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Research Article

Some New Dimensions to Construct Economical Circular Weakly Balanced Neighbor Robust Designs

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Neighbor-balanced designs are used in the experiments where the performance of a treatment is affected by the treatments applied to its neighboring units. These are well-known designs to balance the neighbor effects. Among these designs, minimal neighbor-balanced designs are economical; therefore, these are preferred by the experimenters. For v even, minimal neighbor balanced designs in circular blocks cannot be constructed for most of the cases, where v is number of treatments. In such situations, experimenters would like to relax the neighbor balance property up to some extent and consider the minimal circular weakly balanced neighbor designs as the better alternates to the minimal circular neighbor-balanced designs. In this article, some generators are developed to obtain minimal circular weakly balanced neighbor designs in blocks of equal, two and three different sizes.

1. Introduction

Neighbor-balanced designs (NBDs) are used in the experiments where performance of a treatment is influenced by the treatments applied to its neighboring units. NBDs are well-known designs to balance the neighbor effects. Among the NBDs, minimal NBDs are economical; therefore, these are preferred by the experimenters. If each pair of distinct treatments appears once in adjacent plots, then, the design is called minimal NBD. A block formed in a cycle in such a way that its first and last units are considered as adjacent neighbors is called a circular block. In circular blocks, each unit has one left-neighbor and one right-neighbor. Williams [1] suggested nearest neighbor-balanced designs (NBDs) in linear blocks for first-order autoregressive model. Rees [2] introduced neighbor designs in a technique used in virus research and constructed these designs for odd number of treatments (v). Lawless [3] gave a note on different types of balanced incomplete designs balanced for residual effects. Hwang and Lin [4,5] constructed four classes of NBDs for v [6] discussed that the bias due to neighbor effects can be minimized through NBDs. Azais et al. [6] suggested that NBDs should be used to deal with neighbor effects and if a NBD requires large number of experimental units, a partially NBD is used. Jacroux [7] constructed equineighbored designs in blocks of sizes three. Kunert [8] showed that the bias due to neighbor effects can be minimized through NBDs or partially NBDs. Ahmed and Akhtar [9] constructed some series of NBDs which are not minimal. Akhtar and Ahmed [10] constructed some second-order NBDs for some cases of v [11] constructed some all-order NBDs for$V$ odd. Ahmed and Akhtar [12] constructed some NBDs in circular blocks of sizes six which are not minimal. Akhtar et al. [13] constructed some NBDs in circular blocks of sizes five. Misra and Nutan [14,15] constructed circular generalized neighbor designs (CGNDs) by relaxing the neighbor balance property. The CGNDs will become more economical and better alternate to the minimal CNBDs if most of the unordered pairs appear (i) once as neighbor while other appear no time, or (ii) once as neighbor while other appear twice. GNDs

Table 1: Blocks obtained from S_1.

Blocks																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4	5	6	7	8	9	10	11	12	13	14	15	16	17	0	1	2	3
10	11	12	13	14	15	16	17	0	1	2	3	4	5	6	7	8	9
17	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

possessing this property are also called minimal circular weakly balanced neighbor designs (CWBNDs). Mishra [16] presented some families of proper GNDs. Ahmed et al. [17] constructed economical GNDs in use of serology. Zafaryab et al. and Shehzad et al. [18,19] developed some generators to obtain the minimal CWBNDs (MCWBNDs) for some cases. Iqbal et al. [20] presented GNDs for k (block size) = 3. This review shows that MCWBNDs-II (MCWBNDs in which 3 $v/2$ unordered pairs do not appear as neighbors while all other appear once) are not available in the literature. In this study, therefore, some generators are presented to generate MCWBNDs-II for all $k > 2$. These designs are economical and efficient to balance the neighbor effects.

Neutrosophic statistics is the extension of classical statistics and is applied when the data are coming from a complex process or from an uncertain environment. The proposed study can be extended for neutrosophic statistics. Few researchers [21–23] constructed designs of (i) sampling plan for exponential distribution under the neutrosophic statistical interval method, (ii) a new attribute control chart under neutrosophic statistics, and (iii) variable sampling plan for Pareto distribution using the neutrosophic statistical interval method.

2. Method of Cyclic Shifts

This method was introduced by Iqbal [24] which is simplified here for CBNDs and MCWBNDs-II.

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be i sets of shifts, $j = 1,2, \ldots, i$, $u = 1,2, \ldots,$ k-1, and $1 \leq q_{ju} \leq v$-1.

(i) If all from 1 to v-1 appears exactly once in S $*$, then designs will be MCBND.

(ii) If all from 1 to v-1 appears once in S $*$ except any three elements which do not appear, then designs will be MCWBND-II.

where S $*$ contains

(i) Each element of all S_j.

(ii) Sum (mod v) of all elements in each S_j.

(iii) Complements of all elements in (i) and (ii), here complement of 'a' is 'v-a'.

Logic behind Rule I of this method expresses that

If $m = (v\text{-}2)/2$ for, v even then, S = [1, 2, . . ., m-1] or S = [1, 2, . . ., m-2, m] will provide MCWBNDs-II if sum of S is divisible by v. Otherwise, for this, elements with their complements have to be replaced.

Example 1. $S_1 = [4, 6, 7]$ and $S_2 = [2, 3]$ generate following MCWBND-II for $v = 18$, $k_1 = 4$, and $k_2 = 3$.

For S_1, get v Blocks and assign 0, 1, . . ., v-1 to first units of each block, respectively. Add 4 (mod 18) to the first unit element of each block for second unit elements. Similarly, add 6 (mod 18) and so on (see Table 1).

Take v more blocks for S_2 and obtain the design (see Table 2).

Table 1 and Table 2 jointly present the MCWBND-II for $v = 18$, $k_1 = 4$, and $k_2 = 3$. In this design, 27 unordered pairs do not appear as neighbors while all other appear once.

2.1. Logic behind Rule I. Using the logic behind Rule I, following theorems are developed for MCWBNDs-II.

Theorem 1. *If $m = 4u$, i sets obtained from S = [1, 2, . . ., (m/2), (m+4)/2, (m+6)/2,. . ., (m-1), (3m + 2)/2] will provide MCWBNDs-II for $v = 2ik+4$, where $m = (v\text{-}2)/2$.*

Proof. Let Sum = [1 + 2, . . .+(m/2)+((m+4)/2)+((m+6)/2)+ . . .+(m-1)+(3m + 2)/2]

$= 2(m+1)m/4 = v\ (m/4)$ Since $v = 2(m+1)$.

$m/4$ will be integer for $m = 4u$.

Sum will be divisible of v if $m/4$ is integer. Hence proved. □

Theorem 2. *If $m = 4u+1$, then sets derived from S = [1, 2, . . ., (3m + 1)/4, (3m + 9)/4, (3m + 13)/4, . . ., (m-1), (5m + 3)/4] will provide MCWBNDs-II for $v = 2ik+4$.*

Proof. Let Sum = [1 + 2, . . .+(3m + 1)/4+(3m + 9)/4+(3m + 13)/4+. . .+(m-1)+(5m + 3)/4]

$= 2(m+1)\ (m\text{-}1)/4 = v\ (m\text{-}1)/4$ Since $v = 2(m+1)$.

$(m\text{-}1)/4$ will be integer for $m = 4u+1$.

Sum will be divisible of v if $(m\text{-}1)/4$ is integer. Hence proved. □

Theorem 3. *If $m = 4u+2$, i sets obtained from S = [2, 3, . . ., m-2, m, 2m + 1] will provide MCWBNDs-II for $v = 2ik+4$.*

Proof. Let Sum = 2 + 3+,. . .+(m-2)+m+(2m + 1)

$= 2(m+1)\ (m+2)/4 = v\ (m+2)/4$ Since $v = 2(m+1)$.

$(m+2)/4$ will be integer for $m = 4u+2$.

Sum will be divisible of v if $(m+2)/4$ is integer. Hence proved. □

Theorem 4. *If $m = 4u+3$, then i sets obtained from S = [1, 2, . . ., (m+1)/4, (m+9)/4, (m+13)/4, m-2, m, (7m + 3)/4] produce MCWBNDs-II for $v = 2ik+4$.*

Table 2: Blocks obtained from S_2.

Blocks																	
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	0	1
5	6	7	8	9	10	11	12	13	14	15	16	17	0	1	2	3	4

Proof. Let Sum = [1 + 2, ...+(m+1)/4+(m+9)/4+(m+13)/4+(m-2)+m+(7m + 3)/4]

= 2(m+1) (m+1)/4 = v (m+1)/4 Since v = 2(m+1).

(m+1)/4 will be integer for $m = 4u+3$.

Sum will be divisible of v if (m+1)/4 is integer. Hence proved. □

2.2. Efficiency of Neighbor Effects. The efficiency factor for neighbor effect (residual effect) is the harmonic mean of nonzero Eigen values of their respective information matrix (see [25]). The high value of E_n shows that design is suitable for the estimation of neighbor effects. Our developed generators provide the designs with high values of E_n for v even; therefore, these designs are suitable for the estimation of neighbor effects. Table 3 shows the the E_n of some existing MCBNDs and proposed designs for comparison.

3. Construction of MCWBNDs-II

Here, minimal CWBNDs-II are generated through Rule I. To generate required MCWBNDs-I, sets of shifts will be obtained from the elements of selected S, see Theorem 2–Theorem 4. Here, $m = (v\text{-}2)/2$, and i, u, and w are integers.

(i) If $m = 4u$, then, $S = [1, 2, \ldots, (m/2), (m+4)/2, (m+6)/2, \ldots, (m\text{-}1), (3m + 2)/2]$

(ii) If $m = 4u+1$, then, $S = [1, 2, \ldots, (3m + 1)/4, (3m + 9)/4, (3m + 13)/4, \ldots, (m\text{-}1), (5m + 3)/4]$

(iii) If $m = 4u+2$, then, $S = [2, 3, \ldots, m\text{-}2, m, 2m + 1]$

(iv) If $m = 4u+3$, then, $S = [1, 2, \ldots, (m+1)/4, (m+9)/4, (m+13)/4, m\text{-}2, m, (7m + 3)/4]$.

3.1. MCWBNDs-II in Blocks of Equal Sizes. Here, MCWBNDs-II are generated in equal block sizes from i sets which will be generated in the following manner.

Divide the elements of selected S in i groups of size k in such a way that v is the factor of sum of each group; then, discard one element from each group.

3.1.1. Generator 3.1.1. MCWBNDs-II can be generated for $v = 2i$k+4 and

(i) $k = 4l$, i and l integer and $m = 4u+1$,

(ii) $k = 4l + 2$, i odd, l integer and $m = 4u+3$.

(iii) $k = 4l + 2$, i even, l integer and $m = 4u+1$.

(iv) k (odd) > 3, $i = 4j$ and $m = 4u+1$.

(v) k (odd) > 3, $i = 4j+2$ and m (mod 4) ≡ 3.

(vi) $k = 4w + 1$, $k > 1$, $i = 4j+3$ and $m = 4u$.

(vii) $k = 4w + 1$, $k > 1$, $i = 4j+1$ and $m = 4u+2$.

(viii) $k = 4w + 3$, $k > 3$, $i = 4j+1$ and $m = 4u$.

(ix) $k = 4w + 3$, $k > 3$, $i = 4j+3$ and $m = 4u+2$.

Example 2. MCWBND-II is generated for $v = 18$ and $k = 7$.

$S_1 = [2,3,4,6,7,13]$.

Example 3. MCWBND-II is generated for $v = 28$ and $k = 5$.

$S_1 = [3,4,5,6,8]$, $S_2 = [7,9,10,12,17]$.

3.2. MCWBNDs-II in Blocks of Two Different Sizes. Here, MCWBNDs-II are constructed in two different block sizes from (i+1) sets which will be generated in the following manner.

Divide the elements of selected S in i groups of size k_1 and one of k_2 in such a way that v is the factor of sum of each group; then, discard one element from each group.

3.2.1. Generator 3.2.1. MCWBNDs-II can be generated for $v = 2(i+1)k_1+2$, $k_2 = k_1$-1, and

(i) $k_1 = 4l$, i and l integer and $m = 4u$.

(ii) $k_1 = 4l + 2$, i odd and $m = 4u$.

(iii) $k_1 = 4l + 2$, i even and $m = 4u+2$.

(iv) k_1 (odd) > 3, $i = 4j+3$ and $m = 4u$.

(v) k_1 (odd) > 3, $i = 4j+1$ and $m = 4u+2$.

(vi) $k_1 = 4w + 1$, $k_1 > 1$, $i = 4j+2$ and $m = 4u+3$.

(vii) $k_1 = 4w + 1$, $k_1 > 1$, $i = 4j$ and $m = 4u+1$.

(viii) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j$ and $m = 4u+3$.

(ix) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j+2$ and $m = 4u+1$.

Example 4. MCWBND-II is generated for $v = 44$, $k_1 = 7$, and $k_2 = 6$.

$S_1 = [3, 4, 5, 67, 18]$, $S_2 = [9, 10, 11, 12, 13, 14, 19]$, $S_3 = [8, 15, 16, 20, 27]$.

3.2.2. Generator 3.2.2. MCWBNDs-II can be generated for $v = 2(i+1)k_1$, $k_2 = k_1$-2, and

(i) $k_1 = 4l$, $l > 1$, i and l integer and $m = 4u+3$.

(ii) $k_1 = 4l + 2$, i odd, l integer and $m = 4u+3$.

(iii) $k_1 = 4l + 2$, i even, l integer and $m = 4u+1$.

(iv) k_1 (odd) > 3, $i = 4j+1$, and $m = 4u+1$.

(v) k_1 (odd) > 3, $i = 4j+3$, and $m = 4u+3$.

Table 3: E_n for comparison of existing MCBNDs and proposed MCWBNDs.

Existing MCBNDs				Proposed MCWBNDs given in appendix		
v	k	E_n	Set of shifts	v	k	E_n
17	4	0.79	[3, 7, 8]+[2, 4, 5]	20	4	0.78
25	4	0.78	[2, 4, 8]+[7, 10, 16]+[1, 3, 9]	28	4	0.77
33	4	0.77	[6, 11, 16]+[5, 7, 8]+ [2, 9, 10]+[1, 3, 14]	36	4	0.76
41	5	0.83	[4,6,15,39]+[5, 7–9]+[1, 2, 11, 13]+ [10, 16, 17, 19]	44	5	0.81

(vi) $k_1 = 4w + 1$, $k_1 > 1$, $i = 4j$, and $m = 4u$.

(vii) $k_1 = 4w + 1$, $k_1 > 1$, $i = 4j+2$, and $m = 4u+2$.

(viii) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j$, and $m = 4u+2$.

(ix) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j+2$, and $m = 4u$.

Example 5. MCWBND-II is generated for $v = 20$, $k_1 = 5$, and $k_2 = 3$.

$S_1 = [3, 4, 5, 6]$, $S_2 = [7, 12]$.

3.3. MCWBNDs-II in Blocks of Three Different Sizes. Here, MCWBNDs-II are constructed in three different block sizes using $(i+2)$ sets which will be generated in the following manner.

Divide the elements of selected S in i groups of size k_1 and one each of k_2 and k_3 in such a way that v is factor of sum of each group; then, discard one element from each group.

3.3.1. Generator 3.3.1. MCWBNDs-II can be generated for $v = 2ik_1+2k_2+2k_3+4$, $k_2 = k_1-1$, $k_3 = k_1-2$, and

(*i*) $k_1 = 4l$, $l > 1$, i and l integer and $m = 4u+2$.

(ii) $k_1 = 4l + 2$, i odd, l integer and $m = 4u$.

(iii) $k_1 = 4l + 2$, i even, l integer and $m = 4u+2$.

(iv) k_1 (odd) > 3, $i = 4j$, and $m = 4u$.

(v) k_1 (odd) > 3, $i = 4j+2$, and $m = 4u+2$.

(vi) $k_1 = 4w + 1$, $k_1 > 1$, $i = 4j+1$, and $m = 4u+1$.

(vii) $k_1 = 4w + 1$, $k_1 > 1$, $i = 4j+3$, and $m = 4u+3$.

(viii) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j+1$, and $m = 4u+3$.

(ix) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j+3$, and $m = 4u+1$.

Example 6. MCWBND-II is generated for $v = 28$, $k_1 = 5$, $k_2 = 4$, and $k_3 = 3$.

$S_1 = [2, 3, 5, 17]$, $S_2 = [7, 8, 9]$, $S_3 = [10, 12]$.

Example 7. MCWBND-II is generated for $v = 34$, $k_1 = 6$, $k_2 = 5$, and $k_3 = 4$.

$S_1 = [3, 4, 5, 6, 14]$, $S_2 = [10, 11, 15, 25]$, $S_3 = [8, 12, 13]$.

3.3.2. Generator 3.3.2. MCWBNDs-II can be generated for $v = 2(i+2)k_1-4$, $k_2 = k_1-1$, $k_3 = k_1-3$, and

(i) $k_1 = 4l$, $l > 1$, i and l integer and $m = 4u+1$.

(ii) $k_1 = 4l + 2$, l integer, i odd and $m = 4u+3$.

(iii) $k_1 = 4l + 2$, l integer, i even, and $m = 4u+1$.

(iv) k_1 (odd) > 5, $i = 4j$, and $m = 4u+3$.

(v) k_1 (odd) > 5, $i = 4j+2$, and $m = 4u+1$.

(vi) $k_1 = 4w + 1$, $k_1 > 5$, $i = 4j+1$, and $m = 4u$.

(vii) $k_1 = 4w + 1$, $k_1 > 5$, $i = 4j+3$, and $m = 4u+2$.

(viii) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j+1$, and $m = 4u+2$.

(ix) $k_1 = 4w + 3$, $k_1 > 3$, $i = 4j+3$, and $m = 4u$.

Example 8. MCWBND-II is generated for $v = 44$, $k_1 = 8$, $k_2 = 7$, and $k_3 = 5$.

$S_1 = [3, 4, 5, 6, 7, 8, 10]$, $S_2 = [11, 12, 13, 14, 16, 20]$, $S_3 = [15, 18, 19, 27]$.

Example 9. MCWBND-II is generated for $v = 50$, $k_1 = 9$, $k_2 = 8$, and $k_3 = 6$.

$S_1 = [2, 5, 4, 6, 7, 8, 9, 10]$, $S_2 = [12, 15, 16, 17, 19, 23, 379]$, $S_3 = [14, 18, 20, 21, 22]$.

3.3.3. Generator 3.3.3. MCWBNDs-II can be generated for $v = 2ik_1+2k_2+2k_3+4$, $k_2 = k_1-2$, $k_3 = k_1-3$, and

(*i*) $k_1 = 4l$, $l > 1$, i and l integer and $m = 4u+1$.

(ii) $k_1 = 4l + 2$, l integer, i odd, and $m = 4u+2$.

(iii) $k_1 = 4l + 2$, l integer, i even, and $m = 4u+1$.

(iv) k_1 (odd) > 5, $i = 4j$, and $m = 4u+2$.

(v) $k1$ (odd) > 5, $i = 4j+2$, and $m = 4u$.

(vi) $k_1 = 4w+ 1$, $k_1 > 5$, $i = 4j+1$, and $m = 4u+3$.

(vii) $k_1 = 4w+ 1$, $k_1 > 5$, $i = 4j+3$, and $m = 4u+1$.

(viii) $k_1 = 4w+ 3$, $k_1 > 3$, $i = 4j+1$, and $m = 4u+1$.

(ix) $k_1 = 4w+ 3$, $k_1 > 3$, $i = 4j+3$, and $m = 4u+3$.

Example 10. MCWBND-II is generated for $v = 42$, $k_1 = 8$, $k_2 = 6$, and $k_3 = 5$.

$S_1 = [2, 4, 5, 6, 7, 8, 9]$, $S_2 = [10, 12, 13, 15, 31]$, $S_3 = [16, 17, 18, 19]$.

Catalogues of MCWBNDs-II generated from these generators are presented in blocks of equal, two different and three different sizes as Table 4–6, respectively, in Appendix.

Table 4: MCWBNDs-II for $v \leq 100$ and $4 \leq k \leq 10$.

v	K	Set of shifts	En
17	4	[3,7,8]+[2,4,5]	
21	5	[1,3,4,5]+[6,14,9,11]	
20	4	[3,4,12]+[5,6,7]	0.78
28	4	[4,10,12]+[6,8,9]+[3,7,17]	0.77
36	4	[3,10,22]+[7,8,15]+[9,11,12]+[5,13,16]	0.76
44	4	[4,10,27]+[8,11,19]+[12,14,16]+[9,13,15]+[5,18,20]	0.76
52	4	[8,21,22]+[7,9,32]+[12,13,16]+[10,17,19]+[14,15,18]+[3,23,24]	0.76
60	4	[4,26,27]+[8,9,37]+[12,16,21]+[15,17,18]+[14,19,20]+[13,22,24]+ [5,25,28]	0.76
68	4	[4,30,31]+[5,19,42]+[12,22,24]+[15,16,23]+[17,18,20]+[11,21,28]+ [9,25,27]+[6,29,32]	0.76
76	4	[8,33,34]+[5,20,47]+[14,23,32]+[16,17,28]+[19,21,25]+[18,22,24]+ [13,26,27]+[9,30,31]+[3,35,36]	0.84
84	4	[4,25,52]+[8,33,37]+[12,21,40]+[22,23,34]+[19,20,27]+[13,26,36]+ [16,29,38]+[17,30,35]+[15,24,31]+[10,28,39]	0.76
92	4	[5,41,42]+[8,21,57]+[10,36,37]+[16,30,33]+[18,19,38]+[23,24,25]+ [22,27,29]+[26,28,31]+[15,32,34]+[12,39,40]+[3,43,44]	0.75
100	4	[4,46,47]+[12,40,41]+[11,17,62]+[20,33,34]+[19,31,32]+[24,25,30]+ [22,23,26,29]+[27,28,44]+[14,15,35,36]+[16,37,39]+[6,9,42,43]+ [5,45,48]	0.75
34	5	[4,5,11,12]+[10,13,14,25]+[3,7,8,15]	0.82
44	5	[3,8,11,20]+[6,7,10,16]+[4,12,13,14]+[15,18,19,27]3	0.81
74	5	[3,4,10,55]+[9,11,18,28]+[13,14,15,20]+[26,31,32,35]+[7,17,21,23]+ [5,16,22,30]+[27,29,33,34]	0.81
84	5	[4,5,20,52]+[9,10,17,40]+[12,15,21,30]+[14,16,19,28]+ [13,22,23,25]+[11,18,26,27]+[33,34,35,37]+[31,36,38,39]	0.81
28	6	[3,4,5,6,8]+[7,9,10,12,17]	0.86
52	6	[3,4,5,7,32]+[8,9,10,11,12]+[15,16,17,18,24]+[13,19,21,22,23]	0.84
76	6	[4,5,6,11,47]+[8,9,10,12,30]+[13,14,15,16,17]+[22,23,24,26,36]+ [20,25,27,28,34]+[19,31,32,33,35]	0.84
100	6	[4,5,6,20,62]+[7,9,11,23,48]+[14,16,17,18,22]+[10,12,19,24,34]+ [28,29,31,45,40]+[32,33,36,35,43]+[26,37,39,41,42]+[25,30,44,46,47]	0.84
18	7	[2,3,4,6,7,13]	0.90
60	7	[2,3,4,6,7,37]+[12,13,14,15,27,28]+[21,17,18,19,20,16]+ [8,10,22,24,25,26]	0.87
74	7	[3,4,6,8,17,34]+[9,10,11,12,13,14]+[16,18,20,21,25,33]+ [24,26,28,31,35,55]+[7,22,27,29,30,32]	0.87
20	8	[2,3,4,5,6,7,12]	0.70
36	8	[2,3,4,5,6,7,8]+[10,11,12,13,15,16,22]	0.77
52	8	[2,4,5,6,7,8,19]+[10,11,12,13,15,16,18]+[14,17,21,22,23,24,32]	0.81
68	8	[4,5,6,7,8,17,18]+[12,13,14,15,16,27,28]+[2,19,20,22,23,24,25]+ [10,21,29,30,31,32,42]	0.88
84	8	[4,5,6,7,8,25,26]+[13,14,15,16,20,39,40]+[17,19,21,22,23,24,30]+ [27,28,29,31,33,34,52]+[2,9,10,35,36,37,38]	0.84
100	8	[2,3,4,7,12,30,41]+[9,10,11,13,14,15,20]+[19,22,23,24,25,35,36]+ [21,26,27,28,29,31,32]+[18,37,39,40,43,44,62]+[33,34,42,45,46,47,48]	0.84
58	9	[2,3,4,5,6,8,9,20]+[13,14,16,17,18,21,22,43]+ [11,12,19,23,24,25,26,27]	0.90
76	9	[3,4,5,6,7,8,9,32]+[12,13,14,15,16,17,18,36]+ [20,21,22,23,24,25,47,27]+[10,26,28,30,31,33,34,35]	0.89
44	10	[5,6,7,8,9,10,11]+[2,3,12,14,16,18,19]	0.91
84	10	[3,4,5,6,7,8,9]+[13,14,15,16,17,18,19]+[21,22,23,25,26,27,28]+[31,33,34,35,36,37,38]	0.91

Table 5: MCWBNDs-II for $v \leq 100$, $4 > k1 \leq 10$, and $3 \leq k2 \leq 9$.

v	k1	k2	Sets of shifts	En
16	4	3	[3,4,7]+[1,5]	0.75
24	4	3	[2,10,11]+[5,7,8]+[6,15]	0.75
32	4	3	[2,9,20]+[6,7,15]+[8,10,11]+[13,14]	0.71
40	4	3	[3,17,18]+[6,8,25]+[9,13,14]+[10,11,12]+[5,16]	0.75
48	4	3	[6,20,21]+[7,9,30]+[10,11,19]+[12,14,17]+[13,15,16]+[22,23]	0.75
56	4	3	[3,16,35]+[7,19,24]+[10,11,27]+[12,14,25]+[15,18,22]+[9,17,26]+[13,20]	0.75
64	4	3	[3,19,40]+[9,22,25]+[11,15,28]+[13,14,30]+[16,18,29]+[12,17,31]+[6,26,27]+[20,21]	0.75
72	4	3	[3,22,45]+[7,25,34]+[9,20,35]+[14,15,30]+[17,18,21]+[19,23,29]+[10,24,33]+[11,26,31]+[12,28]	0.75
80	4	3	[3,25,50]+[7,29,38]+[11,28,32]+[14,15,39]+[13,26,37]+[20,21,23]+[22,24,33]+[8,31,36]+[17,18,35]+[19,27]	0.75
88	4	3	[3,28,55]+[12,34,35]+[14,22,39]+[15,24,43]+[16,26,38]+[21,23,25]+[9,36,42]+[18,29,30]+[10,32,41]+[20,27,37]+[17,31]	0.75
96	4	3	[3,31,60]+[7,40,44]+[19,28,39]+[14,27,42]+[17,26,45]+[22,23,30]+[24,25,32]+[20,29,46]+[11,34,47]+[16,33,38]+[12,37,41]+[18,35]	0.75
18	5	3	[2,6,4,5]+[7,8]	0.78
48	5	3	[4,5,17,21]+[8,9,10,14]+[6,11,13,16]+[15,19,20,30]+[22,23]	0.80
58	5	3	[20,25,26,28]+[8,10,13,23]+[11,12,14,15]+[3,16,18,19]+[5,7,21,24]+[22,27]	0.80
88	5	3	[11,21,23,30]+[12,14,26,27]+[10,13,28,31]+[5,18,25,39]+[15,16,19,36]+[8,20,24,29]+[32,34,38,55]+[35,37,40,42]+[41,43]	0.79
98	5	3	[5,6,41,42]+[9,11,37,38]+[17,19,22,24]+[10,26,27,28]+[14,20,21,31]+[13,18,29,30]+[34,35,46,48]+[39,40,45,47]+[2,15,36,44]+[32,43]	0.79
32	6	3	[2,4,5,7,13]+[8,9,10,11,20]+[14,15]	0.82
56	6	3	[3,4,6,7,35]+[11,20,22,23,26]+[5,8,9,15,17]+[14,18,19,24,25]+[16,27]	0.76
80	6	3	[4,6,14,22,32]+[11,12,38,39,50]+[13,15,16,17,18]+[21,23,26,33,37]+[24,28,31,34,35]+[5,7,9,27,29]+[25,36]	0.82
50	7	3	[2,3,5,7,9,23]+[11,13,14,15,20,21]+[10,12,16,17,18,19]+[22,24]	0.85
64	7	3	[3,4,6,7,11,31]+[10,12,13,16,28,40]+[15,17,19,20,21,22]+[8,18,23,25,26,27]+[29,30]	0.76
24	8	3	[2,4,5,6,8,7,15]+[10,11]	0.84
40	8	3	[2,3,4,5,6,7,12]+[10,11,14,16,17,19,25]+[13,18]	0.86
56	8	3	[3,4,6,7,8,12,15]+[13,16,20,22,24,27,35]+[9,10,14,17,18,19,23]+[25,26]	0.86
72	8	3	[2,5,6,7,8,10,33]+[11,12,13,16,18,20,45]+[14,17,19,21,22,23,24]+ [25,26,28,29,30,32,31]+[34,35]	0.87
88	8	3	[4,5,6,7,8,15,41]+[9,10,11,12,13,14,18]+[19,20,21,22,23,26,29]+[27,28,30,31,34,35,55]+[25,32,36,37,38,39,40]+[42,43]	0.87
26	9	3	[2,3,4,6,7,8,9,12]+[10,11]	0.86
80	9	3	[6,7,8,9,9,24,31,34,36]+[11,12,13,14,15,17,18,50]+ [3,19,20,21,22,23,25,26]+[16,27,28,29,32,33,35,38]+[37,39]	0.88
98	9	3	[4,5,6,7,9,10,26,29]+[15,16,17,18,20,23,31,42]+[11,12,13,24,25,34,37,39]+[28,30,32,33,35,36,40,41]+[21,22,27,38,44,45,46,48]+[43,47]	0.88
48	10	3	[4,5,6,7,8,9,11,21,22]+[2,12,13,14,16,17,19,20,30]+[15,23]	0.88
88	10	3	[3,5,6,7,8,9,10,12,26]+[14,15,16,17,18,19,20,21,32]+ [22,24,25,27,28,29,30,31,35]+[23,36,37,38,39,40,41,42,55]+ [34,43]	0.89
40	5	4	[1,17,18,4]+[6,7,8,9]+[16,12,13,14]+[2,3,19]	0.71
50	5	4	[6,2,17,1]+[7,3,18,4]+[8,14,20,22]+[10,15,21,23]+[19,16,24]	0.79
80	5	4	[36,2,3,4]+[32,6,8,9]+[21,12,13,14]+[16,17,18,19]+[11,22,23,24]+[26,27,28,29]+[7,1,33,34]+[37,38,39]	0.75
90	5	4	[5,10,31,41]+[4,27,28,29]+[14,15,20,30]+[16,17,18,38]+[13,21,24,25]+[9,12,26,35]+[33,34,37,44]+[36,39,40,42]+[19,22,43]	0.80
34	6	4	[3,4,5,6,14]+[9,10,12,13,16]+[7,11,15]	0.83
58	6	4	[4,5,6,20,21]+[8,9,10,11,13]+[16,17,18,22,28]+[14,23,24,25,27]+[12,19,26]	0.83
82	6	4	[4,5,6,31,33]+[9,10,11,12,32]+[14,15,16,17,18]+[21,22,23,39,40]+[26,27,28,29,30]+[13,34,35,37,38]+[20,25,36]	0.79
24	7	4	[3,4,6,7,15,11]+[5,8,10]	0.85
66	7	4	[10,16,18,23,27,31]+[5,9,11,12,13,14]+[8,20,21,22,28,32]+[15,17,19,24,25,26]+ [4,29,30]	0.85
80	7	4	[4,5,6,7,25,31]+[9,10,11,13,14,15]+[18,19,20,21,34,36]+[22,23,24,27,28,33]+[29,32,35,38,39,50]+[16,26,37]	0.85
26	8	4	[3,5,6,7,10,8,11]+[4,9,12]	0.86
42	8	4	[2,3,5,6,7,8,10]+[13,14,16,15,17,19,20]+[9,11,18]	0.87

Table 5: Continued.

v	k1	k2	Sets of shifts	En
58	8	4	[4,5,6,7,8,12,13]+[10,11,14,15,16,20,28]+[18,19,23,21,24,25,27]+[9,22,26]	0.87
74	8	4	[4,5,6,7,8,10,33]+[11,12,14,15,17,34,36]+[27,18,19,22,23,24,13]+ [25,26,28,29,30,31,32]+[16,20,35]	0.87
90	8	4	[7,8,17,24,36,41,42]+[9,10,11,12,13,15,16]+[19,20,21,22,23,25,32]+[31,26,27,28,29,30,6]+[33,34,35,37,38,39,40]+[2,43,44]	0.87
64	9	4	[4,5,7,9,17,20,23,40]+[11,12,13,14,15,16,18,19]+[8,21,22,25,26,27,30,31]+[6,28,29]	0.88
82	9	4	[3,4,5,6,6,7,8,9,38]+[13,14,15,16,17,18,19,40]+[24,25,26,27,28,35,34,36]+[21,22,23,29,31,30,33,37]+[10,3239]	0.88
50	10	4	[6,7,8,9,11,12,13,15,14]+[22,10,16,17,18,19,21,20,4]+[2,23,24]	0.89
90	10	4	[5,6,7,8,9,11,12,13,15]+[14,16,17,18,19,21,20,22,23]+[24,26,27,28,29,30,32,31,40]+[33,34,35,36,37,39,38,41,42]+ [2,43,44]	0.89
24	6	5	[6,7,8,10,15]+[3,4,5,11]	0.84
48	6	5	[6,16,19,23,30]+[7,8,9,11,12]+[13,15,17,20,21]+[3,4,5,14]	0.70
72	6	5	[2,3,5,30,31]+[8,9,10,11,28]+[16,17,32,33,34]+[20,22,23,25,35]+[13,24,26,29,45]+[14,15,18,21]	0.84
96	6	5	[3,4,5,22,60]+[9,10,11,12,46]+[14,15,16,17,21]+[7,19,20,23,26]+[28,29,30,42,45]+[31,32,35,33,37]+[25,27,43,44,47]+[38,39,40,41]	0.84
26	7	5	[5,6,7,11,12,8]+[2,4,9,10]	0.86
40	7	5	[3,4,5,6,7,13]+[9,10,12,14,16,18]+[11,17,19,25]	0.86
82	7	5	[3,4,5,7,30,31]+[9,10,11,12,15,24]+[16,20,21,22,32,40]+[18,23,25,26,27,28]+[8,19,29,33,34,35]+[36,37,38,39]	0.86
96	7	5	[3,4,5,7,30,46]+[9,10,11,13,14,31]+[19,20,21,24,45,47]+[23,26,27,28,32,38]+[22,29,33,35,34,37]+[15,17,25,40,41,42]+[39,43,44,60]	0.86
48	9	5	[2,3,4,5,6,8,9,10]+[12,13,14,15,16,20,17,30]+[11,19,21,22]	0.89
66	9	5	[3,4,5,6,7,8,9,22]+[11,12,14,15,16,17,19,18]+[20,21,23,24,25,26,27,31]+[28,29,30,32]	0.89
32	10	5	[3,4,5,6,7,8,9,10,11]+[13,14,15,20]	0.89
72	10	5	[3,4,5,6,7,8,9,10,18]+[11,12,13,14,17,19,16,21,20]+[23,24,26,25,28,29,31,35,45]+[30,32,33,34]	0.89
42	7	6	[3,4,5,6,7,15]+[10,11,12,13,14,16]+[9,17,18,19,20]	0.87
56	7	6	[2,3,4,5,6,35]+[9,10,15,17,26,27]+[13,14,16,18,19,20]+[11,22,23,24,25]	0.79
98	7	6	[3,4,5,6,7,15]+[10,11,12,13,14,16]+[38,18,19,20,21,9]+[22,23,25,26,27,28]+[32,33,34,35,40,48]+[31,36,37,39,42,43]+[41,44,45,46,47]	0.86
32	8	7	[9,10,11,13,14,15,20]+[2,3,5,6,7,8]	0.89
48	8	7	[30,16,20,21,22,23,9]+[10,11,12,13,14,15,17]+[2,5,6,7,8,19]	0.89
64	8	7	[2,3,5,6,7,9,31]+[10,11,12,14,15,18,40]+[17,20,25,27,28,29,30]+[13,19,21,22,23,26]	0.87
80	8	7	[2,3,4,5,7,8,50]+[12,13,14,15,20,36,39]+[16,19,21,23,24,25,26]+[22,32,33,34,35,37,38]+[17,18,27,28,29,31]	0.88
96	8	7	[5,6,7,32,38,41,60]+[10,11,12,13,14,15,20]+[18,19,21,22,23,25,47]+[28,29,31,40,42,45,46]+[4,8,33,34,35,37,39]+[16,24,26,30,43,44]	0.83
34	9	7	[4,5,6,8,9,10,11,12]+[2,7,13,15,14,16]	0.89
88	9	7	[2,4,5,6,8,9,12,41]+[13,14,15,17,18,25,43,28]+[23,26,27,30,32,34,37,39]+[21,22,24,29,31,35,36,55]+[10,19,20,38,40,42]	0.89
72	9	8	[3,4,5,6,7,9]+[11,12,13,14,15,16]+[20,23,24,26,25,31]+[21,22,28,29,30]	0.89
90	9	8	[3,4,6,7,8,9,10,41]+[13,14,15,16,17,18,31,44]+[23,24,25,26,30,39,40,42]+[27,28,29,34,33,35,36,37]+[5,19,20,22,32,38,43]	0.89
58	10	8	[2,3,4,5,6,7,8,9,13]+[14,15,16,17,18,19,20,22,23]+[12,21,24,25,26,27,28]	0.91
98	10	8	[3,4,5,6,7,8,9,10,44]+[13,14,15,16,17,18,19,31,41]+[24,25,26,27,28,29,30,40,42]+[20,21,32,33,34,35,37,38,43]+[22,36,39,45,46,47,48]	0.86
40	10	9	[3,4,5,6,7,8,9,17,19]+[10,11,12,13,14,16,18,25]	0.91
80	10	9	[2,3,4,5,6,7,8,9,35]+[12,13,14,15,16,17,19,18,25]+[20,22,23,24,26,27,28,29,31]+[21,32,33,34,36,37,38,39]	0.91

Table 6: MCWBNDs-II for $v \leq 100$, $5 \leq k1 \leq 10$, $4 < k2 \leq 9$, and $3 \leq k3 \leq 7$.

v	k1	k2	k3	Sets of shifts	En
26	5	4	3	[3,5,7,9]+[4,10,11]+[8,12]	0.76
56	5	4	3	[2,4,14,35]+[9,10,11,19]+[8,12,15,16]+[22,23,24,25]+[13,17,20]+[26,27]	0.79
58	5	4	3	[4,5,19,27]+[7,9,12,24]+[10,13,16,18]+[17,20,22,43]+[8,23,25]+[21,26]	0.79
96	5	4	3	[4,5,40,45]+[10,16,27,35]+[15,19,25,28]+[18,22,26,29]+[34,38,43,44]+[17,21,23,24]+[7,20,31,32]+[37,39,42,60]+[13,30,41]+[46,47]	0.79
98	5	4	3	[4,12,32,47]+[10,16,17,46]+[13,15,21,44]+[18,19,20,33]+[24,34,43,73]+[7,26,29,30]+[11,23,27,35]+[37,38,40,45]+[14,41,42]+ [31,39]	0.80
40	6	4	3	[2,3,7,8,19]+[10,11,12,13,25]+[6,14,16]+[17,18]	0.81
64	6	4	3	[3,6,8,15,31]+[10,11,12,13,14]+[16,17,20,28,40]+[19,21,22,23,25]+ [9,26,27]+[29,30]	0.82
88	6	4	3	[2,4,6,20,55]+[9,10,11,12,38]+[17,18,41,42,43]+[23,24,31,36,40]+[27,28,29,30,37]+ [5,13,14,21,32]+ [16,26,39]+[34,35]	0.82
58	7	4	3	[4,5,6,7,15,18]+[12,14,16,17,21,28]+[9,10,20,23,25,27]+[11,22,24]+[19,26]	0.82
72	7	4	3	[4,5,6,9,22,24]+[8,12,15,26,45,31]+[17,18,19,20,21,33]+[11,13,23,30,25,32]+[14,28,29]+[34,35]	0.84
42	6	5	3	[3,4,6,7,20]+[12,13,15,16,17]+[8,9,10,14]+[18,19]	0.81
66	6	5	3	[3,4,6,20,31]+[9,10,11,12,16]+[18,25,26,30,32]+[19,22,23,24,29]+[7,13,14,27]+[21,28]	0.82
90	6	5	3	[3,4,6,31,44]+[9,10,12,17,34]+[24,27,35,36,40]+[7,19,20,21,22]+[25,26,32,33,41]+[28,29,30,38,39]+[13,14,15,37]+[42,43]	0.82
32	7	5	3	[2,3,5,6,7,8]+[10,11,14,20]+[13,15]	0.82
34	8	5	3	[3,4,8,9,11,15,16]+[5,6,10,12]+[13,14]	0.83
50	8	5	3	[7,8,10,11,15,19,24]+[3,9,12,16,17,20,21]+[4,13,14,18]+[22,23]	0.85
66	8	5	3	[5,6,7,8,10,12,17]+[11,14,15,16,25,21,26]+[9,13,19,20,22,23,24]+[27,28,29,30]+[31,32]	0.86
82	8	5	3	[4,5,6,7,8,25,26]+[10,15,16,27,28,29,30]+[18,19,20,21,22,23,24]+[3,13,14,31,32,34,35]+ [36,38,39,40]+[33,37]	0.81
34	7	6	3	[7,8,9,11,13,14]+[2,4,5,10]+[15,16]	0.83
48	7	6	3	[3,5,6,7,11,15]+[8,10,14,19,21,22]+[9,16,17,20,30]+[13,23]	0.84
74	7	6	3	[3,5,6,7,20,31]+[12,13,14,27,35,36]+[16,17,18,19,29,34]+[10,23,24,25,28,30]+[4,21,22,26]+[32,33]	0.85
90	7	6	3	[3,4,7,10,32,33]+[9,11,13,14,16,21]+[19,20,25,35,36,37]+[23,24,26,27,28,30]+[15,18,29,34,38,44]+[17,31,39,40]+[42,43]	0.86
56	9	6	3	[6,7,8,9,10,11,22,35]+[15,16,17,18,19,20,24,25]+[2,5,12,13,23]+[26,27]	0.86
74	9	6	3	[6,7,8,13,19,29,30,31]+[11,12,14,15,17,18,20,32]+[21,22,23,25,27,26,28,34]+[2,4,10,24,33]+[35,36]	0.90
40	9	7	3	[4,5,6,7,8,9,13,25]+[2,11,14,16,17,19]+[18,10]	0.86
58	9	7	3	[3,4,5,6,7,8,9,14]+[10,11,12,13,15,17,18,19]+[23,24,25,26,27,28]+[20,22]	0.87
42	10	7	3	[3,4,6,8,9,10,11,20,12]+[7,13,14,16,15,17]+[18,19]	0.87
82	10	7	3	[3,4,5,6,7,8,9,10,28]+[13,14,15,16,17,18,19,20,21]+[22,23,24,25,26,27,30,29,39]+[33,34,35,36,37,40]+[32,38]	0.89
42	9	8	3	[6,7,8,9,11,12,13,14]+[2,3,10,15,16,17,20]+[18,19]	0.87
96	9	8	3	[4,5,6,7,8,9,10,45]+[14,15,16,17,18,27,35,37]+[19,20,21,22,23,24,25,26]+[28,29,30,31,32,34,33,60]+[38,39,41,40,42,43,44]+[46,47]	0.88
64	10	8	3	[2,4,5,6,7,8,9,10,12]+[14,15,16,17,18,19,20,22,40]+[21,23,25,26,27,28,29]+[30,31]	0.88
66	10	9	3	[3,4,5,6,7,8,9,10,12]+[14,15,16,17,19,20,21,31,32]+[18,22,23,24,26,27,28,29]+[25,30]	0.89
48	7	5	4	[4,5,6,7,11,12]+[8,9,13,14,20,30]+[17,19,21,23]+[10,15,22]	0.84
34	7	5	4	[7,8,9,12,10,16]+[3,4,11,15]+[5,13,14]	0.83
50	7	6	4	[3,4,5,6,7,23]+[11,12,13,16,14,24]+[15,17,18,20,21]+[8,19,22]	0.76
90	7	5	4	[4,5,6,7,21,44]+[8,11,12,13,14,31]+[18,19,20,22,41,43]+[25,26,27,28,32,33]+[16,24,29,34,35,40]+[36,37,38,39]+[15,23,42]	0.85
50	7	6	4	[3,4,5,6,7,24]+[11,12,13,17,14,23]+[15,16,20,19,21]+[8,18,22]	0.84
64	7	6	4	[4,5,6,7,8,31]+[11,12,13,14,28,40]+[16,17,18,19,21,22]+[20,23,26,27,30]+[9,25,29]	0.90
40	9	6	4	[4,5,6,7,8,9,13,25]+[12,14,16,17,19]+[10,11,18]	0.86
58	9	6	4	[5,7,8,9,14,16,26,27]+[10,11,12,13,15,17,18,19]+[20,21,22,23,24]+[3,25,28]	0.87
42	9	7	4	[5,6,7,8,9,10,17,18]+[11,12,13,14,16,15]+[2,19,20]	0.87
64	10	7	4	[3,4,5,6,7,8,9,10,11]+[15,16,17,18,19,20,21,22,30]+[12,13,23,25,26,27]+[29,31,40]	0.88
80	9	8	4	[3,4,5,6,7,8,9,36]+[13,14,15,16,17,28,18,29]+[20,24,25,26,27,32,33,34]+[21,22,23,35,38,39,50]+[11,31,37]	0.90
96	9	8	4	[4,5,6,7,8,10,26,27]+[13,14,15,16,17,20,42,43]+[2,18,22,23,24,25,38,39]+[60,29,30,31,32,34,35,28]+[37,40,44,41,46,47]+[19,21]	0.85

Table 6: Continued.

v	k1	k2	k3	Sets of shifts	En
98	9	8	4	[4,5,6,7,8,9,27,29]+[13,14,15,16,17,18,45,46]+[19,21,22,23,24,25,35,26]+[28,30,31,32,33,34,38,48]+[36,37,40,41,43,42,44]+[10,39,47]	0.88
66	10	8	4	[3,4,5,6,7,8,9,10,12]+[14,15,16,17,19,18,26,29,31]+[21,23,24,25,27,28,30]+[11,22,32]	0.88
48	10	9	4	[3,5,6,7,9,10,21,11,22]+[12,13,14,15,16,19,17,30]+[4,20,23]	0.89
88	10	9	4	[3,4,5,6,7,8,9,10,34]+[13,14,15,16,17,18,19,20,32]+[23,24,25,26,27,28,29,30,31]+[36,37,38,39,40,42,43,55]+[11,35,41]	0.90
66	7	6	5	[5,6,7,8,9,28]+[2,4,12,13,15,19]+[23,18,20,21,22,17]+[16,24,25,26,27]+[29,30,31,32]	0.85
80	7	6	5	[3,4,5,6,11,50]+[13,14,19,33,35,39]+[17,18,20,21,32,37]+[22,23,25,26,27,28]+[16,29,31,34,38]+[8,10,24,36]	0.86
82	7	6	5	[4,6,7,8,18,36]+[5,9,13,14,15,25]+[10,16,17,20,38,61]+[22,23,24,27,28,29]+[26,30,31,32,33]+[34,35,37,39]	0.86
40	8	6	5	[6,7,8,9,12,14,19]+[2,3,10,11,13]+[16,17,18,25]	0.86
56	8	6	5	[3,4,5,10,26,27,35]+[9,11,13,14,16,17,24]+[15,18,20,22,25]+[6,7,19,23]	0.86
72	8	6	5	[4,5,6,7,8,9,31]+[12,13,15,16,21,28,29]+[19,22,23,24,32,33,45]+[17,25,26,30,35]+[3,14,20,34]	0.87
88	8	6	5	[2,4,5,6,7,8,55]+[13,14,15,16,21,42,43]+[18,19,20,22,23,24,40]+[29,30,31,32,34,39,41]+[27,35,36,37,38]+[11,17,25,26]	0.87
42	9	6	5	[5,6,7,8,9,20,12,13]+[14,15,16,17]+[2,10,11,18]	0.79
90	9	6	5	[4,5,6,7,8,9,10,44]+[13,14,16,17,18,20,42,41]+[24,25,26,28,38,39,40,45]+[27,29,30,31,32,34,33,60]+[2,15,22,19,37]+[35,43,46,47]	0.80
42	8	7	5	[2,4,5,6,7,8,9]+[15,11,12,13,14,16]+[17,18,19,20]	0.87
58	8	7	5	[3,4,5,6,7,8,23]+[10,12,13,14,16,17,25]+[15,18,19,20,21,22]+[24,26,27,28]	0.80
74	8	7	5	[3,4,6,7,8,9,35]+[12,13,14,15,16,31,36]+[24,26,27,28,29,32,34]+[18,19,20,21,23,30]+[5,10,25,33]	0.85
90	8	7	5	[4,5,6,7,8,16,41]+[9,10,11,12,14,15,17]+[19,21,22,23,25,24,28]+[29,30,32,31,34,43,44]+[13,20,35,36,37,38]+[33,39,40,42]	0.87
82	9	7	5	[3,4,5,6,7,9,18,28]+[11,12,13,15,17,19,32,35]+[22,23,24,25,26,27,30,61]+ [16,20,29,31,33,34]+ [36,37,38,39]	0.87
98	9	7	5	[3,4,5,6,7,9,14,48]+[12,13,15,16,17,22,44,47]+[25,26,27,30,39,40,41,42]+[28,29,31,33,35,34,36,45]+[19,20,21,37,38,43]+[8,11,32,46]	0.88
66	10	7	5	[3,4,5,6,7,8,9,10,12]+[11,19,20,22,23,24,25,26,27]+[15,16,17,18,21,32]+[28,29,30,31]	0.82
82	9	8	5	[3,4,5,6,7,8,9,38]+[13,14,15,16,18,26,19,33]+[11,17,20,21,22,23,24,25]+[28,29,30,31,32,34,35]+[36,37,39,40]	0.90
48	10	8	5	[4,5,6,7,8,9,11,21,22]+[13,14,16,17,19,23,30]+[2,10,15,20]	0.88
88	10	8	5	[3,4,5,6,7,8,9,10,34]+[13,14,15,16,17,18,19,20,32]+[22,23,24,25,27,28,29,30,35]+[26,31,36,37,38,40,55]+[39,41,42,43]	0.90
50	10	9	5	[6,7,8,9,11,12,13,15,14]+[2,3,4,16,17,18,19,20]+[21,22,23,24]	0.89
90	10	9	5	[3,4,5,6,7,8,9,10,36]+[13,14,15,16,17,18,19,37,20]+[22,24,25,26,27,28,29,30,38]+[23,31,32,33,34,35,39,42]+[40,41,43,44]	0.90
64	9	7	6	[3,4,5,6,7,8,9,20]+[11,12,13,14,15,16,18,19]+[22,23,25,30,31,40]+[17,26,27,28,29]	0.88
82	9	7	6	[3,4,5,6,7,8,9,38]+[12,14,15,16,17,18,30,31]+[22,24,25,26,27,28,39,34]+ [13,19,23,29,33,37]+[20,32,35,36,40]	0.89
48	9	8	6	[4,5,6,8,11,17,22,20]+[9,10,12,13,14,15,16]+[2,19,21,23,30]	0.89
66	9	8	6	[3,4,5,6,7,8,9,22]+[11,12,14,15,16,17,19,18]+[21,23,24,25,26,28,31]+[13,27,29,30,32]	0.88
50	9	8	7	[5,6,7,8,9,17,21,23]+[10,12,13,14,15,16,18]+[3,11,19,22,20,24]	0.89
74	10	9	7	[3,4,5,6,7,8,9,10,20]+[23,13,14,15,16,17,18,19,12]+[22,24,25,26,27,28,29,30]+[31,32,33,34,35,36]	0.89

4. Conclusions and Future Research

Neighbor effects may arise in experiments of serology for virus research and in agriculture experiments, due to nature of plots, etc. In the presence of neighbor effects, misleading conclusions may be drawn in the variety competition experiments. Minimal NBDs are available in the literature to neutralize these effects economically for v odd. To overcome this problem for v even, complete solution is given in this article to construct proposed MCWBNDs-I for each and every $k > 2$. At least $v(v\text{-}1)$ experimental units are required to satisfy the neighbor balance property for v even. MCWBND-II requires $vv(v\text{-}4)/2$ units. Hence, we lose $[3/(v-1) \times 100]\%$ neighbor balance and save at least $[50(v+2)/v-1]\%$ experimental units. Our proposed designs save more than 50% of experimental material for v even at the cost of losing at most 3% efficiency of neighbour effects, therefore, are efficient to reduce the bias due to neighbor effects.

As a future research, (i) an algorithm coded with R-language can be developed to generate the proposed designs, (ii) the proposed new designs can be applied in experiments of serology and agriculture to obtain the real data for comparison, and (iii) the current study can be extended using neutrosophic statistics [26].

Data Availability

The data used in the article are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

All authors contributed equally.

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