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Research Article

Optimal Online Service Strategy and Price Decision in Omnichannel Retail

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As a new emerging sales promotion tool, various types of online services are increasingly adopted by firms to improve consumers' satisfaction and then increase profit. This paper simulates a two-echelon supply chain where a supplier sells the product through an offline or online retailer. Online channel is characterised by direct selling and reselling. We consider three online service strategies: no online service and preemptive and reactive online service. Several results are obtained. We find that investing in online services can benefit all players in most scenarios more than no-service scenario. In the direct selling case, the offline retailer benefits the most from the reactive service strategy due to the webrooming effect, whereas the supplier performance is best in the preemptive service strategy. However, in the reselling case, we find that the supplier and the online and offline retailers benefit the most from reactive service strategy. Furthermore, we compare the prices and service levels of the three strategies and find that the webrooming effect coefficient can affect the optimal wholesale price, retail prices, and online service level. Finally, the findings indicate that the supplier's choice of the two cases depends on the fixed cost of the online channel.

1. Introduction

Since online shopping has become an immensely viral commerce, many suppliers have started to build online sales channels, such as Huawei, Apple, and Gree. Similar to physical stores, online stores offer consumers a variety of products, whereas they are not restricted by visible physical space. Due to intense competition and high development of streaming technology, firms utilize various types of online services, such as video streaming advertisement, live streaming introduction, and free trial activities, to boost online retail sales. Online assistances can provide more detailed product information and satisfy the social demand of consumers, thereby improving the consumers' satisfaction and increasing online sales [1]. For example, recognising the great advantage of online sales reflected by COVID-19, about 81 thousand firms in China have used live streaming to sell the product online until Dec, 2020 [2]. Clearly, online service can significantly increase a retailer's online sales revenue. On the other hand, it also helps physical offline stores to increase their sales. For example, Jiaqi Li, a Chinese well-known live streamer, sold 150,000 lipsticks in one live broadcast, and several kinds of lipsticks he recommended also were out of stock in physical stores throughout the year [3]. Online service has propelled consumers' webrooming behaviour [4]. As a result of easy access to social media, consumers' webrooming behaviour arises when they search products online for a final assessment and purchase before visiting the store [5]. Most researchers and practitioners focus on the positive influence of the online services.

Nonetheless, Flavián et al. [6] pointed out that online services generate negative and positive implications for retailers. Online services provide the online retailers with an advantage in competing against offline retailers since online service produces a higher perception of service quality for online consumers [7], which results in more purchases [8, 9]. However, offline retailers benefit from free-riding: some offline consumers may enjoy the online service online and then buy the product offline. The reason is that the initiating online retailer covers the cost of online, but the demand enhancement effect spills over to the offline retailer [10, 11]. Therefore, such free-riding behaviour may hurt online retailers. That is, the free-riding behaviour (webrooming behaviour) of offline retailers reduces the revenue of the online channel.

The mixed effect of online service is even more blurred in a supply-chain setting with both offline and online retailers and an upstream supplier because the supplier can strategically charge different wholesale prices to the two retailers according to the online service effort. In response, the online retailer may initiate either reactive or preemptive online service. Following Li et al. [12], we regard the online service initiated before/after the supplier's pricing decision and the offline retailer's price decision as *preemptive/reactive* online service.

Previous studies have assumed that the cost of the online channel is fixed [13]. However, in practice, the cost of online channel is no longer fixed with the prevailing use of We Media. To attract consumers' attention and expand demand, suppliers or online retailers invest more online services to attract consumers either in social or shopping apps. A higher service level can attract relatively more consumers. However, doing so requires the service provider to bear higher online service costs. Hence, the question is "Are these service inputs necessary to improve firm's profits?"

Another problem is how the supplier will encroach on the online channel. Studies have found that supplier encroachment can provide many benefits, such as reducing double marginalization [14], enhancing demand [15, 16], and improving consumers satisfaction [17]. Thus, we focus on two cases: the direct selling case, in which the supplier sells the product via an offline retailer and a self-built online channel, and the reselling case, where the product is sold by an offline retailer and an online retailer. In the direct case, the supplier is the online service provider, and the independent online retailer is the service provider in the reselling case. We consider three strategies of online service for each case:

(i) No online service strategy: no additional online service investment in online channel

(ii) Preemptive online service strategy: the online service level is decided before the supplier's pricing decision

(iii) Reactive online service strategy: the online service level is determined after the price decision.

Our study will answer the following questions:

(1) Does investments in online service increase the supplier and the retailer's profit due to the webrooming behaviour?

(2) When should the online retailer initiate preemptive online service? When does it prefer a reactive online service strategy? How does the webrooming strategy affect the supply chain's performance and the consumer surplus?

(3) In which kind of case will the supplier choose to enter the online channel to achieve the highest profit?

However, price [18] and service strategy [19, 20] in supply chain management have been well explored in various scenarios. For example, Li et al. [20] studied the optimal price and service strategy considering the showrooming behaviour and found that additional service effort can benefit both the supplier and the retailer. Literature about webrooming behaviour mainly focused on consumers' webrooming intensions [21, 22] or investigated whether to introduce the webrooming [19, 23]. However, few studies have examined how the webrooming behaviour affects pricing strategies and online service levels and how to decide the optimal timing of the online service decisions under two supply chain structures. To fill this gap, our paper first analyses two different supply chain structures considering the online service investment and consumers' webrooming behaviour. We investigated whether to invest in the online service and obtain the optimal decisions and the equilibrium outcomes under each case. Additionally, we compared the equilibrium outcomes in various service strategies for two cases.

The rest of the paper is organized as follows. Section 2 reviews and discusses the literature related to our study. Section 3 introduces the model setup, including notations and formulations. Section 4 discusses the influence of the webrooming effect coefficient, presents the main results, and compares the optimal outcomes of the three service strategies in each case. Section 5 summarizes the findings and discussions. Finally, Section 6 provides concluding remarks on managerial insights as well as the limitations. All proofs are provided in the online companion.

2. Literature Review

2.1. Omnichannel Retail. In an omnichannel retailing environment, channel integration makes the channel interchange possible and offers a seamless shopping experience to consumers during the search and purchase processes [24, 25]. The omnichannel is greatly convenient for consumers, but it creates both challenges and opportunities for retailers and suppliers [26, 27]. The consumers' omnichannel shopping choices have become diversified, and there have been many studies of the consumer shopping process, which include phenomena such as showrooming [25, 28] and webrooming [23]. Gensler et al. [29] defined showrooming as a cross-channel free-riding behaviour, which meant consumers searching for information at a physical store and purchasing online from a competing retailer. Chatterjee and Kumar [30] and Herhausen et al. [31] regarded showrooms as a business opportunity for reducing consumers' online shopping uncertainty. Some studies investigated the omnichannel fulfilment strategy, for example, the buy-online-pick-up-in-store (BOPS) strategy [32, 33]. Others investigated the omnichannel operation strategy, considering the product returns [34, 35]. By contrast, we focused on two omnichannel structures considering the online service investment, where the supplier works together with an offline retailer or relies on an offline retailer and an online retailer to provide a seamless shopping experience for consumers. Thus, we examined the optimal price and the service strategy and the optimal timing of the online service investment in the omnichannel environment. We compared two supply chain structures and obtained the conditions in which the dominated supply chain structure exists.

2.2. Webrooming Behaviour. Extant webrooming research tends to analyse what factors will influence the consumers' webrooming intention empirically [36, 37]. Some studies have examined why webrooming has become a common omnichannel shopping behaviour, and they found that searching for information online owes to convenience and ease of comparisons [21], while purchasing offline enhances service quality and reduces purchase risks [29]. Others have investigated whether to introduce the webrooming [19, 23]. Sun et al. [19] investigated the channel strategies for online retailers regarding whether to introduce web showrooms in the face of consumer webrooming behaviour. Sun et al. [23] developed a theoretical supply chain model consisting of a manufacturer and a retailer to investigate the optimal webrooming strategy for dual-channel supply chains in the presence of consumer webrooming behaviour and found that the results depended on the online shopping cost of consumers. Few studies have used game theory to model the consumers' webrooming behaviour to explore how it affects the performance of retailers and suppliers in terms of service investment and online channel selection. To fill this gap, our paper first analyses two different supply chain structures, considering the online service investment and consumers' webrooming behaviour. We investigated whether to invest in the online service and obtain the optimal decisions and the equilibrium outcomes under each case. Additionally, we compared the equilibrium outcomes in various service strategies for two cases.

2.3. Service Strategy. This study also focuses on the literature of service strategies. Most studies have focused on retailers' service strategies. Some studies have shown that designing a reasonable sales effort strategy can increase demand [38–40] and coordinate the supply chain [41]. Others have found that a retailer's service can influence pricing strategies [42, 43]. Moreover, the service level also affects the supplier and retailer's profit [44]. Li et al. [20] investigated showrooming behaviour and found that additional service effort can benefit both suppliers and retailers. Moreover, their results indicated the best timing of service investment. Han et al. [13] studied the service and price competition of online retail channels and found that the retail price and the wholesale price are influenced by the service level. However, few studies examine how the webrooming behaviour affects pricing strategies and online service, not to mention the impact of service strategy timing.

3. Model Setup

3.1. Supply-Chain Structure. Following the prior literature [45], this paper simulates a two-echelon supply chain where a supplier sells the product through an offline or online retailer, and the supplier has now decided to enter the online channel. Following the previous study [46, 47], we design the supply chain structures as direct selling case, as shown in Figure 1, and the reselling case, as shown in Figure 2.

In the direct selling case, the supplier sells products directly to consumers through a self-built online channel and indirectly through an independent offline retailer. In this case, the self-built online channel incurs a fixed entry cost F, and the supplier sets an online price (p_o). Additionally, for the indirect sales, the supplier first sells the product to the retailer at the wholesale price (w). Then, the retailer sells the product to consumers at the offline price (p_r). Because the supplier itself builds the online channel, we call this the direct selling case. The sequence of events in the direct selling case is shown in Figure 3 and will be described in detail in the next section.

In the reselling case (Figure 2), the online and offline retailer order products from the supplier at the wholesale price (w) and then decide the retail price (p_o, p_r) simultaneously. We named this the reselling case because the supplier sells the product through an online retailer on the online channel.

3.2. Online Service and Webrooming. We use s to denote the online service invested by the supplier or the online retailer as an online sales promotion tool. Consumers can know about the product online through an online channel and then purchase from the offline channel. Such behaviour is called "Webrooming". We use λ $(0 < \lambda < 1)$ to represent the degree of webrooming, or the webrooming effect coefficient and spillover effect, which measures the degree of the offline retailer's free-riding on online service. Thus, if the service is raised by one unit, the demand of offline channel will expand by λ units, while the demand in the online channel will increase by only $(1 - \lambda)$ units because of consumers' webrooming behaviour. If $\lambda = 0$, there is no webrooming effect. As λ increases, the webrooming effect is stronger. If λ becomes too large, that is, $\lambda \geq (2 - 2b + \sqrt{2}\sqrt{5 - 8b - 3b^2 + 8b^3 - 2b^4}/3 - 4b + b^2)$, the supplier can reach any positive value, thereby leading to a trivial and uninteresting case. Hence, we focus on the case of $0 < \lambda < \min\{1, (2 - 2b + \sqrt{2}\sqrt{5 - 8b - 3b^2 + 8b^3 - 2b^4}/3 - 4b + b^2)\}$ (which is proven in the following section). In particular, we use a strictly convex service function $C(s)$ to depict the cost of online service with the following properties: $C'(s) > 0$, $C''(s) > 0$. Drawing on previous studies [48, 49], we set $C(s) = (1/2)s^2$.

The subscripts s, o, and r denote the parameters corresponding to the supplier, the online retailer, and the offline retailer, respectively. Additionally, we use the superscripts DN, DB, and DA to denote no, preemptive, and reactive service strategies in the direct selling case, respectively. Similarly, superscripts RN, RB, and RA represent no online, preemptive, and reactive service strategies in the reselling case, respectively.

The notations adopted in this study are summarized in Table 1.

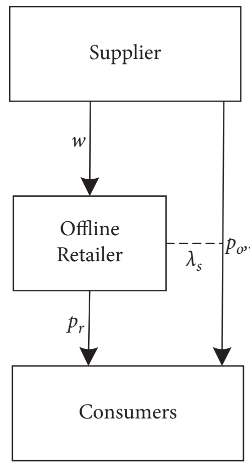

Figure 1: Direct selling supply-chain structure.

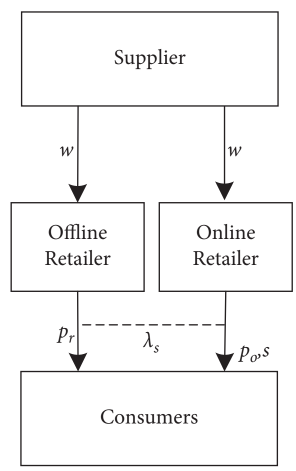

Figure 2: Reselling supply-chain structure.

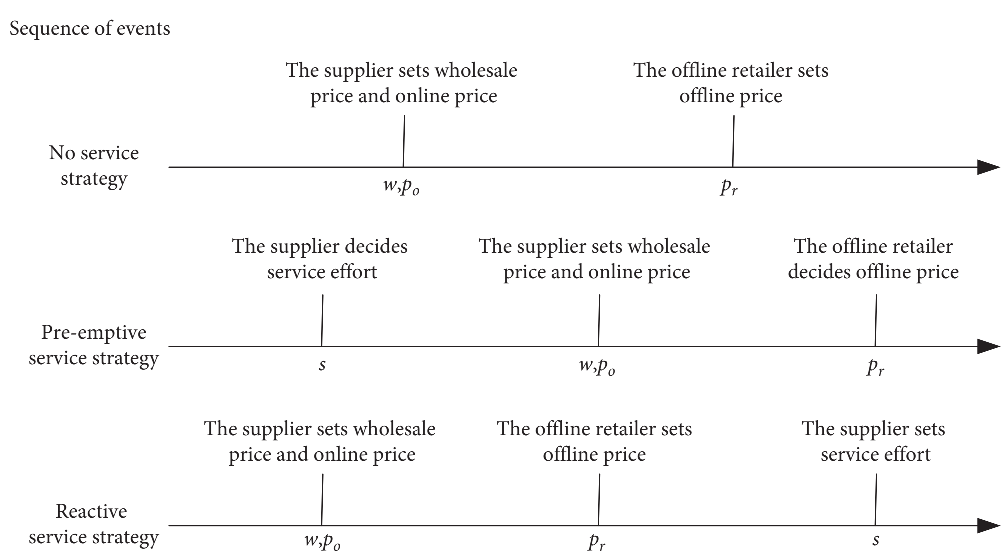

Figure 3: Sequence of events in the direct selling case.

Table 1: Summary of notations.

a: original total market demand
b: cross-price elasticity coefficient
θ: consumers' preference for the offline channel
w: wholesale price
s: online service level
λ: webrooming effect coefficient
$C(s)$: cost of online service
F: fixed online channel cost
p_o: online channel price
p_r: offline channel price
q_o^i: online channel demand, $i \in \{\text{DN, DB, DA, RN, RB, and RA}\}$
q_r^i: offline channel demand
π_s^i: profit of the supplier
π_o^i: profit of the online retailer
π_r^i: profit of the offline retailer

3.3. Demand. Drawing on previous studies [42, 50], we adopt the following linear functions to represent the demand of the offline and online channels.

$$\begin{aligned} q_r &= \theta a - p_r + b p_o + \lambda s, \\ q_o &= (1-\theta)a - p_o + b p_r + (1-\lambda)s. \end{aligned} \tag{1}$$

The total market size is represented by a. Mathematically, a is the base market demand when both prices are zero and the service is not offered. θ $(0<\theta<1)$ and $1-\theta$ represent consumers' preferences for offline and online stores, separately [51]. θa and $(1-\theta)a$ are the demands of the offline and online channels, respectively. In our model, to simplify the calculations and retain generalizability, we assume that the price elasticity is equal to 1. b denotes the two channels' elasticity coefficient. $0<b<1$ signifies that the effect of the ownership price is greater than the effect of the cross price.

4. Model Analysis

4.1. Direct Selling Case. Following Li et al. (2019), we discuss the optimal pricing, service level (if it is considered), and profit of the supplier and retailer in the three service strategies with a webrooming effect. The benchmark model does not consider service investment, and we use superscript DN to denote this strategy. The other two strategies both consider service investment. We use superscripts DB and DA to represent the preemptive and reactive strategies, respectively. The sequence of the game in each strategy is shown in Figure 3.

In the strategy with no online service, the supplier first sets the wholesale price and online price, and then the offline retailer decides on its offline price. In the preemptive strategy, the service decision is taken prior to the pricing decisions. Therefore, in our model, the supplier first decides the online service level and then sets the online price and wholesale price. On this basis, the offline retailer then decides its offline price. In the reactive strategy, the sequence of the other decisions is the same, but the supplier decides the service level last.

4.1.1. No Online Service Strategy. Without service investment, the supplier decides only the wholesale price (w) and online price (p_o). On that basis, the offline retailer then sets its offline price (p_r). The demand functions of the two channels are represented as follows:

$$\begin{aligned} q_r^{DN} &= \theta a - p_r + b p_o, \\ q_o^{DN} &= (1-\theta)a - p_o + b p_r. \end{aligned} \tag{2}$$

The profit functions of the supplier and the retailer are as follows:

$$\begin{aligned} \pi_r^{DN} &= (p_r - w) q_r^{DN}, \\ \pi_s^{DN} &= w q_r^{DN} + p_o q_o^{DN} - F. \end{aligned} \tag{3}$$

We use backward induction to solve this Stackelberg game, and the equilibrium outcomes are demonstrated as Lemma 1.

Lemma 1. *When the supplier invests no online service in the direct selling case, the optimal wholesale price it sets is* $w^{DN*} = (ab + a\theta - ab\theta/2(1-b^2))$. *The optimal online and offline retail prices are* $p_o^{DN*} = (a - a\theta + ab\theta/2(1-b^2))$ *and* $p_r^{DN*} = (a(2b(-1+\theta) - 3\theta + b^2\theta)/4(-1+b^2))$, *respectively. The optimal profit of the supplier is* $\pi_s^{DN*} = (a^2(2 + 4(-1+b)\theta + (3 - 4b + b^2)\theta^2)/8(1-b^2)) - F$, *and the optimal profit of the offline retailer is* $\pi_r^{DN*} = (a^2\theta^2/16)$.

4.1.2. Preemptive Online Service Strategy. This situation considers service investment before pricing decision with the webrooming effect. The supplier first decides its service level (s) and then determines the online price (p_o) and wholesale price (w); finally, the offline retailer sets the offline price (p_r). The profit functions of the supplier and the retailer are as follows:

$$\begin{aligned} \pi_s^{DB} &= w(\theta a - p_r + b p_o + \lambda s) + p_o((1-\theta)a - p_o + b p_r + (1-\lambda)s) - \frac{1}{2}s^2 - F, \\ \pi_r^{DB} &= (p_r - w)(\theta a - p_r + b p_o + \lambda s). \end{aligned} \tag{4}$$

We summarize the equilibrium in Lemma 2.

Lemma 2. *When the online service level is determined before the offline price, the optimal decisions of the supplier are* $s^{DB*} = -(a(2+2(-1+b)\lambda + (-1+b)\theta(2+(-3+b)\lambda))/-2-4\lambda-4b(-1+\lambda)\lambda + 3\lambda^2 + b^2(4+\lambda^2))$, $w^{DB*} = (a(-2(\theta+\lambda+\theta\lambda-\lambda^2) + b(-4-\lambda^2+\theta(4+\lambda)))/-4-8\lambda-8b(-1+\lambda)\lambda + 6\lambda^2 + 2b^2(4+\lambda^2))$ *and* $p_o^{DB*} = (a(-4+\lambda^2-\theta(-4+4b+\lambda))/-4-8\lambda-8b(-1+\lambda)\lambda + 6\lambda^2 + 2b^2(4+\lambda^2))$, *and the offline retailer's optimal decision is* $p_r^{DB*} = (a(2b^2\theta - 3(\theta+\lambda+\theta\lambda-\lambda^2) + 2b(-2-\lambda^2+\theta(2+\lambda)))/-4-8\lambda-8b(-1+\lambda)\lambda + 6\lambda^2 + 2b^2(4+\lambda^2))$. *In addition, the optimal profits of the supplier and the offline retailer are* $\pi_s^{DB*} = (a^2(-4+(-5+8b-2b^2)\theta^2 + \lambda^2 - 2\theta(-4+4b+\lambda))/4(-2-4\lambda-4b(-1+\lambda)\lambda + 3\lambda^2 + b^2(4+\lambda^2))) - F$ *and* $\pi_r^{DB*} = (a^2(\lambda + (-1+b)\lambda^2 + \theta(1-2b^2+\lambda-b\lambda))^2/4(2+4\lambda+4b(-1+\lambda)\lambda - 3\lambda^2 - b^2(4+\lambda^2))^2)$, *respectively.*

Then, we analyse the effect of the webrooming effect coefficient on the decision outcomes.

Proposition 1 (impact of webrooming effect coefficient). *If the supplier invests in the online service before setting the wholesale price, we have the following. (1) The service level and the online retail price will decrease with the webrooming effect coefficient. (2) When θ is small, the wholesale price ($\theta < \theta_1$) and offline retail price ($\theta < \theta_2$) decrease with the webrooming effect coefficient; otherwise, they increase with the webrooming effect coefficient. (3) When θ is small, the offline retailer's profit ($\theta < \theta_4$) and the supplier's profit ($\theta < \theta_5$) decrease with the webrooming effect coefficient; otherwise, they increase.*

The above conclusion shows that investing in service is beneficial for all players and the total supply chain. Proposition 1 indicates that the webrooming behaviour negatively affects the service level (s). As webrooming strengthens, the retailer free rides on more service are invested by the supplier. To prevent the retailer from free-riding on more service, the supplier will reduce the service input. Now, it does not need to increase the online price to cover the cost of service; therefore, it also reduces the online price.

However, the webrooming effect on the wholesale price and the offline price depends on the consumer's preference for the offline channel. If the consumers' offline channel preference is low, the wholesale price (w) decreases with the degree of webrooming. The reason is that for the supplier, the demand enhancement effect and the spillover effect have adverse impacts. A small θ means a small initial offline demand; thus, the supplier will set a lower wholesale price to spur the offline demand. In particular, as λ increases, the spillover effect, which hurts the supplier, strengthens. Therefore, the supplier will also decrease the wholesale price, as λ increases, to reinforce the demand enhancement effect. If θ is large, the original offline demand is large enough, and the effect of the offline demand promotion is weak. Therefore, the supplier has no need to reduce the wholesale price. On the contrary, it will raise the wholesale price to gain a higher profit. In particular, when the spillover effect strengthens, the wholesale price is set much higher. Consequently, the retailer sets a low offline price to attract more consumers. Therefore, when θ is small, the wholesale price (w) also decreases with λ. However, when θ is large, the original online channel demand is small. In response, the retail price also increases with λ. Moreover, the changes of the retailer's and supplier's profit with λ are similar to the wholesale price and offline price, which implies that the profit is influenced mainly by the retail price.

4.1.3. Reactive Online Service Strategy. We suppose that the online service is invested after the supplier makes the price decision in this section. First, the supplier decides the wholesale price (w) and online price (p_o). Second, the retailer sets the offline price (p_r). Finally, the supplier chooses the service investment (s). This decision sequence is shown in Figure 3. The profit functions are as follows:

$$\begin{aligned}\pi_s^{DA} &= w(\theta a - p_r + bp_o + \lambda s) + p_o((1-\theta)a - p_o + bp_r + (1-\lambda)s) - \frac{1}{2}s^2 - F,\\ \pi_r^{DA} &= (p_r - w)(\theta a - p_r + bp_o + \lambda s).\end{aligned} \tag{5}$$

The equilibrium outcomes are given as Lemma 3.

Lemma 3. *When the supplier initiates online service after deciding the price in the direct selling case, it will set the wholesale price with* $w^{DA*} = (a(-b^2\theta\lambda^2 - 2(\theta+\lambda+\theta\lambda-\lambda^2) - 2b(2+\lambda^2) + b\theta(4+\lambda+\lambda^2))/-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda(-4+4\lambda-\lambda^2+\lambda^3) + b^2(8+4\lambda^2+\lambda^4))$, *an online retail price with* $p_o^{DA*} = -(a(4+\theta(-4+\lambda-\lambda^2+b(4+\lambda^2)))/-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda(-4+4\lambda-\lambda^2+\lambda^3) + b^2(8+4\lambda^2+\lambda^4))$ *and the service level with* $s^{DA*} = -(a(2(1+(-1+b)\lambda)(2+\lambda^2) + \theta(-4+7\lambda+\lambda^3+b^2\lambda^3-2b(-2+4\lambda+\lambda^3)))/-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda(-4+4\lambda-\lambda^2+\lambda^3) + b^2(8+4\lambda^2+\lambda^4))$. *The optimal offline retail price is* $p_r^{DA*} = (a(b^2\theta(2+\lambda^2) - (\theta+\lambda+\theta\lambda-\lambda^2)(3+\lambda^2) + b(-4-3\lambda^2-\lambda^4+\theta(4+2\lambda+\lambda^2+\lambda^3)))/-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda(-4+4\lambda-\lambda^2+\lambda^3) + b^2(8+4\lambda^2+\lambda^4))$. *The optimal profits of the online retailer and the supplier can be represented as* $\pi_r^{DA*} = a^2(1+\lambda^2)^2(\lambda+(-1+b)\lambda^2 + \theta(1-2b^2+\lambda-b\lambda))^2/(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda(-4+4\lambda$

$-\lambda^2+\lambda^3)+b^2\ (8+4\lambda^2+\lambda^4))^2$ *and* $\pi_s^{DA\ *}=-(a^2(4+\theta^2(5+\lambda^2+b^2(2+\lambda^2)-2b(4+\lambda^2))+2\theta(-4+\lambda-\lambda^2+b(4+\lambda^2)))/2(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda(-4+4\lambda-\lambda^2+\lambda^3)+b^2\ (8+4\lambda^2+\lambda^4)))-F$, *respectively.*

By analysing the results in Lemma 3, we get the following proposition.

Proposition 2 (impact of webrooming effect coefficient). *If the supplier invests in the online service after deciding the supplier's price, we have the following. (1) The online price and the supplier's profit decrease with the webrooming effect coefficient. (2) When θ is small, the wholesale price ($\theta<\theta_2$) and service level ($\theta<\theta_1$) decrease with the webrooming effect coefficient; otherwise, they increase with it. (3) The offline price and the retailer's profit increase with the degree of webrooming.*

The service level is related to the wholesale price when the online service is invested after the wholesale price is decided. Under the DB scenario, when the consumer's preference on the offline channel is low, the supplier would take measures to expand the offline demand. Thus, it will decide a low wholesale price, which decreases with the degree of webrooming (λ), to maximize the demand as much as possible. In contrast, when θ is large, the offline channel has little potential to enhance the demand. Thus, the supplier will raise the wholesale price as λ increases to offset the spillover effect. Influenced by this, the service has the same change trend. Webrooming behaviour positively affects the offline demand but negatively affects the online demand. Therefore, the supplier will reduce the online price to increase online demand, which decreases as the webrooming effect coefficient increases. Even though the online price is lower, the online demand still decreases with λ because the spillover effect covers the demand enhancement effect. As a result, the supplier's profit also decreases as λ increases. The retailer will increase the offline price to gain a higher profit as λ increases. Because of the spillover effect and the higher offline retail price, the profit of the offline retailer also increases with λ.

4.1.4. Comparison of the Three Strategies in the Direct Selling Case. We then compare the equilibrium outcomes in Lemma 1, Lemma 2, and Lemma 3, obtaining the following theorem and proposition.

Proposition 3 (the direct selling case). *In equilibrium, we have the following:*

(i) The comparison result of the wholesale prices is $w^{DB\ *}>w^{DA\ *}>w^{DN\ *}$.

(ii) The optimal online service level in the reactive service strategy is greater than that in the preemptive scenario. That is, $s^{DA\ *}>s^{DB\ *}$.

(iii) The optimal online price in the preemptive service strategy is greater than in the reactive scenario, while the offline retailer makes a different decision. That is,

$$p_o^{DB\ *}>p_o^{DA\ *}>p_o^{DN\ *},\ \text{and}\ p_r^{DA\ *}>p_r^{DB\ *}>p_r^{DN\ *}. \tag{6}$$

Service investment enhances overall market demand. Hence, the supplier increases its wholesale price with the purpose of earning more profits. The wholesale prices in the preemptive and reactive scenarios are both greater than those in the no-service strategy scenario. In addition, in the preemptive strategy, the service provider is the supplier; furthermore, it makes the service decision before deciding the wholesale price. Therefore, it would set a higher price to offset the online service cost; that is, $w^{DB\ *}>w^{DA\ *}$.

In the preemptive strategy, the supplier would consider the effect of the wholesale price on the overall demand. If it inputs a high service, it will set a high wholesale price, which will damage the total demand and reduce the supplier's profit. However, in the reactive strategy, such a consideration does not apply. After deciding the wholesale price, the supplier will set a high online service level to increase the demand. Therefore, we conclude that the optimal online service level in the reactive service strategy is greater than in the preemptive scenario.

It is an interesting proposition in this paper that the comparisons of the two retail prices have different results. Both the offline and online retail prices in the no-service strategy are the lowest due to the lowest wholesale price. The supplier decides the wholesale price and online price simultaneously, so the changing trend of the online price is in accordance with the wholesale price. However, the offline retailer sets its offline retailer after obtaining the wholesale and the online prices. As shown in Proposition 3(iiMedia), the service level in the DA strategy is much higher than that in the DB scenario, which means that the spillover effect of DA is also stronger than that of DB. Although the reactive strategy has a weak demand enhancement effect with a higher offline price, the lower wholesale price of DA can bring about a larger marginal profit, which will combine with the spillover effect to recover the declining revenue. Therefore, the offline retailer will schedule a higher offline price in DA than in DB.

Theorem 1 (the direct selling case). *In the equilibrium of the direct selling case, the comparison of the optimal profits of the supplier and the offline retailer can be demonstrated as follows: $\pi_s^{DB\ *}>\pi_s^{DA\ *}>\pi_s^{DN\ *}$ and $\pi_r^{DA\ *}>\pi_r^{DB\ *}>\pi_r^{DN\ *}$, respectively. In addition, the comparison result of the supply chain profit is $\pi_T^{DA\ *}>\pi_T^{DB\ *}>\pi_T^{DN\ *}$.*

As shown in Theorem 1, investing in online services can benefit both the supplier and the offline retailer compared with the no-service strategy. Service investment is inevitably beneficial to both the supplier and the retailer. In particular, we compare the preemptive and reactive situations and find that the retailer earns the most in the reactive strategy. In contrast, the supplier earns the most in the preemptive strategy. The retailer decides the highest offline price in the reactive scenario, which covers the decrease in offline demand. In addition, the service spillover effect is also the strongest in DA, which also contributes to the profit increase.

For the supplier, the wholesale and the online prices in DB are the highest, and the spillover effect in DB is weaker than that in DA. Combined with the above effects, the supplier gains the most profit in the preemptive service strategy. Moreover, the total profit of the supply chain in the reactive service is also the highest. The decision maker's optimal strategy may not be consistent with the system's best performance.

Therefore, in practice, the online service level tends to be decided before pricing decisions are made. For example, some firms, especially in the electronics industry, such as Apple and HUAWEI, always hold new product release online conferences to introduce new products, and that conference occurs before the price is published. We also find that the total reactive demand is the highest. In fact, due to the need of expanding sales and capturing the market, suppliers now also hire Internet celebrities to promote the product. Specifically, bloggers on social media (such as Weibo, Xiaohongshu, YouTube, Instagram, and Bilibili) use videos or live streaming to introduce products with detailed explanations. Such promotional forms instill a vivid, immediate, and multidimensional feeling in consumers through product usage sharing and arouses their desire to buy.

4.2. Reselling Case. The supplier can also enter the online channel through an online retailer without a fixed cost. In this scenario, we study three players (the supplier, online retailer, and offline retailer) in the market. We also discuss the optimal pricing, service level (if considered), and profits of the supplier and two retailers under three service strategies with a webrooming effect. Here, we use the superscripts RN, RB, and RA to denote no-service, preemptive, and reactive service strategies, respectively. Figure 4 shows the sequence of events.

As shown in Figure 4, the game sequence in the reselling case is distinct from that in the direct selling case. In this case, the online retailer becomes the service provider. Under the no-service scenario, the supplier decides only the wholesale price. Then, two retailers decide the online and offline prices simultaneously. Under the preemptive scenario, the online retailer first sets its online service level, and the supplier then sets the wholesale price. Finally, two retailers decide the online and offline prices simultaneously. Under the reactive scenario, service level decisions are made after the supplier and two retailers make their pricing decisions.

4.2.1. No Online Service Strategy. Without service investment, the supplier decides only the wholesale price (w). On that basis, two retailers then set the retail prices (p_o, p_r) simultaneously. The demand functions are the same as in the direct selling case. The profit functions are different:

$$\begin{aligned}\pi_r^{RN} &= (p_r - w)(\theta a - p_r + b p_o),\\ \pi_o^{RN} &= (p_o - w)((1-\theta)a - p_o + b p_r),\\ \pi_s^{RN} &= w(\theta a - p_r + b p_o + (1-\theta)a - p_o + b p_r).\end{aligned} \tag{7}$$

By the reverse order method, we calculate the optimal results as Lemma 4.

Lemma 4. *When the online retailer chooses no online service investment, the optimal wholesale price is* $w^{RN*} = (a/\ 4(1-b))$, *and the optimal retail prices are* $p_o^{RN*} = -(a(-10 + b(7-12\theta) + 8\theta + 4b^2\theta)/4(-1+b)(-4+b^2))$ *and* $p_r^{RN*} = (a(2 + b(5-12\theta) + 4b^2(-1+\theta) + 8\theta)/\ 4(-1+b)(-4+b^2))$. *The equilibrium profits of the supplier, the online retailer, and the offline retailer are as follows:* $\pi_s^{RN*} = (a^2/8(2-3b+b^2))$, $\pi_o^{RN*} = (a^2(6-8\theta+b(-1+4\theta))^2/16(-4+b^2)^2)$, *and* $\pi_r^{RN*} = (a^2(2-8\theta+b(-3+4\theta))^2/16(-4+b^2)^2)$.

4.2.2. Preemptive Online Service Strategy. In contrast to the direct selling case, the service provider is an online supplier in the reselling case. The timeline of the game is as follows: firstly, the online retailer decides its service level (s) and the supplier then decides wholesale price (w); after that, the two retailers determine the retail price (p_o, p_r) simultaneously. The profit functions of the partners are as follows:

$$\begin{aligned}\pi_r^{RB} &= (p_r - w)(\theta a - p_r + b p_o + \lambda s),\\ \pi_o^{RB} &= (p_o - w)((1-\theta)a - p_o + b p_r + (1-\lambda)s) - \frac{1}{2}s^2,\\ \pi_s^{RB} &= w(\theta a - p_r + b p_o + \lambda s + (1-\theta)a - p_o + b p_r + (1-\lambda)s).\end{aligned} \tag{8}$$

Thus, we can calculate the equilibrium results in Lemma 5.

Lemma 5. *When the online retailer chooses the preemptive service strategy, the equilibrium decisions are as follows:* $s^{RB*} = (a(6-8\theta+b(-1+4\theta))(6-8\lambda+b(-1+4\lambda))/\ 92+8b^4+96\lambda-64\lambda^2+b^2(-65+8\lambda-16\lambda^2)+4b(3-16\lambda+16\lambda^2))$, $w^{RB*} = -(a(-2+b)\ (-16+4b^2+2b^3+\theta(6-8\lambda)-6\lambda+8\lambda^2+b\ (-8+\lambda-4\lambda^2\ +\theta(-1+4\lambda)))/(-1+b)(92+8b^4+96\lambda-64\lambda^2+b^2(-65+8\lambda-16\lambda^2)\ +4b(3-16\lambda+16\lambda^2)))$, $p_o^{RB*} = -(a(-2+b)(-40+b^2(14-8\theta)+8b^3\theta+\theta(38-8\lambda)-6\lambda+8\lambda^2+b(8+\lambda-4\lambda^2+\theta(-33+4\lambda)))/(-1+b)(92+8b^4+96\lambda-64\lambda^2+b^2(-65+8\lambda-16\lambda^2)+4b(3-16\lambda+16\lambda^2)))$ *and* $p_r^{RB*} = \left(\left(a\left(b^3(10-24\theta)+8b^4\ (-1+\theta)-8(2+3\lambda-4\lambda^2+\theta(5+4\lambda))\ +b^2(36-3\lambda+12\lambda^2-\theta(13+12\lambda))+b(-40+\ 22\lambda-40\lambda^2+\theta(74+40\lambda))\right)\right)/((-1+b)(92+8b^4+96\lambda-64\lambda^2+b^2\ (-65+8\lambda-16\lambda^2)+4b(3-16\lambda+16\lambda^2))))$. *The optimal profit of the supplier is* $\pi_s^{RB*} = (2a^2(-2+b)(-16+4b^2+2b^3+\theta\ (6-8\lambda)-6\lambda+8\lambda^2+b(-8+\lambda-4\lambda^2+\theta(-1+4\lambda)))^2/(-1+b)(92+8b^4+96\lambda-64\ \lambda^2+b^2(-65+8\lambda-16\lambda^2)+4b(3-16\lambda+16\lambda^2))^2)$, *and that of the online retailer and the offline retailer are* $\pi_o^{RB*} = (a^2(6-8\theta+b(-1+4\theta))^2/2(92+8b^4+96\lambda-64\lambda^2+b^2(-65+8\lambda-16\lambda^2)+4b(3-16\lambda+16\lambda^2)))$ *and* $\pi_r^{RB*} = (4a^2(-8+b^2(2-8\theta)+b^3(-3+4\theta)+\ 6\lambda-8\lambda^2+\theta(26+8\lambda)-b(-12+\lambda-4\lambda^2+\theta(15+4\lambda)))^2/(92+8b^4\ +96\lambda-64\lambda^2+b^2(-65+8\lambda-16\lambda^2)\ +4b(3-16\lambda+16\lambda^2))^2)$, *respectively.*

Then, we show how the webrooming coefficient impacts the optimal decisions.

Proposition 4 (Impact of webrooming coefficient). *If the retailer invests in the online service before the supplier determines the wholesale price, we have the following. (1) The service level, online wholesale price, and offline price all*

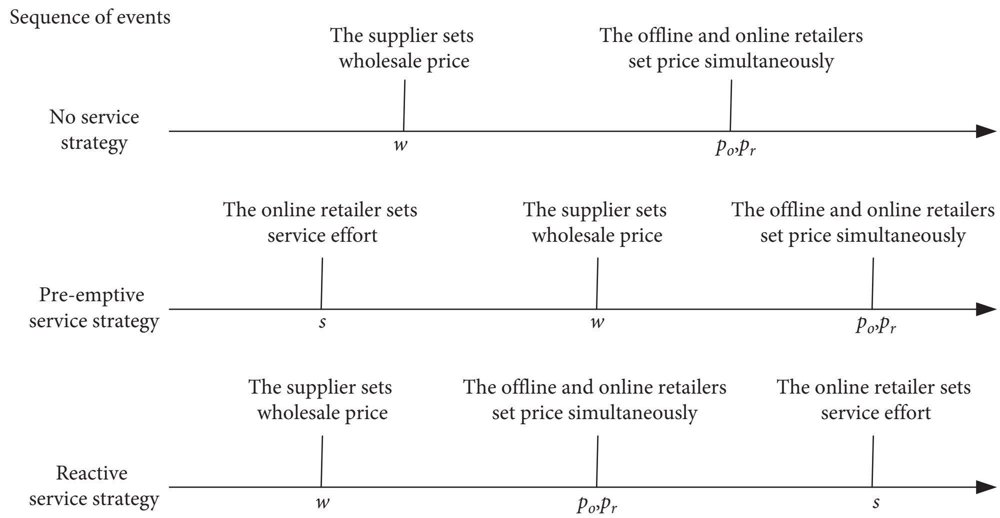

Figure 4: Sequence of events in the reselling case.

decrease with the webrooming effect coefficient. (2) The profits of supplier and online retailer also decrease with the degree of webrooming. (3) Nevertheless, the offline retailer's profit function is nonmonotonic.

In this scenario, the online retailer will set the service level before obtaining the wholesale price. Considering the webrooming effect, the online retailer will reduce its service to prevent the offline retailer from free-riding on more services. Specifically, as the webrooming effect coefficient (λ) increases, the online service level decreases. Influenced by this, the wholesale price also falls to spur demand. The wholesale price can positively affect the retail price. Hence, the online and offline prices also decrease as λ increases. However, the increasing price causes the decreasing demand, and the effect of the latter covers that of the former. Therefore, the supplier and the online retailer's profit also decrease with the webrooming effect coefficient. However, because the offline retailer free rides on the online retailer's service, the comprehensive impact of the spillover effect and the demand decrease effect on the retailer is complex. Hence, the profit of the offline retailer is nonmonotonic rather than strictly decreasing with λ.

4.2.3. Reactive Online Service Strategy. In this situation, the online retailer invests in service after the pricing decision. The supplier first determines the wholesale price (w), and the two retailers then set the retail price (p_o, p_r) simultaneously. Finally, the online retailer decides its service level (s), and the supplier then determines the wholesale price (w). In addition, the profit functions are the same as in Section 4.2.2.

Similarly, the equilibrium under the reactive service strategy is summarized in Lemma 6.

Lemma 6. *When the online retailer invests in online service after obtaining the wholesale price, the optimal decisions are* $w^{RA*} = -(a(2+b+\theta(-1+\lambda)+\lambda-\lambda^2)/2(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2))$, $s^{RA*} = -a(-1+\lambda)(-8+b^2(1-4\theta)-10\lambda+6\lambda^2+2\theta(5+7\lambda-4\lambda^2)+b(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2))/2(3+2b+3\lambda-2\lambda^2)(b^2-b(-1+\lambda)\lambda+2(-1-2\lambda+\lambda^2))$, $p_o^{RA*} = (a(-4b^3\theta+b^2(-7-\lambda+\lambda^2+\theta(6-6\lambda+4\lambda^2))+b(-2-9\lambda+6\lambda^2+2\lambda^3-\lambda^4+\theta(9+20\lambda-14\lambda^2+\lambda^3))+2(6+10\lambda-4\lambda^2-3\lambda^3+\lambda^4-\theta(6+8\lambda-7\lambda^2+\lambda^3))))/(2(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2)(b^2-b(-1+\lambda)\lambda+2(-1-2\lambda+\lambda^2)))$ *and* $p_r^{RA*} = ((a(2+4b^3(-1+\theta)+9\lambda-11\lambda^3+4\lambda^4+\theta(5+12\lambda+3\lambda^2-4\lambda^3)-b^2(2+8\lambda-7\lambda^2+\theta(2-2\lambda+4\lambda^2))+b(7+9\lambda-8\lambda^2+7\lambda^3-3\lambda^4+\theta(-8-16\lambda+5\lambda^2+3\lambda^3))))/(2(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2)(b^2-b(-1+\lambda)\lambda+2(-1-2\lambda+\lambda^2))))$. *The optimal profits of the supplier, the online retailer, and the offline retailer are as follows:* $\pi_s^{RA*} = (a^2(2+b+\theta(-1+\lambda)+\lambda-\lambda^2)^2/4(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2)(b^2-b(-1+\lambda)\lambda+2(-1-2\lambda+\lambda^2)))$, $\pi_o^{RA*} = (-((a^2(-1-2\lambda+\lambda^2)(-8+b^2(1-4\theta)-10\lambda+6\lambda^2+2\theta(5+7\lambda-4\lambda^2)+b(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2))^2)/(8(3+2b+3\lambda-2\lambda^2)^2(b^2-b(-1+\lambda)\lambda+2(-1-2\lambda+\lambda^2))^2)))$ *and* $\pi_r^{RA*} = ((a^2(2+b^2(-3+4\theta)+\lambda-2\lambda^2+5\lambda^3-2\lambda^4+b(-3+\theta-6\lambda+3\theta\lambda+5\lambda^2-4\theta\lambda^2)+\theta(-7-14\lambda+3\lambda^2+2\lambda^3))^2)/(4(3+2b+3\lambda-2\lambda^2)^2(b^2-b(-1+\lambda)\lambda+2(-1-2\lambda+\lambda^2))^2))$.

Then, comparing the above profits with the profits of the no-service strategy, we draw the conclusion in Proposition 5.

Proposition 5 (Impact of webrooming effect coefficient). *If the retailer invests in the service after the supplier decides the wholesale price, we have the following. (1) The service decreases as the webrooming effect coefficient increases. (2) When θ is small, the wholesale price ($\theta < \widehat{\theta}_1$), online retail price ($\theta < \widehat{\theta}_2$), and offline retail price ($\theta < \widehat{\theta}_3$) all decrease with the webrooming effect coefficient (λ); otherwise, they all increase with λ. (3) The online retailer's profit and the supplier's profit decrease with the webrooming effect coefficient. (4) Nevertheless, the offline retailer's profit function is nonmonotonic.*

In this section, the wholesale price (w) decision is not influenced by the service(s). When θ is small, the online channel demand is large, and the online retailer may have little incentive to provide a high service level. The situation will be severe if the spillover effect strengthens (λ increases).

Therefore, the supplier will lessen the wholesale price as λ increases to encourage the online retailer to offer a relatively high online service level. If θ is large, the online retailer has a motivation to spur the demand. The demand enhancement effect caused by service investment is outstanding in this scenario. Notably, because of the spillover effect, the service always decreases with λ. Therefore, the supplier will raise the wholesale price to offset the demand decrease. The wholesale price affects the retail prices. Therefore, the trend of the online and offline price functions is similar to that of the wholesale price function. The online and the overall demand decrease with λ, and the demand enhancement effect prevails. Therefore, the online retailer's profit and the supplier's profit also decrease with λ. However, the offline demand does not change monotonically, and the profit is also affected by the spillover effect. Therefore, the trend of offline profit is complex and nonmonotonic.

4.2.4. Comparison of Three Strategies in the Reselling Case. We then compare the equilibrium outcomes in Lemma 4, Lemma 5, and Lemma 6, and we obtain the following results.

Proposition 6 (strategy in the reselling case). *In equilibrium, we get the following:*

(i) *The comparison results of the wholesale price are If* $\theta < (1/2), w^{RB*} > w^{RA*} > w^{RN*}$; *otherwise*, $w^{RB*} > w^{RN*} > w^{RA*}$.

(ii) *The optimal online service level in the reactive scenario is greater than in the preemptive strategy. That is* , $s^{RA*} > s^{RB*}$.

(iii) *The online and offline prices depend on the original offline demand proportion (θ). When θ is small,* $p_o^{RB*} > p_o^{RA*} > p_o^{RN*}$ $(\theta < \widetilde{\theta}_1)$, *and* $p_r^{RB*} > p_r^{RA*} > p_r^{RN*}$ $(\theta < \widetilde{\theta}_2)$;*otherwise,* $p_o^{RA*} > p_o^{RB*} > p_o^{RN*}$, *and* $p_r^{RA*} > p_r^{RB*} > p_r^{RN*}$.

Proposition 6(i) shows that the wholesale price is the highest in the preemptive strategy. When the online retailer decides to enhance the online service level before setting the price, it will enhance the demand and be unaffected by the wholesale price. Thus, realizing the increased demand, the supplier will set a high wholesale price to maximize its profit. The comparison between w^{RA*} and w^{RN*} depends on consumers' preference for the offline channel (θ). If $\theta > (1/2)$, the original online demand is small. The online retailer has an incentive to invest in service, while it is also afraid that the offline retailer will free ride on most of the service. Therefore, the supplier will set a lower price than that in the no-service scenario to encourage the online retailer to invest in service. Otherwise, when the original online demand is greater than the other channel, the online retailer has no such concerns. There is a high probability that it will invest in service. Hence, the supplier will improve the wholesale price to maximize its profit.

Proposition 6 is the same as Proposition 3(ii). As Proposition 6(i) demonstrates, the wholesale price is always greater in RB than in RA. The online retailer is unwilling to invest a greater service level in the preemptive strategy because it is afraid of facing a higher wholesale price. In the reactive scenario, to improve demand, the supplier will lower the wholesale price to spur the online retailer to invest in more service. Consequently, the service level in the reactive strategy is higher than that in the preemptive strategy.

Proposition 6(iii) shows that the price in the no-service strategy is always the lowest. The reason is that service investment increases the market demand, so both retailers set higher retail prices to acquire more profits. In the reselling case, we also find that the service level in RA is higher than that in RB. When θ is small, the price comparison is consistent with the comparison of the wholesale price. A lower wholesale price in RA results in lower retail prices in RA. However, when θ is large, the demand enhancement effect is noticeable. Because of the higher service level in RA, the two retailers would set a higher reactive strategy than in the preemptive scenario. The two retail prices are decided simultaneously, with a similar trend.

Theorem 2 (strategy in the reselling case). *In equilibrium, if service investment exists, the supplier and the online retailer consistently earn more profit than in the no-service strategy. The offline retailer gains more profit under the preemptive strategy than under the no-service scenario if and only if the webrooming effect coefficient and consumers' offline preference are medium. Moreover, compared with the preemptive and no-service strategies, the reactive service decision results in higher profits for the three players.*

When the three strategies are compared, the reactive strategy is the most beneficial one. The online service investment increases the overall market demand. For the online retailer, the demand enhancement effect covers the spillover effect in RA, which brings more revenue. For the offline retailer, the lower wholesale price and the spillover effect both have a positive impact on profit. The demand enhancement effect covers the negative impact of the wholesale price decrease, so the supplier also gains more profit. Therefore, it achieves a triple-win outcome.

In current new promotions, an increasing number of online retailers are taking reactive service investment measures. For example, some online retailers collaborate with famous live streamers to sell products through studios. In an online studio, they provide the services of field application, product introduction, price comparison, and so on. The effect of such publicity is surprising. The well-known live streamer Viya in China guided annual sales to a level of approximately 40 billion dollars in 2020. More than 33 online rooms generated one-day sales of over 100 million dollars on the first presale day of Double 11 in Tmall in 2020 (http://www.199it.com/archives/1175203.html).

4.3. Comparison of Direct Selling and Reselling Cases. We compare the supplier's profit between the direct selling and reselling cases, i.e., π_s^{DB*} and π_s^{RA*}, to explore the supplier encroachment decision. In what situation will the supplier decide to encroach on the online channel?

4.3.1. Supplier Encroachment. The difference between $\pi_s^{DB\,*}$ and $\pi_s^{RA\,*}$ depends on the fixed cost F: If $F < \overline{F}$, there is $\pi_s^{DB\,*} > \pi_s^{RA\,*}$, with

$$\overline{F} = \frac{1}{4}a^2\begin{pmatrix} -4 + \left(-5 + 8b - 2b^2\right)\theta^2 + \lambda^2 - 2\theta(-4 + 4b + \lambda)/ - 2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2\left(4 + \lambda^2\right) \\ -\left(2 + b + \theta(-1 + \lambda) + \lambda - \lambda^2\right)^2/\left(-3 + b + 2b^2 - 3\lambda + 3b\lambda + 2\lambda^2 - 2b\lambda^2\right)\left(b^2 - b(-1 + \lambda)\lambda + 2\left(-1 - 2\lambda + \lambda^2\right)\right) \end{pmatrix}. \quad (9)$$

Theorem 3. *The supplier profits more in the direct selling case if the fixed cost is small ($F< \overline{F}$); otherwise, it would choose the reselling case to enter the online channel rather than encroaching. In addition, consumers' offline channel preference can affect the threshold, and we obtain* $(\partial\overline{F}/\partial\theta) < 0$.

Theorem 3 indicates that the supplier's choice of encroachment depends on the fixed online channel cost. If the fixed cost is not too high, the supplier chooses the direct selling case for encroachment. More importantly, we also find that the threshold is related to consumers' offline preferences. If consumers tend to shop offline, it is more likely that the supplier will enter the online channel through the reselling case. When consumers' preference for offline channels is strong, the supplier must reduce the wholesale price to stimulate demand. The demand enhancement effect, in turn, leads to more revenue for the supplier. Therefore, the supplier will prefer the reselling case. In contrast, if consumers are willing to shop online, there is a large market on the online channel. Hence, the supplier will choose to sell products through its own direct online channel rather than via an online retailer to earn more profit.

4.3.2. Numerical Studies. The following numerical examples show the influence of the webrooming effect coefficient on service level and wholesale price for the two cases. Let $a = 1, b = 0.3$, and $\theta = 0.4$.

Figures 5 and 6 show that the service level and the wholesale price are greater in the direct selling case than in the reselling case. As the service provider in the direct selling case, the supplier will transfer the service cost to the other, setting a higher wholesale price. In contrast, in the reselling case, it decides a lower wholesale price to encourage the online retailer to increase the online service level. The offline price also increases in the direct selling case based on the wholesale price.

Interestingly, Figure 7 shows that the online price comparison between the direct selling case and the reselling case is affected by the webrooming effect coefficient (λ). When λ is small, the online price in the direct selling case is higher. Otherwise, it has the opposite result. When λ is small, the demand increase caused by service investment mostly goes to the online channel, so the supplier will determine a higher retail price to gain more profits. In contrast, if λ is large, the demand increase caused by service investment mostly goes to the offline channel, and the supplier needs to set a lower online price to attract more consumption.

Finally, we analyse the total consumer surplus of all the scenarios, which is shown in Figure 8. The figure shows that consumer surplus under RA and DA has the best outcomes in the reselling and direct selling case, respectively. In particular, the order of consumer surplus is DA > DB > DN and RA > RB > RN. Although the DA strategy obtains the highest level of consumer surplus in the direct selling case, the supplier will choose the DB strategy. Therefore, we compare only the DB and RA strategies. We find that consumer surplus is notably greater in the RA than in the DB scenario. The reason is that, as shown by Figures 5, 7, and 9, the service level in the RA situation is less than that in the DB situation. The comparison of retail prices has similar results. The sales increase caused by the online service cannot be offset by the sales decrease caused by a retail price increase. Therefore, consumer surplus is lower in DB than in RA. The insight is that online service investment can help the supplier and the retailer increase profits; however, it does not mean a high level of consumer surplus.

5. Findings and Discussion

As an extension of Li et al. [20], who investigated the pricing and three offline service strategies in a dual-channel supply chain under the showrooming effect, we study pricing decisions and the online service strategies considering the webrooming effect. Besides, our study also extends the single structure of the dual-channel supply chain into two structures: direct selling case and reselling case. Their paper found that service investment is always beneficial to the supplier and the offline retailer, and the ex-post one is the optimal service strategy for both players. Our study shows that the optimal strategy is not identical in two different cases. It is worth noting that service investment may hurt the offline retailer in the reselling case. This paper also enriches the related literature. Previous studies about webrooming have mostly adopted empirical methods to study the consumers' motivation to webrooming behaviour. Our paper is the first to use a game theory model to analyse firms' performance based on consumers' webrooming behaviour. Our study also has practical significance. The findings provide directions for suppliers and retailers to make pricing and service decisions based on webrooming behaviour. Moreover, we also guide consumers in the purchase processes. The four substantive findings of this study are discussed below.

First, online service and webrooming behaviour improve both the supplier and retailer's profits in almost all cases except that the preemptive strategy is adopted in the

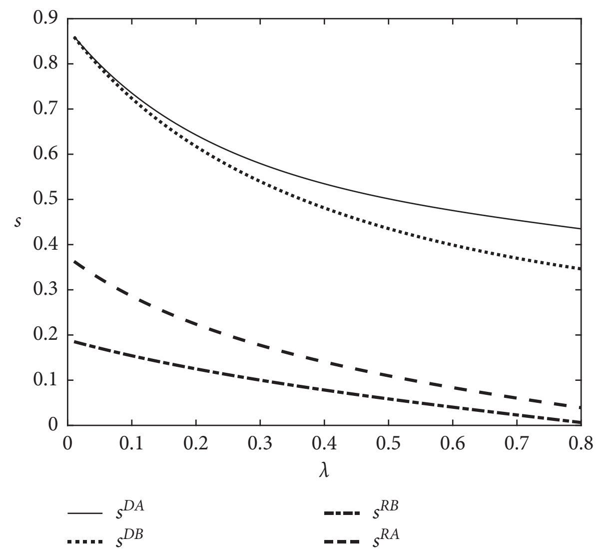

Figure 5: The influence of λ on s.

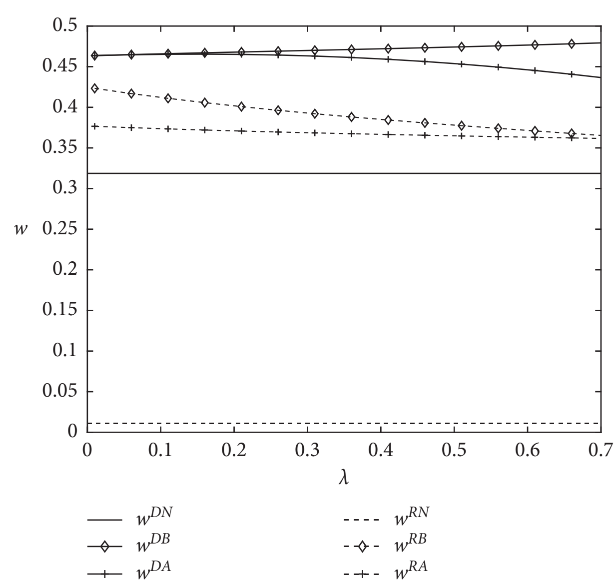

Figure 6: The influence of λ on w.

reselling case. In particular, when consumers' offline preference and the webrooming effect are low enough, the spillover effect and the demand enhancement effect are too weak to offset the profit decrease because of lower offline price.

Second, service providers have different choices in different cases. In the case of direct selling, the supplier, who is the service provider, will adopt the preemptive service strategy. Conversely, in the reselling case, the service provider is the online retailer, who will embrace the reactive service strategy. The supplier and the offline retailer have both competitive and cooperative relationships in the direct selling case. In the preemptive strategy, the lower wholesale price generates a larger demand enhancement effect, which significantly benefits the supplier. However, in the other case, the two retailers are pure competitors. The supplier will decrease the wholesale price to spur more demand in the reactive strategy and the online retailer performs best in this case.

Third, it is interesting that a service provider's self-interest is neither aligned with the supply-chain profit nor consumer surplus under the direct selling case. In particular, the reactive strategy always leads to better supply chain performance and greater consumer surplus, which is not the best decision for the supplier in the direct selling case. We further compare the prices and service levels of online with three strategies. The outcomes in the reselling case relate to consumers' preference for the offline channel, while it is not established in the other case. The reason is that the profits of two players in the reselling case are influenced by each single

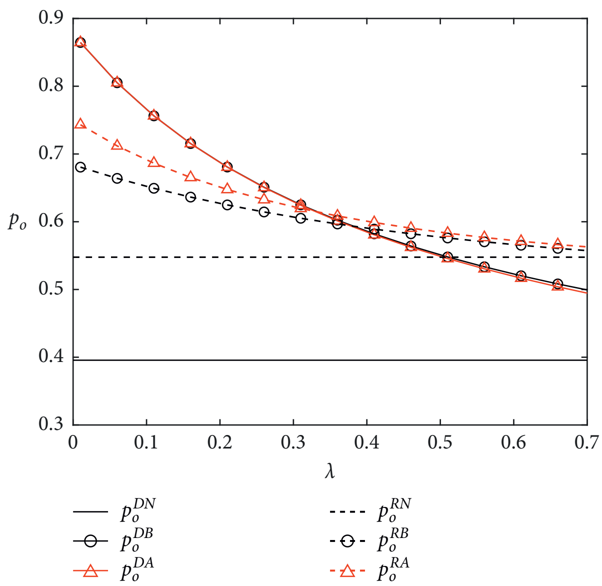

FIGURE 7: The influence of λ on p_o.

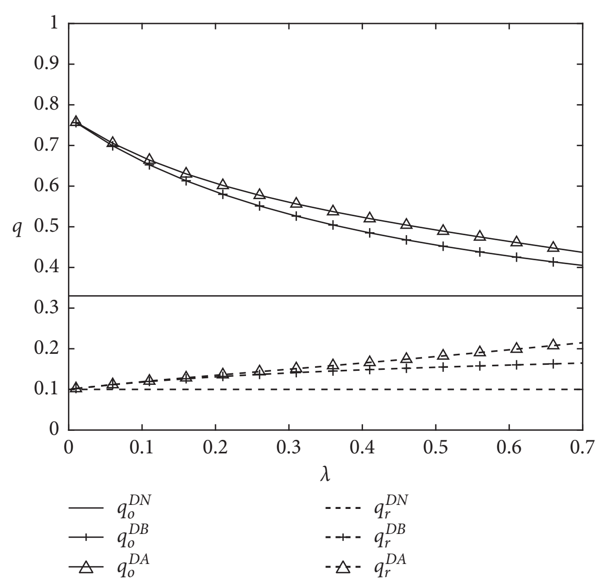

FIGURE 8: The total demand of the two channels in the three strategies of the two cases.

channel's demand. On the contrary, the supplier's revenue in the direct selling case is determined by the total market demand.

Fourth, the supplier's encroachment decision is up to the fixed cost of building the self-direct online channel. When the fixed cost is over the threshold, the supplier adopts the reselling strategy; otherwise, it uses the direct selling strategy. We find that the fixed cost threshold decreases with consumers' preference for the offline channel. This implies that if consumers tend to purchase offline, the supplier is more likely to encroach online channel through the direct selling case. Because consumers' strong offline preference would induce the supplier to lower the wholesale price to

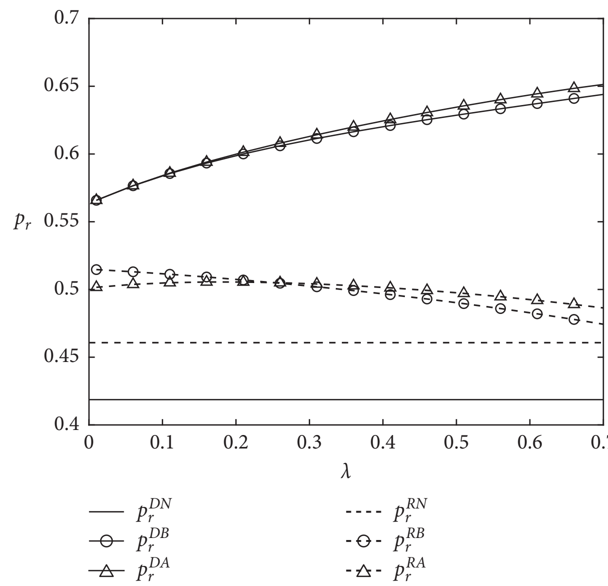

FIGURE 9: The influence of λ on p_r.

stimulate the demand, the demand enhancement effect, in turn, leads to more revenue for the supplier.

6. Conclusions

6.1. Summary. In the e-commerce era, many decision makers adopted online services to promote sales and increase revenue. Our study explicitly reveals how the supplier makes the price and service investment decisions in response to webrooming behaviour and what kind of supply chain structure the supplier will choose to encroach on the online channel. In this paper, we consider a supply chain in which the supplier can decide how to enter the online channel with a choice between selling through a self-built online channel (the direct selling case) and selling wholesale to an online retailer (the reselling case). In the direct selling case, the supplier will invest in the online service before determining the wholesale price; in the reselling case, the online retailer as the service provider will decide its service level after obtaining the wholesale price.

6.2. Managerial Implication. Several managerial implications are indicated. First, most decisions are related to consumers' preference for offline and online channels. This indicates that the supplier or the online retailer should take action to estimate the consumers' channel preference and make reasonable and correct decisions accordingly. They can use methods such as questionnaire surveys or collect mobile data to evaluate the consumers' channel preferences. Second, this paper investigated how the webrooming phenomenon may affect the service strategy and other decisions. We demonstrated the influence of webrooming effect coefficient on the equilibrium outcomes. Therefore, taking measures to utilize and encourage the consumers' webrooming behaviors is necessary for firms, which requires the cooperation of online sellers and offline retailers. Because webrooming is a cross-channel behavior, it is difficult for a single firm to follow it. Third, we have shown that investing in online service always brought more benefits for the service provider than no-service input. Therefore, we suggest that the online seller should try to invest in the online service on its channel, including video streaming advertisements, live streaming introductions, free trial activities, augmented reality (AR) technology, and other advanced technologies, to attract consumers and enhance their shopping experience. For example, cosmetic brands, such as MAC and Lancome, have launched the AR makeup test on the Taobao platform, which have attracted plenty of consumers to patronize this product online. Finally, firms need to encourage consumers to accept and embrace various types of online services, which may provide more details and information about products. Experiencing efficient online services could reduce consumer uncertainty about products and increase their purchase intention. Hence, the sales of the products and the profits of firms may increase.

6.3. Limitations. Despite the importance of the managerial insights into service investment and supplier online entry, our study has a few limitations. First, online selling is not limited to these two cases. In fact, selling products through an online platform is also very common. Thus, future research can expand the online channel choices. Second, in the direct selling case, we consider one supplier and one retailer. Future studies cannot be limited in this way but should consider multiple suppliers or multiple retailers. Similarly, the reselling case assumes that the two retailers are independent, but what if the online retailer and offline retailer are attached to the same brand? Finally, we consider the supplier to be the most powerful player. However, some offline retailers, such as Wal-Mart, can play a more powerful role in practice. Therefore, future studies of the impact of different channel power structures on webrooming behaviour are necessary.

Appendix

Proof of Lemma 1. The profit function of the supplier and the retailer are as follows:

$$\begin{aligned}\pi_r^{DN} &= (p_r - w)q_r^{DN},\\ \pi_s^{DN} &= wq_r^{DN} + p_o q_o^{DN} - F.\end{aligned} \tag{A.1}$$

With backward induction, we first solve the retailer's problem. The retailer maximizes its profit over p_r:

$$\frac{\partial \pi_r^{DN}}{\partial p_r} = w + a\theta + bp_o - 2p_r. \tag{A.2}$$

π_r^{DN} is strictly concave with respect to p_r because $(\partial^2 \pi_r^{DN}/\partial p_r^{\,2}) = -2 < 0$.

Therefore, we get the optimal offline price

$$p_r = \frac{1}{2}\left(w + a\theta + bp_o\right). \tag{A.3}$$

Substituting (A.3) into π_s^{DN} and differentiating π_s^{DN} with respect to w and p_o, we have

$$\begin{aligned}\frac{\partial \pi_s^{DN}}{\partial p_o} &= a + bw + \frac{1}{2}a(-2+b)\theta + \left(-2+b^2\right)p_o,\\ \frac{\partial \pi_s^{DN}}{\partial w} &= \frac{1}{2}\left(-2w + a\theta + 2bp_o\right).\end{aligned} \tag{A.4}$$

The Hessian Matrix of π_s^{DN} in terms of w and p_o is as follows:

$$\begin{aligned}H_1 &= \begin{vmatrix} \dfrac{\partial^2 \pi_s^{DN}}{\partial p_o^{\,2}} & \dfrac{\partial^2 \pi_s^{DN}}{\partial p_o \partial w} \\[2ex] \dfrac{\partial^2 \pi_s^{DN}}{\partial w\, \partial p_o} & \dfrac{\partial^2 \pi_s^{DN}}{\partial w^2} \end{vmatrix}\\ &= \begin{vmatrix} b^2 - 2 & b \\ b & -1 \end{vmatrix}.\end{aligned} \tag{A.5}$$

By calculating, we get

$$\frac{\partial^2 \pi_s^{DN}}{\partial p_o^{\,2}} = b^2 - 2 < 0, \text{ and } \left|H_1\right| = 2\left(1 - b^2\right) > 0. \tag{A.6}$$

So the supplier's profit function is a joint concave function with respect to w and p_o. So the optimal wholesale price, retail prices, and profits are

$$\begin{aligned}w^{DN\,*} &= \frac{ab + a\theta - ab\theta}{2\left(1 - b^2\right)},\\ p_o^{DN\,*} &= \frac{a - a\theta + ab\theta}{2\left(1 - b^2\right)},\\ p_r^{DN\,*} &= \frac{a\left(2b(-1+\theta) - 3\theta + b^2\theta\right)}{4\left(-1 + b^2\right)},\\ \pi_r^{DN\,*} &= \frac{a^2\theta^2}{16},\\ \pi_s^{DN\,*} &= \frac{a^2\left(2 + 4(-1+b)\theta + \left(3 - 4b + b^2\right)\theta^2\right)}{8\left(1 - b^2\right)} - F.\end{aligned} \tag{A.7}$$

□

Proof of Lemma 2. The profit functions of the supplier and the retailer can be obtained as

$$\begin{aligned}\pi_s^{DB} &= w\left(\theta a - p_r + bp_o + \lambda s\right)\\ &\quad + p_o\left((1-\theta)a - p_o + bp_r + (1-\lambda)s\right) - \frac{1}{2}s^2 - F,\\ \pi_r^{DB} &= \left(p_r - w\right)\left(\theta a - p_r + bp_o + \lambda s\right).\end{aligned} \tag{A.8}$$

The optimal online selling price to maximize π_r^{DB} can be given by

$$p_r = \frac{1}{2}\left(w + a\theta + s\lambda + bp_o\right). \tag{A.9}$$

Substituting (A.9) into π_s^{DB} and differentiating π_s^{DB} with respect to w and p_o, we have

$$\begin{aligned}\frac{\partial \pi_s^{DB}}{\partial p_o} &= a + s + bw - a\theta + \frac{ab\theta}{2} - s\lambda + \frac{bs\lambda}{2} + \left(-2 + b^2\right)p_o,\\ \frac{\partial \pi_s^{DB}}{\partial w} &= \frac{1}{2}\left(-2w + a\theta + s\lambda + 2bp_o\right).\end{aligned} \tag{A.10}$$

The Hessian Matrix of π_s^{DB} in terms of w and p_o is as follows:

$$H_2 = \begin{vmatrix} \dfrac{\partial^2 \pi_s^{DB}}{\partial p_o^{\,2}} & \dfrac{\partial^2 \pi_s^{DB}}{\partial p_o \partial w} \\ \dfrac{\partial^2 \pi_s^{DB}}{\partial w\, \partial p_o} & \dfrac{\partial^2 \pi_s^{DB}}{\partial w^2} \end{vmatrix} = \begin{vmatrix} b^2 - 2 & b \\ b & -1 \end{vmatrix}. \tag{A.11}$$

Through calculating, we get

$$\frac{\partial^2 \pi_s^{DB}}{\partial p_o^{\,2}} = b^2 - 2 < 0, \text{ and } \left|H_2\right| = 2\left(1 - b^2\right) > 0. \tag{A.12}$$

The supplier's profit function is a joint concave function with respect to w and p_o, so we have

$$w = \frac{a(b + \theta - b\theta) + s(b + \lambda - b\lambda)}{2 - 2b^2}, \tag{A.13}$$

$$p_o = \frac{a + s + a(-1 + b)\theta + (-1 + b)s\lambda}{2\left(1 - b^2\right)}. \tag{A.14}$$

Inserting (A.13) and (A.14) into π_s^{DB} and we calculate

$$\begin{aligned}\frac{\partial \pi_s^{DB}}{\partial s} &= \frac{a\left(2 + 2(-1 + b)\lambda + (-1 + b)\theta(2 + (-3 + b)\lambda)\right) + s\left(-2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2\left(4 + \lambda^2\right)\right)}{4\left(1 - b^2\right)},\\ \frac{\partial^2 \pi_s^{DB}}{\partial s^2} &= \frac{-2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2\left(4 + \lambda^2\right)}{4\left(1 - b^2\right)}.\end{aligned} \tag{A.15}$$

Only if $-2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2(4 + \lambda^2) < 0$, that is $(\partial^2 \pi_s^{DB}/\partial s^2) < 0$, we can get the optimal service effort level, which means λ cannot be too large. Otherwise, if it is too large $(\lambda > (2 - 2b + \sqrt{2}\sqrt{5 - 8b - 3b^2 + 8b^3 - 2b^4}/3 - 4b + b^2))$, that is $(\partial^2 \pi_s^{DB}/\partial s^2) < 0$, the supplier's profit will always increase as the price increases, which indicates that the supplier's profit will be positively infinite. Thereby, this study only analyzes the case when $0 < \lambda < (2 - 2b + \sqrt{2}\sqrt{5 - 8b - 3b^2 + 8b^3 - 2b^4}/3 - 4b + b^2)$ to obtain the optimal decisions and equilibrium results of the supplier and retailers.

All the optimal results are obtained as follows:

$$\begin{aligned} s^{DB*} &= -\frac{a(2 + 2(-1 + b)\lambda + (-1 + b)\theta(2 + (-3 + b)\lambda))}{-2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2\left(4 + \lambda^2\right)},\\ w^{DB*} &= \frac{a\left(-2\left(\theta + \lambda + \theta\lambda - \lambda^2\right) + b\left(-4 - \lambda^2 + \theta(4 + \lambda)\right)\right)}{-4 - 8\lambda - 8b(-1 + \lambda)\lambda + 6\lambda^2 + 2b^2\left(4 + \lambda^2\right)},\\ p_o^{DB*} &= \frac{a\left(-4 + \lambda^2 - \theta(-4 + 4b + \lambda)\right)}{-4 - 8\lambda - 8b(-1 + \lambda)\lambda + 6\lambda^2 + 2b^2\left(4 + \lambda^2\right)},\end{aligned}$$

$$
\begin{aligned}
p_r^{DB\,*} &= \frac{a\left(2b^2\theta - 3\left(\theta + \lambda + \theta\lambda - \lambda^2\right) + 2b\left(-2 - \lambda^2 + \theta(2+\lambda)\right)\right)}{-4 - 8\lambda - 8b(-1+\lambda)\lambda + 6\lambda^2 + 2b^2\left(4+\lambda^2\right)},\\
\pi_r^{DB\,*} &= \frac{a^2\left(\lambda + (-1+b)\lambda^2 + \theta\left(1 - 2b^2 + \lambda - b\lambda\right)\right)^2}{4\left(2 + 4\lambda + 4b(-1+\lambda)\lambda - 3\lambda^2 - b^2\left(4+\lambda^2\right)\right)^2},\\
\pi_s^{DB\,*} &= \frac{a^2\left(-4 + \left(-5 + 8b - 2b^2\right)\theta^2 + \lambda^2 - 2\theta(-4 + 4b + \lambda)\right)}{4\left(-2 - 4\lambda - 4b(-1+\lambda)\lambda + 3\lambda^2 + b^2\left(4+\lambda^2\right)\right)} - F.
\end{aligned}
\tag{A.16}
$$

□

Proof of Proposition 1. We then analyse how does the webrooming effect coefficient impacts the optimal decisions in DB scenario. The first-order derivation of the equilibrium outcomes with respect to λ are shown as

$$
\begin{aligned}
\frac{\partial s^{DB\,*}}{\partial\lambda} &= \frac{\left(a(-1+b)\left(\begin{array}{c}\theta\left(-14 + 12\lambda - 9\lambda^2\right) + b^3\theta\left(-4+\lambda^2\right) + 6\left(2 - 2\lambda + \lambda^2\right)\\ +b^2\left(\theta\left(12 + 4\lambda - 7\lambda^2\right) + 2\left(-4+\lambda^2\right)\right) + b\left(4(1-2\lambda)\lambda + \theta\left(10 - 16\lambda + 15\lambda^2\right)\right)\end{array}\right)\right)}{\left(2 + 4\lambda + 4b(-1+\lambda)\lambda - 3\lambda^2 - b^2\left(4+\lambda^2\right)\right)^2} < 0,\\
\frac{\partial w^{DB\,*}}{\partial\lambda} &= \frac{-\left(a\left(\begin{array}{c}b^3\theta\left(-4 + 8\lambda + \lambda^2\right) + 2\left(-2 + 4\lambda + \lambda^2 + \theta\left(2 - 6\lambda - 3\lambda^2\right)\right)\\ +2b^2\left(-4 + 8\lambda + \lambda^2 - 3\theta\left(-4 + 6\lambda + \lambda^2\right)\right) + b\left(-4\left(-4 + 7\lambda + \lambda^2\right) + \theta\left(-22 + 40\lambda + 11\lambda^2\right)\right)\end{array}\right)\right)}{\left(2\left(2 + 4\lambda + 4b(-1+\lambda)\lambda - 3\lambda^2 - b^2\left(4+\lambda^2\right)\right)^2\right)},\\
\theta_1 &= -\frac{2\left(-2 + 8b - 4b^2 + 4\lambda - 14b\lambda + 8b^2\lambda + \lambda^2 - 2b\lambda^2 + b^2\lambda^2\right)}{4 - 22b + 24b^2 - 4b^3 - 12\lambda + 40b\lambda - 36b^2\lambda + 8b^3\lambda - 6\lambda^2 + 11b\lambda^2 - 6b^2\lambda^2 + b^3\lambda^2}.
\end{aligned}
\tag{A.17}
$$

If and only if $\theta > \theta_1$, $(\partial w^{DB\,*}/\partial\lambda) > 0$; otherwise, $(\partial w^{DB\,*}/\partial\lambda) < 0$.

$$
\begin{aligned}
\frac{\partial p_o^{DB\,*}}{\partial\lambda} &= \frac{\left(a\left(\begin{array}{c}8b^3\theta\lambda + 3\theta\left(6 - 8\lambda + \lambda^2\right) - 4\left(4 - 5\lambda + \lambda^2\right) + b^2\left(16\lambda + \theta\left(12 - 40\lambda + \lambda^2\right)\right)\\ -4b\left(-4 + 8\lambda - \lambda^2 + \theta\left(8 - 14\lambda + \lambda^2\right)\right)\end{array}\right)\right)}{\left(2\left(2 + 4\lambda + 4b(-1+\lambda)\lambda - 3\lambda^2 - b^2\left(4+\lambda^2\right)\right)\right)^2} < 0,\\
\frac{\partial p_r^{DB\,*}}{\partial\lambda} &= \frac{-\left(a\left(\begin{array}{c}4b^4\theta\lambda + 2b^3(4 + \theta(-4+\lambda))\lambda - 8b\left(-2 + 4\lambda + \lambda^2\right) + 6b\theta\left(-4 + 8\lambda + 3\lambda^2\right)\\ +b^2\left(-4 + 8\lambda + 5\lambda^2 + \theta\left(20 - 26\lambda - 11\lambda^2\right)\right) + 3\left(-2 + 4\lambda + \lambda^2 + \theta\left(2 - 6\lambda - 3\lambda^2\right)\right)\end{array}\right)\right)}{\left(2\left(2 + 4\lambda + 4b(-1+\lambda)\lambda - 3\lambda^2 - b^2\left(4+\lambda^2\right)\right)^2\right)},\\
\theta_2 &= \frac{-8b^3\lambda + b^2\left(4 - 8\lambda - 5\lambda^2\right) - 3\left(-2 + 4\lambda + \lambda^2\right) + 8b\left(-2 + 4\lambda + \lambda^2\right)}{6 - 18\lambda + 4b^4\lambda + 2b^3(-4+\lambda)\lambda - 9\lambda^2 + b^2\left(20 - 26\lambda - 11\lambda^2\right) + 6b\left(-4 + 8\lambda + 3\lambda^2\right)}.
\end{aligned}
\tag{A.18}
$$

If $\theta > \theta_2$, $(\partial p_r^{DB\,*}/\partial\lambda) > 0$; otherwise, $\partial p_r^{DB\,*}/\partial\lambda < 0$.

$$\frac{\partial q_r^{DB\,*}}{\partial\lambda} = \frac{-\left(a\left(\begin{array}{c} -2+4\lambda+4b^4\theta\lambda+\lambda^2+\theta\left(2-6\lambda-3\lambda^2\right)+b^2\left(4-8\lambda+3\lambda^2+\theta\left(-4+10\lambda-5\lambda^2\right)\right) \\ +b^3\left(8\lambda+\theta\left(4-16\lambda+\lambda^2\right)\right)+b\left(-4\lambda(1+\lambda)+\theta\left(-2+8\lambda+7\lambda^2\right)\right)\end{array}\right)\right)}{\left(2\left(2+4\lambda+4b(-1+\lambda)\lambda-3\lambda^2-b^2\left(4+\lambda^2\right)\right)^2\right)},$$
$$\theta_3 = \frac{2-4\lambda-8b^3\lambda-\lambda^2+4b\lambda(1+\lambda)+b^2\left(-4+8\lambda-3\lambda^2\right)}{(-1+b)\left(-2+6\lambda+4b^3\lambda+3\lambda^2-2b\lambda(1+2\lambda)+b^2\left(4-12\lambda+\lambda^2\right)\right)}. \tag{A.19}$$

If and only if $\theta > \theta_3$, $(\partial q_r^{DB\,*}/\partial\lambda) < 0$; otherwise, $(\partial q_r^{DB\,*}/\partial\lambda) > 0$.

$$\frac{\partial q_o^{DB\,*}}{\partial\lambda} = \frac{-\left(\left(\begin{array}{c} a\left(4b^5\theta\lambda+8b^4(\theta+\lambda-3\theta\lambda)-3\theta\left(6-8\lambda+\lambda^2\right)+4\left(4-5\lambda+\lambda^2\right)+b^2\left(8(-2+\lambda)+\theta\left(8+8\lambda-5\lambda^2\right)\right)\right. \\ \left.+b^3\left(12-24\lambda+\lambda^2+\theta\left(0-28+38\lambda+\lambda^2\right)\right)+b\left(-14+28\lambda-5\lambda^2+\theta\left(30-50\lambda+7\lambda^2\right)\right)\right)\end{array}\right)\right)}{\left(2\left(-2+4b\lambda+\lambda^2+b^2\left(4+\lambda^2\right)\right)^2\right)} < 0,$$
$$\frac{\partial \pi_r^{DB\,*}}{\partial\lambda} = \frac{\left(a^2\left(\begin{array}{c} \lambda+(-1+b)\lambda^2+\theta\left(1-2b^2+\lambda-b\lambda\right)\right)\left(-2+4\lambda+4b^4\theta\lambda+\lambda^2+\theta\left(2-6\lambda-3\lambda^2\right)+b^2\left(4-8\lambda+3\lambda^2+\theta\left(-4+10\lambda-5\lambda^2\right)\right)\right. \\ \left.+b^3\left(8\lambda+\theta\left(4-16\lambda+\lambda^2\right)\right)+b\left(-4\lambda(1+\lambda)+\theta\left(-2+8\lambda+7\lambda^2\right)\right)\right)\end{array}\right)\right)}{\left(2\left(-2-4\lambda-4b(-1+\lambda)\lambda+3\lambda^2+b^2\left(4+\lambda^2\right)\right)^3\right)},$$
$$\theta_4 = \frac{2-4\lambda-8b^3\lambda-\lambda^2+4b\lambda(1+\lambda)+b^2\left(-4+8\lambda-3\lambda^2\right)}{(-1+b)\left(-2+6\lambda+4b^3\lambda+3\lambda^2-2b\lambda(1+2\lambda)+b^2\left(4-12\lambda+\lambda^2\right)\right)}. \tag{A.20}$$

If $\theta > \theta_4$, $(\partial \pi_r^{DB\,*}/\partial\lambda) > 0$, and when $\theta < \theta_4$, $(\partial \pi_r^{DB\,*}/\partial\lambda) < 0$.

$$\frac{\partial \pi_s^{DB\,*}}{\partial\lambda} = \frac{\left(a^2\left(\begin{array}{c} 2(-1+b)(2+(-3+b)\lambda)\left(4+\left(5-8b+2b^2\right)\theta^2-\lambda^2+2\theta(-4+4b+\lambda)\right) \\ +(-2\theta+2\lambda)\left(-2-4\lambda-4b(-1+\lambda)\lambda+3\lambda^2+b^2\left(4+\lambda^2\right)\right)\end{array}\right)\right)}{\left(4\left(2+4\lambda+4b(-1+\lambda)\lambda-3\lambda^2-b^2\left(4+\lambda^2\right)\right)^2\right)},$$
$$\theta_5 = \frac{4-4b-\lambda}{5-8b+2b^2}. \tag{A.21}$$

If $\theta > \theta_5$, $(\partial \pi_r^{DB\,*}/\partial\lambda) > 0$; otherwise, $(\partial \pi_r^{DB\,*}/\partial\lambda) < 0$.

To sum up, for all λ, there are $(\partial s^{DB\,*}/\partial\lambda) < 0$, $(\partial p_o^{DB\,*}/\partial\lambda) < 0$, and $(\partial q_o^{DB\,*}/\partial\lambda) < 0$, while $(\partial w^{DB\,*}/\partial\lambda)$, $(\partial p_r^{DB\,*}/\partial\lambda)$, $(\partial q_r^{DB\,*}/\partial\lambda)$, $\partial \pi_r^{DB\,*}/\partial\lambda$ and $(\partial \pi_s^{DB\,*}/\partial\lambda)$ depend on consumers' offline preference. □

Proof of Lemma 3. The profit functions of the retailer and the supplier are given by

$$\pi_r^{DA} = (p_r - w)(\theta a - p_r + bp_o + \lambda s),$$
$$\pi_s^{DA} = w(\theta a - p_r + bp_o + \lambda s) + p_o((1-\theta)a - p_o + bp_r + (1-\lambda)s) - \frac{1}{2}s^2 - F. \tag{A.22}$$

The supplier maximizes its profit over

$$\frac{\partial \pi_s^{DA}}{\partial s} = -s + w\lambda + (1-\lambda) p_o. \tag{A.23}$$

So we can get

$$s = w\lambda + (1-\lambda) p_o. \tag{A.24}$$

Substituting (A.24) into π_r^{DA} and taking the first-order partial derivatives of π_r^{DA} with respect to p_r, we obtain

$$\frac{\partial \pi_r^{DA}}{\partial p_r} = w + a\theta + w\lambda^2 + \left(b + \lambda - \lambda^2\right) p_o - 2p_r. \tag{A.25}$$

With the reason of $(\partial^2 \pi_r^{DA}/\partial {p_r}^2) = -2 < 0$, π_r^{DA} is strictly concave with respect to p_r. And we have

$$p_r = \frac{1}{2}\left(w + a\theta + w\lambda^2 + \left(b + \lambda - \lambda^2\right) p_o\right). \tag{A.26}$$

Substituting (A.26) into π_r^{DA} and taking the first-order partial derivatives of π_r^{DA} with respect to p_o and w, we obtain

$$\begin{aligned} \frac{\partial \pi_s^{DA}}{\partial p_o} &= \frac{1}{2}\left(2a + a(-2+b)\theta - w(-1+\lambda)\lambda + bw\left(2+\lambda^2\right) + 2\left(-1 + b^2 - 2\lambda - b(-1+\lambda)\lambda + \lambda^2\right) p_o\right), \\ \frac{\partial \pi_s^{DA}}{\partial w} &= \frac{1}{2}\left(-2w + a\theta + \left(-(-1+\lambda)\lambda + b\left(2+\lambda^2\right)\right) p_o\right). \end{aligned} \tag{A.27}$$

The Hessian Matrix of π_s^{DA} in terms of w and p_o is as follows:

$$\begin{aligned} |H_3| &= \begin{vmatrix} \dfrac{\partial^2 \pi_s^{DA}}{\partial {p_o}^2} & \dfrac{\partial^2 \pi_s^{DA}}{\partial p_o \partial \mathrm{w}} \\[2ex] \dfrac{\partial^2 \pi_s^{DA}}{\partial \mathrm{w} \partial p_o} & \dfrac{\partial^2 \pi_s^{DA}}{\partial \mathrm{w}^2} \end{vmatrix} \\ &= \begin{vmatrix} \left(-1 + b^2 - 2\lambda - b(-1+\lambda)\lambda + \lambda^2\right) & \dfrac{1}{2}\left(-(-1+\lambda)\lambda + b\left(2+\lambda^2\right)\right) \\[2ex] \dfrac{1}{2}\left(-(-1+\lambda)\lambda + b\left(2+\lambda^2\right)\right) & -1 \end{vmatrix}. \end{aligned} \tag{A.28}$$

With the assumption of $0 < b < 1$, $0 < \lambda < 1$, and $\lambda > (2 - 2b + \sqrt{2}\sqrt{5 - 8b - 3b^2 + 8b^3 - 2b^4}/3 - 4b + b^2)$, we have $(\partial^2 \pi_s^{DA}/\partial {p_o}^2) < 0$, $|H_3| > 0$.

We obtain all the optimal results:

$$\begin{aligned} w^{DA*} &= \frac{a\left(-b^2\theta\lambda^2 - 2\left(\theta + \lambda + \theta\lambda - \lambda^2\right) - 2b\left(2+\lambda^2\right) + b\theta\left(4 + \lambda + \lambda^2\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)}, \\ p_o^{DA*} &= -\frac{a\left(4 + \theta\left(-4 + \lambda - \lambda^2 + b\left(4 + \lambda^2\right)\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)}, \\ p_r^{DA*} &= \frac{a\left(b^2\theta\left(2+\lambda^2\right) - \left(\theta + \lambda + \theta\lambda - \lambda^2\right)\left(3 + \lambda^2\right) + b\left(-4 - 3\lambda^2 - \lambda^4 + \theta\left(4 + 2\lambda + \lambda^2 + \lambda^3\right)\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)}, \end{aligned}$$

$$
\begin{aligned}
s^{DA*} &= -\frac{a\left(2(1+(-1+b)\lambda)\left(2+\lambda^{2}\right)+\theta\left(-4+7\lambda+\lambda^{3}+b^{2}\lambda^{3}-2b\left(-2+4\lambda+\lambda^{3}\right)\right)\right)}{-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda\left(-4+4\lambda-\lambda^{2}+\lambda^{3}\right)+b^{2}\left(8+4\lambda^{2}+\lambda^{4}\right)},\\
\pi_r^{DA*} &= \frac{a^{2}\left(1+\lambda^{2}\right)^{2}\left(\lambda+(-1+b)\lambda^{2}+\theta\left(1-2b^{2}+\lambda-b\lambda\right)\right)^{2}}{\left(-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda\left(-4+4\lambda-\lambda^{2}+\lambda^{3}\right)+b^{2}\left(8+4\lambda^{2}+\lambda^{4}\right)\right)^{2}},\\
\pi_s^{DA*} &= -\frac{a^{2}\left(4+\theta^{2}\left(5+\lambda^{2}+b^{2}\left(2+\lambda^{2}\right)-2b\left(4+\lambda^{2}\right)\right)+2\theta\left(-4+\lambda-\lambda^{2}+b\left(4+\lambda^{2}\right)\right)\right)}{2\left(-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda\left(-4+4\lambda-\lambda^{2}+\lambda^{3}\right)+b^{2}\left(8+4\lambda^{2}+\lambda^{4}\right)\right)}-F.
\end{aligned}
\tag{A.29}
$$

□

Proof of Proposition 2. Similarly, we then analyse how does the webrooming effect coefficient impact the optimal decisions in DA scenario. The first-order derivation of the equilibrium outcomes with respect to λ are shown as

$$
\begin{aligned}
\frac{\partial s^{DA*}}{\partial\lambda} &= \frac{\left(a\left(\begin{array}{c} -(-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda(-4+4\lambda-\lambda^{2}+\lambda^{3})+b^{2}(8+4\lambda^{2}+\lambda^{4}))(-4+4\lambda-6\lambda^{2}+3b^{2}\theta\lambda^{2}+\theta(7+3\lambda^{2})) \\ +b(4+6\lambda^{2}-2\theta(4+3\lambda^{2}))+2(-4+5\lambda-3\lambda^{2}+2\lambda^{3}+2b^{2}\lambda(2+\lambda^{2})+b(4-8\lambda+3\lambda^{2}-4\lambda^{3})) \\ (2(1+(-1+b)\lambda)(2+\lambda^{2})+\theta(-4+7\lambda+\lambda^{3}+b^{2}\lambda^{3}-2b(-2+4\lambda+\lambda^{3}))) \end{array}\right)\right)}{\left(-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda\left(-4+4\lambda-\lambda^{2}+\lambda^{3}\right)+b^{2}\left(8+4\lambda^{2}+\lambda^{4}\right)\right)^{2}},\\
\breve{\theta}_1 &= \frac{-\left(\left(\begin{array}{c} 2(-24+28\lambda-26\lambda^{2}-3\lambda^{4}+2\lambda^{5}-\lambda^{6}+b^{2}(16-6\lambda^{4}+2\lambda^{5}-3\lambda^{6}) \\ +b^{3}(-16-16\lambda^{2}+2\lambda^{4}+\lambda^{6})+b(24-32\lambda+42\lambda^{2}+7\lambda^{4}-4\lambda^{5}+3\lambda^{6}) \end{array}\right)\right)}{\left(\begin{array}{c} 60-40\lambda+71\lambda^{2}-28\lambda^{3}+16\lambda^{4}+\lambda^{6}+b^{4}\lambda^{2}(-24-4\lambda^{2}+\lambda^{4})-4b^{3}(-16-8\lambda-4\lambda^{2}+2\lambda^{4}+\lambda^{6}) \\ -4b(24-26\lambda+42\lambda^{2}-15\lambda^{3}+12\lambda^{4}+\lambda^{6})+2b^{2}(-12-48\lambda+52\lambda^{2}-16\lambda^{3}+22\lambda^{4}+3\lambda^{6})) \end{array}\right)}.
\end{aligned}
\tag{A.30}
$$

When $\theta > \breve{\theta}_1$, there exists $(\partial s^{DB*}/\partial\lambda) >$; otherwise, $(\partial s^{DB*}/\partial\lambda) < 0$.

$$
\begin{aligned}
\frac{\partial w^{DA*}}{\partial\lambda} &= \frac{\left(a\left(\begin{array}{c} 2(b^{2}\theta\lambda^{2}+2(\theta+\lambda+\theta\lambda-\lambda^{2})+2b(2+\lambda^{2})-b\theta(4+\lambda+\lambda^{2}))(-4+5\lambda-3\lambda^{2}+2\lambda^{3}+2b^{2}\lambda(2+\lambda^{2})+b(4-8\lambda+3\lambda^{2}-4\lambda^{3})) \\ +(-2-4(-1+b)\lambda+\theta(-2+b+2b\lambda-2b^{2}\lambda))(-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda(-4+4\lambda-\lambda^{2}+\lambda^{3})+b^{2}(8+4\lambda^{2}+\lambda^{4})) \end{array}\right)\right)}{\left(-4-8\lambda+5\lambda^{2}-2\lambda^{3}+\lambda^{4}-2b\lambda\left(-4+4\lambda-\lambda^{2}+\lambda^{3}\right)+b^{2}\left(8+4\lambda^{2}+\lambda^{4}\right)\right)^{2}},\\
\breve{\theta}_2 &= \frac{-((2(4-8\lambda-3\lambda^{2}-4\lambda^{3}+5\lambda^{4}-2\lambda^{5}+2b^{3}\lambda^{3}(4+\lambda^{2})+b^{2}(8-16\lambda+8\lambda^{2}-16\lambda^{3}+5\lambda^{4}-6\lambda^{5})+2b(-8+14\lambda-2\lambda^{2}+6\lambda^{3}-5\lambda^{4}+3\lambda^{5})))}{\left(\begin{array}{c} -8+20\lambda-2\lambda^{2}+6\lambda^{4}+2b^{4}\lambda(-8+\lambda^{4})-b(-44+80\lambda-7\lambda^{2}+20\lambda^{3}+13\lambda^{4}+2\lambda^{5}) \\ -b^{3}(-8+16\lambda+12\lambda^{2}+16\lambda^{3}+\lambda^{4}+6\lambda^{5})+b^{2}(-48+88\lambda+8\lambda^{2}+36\lambda^{3}+8\lambda^{4}+6\lambda^{5})) \end{array}\right)}.
\end{aligned}
\tag{A.31}
$$

When $\theta > \breve{\theta}_2$, $(\partial w^{DB*}/\partial\lambda) > 0$; otherwise, $(\partial w^{DB*}/\partial\lambda) < 0$.

$$\frac{\partial p_o^{DA\ *}}{\partial \lambda}=\frac{\left(a\left(\begin{array}{c}-\theta(1+2(-1+b)\lambda)\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\\ +2\left(-4+5\lambda-3\lambda^2+2\lambda^3+2b^2\lambda\left(2+\lambda^2\right)+b\left(4-8\lambda+3\lambda^2-4\lambda^3\right)\right)\left(4+\theta\left(-4+\lambda-\lambda^2+b\left(4+\lambda^2\right)\right)\right)\end{array}\right)\right)}{\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2}<0,$$

$$\frac{\partial p_r^{DA\ *}}{\partial \lambda}=\frac{\left(a\left(\begin{array}{c}\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\\ \left(\begin{array}{c}-3-6(-1+b)\lambda-3\lambda^2-4(-1+b)\lambda^3\\ +\theta\left(-3-2\lambda+2b^2\lambda-3\lambda^2+b\left(2+2\lambda+3\lambda^2\right)\right)\end{array}\right)\\ -2\left(-4+5\lambda-3\lambda^2+2\lambda^3+2b^2\lambda\left(2+\lambda^2\right)+b\left(4-8\lambda+3\lambda^2-4\lambda^3\right)\right)\\ \left(\begin{array}{c}b^2\theta\left(2+\lambda^2\right)\\ -\left(\theta+\lambda+\theta\lambda-\lambda^2\right)\left(3+\lambda^2\right)+b\left(-4-3\lambda^2-\lambda^4+\theta\left(4+2\lambda+\lambda^2+\lambda^3\right)\right)\end{array}\right)\end{array}\right)\right)}{\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2}>0,$$

$$\frac{\partial q_r^{DA\ *}}{\partial \lambda}=\frac{\left(a\left(\begin{array}{c}2\left(1+\lambda^2\right)\left(\lambda+(-1+b)\lambda^2+\theta\left(1-2b^2+\lambda-b\lambda\right)\right)\left(-4+5\lambda-3\lambda^2+2\lambda^3+2b^2\lambda\left(2+\lambda^2\right)+b\left(4-8\lambda+3\lambda^2-4\lambda^3\right)\right)\\ -(1+\theta-b\theta+2(-1+b)\lambda)\left(1+\lambda^2\right)\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\\ -2\lambda\left(\lambda+(-1+b)\lambda^2+\theta\left(1-2b^2+\lambda-b\lambda\right)\right)\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\end{array}\right)\right)}{\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2}>0.$$

$$\frac{\partial q_o^{DA\ *}}{\partial \lambda}=\frac{\left(a\left(\begin{array}{c}\left(2b^3\theta\lambda+2b^2(\theta+\lambda-3\theta\lambda)-2\lambda\left(1-3\lambda+2\lambda^2\right)+\theta\left(-2+2\lambda+3\lambda^2\right)+b\left(1+\theta-2\lambda-3\lambda^2-3\theta\lambda^2+4\lambda^3\right)\right)\\ \left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)-2\left(\begin{array}{c}-4+5\lambda-3\lambda^2+2\lambda^3+2b^2\lambda\left(2+\lambda^2\right)\\ +b\left(4-8\lambda+3\lambda^2-4\lambda^3\right)\end{array}\right)\\ \left(\begin{array}{c}-4-\lambda^2+2\lambda^3-\lambda^4+b^3\theta\left(2+\lambda^2\right)+\theta\left(4-2\lambda+\lambda^2+\lambda^3\right)+b^2\left(4+\lambda^2+\theta\left(-4+2\lambda-3\lambda^2\right)\right)\\ +b\left((-1+\lambda)^2\lambda(1+\lambda)+\theta\left(-3+\lambda-\lambda^3\right)\right)\end{array}\right)\end{array}\right)\right)}{\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2}<0,$$

$$\frac{\partial \pi_r^{DA\ *}}{\partial \lambda}=\frac{\left(\begin{array}{c}2a^2\left(1+\lambda^2\right)\left(\lambda+(-1+b)\lambda^2+\theta\left(1-2b^2+\lambda-b\lambda\right)\right)\\ \left(\begin{array}{c}\left(-2\left(1+\lambda^2\right)\lambda+(-1+b)\lambda^2+\theta\left(1-2b^2+\lambda-b\lambda\right)\right)\left(-4+5\lambda-3\lambda^2+2\lambda^3+2b^2\lambda\left(2+\lambda^2\right)\right.\\ \left.+b\left(4-8\lambda+3\lambda^2-4\lambda^3\right)\right)\end{array}\right)\\ +(1+\theta-b\theta+2(-1+b)\lambda)\left(1+\lambda^2\right)\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\\ +2\lambda\left(\lambda+(-1+b)\lambda^2+\theta\left(1-2b^2+\lambda-b\lambda\right)\right)\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\end{array}\right)}{\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^3}. \tag{A.32}$$

For all λ, there holds $(\partial\pi_r^{DA\ *}/\partial\lambda)>0$.

$$\frac{\partial \pi_s^{DA\ *}}{\partial \lambda}=\frac{\left(a^2\left(\begin{array}{c}-2\theta(1+(-1+b)(2+(-1+b)\theta)\lambda)\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)\\ +2\left(-4+5\lambda-3\lambda^2+2\lambda^3+2b^2\lambda\left(2+\lambda^2\right)+b\left(4-8\lambda+3\lambda^2-4\lambda^3\right)\right)\left(4+\theta^2\left(5+\lambda^2+b^2\left(2+\lambda^2\right)-2b\left(4+\lambda^2\right)\right)+2\theta\left(-4+\lambda-\lambda^2+b\left(4+\lambda^2\right)\right)\right)\end{array}\right)\right)}{\left(2\left(-4-8\lambda+5\lambda^2-2\lambda^3+\lambda^4-2b\lambda\left(-4+4\lambda-\lambda^2+\lambda^3\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2\right)}. \tag{A.33}$$

For all λ, there holds $(\partial\pi_s^{DA\,*}/\partial\lambda) < 0$. □

Proof of Proposition 3. We compare the equilibrium wholesale price, service level, and retail prices under direct selling case.

Case 3(i)

The difference between wholesale price under any two strategies of DN, DB, and DA is

$$
\begin{aligned}
w^{DB\,*} - w^{DN\,*} &= \frac{ab - ab\theta + a\theta}{2\left(-1 + b^2\right)} + \frac{a\left(-2\left(\theta + \lambda + \theta\lambda - \lambda^2\right) + b\left(-4 - \lambda^2 + \theta(4 + \lambda)\right)\right)}{-4 - 8\lambda - 8b(-1 + \lambda)\lambda + 6\lambda^2 + 2b^2\left(4 + \lambda^2\right)} > 0,\\
w^{DA\,*} - w^{DN\,*} &= \frac{ab - ab\theta + a\theta}{2\left(-1 + b^2\right)} + \frac{a\left(-b^2\theta\lambda^2 - 2\left(\theta + \lambda + \theta\lambda - \lambda^2\right) - 2b\left(2 + \lambda^2\right) + b\theta\left(4 + \lambda + \lambda^2\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)} > 0,\\
w^{DA\,*} - w^{DB\,*} &= \frac{a\left(-b^2\theta\lambda^2 - 2\left(\theta + \lambda + \theta\lambda - \lambda^2\right) - 2b\left(2 + \lambda^2\right) + b\theta\left(4 + \lambda + \lambda^2\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)}\\
&\quad + \frac{a\left(2\left(\theta + \lambda + \theta\lambda - \lambda^2\right) + b\left(4 + \lambda^2 - \theta(4 + \lambda)\right)\right)}{-4 - 8\lambda - 8b(-1 + \lambda)\lambda + 6\lambda^2 + 2b^2\left(4 + \lambda^2\right)} < 0.
\end{aligned}
\tag{A.34}
$$

So we obtain $w^{DB\,*} > w^{DA\,*} > w^{DN\,*}$

Case 3 (iiMedia).

The difference between live streaming service level under DA and under DB is

$$
s^{DA\,*} - s^{DB\,*} = \frac{\left(2a\lambda\left(1 - 2b^2 - 2(-1 + \lambda)\lambda + 2b(-1 + \lambda)\lambda\right)\left(\lambda + (-1 + b)\lambda^2 + \theta(1 + \lambda - b(2b + \lambda))\right)\right)}{\left(\left(-2 + 4b^2 + 4(-1 + b)\lambda + (-3 + b)(-1 + b)\lambda^2\right)\left(-2b(-1 + \lambda)\lambda\left(4 + \lambda^2\right) + (-2 + \lambda)\left(2 + 5\lambda + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)\right)\right)} > 0.
\tag{A.35}
$$

So we have $s^{DA\,*} > s^{DB\,*}$

Case 3(iii)

We also compare the optimal online retail price between any two strategies in Direct Selling Case.

$$
\begin{aligned}
p_o^{DB\,*} - p_o^{DN\,*} &= \frac{a(1 + (-1 + b)\lambda)(2 + 2(-1 + b)\lambda + (-1 + b)\theta(2 + (-3 + b)\lambda))}{2\left(-1 + b^2\right)\left(-2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2\left(4 + \lambda^2\right)\right)} > 0,\\
p_o^{DA\,*} - p_o^{DN\,*} &= \frac{\left(a(1 + (-1 + b)\lambda)\left((1 + (-1 + b)\lambda)\left(4 + \lambda^2\right) + \theta\left(-4 + 6\lambda - \lambda^2 + \lambda^3 + b^2\lambda\left(2 + \lambda^2\right) + b\left(4 - 8\lambda + \lambda^2 - 2\lambda^3\right)\right)\right)\right)}{\left(2\left(-1 + b^2\right)\left(-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)\right)\right)} > 0,\\
p_o^{DB\,*} - p_o^{DA\,*} &= \frac{\left(a\lambda^3(1 + (-1 + b)\lambda)\left(\lambda + (-1 + b)\lambda^2 + \theta\left(1 - 2b^2 + \lambda - b\lambda\right)\right)\right)}{\left(2\left(-2 - 4\lambda - 4b(-1 + \lambda)\lambda + 3\lambda^2 + b^2\left(4 + \lambda^2\right)\right)\left(-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)\right)\right)} > 0.
\end{aligned}
\tag{A.36}
$$

By comparing, we can get $p_o^{DB\,*} > p_o^{DA\,*} > p_o^{DN\,*}$.

At the same time, we also compare the offline prices and get the following results.

$$
\begin{aligned}
p_r^{DB*} - p_r^{DN*} &= -\frac{a\left(2b(-1+\lambda) - 3\lambda + b^2\lambda\right)(2 + 2(-1+b)\lambda + (-1+b)\theta(2 + (-3+b)\lambda))}{4\left(-1+b^2\right)\left(-2 - 4\lambda - 4b(-1+\lambda)\lambda + 3\lambda^2 + b^2\left(4+\lambda^2\right)\right)} > 0,\\
p_r^{DA*} - p_r^{DN*} &= \frac{a\left(b^2\theta\left(2+\lambda^2\right) - \left(\theta + \lambda + \theta\lambda - \lambda^2\right)\left(3+\lambda^2\right) + b\left(-4 - 3\lambda^2 - \lambda^4 + \theta\left(4 + 2\lambda + \lambda^2 + \lambda^3\right)\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)} - \frac{a\left(2b(-1+\theta) - 3\theta + b^2\theta\right)}{4\left(-1+b^2\right)} > 0,\\
p_r^{DB*} - p_r^{DA*} &= \frac{-\left(\left(a\lambda^2\left(1 - 2b^2 + 2\lambda + 3b(-1+\lambda)\lambda - 3\lambda^2\right)\left(\lambda + (-1+b)\lambda^2 + \theta\left(1 - 2b^2 + \lambda - b\lambda\right)\right)\right)\right)}{\left(2\left(-2 - 4\lambda - 4b(-1+\lambda)\lambda + 3\lambda^2 + b^2\left(4+\lambda^2\right)\right)\left(-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)\right)\right)} < 0.
\end{aligned}
\tag{A.37}
$$

So we obtain $p_r^{DA*} > p_r^{DB*} > p_r^{DN*}$ □

Proof of Theorem 1. We compare the profit of the offline retailer and the supplier between strategies DB and DN, and the results are as follows, respectively.

$$
\begin{aligned}
\pi_r^{DB*} - \pi_r^{DN*} &= \frac{1}{16}a^2\left(-\theta^2 + \frac{4\left(\lambda + (-1+b)\lambda^2 + \theta\left(1 - 2b^2 + \lambda - b\lambda\right)\right)^2}{\left(2 + 4\lambda + 4b(-1+\lambda)\lambda - 3\lambda^2 - b^2\left(4+\lambda^2\right)\right)^2}\right) > 0,\\
\pi_s^{DB*} - \pi_s^{DN*} &= \frac{a^2(2 + 2(-1+b)\lambda + (-1+b)\theta(2 + (-3+b)\lambda))^2}{8\left(-1+b^2\right)\left(-2 - 4\lambda - 4b(-1+\lambda)\lambda + 3\lambda^2 + b^2\left(4+\lambda^2\right)\right)} > 0.
\end{aligned}
\tag{A.38}
$$

Thus, we obviously have $\pi_T^{DB*} - \pi_T^{DN*} > 0$

The difference between the offline retailer's optimal profits in strategy DA and strategy DN is

$$
\pi_r^{DA*} - \pi_r^{DN*} = -\frac{1}{16}a^2\theta^2 + \frac{a^2\left(1+\lambda^2\right)^2\left(\lambda + (-1+b)\lambda^2 + \theta(1 + \lambda - b(2b+\lambda))\right)^2}{\left(-2b(-1+\lambda)\lambda\left(4+\lambda^2\right) + (-2+\lambda)\left(2 + 5\lambda + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)\right)^2} > 0. \tag{A.39}
$$

And the difference between the supplier's optimal profits in strategy DA and strategy DN is

$$
\begin{aligned}
\pi_s^{DA*} - \pi_s^{DN*} = \frac{1}{8}a^2\Bigg(&\frac{2 + 4(-1+b)\theta + \left(3 - 4b + b^2\right)\theta^2}{-1+b^2}\\
&-\frac{4\left(4 + \theta^2\left(5 + \lambda^2 + b^2\left(2+\lambda^2\right) - 2b\left(4+\lambda^2\right)\right) + 2\theta\left(-4 + \lambda - \lambda^2 + b\left(4+\lambda^2\right)\right)\right)}{-4 - 8\lambda + 5\lambda^2 - 2\lambda^3 + \lambda^4 - 2b\lambda\left(-4 + 4\lambda - \lambda^2 + \lambda^3\right) + b^2\left(8 + 4\lambda^2 + \lambda^4\right)}\Bigg) > 0.
\end{aligned}
\tag{A.40}
$$

In consequence, there is $\pi_T^{DA*} - \pi_T^{DN*} > 0$.

Similarly, we also compare the equilibrium profit under DA and DB.

$$\pi_r^{DA\,*} - \pi_r^{DB\,*} = \frac{\left(a^2\lambda^2\left(-3+6b\lambda+2\lambda^2+b^2\left(6+\lambda^2\right)\right)\left(-8-\lambda^2+2\lambda^4+b^2\left(2+\lambda^2\right)\left(8+3\lambda^2\right)+2b\lambda\left(8+5\lambda^2\right)\right)\left(\lambda+b\lambda^2-\theta\left(-1+\lambda+b\left(2b+\lambda+\lambda^2\right)\right)\right)^2\right)}{\left(4\left(-2+4b\lambda+\lambda^2+b^2\left(4+\lambda^2\right)\right)^2\left(-4+\lambda^2+2b\lambda\left(4+\lambda^2\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2\right)}. \tag{A.41}$$

For all λ, $\pi_r^{DA\,*} > \pi_r^{DB\,*}$.

$$\pi_s^{DA\,*} - \pi_s^{DB\,*} = -\frac{a^2\lambda^2\left(\lambda(1+b\lambda)-\theta\left(-1+2b^2+\lambda+b\lambda(1+\lambda)\right)\right)^2}{4\left(-2+4b\lambda+\lambda^2+b^2\left(4+\lambda^2\right)\right)\left(-4+\lambda^2+2b\lambda\left(4+\lambda^2\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)} < 0,$$
$$\pi_T^{DB\,*} - \pi_T^{DB\,*} = \frac{\left(a^2\lambda^2\begin{pmatrix}16-7\lambda^2-9\lambda^4+4\lambda^6+2b^3\lambda\left(64+64\lambda^2+11\lambda^4\right)+2b\lambda\left(-32-6\lambda^2+15\lambda^4\right)+2b^4\left(32+38\lambda^2+12\lambda^4+\lambda^6\right)\\+b^2\left(-64+40\lambda^2+79\lambda^4+7\lambda^6\right)\end{pmatrix}\left(\lambda(1+b\lambda)-\theta\left(-1+2b^2+\lambda+b\lambda(1+\lambda)\right)\right)^2\right)}{\left(4\left(-2+4b\lambda+\lambda^2+b^2\left(4+\lambda^2\right)\right)^2\left(-4+\lambda^2+2b\lambda\left(4+\lambda^2\right)+b^2\left(8+4\lambda^2+\lambda^4\right)\right)^2\right)} > 0. \tag{A.42}$$

In general, there exists $\pi_r^{DA\,*} > \pi_r^{DB\,*} > \pi_r^{DN\,*}$, $\pi_s^{DB\,*} > \pi_s^{DA\,*} > \pi_s^{DN\,*}$ and $\pi_T^{DB\,*} > \pi_T^{DB\,*} > \pi_T^{DN\,*}$. □

Proof of Lemma 4. In the reselling case, the revenue function can be demonstrated as follows:

$$\begin{aligned}\pi_r^{RN} &= (p_r - w)(\theta a - p_r + b p_o),\\ \pi_o^{RN} &= (p_o - w)((1-\theta)a - p_o + b p_r),\\ \pi_s^{RN} &= w(\theta a - p_r + b p_o + (1-\theta)a - p_o + b p_r).\end{aligned} \tag{A.43}$$

We solve this problem by reverse order method. First of all, we differentiate π_r^{RN} with respect to p_r and π_o^{RN} with respect to p_o, respectively.

$$\begin{aligned}\frac{\partial \pi_r^{RN}}{\partial p_r} &= w + a\theta + b p_o - 2p_r,\\ \frac{\partial \pi_o^{RN}}{\partial p_o} &= w + a(1-\theta) - 2p_o + b p_r.\end{aligned} \tag{A.44}$$

Because we have $(\partial^2\pi_r^{RN}/\partial {p_r}^2) = -2 < 0$ and $(\partial^2\pi_o^{RN}/\partial {p_o}^2) = -2 < 0$, we can let the two above-mentioned derivatives be zero and yield

$$p_o = \frac{2a(1-\theta) + (2+b)w + ab\theta}{4-b^2}, \tag{A.45}$$

$$p_r = \frac{ab(1-\theta) + (2+b)w + 2a\theta}{4-b^2}. \tag{A.46}$$

Then substituting (A.42) and (A.43) into π_s^{RN} and taking the first-order partial derivative of π_s^{RN} with respect to w, we obtain

$$\frac{\partial \pi_s^{RN}}{\partial w} = -\frac{a + 4(-1+b)w}{-2+b}. \tag{A.47}$$

Finally, we calculate all the optimal results:

$$\begin{aligned}w^{RN*} &= \frac{a}{4(1-b)},\\ p_o^{RN\,*} &= -\frac{a\left(-10 + b(7-12\theta) + 8\theta + 4b^2\theta\right)}{4(-1+b)\left(-4+b^2\right)},\\ p_r^{RN\,*} &= \frac{a\left(2 + b(5-12\theta) + 4b^2(-1+\theta) + 8\theta\right)}{4(-1+b)\left(-4+b^2\right)},\\ \pi_r^{RN\,*} &= \frac{a^2(2-8\theta+b(-3+4\theta))^2}{16\left(-4+b^2\right)^2},\\ \pi_o^{RN\,*} &= \frac{a^2(6-8\theta+b(-1+4\theta))^2}{16\left(-4+b^2\right)^2},\\ \pi_s^{RN\,*} &= \frac{a^2}{8\left(2-3b+b^2\right)}.\end{aligned} \tag{A.48}$$

□

Proof of Lemma 5.

$$\begin{aligned}\pi_r^{RB} &= (p_r - w)(\theta a - p_r + b p_o + \lambda s),\\ \pi_o^{RB} &= (p_o - w)((1-\theta)a - p_o + b p_r + (1-\lambda)s) - \frac{1}{2}s^2,\\ \pi_s^{RB} &= w(\theta a - p_r + b p_o + \lambda s + (1-\theta)a - p_o + b p_r + (1-\lambda)s).\end{aligned} \tag{A.49}$$

Similar to the situation of no service, we first differentiate π_r^{RN} with respect to p_r and π_o^{RN} with respect to p_o, respectively.

$$\begin{aligned}\frac{\partial \pi_r^{RB}}{\partial p_r} &= w + a\theta + s\lambda + b p_o - 2p_r,\\ \frac{\partial \pi_s^{RB}}{\partial p_o} &= w + a(1-\theta) + s(1-\lambda) - 2p_o + b p_r.\end{aligned} \tag{A.50}$$

Next, we yield the following optimal solutions for given s and w:

$$p_o = -\frac{2a + 2s + 2w + bw - 2a\theta + ab\theta - 2s\lambda + bs\lambda}{-4 + b^2}, \tag{A.51}$$

$$p_r = -\frac{ab + bs + 2w + bw + 2a\theta - ab\theta + 2s\lambda - bs\lambda}{-4 + b^2}. \tag{A.52}$$

Substituting (A.48) and (A.49) into π_s^{RB} and taking the first-order partial derivative of π_s^{RB} with respect to w, we obtain

$$\frac{\partial \pi_s^{RB}}{\partial w} = -\frac{a + s + 4(-1 + b)w}{-2 + b}. \tag{A.53}$$

We have $(\partial^2 \pi_s^{RB}/\partial w^2) = (4(-1 + b)/2 - b) < 0$, so π_s^{RB} is strictly concave with respect to w.

$$w = \frac{a + s}{4(1 - b)}. \tag{A.54}$$

Substituting the above w into π_o^{RB}, we solve the optimal service effort (s). The process is as follows:

$$\frac{\partial \pi_o^{RB}}{\partial s} = \frac{\left(a(6 - 8\theta + b(-1 + 4\theta))(6 - 8\lambda + b(-1 + 4\lambda)) - s\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)\right)}{8\left(-4 + b^2\right)^2},$$

$$\frac{\partial^2 \pi_s^{RB}}{\partial s^2} = \frac{-92 - 8b^4 - 96\lambda + 64\lambda^2 - b^2\left(-65 + 8\lambda - 16\lambda^2\right) - 4b\left(3 - 16\lambda + 16\lambda^2\right)}{8\left(-4 + b^2\right)^2} < 0. \tag{A.55}$$

The following results is the optimal.

$$s^{RB*} = \frac{a(6 - 8\theta + b(-1 + 4\theta))(6 - 8\lambda + b(-1 + 4\lambda))}{92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)},$$

$$w^{RB*} = -\frac{a(-2 + b)\left(-16 + 4b^2 + 2b^3 + \theta(6 - 8\lambda) - 6\lambda + 8\lambda^2 + b\left(-8 + \lambda - 4\lambda^2 + \theta(-1 + 4\lambda)\right)\right)}{(-1 + b)\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)},$$

$$p_o^{RB*} = -\frac{a(-2 + b)\left(-40 + b^2(14 - 8\theta) + 8b^3\theta + \theta(38 - 8\lambda) - 6\lambda + 8\lambda^2 + b\left(8 + \lambda - 4\lambda^2 + \theta(-33 + 4\lambda)\right)\right)}{(-1 + b)\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)},$$

$$p_r^{RB*} = \frac{\left(a\left(b^3(10 - 24\theta) + 8b^4(-1 + \theta) - 8\left(2 + 3\lambda - 4\lambda^2 + \theta(5 + 4\lambda)\right) + b^2\left(36 - 3\lambda + 12\lambda^2 - \theta(13 + 12\lambda)\right) + b\left(-40 + 22\lambda - 40\lambda^2 + \theta(74 + 40\lambda)\right)\right)\right)}{\left((-1 + b)\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)\right)},$$

$$\pi_r^{RB*} = \frac{4a^2\left(-8 + b^2(2 - 8\theta) + b^3(-3 + 4\theta) + 6\lambda - 8\lambda^2 + \theta(26 + 8\lambda) - b\left(-12 + \lambda - 4\lambda^2 + \theta(15 + 4\lambda)\right)\right)^2}{\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)^2},$$

$$\pi_o^{RB*} = \frac{a^2(6 - 8\theta + b(-1 + 4\theta))^2}{2\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)},$$

$$\pi_s^{RB*} = \frac{2a^2(-2 + b)\left(-16 + 4b^2 + 2b^3 + \theta(6 - 8\lambda) - 6\lambda + 8\lambda^2 + b\left(-8 + \lambda - 4\lambda^2 + \theta(-1 + 4\lambda)\right)\right)^2}{(-1 + b)\left(92 + 8b^4 + 96\lambda - 64\lambda^2 + b^2\left(-65 + 8\lambda - 16\lambda^2\right) + 4b\left(3 - 16\lambda + 16\lambda^2\right)\right)^2}. \tag{A.56}$$

□

Proof of Proposition 4. The first-order derivation of the equilibrium outcomes in RB strategy with respect to λ are shown as

$$\frac{\partial s^{RB*}}{\partial \lambda} = \frac{\left(4a(-2+b)(6-8\theta+b(-1+4\theta))\left(164+8b^4-96\lambda+64\lambda^2-4b\left(3-16\lambda+16\lambda^2\right)+b^2\left(-63-8\lambda+16\lambda^2\right)\right)\right)}{\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^2}<0,$$

$$\frac{\partial w^{RB*}}{\partial \lambda} = \frac{-\left(\left(a(-2+b)(6-8\theta+b(-1+4\theta))\left(164+8b^4-96\lambda+64\lambda^2-4b\left(3-16\lambda+16\lambda^2\right)\right)+b^2\left(-63-8\lambda+16\lambda^2\right)\right)\right)}{\left((-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^2\right)\right)}<0,$$

$$\frac{\partial p_o^{RB*}}{\partial \lambda} = \frac{-\left(\left(\begin{array}{c} a(-2+b)(6-8\theta+b(-1+4\theta))548-608\lambda+64\lambda^2+8b^4(-1+8\lambda)-16b^3(-7+12\lambda) \\ +b\left(-460+832\lambda-64\lambda^2\right)+b^2\left(-95-136\lambda+16\lambda^2\right) \end{array}\right)\right)}{\left((-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^2\right)\right)}<0,$$

$$\frac{\partial p_r^{RB*}}{\partial \lambda} = \frac{\left(\begin{array}{c} a(6-8\theta+b(-1+4\theta)) \\ \left(\begin{array}{c} b^4(176-320\lambda)+8b^5(-5+8\lambda)+b^3\left(-67+280\lambda-48\lambda^2\right) \\ +16\left(-7+40\lambda+16\lambda^2\right)+8b^2\left(-75+100\lambda+32\lambda^2\right) \\ -4b\left(-185+376\lambda+112\lambda^2\right) \end{array}\right) \end{array}\right)}{\left((-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^2\right)}<0,$$

$$\frac{\partial \pi_r^{RB*}}{\partial \lambda} = \frac{\left(\begin{array}{c} 8a^2(6-8\theta+b(-1+4\theta)) \\ \left(\begin{array}{c} 220-b^2(1-4\lambda)^2-416\lambda-64\lambda^2-64b^3(-1+2\lambda)+16b^4(-1+2\lambda) \\ +4b\left(-61+112\lambda+16\lambda^2\right)\right)(-8+b^2(2-8\theta) \\ +b^3(-3+4\theta)+6\lambda-8\lambda^2+\theta(26+8\lambda)-b\left(-12+\lambda-4\lambda^2+\theta(15+4\lambda)\right) \end{array}\right) \end{array}\right)}{\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^3}. \tag{A.57}$$

It is nonmonotonic.

$$\frac{\partial \pi_o^{RB*}}{\partial \lambda} = \frac{4a^2(-2+b)(6-8\theta+b(-1+4\theta))^2(6-8\lambda+b(-1+4\lambda))}{\left(92+8b^4+96\lambda-64\lambda^2+b^2(-65+8\lambda-16\lambda^2)+4b(3-16\lambda+16\lambda^2)\right)^2}<0,$$

$$\frac{\partial \pi_s^{RB*}}{\partial \lambda} = \frac{\left(\begin{array}{c} 4a^2(-2+b)(6-8\theta+b(-1+4\theta))\left(\begin{array}{c} 164+8b^4-96\lambda+64\lambda^2-4b\left(3-16\lambda+16\lambda^2\right) \\ +b^2\left(-63-8\lambda+16\lambda^2\right) \end{array}\right) \\ \left(-16+4b^2+2b^3+\theta(6-8\lambda)-6\lambda+8\lambda^2+b\left(-8+\lambda-4\lambda^2+\theta(-1+4\lambda)\right)\right) \end{array}\right)}{\left((-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^3\right)}<0. \tag{A.58}$$

□

Proof of Lemma 6. The profit functions are as same as those Proof of Lemma 5. And we also use the backward induction to solve the problem.

The supplier maximizes its profit over s:

$$\frac{\partial \pi_o^{RA}}{\partial s} = -s-w+p_o. \tag{A.59}$$

We can have

$$s=(1-\lambda)(p_o-w). \tag{A.60}$$

Substituting the above s into π_o^{RA}, we differentiate π_r^{RA} with respect to p_r and π_o^{RA} with respect to p_o, respectively.

$$\frac{\partial \pi_r^{RA}}{\partial p_r} = w + a\theta + (-1+\lambda)\lambda(w - p_o) + bp_o - 2p_r,$$
$$\frac{\partial \pi_o^{RA}}{\partial p_o} = a - a\theta + 2w\lambda - w\lambda^2 + \left(-1 - 2\lambda + \lambda^2\right)p_o + bp_r. \tag{A.61}$$

Similarly, because there are $(\partial^2 \pi_r^{RA}/\partial {p_r}^2) = -2 < 0$ and $(\partial^2 \pi_o^{RA}/\partial {p_o}^2) = -1 - 2\lambda + \lambda^2 < 0$, we can obtain

$$p_o = -\frac{2a + bw - 2a\theta + ab\theta + 4w\lambda - bw\lambda - 2w\lambda^2 + bw\lambda^2}{-2 + b^2 - 4\lambda + b\lambda + 2\lambda^2 - b\lambda^2}, \tag{A.62}$$

$$p_r = -\frac{ab + w + a\theta - ab\theta + a\lambda + w\lambda + 2bw\lambda + a\theta\lambda - a\lambda^2 - bw\lambda^2}{-2 + b^2 - 4\lambda + b\lambda + 2\lambda^2 - b\lambda^2}. \tag{A.63}$$

Substituting A.59 and (A.60) into π_s^{RA}, we solve the optimal service effort (w). The process is as follows:

$$\frac{\partial \pi_s^{RA}}{\partial \mathrm{w}} = -\frac{a\left(2 + b + \theta(-1+\lambda) + \lambda - \lambda^2\right) + 2w\left(-3 + b + 2b^2 - 3\lambda + 3b\lambda + 2\lambda^2 - 2b\lambda^2\right)}{b^2 - b(-1+\lambda)\lambda + 2\left(-1 - 2\lambda + \lambda^2\right)},$$
$$\frac{\partial^2 \pi_s^{RA}}{\partial w^2} = -\frac{2\left(-3 + b + 2b^2 - 3\lambda + 3b\lambda + 2\lambda^2 - 2b\lambda^2\right)}{b^2 - b(-1+\lambda)\lambda + 2\left(-1 - 2\lambda + \lambda^2\right)} < 0. \tag{A.64}$$

All optimal results are represented as follows:

$$w^{RA*} = -\frac{a\left(2 + b + \theta(-1+\lambda) + \lambda - \lambda^2\right)}{2\left(-3 + b + 2b^2 - 3\lambda + 3b\lambda + 2\lambda^2 - 2b\lambda^2\right)},$$

$$p_o^{RA*} = \frac{\left(a\left(\begin{array}{c} -4b^3\theta + b^2\left(-7 - \lambda + \lambda^2 + \theta\left(6 - 6\lambda + 4\lambda^2\right)\right) + b\left(-2 - 9\lambda + 6\lambda^2 + 2\lambda^3 - \lambda^4 + \theta\left(9 + 20\lambda - 14\lambda^2 + \lambda^3\right)\right) \\ +2\left(6 + 10\lambda - 4\lambda^2 - 3\lambda^3 + \lambda^4 - \theta\left(6 + 8\lambda - 7\lambda^2 + \lambda^3\right)\right) \end{array}\right)\right)}{\left(2\left(-3 + b + 2b^2 - 3\lambda + 3b\lambda + 2\lambda^2 - 2b\lambda^2\right)\left(b^2 - b(-1+\lambda)\lambda + 2\left(-1 - 2\lambda + \lambda^2\right)\right)\right)},$$

$$p_r^{RA*} = \frac{\left(a\left(\begin{array}{c} 2 + 4b^3(-1+\theta) + 9\lambda - 11\lambda^3 + 4\lambda^4 + \theta\left(5 + 12\lambda + 3\lambda^2 - 4\lambda^3\right) - b^2\left(2 + 8\lambda - 7\lambda^2 + \theta\left(2 - 2\lambda + 4\lambda^2\right)\right) \\ +b\left(7 + 9\lambda - 8\lambda^2 + 7\lambda^3 - 3\lambda^4 + \theta\left(-8 - 16\lambda + 5\lambda^2 + 3\lambda^3\right)\right) \end{array}\right)\right)}{\left(2\left(-3 + b + 2b^2 - 3\lambda + 3b\lambda + 2\lambda^2 - 2b\lambda^2\right)\left(b^2 - b(-1+\lambda)\lambda + 2\left(-1 - 2\lambda + \lambda^2\right)\right)\right)},$$

$$s^{RA*} = \frac{a(1-\lambda)\left(-8 + b^2(1 - 4\theta) - 10\lambda + 6\lambda^2 + 2\theta\left(5 + 7\lambda - 4\lambda^2\right) + b\left(-4 + \theta + \lambda - 5\theta\lambda - \lambda^2 + 4\theta\lambda^2\right)\right)}{2\left(3 + 2b + 3\lambda - 2\lambda^2\right)\left(b^2 - b(-1+\lambda)\lambda + 2\left(-1 - 2\lambda + \lambda^2\right)\right)},$$

$$\pi_r^{RA*} = \frac{\left(a^2\left(2+b^2(-3+4\theta)+\lambda-2\lambda^2+5\lambda^3-2\lambda^4+b\left(-3+\theta-6\lambda+3\theta\lambda+5\lambda^2-4\theta\lambda^2\right)+\theta\left(-7-14\lambda+3\lambda^2+2\lambda^3\right)\right)^2\right)}{\left(4\left(3+2b+3\lambda-2\lambda^2\right)^2\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^2\right)},$$

$$\pi_o^{RA*} = \frac{-\left(\left(a^2\left(-1-2\lambda+\lambda^2\right)\left(-8+b^2(1-4\theta)-10\lambda+6\lambda^2+2\theta\left(5+7\lambda-4\lambda^2\right)+b\left(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2\right)\right)^2\right)\right.}{\left(8\left(3+2b+3\lambda-2\lambda^2\right)^2\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^2\right)},$$

$$\pi_s^{RA*} = \frac{a^2\left(2+b+\theta(-1+\lambda)+\lambda-\lambda^2\right)^2}{4\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)}, \tag{A.65}$$

$\square$

Proof of Proposition 5. We then analyse how does the webrooming effect coefficient impacts the optimal decisions in RA scenario. The first-order derivation of the equilibrium outcomes with respect to λ are shown as

$$\frac{\partial w^{RA*}}{\partial \lambda} = -\frac{a(-1+2\theta)\left(3+b-2\lambda+\lambda^2\right)}{2(-1+b)\left(3+2b+3\lambda-2\lambda^2\right)^2},$$
$$\widehat{\theta}_1 = \frac{a-\left(3+b-2\lambda+\lambda^2\right)}{2a}. \tag{A.66}$$

If and only if $\theta > \widehat{\theta}_1$, $(\partial w^{RA*}/)\partial\lambda > 0$. While if $0 < \theta < \widehat{\theta}_1$, $(\partial w^{RA*}/\partial\lambda) < 0$.

$$\frac{\partial p_o^{RA*}}{\partial \lambda} = \frac{-\left(\left(\begin{array}{c} a\left(\begin{array}{c} 2b^5(-1+4\theta)(-1+2\lambda)+b^4\left(-14+26\lambda-13\lambda^2+8\lambda^3+\theta\left(28-48\lambda+58\lambda^2-32\lambda^3\right)\right) \\ +b^3\left(8-36\lambda+130\lambda^2-70\lambda^3+12\lambda^4-4\lambda^5+\theta\left(35+52\lambda-269\lambda^2+162\lambda^3-52\lambda^4+16\lambda^5\right)\right) \end{array}\right) \\ +4\left(-24-36\lambda+53\lambda^2+50\lambda^3-64\lambda^4+18\lambda^5-\lambda^6+2\theta\left(15+27\lambda-32\lambda^2-41\lambda^3+48\lambda^4-14\lambda^5+\lambda^6\right)\right) \\ +b^2\left(94+24\lambda-212\lambda^2+198\lambda^3-135\lambda^4+40\lambda^5-\lambda^6+\theta\left(-121-4\lambda+235\lambda^2-362\lambda^3+306\lambda^4-88\lambda^5+2\lambda^6\right)\right) \\ -2b\left(-1-56\lambda+67\lambda^2+152\lambda^3-178\lambda^4+50\lambda^5-2\lambda^6+\theta\left(23+94\lambda-133\lambda^2-248\lambda^3+296\lambda^4-84\lambda^5+4\lambda^6\right)\right) \end{array}\right)\right)}{\left(2(-1+b)\lambda\left(3+2b+3\lambda-2\lambda^2\right)\lambda^2\lambda\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\lambda 2\right)},$$

$$\theta_2 = \frac{\left(\begin{array}{c} \left(b^5(-2+4\lambda)\right)+b^4\left(14-26\lambda+13\lambda^2-8\lambda^3\right)+2b^3\left(-4+18\lambda-65\lambda^2+35\lambda^3-6\lambda^4+2\lambda^5\right) \\ +b^2\left(-94-24\lambda+212\lambda^2-198\lambda^3+135\lambda^4-40\lambda^5+\lambda^6\right)+4\left(24+36\lambda-53\lambda^2-50\lambda^3+64\lambda^4-18\lambda^5+\lambda^6\right) \\ -2b\left(1+56\lambda-67\lambda^2-152\lambda^3+178\lambda^4-50\lambda^5+2\lambda^6\right) \end{array}\right)}{\left(\begin{array}{c} 8b^5(-1+2\lambda)+b^4\left(28-48\lambda+58\lambda^2-32\lambda^3\right)+b^3\left(35+52\lambda-269\lambda^2+162\lambda^3-52\lambda^4+16\lambda^5\right) \\ +8\left(15+27\lambda-32\lambda^2-41\lambda^3+48\lambda^4-14\lambda^5+\lambda^6\right)+b^2\left(-121-4\lambda+235\lambda^2-362\lambda^3+306\lambda^4-88\lambda^5+2\lambda^6\right) \\ -2b\left(23+94\lambda-133\lambda^2-248\lambda^3+296\lambda^4-84\lambda^5+4\lambda^6\right) \end{array}\right)}. \tag{A.67}$$

If $\theta > \widehat{\theta}_2$, $(\partial p_o^{RA*}/\partial\lambda) > 0$; otherwise, $(\partial p_o^{RA*}/\partial\lambda) < 0$.

$$\frac{\partial p_r^{RA*}}{\partial\lambda} = \frac{\left(a \left(\begin{array}{c} 4b^5(-1+4\theta)(-1+\lambda) + b^4\left(-16+26\lambda-15\lambda^2+8\lambda^3+\theta\left(4-52\lambda+66\lambda^2-32\lambda^3\right)\right) \\ +2b^3\begin{pmatrix} -20-24\lambda+64\lambda^2-31\lambda^3+5\lambda^4-2\lambda^5+\theta \\ \left(35+21\lambda-111\lambda^2+65\lambda^3-22\lambda^4+8\lambda^5\right) \end{pmatrix} \\ +b^2\begin{pmatrix} 28-68\lambda-96\lambda^2+142\lambda^3-111\lambda^4+32\lambda^5+\lambda^6+\theta \\ \left(-13+122\lambda+70\lambda^2-254\lambda^3+229\lambda^4-52\lambda^5-6\lambda^6\right) \end{pmatrix} \\ -2\begin{pmatrix} 9-4\lambda-66\lambda^2-0\lambda^3+43\lambda^4+8V^5-6\lambda^6+\theta \\ \left(-9+8\lambda+84\lambda^2+56\lambda^3-55\lambda^4-12\lambda^5+8\lambda^6\right) \end{pmatrix} \\ +b\left(-2\left(-23-54\lambda+66\lambda^2+100\lambda^3-105\lambda^4+10\lambda^5+6\lambda^6\right)+\theta\left(-71-156\lambda+220\lambda^2+332\lambda^3-341\lambda^4+28\lambda^5+20\lambda^6\right)\right) \end{array} \right)\right)}{\left(2(-1+b)\lambda\left(3+2b+3\lambda-2\lambda^2\right)\lambda^2\lambda\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\lambda^2\right)},$$

$$\widehat{\theta}_3 = \frac{\left(\begin{array}{c} 4b^5(-1+\lambda) + b^4\left(16-26\lambda+15\lambda^2-8\lambda^3\right) \\ +2b^3\left(20+24\lambda-64\lambda^2+31\lambda^3-5\lambda^4+2\lambda^5\right) \\ -b^2\begin{pmatrix} 28-68\lambda-96\lambda^2+142\lambda^3 \\ -111\lambda^4+32\lambda^5+\lambda^6 \end{pmatrix} \\ -2\left(-9+4\lambda+66\lambda^2+40\lambda^3-43\lambda^4-8\lambda^5+6\lambda^6\right) \\ +2b\left(-23-54\lambda+66\lambda^2+100\lambda^3-105\lambda^4+10\lambda^5+6\lambda^6\right) \end{array} \right)}{\left(\begin{array}{c} 16b^5(-1+\lambda) + b^4\left(4-52\lambda+66\lambda^2-32\lambda^3\right) \\ +2b^3\left(35+21\lambda-111\lambda^2+65\lambda^3-22\lambda^4+8\lambda^5\right) \\ +b^2\begin{pmatrix} -13+122\lambda+70\lambda^2-254\lambda^3+ \\ 229\lambda^4-52\lambda^5-6\lambda^6 \end{pmatrix} \\ -2\left(-9+8\lambda+84\lambda^2+56\lambda^3-55\lambda^4-12\lambda^5+8\lambda^6\right) \\ +b\left(-71-156\lambda+220\lambda^2+332\lambda^3-341\lambda^4+28\lambda^5+20\lambda^6\right) \end{array} \right)}. \tag{A.68}$$

If $\theta > \widehat{\theta}_3$, $(\partial p_r^{RA*}/\partial\lambda) > 0$; and if $\theta < \widehat{\theta}_3$, $(\partial p_r^{RA*}/\partial\lambda) < 0$.

$$\frac{\partial s^{RA*}}{\partial\lambda} = \frac{\left(a\left(3+b-2\lambda+\lambda^2\right) \left(\begin{array}{c} b^4(-2+8\theta) - 4b^3\left(-2+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2\right) + 4\left(-11-26\lambda-\lambda^2+20\lambda^3-6\lambda^4+\theta\left(13+34\lambda+5\lambda^2-28\lambda^3+8\lambda^4\right)\right) \\ -2b\left(15+2\lambda-17\lambda^2+24\lambda^3-8\lambda^4+2\theta\left(-3+10\lambda+9\lambda^2-24\lambda^3+8\lambda^4\right)\right) + b^2\left(16+40\lambda-26\lambda^2+4\lambda^3-2\lambda^4+\theta\left(-39-62\lambda+49\lambda^2-20\lambda^3+8\lambda^4\right)\right) \end{array} \right)\right)}{\left(2\left(3+2b+3\lambda-2\lambda^2\right)^2\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^2\right)}. \tag{A.69}$$

For all λ, $(\partial s^{RA*}/\partial\lambda) < 0$ holds.

$$\frac{\partial \pi_r^{RA*}}{\partial\lambda} = \frac{\left(\begin{array}{c} a^2\left(\begin{array}{c} 2+b^2\,(-3+4\theta)+\lambda-2\lambda^2+5\lambda^3-2\lambda^4+b\left(-3+\theta-6\lambda+3\theta\lambda+5\lambda^2-4\theta\lambda^2\right) \\ +\theta\left(-7-14\lambda+3\lambda^2+2\lambda^3\right) \end{array}\right) \\ \left(\begin{array}{c} b^4\,(3-4\lambda+2\lambda\,(-7+8\lambda)) \\ -2b^3\left(8-11\lambda+7\lambda^2-4\lambda^3+2\lambda\left(1+9\lambda-16\lambda^2+8\lambda^3\right)\right) \\ +b^2\left(\begin{array}{c} -52-24\lambda+119\lambda^2-58\lambda^3+ \\ 11\lambda^4-4\lambda^5+2\theta\left(29+\lambda-84\lambda^2+53\lambda^3-23\lambda^4+8\lambda^5\right) \end{array}\right) \\ +2\,(15+16\lambda-68\lambda^2-64\lambda^3+69\lambda^4-4\lambda^5-4\lambda^6 \\ +\theta\left(\begin{array}{c} -21-32\lambda+88\lambda^2+ \\ 104\lambda^3-107\lambda^4+12\lambda^5+4\lambda^6 \end{array}\right) \end{array}\right) \\ -b\left(\begin{array}{c} 4\left(3+18\lambda-24\lambda^3+27\lambda^4-8\lambda^5\right) \\ +\theta\left(-21-84\lambda+52\lambda^2+172\lambda^3-199\lambda^4+44\lambda^5+4\lambda^6\right) \end{array}\right) \end{array}\right)}{\left(2\lambda\left(3+2b+3\lambda-2\lambda^2\right)^3\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^3\right)}. \tag{A.70}$$

It is nonmonotonic.

$$\begin{aligned} \frac{\partial \pi_o^{RA*}}{\partial\lambda} &= \frac{-\left(\begin{array}{c} a^2\left(8+b^2\,(-1+4\theta)+10\lambda-6\lambda^2+2\theta\left(-5-7\lambda+4\lambda^2\right)-b\left(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2\right)\right) \\ \left(\begin{array}{c} 2b^5\,(-1+4\theta)\,(-1+\lambda)-2b^4\left(4+(-5+8\theta)\lambda+(1-6\theta)\lambda^2+(-1+4\theta)\lambda^3\right) \\ -b^3\left(24+16\lambda-38\lambda^2+6\lambda^3+10\lambda^4-2\lambda^5+\theta\left(-27-7\lambda+71\lambda^2+11\lambda^3-40\lambda^4+8\lambda^5\right)\right) \\ +4\left(-1-2\lambda+\lambda^2\right)\left(9+11\lambda-23\lambda^2-21\lambda^3+26\lambda^4-6\lambda^5+\theta\left(-9-13\lambda+25\lambda^2+33\lambda^3-36\lambda^4+8\lambda^5\right)\right) \\ -2b\left(11+27\lambda-12\lambda^2+22\lambda^3+29\lambda^4-85\lambda^5+48\lambda^6-8\lambda^7+2\theta\left(1+6\lambda-2\lambda^2-57\lambda^3-5\lambda^4+81\lambda^5-48\lambda^6+8\lambda^7\right)\right) \\ +b^2\left(-2\left(5+17\lambda+15^2+24\lambda^3-29\lambda^4+12\lambda^5-5\lambda^6+\lambda^7\right)+\theta\left(-5-5\lambda+54\lambda^2+76\lambda^3-105\lambda^4+85\lambda^5-44\lambda^6+8\lambda^7\right)\right) \end{array}\right) \end{array}\right)}{\left(4\lambda\left(3+2b+3\lambda-2\lambda^2\right)\lambda^3\,\lambda\left(b^2-b\,(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\lambda^3\right)} < 0, \\ \frac{\partial \pi_s^{RA*}}{\partial\lambda} &= \frac{\left(a^2\left(2+b+\theta\,(-1+\lambda)+\lambda-\lambda^2\right)\left(3+b-2\lambda+\lambda^2\right)\left(8+b^2\,(-1+4\theta)+10\lambda-6\lambda^2+2\theta\left(-5-7\lambda+4\lambda^2\right)-b\left(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2\right)\right)\right)}{\left(4\,(-1+b)\left(3+2b+3\lambda-2\lambda^2\right)^2\left(b^2-b\,(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^2\right)}. \end{aligned} \tag{A.71}$$

There always holds $(\partial\pi_s^{RA*}/\partial\lambda) < 0$.

Proof of Proposition 6. We compare the equilibrium wholesale price, live streaming service level, and retail prices under reselling case.

Case 6(i)

The difference between wholesale price under any two strategies of RN, RB, and RA is

$$
\begin{aligned}
w^{RB*} - w^{RN*} &= -\frac{a(6-8\theta+b(-1+4\theta))(6-8\lambda+b(-1+4\lambda))}{4(-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)} > 0,\\
w^{RA*} - w^{RN*} &= \frac{a(-1+2\theta)(-1+\lambda)}{4(1-b)\left(3+2b+3\lambda-2\lambda^2\right)}.
\end{aligned}
\tag{A.72}
$$

There is $\{\theta < (1/2), w^{RA*} - w^{RN*} > 0; \theta > (1/2), w^{RA*} - w^{RN*} < 0\}$.

$$
w^{RA*} - w^{RB*} = \frac{-\left(\left(\begin{array}{l} a(2+b)(4b^3(-1+2\theta)(-1+\lambda) - b\left(4+\theta+15\lambda-37\theta\lambda+5\lambda^2+4\theta\lambda^2-4\lambda^3\right) \\ +b^2(-9+12\lambda-4\theta(-5+8\lambda)) + 2\left(-2+5\lambda+7\lambda^2-4\lambda^3+\theta\left(-5-7\lambda+4\lambda^2\right)\right)\end{array}\right)\right)}{\left(2(-1+b)\left(3+2b+3\lambda-2\lambda^2\right)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b3-16\lambda+16\lambda^2\right)\right)\right)} < 0.
\tag{A.73}
$$

So if $\theta < (1/2)$, there is $w^{RB*} > w^{RA*} > w^{RN*}$; otherwise, $w^{RB*} > w^{RN*} > w^{RA*}$.

Case 6 (iiMedia)

$$
\begin{aligned}
s^{RA*} - s^{RB*} &= -\frac{a(6-8\theta+b(-1+4\theta))(6-8\lambda+b(-1+4\lambda))}{92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)}\\
&\quad - \frac{a(-1+\lambda)\left(-8+b^2(1-4\theta)-10\lambda+6\lambda^2+2\theta\left(5+7\lambda-4\lambda^2\right)+b\left(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2\right)\right)}{2\left(3+2b+3\lambda-2\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)} > 0.
\end{aligned}
\tag{A.74}
$$

So we get $s^{RA*} > s^{RB*}$.

Case 6(iii)

We then compare the optimal online retail price between any two strategies in the reselling case.

$$p_o^{RB*}-p_o^{RN*}=\frac{-\left(\left(a(6-8\theta+b(-1+4\theta))\left(-60+128\lambda-64\lambda^2+4b^3\lambda(-1+4\lambda)+b^2\left(-7+64\lambda-80\lambda^2\right)+4b\left(13-44\lambda+32\lambda^2\right)\right)\right)\right)}{\left(4(-1+b)\left(-4+b^2\right)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)\right)}>0,$$

$$p_o^{RA*}-p_o^{RN*}=\frac{1}{4}a\left(\frac{-10+b(7-12\theta)+8\theta+4b^2\theta}{(-1+b)\left(-4+b^2\right)}+\frac{2\left(\begin{array}{c}-4b^3\theta+b^2\left(-7-\lambda+\lambda^2+\theta\left(6-6\lambda+4\lambda^2\right)\right)\\+b\left(-2-9\lambda+6\lambda^2+2\lambda^3-\lambda^4+\theta\left(9+20\lambda-14\lambda^2+\lambda^3\right)\right)\\+2\left(6+10\lambda-4\lambda^2-3\lambda^3+\lambda^4-\theta\left(6+8\lambda-7\lambda^2+\lambda^3\right)\right)\end{array}\right)}{\left(\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\right)}\right)>0,$$

$$p_o^{RA*}-p_o^{RB*}=\frac{a(-2+b)\left(-40+b^2(14-8\theta)+8b^3\theta+\theta(38-8\lambda)-6\lambda+8\lambda^2+b\left(8+\lambda-4\lambda^2+\theta(-33+4\lambda)\right)\right)}{(-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)}$$
$$+\frac{\left(a\left(\begin{array}{c}-4b^3\theta+b^2\left(-7-\lambda+\lambda^2+\theta\left(6-6\lambda+4\lambda^2\right)\right)+b\left(-2-9\lambda+6\lambda^2+2\lambda^3-\lambda^4+\theta\left(9+20\lambda-14\lambda^2+\lambda^3\right)\right)\\+2\left(6+10\lambda-4\lambda^2-3\lambda^3+\lambda^4-\theta\left(6+8\lambda-7\lambda^2+\lambda^3\right)\right)\end{array}\right)\right)}{\left(2\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\right)}>0,$$

$$\tilde{\theta}_1=\frac{\left(\begin{array}{c}8b^6(-1+\lambda)\lambda-4b^5\left(3-18\lambda+20\lambda^2-4\lambda^3+2\lambda^4\right)+b^4\left(95-197\lambda+297\lambda^2-212\lambda^3+88\lambda^4\right)+b^2\left(-280+256\lambda+432\lambda^2-1110\lambda^3+446\lambda^4-8\lambda^5\right)\\+b^3\left(22+133\lambda-596\lambda^2+798\lambda^3-313\lambda^4+4\lambda^5\right)+8\left(18-4\lambda-18\lambda^2+21\lambda^3-15\lambda^4+4\lambda^5\right)-4b\left(2+83\lambda-44\lambda^2-100\lambda^3+39\lambda^4+4\lambda^5\right)\end{array}\right)}{\left(\begin{array}{c}32b^6(-1+\lambda)\lambda+b^3\left(137+58\lambda-926\lambda^2+1479\lambda^3-572\lambda^4\right)-8b^5\left(7-22\lambda+26\lambda^2-9\lambda^3+4\lambda^4\right)+8\left(24+10\lambda-31\lambda^2+\lambda^3+4\lambda^4\right)\\+4b^4\left(31-77\lambda+149\lambda^2-136\lambda^3+56\lambda^4\right)-4b\left(11+90\lambda-32\lambda^2-153\lambda^3+68\lambda^4\right)+2b^2\left(-178+190\lambda+313\lambda^2-833\lambda^3+316\lambda^4\right)\end{array}\right)}. \tag{A.75}$$

If $\theta<\tilde{\theta}_1$, $p_o^{RA*}-p_o^{RB*}>0$; otherwise, $p_o^{RA*}-p_o^{RB*}<0$. So if $\theta<\tilde{\theta}_1$, $p_o^{RA*}>p_o^{RB*}>p_o^{RN*}$ and when $\theta>\tilde{\theta}_1$, $p_o^{RB*}>p_o^{RA*}>p_o^{RN*}$.

Similarly, the comparison of the optimal offline price under the reselling case is presented as follows:

$$p_r^{RB*}-p_r^{RN*}=\frac{\left(a\left(b^3(10-24\theta)+8b^4(-1+\theta)-8\left(2+3\lambda-4\lambda^2+\theta(5+4\lambda)\right)+b^2\left(36-3\lambda+12\lambda^2-\theta(13+12\lambda)\right)+b\left(-40+22\lambda-40\lambda^2+\theta(74+40\lambda)\right)\right)\right)}{\left((-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)\right)}$$
$$-\frac{a\left(2+b(5-12\theta)+4b^2(-1+\theta)+8\theta\right)}{4(-1+b)\left(-4+b^2\right)}>0,$$

$$p_r^{RA*}-p_r^{RN*}=\frac{\left(a\left(\begin{array}{c}-4b^3\theta+b^2\left(-7-\lambda+\lambda^2+\theta\left(6-6\lambda+4\lambda^2\right)\right)+b\left(-2-9\lambda+6\lambda^2+2\lambda^3-\lambda^4+\theta\left(9+20\lambda-14\lambda^2+\lambda^3\right)\right)\\+2\left(6+10\lambda-4\lambda^2-3\lambda^3+\lambda^4-\theta\left(6+8\lambda-7\lambda^2+\lambda^3\right)\right)\end{array}\right)\right)}{\left(2\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\right)}$$
$$-\frac{a\left(2+b(5-12\theta)+4b^2(-1+\theta)+8\theta\right)}{4(-1+b)\left(-4+b^2\right)}>0,$$

$$p_r^{RA*}-p_r^{RB*}=\frac{\left(a\left(\begin{array}{c}-4b^3\theta+b^2\left(-7-\lambda+\lambda^2+\theta\left(6-6\lambda+4\lambda^2\right)\right)+b\left(-2-9\lambda+6\lambda^2+2\lambda^3-\lambda^4+\theta\left(9+20\lambda-14\lambda^2+\lambda^3\right)\right)\\+2\left(6+10\lambda-4\lambda^2-3\lambda^3+\lambda^4-\theta\left(6+8\lambda-7\lambda^2+\lambda^3\right)\right)\end{array}\right)\right)}{\left(2\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)\right)}$$
$$-\frac{\left(a\left(b^3(10-24\theta)+8b^4(-1+\theta)-8\left(2+3\lambda-4\lambda^2+\theta(5+4\lambda)\right)+b^2\left(36-3\lambda+12\lambda^2-\theta(13+12\lambda)\right)+b\left(-40+22\lambda-40\lambda^2+\theta(74+40\lambda)\right)\right)\right)}{\left((-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)\right)},$$

$$\tilde{\theta}_2=\frac{\left(\begin{array}{c}8b^6(-1+\lambda)^2-8b^5\left(6-16\lambda+12\lambda^2-3\lambda^3+\lambda^4\right)+4\left(2-39\lambda-48\lambda^2+21\lambda^3+4\lambda^4\right)+b^4\left(-26-214\lambda+449\lambda^2-276\lambda^3+88\lambda^4\right)\\+b^2\left(22+497\lambda-140\lambda^2-851\lambda^3+416\lambda^4\right)+b^3\left(135-179\lambda-594\lambda^2+823\lambda^3-317\lambda^4+4\lambda^5\right)-4b\left(15-12\lambda-118\lambda^2-34\lambda^3+33\lambda^4+4\lambda^5\right)\end{array}\right)}{\left(\begin{array}{c}32b^6(-1+\lambda)^2+4\left(-5-60\lambda-51\lambda^2+36\lambda^3\right)+b^3\left(188-430\lambda-805\lambda^2+1443\lambda^3-572\lambda^4\right)-8b^5\left(8-34\lambda+35\lambda^2-13\lambda^3+4\lambda^4\right)\\+4b^4\left(-24-54\lambda+207\lambda^2-164\lambda^3+56\lambda^4\right)-4b\left(27-18\lambda-190\lambda^2-63\lambda^3+68\lambda^4\right)+b^2\left(71+612\lambda-331\lambda^2-1248\lambda^3+640\lambda^4\right)\end{array}\right)},$$

$$\cdot\,\theta<\tilde{\theta}_2,\ p_r^{RA*}-p_r^{RB*}>0;\ \theta>\tilde{\theta}_1,\ p_r^{RA*}-p_r^{RB*}<0. \tag{A.76}$$

So we have $\{\theta<\widetilde{\theta}_2,\ p_o^{RA*}>p_o^{RB*}>p_o^{RN*};\ \theta>\widetilde{\theta}_2,\ p_o^{RB*}>p_o^{RA*}>p_o^{RN*}\}$. □

Proof of Theorem 2. The difference of each firm's optimal profit between preemptive strategy and no-service strategy is

$$\begin{aligned}\pi_r^{RB*}-\pi_r^{RN*}&=-\frac{a^2(2-8\theta+b(-3+4\theta))^2}{16(-4+b^2)^2}\\&+\frac{4a^2\left(-8+b^2(2-8\theta)+b^3(-3+4\theta)+6\lambda-8\lambda^2+\theta(26+8\lambda)-b\left(-12+\lambda-4\lambda^2+\theta(15+4\lambda)\right)\right)^2}{\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^2}.\end{aligned}\tag{A.77}$$

The function $(\pi_r^{RB*}-\pi_r^{RN*})$ is nonmonotonic; only if λ and θ are medium, there is $\pi_r^{RB*}-\pi_r^{RN*}>0$, otherwise, $\pi_r^{RB*}-\pi_r^{RN*}<0$.

$$\begin{aligned}\pi_o^{RB*}-\pi_o^{RN*}&=\frac{a^2(6-8\theta+b(-1+4\theta))^2(6-8\lambda+b(-1+4\lambda))^2}{16\left(-4+b^2\right)^2\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)}>0,\\ \pi_s^{RB*}-\pi_s^{RN*}&=-\frac{a^2}{8\left(2-3b+b^2\right)}+\frac{2a^2(-2+b)\left(-16+4b^2+2b^3+\theta(6-8\lambda)-6\lambda+8\lambda^2+b\left(-8+\lambda-4\lambda^2+\theta(-1+4\lambda)\right)\right)^2}{(-1+b)\left(92+8b^4+96\lambda-64\lambda^2+b^2\left(-65+8\lambda-16\lambda^2\right)+4b\left(3-16\lambda+16\lambda^2\right)\right)^2}>0.\end{aligned}\tag{A.78}$$

Similarly, the difference of each firm's optimal profit between reactive strategy and no-service strategy is

$$\begin{aligned}\pi_r^{RA*}-\pi_r^{RN*}&=\frac{1}{16}a^2\Bigg(-\frac{(2-8\theta+b(-3+4\theta))^2}{\left(-4+b^2\right)^2}\\&+\frac{\left(4\left(2+b^2(-3+4\theta)+\lambda-2\lambda^2+5\lambda^3-2\lambda^4+b\left(-3+\theta-6\lambda+3\theta\lambda+5\lambda^2-4\theta\lambda^2\right)+\theta\left(-7-14\lambda+3\lambda^2+2\lambda^3\right)\right)^2\right)}{\left(\left(3+2b+3\lambda-2\lambda^2\right)^2\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^2\right)}\Bigg)>0,\\ \pi_o^{RA*}-\pi_o^{RN*}&=\frac{1}{16}a^2\Bigg(-\frac{(6-8\theta+b(-1+4\theta))^2}{\left(-4+b^2\right)^2}\\&-\frac{\left(2\left(-1-2\lambda+\lambda^2\right)\left(-8+b^2(1-4\theta)-10\lambda+6\lambda^2+2\theta\left(5+7\lambda-4\lambda^2\right)+b\left(-4+\theta+\lambda-5\theta\lambda-\lambda^2+4\theta\lambda^2\right)\right)^2\right)}{\left(\left(3+2b+3\lambda-2\lambda^2\right)^2\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)^2\right)}\Bigg)>0,\\ \pi_s^{RA*}-\pi_s^{RN*}&=\frac{1}{8}a^2\Bigg(-\frac{1}{2-3b+b^2}\\&+\frac{2\left(2+b+\theta(-1+\lambda)+\lambda-\lambda^2\right)^2}{\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)}\Bigg)>0.\end{aligned}\tag{A.79}$$

And the comparison of each firm's optimal profit under strategy RA and RB is

$$\pi_r^{RA*}-\pi_r^{RB*}=\frac{1}{4}a^2\left(\frac{\left(2+b^2(-3+4\theta)-3\lambda-\lambda^2+b(-3+\theta-4\lambda+5\theta\lambda)+\theta\left(-7+2\lambda+\lambda^2\right)\right)^2}{(3+2b+\lambda)^2\left(-2+b^2+b\lambda\right)^2}-\frac{\left(4a^2\left(b^2(2-8\theta)+b^3(-3+4\theta)-2\left(4-3\lambda+\lambda^2\right)+2\theta\left(13-2\lambda+\lambda^2\right)-b\left(-12+\lambda-3\lambda^2+\theta\left(15+2\lambda+3\lambda^2\right)\right)\right)^2\right)}{\left(92+8b^4+24\lambda-4\lambda^2+b^2\left(-65+6\lambda-9\lambda^2\right)+4b\left(3-10\lambda+3\lambda^2\right)\right)^2}\right)>0,$$

$$\pi_o^{RA*}-\pi_o^{RB*}=\frac{1}{8}a^2\left(-\frac{4(6-8\theta+b(-1+4\theta))^2}{92+8b^4+24\lambda-4\lambda^2+b^2\left(-65+6\lambda-9\lambda^2\right)+4b\left(3-10\lambda+3\lambda^2\right)}+\frac{\left(b^2(-1+4\theta)+2(4+\lambda-\theta(5+\lambda))+b(4-\lambda+\theta(-1+3\lambda))\right)^2}{(3+2b+\lambda)^2\left(-2+b^2+b\lambda\right)^2}\right)>0,$$

$$\pi_s^{RA*}-\pi_s^{RB*}=\frac{a^2}{4(-1+b)}\left(\frac{(2+b+\lambda-\theta(1+\lambda))^2}{(3+2b+\lambda)\left(-2+b^2+b\lambda\right)}-\frac{8(-2+b)\left(4b^2+2b^3+2\left(-8-3\lambda+\lambda^2+\theta\left(3+2\lambda-\lambda^2\right)\right)+b\left(-8+\lambda-3\lambda^2+\theta\left(-1+2\lambda+3\lambda^2\right)\right)\right)^2}{\left(92+8b^4+24\lambda-4\lambda^2+b^2\left(-65+6\lambda-9\lambda^2\right)+4b\left(3-10\lambda+3\lambda^2\right)\right)^2}\right)>0. \tag{A.80}$$

So for any firm, the reactive service strategy is always the optimal one, which can bring them the most profit. □

Proof of Theorem 3.

$$\pi_s^{DB*}-\pi_s^{RA*}=\frac{a^2\left(-4+\left(-5+8b-2b^2\right)\theta^2+\lambda^2-2\theta(-4+4b+\lambda)\right)}{4\left(-2-4\lambda-4b(-1+\lambda)\lambda+3\lambda^2+b^2\left(4+\lambda^2\right)\right)}-F-\frac{a^2\left(2+b+\theta(-1+\lambda)+\lambda-\lambda^2\right)^2}{4\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)}. \tag{A.81}$$

When $\pi_s^{DB*}=\pi_s^{RA*}$,

$$\mathrm{F}=\overline{F}=\frac{1}{4}a^2\left(\frac{-4+\left(-5+8b-2b^2\right)\theta^2+\lambda^2-2\theta(-4+4b+\lambda)}{-2-4\lambda-4b(-1+\lambda)\lambda+3\lambda^2+b^2\left(4+\lambda^2\right)}-\frac{\left(2+b+\theta(-1+\lambda)+\lambda-\lambda^2\right)^2}{\left(-3+b+2b^2-3\lambda+3b\lambda+2\lambda^2-2b\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)}\right), \tag{A.82}$$

Thus, if and only if $\mathrm{F}<\overline{F}$ (that is $\pi_s^{DB*}>\pi_s^{RA*}$), the supplier will encroach the online channel by the Direct Selling Case.

In addition,

$$\frac{\partial\overline{F}}{\partial\theta}=\frac{1}{4}a^2\left(-\frac{2\left(-4+b(4-8\theta)+5\theta+2b^2\theta+\lambda\right)}{-2-4\lambda-4b(-1+\lambda)\lambda+3\lambda^2+b^2\left(4+\lambda^2\right)}-\frac{2(-1+\lambda)\left(2+b+\theta(-1+\lambda)+\lambda-\lambda^2\right)}{(-1+b)\left(3+2b+3\lambda-2\lambda^2\right)\left(b^2-b(-1+\lambda)\lambda+2\left(-1-2\lambda+\lambda^2\right)\right)}\right)<0. \tag{A.83}$$

□

Data Availability

The model verification data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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