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Research Article

An Approach of Decision-Making under the Framework of Fermatean Fuzzy Sets

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Because of its influence on various elements of human life experiences and conditions, the building industry is a significant business. In the recent past, environmental considerations have been incorporated in the design and planning stages of building supply chains. The process of evaluating and selecting suppliers is one of the most important issues in supply chain management. A multicriteria decision-making (MCDM) problem can be utilized to handle such issues. The goal of this research is to present a new and efficient technique for selecting suppliers with ambiguous data. The suggested methodology's structure is based on technology for order of preference by similarity to ideal solution (TOPSIS), with Fermatean fuzzy sets (F_r FSs) employed to cope with information uncertainty. In this article, authors modified the distance between F_r FSs to propose the similarity measure and implemented it to form the MCDM model to resolve the vague and uncertain data. Moreover, we used this similarity measure to choose the optimal alternative. A practical example for alternative selection is provided, along with a comparison of the acquired findings to existing approach. Finally, to strengthen the outcome obtained through the proposed model, sensitivity analysis and time complexity analysis are performed.

1. Introduction

In real-world situations, we frequently encounter tasks and activities that necessitate the usage of decision-making (DM_g) procedures. DM_g may be viewed as a problem-solving process that yields an ideal, or at the very least reasonable, solution. In general, DM_g is a mental and reasoning process that leads to choose an ideal option from a collection of possible alternatives in a DM_g circumstance. TOPSIS is a valuable method for MCDM issues in the real world. Hwang and Yoon [1] first proposed this strategy in 1981, with Yoon continuing the process in 1987. TOPSIS rates options and determines the best compromise between them and the ideal solution. TOPSIS is an effective approach for ranking and picking a number of generally recognized alternatives using distance metrics that is both practical and helpful. TOPSIS is the best compromise choice, having the lowest distance from the positive-ideal solution and the greatest distance from the negative-ideal solution [2–4]. So far, TOPSIS has been thoroughly investigated by explorers and experts, and it has been successfully applied to a wide range of DM_g situations [5–8].

The DM_g procedure demands the analysis of a small number of possibilities stated in terms of evaluative criteria for the most part. Instead, when analyzing all of the criteria at once, the issue may be to rank these possibilities in terms of how desirable they are to the decision-maker. If all parameters are assessed at the same time, another goal would be to discover the best choice or to estimate the relative over all preferences of each alternative. The basic goal of MCDM is to solve challenges like these: (1) PROMETHEE, (2) ELECTRE, (3) AHP, (4) VIKOR, (5) Fuzzy AHP, (6) TOPSIS, and (7) Fuzzy TOPSIS are the seven most significant MCDM approaches. Hundreds of experts have implemented TOPSIS in many domains, updated or modified the TOPSIS approach to meet unique issues.

One of the inevitabilities of dealing with DM_g challenges is the ambiguity of information. The opinions and expressions of decision-makers are frequently the source of this ambiguity. We may describe and capture information uncertainty in a variety of ways. Fuzzy sets (FSs) theory has been a popular method for dealing with uncertainty in DM_g situations in recent years. Furthermore, the linear programming (LP) presented in [9] was used to calculate the weights of criteria [10–12] based on decision-makers' evaluations. The F_r F-TOPSIS approach was created in a variety of fuzzy situations. The study's key contribution is the use of F_r FSs to expand the F_r F-TOPSIS approach and apply the enlarged methodology to evaluating green building suppliers.

2. Literature Review

In the middle of the 1960s, Zadeh [13] proposed the concept of FSs, which ushered in a new era for scholars. In real-world situations, FSs typically reflect uncertainty and ambiguity. The majority of the experts have concentrated on FS expansions and applications. In 1986, Atanassov [14] proposed the notion of intuitionistic fuzzy sets (IFSs), which is one of the most important extensions of FSs and have two number of degrees named, membership degree (MD), and nonmembership degree (NMD) such that $0 \leq MD + NMD \leq 1$.

Recently, Pythagorean fuzzy sets (P_g FS) [15] have gotten more concentration from the experts and implemented in different fields of DM_g procedures. When comparing two items based on their unequal content, distance measures are quite useful. Zeng et al. [16] demonstrated the use of various P_g F distance and similarity measurements in MCDM. Hussain and Yang [17] provided various Hausdorff metric-based P_g F distance and similarity measures with P_g F-TOPSIS applicability. Li and Lu [18] presented some generalized distance measurements and their continuous versions for P_g FSs. Ejegwa [19] provided several distance and similarity measurements for P_g FSs based on membership grades. Wei and Wei [20] proposed some cosine function-based P_g F similarity measurements. Peng et al. [21] presented 12 P_g F distance and similarity measurements, along with their applicability (2017). Although P_g FSs have a wide spectrum of uses, they are unable to handle circumstances, where $MD^2 + NMD^2 > 1$, for instance, if $\mathrm{MD} = 0.8$ and $\mathrm{NMD} = 0.7$, then $0.8^2 + 0.7^2 = 0.64 + 0.49 = 1.13 > 1$. To overcome such situations, Senapati and Yager [22] introduced as a new sort of FSs recently, named F_r FSs. F_r FSs make up of both MD and NMD which satisfies the condition $MD^3 + NMD^3 < 1$, so it handles the abovementioned circumstances accurately. F_r FSs are derived from the ideas of IFS and P_g FS. F_r FSs, on the other hand, use novel concepts to manage uncertain data that make them more flexible and efficient than IFSs and P_g FS [23, 24]. Because they are all confined within the space of F_r FSs, F_r FSs are more powerful than FSs, IFSs, and P_g FSs. Senapati and Yager [24] presented certain F_r FS aggregation operators and their application in decision-making. Mishra and Rani [25] proposed the weighted aggregated sum product assessment (WASPAS) method in the Fermatean fuzzy (F_r F) environment. Garg et al. [26] demonstrated the use of FF aggregating functions in the COVID-19 testing facility. The continuities and derivatives of FF functions were investigated by Yang et al. [27]. Sergi and Sari [28] proposed some FF capital budgeting approaches. Sahoo [29] suggested some FFS scoring functions and their application to transportation issues and decision-making.

The major reason we used F_r FSs in designing the current study's strategy is because of its flexibility in dealing with unclear information. The goal of this research is to develop a new and efficient system for evaluating and selecting green suppliers in a building supply chain where there is uncertainty. In the evaluation process, the technique described in this study takes into account the ambiguity of information given by decision-makers. To deal with information uncertainty, we employed F_r FSs. The suggested technique is based on the extended TOPSIS (E-TOPSIS) and LP methods, which is both efficient and helpful.

Failure mode effect analysis (FMEA) is a common and effective technique that may be used to assess risk and improve the safety of a repairable engineering system, according to Kushwaha et al. [30]. Yorulmaz et al. [31] proposed TOPSIS based on modified Mahalanobis distance measure to rank the 81 Turkish provinces by considering distinct levels of development. One of the most important activities in the purchasing department is supplier selection. By assisting in the selection of the most suitable supplier, choosing the correct supplier makes a strategic difference in an organization's capacity to decrease costs and improve product quality. Cakar and Cavus [32] implemented fuzzy TOPSIS to select the best supplier. The criteria for choosing an air traffic control (ATC) radar station that effectively fulfills the job of radar in air traffic management are developed and assessed in [33]. Picture fuzzy set and rough setbased approaches are proposed in this study to consider the unclear concerns linked with students' job decision since they are shown to be appropriate due to their inherent qualities to cope with incomplete and imprecise information [34]. To select the construction machinery, Bozanic et al. [35] offered the Neuro-Fuzzy System as a decision-making aid.

There has been no previous study employing the F_r F-TOPSIS approach with F_r FSs to deal with MCDM, to the best of the authors' knowledge. The primary contributions of this study can be summarized as follows:

(1) To tackle MCDM situations with ambiguous knowledge that may be stated by a number of decision-makers, a novel D M_g technique based on F_r F-TOPSIS and F_r FSs is proposed.

(2) An example demonstrates the effectiveness of the proposed technique for evaluating green building providers.

The remainder of the paper is arranged as follows: Section 2 contains some fundamental and relevant knowledge. In Section 3, the features of novel F_r FSs are thoroughly examined. To address the ambiguous information, an MCDM model based on F_r F-TOPSIS is created. An MCDM issues relevant to select the supplier is provided in Section 5. The validity of the suggested model is explored in Section 6. Subsection 7.1 examines a complete comparison based on TC. Figure 1 represents the research process of this article.

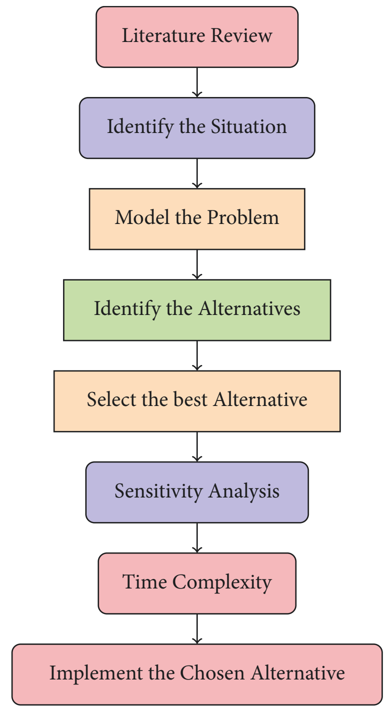

Figure 1: Research process.

3. Basic Concepts

Some basic ideas connected to the present work such as FSs, IFSs, F_r FSs, and LP are briefly penned in this section.

Definition 1. [13] A FS $\mathscr{F}$ over $Y = \{y_1, y_2, \ldots, y_n\}$ can be illustrated as follows:

$$\mathscr{F} = \{(y, \mu_{\mathscr{F}}(y)) | y \in Y\}. \tag{1}$$

where $\mu_{\mathscr{F}}(y))$: $X \longrightarrow [0, 1]$ is a MD so that $y \in Y$ to $\mathscr{F}$.

Definition 2. [14] Let Y be a fixed set, an IFS $\mathscr{I}$ on Y is characterized as follows:

$$\mathscr{I} = \{\langle y, \alpha_{\mathscr{I}}(y), \beta_{\mathscr{I}}(y)\rangle | y \in Y\}, \tag{2}$$

where $\alpha_{\mathscr{I}}(y), \beta_{\mathscr{I}}(y) \in [0, 1]$ are called the MD and NMD of $y \in Y$ to set $\mathscr{I}$ with the following condition: $0 \le \alpha_{\mathscr{I}}(y) + \beta_{\mathscr{I}}(y) \le 1$, for all $y \in Y$.

For all $y \in Y$, $\omega_{\mathscr{I}}(y)$ is known as hesitancy degree of $y \in \mathscr{I}$, where $\omega_{\mathscr{I}}(y) = 1 - \alpha_{\mathscr{I}}(y) - \beta_{\mathscr{I}}(y)$.

Definition 3. [36] A P_g FS $\mathscr{P}$ over Y is given by

$$\mathscr{P} = \{(y, < \alpha_{\mathscr{P}}(y), \beta_{\mathscr{P}}(y) >) | y \in Y\}, \tag{3}$$

where $\alpha_{\mathscr{P}}(y), \beta_{\mathscr{P}}(y) \in [0.1]$ are the MD and NMD of y to $\mathscr{P}$ such that $0 \le \alpha^2_{\mathscr{P}}(y) + \beta^2_{\mathscr{P}}(y) \le 1$. The degree of hesitancy or indeterminacy represented by $\eta_{\mathscr{P}}(y)$ is written as $\eta_{\mathscr{P}}(y) = \sqrt{1 - \alpha^2_{\mathscr{P}}(y) - \beta^2_{\mathscr{P}}(y)}$.

Definition 4. [22] A Fermatean fuzzy set over the set $Y = \{y_1, y_2, \ldots, y_n\}$ is defined as follows:

$$F = \{\langle y, \alpha_F(y), \eta_F(y)\rangle | y \in Y\}, \tag{4}$$

where $\alpha_F(y), \eta_F(y) \in [0, 1]$ and are called the MD, NMD of $y \in Y$ to the set F, respectively and $\alpha_F(y), \eta_F(y)$ fulfil the condition: $0 \le \alpha^3_F(y) + \eta^3_F(y) \le 1$, for all $y \in Y$. Also $\zeta_F(y) = \sqrt[3]{1 - \alpha^3_F(y) - \eta^3_F(y)}$, then $\zeta_F(y)$ is supposed to be an indeterminacy membership degree (IMD) of $y \in Y$ in F. For simplicity, F_r FSs over Y is read as F_r FSs(Y).

Definition 5. Reference [9]. The following is the formula of an LP model:

$$\begin{aligned} \text{Maximize} : S &= c_1 t_1 + c_2 t_2 + c_3 t_3 + \cdots + c_n t_n, \\ \text{Subject to} : & a_{11} t_1 + a_{12} t_2 + a_{13} t_3 + \cdots + a_{1n} t_n \le b_1, \\ & a_{21} t_1 + a_{22} t_2 + a_{23} t_3 + \cdots + a_{2n} t_n \le b_2, \\ & \vdots \\ & a_{m1} t_1 + a_{m2} t_2 + a_{m3} t_3 + \cdots + a_{mn} t_n \le b_m, \\ & t_1, t_2, \ldots, t_n \ge 0. \end{aligned} \tag{5}$$

In LP model, m indicates the cardinality of constraints and n shows the number of decision variables.

4. A Modified Distance Measure between F_r FSs

A modified Hamming distance measure between two F_r FSs is presented to tackle the vague data in this section.

Definition 6. Suppose that F_1 and F_2 be two F_r FSs defined on a fixed set $Y = \{y_1, y_2, y_3\}$, then the distance $D_F(F_1, F_2)$ is defined as follows:

$$D_F(F_1, F_2) = \frac{1}{3n} \sum_{i=1}^{n} \left(\begin{array}{c} \left[\left| \alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i) \right| + \left| \eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i) \right| \right] + \\ \max \left[\left| \alpha^3_{F_1}(y_i) - \alpha^3_{F_2}(y_i) \right|, \left| \eta^3_{F_1}(y_i) - \eta^3_{F_2}(y_i) \right| \right] \end{array} \right). \tag{6}$$

Example 1. Let F_1 and F_2 be two F_r FSs over $Y = \{y_1, y_2, y_3\}$ given by $F_1 = \{(y_1, (0.8, 0.7)), (y_2, (0.9, 0.8), (y_3, (0.5, 0.9))\}$ and $F_2 = \{(y_1, (0.8, 0.62)), (y_2, (0.7, 0.6)), (y_3, (0.9, 0.6))\}$, based on Definition 6, we get, $D_F(F_1, F_2) = 0.2974$.

Theorem 1. *Let D be a mapping such that D: $F_rFSs(X)\times F_rFSs(X)\longrightarrow[0,1]$. If the requirements below are achieved, then $D_F(F_1,F_2)$ is a distance measure.*

(1) $0\le D_F(F_1,F_2)\le 1$;

(2) $D_F(F_1,F_2)=0\sqrt{b^2-4ac}$ iff $F_1=F_2$;

(3) $D_F(F_1,F_2)=D_F(F_2,F_1)$;

(4) $D_F(F_1,F_3)\ge D_F(F_1,F_2)$ and $D_F(F_1,F_3)\ge D_F(F_2,F_3)$, for any $F_1,F_2,F_3\in F_r$ FSs(X).

Proof. As, (6) is easy to prove, however, the last condition (4) is proved as follows: For any $F_1,F_2,F_3\in F_r$ FSs(X), and $F_1\subseteq F_2\subseteq F_3$, then on the basis of Definition 5, we get

$$\begin{aligned}&\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_3}(x_i)\right|\ge\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_2}(x_i)\right|,\\&\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_3}(x_i)\right|\ge\left|\alpha_{F_1}(x_i)-\eta^3_{F_2}(x_i)\right|.\end{aligned}\tag{7}$$

By adding equation (7), we get

$$\begin{aligned}&\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_3}(x_i)\right|+\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_3}(x_i)\right|\\&\ge\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_2}(x_i)\right|+\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_2}(x_i)\right|,\\&\Rightarrow\\&\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_3}(x_i)\right|+\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_3}(x_i)\right|\\&+\max\left\{\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_3}(x_i)\right|,\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_3}(x_i)\right|\right\}\\&\ge\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_2}(x_i)\right|+\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_2}(x_i)\right|\\&+\max\left\{\left|\alpha^3_{F_1}(x_i)-\alpha^3_{F_2}(x_i)\right|,\left|\alpha^3_{F_1}(x_i)-\eta^3_{F_2}(x_i)\right|\right\},\end{aligned}\tag{8}$$

$\Rightarrow D_F(F_1,F_3)\ge D_F(F_1,F_3)$, similarly, we can show, $D_F(F_1,F_3)\ge D_F(F_2,F_3)$. □

Since, criteria's weights have great impact in DM_g, we transform the Definition 2.6 into a weighted distance measure (WDM) between two F_r FSs as follows:where $w_j(1\le j\le m)$ denotes the m criteria weights such that $\sum_{j=1}^m w_j=1$.

Definition 7. Suppose that F_1 and F_2 are two F_r FSs over $Y=\{y_1,y_2,\ldots,y_n\}$ and w_j are the m criteria's weights satisfying the condition $\sum_{j=1}^m w_j=1$. Then the WDM $D^w_F(F_1,F_2)$ is penned as below:

$$\begin{aligned}&D^w_F(F_1,\mathscr{B})\\&=\sum_{i=1}^n w_j\left(\begin{array}{c}\left[\left|\alpha^3_{F_1}(y_i)-\alpha^3_{F_2}(y_i)\right|+\left|\eta^3_{F_1}(y_i)-\eta^3_{F_2}(y_i)\right|\right]+\\\max\left[\left|\alpha^3_{F_1}(y_i)-\alpha^3_{F_2}(y_i)\right|,\left|\eta^3_{F_1}(y_i)-\eta^3_{F_2}(y_i)\right|\right]\end{array}\right).\end{aligned}\tag{9}$$

Example 2. Let. F_1. and F_1 be two F_r FSs on a set $Y=\{y_1,y_2,y_3\}$. Example 1 takes the result by using the weights of y_1,y_2 and y_3 as $w_1=0.25,w_2=0.35$ and $w_3=0.4$, respectively. based on Definition 2.7, $D^w_F(F_1,F_2)=0.7539$.

Theorem 2. *The WDM $D^w_F(F_1,F_2)$ between two F_r FSs F_1 and F_2 satisfy the following four conditions:*

(1) $0\le D^w_F(F_1,F_2)\le 1$;

(2) $D^w_F(F_1,F_2)=0$ iff $F_1=F_2$;

(3) $D^w_F(F_1,F_2)=D_F(F_2,F_1)$;

(4) $D^w_F(F_1,F_3)\ge D^w_F(F_1,F_2)$ and $D^w_F(F_1,F_3)\ge D^w_F(F_2,F_3)$, for any $F_1,F_2,F_3\in F_r$ FSs(X).

Proof. In order to prove Theorem 2, follow the same strategy as Theorem 1. □

Definition 8. Suppose that F_1 and F_2 are two F_r FSs over $Y=\{y_1,y_2,\ldots,y_n\}$. Then measure of similarity $S_p(F_1,F_2)$ on the basis of Definition 7 is penned as follows:

$$\begin{aligned}&S_F(F_1,F_2)\\&=1-\sum_{i=1}^n w_j\left(\begin{array}{c}\left[\left|\alpha^3_{F_1}(y_i)-\alpha^3_{F_2}(y_i)\right|+\left|\eta^3_{F_1}(y_i)-\eta^3_{F_2}(y_i)\right|\right]+\\\max\left[\left|\alpha^3_{F_1}(y_i)-\alpha^3_{F_2}(y_i)\right|,\left|\eta^3_{F_1}(y_i)-\eta^3_{F_2}(y_i)\right|\right]\end{array}\right).\end{aligned}\tag{10}$$

Definition 9. A mapping S: $F_rFSs(X)\times F_rFSs(X)\longrightarrow[0,1]$. $S_F(F_1,F_2)$ is supposed to be a measure of similarity if $S_F(F_1,F_2)$ fulfills the following four axioms:

(1) $0\le S_F(F_1,F_2)\le 1$;

(2) $S_F(F_1,F_2)=1$ iff $F_1=F_2$;

(3) $S_F(F_1,F_2)=S_F(F_2,F_1)$;

(4) $S_F(F_1,F_3)\le S_F(F_1,F_2)$ and $S_F(F_1,F_3)\le S_F(F_2,F_3)$, for any $F_1,F_2,F_3\in F_r$ FSs(X) and $F_1\subseteq F_2\subseteq F_3$.

5. MCDM Model Based on Fermatean Fuzzy TOPSIS (F_r F-Topsis)

We suggested an MCDM using F_r F information based on TOPSIS employing LP methodology in this part. The LP model is used to assess the weights of criteria under various restrictions. Suppose that $H=\{H_1,H_2,\ldots,H_n\}$ be a collection of alternatives, and $G=\{G_1,G_2,\ldots,G_m\}$ be the collection of criteria with $\mu=\{\mu_1,\mu_2,\ldots,\mu_m\}$, where $\sum_{j=1}^m\mu_j=1$ as the weight vector of the criteria G_j, where $j=1,2,3,\ldots,m$. A F_r F decision matrix denoted by $\mathscr{F}=[\Omega_{ij}]_{n\times m}=[(\alpha_{ij},\eta_{ij})]_{n\times m}$ with α_{ij} as MD and η_{ij} NMD that the alternatives $A_i(i=1,2,\ldots,n)$ fulfills, respectively. To reach the optimal solution, follow the steps of proposed MCDM model.

Step 1. Developed a F_r F decision matrix denoted by $\mathscr{F}=[\Omega_{ij}]_{n\times m}$ according to the given information presented by the DM.

Step 2. Figure out the F_r F positive-ideal solution (F_r FPIS), Ω^+_p and F_r F negative-ideal solution (F_r FNIS), Ω^-_p as follows:

Table 1: F_r F decision matrix.

Alternatives	
Q_1	$\{(y_1, 0.7, 0.3), (y_2, 0.4, 0.6), (y_3, 0.5, 0.5), (y_4, 0.8, 0.2), (y_5, 0.8, 0.4)\}$
Q_2	$\{(y_1, 0.5, 0.8), (y_2, 0.8, 0.6), (y_3, 0.4, 0.5), (y_4, 0.7, 0.4), (y_5, 0.6, 0.5)\}$
Q_3	$\{(y_1, 0.9, 0.6), (y_2, 0.8, 0.1), (y_3, 0.6, 0.4), (y_4, 0.7, 0.5), (y_5, 0.9, 0.3)\}$
Q_4	$\{(y_1, 0.6, 0.7), (y_2, 0.8, 0.3), (y_3, 0.7, 0.2), (y_4, 0.5, 0.3), (y_5, 0.7, 0.3)\}$

$$\Omega_p^+ = \{(\alpha_{ij}^+, \eta_{ij}^+)\} = \begin{pmatrix} \left\{\left(\max_j(\alpha_{ij}), \max_j(\eta_{ij})\right)\right\}: U_j \in \xi_1 \\ \left\{\left(\min_j(\alpha_{ij}), \min_j(\eta_{ij})\right)\right\}: U_j \in \xi_2 \end{pmatrix}. \quad (11)$$

$$\Omega_p^- = \{(\alpha_{ij}^-, \eta_{ij}^-)\} = \begin{pmatrix} \left\{\left(\min_j(\alpha_{ij}), \min_j(\eta_{ij})\right)\right\}: U_j \in \xi_1 \\ \left\{\left(\min_j(\alpha_{ij}), \min_j(\eta_{ij})\right)\right\}: U_j \in \xi_2 \end{pmatrix}, \quad (12)$$

where ξ_1 and ξ_2 are subcollections of beneficial and cost criteria, respectively, so that $\xi_1 \cap \xi_2 = \varnothing$.

Step 3. Compute the weighted similarity degree (WSD) $S_{F_i}^{+w}$ between F_r FPIS Ω_F^+ and each alternative likewise the WSD $S_{F_i}^{-w}$ between F_r FNIS Ω_F^- by using equation (12), respectively:

$$S_{Fi}^{+w}(H_i, \Omega_F^+) = 1 - \sum_{j=1}^{m} w_j \begin{pmatrix} \left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^+\right| + \left|\eta_{F_1}(x_i) - \eta_{ij}^+\right|\right] + \\ \max\left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^+\right|, \left|\eta_{F_1}(x_i) - \eta_{ij}^+\right|\right] \end{pmatrix}. \quad (13)$$

$$S_{Fi}^{-w}(H_i, \Omega_F^-) = 1 - \sum_{j=1}^{m} w_j \begin{pmatrix} \left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^-\right| + \left|\eta_{F_1}(x_i) - \eta_{ij}^-\right|\right] + \\ \max\left[\left|\alpha_{F_1}(x_i) - \alpha_{ij}^-\right|, \left|\eta_{F_1}(x_i) - \eta_{ij}^-\right|\right] \end{pmatrix}, \quad (14)$$

where, $1 \le i \le n$.

Step 4. Based on equations (15) and (16), construct the model to find the objective function Z for the weights of criteria as follows:

$$Z = \left(S_{Fi}^{+w}(H_i, \Omega_F^+) - S_{Fi}^{-w}(H_i, \Omega_F^-)\right). \quad (15)$$

Step 5. We derive the weights μ_j of the criterion $G_j\,(j = 1, 2, 3, \ldots, m)$ by solving the LP model described in [30], so that the objective function Z produced in Step 4 is maximized.

Step 6. Based on equations (15) and (16), calculate the degree of similarity and evaluate $S_{F_i}^{+w}$ and $S_{F_i}^{-w}$ on the basis of equations (9) and (10) between each option and the components achieved in F_r FPIS Ω_p^+ and F_r FNIS Ω_p^-, respectively.

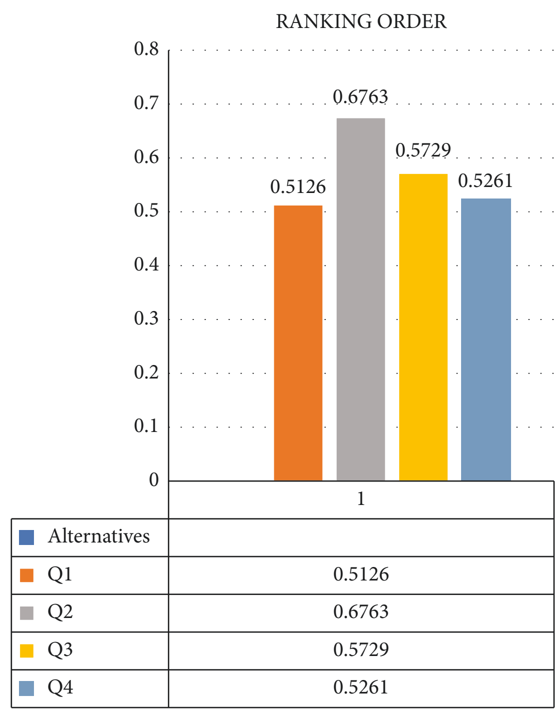

Figure 2: Ranking of alternatives.

Step 7. Determine the coefficient of relative closeness $\mathscr{R}^C{}_i$ of each alternative H_i with respect to the F_r FPIS Ω_F^+ as follows:

$$\mathscr{R}_i^C = \frac{S_{F_i}^{+w}}{S_{F_i}^{+w} + S_{F_i}^{-w}}. \quad (16)$$

The greater the value $\mathscr{R}^C{}_i$ of the alternatives to F_r FPIS (Ω_p^{+w}), the more likely we are to find the greatest choice from a set of alternatives H_i, where $1 \le i \le n$.

6. Solution of Problems Based on F_r F-Topsis

The authors used the proposed MCDM model to recognize the pattern and breakout of dengue disease in this section.

Step 1. F_r F decision matrix $P_c = [\Omega_{ij}]_{4\times 5}$ denoted in Table 1.

Step 2. The ideal solution $\Omega_F^+ = \{(y_1, 0.9000, 0.8000), (y_2, 0.8000, 0.6000), (y_3, 0.7000, 0.5000), (y_4, 0.8000, 0.5000), (y_5, 0.9000, 0.5000)\}$ $\Omega_F^- = \{(y_1, 0.5000, 0.3000), (y_2, 0.4000, 0.1000), (y_3, 0.4000, 0.2000), (y_4, 0.5000, 0.2000), (y_5, 0.6000, 0.3000)\}$

Step 3. The WSD $S_{F_i}^{+w}$ between F_r FPIS Ω_F^+ and each alternative as well as the WSD $S_{F_i}^{-w}$ between F_r FNIS Ω_F^-

Table 2: Results obtained for altering the weights of criteria.

Alternatives	Original	Increment in w_1	Increment in w_2	Increment in w_3	Increment in w_4	Increment in w_5
Q_1	0.5126	0.5013	0.5214	0.5180	0.5127	0.5210
Q_2	0.6763	0.6600	0.6508	0.6691	0.6707	0.6700
Q_3	0.5729	0.5630	0.5621	0.5592	0.5730	0.5745
Q_4	0.5261	0.5187	0.5201	0.5150	0.5271	0.5268

by using equations (13) and (14), respectively, in terms of weights.

Step 4. Based on equations (13) and (14), evaluate $Z = -0.0250w_1 - 0.3650w_2 - 0.1450w_3 - 0.0550w_4$ which is written in equation (17).

Step 5. Based on LP model penned in [9], the weights w_j of the criteria P_j, where $j = 1, 2, 3, 4, 5$ are obtained as follows:

$$w_1 = 0.2, w_2 = 0.3, w_3 = 0.25, w_4 = 0.1 \text{ and } w_5 = 0.15. \tag{17}$$

Step 6. Degree of positive and negative weighted similarities $S_+^{f_{ri}}$ and $S_-^{f_{ri}}$ are obtained by using equations (7) and (8) as follows:

$$\begin{aligned}
&S_+^{f_{r1}}(Q_1, \Omega_F^+) = 0.5100, S_+^{f_{r2}}(Q_2, \Omega_F^+) = 0.7000,\\
&S_+^{f_{r3}}(Q_3, \Omega_F^+) = 0.5700, S_+^{f_{r4}}(Q_4, \Omega_F^+) = 0.5550, \text{ and}\\
&S_-^{f_{r1}}(Q_1, \Omega_F^-) = 0.4850, S_-^{f_{r2}}(Q_2, \Omega_F^-) = 0.3350,\\
&S_-^{f_{r3}}(Q_3, \Omega_F^-) = 0.4250, S_-^{f_{r4}}(Q_4, \Omega_F^-) = 0.5000.
\end{aligned} \tag{18}$$

Step 7. Values of $\mathscr{R}^C{}_i$ of each alternative is the following:

$$\begin{aligned}
\mathscr{R}_1^C &= 0.5126,\\
\mathscr{R}_2^C &= 0.6763,\\
\mathscr{R}_3^C &= 0.5729,\\
\mathscr{R}_4^C &= 0.5261.
\end{aligned} \tag{19}$$

Step 8. Arrange the alternatives according to the values of R_i^C as obtained in Step 4. We get, $Q_2 \prec Q_3 \prec Q_4 \prec Q_1$. Hence, the optimal alternative attained is Q_2 which is illustrated in Figure 2.

Example 3. A construction company wanted to select four suppliers, Q_1, Q_2, Q_3, and Q_5 according to certain criteria. Suppliers are evaluated against five parameters, P_1, P_2, P_3, and p_5. Weights of criteria have great impact in decisions, authors have used LP model to compute the weights. Assume that the evaluation values of the alternatives in relation to each criterion provided by the committee are represented by F_r FNs, as shown in the F_r F decision matrix given in Table 1.

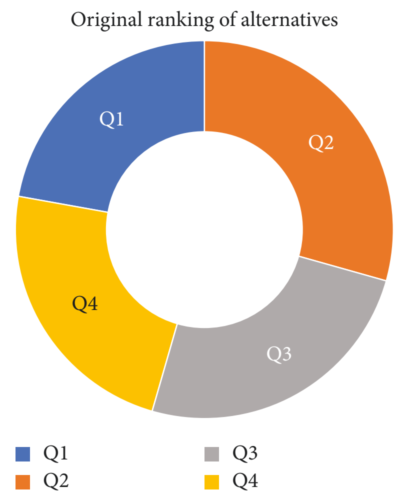

Figure 3: Original result.

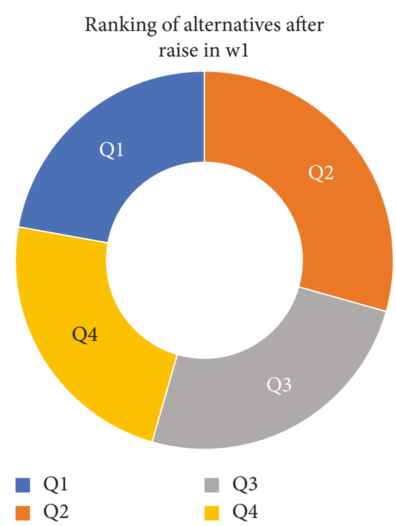

Figure 4: 0.05 to 0.1 raise in w_1.

7. Sensitivity Analysis

Because the information for MCDM problems is frequently uncertain and ambiguous, there is a need for a tool that can assist us make more correct decisions. Sensitivity analysis (SA) can help in this regard. In this part, weighted SA is used to evaluate the impact of changing the weights of criteria on the results provided by the proposed model. A formula described in [37] is used to generate a new vector for criterion weights, and the behavior of the findings obtained by the suggested model is then examined. We changed the

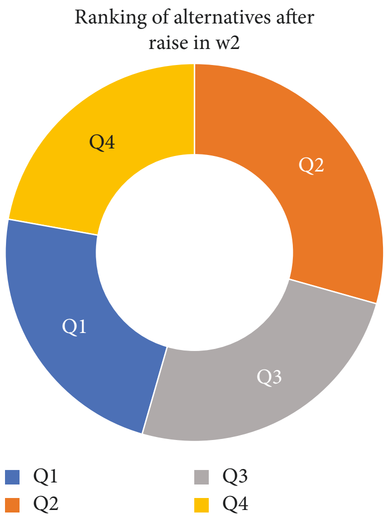

Figure 5: 0.05 to 0.1 raise in w_2.

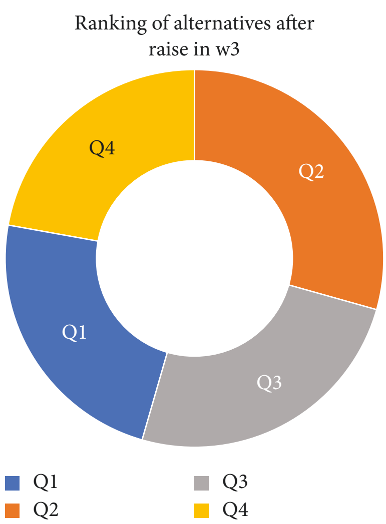

Figure 6: 0.05 to 0.1 raise in w_3.

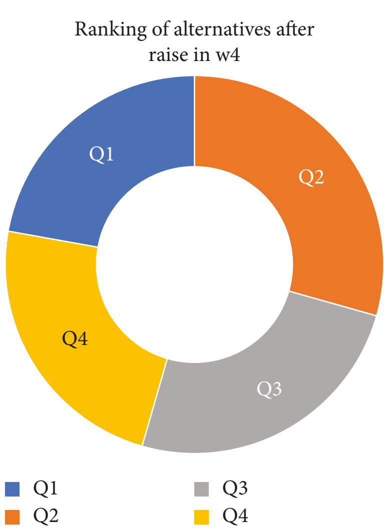

Figure 7: 0.05 to 0.1 raise in w_4.

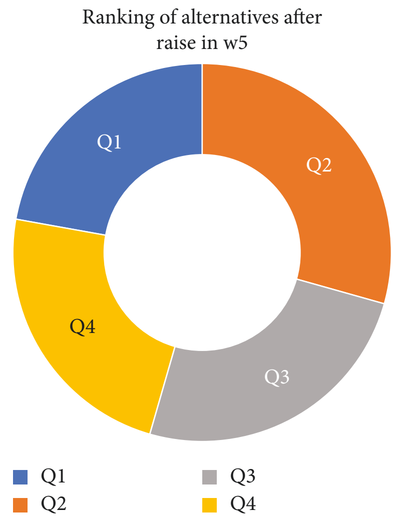

Figure 8: 0.05 to 0.1 raise in w_5.

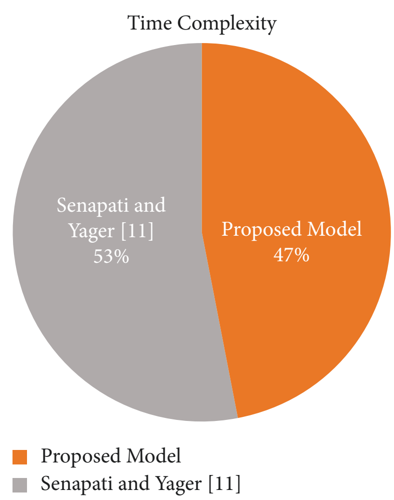

Figure 9: Graphical view of TC analysis.

Table 3: TC among the proposed and existing technique.

Techniques	Executing time
Proposed model	0.4513 seconds
Senapati and Yager [23]	0.510 seconds

weights of individual criteria by raising different ratios and looked at the effect on the final findings. Table 2 shows the outcomes achieved by varying the weights of criterion. Figures 3 to 8 show that raising 0.05 to 0.1 in each weight results in a little change in the numeric values, but the ranking orders remain same, demonstrating the usefulness and strength of our suggested model.

7.1. Comparison Based on Time Complexity (TC). In order to strengthen the results obtained from the proposed MCDM model, TC analysis is performed in the present subsection. TC is the time required to execute an algorithm to reach the final result. TC is measured among the proposed and the existing techniques presented by Senapati and Yager [23]. The executing time of each technique is evaluated with the help of MATLAB which is presented in Table 3 and its graphical view is illustrated in Figure 9. From Table 3, it can be seen that our approach takes less time as compared to others; hence, the proposed MCDM model is more effective and resolves the issues rapidly.

8. Conclusions

TOPSIS is one of the most well-known MCDM approaches. The focus of this research was on TOPSIS extensions named F_r. F-TOPSIS is used in complicated decision scenarios with uncertainty. The total of squares of MD and NMD to which an item meeting a criteria supplied by expert is subjected in some real-world situations may be greater than one, but their cube sum may be less than or equal to one. As a result, P_g FS is unable to handle such a situation. From this perspective, the F_r FS might be used to mimic some D M_g scenarios that P_g FS cannot handle. In this study, we offer an MCDM technique based on TOPSIS in a F_r FS environment. Finally, we provided an example to demonstrate how this method might be utilized efficiently.

In light of the foregoing, future research could concentrate on:

(1) Using other traditional objective and subjective multicriteria decision-making methods in conjunction with F_r FS to determine and evaluate criteria for the selection of the alternative.

(2) Aside from that, the benefits of the current strategy can be enhanced by considering the objective weight of risk factors, which are not taken into account in this study.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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