Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 8600591, 14 pages
https://doi.org/10.1155/2022/8600591

Research Article

Analysis of Fractional Bioconvection with Hybrid Nanoparticles in Channel Flow

Muhammad Imran Asjad ,[1] Muhammad Danish Ikram,[1] Noman Sarwar,[1] Taseer Muhammad ,[2] Sivanandam Sivasankaran,[3] and Sami Ahmed Abdu Subaihi[3]

[1]*Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan*
[2]*Department of Mathematics, College of Sciences, King Khalid University, Abha 61413, Saudi Arabia*
[3]*Mathematical Modelling and Applied Computation (MMAC) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*

Correspondence should be addressed to Taseer Muhammad; taseer_qau@yahoo.com

Received 3 March 2022; Revised 9 May 2022; Accepted 18 June 2022; Published 23 August 2022

Academic Editor: Ardashir Mohammadzadeh

In this paper, MHD Brinkman-type fluid flow containing titanium dioxide and silver nanoparticle hybrid nanoparticles with generalized Mittag–Leffler kernel-based fractional derivative is investigated in the presence of bioconvection. The governing equations with dimensional analysis and fractional approach are obtained by using the fractional Fourier's law for heat flux and Fick's law for diffusion. As a result, the bioconvection Rayleigh number, which is responsible for the declining in the fluid velocity and fractional parameters used to control the thermal and momentum boundary layers thickness of fluid properties. The obtained solutions can be beneficial for proper analysis of real data and provide a tool for testing possible approximate solutions where needed.

1. Introduction

The concepts of fractional Brinkman form models with hybrid nanoparticles through an oscillating vertical plate and a magnetic field having variable direction are not detailed, however. To fill this void, a fractional Brinkman sorting fluid show is used to blend a flow of hybrid nanofluids over a swaying vertical plate. Brinkman's sort of fluid show was created by Brinkman in his pioneering work while investigating fluid flow due to thick constraint on the surface of a thick swarm particle [1, 2]. Saqib et al. [3] discussed the shape impact on the MHD flow of time-fractional, Ferro-Brinkman-sorted nanofluid having slope warming. Asjad et al. [4] introduced non-Newtonian fractional derivatives in a convective channel containing hybrid nanoparticles by Prabhakar. Khan et al. [5] investigated the chemical response and heat era impact of nonofluids of the Brinkman-type H_2O-Cu, Ag, TiO_2, and Al_2O_3 in a porous medium with an MHD flow. Nanjundappa et al. [6] explored the impact of dust particles on Darcy–Brinkman gravity-driven ferrothermal-convection in a ferrofluid soaked porous layer with an inside heat source: the impact of boundaries. Sarwar et al. [7] explained a comparative ponder on a non-Newtonian fractional-order Brinkman sorted fluid with two different parts. Shafie et al. [8] blended the convection flow of Brinkman sorted hybrid nanofluid based on Atangana–Baleanu fractional demonstration. Ali et al. [9] presented the Caputo–Fabrizio fractional derivative modelling of transitory MHD Brinkman nanofluid. Saqib et al. [10] examined the attractive resistive flow of a generalized Brinkman sort of nanofluid containing carbon nanotubes with inclined warming.

In many circumstances, mathematical models of integer order derivatives, including nonlinear models, do not operate satisfactorily. Fractional calculus has numerous applications in the domains of electromagnetics, viscoelasticity, fluid mechanics, signal processing, and optics. It has been used to demonstrate the physical and design shapes that can be found in a model to be best described by fractional differential conditions. The fractional derivative models are utilized for exact modelling of those frameworks that require exact modelling of damping. Boutiara et al. [11] debated a lesson of Langevin conditions within the outline of Caputo function-dependent kernel fractional derivatives through antiperiodic edge situations. Ali et al. [12] introduced controlled regulation retention of the common convection flow of hybrid nanofluids with a consistent relative Caputo-fractional derivative due to weight angle. Imran et al. [13] studied the mass and heat transport of a differentially sorted fluid with a noninteger time-fractional Caputo derivative. Ahmad et al. [14] announced the numerical modelling of ($Cu\text{-}Al_2O_3$) water-based Maxwell hybrid nanofluids with Caputo–Fabrizio fractional derivative. Mirza and Vieru [15] considered fundamental arrangements for an advection-diffusion condition with a time-fractional Caputo–Fabrizio derivative. Gul et al. [16] explored subjective investigations of certain Dirichlet boundary value problems aimed at Caputo–Fabrizio fractional differential conditions. Abdo et al. [17] offered a definite Atangana–Baleanu–Caputo derivative with nonlinear pantograph fractional differential conditions. Sweilam et al. [18] mathematically demonstrated that the Atangana–Baleanu–Caputo fractional derivative is an ideal control for cancer treatment. Sarwar et al. [19] investigated the Prabhakar derivative for convection flow Casson fluid over fluctuating plate based on the generalized Fourier law. Shah et al. [20] studied the common convection flow of Prabhakar fractional Maxwell fluid by generalized heat transportation. Elnaqeeb et al. [21] analyzed the normal convection flow of carbon nanotube Prabhakar-like fractional second-grade nanofluids over an infinite plate with Newtonian heating. Garrappa and Kaslik [22] considered the steadiness of fractional-order frameworks with Prabhakar derivatives.

Choi was one of the first to show nanofluids containing nanoparticles in 1995. Nanofluids can provide various benefits, including thermal conductivity. For the most part, these are intended for biomedical engineering, mechanical design, and fluid mechanics by Choi and Eastman [23]. Ali et al. [24] presented an investigation of a scientific fragmentary demonstration of hybrid viscous nanofluids and their application in heat and mass transfer. Ahmad et al. [25] verified expository arrangements for a complimentary convection flow of Casson nanofluid over an infinite vertical plate. Gul et al. [26] studied the hybrid nanofluid flow inside the cone-shaped hole between the cone and the surface of a spinning disk. Rafique et al. [27] defined the Casson nanofluid flow over a slanted permeable inclined surface with energy and mass transport. Khan et al. [28] discussed the effects of interfacial electro kinetic MHD radiative nanofluids flow on porous microchannels by thermophoresis and Brownian movement impacts. Ali Lund et al. [29] conferred hybrid nanofluid on the nonlinear contracting sheet with double branches of an MHD three-dimensional pivoting stream. Shah et al. [30] debated the recreation of entropy optimization and the warm behavior of nanofluids through the permeable media. Kumar et al. [31] deliberated a novel approach for the examination of warm exchange development through ferromagnetic hybrid nanofluid via considering sun-oriented radiation. Dadheech et al. [32] discussed natural convection and an angled magnetic field being used to compare the heat transfer of MoS2/C2H6O2 and SiO2-MoS2/C2H6O2 nanofluids. Dhif et al. [33] conducted a deliberate thermal study of a hybrid nanofluids solar collector and storage system. Khan et al. [34] examined the squeezing flow of nanofluids using mixed convection. Bu et al. [35] conducted a squeezing flow of nanofluids in a rotating channel with mixed convection and thermal radiation. By considering diverse physical flow conditions, thermal transport in aluminum alloy nanomaterials based on radiative nanofluids was investigated by Ullah Khan et al. [36]. In a triangular enclosure with zigzags and an elliptic obstacle, MHD flow of a hybrid nanofluid was introduced by Chabani et al. [37]. According to Rajashekhar et al. [38], the peristaltic flow of a Ree–Eyring liquid is affected by the different qualities of hemodynamic flow, mass, and heat transport. In a porous lid-driven hollow with a magnetic field, the entropy formation and heat transmission of Cu–water nanofluid were described by Marzougui et al. [39].

Bioconvection is defined as the wonder of macroscopic convection motion of fluid caused by the thickness angle established by the directional collective swimming of microorganisms. Plat proposed the concept of bioconvection in 1961. Bioconvection applications include biological polymer manufacture, biotechnology, and bio-sensors, as well as the testing and laboratory industries, among others [40, 41]. Ge-Jile et al. [42] analyzed nanofluid with motile microorganisms through the three-dimensional radiative bioconvective stream of a Sisko. Ramzan et al. [43] present bioconvection as a component in a three-dimensional digression hyperbolic partially ionized magnetized nanofluid stream with Cattaneo–Christov heat flux and activation vitality. Alhussain et al. [44] analyzed the warm conductivity and magneto-bioconvective augmentation in a nanofluid stream holding gyrotactic microorganisms. Farooq et al. [45] adjusted exponential space-based heat sources and Cattaneo–Christov expressions with a thermally radioactive bioconvection flow of Carreau nanofluid. Yousuf et al. [46] discussed the magneto-bioconvection flow of Williamson nanofluids through an inclined plate by entropy generation and gyrotactic microorganisms. Saqib et al. [47] inspected a Brinkman-type fluid (BTF) fractional model with hybrid nanoparticles. Danish Ikram et al. [48] calculated the heat transfer of viscous fluid with clay nanoparticles over an exponentially moving upright plate. Asjad Imran et al. [49] analyzed the thermophysical properties of clay nanofluids using a hybrid fractional operator. Ikram et al. [50] discussed the fractional model of Brinkman-type fluid (BTF) holding hybrid nanoparticles in a bounded microchannel via a constant proportional Caputo fractional operator.

Channel flow is used in a wide range of industrial applications, including heat exchangers in power plants and chemical reactors in the pharmaceutical industry. Although many actual processes with Newtonian behavior in both phases can be termed two-phase flows, there are a huge number of related applications where the continuous liquid phase exhibits non-Newtonian flow properties. There are several instances in the biochemical and biomedical industries, as well as in the food processing industry [51]. The impact of a vortex generator shape on liquids and the heat transition of hybrid nanofluids in a channel were examined by Zheng et al. [52]. On the scientific scale, D'Ippolito et al. [53] investigated the resistance of open channel flow due to vegetation. Haq et al. [54] demonstrated the fractional viscous liquid influence of MHD channel flow across a porous medium using the Caputo–Fabrizio time-fractional derivative.

In the absence of fractional bioconvection, the analyses above were conducted with or without fractional derivatives. The main objective, on the other hand, is to merge these two interesting subjects, fractional derivatives, and bioconvection. Recently, Asjad et al. [55] presented fractional bioconvection properties for sticky fluid over an infinite perpendicular plate with Caputo fractional derivative. In the above-mentioned literature, there is not a single study on the subject of fractional bioconvection between two parallel plates with a Prabhakar fractional derivative. As a result, our motivation is to use the Laplace transform approach to solve the fluid flow and heat transfer problems of bioconvection. Graphics are used to offer a graphical explanation of flow parameters.

2. Mathematical Formulation

Consider an MHD convection flow happening within a microchannel of a generalized, electrically conductive, $(Ag - TiO_2 - H_2O)$ hybrid nanoparticles situated in xy-plane as shown in Figure 1.

The assumptions are as follows:

(i) Microchannel is measured of length infinite by width L
(ii) At $t \leq 0$, the temperature of system is T_0
(iii) The channel is along $x-$ axis and normal to $y-$ axis
(iv) Fluid flow occurs in the x-direction
(v) After $t = 0^+$, the temperature and concentration level of microorganism raised from T_0 to T_w and N_∞ to N_w, respectively
(vi) Magnetics field of strength B_0 is applied normal to the plate

The stream of electrically conductive $(Ag - TiO_2 - H_2O)$ hybrid nanofluids endures electromotive drive, which yields current. The initiated attractive fields are disregarded since there is speculation about really small Reynolds numbers. The electromagnetic force activates the electric flux concentration [56]. The relation of thermophysical properties of nano and hybrid nanofluids is defined in Tables 1 and 2, respectively.

The governing equations of momentum and energy are as follows [47]:

$$\left(\frac{\partial u(y,t)}{\partial t} + \beta_{b^*} u(y,t)\right)\rho_{hnf} = \mu_{hnf}\frac{\partial^2 u(y,t)}{\partial y^2} - \sigma_{hnf} B_0^2 u(y,t) + g\left[(\rho\beta_T)_{hnf}(T - T_0) - \gamma(\rho_m - \rho)(N - N_0)\right]. \quad (1)$$

The energy equation is as follows:

$$\left(\rho C_p\right)_{hnf}\frac{\partial T(y,t)}{\partial t} = -\frac{\partial q(y,t)}{\partial y}. \quad (2)$$

The generalized Fourier's law for thermal flux is as follows:

$$q(y,t) = -k_{hnf}^{C} D_{\alpha,\beta,a}^{\gamma}\frac{\partial T(y,t)}{\partial y}. \quad (3)$$

The diffusion balance equation is as follows:

$$\frac{\partial N(y,t)}{\partial t} = -\frac{\partial L(y,t)}{\partial y}. \quad (4)$$

The generalized Fick's law for diffusion equation is as follows:

$$L = -D_{hnf}^{C} D_{\alpha,\beta,a}^{\gamma}\frac{\partial N(y,t)}{\partial y}, \quad (5)$$

where $cD_{\alpha,\beta,a}^{\gamma}$ denotes the Prabhakar fractional derivative and is defined as [57, 58].

For (1)–(5), we consider the following initial and boundary conditions:

$$\begin{aligned} u(y,0) &= 0,\\ T(y,0) &= T_0,\\ N(y,0) &= N_0,\\ 0 &\leq y \leq L, \end{aligned} \quad (6)$$

$$\begin{aligned} u(y,t) &= 0,\\ T(0,t) &= T_0,\\ N(0,t) &= N_0,\\ t &> 0, \end{aligned} \quad (7)$$

$$\begin{aligned} u(L,t) &= 0,\\ T(L,t) &= T_w,\\ N(L,t) &= N_w,\\ t &> 0. \end{aligned} \quad (8)$$

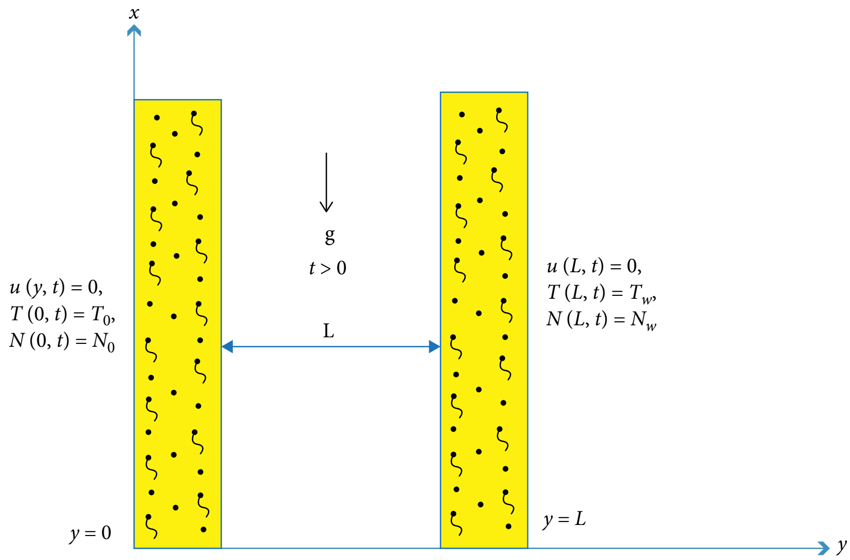

FIGURE 1: Geometry of the problem.

TABLE 1: The thermophysical properties of hybrid nanofluid and nanofluid under consideration are given in [50].

Nanofluid	Hybrid nanofluid
$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$	$\rho_{hnf} = (1-\phi_{hnf})\rho_f + \phi_{TiO_2}\rho_{TiO_2} + \phi_{Ag}\rho_{Ag}$
$\mu_{nf} = \mu_f/(1-\phi)^{5/2}$	$\mu_{hnf} = \mu_f/[1-(\phi_{Ag}+\phi_{TiO_2})]^{5/2}$
$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$	$(\rho C_p)_{hnf} = (1-\phi_{hnf})(\rho C_p)_f + \phi_{Ag}(\rho c_p)_{Ag} + \phi_{TiO_2}(\rho C_p)_{TiO_2}$
$(\rho B_T)_{nf} = (1-\phi)(\rho B_T)_f + \phi(\rho B_T)_s$	$(\rho B_T)_{hnf} = (1-\phi_{hnf})(\rho B_T)_f + \phi_{Ag}(\rho B_T)_{Ag} + \phi_{TiO_2}(\rho B_T)_{TiO_2}$
$\sigma_{nf}/\sigma_f = 1 + 3(\sigma_s/\sigma_f - 1)\phi/(\sigma_s/\sigma_f + 2) - (\sigma_s/\sigma_f - 1)\phi$	$\sigma_{hnf}/\sigma_f = 1 + 3(\phi_{Ag}\sigma_{Ag} + \phi_{TiO_2}\sigma_{TiO_2}/\sigma_f - \phi_{hnf})/$ $(\phi_{Ag}\sigma_{Ag} + \phi_{TiO_2}\sigma_{TiO_2}/\phi_{hnf}\sigma_f + 2) - (\phi_{Ag}\sigma_{Ag} + \phi_{TiO_2}\sigma_{TiO_2}/\sigma_f - \phi_{hnf})$
$K_{nf}/K_f = k_s + 2k_f - 2\phi(k_s - k_f)/k_s + 2k_f + \phi(k_s - k_f)$	$K_{hnf}/K_f = \phi_{Ag}k_{Ag} + \phi_{TiO_2}k_{TiO_2}/\phi_{hnf} + 2k_f + 2(\phi_{Ag}k_{Ag} + \phi_{TiO_2}k_{TiO_2})$ $-2\phi_{hnf}K_f/\phi_{Ag}k_{Ag} + \phi_{TiO_2}k_{TiO_2}/\phi_{hnf} + 2k_f + (\phi_{Ag}k_{Ag} + \phi_{TiO_2}k_{TiO_2}) - \phi_{hnf}K_f$

TABLE 2: Thermophysical possessions of nanoparticles and base fluid [50].

Material	Base fluid water (H_2O)	Nanoparticles silver (Ag)	Nanoparticles titanium dioxide (TiO_2)
ρ	997.1	10500	425
$\mathbf{C_p}$	4179	235	6862
k	0.613	429	8.9538
$\beta_T \times 10^{-5}$	21	1.89	0.9
σ	0.05	3.6×10^7	1×10^{-12}
Pr	6.2	—	—

By introducing dimensionless variables,

$$\tau = \frac{\nu}{L^2} t, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_0}{T_w - T_0}, \quad N^* = \frac{N - N_0}{N_w - N_0}, \quad q^* = \frac{q}{q_o}, \quad L^* = \frac{L}{L_0}, \quad q_0 = \frac{k_{hnf}(T_w - T_0) u_0}{\nu_{hnf}}, \quad L_0 = \frac{D_{hnf}(N_w - N_0) u_0}{\nu_{hnf}}. \tag{9}$$

The dimensionless fundamental equations are obtained by substituting from (9) into (1)–(8) and ignoring the star documentation.

Momentum and bioconvection equations in dimensionless form are as follows:

$$A_o^{\bullet} \frac{\partial V(Y,\tau)}{\partial \tau} + \beta_b^{\bullet} V(Y,\tau) = A_1^{\bullet} \frac{\partial^2 V(Y,\tau)}{\partial Y^2} - A_2^{\bullet} M V(Y,\tau) + Gr\left[A_3^{\bullet} \theta(Y,\tau) - Ra N(Y,\tau)\right]. \tag{10}$$

Dimensionless form of energy equation is as follows:

$$B_0 \frac{\partial \theta(Y,\tau)}{\partial \tau} = -\frac{\partial q(Y,\tau)}{\partial Y}. \tag{11}$$

In dimensionless form, the generalized Fourier's law for thermal flux [57, 58] is as follows:

$$q(Y,\tau) = -{}^{C}D^{\gamma}_{\alpha,\beta,a} \frac{\partial \theta(y,\tau)}{\partial Y}. \tag{12}$$

Dimensionless diffusion balance equation is as follows:

$$Lb \frac{\partial N(Y,\tau)}{\partial \tau} = -\frac{\partial L(Y,\tau)}{\partial Y}. \tag{13}$$

Dimensionless form of Fick's law [57, 58] is as follows:

$$L(Y,\tau) = -{}^{C}D^{\gamma}_{\alpha,\beta,a} \frac{\partial N(Y,\tau)}{\partial y}. \tag{14}$$

Constraints are associated with

$$V(Y,0) = 0, \quad \theta(Y,0) = 0,\ N(Y,0) = 0,\ Y \geq 0, \tag{15}$$

$$V(0,\tau) = 0, \quad \theta(0,\tau) = 0,\ N(0,\tau) = 0,\ \tau > 0, \tag{16}$$

$$V(1,\tau) = 0, \quad \theta(1,\tau) = 1,\ N(1,\tau) = 1,\ \tau > 0. \tag{17}$$

Variables are as follows:

$$\beta_b^{\bullet} = \frac{L^2 \beta_{b^*} \rho_f}{\mu_f}, \quad M = \frac{L^2 \sigma_f B_0^2}{\mu_f}, \quad Gr = \frac{L^3 g (B_T)_f (T_w - T_0)}{\nu_f^2}, \quad \Pr = \frac{(\mu C_p)_f}{k_f}, \quad Lb = \frac{\nu_f}{D_{hnf}},$$
$$Ra = \frac{\gamma(\rho_m - \rho)(N_w - N_0)}{\rho_f (B_T)_f (T_w - T_0)}, \quad \lambda_{hnf} = \frac{k_{hnf}}{k_f}, \quad A_0^{\bullet} = 1 - \phi_{hnf} + \frac{\phi_{Ag}\rho_{Ag} + \phi_{Tio_2}\rho_{Tio_2}}{\rho_f}, \quad A_2^{\bullet} = \frac{\sigma_{hnf}}{\sigma_f},$$
$$A_1^{\bullet} = \frac{1}{\left[1 - \left(\phi_{Ag} + \phi_{Tio_2}\right)\right]^{2.5}}, \quad A_3^{\bullet} = 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho B_T)_{Ag} + \phi_{Tio_2}(\rho B_T)_{Tio_2}}{(\rho B_T)_f}, \quad B_0 = \frac{\Pr A_4^*}{\lambda_{hnf}},$$
$$A_4^{\bullet} = 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho C_p)_{Ag} + \phi_{Tio_2}(\rho C_p)_{Tio_2}}{(\rho C_p)_f}, \quad B_1 = \frac{A_0^*}{A_1^*}, \quad B_5 = \frac{B_3}{B_2}, \quad B_2 = \frac{A_2^* M + A_o^* \beta_b}{A_1^*},$$
$$B_3 = \frac{A_3^* Gr}{A_1^*}, \quad B_4 = \frac{Gr Ra}{A_1^*}, \quad B_6 = \frac{B_4}{B_2}. \tag{18}$$

In the above equations, β_b^* is the Brinkman parameter, M is the dimensionless magnetic field parameter, Pr is dimensionless Prandtl number, Gr is dimensionless Grashof number, and Ra is dimensionless bioconvection Rayleigh number, respectively.

3. Solution of the Problem

3.1. Solution of Temperature Field. We deliberate $\beta \in [0, 1)$ along these lines, and in the above formularies, the boundary m remains equivalent near zero. By utilizing the Laplace change strategy and applying it to equations (11) and (12) through requirements (16) and (17), and using the fractional derivative of Prabhakar, we acquire changed issue for temperature field:

$$sB_0\,\overline{\theta}(Y,s) = -\frac{\partial \overline{q}(Y,s)}{\partial Y}, \tag{19}$$

$$\overline{q}(Y,s) = -s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}\frac{\partial \overline{\theta}(Y,s)}{\partial Y}. \tag{20}$$

Using (19) and (20), we have

$$\frac{\partial^2 \overline{\theta}(Y,s)}{\partial Y^2} - \frac{B_0 s\,\overline{\theta}(Y,s)}{s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}} = 0. \tag{21}$$

Subject to the constraints,

$$\overline{\theta}(1,s) = \frac{1}{s}, \quad \overline{\theta}(0,s) = 0, \tau > 0. \tag{22}$$

The general solution of equation (21) with equation (22) is as follows:

$$\overline{\theta}(Y,s) = \frac{1}{s}\left(\frac{1}{\left(e^{-\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}} - e^{\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}}\right)/2}\right) \cdot \left(\frac{e^{-Y\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}} - e^{Y\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}}}{2}\right). \tag{23}$$

Or,

$$\overline{\theta}(Y,s) = \frac{1}{s}\left[\frac{\sinh Y\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}}{\sinh\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}}\right]. \tag{24}$$

It is important that equation (24) can be written in the equivalent form

$$\overline{\theta}(Y,s) = \frac{1}{s}\sum_{n=0}^{\infty}\left[e^{-(2n+1-Y)\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}} - e^{-(2n+1+Y)\sqrt{B_0 s/s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}}}\right]. \tag{25}$$

Moreover, equation (25) can also be expressed as a series approach, allowing us to rationally determine the inverse Laplace transform.

$$\overline{\theta}(Y,s) = \frac{1}{s} + \left[\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\frac{(Y-2n-1)^m (a)^k (B_0)^{m/2}\Gamma(\gamma m/2+k)}{m!k!\Gamma(\gamma m/2)}\frac{1}{s^{\alpha k - m/2+\beta m/2+1}}\right] - \left[\sum_{p=0}^{\infty}\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-2n-1-Y)^l (a)^p (B_0)^{l/2}\Gamma(\gamma l/2+p)}{l!p!\Gamma(\gamma l/2)}\frac{1}{s^{\alpha p+\beta l/2-l/2+1}}\right]. \tag{26}$$

Taking inverse Laplace transform on equation (26), we get

$$\theta(Y,\tau) = 1 + \sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\frac{(Y-2n-1)^m (a)^k (B_0)^{m/2}\Gamma(\gamma m/2+k)}{m!k!\Gamma(\gamma m/2)}\frac{\tau^{\alpha k+\beta m/2-m/2}}{\Gamma(\alpha k+\beta m/2-m/2+1)} - \sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\frac{(-2n-1-Y)^l (B_0)^{l/2}(a)^p\Gamma(\gamma l/2+p)}{l!p!\Gamma(\gamma l/2)}\frac{\tau^{\alpha p-l/2+\beta l/2}}{\Gamma(\alpha p+\beta l/2-l/2+1)}. \tag{27}$$

3.2. Solution of Bioconvection Field. By utilizing the Laplace change approach and applying it to equations (13) and (14) with requirements (16) and (17) and utilizing the Prabhakar derivative, we acquire the changed issue for the bioconvection field.

$$Lbs\,\overline{N}(Y,s) = -\frac{\partial \overline{L}(Y,s)}{\partial Y}, \tag{28}$$

$$\overline{L}(Y,s) = -s^{\beta}\left(1-as^{-\alpha}\right)^{\gamma}\frac{\partial \overline{N}(Y,s)}{\partial Y}. \tag{29}$$

Using equations (28) and (29), we have

$$\frac{\partial^2 \overline{N}(Y,s)}{\partial Y^2} - \frac{sLb}{s^{\beta}(1-as^{-\alpha})^{\gamma}}\overline{N}(Y,s) = 0. \tag{30}$$

Subject to the constraints,

$$\overline{N}(1,s) = \frac{1}{s}, \ \overline{N}(0,s) = 0. \tag{31}$$

The general solution of equation (30) with equation (31) is as follows:

$$\overline{N}(Y,s) = \frac{1}{s}\left(\frac{1}{\left(e^{-\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}\right)/2}\right) \cdot \left(\frac{e^{-Y\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{Y\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}}{2}\right). \tag{32}$$

Or,

$$\overline{N}(Y,s) = \frac{1}{s}\left[\frac{\sinh Y\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}{\sinh\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}\right]. \tag{33}$$

It is important that equation (33) can be composed within the comparable form

$$\overline{N}(Y,s) = \frac{1}{s}\sum_{d=0}^{\infty}\left[e^{-(2d+1-Y)\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{-(2d+1+Y)\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}\right]. \tag{34}$$

Moreover, equation (34) can be communicated in an arrangement shape so that we are able to discover the Laplace inverse transform logically.

$$\begin{aligned}\overline{N}(Y,s) = \frac{1}{s} + &\left[\sum_{d=0}^{\infty}\sum_{s_1=1}^{\infty}\sum_{s_2=0}^{\infty}\frac{(-2d-1+Y)^{s_1}\,(a)^{s_2}\,(Lb)^{s_1/2}\,\Gamma(\gamma s_1/2+s_2)}{s_1!s_2!\Gamma(\gamma s_1/2)}\frac{1}{s^{\alpha s_2+\beta s_1/2-s_1/2+1}}\right] \\ &-\left[\sum_{d=0}^{\infty}\sum_{p_1=0}^{\infty}\sum_{p_2=0}^{\infty}\frac{(-2d-1-Y)^{p_1}\,(a)^{p_2}\,(Lb)^{p_1/2}\,\Gamma(\gamma p_1/2+p_2)}{p_1!p_2!\Gamma(\gamma p_1/2)}\frac{1}{s^{\alpha p_2+\beta p_1/2-p_1/2+1}}\right].\end{aligned} \tag{35}$$

Taking inverse Laplace transform on equation (35), we get

$$\begin{aligned}N(Y,\tau) = 1 + &\sum_{d=0}^{\infty}\sum_{s_1=1}^{\infty}\sum_{s_2=0}^{\infty}\frac{(-2d-1+Y)^{s_1}\,(a)^{s_2}\,(Lb)^{s_1/2}\,\Gamma(\gamma s_1/2+s_2)}{s_1!s_2!\Gamma(\gamma s_1/2)}\frac{\tau^{\alpha s_2+\beta s_1/2-s_1/2}}{\Gamma(\alpha s_2+\beta s_1/2-s_1/2+1)} \\ &-\sum_{d=0}^{\infty}\sum_{p_1=0}^{\infty}\sum_{p_2=0}^{\infty}\frac{(-2d-1-Y)^{p_1}\,(a)^{p_2}\,(Lb)^{p_1/2}\,\Gamma(\gamma p_1/2+p_2)}{p_1!p_2!\Gamma(\gamma p_1/2)}\frac{\tau^{\alpha p_2+\beta p_1/2-p_1/2}}{\Gamma(\alpha p_2+\beta p_1/2-p_1/2+1)}.\end{aligned} \tag{36}$$

3.3. Solution of Velocity Field. The Laplace transform is used in equation (10) using expressions from (15)–(17), and we attain

$$\left[\frac{\partial^2}{\partial Y^2} - B_1 s - B_2\right]\overline{V}(Y,s) = -B_3\overline{\theta}(Y,s) + B_4\overline{N}(Y,s), \tag{37}$$

which satisfies the following constraints:

$$\overline{V}(1,\ s) = 0, \overline{V}(0,\ s) = 0. \tag{38}$$

The solution of equation (37) subject to constraints (38), we have

$$\begin{aligned}\overline{V}(Y,s) = &-\frac{B_5}{s}\sum_{n=0}^{\infty}\left[\frac{e^{-(2n)\sqrt{B_0 s/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{-(2n+2)\sqrt{B_0 s/s^{\beta}(1-as^{-\alpha})^{\gamma}}}}{\left[1+\left(B_1 s/B_2 - B_0 s/B_2 s^{\beta}(1-as^{-\alpha})^{\gamma}\right)\right]}\right]\times\left[\sum_{m=0}^{\infty} e^{-(2m+1-Y)\sqrt{B_2+B_1 s}} - \sum_{m=0}^{\infty} e^{-(2m+1+Y)\sqrt{B_2+B_1 s}}\right]\\ &+\frac{B_6}{s}\sum_{d=0}^{\infty}\left[\frac{e^{-(2d)\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{-(2d+2)\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}}{\left[1+\left(B_1 s/B_2 - sLb/B_2 s^{\beta}(1-as^{-\alpha})^{\gamma}\right)\right]}\right]\times\left[\sum_{m=0}^{\infty} e^{-(2m+1-Y)\sqrt{B_2+B_1 s}} - \sum_{m=0}^{\infty} e^{-(2m+1+Y)\sqrt{B_2+B_1 s}}\right]\\ &+\frac{B_5}{s}\sum_{n=0}^{\infty}\left[\frac{e^{-(2n+1-Y)\sqrt{B_0 s/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{-(2n+1+Y)\sqrt{B_0 s/s^{\beta}(1-as^{-\alpha})^{\gamma}}}}{\left[1+\left(B_1 s/B_2 - B_0 s/B_2 s^{\beta}(1-as^{-\alpha})^{\gamma}\right)\right]}\right]\\ &-\frac{B_6}{s}\sum_{d=0}^{\infty}\left[\frac{e^{-(2d+1-Y)\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}} - e^{-(2d+1+Y)\sqrt{sLb/s^{\beta}(1-as^{-\alpha})^{\gamma}}}}{\left[1+\left(B_1 s/B_2 - sLb/B_2 s^{\beta}(1-as^{-\alpha})^{\gamma}\right)\right]}\right].\end{aligned} \tag{39}$$

Equation (39) can be written in component form as follows:

$$\begin{aligned}\overline{V}(Y,\ s) = &\overline{V}_1(Y,\ s) + \overline{V}_2(Y,\ s) + \overline{V}_3(Y,\ s) + \overline{V}_4(Y,\ s) + \overline{V}_5(Y,\ s) + \overline{V}_6(Y,\ s) + \overline{V}_7(Y,\ s) + \overline{V}_8(Y,\ s) + \overline{V}_9(Y,\ s)\\ &+ \overline{V}_{10}(Y,\ s) + \overline{V}_{11}(Y,\ s) + \overline{V}_{12}(Y,\ s).\end{aligned} \tag{40}$$

It is challenging to find the inverse Laplace transform of equation (40), so we can rewrite it in a suitable series.

$$\begin{aligned}V(Y,\tau) = &V_1(Y,\tau) + V_2(Y,\tau) + V_3(Y,\tau) + V_4(Y,\tau) + V_5(Y,\tau) + V_6(Y,\tau) + V_7(Y,\tau) + V_8(Y,\tau) + V_9(Y,\tau)\\ &+ V_{10}(Y,\tau) + V_{11}(Y,\tau) + V_{12}(Y,\tau).\end{aligned} \tag{41}$$

Next, taking the inverse Laplace of equation (41), component wise, we have

$$\begin{aligned}V_1(Y,\tau) = &-B_5\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{\delta_2=0}^{\infty}\sum_{\delta_1=0}^{\infty}\sum_{w_2=0}^{\infty}\sum_{w_1=0}^{\infty}\sum_{z_1=0}^{\infty}\sum_{z_2=0}^{\infty}\sum_{z_3=0}^{\infty}\frac{(Y-2m-1)^{w_1}(-B_1)^{z_1}(B_0)^{\delta_1/2}}{\delta_1!\delta_2!w_1!w_2!z_2!z_3!(B_1)^{z_2-w_2}(B_2)^{w_2+z_1-w_1/2}}\\ &\times\frac{(-2n)^{\delta_1}(-B_0)^{z_2}(a)^{\delta_2+z_3}\tau^{\alpha\delta_2+\beta\delta_1/2-\delta_1/2-w_2+\alpha z_3+\beta z_2-z_1}}{\Gamma(\alpha\delta_2+\beta\delta_1/2-\delta_1/2-w_2+\alpha z_3+\beta z_2-z_1+1)}\,\frac{\Gamma(\gamma\delta_1/2+\delta_2)\Gamma(w_1/2+1)\Gamma(z_1+1)\Gamma(\gamma z_2+z_3)}{\Gamma(\gamma\delta_1/2)\Gamma(w_1/2+1-w_2)\Gamma(z_1+1-z_2)\Gamma(\gamma z_2)},\end{aligned}$$

$$\begin{aligned}V_2(Y,\tau) = &B_5\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{\delta_1=0}^{\infty}\sum_{\delta_2=0}^{\infty}\sum_{q_1=0}^{\infty}\sum_{q_2=0}^{\infty}\sum_{z_1=0}^{\infty}\sum_{z_2=0}^{\infty}\sum_{z_3=0}^{\infty}\frac{(-(Y+2m+1))^{q_1}(-B_1)^{z_1}(B_0)^{\delta_1/2}}{\delta_1!\delta_2!q_1!q_2!z_2!z_3!(B_1)^{z_2-q_2}(B_2)^{q_2+z_1-q_1/2}}\\ &\times\frac{(-2n)^{\delta_1}(-B_0)^{z_2}(a)^{\delta_2+z_3}\tau^{\alpha\delta_2+\beta\delta_1/2-\delta_1/2-q_2+\alpha z_3+\beta z_2-z_1}}{\Gamma(\alpha\delta_2+\beta\delta_1/2-\delta_1/2-q_2+\alpha z_3+\beta z_2-z_1+1)}\,\frac{\Gamma(\gamma\delta_1/2+\delta_2)\Gamma(q_1/2+1)\Gamma(z_1+1)\Gamma(\gamma z_2+z_3)}{\Gamma(\gamma\delta_1/2)\Gamma(q_1/2+1-q_2)\Gamma(z_1+1-z_2)\Gamma(\gamma z_2)},\end{aligned}$$

$$\begin{aligned}V_3(Y,\tau) = &B_5\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{w_1=0}^{\infty}\sum_{w_2=0}^{\infty}\sum_{z_1=0}^{\infty}\sum_{z_2=0}^{\infty}\sum_{z_3=0}^{\infty}\frac{(Y-2m-1)^{w_1}(-B_1)^{z_1}(\mathrm{Pr})^{j_1/2}}{j_1!j_2!w_1!w_2!z_2!z_3!(B_1)^{z_2-w_2}(B_2)^{w_2+z_1-w_1/2}}\\ &\times\frac{(-2n-2)^{j_1}(-B_0)^{z_2}(a)^{j_2+z_3}\tau^{\alpha j_2+\beta j_1/2-j_1/2-w_2+\alpha z_3+\beta z_2-z_1}}{\Gamma(\alpha j_2+\beta j_1/2-j_1/2-w_2+\alpha z_3+\beta z_2-z_1+1)}\,\frac{\Gamma(\gamma j_1/2+j_2)\Gamma(w_1/2+1)\Gamma(z_1+1)\Gamma(\gamma z_2+z_3)}{\Gamma(\gamma j_1/2)\Gamma(w_1/2+1-w_2)\Gamma(z_1+1-z_2)\Gamma(\gamma z_2)},\end{aligned}$$

$$V_4(Y,\tau) = -B_5 \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{q_1=0}^{\infty}\sum_{q_2=0}^{\infty}\sum_{z_1=0}^{\infty}\sum_{z_2=0}^{\infty}\sum_{z_3=0}^{\infty} \frac{(-(Y+2m+1))^{q_1}(-B_1)^{z_1}(B_0)^{j_1/2}}{j_1!j_2!q_1!q_2!z_2!z_3!(B_1)^{z_2-q_2}(B_2)^{q_2+z_1-q_1/2}} \times \frac{(-2n-2)^{j_1}(-B_0)^{z_2}(a)^{j_2+z_3}\tau^{\alpha j_2+\beta j_1/2-j_1/2-q_2+\alpha z_3+\beta z_2-z_1}}{\Gamma(\alpha j_2+\beta j_1/2-j_1/2-q_2+\alpha z_3+\beta z_2-z_1+1)} \frac{\Gamma(\gamma j_1/2+j_2)\Gamma(q_1/2+1)\Gamma(z_1+1)\Gamma(\gamma z_2+z_3)}{\Gamma(\gamma j_1/2)\Gamma(q_1/2+1-q_2)\Gamma(z_1+1-z_2)\Gamma(\gamma z_2)},$$

$$V_5(Y,\tau) = B_6 \sum_{d=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{w_1=0}^{\infty}\sum_{w_2=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty}\sum_{h_3=0}^{\infty} \frac{(Y-2m-1)^{w_1}(-B_1)^{h_1}(Lb)^{n_1/2}}{n_1!n_2!w_1!w_2!h_2!h_3!(B_1)^{h_2-w_2}(B_2)^{w_2+h_1-w_1/2}} \times \frac{(-2d)^{n_1}(-Lb)^{h_2}(a)^{n_2+h_3}\tau^{\alpha n_2+\beta n_1/2-n_1/2-w_2+\alpha h_3+\beta h_2-h_1}}{\Gamma(\alpha n_2+\beta n_1/2-n_1/2-w_2+\alpha h_3+\beta h_2-h_1+1)} \frac{\Gamma(\gamma n_1/2+n_2)\Gamma(w_1/2+1)\Gamma(h_1+1)\Gamma(\gamma h_2+h_3)}{\Gamma(\gamma n_1/2)\Gamma(w_1/2+1-w_2)\Gamma(h_1+1-h_2)\Gamma(\gamma h_2)},$$

$$V_6(Y,\tau) = -B_6 \sum_{d=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{q_1=0}^{\infty}\sum_{q_2=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty}\sum_{h_3=0}^{\infty} \frac{(-(Y+2m+1))^{q_1}(-B_1)^{h_1}(Lb)^{n_1/2}}{n_1!n_2!q_1!q_2!h_2!h_3!(B_1)^{h_2-q_2}(B_2)^{q_2+h_1-q_1/2}} \times \frac{(-2d)^{n_1}(-Lb)^{h_2}(a)^{n_2+h_3}\tau^{\alpha n_2+\beta n_1/2-n_1/2-q_2+\alpha h_3+\beta h_2-h}}{\Gamma(\alpha n_2+\beta n_1/2-n_1/2-q_2+\alpha h_3+\beta h_2-h_1+1)} \frac{\Gamma(\gamma n_1/2+n_2)\Gamma(q_1/2+1)\Gamma(z_1+1)\Gamma(\gamma h_2+h_3)}{\Gamma(\gamma n_1/2)\Gamma(q_1/2+1-q_2)\Gamma(h_1+1-h_2)\Gamma(\gamma z_2)},$$

$$V_7(Y,\tau) = -B_6 \sum_{d=0}^{\infty}\sum_{m=0}^{\infty}\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\sum_{w_1=0}^{\infty}\sum_{w_2=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty}\sum_{h_3=0}^{\infty} \frac{(Y-2m-1)^{w_1}(-B_1)^{h_1}(Lb)^{m_1/2}}{m_1!m_2!w_1!w_2!h_2!h_3!(B_1)^{h_2-w_2}(B_2)^{w_2+h_1-w_1/2}} \times \frac{(-2d-2)^{m_1}(-B_0)^{h_2}(a)^{m_2+h_3}\tau^{\alpha m_2+\beta m_1/2-m_1/2-w_2+\alpha h_3+\beta h_2-h_1}}{(\alpha m_2+\beta m_1/2-m_1/2-w_2+\alpha h_3+\beta h_2-h_1+1)} \frac{\Gamma(\gamma m_1/2+m_2)\Gamma(w_1/2+1)\Gamma(h_1+1)\Gamma(\gamma h_2+h_3)}{\Gamma(\gamma m_1/2)\Gamma(w_1/2+1-w_2)\Gamma(h_1+1-h_2)\Gamma(\gamma h_2)},$$

$$V_8(Y,\tau) = B_6 \sum_{d=0}^{\infty}\sum_{m=0}^{\infty}\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\sum_{q_1=0}^{\infty}\sum_{q_2=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty}\sum_{h_3=0}^{\infty} \frac{(-(Y+2m+1))^{q_1}(-B_1)^{h_1}(Lb)^{m_1/2}}{m_1!m_2!q_1!q_2!h_2!h_3!(B_1)^{h_2-q_2}(B_2)^{q_2+h_1-q_1/2}} \times \frac{(-2d-2)^{m_1}(-Lb)^{h_2}(a)^{m_2+h_3}\tau^{\alpha m_2+\beta m_1/2-m_1/2-q_2+\alpha h_3+\beta h_2-h_1}}{\Gamma(\alpha m_2+\beta m_1/2-m_1/2-q_2+\alpha h_3+\beta h_2-h_1+1)} \frac{\Gamma(\gamma m_1/2+m_2)\Gamma(q_1/2+1)\Gamma(h_1+1)\Gamma(\gamma h_2+h_3)}{\Gamma(\gamma m_1/2)\Gamma(q_1/2+1-q_2)\Gamma(h_1+1-h_2)\Gamma(\gamma h_2)},$$

$$V_9(Y,\tau) = B_5 \sum_{n=0}^{\infty}\sum_{t_1=0}^{\infty}\sum_{t_2=0}^{\infty}\sum_{z_1=0}^{\infty}\sum_{z_2=0}^{\infty}\sum_{z_3=0}^{\infty} \frac{(Y-2n-1)^{t_1}(-B_1)^{z_1}(B_0)^{t_1/2}}{t_1!t_2!z_2!z_3!(B_1)^{z_2}(B_2)^{z_1}} \times \frac{(-B_0)^{z_2}(a)^{t_2+z_3}\tau^{\alpha t_2+\beta t_1/2-t_1/2+\alpha z_3+\beta z_2-z_1}}{\Gamma(\alpha t_2+\beta t_1/2-t_1/2+\alpha z_3+\beta z_2-z_1+1)} \frac{\Gamma(\gamma t_1/2+t_2)\Gamma(z_1+1)\Gamma(\gamma z_2+z_3)}{\Gamma(\gamma t_1/2)\Gamma(z_1+1-z_2)\Gamma(\gamma z_2)},$$

$$V_{10}(Y,\tau) = -B_5 \sum_{n=0}^{\infty}\sum_{v_1=0}^{\infty}\sum_{v_2=0}^{\infty}\sum_{z_1=0}^{\infty}\sum_{z_2=0}^{\infty}\sum_{z_3=0}^{\infty} \frac{(-(Y+2n+1))^{v_1}(-B_1)^{z_1}(B_0)^{v_1/2}}{v_1!v_2!z_2!z_3!(B_1)^{z_2}(B_2)^{z_1}} \times \frac{(-B_0)^{z_2}(a)^{v_2+z_3}\tau^{\alpha v_2+\beta v_1/2-v_1/2+\alpha z_3+\beta z_2-z_1}}{\Gamma(\alpha v_2+\beta v_1/2-v_1/2+\alpha z_3+\beta z_2-z_1+1)} \frac{\Gamma(\gamma v_1/2+v_2)\Gamma(z_1+1)\Gamma(\gamma z_2+z_3)}{\Gamma(\gamma v_1/2)\Gamma(z_1+1-z_2)\Gamma(\gamma z_2)},$$

$$V_{11}(Y,\tau) = -B_6 \sum_{d=0}^{\infty}\sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty}\sum_{h_3=0}^{\infty} \frac{(Y-2d-1)^{l_1}(-B_1)^{h_1}(Lb)^{l_1/2}}{l_1!l_2!h_2!h_3!(B_1)^{h_2}(B_2)^{h_1}} \times \frac{(-Lb)^{h_2}(a)^{l_2+h_3}\tau^{\alpha l_2+\beta l_1/2-l_1/2+\alpha h_3+\beta h_2-h_1}}{\Gamma(\alpha l_2+\beta l_1/2-l_1/2+\alpha h_3+\beta h_2-h_1+1)} \frac{\Gamma(\gamma l_1/2+l_2)\Gamma(h_1+1)\Gamma(\gamma h_2+h_3)}{\Gamma(\gamma l_1/2)\Gamma(h_1+1-h_2)\Gamma(\gamma h_2)},$$

$$V_{12}(Y,\tau) = B_6 \sum_{d=0}^{\infty}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{h_2=0}^{\infty}\sum_{h_3=0}^{\infty} \frac{(-(Y+2d+1))^{k_1}(-B_1)^{h_1}(Lb)^{k_1/2}}{k_1!k_2!h_2!h_3!(B_1)^{h_2}(B_2)^{h_1}} \times \frac{(-Lb)^{h_2}(a)^{k_2+h_3}\tau^{\alpha k_2+\beta k_1/2-k_1/2+\alpha h_3+\beta h_2-h_1}}{\Gamma(\alpha k_2+\beta k_1/2-k_1/2+\alpha h_3+\beta h_2-h_1+1)} \frac{\Gamma(\gamma k_1/2+k_2)\Gamma(h_1+1)\Gamma(\gamma h_2+h_3)}{\Gamma(\gamma k_1/2)\Gamma(h_1+1-h_2)\Gamma(\gamma h_2)}.$$

(42)

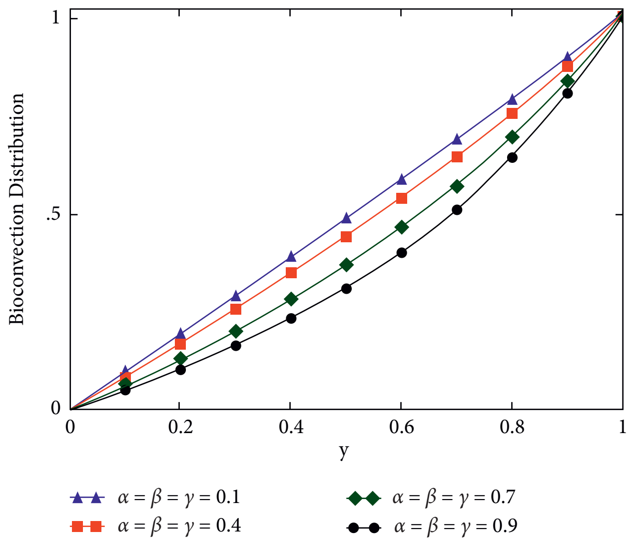

FIGURE 2: The effects of fractional parameters on bioconvection field, when $t = 3, Lb = 3$, and $a = 0.2$.

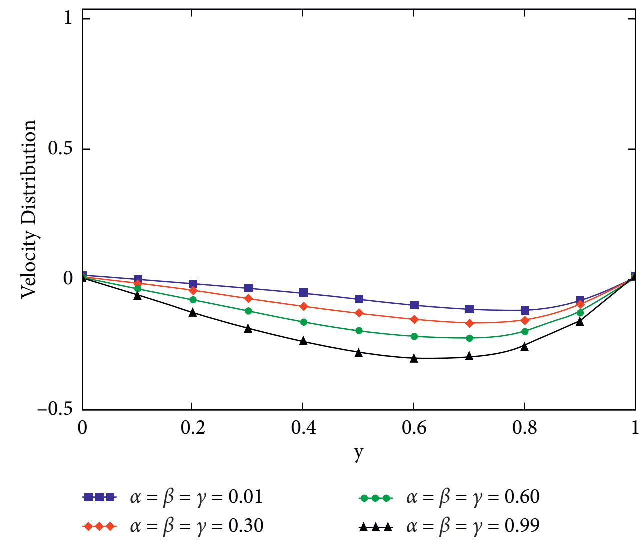

FIGURE 4: The effects of fractional parameters on velocity field for large time, when. $t = 2, \Pr = 6.2, Gr = 12, a = 0.2, Lb = 30, M = 0.01, \beta^* = 0.006, \phi_{hnf} = 0.04$, and $Ra = 4$.

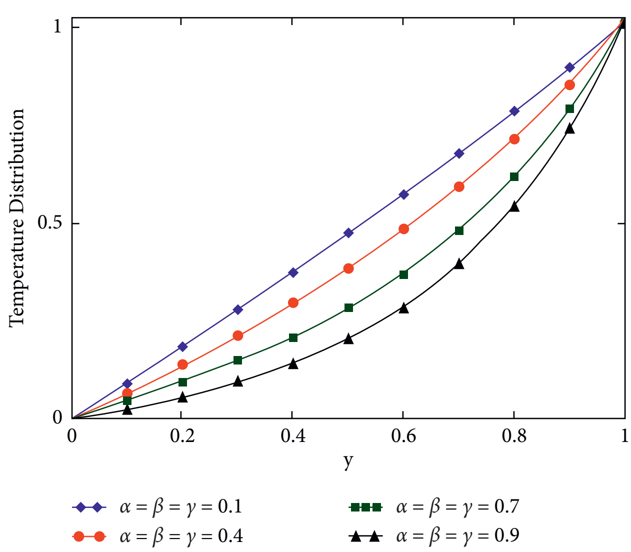

FIGURE 3: The effects of fractional parameters on temperature field for large time, when $t = 3, \Pr = 6.2$, and $a = 0.2$.

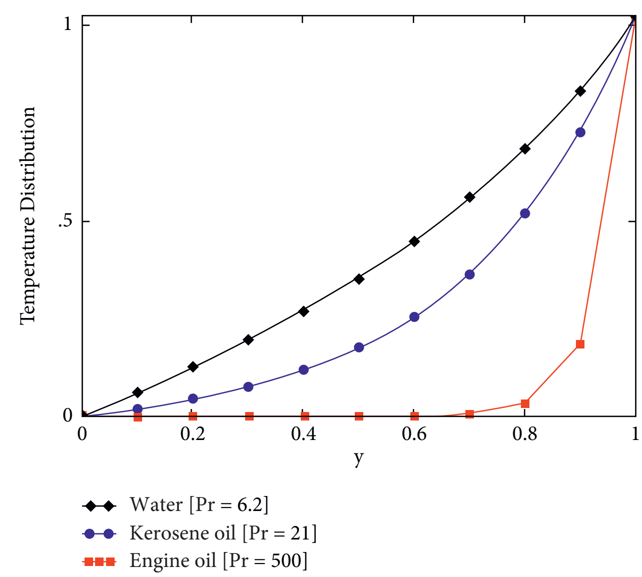

FIGURE 5: The comparison between different base fluids (water, kerosene oil, and engine oil) on temperature field for small time, when $t = 3, \alpha = \beta = \gamma = 0.5$, and $a = 0.2$.

4. Graphical Results and Discussion

Bioconvection has been studied using an MHD effect and thermal transfer model with a Prabhakar fractional approach. Laplace transform procedures are used to provide exact solutions for dimensionless governing equations. Graphical illustrations have been used to explain some of the physical effects of flow parameters.

Figures 2–4 are projected to show the impact of the fractional parameters α, β, γ on bioconvection, temperature, and velocity fields. For a large time, bioconvection, temperature, and velocity decreased by increasing values of α, β, γ. This is due to the boundary layer becoming wider; therefore, bioconvection, temperature, and velocity decrease. Usually, we can say that in fluid dynamics, a fractional approach is better for controlling the boundary layer thickness of the fluid properties.

Figure 5 shows the comparison between several base fluids (water, engine oil, and kerosene oil) on the temperature field. It is clearly noticed that the temperature of water is higher than all other Newtonian liquids, such as kerosene and engine oil. Meanwhile, viscosity and Prandtl numbers are very low for water compared to the other two, so water heats up faster than them physically.

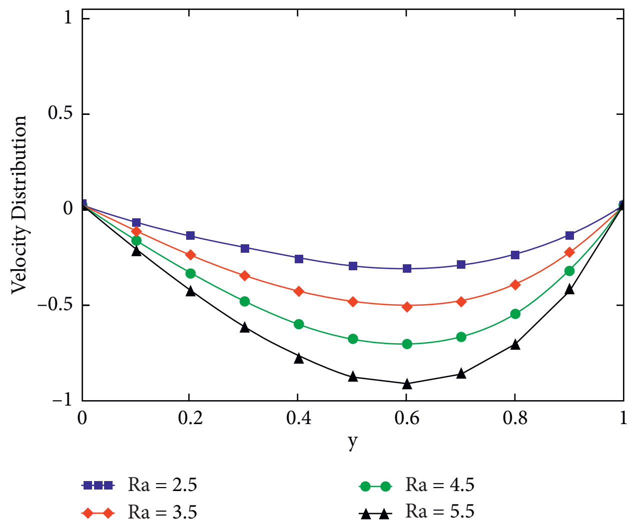

FIGURE 6: The effects of Rayleigh number on velocity field, when. $\Pr = 6.2, t = 5, \alpha = 0.5, \beta = 0.5, \gamma = 0.5, Lb = 5, Gr = 5, \beta^* = 0.006, \phi_{hnf} = 0.04, a = 0.2,$ and $M = 0.2$.

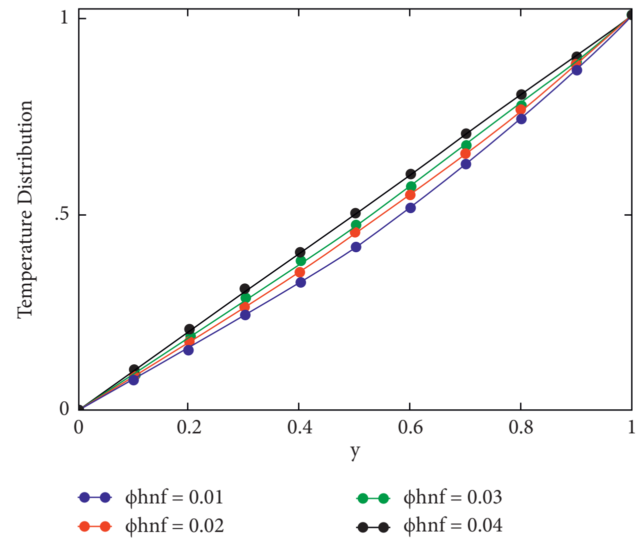

FIGURE 8: The effects of ϕ_{hnf} on temperature field, when $\Pr = 6.2, t = 3, \alpha = \beta = \gamma = 0.5,$ and $a = 0.2$.

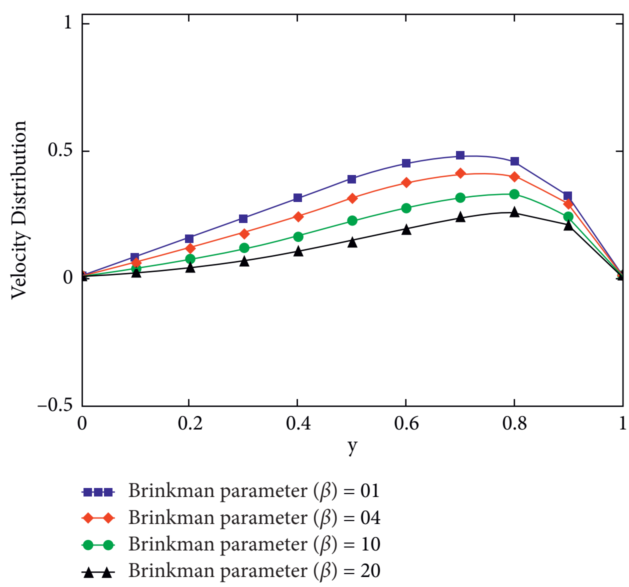

FIGURE 7: The effects of Brinkman parameter on velocity field, when $\Pr = 6.2, t = 5, \alpha = \beta = \gamma = 0.5, Lb = 30, Gr = 12, a = 0.2, Ra = 4, \phi_{hnf} = 0.08,$ and $M = 0.01$.

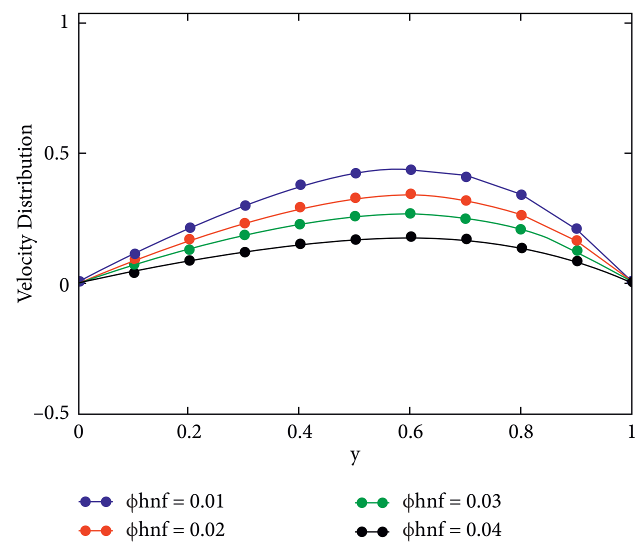

FIGURE 9: The effects of ϕ_{hnf} on velocity field, when. $\Pr = 6.2, t = 4, \alpha = \beta = \gamma = 0.5, Lb = 30, a = 0.2,$ $Gr = 15, Ra = 4,$ and $M = 2.0$.

To see the impact of bioconvection Rayleigh number Ra on velocity, Figure 6 is plotted. It is proved that velocity near the plate decreases for greater values of Ra. Ra decreases the fluid velocity since the buoyancy influenced by the transference of microorganisms is decreased by Ra. The consequences of the Brinkman parameter on the velocity field will be shown in Figure 7. Velocity decreased as the Brinkman parameter's value increased. This is because, by increasing the values of the Brinkman parameter, the drag forces become stronger, so velocity reduces.

Figures 8 and 9 show the effects of the ϕ_{hnf} volume fraction on hybrid nanoparticles. It is discovered that for higher values of ϕ_{hnf} and velocity indicated drops, the temperature can be increased. The nanofluid density has important significance in the velocity field. By mixing nanoparticles through base fluid, the consequent hybrid nanofluids improve considerably thicker which decreases velocity and increases temperature.

In the end, we have presented a comparison between our results and those of Saqib et al. [47]. It is clearly evident that the solution obtained with generalized Mittage–Leffler kernel in the presence of bioconvection shows stronger memory rather than exponential kernel that appeared in Caputo-Fabrizio fractional derivative as presented in Figure 10. It is concluded with the remark that our results can be enhanced in terms of memory.

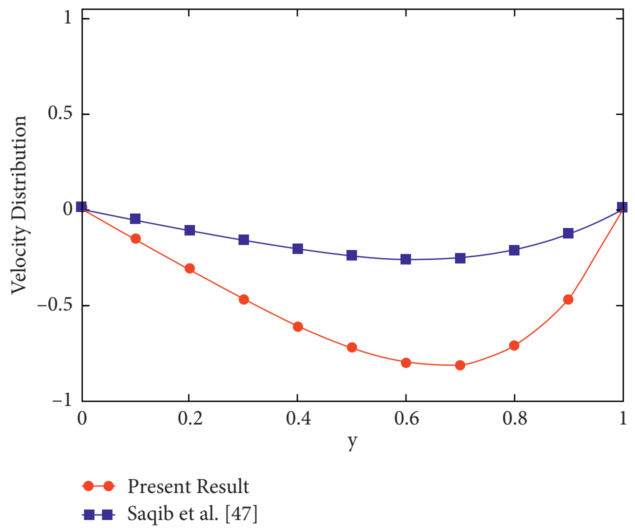

FIGURE 10: The comparison among our results and Saqib et al. [47] for velocity field, when $\Pr = 6.2, t = 2, \beta = \gamma = 0.5, Lb = 10, \beta^* = 0.006, Ra = 5, Gr = 10, \phi_{hnf} = 0.04$, and $M = 0.02$.

5. Conclusions

The current study investigated bioconvection using a heat transfer and MHD effects model, as well as a Prabhakar fractional approach. Laplace transform techniques are used to provide exact solutions for dimensionless governing equations. Graphical illustrations have been used to explain some of the physical effects of flow parameters. The followings are the significant outcomes:

(i) Obtained solutions are predicted for different values of fractional parameters based on generalized Fourier's law are responsible to attain better memory instead of artificial replacement

(ii) For a large time, fluid properties such as temperature, bioconvection, and velocity depict history/memory.

(iii) The temperature of water base nanoparticles is comparatively higher than kerosene and engine oil.

(iv) Bioconvection Rayleigh is responsible for the rapid decline in the momentum equation.

(v) The obtained solutions can be beneficial for proper analysis of real data and provide a tool for testing possible approximate solutions where needed.

For the future direction of readers, this work can be extended to include a large class of fluids of non-Newtonian nature and different thermal and mechanical boundary conditions. Also, you can extend this work with the fuzzy boundary conditions.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All of the authors contributed substantially to the study.

Acknowledgments

The Deanship of Scientific Research (DSR) at King Abdulaziz University (KAU), Jeddah, Saudi Arabia has funded this project, under grant no. (KEP-MSc: 111-130-1443).

References

[1] H. C. Brinkman, "A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles," *Flow, Turbulence and Combustion*, vol. 1, no. 1, p. 27, 1949.

[2] H. C. Brinkman, "On the permeability of media consisting of closely packed porous particles," *Flow, Turbulence and Combustion*, vol. 1, no. 1, p. 81, 1949.

[3] M. Saqib, I. Khan, S. Shafie, and A. Q. Mohamad, "Shape effect on MHD flow of time fractional Ferro-Brinkman type nanofluid with ramped heating," *Scientific Reports*, vol. 11, no. 1, p. 3725, 2021.

[4] M. I. Asjad, N. Sarwar, M. B. Hafeez, W. Sumelka, and T. Muhammad, "Advancement of non-Newtonian fluid with hybrid nanoparticles in a convective channel and prabhakar's fractional derivative—analytical solution," *Fractal and Fractional*, vol. 5, no. 3, p. 99, 2021.

[5] A. Khan, D. Khan, I. Khan, F. Ali, F. u. Karim, and K. S. Nisar, "MHD flow of brinkman type H_2O-Cu, Ag, TiO_2 and Al_2O_3 nanofluids with chemical reaction and heat generation effects in a porous medium," *Journal of Magnetics*, vol. 24, no. 2, pp. 262–270, 2019.

[6] C. E. Nanjundappa, A. Pavithra, and I. S. Shivakuamara, "Effect of dusty particles on Darcy-brinkman gravity-driven ferro-thermal-convection in a ferrofluid saturated porous layer with internal heat source: influence of boundaries," *International Journal of Algorithms, Computing and Mathematics*, vol. 7, no. 1, p. 21, 2021.

[7] S. Sarwar, M. Aleem, M. A. Imran, and A. Akgül, "A comparative study on non-Newtonian fractional order Brinkman type fluid with two different kernels," *Numerical Methods for Partial Differential Equations*, vol. 4, 2020.

[8] S. Shafie, M. Saqib, I. Khan, and A. Qushairi, "Mixed convection flow of brinkman type hybrid nanofluid based on atangana-baleanu fractional model," *Journal of Physics: Conference Series*, vol. 1366, no. 1, 2019.

[9] F. Ali, F. Ali, N. A. Sheikh, I. Khan, and K. S. Nisar, "Caputo–Fabrizio fractional derivatives modeling of transient MHD Brinkman nanoliquid: applications in food technology," *Chaos, Solitons & Fractals*, vol. 131, 2020.

[10] M. Saqib, I. Khan, S. Shafie, A. Qushairi Mohamad, and E. S. M Sherif, "Analysis of magnetic resistive flow of generalized brinkman type nanofluid containing carbon nanotubes with ramped heating," *Computers, Materials & Continua*, vol. 67, no. 1, pp. 1069–1084, 2021.

[11] A. Boutiara, M. S. Abdo, M. A. Alqudah, and T. Abdeljawad, "On a class of Langevin equations in the frame of Caputo function-dependent-kernel fractional derivatives with

antiperiodic boundary conditions," *AIMS Mathematics*, vol. 6, pp. 5518–5534, 2021.
[12] R. Ali, A. Akgül, and M. I. Asjad, "Power law memory of natural convection flow of hybrid nanofluids with constant proportional Caputo fractional derivative due to pressure gradient," *Pramana*, vol. 94, no. 1, p. 131, 2020.
[13] M. A. Imran, I. Khan, M. Ahmad, N. A. Shah, and M. Nazar, "Heat and mass transport of differential type fluid with non-integer order time-fractional Caputo derivatives," *Journal of Molecular Liquids*, vol. 229, pp. 67–75, 2017.
[14] M. Ahmad, M. A. Imran, and M. Nazar, "Mathematical modeling of (Cu– A l 2 O 3) water-based Maxwell hybrid nanofluids with Caputo-Fabrizio fractional derivative," *Advances in Mechanical Engineering*, vol. 12, no. 9, 2020.
[15] I. A. Mirza and D. Vieru, "Fundamental solutions to advection–diffusion equation with time-fractional Caputo–Fabrizio derivative," *Computers & Mathematics with Applications*, vol. 73, no. 1, pp. 1–10, 2017.
[16] R. Gul, M. Sarwar, K. Shah, T. Abdeljawad, and F. Jarad, "Qualitative analysis of implicit dirichlet boundary value problem for caputo-fabrizio fractional differential equations," *Journal of Function Spaces*, vol. 22, pp. 1–9, 2020.
[17] M. S. Abdo, T. Abdeljawad, K. D. Kucche, M. A. Alqudah, S. M. Ali, and M. B. Jeelani, "On nonlinear pantograph fractional differential equations with Atangana–Baleanu–Caputo derivative," *Advances in Difference Equations*, vol. 65, no. 1, 2021.
[18] N. H. Sweilam, S. M. Al-Mekhlafi, T. Assiri, and A. Atangana, "Optimal control for cancer treatment mathematical model using Atangana–Baleanu–Caputo fractional derivative," *Advances in Difference Equations*, vol. 34, no. 1, p. 334, 2020.
[19] N. Sarwar, M. I. Asjad, T. Sitthiwirattham, N. Patanarapeelert, and T. Muhammad, "A prabhakar fractional approach for the convection flow of Casson fluid across an oscillating surface based on the generalized fourier law," *Symmetry*, vol. 13, no. 11, p. 2039, 2021.
[20] N. A. Shah, C. Fetecau, and D. Vieru, "Natural convection flows of Prabhakar-like fractional Maxwell fluids with generalized thermal transport," *Journal of Thermal Analysis and Calorimetry*, vol. 143, no. 3, pp. 2245–2258, 2021.
[21] T. Elnaqeeb, N. A. Shah, and A. Rauf, "Natural convection flows of carbon nanotube Prabhakar-like fractional second-grade nanofluids over an infinite plate with Newtonian heating," *Mathematical Methods in the Applied Sciences*, vol. 27, 2020.
[22] R. Garrappa and E. Kaslik, "Stability of fractional-order systems with Prabhakar derivatives," *Nonlinear Dynamics*, vol. 102, no. 1, pp. 567–578, 2020.
[23] S. U. Choi and J. A. Eastman, "Enhancing thermal conductivity of fluids with nanoparticles," *ASME International Mechanical Engineering Congress and Exposition*, vol. 66, pp. 99–105, 1995.
[24] R. Ali, M. I. Asjad, and A. Akgül, "An analysis of a mathematical fractional model of hybrid viscous nanofluids and its application in heat and mass transfer," *Journal of Computational and Applied Mathematics*, vol. 383, 2021.
[25] M. Ahmad, M. I. Asjad, A. Akgül, and D. Baleanu, "Analytical solutions for free convection flow of Casson nanofluid over an infinite vertical plate," *AIMS Mathematics*, vol. 6, no. 3, pp. 2344–2358, 2020.
[26] T. Gul, M. Bilal, M. Bilal, W. Alghamdi, M. I. Asjad, and T. Abdeljawad, "Hybrid nanofluid flow within the conical gap between the cone and the surface of a rotating disk," *Scientific Reports*, vol. 11, no. 1, p. 1180, 2021.
[27] K. Rafique, M. A. Imran, M. I. Anwar, M. Misiran, and A. Ahmadian, "Energy and mass transport of Casson nanofluid flow over a slanted permeable inclined surface," *Journal of Thermal Analysis and Calorimetry*, vol. 144, no. 6, pp. 2031–2042, 2021.
[28] A. S. Khan, Y. Nie, Z. Shah et al., "Influence of interfacial electrokinetic on MHD radiative nanofluid flow in a permeable microchannel with Brownian motion and thermophoresis effects," *Open Physics*, vol. 18, no. 1, pp. 726–737, 2020.
[29] L. Ali Lund, Z. Omar, I. Khan, and E. M Sherif, "Dual branches of MHD three-dimensional rotating flow of hybrid nanofluid on nonlinear shrinking sheet," *Computers, Materials & Continua*, vol. 66, no. 1, pp. 127–139, 2020.
[30] Z. Shah, M. Sheikholeslami, P. Kumam, and P. Kumam, "Simulation of entropy optimization and thermal behavior of nanofluid through the porous media," *International Communications in Heat and Mass Transfer*, vol. 120, 2021.
[31] K. G. Kumar, E. H. B. Hani, M. E. H. Assad, M. Rahimi-Gorji, and S. Nadeem, "A novel approach for investigation of heat transfer enhancement with ferromagnetic hybrid nanofluid by considering solar radiation," *Microsystem Technologies*, vol. 27, no. 1, pp. 97–104, 2021.
[32] P. K. Dadheech, P. Agrawal, F. Mebarek-Oudina, N. H. Abu-Hamdeh, and A. Sharma, "Comparative heat transfer analysis of MoS2/C2H6O2 and SiO2-MoS2/C2H6O2 nanofluids with natural convection and inclined magnetic field," *Journal of Nanofluids*, vol. 9, no. 3, pp. 161–167, 2020.
[33] K. Dhif, F. Mebarek-Oudina, S. Chouf, H. Vaidya, and A. J. Chamkha, "Thermal analysis of the solar collector cum storage system using a hybrid-nanofluids," *Journal of Nanofluids*, vol. 10, no. 4, pp. 616–626, 2021.
[34] S. I. U. Khan, E. Alzahrani, U. Khan, N. Zeb, and A. Zeb, "On mixed convection squeezing flow of nanofluids," *Energies*, vol. 13, no. 12, p. 3138, 2020.
[35] W. Bu, H. Xu, I. Khan, S. I. Ullah Khan, and A. Zeb, "Mixed convection squeezing flow of nanofluids in a rotating channel with thermal radiation," *Journal of Mathematics*, vol. 15, p. 1, 2022.
[36] S. I. Ullah Khan, U. Khan, N. Ahmed, S. T. Mohyud-Din, I. Khan, and K. S. Nisar, "Thermal transport investigation in AA7072 and AA7075 aluminum alloys nanomaterials based radiative nanofluids by considering the multiple physical flow conditions," *Scientific Reports*, vol. 11, no. 1, pp. 1–11, 2021.
[37] I. Chabani, F. Mebarek-Oudina, and A. A. I. Ismail, "MHD flow of a hybrid nano-fluid in a triangular enclosure with zigzags and an elliptic obstacle," *Micromachines*, vol. 13, no. 2, p. 224, 2022.
[38] C. Rajashekhar, F. Mebarek-Oudina, H. Vaidya, K. V. Prasad, G. Manjunatha, and H. Balachandra, "Mass and heat transport impact on the peristaltic flow of a Ree–Eyring liquid through variable properties for hemodynamic flow," *Heat Transfer*, vol. 50, no. 5, pp. 5106–5122, 2021.
[39] S. Marzougui, F. Mebarek-Oudina, M. Magherbi, and A. Mchirgui, "Entropy generation and heat transport of Cu–water nanoliquid in porous lid-driven cavity through magnetic field," *International Journal of Numerical Methods for Heat and Fluid Flow*, vol. 32, no. 6, pp. 2047–2069, 2021.
[40] M. I. Asjad, N. Sarwar, B. Ali, S. Hussain, T. Sitthiwirattham, and J. Reunsumrit, "Impact of bioconvection and chemical reaction on MHD nanofluid flow due to exponential stretching sheet," *Symmetry*, vol. 13, no. 12, p. 2334, 2021.

[41] J. R. Platt, “Bioconvection patterns” in cultures of free-swimming organisms,” *Science*, vol. 133, no. 3466, pp. 1766-1767, 1961.
[42] H. Ge-Jile, H. Waqas, S. U. Khan, M. I. Khan, S. Farooq, and S. Hussain, “Three-dimensional radiative bioconvective flow of a sisko nanofluid with motile microorganisms,” *Coatings*, vol. 11, no. 3, p. 335, 2021.
[43] M. Ramzan, H. Gul, S. Kadry, and Y. M. Chu, “Role of bioconvection in a three dimensional tangent hyperbolic partially ionized magnetized nanofluid flow with Cattaneo-Christov heat flux and activation energy,” *International Communications in Heat and Mass Transfer*, vol. 120, 2021.
[44] Z. A. Alhussain, A. Renuka, and M. Muthtamilselvan, “A magneto-bioconvective and thermal conductivity enhancement in nanofluid flow containing gyrotactic microorganism,” *Case Studies in Thermal Engineering*, vol. 23, 2021.
[45] U. Farooq, H. Waqas, M. I. Khan, S. U. Khan, Y. M. Chu, and S. Kadry, “Thermally radioactive bioconvection flow of Carreau nanofluid with modified Cattaneo-Christov expressions and exponential space-based heat source,” *Alexandria Engineering Journal*, vol. 60, no. 3, pp. 3073–3086, 2021.
[46] T. A. Yusuf, F. Mabood, B. C. Prasannakumara, and I. E. Sarris, “Magneto-bioconvection flow of Williamson nanofluid over an inclined plate with gyrotactic microorganisms and entropy generation,” *Fluid*, vol. 6, no. 3, p. 109, 2021.
[47] M. Saqib, S. Shafie, I. Khan, Y. M. Chu, and K. S. Nisar, “Symmetric MHD channel flow of nonlocal fractional model of BTF containing hybrid nanoparticles,” *Symmetry*, vol. 12, no. 4, p. 663, 2020.
[48] M. Danish Ikram, M. Imran Asjad, A. Ahmadian, and M. Ferrara, “A new fractional mathematical model of extraction nanofluids using clay nanoparticles for different based fluids,” *Mathematical Methods in the Applied Sciences*, vol. 66, 2020.
[49] M. I. Asjad, M. D. Ikram, R. Ali, D. Baleanu, and A. S. Alshomrani, “New analytical solutions of heat transfer flow of clay-water base nanoparticles with the application of novel hybrid fractional derivative,” *Thermal Science*, vol. 24, no. Suppl. 1, pp. 343–350, 2020.
[50] M. D. Ikram, M. I. Asjad, A. Akgül, and D. Baleanu, “Effects of hybrid nanofluid on novel fractional model of heat transfer flow between two parallel plates,” *Alexandria Engineering Journal*, vol. 60, no. 4, pp. 3593–3604, 2021.
[51] F. Bräuer, E. Trautner, J. Hasslberger, P. Cifani, and M. Klein, “Turbulent bubble-laden channel flow of power-law fluids: a direct numerical simulation study,” *Fluid*, vol. 6, no. 1, p. 40, 2021.
[52] Y. Zheng, H. Yang, H. Mazaheri, A. Aghaei, N. Mokhtari, and M. Afrand, “An investigation on the influence of the shape of the vortex generator on fluid flow and turbulent heat transfer of hybrid nanofluid in a channel,” *Journal of Thermal Analysis and Calorimetry*, vol. 143, no. 2, pp. 1425–1438, 2021.
[53] A. D’Ippolito, F. Calomino, G. Alfonsi, and A. Lauria, “Flow resistance in open channel due to vegetation at reach scale: a review,” *Water*, vol. 13, no. 2, p. 116, 2021.
[54] S. U. Haq, M. A. Khan, Z. A. Khan, and F. Ali, “MHD effects on the channel flow of a fractional viscous fluid through a porous medium: an application of the Caputo-Fabrizio time-fractional derivative,” *Chinese Journal of Physics*, vol. 65, pp. 14–23, 2020.
[55] M. I. Asjad, S. U. Ur Rehman, A. Ahmadian, S. Salahshour, and M. Salimi, “First solution of fractional bioconvection with power law kernel for a vertical surface,” *Mathematics*, vol. 9, no. 12, p. 1366, 2021.
[56] B. Tashtoush and A. Magableh, “Magnetic field effect on heat transfer and fluid flow characteristics of blood flow in multi-stenosis arteries,” *Heat and Mass Transfer*, vol. 44, no. 3, pp. 297–304, 2007.
[57] F. Polito and Z. Tomovski, “Some properties of Prabhakar-type fractional calculus operators,” *Fractional Differential Calculus*, vol. 6, no. 1, pp. 73–94, 2015.
[58] A. Giusti and I. Colombaro, “Prabhakar-like fractional viscoelasticity,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 56, pp. 138–143, 2018.