{d}_{12}} = \left(\left[\kappa^l_{\check{d}_{11}} + \kappa^l_{\check{d}_{12}} - \kappa^l_{\check{d}_{11}}\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{11}} + \kappa^u_{\check{d}_{12}} - \kappa^u_{\check{d}_{11}}\kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{11}}\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{11}}\delta^u_{\check{d}_{12}}\right]\right),\\
&2.\ \mathscr{F}_{\check{d}_{11}} \otimes \mathscr{F}_{\check{d}_{12}} = \left(\left[\kappa^l_{\check{d}_{11}}\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{11}}\kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{11}} + \delta^l_{\check{d}_{12}} - \delta^l_{\check{d}_{11}}\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{11}} + \delta^u_{\check{d}_{12}} - \delta^u_{\check{d}_{11}}\delta^u_{\check{d}_{12}}\right]\right),\\
&3.\ \beta\mathscr{F}_{\check{d}_k} = \left(\left[1 - \left(1 - \kappa^l_{\check{d}_k}\right)^\beta, 1 - \left(1 - \kappa^u_{\check{d}_k}\right)^\beta\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right) = \left(1 - \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^\beta, \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right),\\
&4.\ \mathscr{F}_{\check{d}_k} = \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[1 - \left(1 - \delta^l_{\check{d}_k}\right)^\beta, 1 - \left(1 - \delta^u_{\check{d}_k}\right)^\beta\right]\right) = \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], 1 - \left(1 - \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)^\beta\right).
\end{aligned} \quad (12)$$

Definition 8. Let $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be a collection of IVIFHSNs, and ω_i and ν_j are the weight vector for experts and multi sub-parameters, respectively, with given conditions $\omega_i > 0, \sum_{i=1}^n \omega_i = 1; \nu_j > 0, \sum_{j=1}^m \nu_j = 1$. Then, the IVIFHSWA operator is defined as IVIFHSWA: $\Psi^n > \Psi$.

$$\text{IVIFHSWA}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) = \oplus_{j=1}^m \nu_j\left(\oplus_{i=1}^n \omega_i \mathscr{F}_{\check{d}_{ij}}\right). \quad (13)$$

Theorem 1. *Let $\mathscr{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be a collection of IVIFHSNs, where $(i = 1,2,3\ldots\ldots, n$ and $j = 1,2,3,\ldots\ldots\ldots m)$ And the aggregated value is also an IVIFHSN, such as*

$$\text{IVIFHSWA}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) = \left(1 - \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^m\left(\prod_{i=1}^n\left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right). \quad (14)$$

ω_i and ν_j shows the expert's and multi-sub-attributes weights, respectively, such as $\omega_i > 0, \sum_{i=1}^n \omega_i = 1; \nu_j > 0, \sum_{j=1}^m \nu_j = 1$.

Proof. The proof of the above presented IVIFHSWA operator can be proved by mathematical induction: For $n = 1$, we get $\omega_1 = 1$. Then, we have

$$\begin{aligned}\text{IVIFHSWA}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right) &= \oplus_{j=1}^{m}\nu_j \\ &\mathcal{F}_{\check{d}_{1j}}\text{IVIFHSWA}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right)\\ &= \left(1-\prod_{j=1}^{m}\left(1-\left[\kappa^{l}_{\check{d}_{1j}},\kappa^{u}_{\check{d}_{1j}}\right]\right)^{\nu_j},\prod_{j=1}^{m}\left(\left[\delta^{l}_{\check{d}_{1j}},\delta^{u}_{\check{d}_{1j}}\right]\right)^{\nu_j}\right)\\ &= \left(1-\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{15}$$

For $m = 1$, we get $\nu_1 = 1$. Then, we have

$$\begin{aligned}\text{IVIFHSWA}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right) &= \oplus_{i=1}^{n}\omega_i\mathcal{F}_{\check{d}_{i1}}\\ &= \left(1-\prod_{i=1}^{n}\left(1-\left[\kappa^{l}_{\check{d}_{i1}},\kappa^{u}_{\check{d}_{i1}}\right]\right)^{\omega_i},\prod_{i=1}^{n}\left(\left[\delta^{l}_{\check{d}_{i1}},\delta^{u}_{\check{d}_{i1}}\right]\right)^{\omega_i}\right)\\ &= \left(1-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{16}$$

So, the above theorem is proved for $n = 1$ and. $m = 1$.

Assume that for $m = \alpha_1 + 1, n = \alpha_2$ and $m = \alpha_1$, $n = \alpha_2 + 1$, the above theorem holds. Such as

$$\begin{aligned}\oplus_{j=1}^{\alpha_1+1}\nu_j\left(\oplus_{i=1}^{\alpha_2}\omega_i\mathcal{F}_{\check{d}_{ij}}\right) &= \left(1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right),\\ \oplus_{j=1}^{\alpha_1}\nu_j\left(\oplus_{i=1}^{\alpha_2+1}\omega_i\mathcal{F}_{\check{d}_{ij}}\right) &= \left(1-\prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{17}$$

For $m = \alpha_1 + 1$ and $n = \alpha_2 + 1$, we have

$$\begin{aligned}\oplus_{j=1}^{\alpha_1+1}\nu_j\left(\oplus_{i=1}^{\alpha_2+1}\omega_i\mathcal{F}_{\check{d}_{ij}}\right) &= \oplus_{j=1}^{\alpha_1+1}\nu_j\left(\oplus_{i=1}^{\alpha_2}\omega_i\mathcal{F}_{\check{d}_{ij}}\oplus\omega_{\alpha_2+1}\mathcal{F}_{\check{d}_{(\alpha_2+t1)j}}\right)\\ &= \oplus_{j=1}^{\alpha_1+1}\oplus_{i=1}^{\alpha_2}\nu_j\omega_i\mathcal{F}_{\check{d}_{ij}}\oplus_{j=1}^{\alpha_1+1}\nu_j\omega_{\alpha_2+1}\mathcal{F}_{\check{d}_{(\alpha_2+t1)j}}\left(1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right.\\ &\quad\oplus 1-\prod_{j=1}^{\alpha_1+1}\left(\left(1-\left[\kappa^{l}_{\check{d}_{(\alpha_2+t1)j}},\kappa^{u}_{\check{d}_{(\alpha_2+t1)j}}\right]\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j},\\ &\quad\left.\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\oplus\prod_{j=1}^{\alpha_1+1}\left(\left(\left[\delta^{l}_{\check{d}_{(\alpha_2+t1)j}},\delta^{u}_{\check{d}_{(\alpha_2+t1)j}}\right]\right)^{\omega(\alpha_2+1)}\right)^{\nu_j}\right)\\ &= \left(1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_{2+1}}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{\alpha_{1+1}}\left(\prod_{i=1}^{\alpha_2+1}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{18}$$

Hence, it holds for $m = \alpha_1 + 1$ and $n = \alpha_2 + 1$. So, we can say that Theorem 1 holds for all values of m and. n.

Example 1. Let $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3\}$ be a set of experts with the given weight vector $\omega_i = (0.38, 0.45, 0.17)^T$. The group of experts describes the beauty of a house under-considered attributes $\text{Å} = \{e_1 = lawn,\ e_2 = security\ system\}$ with their corresponding sub-attributes Lawn = e_1 = $\{e_{11} = with\ grass,\ e_{12} = without\ grass\}$ Security system = e_2 = $\{e_{21} = guar\ ds\ ,\ e_{22} = cameras\}$. Let $\text{Å} = e_1 \times e_2$ be a set of sub-attributes

$$\begin{aligned}\text{Å} &= e_1 \times e_2 = \{e_{11}, e_{12}\} \times \{e_{21}, e_{22}\} \\ &= \{(e_{11}, e_{21}), (e_{11}, e_{22}), (e_{12}, e_{21}), (e_{12}, e_{22})\}.\end{aligned} \tag{19}$$

Let $\text{Å} = \{\breve{d}_1, \breve{d}_2, \breve{d}_3, \breve{d}_4\}$ be a set of multi-sub-attributes with weights $\nu_j = (0.2, 0.2, 0.2, 0.4)^T$. The rating values for each alternative in the form of IVIFHSN $(\mathscr{F}, \text{Å}) = ([\kappa^l_{\breve{d}_{ij}}, \kappa^u_{\breve{d}_{ij}}], [\delta^l_{\breve{d}_{ij}}, \delta^u_{\breve{d}_{ij}}])_{3\times 4}$ given as:

$$(\mathscr{F}, \text{Å}) = \begin{bmatrix} ([0.3, 0.5], [0.4, 0.5]) & ([0.4, 0.6], [0.3, 0.4]) & ([0.5, 0.7], [0.1, 0.3]) & ([0.4, 0.5], [0.3, 0.4]) \\ ([0.1, 0.5], [0.2, 0.3]) & ([0.3, 0.4], [0.5, 0.6]) & ([0.2, 0.4], [0.2, 0.3]) & ([0.1, 0.3], [0.6, 0.7]) \\ ([0.2, 0.6], [0.2, 0.3]) & ([0.5, 0.6], [0.2, 0.4]) & ([0.2, 0.4], [0.2, 0.6]) & ([0.3, 0.4], [0.5, 0.6]) \end{bmatrix}$$

$$\begin{aligned}&\text{IVIFHSWA}\left(\mathscr{F}_{\breve{d}_{11}}, t\mathscr{F}_{\breve{d}_{12}} n, q \ldots h \ldots_{\ldots} x, 7\mathscr{F}_{\breve{d}_{34}}\right) \\ &= \left(1 - \prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1 - \left[\kappa^l_{\breve{d}_{ij}}, \kappa^u_{\breve{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(\left[\delta^l_{\breve{d}_{ij}}, \delta^u_{\breve{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right) \\ &= \left(\begin{array}{c} \left(\begin{array}{c} \left\{\begin{array}{c}[0.3, 0.5]^{0.38}[0.1, 0.5]^{0.45} \\ [0.2, 0.6]^{0.17}\end{array}\right\}^{0.2} \left\{\begin{array}{c}[0.4, 0.6]^{0.38}[0.3, 0.4]^{0.45} \\ [0.5, 0.6]^{0.17}\end{array}\right\}^{0.2} \\ \left\{\begin{array}{c}[0.5, 0.7]^{0.38}[0.2, 0.4]^{0.45} \\ [0.2, 0.4]^{0.17}\end{array}\right\}^{0.2} \left\{\begin{array}{c}[0.4, 0.5]^{0.38}[0.1, 0.3]^{0.45} \\ [0.3, 0.4]^{0.17}\end{array}\right\}^{0.4} \end{array}\right), \\ 1 - \left(\begin{array}{c} \left\{\begin{array}{c}[0.5, 0.6]^{0.38}[0.7, 0.8]^{0.45} \\ [0.7, 0.8]^{0.17}\end{array}\right\}^{0.2} \left\{\begin{array}{c}[0.6, 0.7]^{0.38} \\ [0.4, 0.5]^{0.45}[0.6, 0.8]^{0.17}\end{array}\right\}^{0.2} \\ \left\{\begin{array}{c}[0.7, 0.9]^{0.38}[0.7, 0.8]^{0.45} \\ [0.4, 0.8]^{0.17}\end{array}\right\}^{0.2} \left\{\begin{array}{c}[0.6, 0.7]^{0.38}[0.3, 0.4]^{0.45} \\ [0.4, 0.5]^{0.17}\end{array}\right\}^{0.4} \end{array}\right) \end{array}\right) \\ &= ([0.3198, 0.4719], [0.2798, 0.5617]).\end{aligned} \tag{20}$$

3.1. Properties of IVIFHSWA Operator

3.1.1. Idempotency. If $\mathscr{F}_{\breve{d}_{ij}} = \mathscr{F}_{\breve{d}_k} = ([\kappa^l_{\breve{d}_{ij}}, \kappa^u_{\breve{d}_{ij}}], [\delta^l_{\breve{d}_{ij}}, \delta^u_{\breve{d}_{ij}}])$ $\forall\, i, j$, then

$$\text{IVIFHSWA}\left(\mathscr{F}_{\breve{d}_{11}}, \mathscr{F}_{\breve{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\breve{d}_{nm}}\right) = \mathscr{F}_{\breve{d}_k}. \tag{21}$$

Proof. As we know that all $\mathscr{F}_{\breve{d}_{ij}} = \mathscr{F}_{\breve{d}_k} = ([\kappa^l_{\breve{d}_{ij}}, \kappa^u_{\breve{d}_{ij}}], [\delta^l_{\breve{d}_{ij}}, \delta^u_{\breve{d}_{ij}}])$, then, we have

$$\begin{aligned}&\text{IVIFHSWA}\left(\mathscr{F}_{\breve{d}_{11}}, \mathscr{F}_{\breve{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\breve{d}_{nm}}\right) \\ &= \left(1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\breve{d}_{ij}}, \kappa^u_{\breve{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\delta^l_{\breve{d}_{ij}}, \delta^u_{\breve{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right) \\ &= \left(1 - \left(\left(1 - \left[\kappa^l_{\breve{d}_{ij}}, \kappa^u_{\breve{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}, \left(\left(\left[\delta^l_{\breve{d}_{ij}}, \delta^u_{\breve{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}\right).\end{aligned} \tag{22}$$

As $\sum_{j=1}^{m} \nu_j = 1$ and $\sum_{i=1}^{n} \omega_i = 1$, then we have

$$\begin{aligned} &IVIFHSWA\left(\mathcal{F}_{\check{d}11}, \mathcal{F}_{\check{d}12},, \ldots\ldots.., \mathcal{F}_{\check{d}nm}\right) \\ &= \left(1-\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right), \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right) \\ &= \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right) \\ &= \mathcal{F}_{\check{d}_k}. \end{aligned} \tag{23}$$

□

3.1.2. Boundedness. Let $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be a collection of IVIFHSNs where $\mathcal{F}^-_{\check{d}_{ij}} = \left(\min_j \min_i \left\{[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}]\right\}, \max_j \max_i \left\{[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]\right\}\right)$ and $\mathcal{F}^+_{\check{d}_{ij}} = \left(\max_j \max_i \left\{[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}]\right\}, \min_j \min_i \left\{[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]\right\}\right)$, then

$$\mathcal{F}^-_{\check{d}_{ij}} \leq \mathrm{IVIFHSWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) \leq \mathcal{F}^+_{\check{d}_{ij}}. \tag{24}$$

Proof. As we know that $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be an IVIFHSN, then

$$\begin{aligned} &\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \leq \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] \leq \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \\ &\Rightarrow 1-\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \leq 1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] \leq 1-\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \\ &\Leftrightarrow \left(1-\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i} \leq \left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \leq \left(1-\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i} \\ &\Leftrightarrow \left(1-\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i} \leq \left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \leq \left(1-\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i} \\ &\Leftrightarrow \left(1-\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{i=1}^n \omega_i} \leq \prod_{i=1}^n \left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \leq \left(1-\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{i=1}^n \omega_i} \\ &\Leftrightarrow \left(1-\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{j=1}^n \nu_j} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq \left(1-\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{j=1}^n \nu_j} \\ &\Leftrightarrow 1-\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq 1-\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \\ &\Leftrightarrow \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \leq 1-\prod_{j=1}^m \left(\prod_{i=1}^n \left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}. \end{aligned} \tag{25}$$

Similarly,

$$\min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}. \tag{26}$$

Let $\mathrm{IVIFHSWA}(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}) = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]) = \mathcal{F}_{\check{d}_{ij}}$. So, (a) and (b) can be transferred into the form:

$\min_j \min_i \left\{[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}]\right\} \leq \mathscr{F}_{\check{d}_k} \leq \max_j \max_i \left\{[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}]\right\}$ and $\min_j \min_i \left\{[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]\right\} \leq \mathscr{F}_{\check{d}_k} \leq \max_j \max_i \left\{[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]\right\}$ respectively.

Using the score function, we have

$$S\left(\mathscr{F}_{\check{d}_k}\right) = \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4} \leq \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} = S\left(\mathscr{F}^-_{\check{d}_k}\right),$$
$$S\left(\mathscr{F}_{\check{d}_k}\right) = \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4} \geq \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} = S\left(\mathscr{F}^+_{\check{d}_k}\right). \quad (27)$$

Using order relation among two IVIFHSNs, we have

$$\mathscr{F}^-_{\check{d}_k} \leq \text{IVIFHSWA}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \leq \mathscr{F}^+_{\check{d}_k}. \quad (28)$$

□

3.1.3. Shift Invariance. Let $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be an IVIFHSN. Then

$$IVIFHSWA\left(\mathscr{F}_{\check{d}_{11}} \oplus \mathscr{F}_{\check{d}_k}, \mathscr{F}_{\check{d}_{12}} \oplus \mathscr{F}_{\check{d}_k}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}} \oplus \mathscr{F}_{\check{d}_k}\right) = IVIFHSWA\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \oplus \mathscr{F}_{\check{d}_k}. \quad (29)$$

Proof. Let $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ and $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be two IVIFHSNs. Then, using Definition 7 (1)

$$\begin{aligned}
\mathscr{F}_{\check{d}_k} \oplus \mathscr{F}_{\check{d}_{ij}} &= \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right] + \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right), So, \\
&IVIFHSWA\left(\mathscr{F}_{\check{d}_{11}} \oplus \mathscr{F}_{\check{d}_k}, \mathscr{F}_{\check{d}_{12}} \oplus \mathscr{F}_{\check{d}_k}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}} \oplus \mathscr{F}_{\check{d}_k}\right) \\
&= \oplus_{j=1}^m \nu_j \left(\oplus_{i=1}^n \omega_i \left(\mathscr{F}_{\check{d}_{ij}} \oplus \mathscr{F}_{\check{d}_k}\right)\right) \\
&= \left(1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^m \left(\prod_{i=1}^n \left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \left(\left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j}\right) \\
&= \left(1 - \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right) \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right] \prod_{j=1}^m \left(\prod_{i=1}^n \left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right) \\
&= \left(\left(1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^m \left(\prod_{i=1}^n \left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right) \oplus \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)\right) \\
&= IVIFHSWA\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \oplus \mathscr{F}_{\check{d}_k}.
\end{aligned} \quad (30)$$

□

3.1.4. Homogeneity. Prove that IVIFHSWA $(\beta\mathscr{F}_{\check{d}_{11}}, \beta\mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \beta\mathscr{F}_{\check{d}_{nm}}) = \beta$ IVIFHSWA $(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}})$ for any positive real number β.

Proof. Let $\mathscr{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}ij}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be an IVIFHSN and $\beta > 0$. Then using Definition 7, we have

$$\beta\mathscr{F}_{\check{d}_{ij}} = \left(1 - \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\beta}, \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]^{\beta}\right) \tag{31}$$

So,

$$\begin{aligned}
&\left(\beta\mathscr{F}_{\check{d}_{11}}, \beta\mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \beta\mathscr{F}_{\check{d}_{nm}}\right) \\
&= \left(1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\beta\omega_i}\right)^{\nu_j}, \dot{\prod}_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\beta\omega_i}\right)^{\nu_j}\right) \\
&= \left(1 - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}, \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}\right) \\
&= \beta\,\text{IVIFHSWA}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right).
\end{aligned} \tag{32}$$

□

Definition 9. Let $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be a collection of IVIFHSNs, and ω_i and ν_j are the weight vector for experts and multi sub-parameters, respectively, with given conditions $\omega_i > 0, \sum_{i=1}^{n}\omega_i = 1$; $\nu_j > 0, \sum_{j=1}^{m}\nu_j = 1$. Then, the IVIFHSWG operator is defined as IVIFHSWG: $\Psi^n > \Psi$.

$$\begin{aligned}
&\text{IVIFHSWG}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \\
&\quad = \otimes_{j=1}^{m}\nu_j\left(\otimes_{i=1}^{n}\omega_i\mathscr{F}_{\check{d}_{ij}}\right).
\end{aligned} \tag{33}$$

Theorem 2. *Let $\mathscr{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be a collection of IVIFHSNs, where $(i = 1, 2, 3 \ldots\ldots, n$ and $j = 1, 2, 3, \ldots\ldots\ldots m)$ and the aggregated value is also an IVIFHSN, such as*

$$\begin{aligned}
&\text{IVIFHSWG}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \\
&= \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right),
\end{aligned} \tag{34}$$

ω_i and ν_j are expert's and multi-sub-attributes weights respectively, such as. $\omega_i > 0, \sum_{i=1}^{n}\omega_i = 1$; $\nu_j > 0, \sum_{j=1}^{m}\nu_j = 1$.

Proof. The proof of the above theorem can be proved using mathematical induction. For $n = 1$, we get $\omega_1 = 1$. Then, we have

$$\begin{aligned}
&\text{IVIFHSWG}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) = \otimes_{j=1}^{m}\mathscr{F}^{\nu_j}_{\check{d}_{1j}} \\
&\text{IVIFHSWG}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \\
&= \left(\prod_{j=1}^{m}\left(\left[\kappa^l_{\check{d}_{1j}}, \kappa^u_{\check{d}_{1j}}\right]\right)^{\nu_j}, 1 - \prod_{j=1}^{m}\left(1 - \left[\delta^l_{\check{d}_{1j}}, \delta^u_{\check{d}_{1j}}\right]\right)^{\nu_j}\right) \\
&= \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).
\end{aligned} \tag{35}$$

For $m = 1$, we get $\nu_1 = 1$. Then, we have

$$\begin{aligned}&\text{IVIFHSWG}\left(\mathcal{F}_{\check{d}_{11}}, t\mathcal{F}_{\check{d}_{21}} n, q \ldots h \ldots_{\ldots} x, 7\mathcal{F}_{\check{d}_{n1}}\right) = \otimes_{i=1}^{n}\left(\mathcal{F}_{\check{d}_{n1}}\right)^{\omega_i}\\ &= \left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{i1}}, \kappa^u_{\check{d}_{i1}}\right]\right)^{\omega_i}, 1-\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{i1}}, \delta^u_{\check{d}_{i1}}\right]\right)^{\omega_i}\right)\\ &= \left(\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, 1-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{36}$$

So, for $n = 1$ and $m = 1$ the IVIFHSWG operators hold.

Now, for $m = \alpha_1 + 1, n = \alpha_2$ and $m = \alpha_1, n = \alpha_2 + 1$, such as

$$\begin{aligned}&\otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\\ &= \left(\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}, 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\right),\\ &\otimes_{j=1}^{\alpha_1}\left(\otimes_{i=1}^{\alpha_2+1}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\\ &= \left(\prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, 1-\prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{37}$$

For $m = \alpha_1 + 1$ and $n = \alpha_2 + 1$, we have

$$\begin{aligned}\otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2+1}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} &= \otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i} \otimes \left(\mathcal{F}_{\check{d}_{(\alpha_2+t1)j}}\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j}\\ &= \otimes_{j=1}^{\alpha_1+1} \otimes_{i=1}^{\alpha_2}\left(\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \otimes_{j=1}^{\alpha_1+1}\left(\left(\mathcal{F}_{\check{d}_{(\alpha_2+t1)j}}\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j}\\ &= \begin{pmatrix}\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \otimes \prod_{j=1}^{\alpha_1+1}\left(\left(\left[\kappa^l_{\check{d}_{(\alpha_2+1)j}}, \kappa^u_{\check{d}_{(\alpha_2+1)j}}\right]\right)^{\omega_{(\alpha_2+1)}}\right)^{\nu_j},\\ 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \otimes 1-\prod_{j=1}^{\alpha_1+1}\left(\left(1-\left[\delta^l_{\check{d}_{(\alpha_2+t1)j}}, \delta^u_{\check{d}_{(\alpha_2+t1)j}}\right]\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j}\end{pmatrix}\\ &= \left(\prod_{j=1}^{\alpha_{1+1}}\left(\prod_{i=1}^{\alpha_2+1}\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_{2+1}}\left(1-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right).\end{aligned} \tag{38}$$

So, it is proved the for $m = \alpha_1 + 1$ and $n = \alpha_2 + 1$ holds. So, the IVIFHSWG operator holds for all values of m and n. □

Example 2. Let $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3\}$ be a set of experts with the given weight vector $\omega_i = (0.38, 0.45, 0.17)^T$. The group of experts describes the beauty of a house under-considered attributes $\text{Å} = \{e_1 = lawn,\ e_2 = security\,system\}$ with their corresponding sub-attributes Lawn $= e_1 = \{e_{11} = with\,grass, e_{12} = without\,grass\}$ Security system $= e_2 = \{e_{21} = guar\ ds\ , e_{22} = cameras\}$. Let $\text{Å} = e_1 \times e_2$ be a set of sub-attributes

$$\text{Å} = e_1 \times e_2 = \{e_{11}, e_{12}\} \times \{e_{21}, e_{22}\} = \{(e_{11}, e_{21}), (e_{11}, e_{22}), (e_{12}, e_{21}), (e_{12}, e_{22})\}. \tag{39}$$

Let $\text{Å} = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ be a set of multi-sub-attributes with weights $\nu_j = (0.2, 0.2, 0.2, 0.4)^T$. The rating values for each alternative in the form of IVIFHSN $(\mathcal{F}, \text{Å}) = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])_{3\times4}$ given as

$$(\mathcal{F},\text{Å})=\begin{bmatrix}([0.3,0.5],[0.4,0.5]) & ([0.4,0.6],[0.3,0.4]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.5],[0.3,0.4])\\ ([0.1,0.5],[0.2,0.3]) & ([0.3,0.4],[0.5,0.6]) & ([0.2,0.4],[0.2,0.3]) & ([0.1,0.3],[0.6,0.7])\\ ([0.2,0.6],[0.2,0.3]) & ([0.5,0.6],[0.2,0.4]) & ([0.2,0.4],[0.2,0.6]) & ([0.3,0.4],[0.5,0.6])\end{bmatrix}$$

$$\begin{aligned}&\text{IVIFHSWG}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{34}}\right)\\&=\left(\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},1-\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1-\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\\&=\left(\begin{array}{c}\left(\begin{array}{c}\left\{\begin{array}{c}[0.3,0.5]^{0.38}[0.1,0.5]^{0.45}\\ [0.2,0.6]^{0.17}\end{array}\right\}^{0.2}\left\{\begin{array}{c}[0.4,0.6]^{0.38}[0.3,0.4]^{0.45}\\ [0.5,0.6]^{0.17}\end{array}\right\}^{0.2}\\ \left\{\begin{array}{c}[0.5,0.7]^{0.38}[0.2,0.4]^{0.45}\\ [0.2,0.4]^{0.17}\end{array}\right\}^{0.2}\left\{\begin{array}{c}[0.4,0.5]^{0.38}[0.1,0.3]^{0.45}\\ [0.3,0.4]^{0.17}\end{array}\right\}^{0.4}\end{array}\right),\\ 1-\left(\begin{array}{c}\left\{\begin{array}{c}[0.4,0.5]^{0.38}[0.2,0.3]^{0.45}\\ [0.2,0.3]^{0.17}\end{array}\right\}^{0.2}\left\{\begin{array}{c}[0.3,0.4]^{0.38}\\ [0.5,0.6]^{0.45}[0.2,0.4]^{0.17}\end{array}\right\}^{0.2}\\ \left\{\begin{array}{c}[0.1,0.3]^{0.38}[0.2,0.3]^{0.45}\\ [0.2,0.6]^{0.17}\end{array}\right\}^{0.2}\left\{\begin{array}{c}[0.3,0.4]^{0.38}[0.6,0.7]^{0.45}\\ [0.5,0.6]^{0.17}\end{array}\right\}^{0.4}\end{array}\right)\end{array}\right)\\&=([0.2798,0.5617],[0.3198,0.4719]).\end{aligned}\tag{40}$$

3.2. Properties of IVIFSWG

3.2.1. Idempotency. If $\mathcal{F}_{\check{d}_{ij}}=\mathcal{F}_{\check{d}_k}=([\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}],[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}])$ $\forall i,j$, then

$$\text{IVIFHSWG}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right)=\mathcal{F}_{\check{d}_k}.\tag{41}$$

Proof. As we know that all $\mathcal{F}_{\check{d}_{ij}}=\mathcal{F}_{\check{d}_k}=([\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}],[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}])$, then we have

$$\begin{aligned}&\text{IVIFHSWG}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right)\\&=\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\\&=\left(\left(\left(\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j},1-\left(\left(1-\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}\right).\end{aligned}\tag{42}$$

As $\sum_{j=1}^{m}\nu_j=1$ and $\sum_{i=1}^{n}\omega_i=1$, then we have

$$\begin{aligned}&=\left(\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right],1-\left(1-\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)\right)\\&=\left(\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right],\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)\\&=\mathcal{F}_{\check{d}_k}.\end{aligned}\tag{43}$$

□

3.2.2. Boundedness. Let $\mathcal{F}_{\check{d}_{ij}}$ be a collection of IVIFHSNs where $\mathcal{F}^{-}_{\check{d}_{ij}}=\left(\min_j\min_i\left\{[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}]\right\},\max_j\max_i\left\{[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}]\right\}\right)$ and $\mathcal{F}^{+}_{\check{d}_{ij}}=\left(\max_j\max_i\left\{[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}]\right\},\min_j\min_i\left\{[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}]\right\}\right)$, then

$$\mathcal{F}^{-}_{\check{d}_{ij}}\leq\text{IVIFHSWG}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right)\leq\mathcal{F}^{+}_{\check{d}_{ij}}.\tag{44}$$

Proof. As we know that $\mathscr{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be an IVIFHSN, then

$$\begin{aligned}
&\min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \leq \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right] \leq \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\\
&\Rightarrow 1 - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \leq 1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right] \leq 1 - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\\
&\Leftrightarrow \left(1 - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i} \leq \left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \leq \left(1 - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i}\\
&\Leftrightarrow \left(1 - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{i=1}^n \omega_i} \leq \prod_{i=1}^n \left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \leq \left(1 - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{i=1}^n \omega_i}\\
&\Leftrightarrow \left(1 - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{j=1}^n \nu_j} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq \left(1 - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{j=1}^n \nu_j}\\
&\Leftrightarrow 1 - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq 1 - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\\
&\Leftrightarrow \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \leq 1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}.
\end{aligned} \tag{45}$$

Similarly,

$$\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \leq \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}. \tag{46}$$

If $\text{IVIFHSWG}(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}) = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]) = \mathscr{F}_{\check{d}_k}$, then inequalities (C) and (D) can be transferred into the form: $\min_j \min_i \{[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}]\} \leq \mathscr{M}_\sigma \leq \max_j \max_i \{[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}]\}$ and $\min_j \min_i \{[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]\} \leq \mathscr{F}_{\check{d}_k} \leq \max_j \max_i \{[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]\}$ respectively.

Using the score function,

$$\begin{aligned}
S\left(\mathscr{F}_{\check{d}_k}\right) &= \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4} \leq \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} - \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} = S\left(\mathscr{F}^-_{\check{d}_{ij}}\right),\\
S\left(\mathscr{F}_{\check{d}_k}\right) &= \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4} \geq \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} - \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} = S\left(\mathscr{F}^+_{\check{d}_{ij}}\right).
\end{aligned} \tag{47}$$

By order relation between two IVIFHSNs, we have

$$\mathscr{F}^-_{\check{d}_k} \leq \text{IVIFHSWG}\left(\mathscr{F}_{\check{d}_{11}}, \mathscr{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\check{d}_{nm}}\right) \leq \mathscr{F}^+_{\check{d}_k} \tag{48}$$

□

3.2.3. Shift Invariance. Let $\mathscr{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be an IVIFHSN. Then,

$$IVIFHSWG\left(\mathscr{F}_{\check{d}_{11}}\otimes\mathscr{F}_{\check{d}_{k}},\mathscr{F}_{\check{d}_{12}}\otimes\mathscr{F}_{\check{d}_{k}},\ldots\ldots\ldots,\mathscr{F}_{\check{d}_{nm}}\otimes\mathscr{F}_{\check{d}_{k}}\right)=IVIFHSWG\left(\mathscr{F}_{\check{d}_{11}},\mathscr{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathscr{F}_{\check{d}_{nm}}\right)\otimes\mathscr{F}_{\check{d}_{k}}. \tag{49}$$

Proof. Let $\mathscr{F}_{\check{d}_k}=([\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}],[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}])$ and $\mathscr{F}_{\check{d}_{ij}}=([\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}],[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}])$ be two IVIFHSNs. Then, using Definition 7 (2)

$$\mathscr{F}_{\check{d}_k}\otimes\mathscr{F}_{\check{d}_{ij}}=\left(\left[\kappa^l_{\check{d}_k}\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_k}\kappa^u_{\check{d}_{ij}}\right],\left[\delta^l_{\check{d}_k}+\delta^l_{\check{d}_{ij}}-\delta^l_{\check{d}_k}\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_k}+\delta^u_{\check{d}_{ij}}-\delta^u_{\check{d}_k}\delta^u_{\check{d}_{ij}}\right]\right). \tag{50}$$

So,

$$\begin{aligned}
&IVIFHSWG\left(\mathscr{F}_{\check{d}_{11}}\otimes\mathscr{F}_{\check{d}_{k}},\mathscr{F}_{\check{d}_{12}}\otimes\mathscr{F}_{\check{d}_{k}},\ldots\ldots\ldots,\mathscr{F}_{\check{d}_{nm}}\otimes\mathscr{F}_{\check{d}_{k}}\right)\\
&=\otimes_{j=1}^{m}\nu_j\left(\otimes_{i=1}^{n}\omega_i\left(\mathscr{F}_{\check{d}_{ij}}\otimes\mathscr{F}_{\check{d}_k}\right)\right)\\
&=\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j},1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\left(1-\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\\
&=\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right]\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},1-\left(1-\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\\
&=\left(\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\otimes\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right],\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)\right)\\
&\quad IVIFHSWG\left(\mathscr{F}_{\check{d}_{11}},\mathscr{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathscr{F}_{\check{d}_{nm}}\right)\otimes\mathscr{F}_{\check{d}_k}.
\end{aligned} \tag{51}$$

□

3.2.4. Homogeneity. Prove that IVIFHSWG $(\beta\mathscr{F}_{\check{d}_{11}},\beta\mathscr{F}_{\check{d}_{12}},\ldots\ldots\ldots,\beta\mathscr{F}_{\check{d}_{nm}})=\beta$ IVIFHSWG $(\mathscr{F}_{\check{d}_{11}},\mathscr{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathscr{F}_{\check{d}_{nm}})$ for any positive real number β.

Proof. Let $\mathscr{F}_{\check{d}_{ij}}=([\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}],[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}])$ be an IVIFHSN and $\beta>0$. Then using Definition 7, we have

$$\mathscr{F}_{\check{d}_k}=\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right],1-\left(1-\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)^{\beta}\right). \tag{52}$$

So,

$$\begin{aligned}
&\text{IVIFHSWG}\left(\beta\mathscr{F}_{\check{d}_{11}},\beta\mathscr{F}_{\check{d}_{12}},\ldots\ldots\ldots,\beta\mathscr{F}_{\check{d}_{nm}}\right)\\
&=\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left(1-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\beta}\right)^{\nu_j}\right)\\
&=\left(\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\beta\omega_i}\right)^{\nu_j}\right),1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}\right)\\
&=\left(\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta},1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}\right)\\
&=\beta\,\text{IVIFHSWG}\left(\mathscr{F}_{\check{d}_{11}},\mathscr{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathscr{F}_{\check{d}_{nm}}\right).
\end{aligned} \tag{53}$$

□

4. Multi-Criteria Group Decision-Making Approach Based on Proposed Operators

To validate the implications of planned AOs, a DM approach is developed to remove MCGDM obstacles. In addition, numerical illustration is provided to endorse the convenience of the proposed method.

4.1. Proposed MCGDM Approach. Let $\mathfrak{F} = \{\mathfrak{F}^1, \mathfrak{F}^2, \mathfrak{F}^3, \ldots, \mathfrak{F}^s\}$ and $\mathcal{U} = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \ldots, \mathcal{U}_r\}$ be the set of alternatives and experts, respectively. The weights of experts are given as $\omega_i = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)^T$ such that $\omega_i > 0, \sum_{i=1}^n \omega_i = 1$; $\nu_j > 0, \sum_{j=1}^m \nu_j = 1$. Suppose Let $\mathfrak{L} = \{e_1, e_2, e_3, \ldots, e_m\}$ be the set of attributes with their corresponding multi-sub-attributes such as $\mathfrak{L}' = \{(e_{1\rho} \times e_{2\rho} \times \cdots \times e_{m\rho}) \text{ for all } \rho \in \{1, 2, \ldots, t\}\}$ with weights $\nu = (\nu_1, \nu_2, \nu_3, \ldots, \nu_n)^T$ such that $\nu_i > 0, \sum_{i=1}^n \nu_i = 1$. And can be stated as $\mathfrak{L}' = \{\breve{d}_\partial \colon \partial \in \{1, 2, \ldots, m\}\}$. The group of experts $\{\kappa^i \colon i = 1, 2, \ldots, n\}$ assess the alternatives $\{\mathfrak{H}^{(z)} \colon z = 1, 2, \ldots, s\}$ under the chosen sub-attributes $\{\breve{d}_\partial \colon \partial = 1, 2, \ldots, k\}$ in the form of IVIFHSNs such as $(\mathfrak{F}^{(z)}_{\breve{d}_{ik}})_{n\times m} = ([\kappa^l_{\breve{d}_{ik}}, \kappa^u_{\breve{d}_{ik}}], [\delta^l_{\breve{d}_{ik}}, \delta^u_{\breve{d}_{ik}}])_{n*m}$. Where $0 \le \kappa^l_{\breve{d}_{ik}}, \kappa^u_{\breve{d}_{ik}}, \delta^l_{\breve{d}_{ik}}, \delta^u_{\breve{d}_{ik}} \le 1$ and $0 \le (\kappa^u_{\breve{d}_{ik}})^2 + (\delta^u_{\breve{d}_{ik}})^2 \le 1$ for all i, k. The group of experts gives their opinion on each alternative in IVIFHSNs. The algorithmic rule-based on developed operators is given as follows:

Step 1: Expert's opinion for each alternative in the form of IVIFHSNs.

$$\left(\mathfrak{F}^{(z)}_{\breve{d}_{ik}}\right)_{n\times m} = \left(\left[\kappa^l_{\breve{d}_{ik}}, \kappa^u_{\breve{d}_{ik}}\right], \left[\delta^l_{\breve{d}_{ik}}, \delta^u_{\breve{d}_{ik}}\right]\right)_{n*m}$$
$$= \begin{bmatrix} \left(\left[\kappa^l_{\breve{d}_{11}}, \kappa^u_{\breve{d}_{11}}\right], \left[\delta^l_{\breve{d}_{11}}, \delta^u_{\breve{d}_{11}}\right]\right) & \left(\left[\kappa^l_{\breve{d}_{12}}, \kappa^u_{\breve{d}_{12}}\right], \left[\delta^l_{\breve{d}_{12}}, \delta^u_{\breve{d}_{12}}\right]\right) & \cdots & \left(\left[\kappa^l_{\breve{d}_{1m}}, \kappa^u_{\breve{d}_{1m}}\right], \left[\delta^l_{\breve{d}_{1m}}, \delta^u_{\breve{d}_{1m}}\right]\right) \\ \left(\left[\kappa^l_{\breve{d}_{21}}, \kappa^u_{\breve{d}_{21}}\right], \left[\delta^l_{\breve{d}_{21}}, \delta^u_{\breve{d}_{21}}\right]\right) & \left(\left[\kappa^l_{\breve{d}_{22}}, \kappa^u_{\breve{d}_{22}}\right], \left[\delta^l_{\breve{d}_{22}}, \delta^u_{\breve{d}_{22}}\right]\right) & & \left(\left[\kappa^l_{\breve{d}_{2n}}, \kappa^u_{\breve{d}_{2n}}\right], \left[\delta^l_{\breve{d}_{2m}}, \delta^u_{\breve{d}_{2m}}\right]\right) \\ \vdots & \ddots & \vdots & \\ \left(\left[\kappa^l_{\breve{d}_{n1}}, \kappa^u_{\breve{d}_{n1}}\right], \left[\delta^l_{\breve{d}_{n1}}, \delta^u_{\breve{d}_{n1}}\right]\right) & \left(\left[\kappa^l_{\breve{d}_{n2}}, \kappa^u_{\breve{d}_{n2}}\right], \left[\delta^l_{\breve{d}_{n2}}, \delta^u_{\breve{d}_{n2}}\right]\right) & \cdots & \left(\left[\kappa^l_{\breve{d}_{nm}}, \kappa^u_{\breve{d}_{nm}}\right], \left[\delta^l_{\breve{d}_{nm}}, \delta^u_{\breve{d}_{nm}}\right]\right) \end{bmatrix}. \tag{54}$$

Step 2: Develop the normalized decision matrices for each alternative by converting the cost type attributes to benefit type using the normalization rule.

$$\mathscr{F}_{\breve{d}_{ik}} = \begin{cases} \mathscr{F}^c_{\breve{d}_{ij}} = \left(\left[\delta^l_{\breve{d}_{ik}}, \delta^u_{\breve{d}_{ik}}\right], \left[\kappa^l_{\breve{d}_{ik}}, \kappa^u_{\breve{d}_{ik}}\right]\right)_{n\times m} \text{cost type parameter}, \\ \mathscr{F}_{\breve{d}_{ij}} = \left(\left[\kappa^l_{\breve{d}_{ik}}, \kappa^u_{\breve{d}_{ik}}\right], \left[\delta^l_{\breve{d}_{ik}}, \delta^u_{\breve{d}_{ik}}\right]\right)_{n\times m} \text{benefit type parameter}. \end{cases} \tag{55}$$

Step 3: Compute the aggregated values using IVIFHSWA and IVIFHSWG operators for each alternative.

Step 4: Compute the score values for each alternative using the score function.

Step 5: Determine the most suitable alternative.

Step 6: Alternatives ranking.

4.2. Numerical Example. It is an intelligent transformation of fossil waste energy, such as natural gas first converted into hydrogen. The energy content per kilogram of hydrogen is 120 MJ. The advantage of methanol is an extraordinary six times [47]. Hydrogen has a bit of volumetric energy density associated with its particular gravimetric density. A stable thickness of up to 700 bar is not a large enough property for hydrocarbons like gasoline and diesel. Only liquid hydrogen can affect a realistic extent, still less than a quarter of the quantity of gasoline. Therefore, hydrogen vessels for motor tenders will surmount more than used fluid hydrocarbon containers [48]. Cryogenic storage containers are also considered cryogenic storage containers. The Dewar is a double-walled super-insulated container. The vehicles fluid oxygen, nitrogen, hydrogen, helium, and argon, temperatures <110 K/163°C. The assortment method begins with a preliminary screening of the material used for the dashboard and is captivated by the validation configuration built into the application. Defining the ingredients used by the preliminary MS of the dashboard fashioning is serious. Then select from four material assessment abilities: $\mathfrak{F}^1$ = Ti–6Al–4V, $\mathfrak{F}^2$ = SS301–FH, $\mathfrak{F}^3$ = 70Cu–30Zn, and $\mathfrak{F}^4$ = Inconel 718. The aspect of material assortment is specified as follows: $L = \{d_1 =$ Specific gravity = attaining data around the meditation of resolutions of numerous materials,

Table 1: Decision Matrix for $\mathfrak{I}^1$ in the form of IVIFHSN.

	$\check{\mathbf{d}}_1$	$\check{\mathbf{d}}_2$	$\check{\mathbf{d}}_3$	$\check{\mathbf{d}}_4$
$\mathcal{U}_1$	([0.4, 0.5], [0.2, 0.5])	([0.2, 0.4], [0.5, 0.6])	([0.1, 0.3], [0.2, 0.5])	([0.2, 0.4], [0.2, 0.6])
$\mathcal{U}_2$	([0.2, 0.4], [0.2, 0.6])	([0.1, 0.3], [0.4, 0.5])	([0.2, 0.3], [0.3, 0.7])	([0.2, 0.4], [0.2, 0.5])
$\mathcal{U}_3$	([0.3, 0.5], [0.1, 0.4])	([0.4, 0.5], [0.2, 0.4])	([0.4, 0.5], [0.3, 0.4])	([0.2, 0.6], [0.2, 0.4])
$\mathcal{U}_4$	([0.4, 0.6], [0.3, 0.4])	([0.1, 0.3], [0.3, 0.6])	([0.3, 0.4], [0.3, 0.5])	([0.3, 0.4], [0.3, 0.5])

d_2 = Toughness index, d_3 = Yield stress, d_4 = Easily accessible}. The corresponding sub-attributes of the considered parameters, Specific gravity = attaining data around the meditation of resolutions of numerous materials = d_1 = $\{d_{11}$ = assess corporal variations, d_{12} = govern the degree of regularity among tasters}, Toughness index = d_2 = $\{d_{21}$ = Charpy V − Notch Impact Energy, d_{22} = Plane Strain Fracture Toughness}, Yield stress = $d_3\{d_{31}$ = Yield stress}, Easily accessible = d_4 = $\{d_{41}$ = Easily accessible}. Let $\mathfrak{L}' = d_1 \times d_2 \times d_3 \times d_4$ be a set of sub-attributes.

$$\begin{aligned}\mathfrak{L}' &= d_1 \times d_2 \times d_3 \times d_4 \\ &= \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} \times \{d_{31}\} \times \{d_{41}\} \\ &= \left\{\begin{array}{l}(d_{11}, d_{21}, d_{31}\, d_{41}),\ (d_{11}, d_{22}, d_{31}, d_{41}), \\ (d_{12}, d_{21}, d_{31}, d_{41}),\ (d_{12}, d_{22}, d_{31}, d_{41})\end{array}\right\} \\ &= L=' \left\{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\right\}\end{aligned}$$

be a set of all sub-attributes with weights $(0.3, 0.1, 0.2, 0.4)^T$. Let $\{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4\}$ be a set of four experts with weights $(0.1, 0.2, 0.4, 0.3)^T$. To judge the optimal alternative, experts deliver their preferences in IVIFHSNs.

4.2.1. By IVIFHSWA Operator

Step 1: The expert's opinion in the IVIFHSNs form for each alternative is given in Tables 1–4.

Step 2: All parameters are of the same type. So, no need to normalize.

Step 3: Compute the aggregated values for each alternative using the IVIFHSWA operator.

$$\Theta_1 = \left(1 - \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)\frac{1}{2}$$

$$= \left(\begin{array}{c} 1 - \left(\begin{array}{c} \left\{\begin{array}{l}[0.5, 0.6]^{0.1}[0.6, 0.8]^{0.2} \\ [0.5, 0.7]^{0.4}[0.4, 0.6]^{0.3}\end{array}\right\}^{0.3} \left\{\begin{array}{l}[0.6, 0.8]^{0.1}[0.7, 0.9]^{0.2} \\ [0.5, 0.6]^{0.4}[0.7, 0.9]^{0.3}\end{array}\right\}^{0.1} \\ \left\{\begin{array}{l}[0.7, 0.9]^{0.1}[0.7, 0.8]^{0.2} \\ [0.5, 0.6]^{0.4}[0.6, 0.7]^{0.3}\end{array}\right\}^{0.2} \left\{\begin{array}{l}[0.6, 0.8]^{0.1}[0.6, 0.8]^{0.2} \\ [0.4, 0.8]^{0.4}[0.6, 0.7]^{0.3}\end{array}\right\}^{0.4} \end{array}\right), \\ \left(\begin{array}{c} \left\{\begin{array}{l}[0.2, 0.5]^{0.1}[0.2, 0.6]^{0.2} \\ [0.1, 0.4]^{0.4}[0.3, 0.4]^{0.3}\end{array}\right\}^{0.3} \left\{\begin{array}{l}[0.5, 0.6]^{0.1}[0.4, 0.5]^{0.2} \\ [0.2, 0.4]^{0.4}[0.3, 0.6]^{0.3}\end{array}\right\}^{0.1} \\ \left\{\begin{array}{l}[0.2, 0.5]^{0.1}[0.3, 0.7]^{0.2} \\ [0.3, 0.4]^{0.4}[0.3, 0.5]^{0.3}\end{array}\right\}^{0.2} \left\{\begin{array}{l}[0.2, 0.6]^{0.1}[0.2, 0.5]^{0.2} \\ [0.2, 0.4]^{0.4}[0.3, 0.5]^{0.3}\end{array}\right\}^{0.4} \end{array}\right) \end{array}\right)$$

$$= \left(\left[0.4401,\ 0.5121\right],\ \left[0.2615,\ 0.5173\right]\right),$$

$$\Theta_2 = \left(1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)$$

$$= \begin{pmatrix} 1-\begin{pmatrix} \begin{Bmatrix} [0.6,\ 0.7]^{0.1}[0.5,\ 0.7]^{0.2} \\ [0.4,\ 0.8]^{0.4}[0.7,\ 0.8]^{0.3} \end{Bmatrix}^{0.3} \begin{Bmatrix} [0.6,\ 0.8]^{0.1}[0.6,\ 0.9]^{0.2} \\ [0.8,\ 0.9]^{0.4}[0.5,\ 0.7]^{0.3} \end{Bmatrix}^{0.1} \\ \begin{Bmatrix} [0.6,\ 0.8]^{0.1}[0.5,\ 0.9]^{0.2} \\ [0.5,\ 0.6]^{0.4}[0.6,\ 0.7]^{0.3} \end{Bmatrix}^{0.2} \begin{Bmatrix} [0.5,\ 0.6]^{0.1}[0.5,\ 0.6]^{0.2} \\ [0.3,\ 0.7]^{0.4}[0.7,\ 0.9]^{0.3} \end{Bmatrix}^{0.4} \end{pmatrix}, \\ \begin{pmatrix} \begin{Bmatrix} [0.5,\ 0.5]^{0.1}[0.3,\ 0.4]^{0.2} \\ [0.1,\ 0.4]^{0.4}[0.3,\ 0.6]^{0.3} \end{Bmatrix}^{0.3} \begin{Bmatrix} [0.4,\ 0.5]^{0.1}[0.4,\ 0.5]^{0.2} \\ [0.2,\ 0.8]^{0.4}[0.1,\ 0.4]^{0.3} \end{Bmatrix}^{0.1} \\ \begin{Bmatrix} [0.4,\ 0.5]^{0.1}[0.3,\ 0.4]^{0.2} \\ [0.3,\ 0.5]^{0.4}[0.2,\ 0.6]^{0.3} \end{Bmatrix}^{0.2} \begin{Bmatrix} [0.3,\ 0.5]^{0.1}[0.3,\ 0.4]^{0.2} \\ [0.2,\ 0.4]^{0.4}[0.3,\ 0.6]^{0.3} \end{Bmatrix}^{0.4} \end{pmatrix} \end{pmatrix}$$

$$= \left(\left[0.3069,\ 0.6112\right],\ \left[0.3416,\ 0.4851\right]\right),$$

$$\Theta_3 = \left(1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)$$

$$= \begin{pmatrix} 1-\begin{pmatrix} \begin{Bmatrix} [0.6,\ 0.7]^{0.1}[0.4,\ 0.6]^{0.2} \\ [0.6,\ 0.8]^{0.4}[0.5,\ 0.7]^{0.3} \end{Bmatrix}^{0.3} \begin{Bmatrix} [0.6,\ 0.7]^{0.1}[0.5,\ 0.7]^{0.2} \\ [0.6,\ 0.7]^{0.4}[0.5,\ 0.8]^{0.3} \end{Bmatrix}^{0.1} \\ \begin{Bmatrix} [0.6,\ 0.7]^{0.1}[0.5,\ 0.7]^{0.2} \\ [0.5,\ 0.7]^{0.4}[0.5,\ 0.8]^{0.3} \end{Bmatrix}^{0.2} \begin{Bmatrix} [0.6,\ 0.7]^{0.1}[0.4,\ 0.8]^{0.2} \\ [0.7,\ 0.9]^{0.4}[0.6,\ 0.7]^{0.3} \end{Bmatrix}^{0.4} \end{pmatrix}, \\ \begin{pmatrix} \begin{Bmatrix} [0.2,\ 0.5]^{0.1}[0.3,\ 0.4]^{0.2} \\ [0.3,\ 0.5]^{0.4}[0.3,\ 0.4]^{0.3} \end{Bmatrix}^{0.3} \begin{Bmatrix} [0.4,\ 0.6]^{0.1}[0.2,\ 0.3]^{0.2} \\ [0.3,\ 0.6]^{0.4}[0.2,\ 0.4]^{0.3} \end{Bmatrix}^{0.1} \\ \begin{Bmatrix} [0.5,\ 0.6]^{0.1}[0.5,\ 0.7]^{0.2} \\ [0.6,\ 0.7]^{0.4}[0.6,\ 0.7]^{0.3} \end{Bmatrix}^{0.2} \begin{Bmatrix} [0.4,\ 0.7]^{0.1}[0.6,\ 0.8]^{0.2} \\ [0.4,\ 0.5]^{0.4}[0.3,\ 0.7]^{0.3} \end{Bmatrix}^{0.4} \end{pmatrix} \end{pmatrix}$$

$$= \left(\left[0.4343,\ 0.5256\right],\ \left[0.3719,\ 0.5228\right]\right),$$

$$\Theta_4 = \left(1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^{l}_{\check{d}_{ij}},\kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(\left[\delta^{l}_{\check{d}_{ij}},\delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)$$

$$= \begin{pmatrix} 1-\begin{pmatrix} \begin{Bmatrix} [0.5,\ 0.7]^{0.1}[0.3,\ 0.8]^{0.2} \\ [0.5,\ 0.8]^{0.4}[0.6,\ 0.8]^{0.3} \end{Bmatrix}^{0.3} \begin{Bmatrix} [0.4,\ 0.8]^{0.1}[0.5,\ 0.9]^{0.2} \\ [0.5,\ 0.8]^{0.4}[0.5,\ 0.8]^{0.3} \end{Bmatrix}^{0.1} \\ \begin{Bmatrix} [0.5,\ 0.8]^{0.1}[0.5,\ 0.7]^{0.2} \\ [0.6,\ 0.8]^{0.4}[0.6,\ 0.8]^{0.3} \end{Bmatrix}^{0.2} \begin{Bmatrix} [0.6,\ 0.7]^{0.1}[0.5,\ 0.8]^{0.2} \\ [0.5,\ 0.7]^{0.4}[0.5,\ 0.8]^{0.3} \end{Bmatrix}^{0.4} \end{pmatrix}, \\ \begin{pmatrix} \begin{Bmatrix} [0.2,\ 0.4]^{0.1}[0.1,\ 0.3]^{0.2} \\ [0.1,\ 0.4]^{0.4}[0.5,\ 0.5]^{0.3} \end{Bmatrix}^{0.3} \begin{Bmatrix} [0.1,\ 0.4]^{0.1}[0.4,\ 0.5]^{0.2} \\ [0.1,\ 0.5]^{0.4}[0.2,\ 0.4]^{0.3} \end{Bmatrix}^{0.1} \\ \begin{Bmatrix} [0.3,\ 0.4]^{0.1}[0.4,\ 0.5]^{0.2} \\ [0.2,\ 0.6]^{0.4}[0.3,\ 0.6]^{0.3} \end{Bmatrix}^{0.2} \begin{Bmatrix} [0.4,\ 0.5]^{0.1}[0.3,\ 0.4]^{0.2} \\ [0.1,\ 0.5]^{0.4}[0.4,\ 0.5]^{0.3} \end{Bmatrix}^{0.4} \end{pmatrix} \end{pmatrix}$$

$$= \left(\left[0.2956,\ 0.6754\right],\ \left[0.3729,\ 0.6935\right]\right). \quad (56)$$

Table 2: Decision Matrix for $\mathfrak{I}^2$ in the form of IVIFHSN.

	$\check{\mathbf{d}}_1$	$\check{\mathbf{d}}_2$	$\check{\mathbf{d}}_3$	$\check{\mathbf{d}}_4$
$\mathcal{U}_1$	([0.3, 0.4], [0.5, 0.5])	([0.2, 0.4], [0.4, 0.5])	([0.2, 0.4], [0.4, 0.5])	([0.4, 0.5], [0.3, 0.5])
$\mathcal{U}_2$	([0.3, 0.5], [0.3, 0.4])	([0.1, 0.4], [0.4, 0.5])	([0.1, 0.5], [0.3, 0.4])	([0.4, 0.5], [0.3, 0.4])
$\mathcal{U}_3$	([0.2, 0.6], [0.1, 0.4])	([0.1, 0.2], [0.2, 0.8])	([0.4, 0.5], [0.3, 0.5])	([0.3, 0.6], [0.2, 0.4])
$\mathcal{U}_4$	([0.2, 0.3], [0.3, 0.6])	([0.3, 0.5], [0.1, 0.4])	([0.3, 0.4], [0.2, 0.6])	([0.1, 0.3], [0.3, 0.6])

Table 3: Decision Matrix for $\mathfrak{I}^3$ in the form of IVIFHSN.

	$\check{\mathbf{d}}_1$	$\check{\mathbf{d}}_2$	$\check{\mathbf{d}}_3$	$\check{\mathbf{d}}_4$
$\mathcal{U}_1$	([0.3, 0.4], [0.2, 0.5])	([0.3, 0.4], [0.4, 0.6])	([0.3, 0.4], [0.4, 0.5])	([0.3, 0.4], [0.3, 0.6])
$\mathcal{U}_2$	([0.4, 0.6], [0.3, 0.4])	([0.2, 0.5], [0.2, 0.3])	([0.3, 0.5], [0.3, 0.5])	([0.2, 0.6], [0.2, 0.4])
$\mathcal{U}_3$	([0.2, 0.4], [0.3, 0.5])	([0.3, 0.4], [0.3, 0.6])	([0.3, 0.5], [0.3, 0.4])	([0.1, 0.3], [0.4, 0.5])
$\mathcal{U}_4$	([0.3, 0.6], [0.3, 0.4])	([0.3, 0.5], [0.2, 0.4])	([0.2, 0.5], [0.3, 0.4])	([0.3, 0.4], [0.3, 0.6])

Table 4: Decision Matrix for $\mathfrak{I}^4$ in the form of IVIFHSN.

	$\check{\mathbf{d}}_1$	$\check{\mathbf{d}}_2$	$\check{\mathbf{d}}_3$	$\check{\mathbf{d}}_4$
$\mathcal{U}_1$	([0.3, 0.5], [0.2, 0.4])	([0.2, 0.6], [0.1, 0.4])	([0.2, 0.5], [0.3, 0.4])	([0.3, 0.4], [0.4, 0.5])
$\mathcal{U}_2$	([0.2, 0.7], [0.1, 0.3])	([0.1, 0.5], [0.4, 0.5])	([0.3, 0.5], [0.4, 0.5])	([0.2, 0.5], [0.3, 0.4])
$\mathcal{U}_3$	([0.2, 0.5], [0.1, 0.4])	([0.2, 0.5], [0.1, 0.5])	([0.2, 0.4], [0.2, 0.6])	([0.3, 0.5], [0.1, 0.5])
$\mathcal{U}_4$	([0.2, 0.4], [0.5, 0.5])	([0.2, 0.5], [0.2, 0.4])	([0.2, 0.4], [0.3, 0.6])	([0.2, 0.5], [0.4, 0.5])

Table 5: Feature analysis of different models with a proposed model.

	Membership information	Non-membership information	Aggregated attributes information	Aggregated information in intervals form	Aggregated sub-attributes information of any attribute
IVFS [2]	✓	×	×	✓	×
IVIFWA [13]	✓	✓	×	✓	×
IVIFWG [16]	✓	✓	×	✓	×
IFSWA [36]	✓	✓	✓	×	×
IFSWG [36]	✓	✓	✓	×	×
IVIFSWA [40]	✓	✓	✓	✓	×
IVIFSWG [40]	✓	✓	✓	✓	×
IFHSWA [43]	✓	✓	✓	×	✓
IFHSWG [43]	✓	✓	✓	×	✓
Proposed IVIFHSWA	✓	✓	✓	✓	✓
Proposed IVIFHSWG	✓	✓	✓	✓	✓

Table 6: Comparison of planned operators with some prevailing operators.

AO	I^1	I^2	I^3	I^4	Alternatives ranking	Optimal choice
IVIFWA [13]	0.3681	0.2116	0.3509	0.4573	$\mathfrak{I}^4 > \mathfrak{I}^1 > \mathfrak{I}^3 > \mathfrak{I}^2$	$\mathfrak{I}^4$
IVIFWG [16]	0.3104	0.2753	0.2914	0.3952	$\mathfrak{I}^4 > \mathfrak{I}^1 > \mathfrak{I}^3 > \mathfrak{I}^2$	$\mathfrak{I}^4$
IVIFSWA [40]	0.0235	0.0253	0.0584	0.0723	$\mathfrak{I}^4 > \mathfrak{I}^3 > \mathfrak{I}^2 > \mathfrak{I}^1$	$\mathfrak{I}^4$
IVIFSWG [40]	0.2365	0.3734	0.5840	0.7134	$\mathfrak{I}^4 > \mathfrak{I}^3 > \mathfrak{I}^2 > \mathfrak{I}^1$	$\mathfrak{I}^4$
IVIFHSWA	0.4328	0.4362	0.4637	0.5094	$\mathfrak{I}^4 > \mathfrak{I}^3 > \mathfrak{I}^2 > \mathfrak{I}^1$	$\mathfrak{I}^4$
IVIFHSWG	0.4128	0.6819	0.5903	0.7631	$\mathfrak{I}^4 > \mathfrak{I}^2 > \mathfrak{I}^3 > \mathfrak{I}^1$	$\mathfrak{I}^4$

Step 4: Applying the score function $S = \kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}/4$ to determine the score values for all alternatives. $S(\Theta_1) = 0.4328$, $S(\Theta_2) = 0.4362$, $S(\Theta_3) = 0.4637$, and $S(\Theta_4) = 0.5094$.

Step 5: From the above calculation, we get $S(\Theta_4) > S(\Theta_3) > S(\Theta_2) > S(\Theta_1)$. Which shows that $\mathfrak{I}^4$ is the best alternative.

Step 6: So, $\mathfrak{I}^4 > \mathfrak{I}^3 > \mathfrak{I}^2 > \mathfrak{I}^1$ is the obtained ranking of alternatives.

4.2.2. By IVIFHSWG Operator

Step 1 and step 2 are similar to section 4.2.1.

Step 3: Utilized the developed IVIFHSWG operator to compute the aggregated values for each alternative.

$$
\begin{aligned}
\Theta_1 &= \left(\prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(\left[\kappa^{l}_{\tilde{d}_{ij}}, \kappa^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j}, 1 - \prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(1 - \left[\delta^{l}_{\tilde{d}_{ij}}, \delta^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j} \right) \\
&= \left(\begin{array}{c}
\left(\begin{array}{c}
\left\{ \begin{array}{c} [0.4, 0.5]^{0.1} [0.2, 0.4]^{0.2} \\ [0.3, 0.5]^{0.4} [0.4, 0.6]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.2, 0.4]^{0.1} [0.1, 0.3]^{0.2} \\ [0.4, 0.5]^{0.4} [0.1, 0.3]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.1, 0.3]^{0.1} [0.2, 0.3]^{0.2} \\ [0.4, 0.5]^{0.4} [0.3, 0.4]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.2, 0.4]^{0.1} [0.2, 0.4]^{0.2} \\ [0.2, 0.6]^{0.4} [0.3, 0.4]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right), \\
1 - \left(\begin{array}{c}
\left\{ \begin{array}{c} [0.5, 0.8]^{0.1} [0.4, 0.8]^{0.2} \\ [0.6, 0.9]^{0.4} [0.6, 0.7]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.4, 0.5]^{0.1} [0.5, 0.6]^{0.2} \\ [0.6, 0.8]^{0.4} [0.4, 0.7]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.5, 0.8]^{0.1} [0.3, 0.7]^{0.2} \\ [0.6, 0.7]^{0.4} [0.5, 0.7]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.4, 0.8]^{0.1} [0.5, 0.8]^{0.2} \\ [0.6, 0.8]^{0.4} [0.5, 0.7]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right)
\end{array} \right) \\
&= \left([0.4525, 0.5469], [0.1253, 0.5263] \right), \\
\Theta_2 &= \left(\prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(\left[\kappa^{l}_{\tilde{d}_{ij}}, \kappa^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j}, 1 - \prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(1 - \left[\delta^{l}_{\tilde{d}_{ij}}, \delta^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j} \right) \\
&= \left(\begin{array}{c}
\left(\begin{array}{c}
\left\{ \begin{array}{c} [0.3, 0.4]^{0.1} [0.3, 0.5]^{0.2} \\ [0.2, 0.6]^{0.4} [0.2, 0.3]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.2, 0.4]^{0.1} [0.1, 0.4]^{0.2} \\ [0.1, 0.2]^{0.4} [0.3, 0.5]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.2, 0.4]^{0.1} [0.1, 0.5]^{0.2} \\ [0.4, 0.5]^{0.4} [0.3, 0.4]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.4, 0.5]^{0.1} [0.4, 0.5]^{0.2} \\ [0.3, 0.6]^{0.4} [0.1, 0.3]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right), \\
1 - \left(\begin{array}{c}
\left\{ \begin{array}{c} [0.5, 0.5]^{0.1} [0.6, 0.7]^{0.2} \\ [0.6, 0.9]^{0.4} [0.4, 0.7]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.5, 0.6]^{0.1} [0.5, 0.6]^{0.2} \\ [0.2, 0.8]^{0.4} [0.6, 0.9]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.5, 0.6]^{0.1} [0.6, 0.7]^{0.2} \\ [0.5, 0.7]^{0.4} [0.4, 0.8]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.5, 0.7]^{0.1} [0.6, 0.7]^{0.2} \\ [0.6, 0.8]^{0.4} [0.4, 0.7]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right)
\end{array} \right) \\
&= \left([0.5643, 0.8978], [0.5206, 0.7452] \right), \\
\Theta_3 &= \left(\prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(\left[\kappa^{l}_{\tilde{d}_{ij}}, \kappa^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j}, 1 - \prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(1 - \left[\delta^{l}_{\tilde{d}_{ij}}, \delta^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j} \right) \\
&= \left(\begin{array}{c}
\left(\begin{array}{c}
\left\{ \begin{array}{c} [0.3, 0.4]^{0.1} [0.4, 0.6]^{0.2} \\ [0.2, 0.4]^{0.4} [0.3, 0.6]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.3, 0.4]^{0.1} [0.2, 0.5]^{0.2} \\ [0.3, 0.4]^{0.4} [0.3, 0.5]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.3, 0.4]^{0.1} [0.3, 0.5]^{0.2} \\ [0.3, 0.5]^{0.4} [0.2, 0.5]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.3, 0.4]^{0.1} [0.2, 0.6]^{0.2} \\ [0.1, 0.3]^{0.4} [0.3, 0.4]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right), \\
1 - \left(\begin{array}{c}
\left\{ \begin{array}{c} [0.5, 0.8]^{0.1} [0.6, 0.7]^{0.2} \\ [0.5, 0.7]^{0.4} [0.6, 0.7]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.4, 0.6]^{0.1} [0.7, 0.8]^{0.2} \\ [0.4, 0.7]^{0.4} [0.6, 0.8]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.5, 0.6]^{0.1} [0.5, 0.7]^{0.2} \\ [0.6, 0.7]^{0.4} [0.6, 0.7]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.4, 0.7]^{0.1} [0.6, 0.8]^{0.2} \\ [0.5, 0.6]^{0.4} [0.4, 0.7]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right)
\end{array} \right) \\
&= \left([0.6325, 0.9658], [0.2365, 0.5263] \right), \\
\Theta_4 &= \left(\prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(\left[\kappa^{l}_{\tilde{d}_{ij}}, \kappa^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j}, 1 - \prod_{j=1}^{4} \left(\prod_{i=1}^{4} \left(1 - \left[\delta^{l}_{\tilde{d}_{ij}}, \delta^{u}_{\tilde{d}_{ij}} \right] \right)^{\omega_i} \right)^{\nu_j} \right) \\
&= \left(\begin{array}{c}
\left(\begin{array}{c}
\left\{ \begin{array}{c} [0.3, 0.5]^{0.1} [0.2, 0.7]^{0.2} \\ [0.2, 0.5]^{0.4} [0.2, 0.4]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.2, 0.6]^{0.1} [0.1, 0.5]^{0.2} \\ [0.2, 0.5]^{0.4} [0.2, 0.5]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.2, 0.5]^{0.1} [0.3, 0.5]^{0.2} \\ [0.2, 0.4]^{0.4} [0.2, 0.4]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.3, 0.4]^{0.1} [0.2, 0.5]^{0.2} \\ [0.3, 0.5]^{0.4} [0.2, 0.5]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right), \\
1 - \left(\begin{array}{c}
\left\{ \begin{array}{c} [0.6, 0.8]^{0.1} [0.7, 0.9]^{0.2} \\ [0.6, 0.9]^{0.4} [0.5, 0.5]^{0.3} \end{array} \right\}^{0.3} \left\{ \begin{array}{c} [0.6, 0.9]^{0.1} [0.5, 0.6]^{0.2} \\ [0.5, 0.9]^{0.4} [0.6, 0.8]^{0.3} \end{array} \right\}^{0.1} \\
\left\{ \begin{array}{c} [0.6, 0.7]^{0.1} [0.5, 0.6]^{0.2} \\ [0.4, 0.8]^{0.4} [0.4, 0.7]^{0.3} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.5, 0.6]^{0.1} [0.6, 0.7]^{0.2} \\ [0.5, 0.9]^{0.4} [0.5, 0.6]^{0.3} \end{array} \right\}^{0.4}
\end{array} \right)
\end{array} \right) \\
&= \left([0.7975, 0.8569], [0.6395, 0.7586] \right).
\end{aligned} \tag{57}
$$

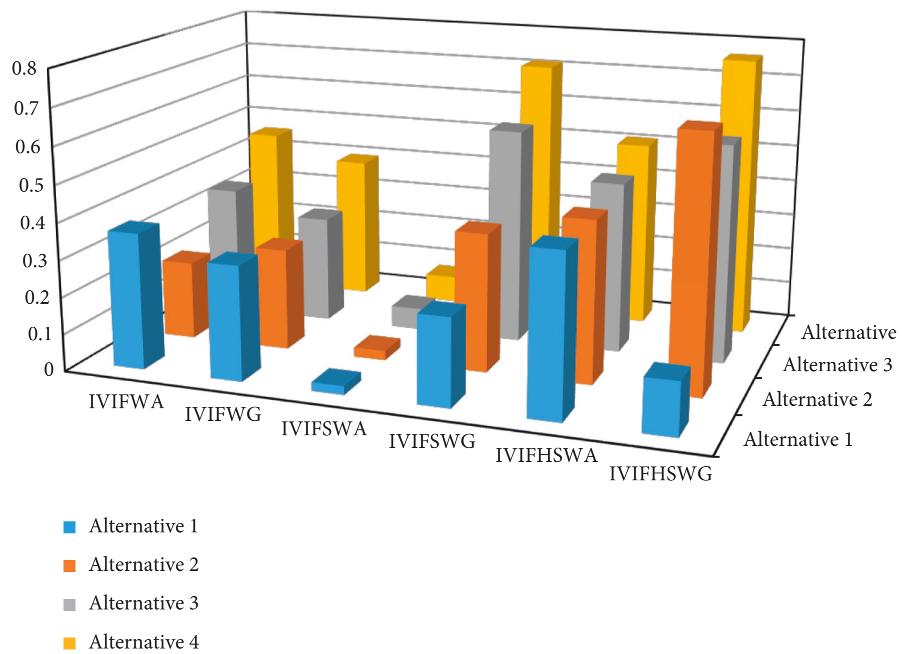

Figure 1: Comparative analysis of the proposed approach with existing models.

Step 4: Applying the score function $S = \kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}/4$ to determine the score values for all alternatives such as $S(\Theta_1) = 0.4128$, $S(\Theta_2) = 0.6819$, $S(\Theta_3) = 0.5903$, and $S(\Theta_4) = 0.7631$.

Step 5: From the above calculation, we get the ranking of alternatives $S(\Theta_4) > S(\Theta_2) > S(\Theta_3) > S(\Theta_1)$. Which shows that $\mathfrak{I}^4$ is the best alternative.

Step 6: So, $\mathfrak{I}^4 > \mathfrak{I}^2 > \mathfrak{I}^3 > \mathfrak{I}^1$ is the obtained ranking of alternatives.

The material assessment through the intended imagery phase is excellent on a hypothetical level. Specific content is more likely to be accurate. Face-centered cube materials are typically used at minor temperatures -163°C and $\mathfrak{I}^1$ = Ti–6Al–4V ratings first. This is steadfast in employing initial investigations and real-world maneuvers. Austenitic steels are still classically used in melted nitrogen or hydrogen storing vessels [49].

5. Comparative Studies

To validate the usefulness of the proposed technique, a comparison between the proposed model and the prevailing methods is planned in the next section.

5.1. Supremacy of the Proposed Technique. The proposed method competently delivers realistic decisions in the DM procedure. We introduced the MCGDM approach using our developed IVIFHSWA and IVIFHSWG operators. Our plan MCGDM technique provides the most subtle and precise information on DM complications. The proposed model is multi-purpose and communicative, adapting to changing instability, commitment, and productivity. Different models have specific classification processes, so there is a direct change in the classification of expected methods according to their expectations. This systematic study and evaluation determined that the results obtained from the conventional method are erroneously equal to the hybrid structure. In addition, due to some favorable conditions, many composite structures of FS such as IVFS, IVIFS, and IVIFSS concentrate in IVIFHSS. It is easy to syndicate insufficient and obscure data in the DM method. Data about the matter can be described more accurately and rationally. Therefore, our proposed method is more efficient, meaningful, superior, and better than multiple mixed FS structures. Table 5 below provides an analysis of the technique presented and the features of some existing models.

5.2. Comparative Analysis. To prove the utility of the planned method, we equate the attained consequences with some prevailing approaches under IVPFS, IVIFSS, and IVPFSS. A summary of all values is specified in Table 6. Wang and Liu [13] developed IVIFWA, and Xu and Chen [16] presented that IVIFWG operators cannot compute the parametrized values of the alternatives. Furthermore, if any expert considers the MD and NMD whose sum exceeds 1, the aforementioned AOs fail to accommodate the scenario. Zulqarnain et al. [40] established AOs for IVIFSS that cannot accommodate the decision-maker's selection when the sum of upper MD and NMD parameters surpasses one. It is detected that, in certain conditions, the existing AOs provide some unattractive outcomes. So, to resolve such complications, we developed the AOs for IVIFHSS, which capably deal with the multi-sub attributes compared to existing AOs. Thus, IVIFHSS is the most generalized form of IVIFSS. Hence, based on the abovementioned details, the anticipated operators in this paper are more influential, consistent, and prosperous. A comparison of the proposed model with prevailing models is given in the following Table 6.

The graphical demonstration of Table 6 is given in the following Figure 1.

6. Conclusion

Decision-making is a pre-planned process for arranging and choosing logical preferences from multiple alternatives. DM is a multifaceted procedure because it can switch from one scene to another. It is serious about differentiating how much real perspective data decision-makers need. The most operational approach in DM is paying close attention and focusing on your goals. In manufacturing, the better stability of manipulation is neutral; Authoritative material and fabricated surround extensive content. In a real DM, assessing alternative facts as told by a professional is permanently incorrect, irregular, and impressive. Therefore, IVIFHSNs can be used to match this uncertain data. The main determination of this work is to extend the AOs for interval-valued intuitionistic fuzzy hypersoft sets. First, we introduce the operational laws for the interval-valued intuitionistic fuzzy hypersoft environment. Considering the developed operational laws, we introduced IVIFHSWA and IVIFHSWG operators with their fundamental properties. Also, a DM method is planned to deal with the complications of MCGDM based on established operators. To illustrate the strength of the developed method, we present a comprehensive mathematical example for MS in manufacturing engineering. A comprehensive analysis and some existing methods are presented to confirm the practicality of the intended approach. Finally, based on the results obtained, it is determined that the method proposed in this study is the most practical and operative to resolve MCGDM obstacles compared to existing techniques. Future investigations highlight emergent DM methods, such as Einstein's hybrid AOs in the IVIFHSS setting. We are confident that these extensive growths and conjectures will support considered professional consideration extents convoluted in the world's environment.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] I. B. Turksen, "Interval valued fuzzy sets based on normal forms," *Fuzzy Sets and Systems*, vol. 20, no. 2, pp. 191–210, 1986.

[3] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[4] H. Garg and D. Rani, "Novel distance measures for intuitionistic fuzzy sets based on various triangle centers of isosceles triangular fuzzy numbers and their applications," *Expert Systems with Applications*, vol. 191, Article ID 116228, 2022.

[5] W. Wang and X. Liu, "Intuitionistic fuzzy geometric aggregation operators based on Einstein operations," *International Journal of Intelligent Systems*, vol. 26, no. 11, pp. 1049–1075, 2011.

[6] H. Garg, "An improved cosine similarity measure for intuitionistic fuzzy sets and their applications to decision-making process," *Hacettepe Journal of Mathematics and Statistics*, vol. 47, no. 6, pp. 1578–1594, 2018.

[7] K. T. Atanassov, "New topological operator over intuitionistic fuzzy sets," *Journal of Computational and Cognitive Engineering*, 2022.

[8] H. Garg and K. Kumar, "An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making," *Soft Computing*, vol. 22, no. 15, pp. 4959–4970, 2018.

[9] P. A. Ejegwa and J. M. Agbetayo, "Similarity-distance decision-making technique and its applications via intuitionistic fuzzy pairs," *Journal of Computational and Cognitive Engineering*, 2022.

[10] H. Garg and D. Rani, "A robust correlation coefficient measure of complex intuitionistic fuzzy sets and their applications in decision-making," *Applied Intelligence*, vol. 49, no. 2, pp. 496–512, 2019.

[11] R. Khan, K. Ullah, D. Pamucar, and M. Bari, "Performance measure using a multi-attribute decision making approach based on Complex T-spherical fuzzy power aggregation operators," *Journal of Computational and Cognitive Engineering*, 2022.

[12] K. T. Atanassov, "Interval valued intuitionistic fuzzy sets," in *Intuitionistic Fuzzy Sets*, pp. 139–177, Physica, Heidelberg, 1999.

[13] W. Wang, X. Liu, and Y. Qin, "Interval-valued intuitionistic fuzzy aggregation operators," *Journal of Systems Engineering and Electronics*, vol. 23, no. 4, pp. 574–580, 2012.

[14] R. Arora and H. Garg, "Group decision-making method based on prioritized linguistic intuitionistic fuzzy aggregation operators and its fundamental properties," *Computational and Applied Mathematics*, vol. 38, no. 2, p. 36, 2019.

[15] H. Garg and D. Rani, "An efficient intuitionistic fuzzy MULTIMOORA approach based on novel aggregation operators for the assessment of solid waste management techniques," *Applied Intelligence*, vol. 52, no. 4, pp. 4330–4363, 2022.

[16] Z. Xu and J. Chen, "On geometric aggregation over interval-valued intuitionistic fuzzy information," in *Fourth international conference on fuzzy systems and knowledge discovery (FSKD 2007)*, vol. 2, pp. 466–471, IEEE, 2007.

[17] Z. Jia and Y. Zhang, "Interval-valued intuitionistic fuzzy multiple attribute group decision making with uncertain weights," *Mathematical Problems in Engineering*, vol. 2019, pp. 1–9, 2019.

[18] Z. Xu and X. Gou, "An overview of interval-valued intuitionistic fuzzy information aggregations and applications," *Granular Computing*, vol. 2, no. 1, pp. 13–39, 2017.

[19] X. Ze-Shui, "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision

making," *Control and Decision*, vol. 22, no. 2, pp. 215–219, 2007.
[20] Z. Mu, S. Zeng, and Q. Liu, "Some interval-valued intuitionistic fuzzy Zhenyuan aggregation operators and their application to multi-attribute decision making," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 26, no. 04, pp. 633–653, 2018.
[21] Z. Zhang, "Geometric Bonferroni means of interval-valued intuitionistic fuzzy numbers and their application to multiple attribute group decision making," *Neural Computing & Applications*, vol. 29, no. 11, pp. 1139–1154, 2018.
[22] D. G. Park, Y. C. Kwun, J. H. Park, and I. Y. Park, "Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems," *Mathematical and Computer Modelling*, vol. 50, no. 9-10, pp. 1279–1293, 2009.
[23] P. Gupta, M. K. Mehlawat, N. Grover, and W. Pedrycz, "Multi-attribute group decision making based on extended TOPSIS method under interval-valued intuitionistic fuzzy environment," *Applied Soft Computing*, vol. 69, pp. 554–567, 2018.
[24] H. Garg and K. Kumar, "Linguistic interval-valued atanassov intuitionistic fuzzy sets and their applications to group decision making problems," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 12, pp. 2302–2311, 2019.
[25] X. Peng and Y. Yang, "Fundamental properties of interval-valued pythagorean fuzzy aggregation operators," *International Journal of Intelligent Systems*, vol. 31, no. 5, pp. 444–487, 2016.
[26] K. Rahman, S. Abdullah, M. Shakeel, M. S. Ali Khan, and M. Ullah, "Interval-valued Pythagorean fuzzy geometric aggregation operators and their application to group decision making problem," *Cogent Mathematics*, vol. 4, no. 1, Article ID 1338638, 2017.
[27] D. Molodtsov, "Soft set theory—first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.
[28] F. Fatimah, D. Rosadi, R. B. F. HakimHakim, and J. C. R. Alcantud, "N-soft sets and their decision making algorithms," *Soft Computing*, vol. 22, no. 12, pp. 3829–3842, 2018.
[29] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562, 2003.
[30] S. Yuksel, T. Dizman, G. Yildizdan, and U. Sert, "Application of soft sets to diagnose the prostate cancer risk," *Journal of Inequalities and Applications*, vol. 2013, no. 1, p. 229, 2013.
[31] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, pp. 589–602, 2001.
[32] F. Fatimah and J. C. R. Alcantud, "The multi-fuzzy N-soft set and its applications to decision-making," *Neural Computing & Applications*, vol. 33, no. 17, Article ID 11437, 2021.
[33] H. Garg, F. Perveen Pa, S. J. John, and L. Perez-Dominguez, "Spherical Fuzzy Soft Topology and its Application in Group Decision-Making Problems," *Mathematical Problems in Engineering*, vol. 2022, Article ID 1007133, 19 pages, 2022.
[34] P. K. Maji, R. Biswas, and A. R. Roy, "Intuitionistic fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, pp. 677–692, 2001.
[35] H. Garg and R. Arora, "TOPSIS method based on correlation coefficient for solving decision-making problems with intuitionistic fuzzy soft set information," *AIMS Mathematics*, vol. 5, no. 4, pp. 2944–2966, 2020.
[36] R. Arora and H. Garg, "A robust aggregation operators for multi-criteria decision-making with intuitionistic fuzzy soft set environment," *Scientia Iranica*, vol. 25, no. 2, pp. 931–942, 2018.
[37] H. Garg and R. Arora, "Generalized Maclaurin symmetric mean aggregation operators based on Archimedean t-norm of the intuitionistic fuzzy soft set information," *Artificial Intelligence Review*, vol. 54, no. 4, pp. 3173–3213, 2021.
[38] H. Garg and R. Arora, "Generalized and group-based generalized intuitionistic fuzzy soft sets with applications in decision-making," *Applied Intelligence*, vol. 48, no. 2, pp. 343–356, 2018.
[39] Y. Jiang, Y. Tang, Q. Chen, H. Liu, and J. Tang, "Interval-valued intuitionistic fuzzy soft sets and their properties," *Computers & Mathematics with Applications*, vol. 60, no. 3, pp. 906–918, 2010.
[40] R. M. Zulqarnain, X. L. Xin, M. Saqlain, and W. A. Khan, "TOPSIS method based on the correlation coefficient of interval-valued intuitionistic fuzzy soft sets and aggregation operators with their application in decision-making," *Journal of Mathematics*, vol. 202116 pages, 2021.
[41] F. Smarandache, "Extension of soft set to hypersoft set, and then to plithogenic hypersoft set," *Neutrosophic Sets and Systems*, vol. 22, pp. 168–170, 2018.
[42] R. M. Zulqarnain, X. L. Xin, M. Xin, and M. Saeed, "Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem," *AIMS Mathematics*, vol. 6, no. 3, pp. 2732–2755, 2021.
[43] R. M. Zulqarnain, I. Siddique, R. Ali, D. Pamucar, D. Marinkovic, and D. Bozanic, "Robust aggregation operators for intuitionistic fuzzy hypersoft set with their application to solve MCDM problem," *Entropy*, vol. 23, no. 6, p. 688, 2021.
[44] M. N. Jafar, M. Saeed, M. Haseeb, and A. Habib, "Matrix theory for intuitionistic fuzzy hypersoft sets and its application in multi-attributive decision-making problems," *Theory and Application of Hypersoft Set*, pp. 65–84, Pons Publishing House, Brussel, 2021.
[45] S. Debnath, "Interval-valued intuitionistic hypersoft sets and their algorithmic approach in multi-criteria decision making," *Neutrosophic Sets and Systems*, vol. 48, pp. 226–250, 2022.
[46] P. Sunthrayuth, F. Jarad, J. Majdoubi, R. M. Zulqarnain, A. Iampan, and I. Siddique, "A novel multicriteria decision-making approach for Einstein weighted average operator under pythagorean fuzzy hypersoft environment," *Journal of Mathematics*, vol. 2022, Article ID 1951389, 24 pages, 2022.
[47] R. Choudhury, R. Wurster, T. Weber et al., "GM Well-To-Wheel Analysis of Energy Use and Greenhouse Gas Emissions of Advanced Fuel/vehicle Systems-A European Study," *Ottobrunn*, 2002.
[48] R. Edwards, V. Mahieu, J. C. Griesemann, J. F. Larivé, and D. J. Rickeard, "Well-to-wheels analysis of future automotive fuels and powertrains in the European context," *SAE Transactions*, pp. 1072–1084, 2004.
[49] A. Godula-Jopek, W. Jehle, and J. Wellnitz, *Hydrogen Storage Technologies: New Materials, Transport, and Infrastructure*, John Wiley & Sons, Hoboken, New Jersey, 2012.