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Research Article

The Limited Aperture Sparse Array for DOA Estimation of Coherent Signals: A Mutual-Coupling-Optimized Array and Coherent DOA Estimation Algorithm

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The paper investigates DOA estimation of coherent Signals with the limited aperture sparse array. Mutual coupling between the sensors of the array cannot be ignored in practical radar with a limited aperture of array sensors, which will result in a degradation in the performance of Direction of Arrival (DOA) estimation. This paper proposes a Mutual-coupling-optimized array (MCOA) with a limited aperture in this scenario to reduce the mutual coupling effect. Firstly, we prove the sparse uniform linear array (SULA) has the smallest mutual coupling leakage when the array aperture and the number of sensors is determined. Secondly, we modify the spacing of the array sensors in SULA to make sure that the spacing between all array sensors and the reference sensor are coprime aiming to estimate DOA without spatial aliasing. Thirdly, we give an expression for the array element spacing arrangement with reduced mutual coupling leakage. Finally, the coherent signals are well resolved by the Sparse Bayesian Learning (SBL) algorithm. Numerous simulations are conducted to validate the advantages of the proposed array compared to several sparse arrays for estimating coherent signals in the presence of mutual coupling.

1. Introduction

The problem of Direction of Arrival (DOA) estimation has attracted a lot of attention in the fields of radar, sonar, navigation, and astronomy [1–7], where the antenna arrays are utilized for collecting the spatial sampling of incident signals. Scholars propose many DOA estimation algorithms based on the uniform linear array (ULA), such as multiple signal classification (MUSIC) [5], estimation of signal parameters via rotational invariance techniques (ESPRIT) [6], propagator method (PM) [8], and parallel factor (PARAFAC) technique [9].

However, the above-given algorithm is predicated on the assumption that the incident signals are uncorrelated. The received signals are typically coherent and the rank of the covariance matrix is insufficient due to the impact of the transmission environment in actual applications. The aforementioned DOA estimation algorithm will be invalid at this time [10]. To tackle this problem, some decoherence algorithms are proposed to deal with coherent signals. The most representative method is the spatial smoothing (SS) method [11], which regards ULA as many subarrays with the same array flow type and then averages the covariance matrix of each subarray to obtain the full rank covariance matrix. Later, people proposed the Forward/backward spatial smoothing techniques [12], improved spatial smoothing techniques [13] on this basis of SS and make a series of improvement on these algorithms [14]. In [15], the authors were devoted to establishing more accurate conditions by studying the positive definiteness of smoothed target covariance matrix. There are also algorithms that reconstruct the covariance matrix of the received signal, such as SVD algorithms and the Toeplitz decoherence method [16]. These methods estimate coherent signals at the expense of array aperture, which reduces DOA estimation performance. The compressed sensing (CS) algorithms [17–19] can estimate DOA by exploiting the sparsity of the target in the spatial domain without taking into account the coherence of the signals. In [20], an iterative adaptive approach (IAA) is given for the beamforming design based on the sparsity. In [21], the Orthogonal Matching Pursuit (OMP) is used to recover the spare signal with high probability, but the accuracy of OMP is lower than that of MUSIC. In [22], both the Sparse Bayesian Learning (SBL) and the relevance vector machine (RVM) are proposed. The weakness of CS algorithms is that they are more complex than the previously described DOA estimating techniques. The traditional DOA estimation algorithms are generally considered based on ULA, but sparse linear arrays are seldom utilized.

Recently, sparse arrays such as Nested arrays (NA) [23] and coprime arrays (CA) [24] have attracted wide attention because such sparse arrays can achieve $O(M^2)$ degrees of freedoms (DOFs) with only M antenna sensors. Though the DOA estimation performance of NA is better than that of CA, the mutual coupling leakage of NA is much greater than that of CA due to the influence of the dense ULA subarray, which reduces performance in the presence of mutual coupling. Despite the array positions of CA and NA can be expressed in closed-form, their continuous degrees of freedom are not the greatest. In comparison to CA and NA, the minimum redundant array (MRA) [25] has the most continuous degrees of freedom, allowing it to use more virtual arrays. However, MRA lacks a closed-form expression for the locations of its sensors, and its array design requires a significant amount of complicated calculations.

The aperture of the array is usually limited in most applications, and the number of array elements is fixed. Because the spacing between the array elements is relatively close, the mutual coupling effect cannot be ignored. ULA and traditional sparse arrays will fail to estimate the DOA of coherent signals and the research of DOA estimation in this situation is relatively few. Though there are some methods [26–28] proposed to mitigate the mutual coupling effects by utilizing a complex mutual coupling model, these methods estimate mutual coupling coefficients at the cost of increased complexity and decreased degree of freedom (DOFs). Therefore, it is a good choice to consider how to reduce the mutual coupling effect when designing the array. Under the restrictions of a set array aperture and a number of array sensors, this paper determines the array design approach with the least mutual coupling leakage, and we further propose a Mutual-coupling-Optimized Array (MCOA) based on the theory of estimating DOA without spatial aliasing [29, 30]. To estimate the DOA of coherent signals, we use the sparse Bayesian learning-based (SBL) compressed sensing algorithm. In particular, we summarize our main contributions as follows:

(1) We propose a mutual-coupling-optimized array under the restriction of a fixed number of sensors and fixed array aperture. Then, we prove that the mutual coupling leakage of the suggested array is smaller than that of conventional sparse arrays and that it can estimate DOA without spatial aliasing.

(2) We employ the SBL algorithm to estimate coherent DOA for the proposed array and compare the SBL algorithm with other algorithms to demonstrate that the estimation performance of the SBL algorithm is better than other algorithms including OMP and IAA.

The remainder of this paper is given as follows: we provide the mathematical model of the sparse array and the definition of mutual coupling matrix in Section 2. In Section 3, we present how to design the mutual coupling optimized array with a limited aperture. Section 4 introduces the specific steps of the sparse Bayesian learning algorithm. Section 5 analyses the CRB of the array in this context. Section 6 verifies the theoretical performance of the proposed array through simulation analysis while Section 7 concludes this paper.

Notations: scalars, vectors, matrices, and sets are denoted by lowercase letters a, lowercase letters in boldface $\mathbf{a}$, uppercase letters in boldface $\mathbf{A}$, and letters in blackboard boldface $\mathbb{A}$, respectively. $\mathbf{A}^T$, $\mathbf{A}^*$, and $\mathbf{A}^H$ are the transpose, complex conjugate, and complex conjugate transpose of $\mathbf{A}$. $\mathrm{Tr}[\cdot]$ denotes the trace operator for a matrix. $\|\cdot\|_F$ represents the Frobenius norm and $\mathrm{diag}(\cdot)$ represents the matrix formed by the diagonal elements of the matrix. $[\mathbf{A}]_{i,j}$ denotes the (i, j) entry of $\mathbf{A}$. $\gcd(n_1, n_2, \cdots, n_M)$ denotes the greatest common divisor of the elements.

2. Mathematical Model

Consider a sparse array consisting of M sensors as shown in Figure 1, and the position of the ith sensor is denoted by $z_i d$ with $d = \lambda/2$, z_i represents the distance of the i- th sensor relative to the reference sensor and λ stands for the wavelength. Suppose that there are K far-field narrowband coherent signals from different directions $\theta = [\theta_1, \theta_2, \cdots, \theta_K]$ $\sqrt{b^2 - 4ac}$ with powers $\mathbf{p} = [\sigma_1^2, \sigma_2^2, \cdots, \sigma_K^2]$ impinge on this sparse array. The received signal of the array can be expressed as follows [2]:

$$\mathbf{X}_0 = \mathbf{AS} + \mathbf{N}, \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)]$ represents the direction matrix and $\mathbf{a}(\theta_i) = [1, e^{j2\pi z_2 d \sin\theta_i/\lambda}, \cdots, e^{j2\pi z_M d \sin\theta_i/\lambda}]^T$ denotes the direction vector of the ith signal. λ is the wavelength of the signal. $\mathbf{S} = [\alpha_1, \alpha_2, \cdots, \alpha_K]\mathbf{s}_0 \in \mathbb{C}^{M\times J}$ means the narrowband coherent signals with J snapshots, where $\mathbf{s}_0$ is the generate signals and α_i stands for the complex constant. $\mathbf{N} \in \mathbb{C}^{M\times J}$ represents the additive white Gaussian noise vector with noise variance σ^2.

There is coupling between the array elements. The received signal model is expressed as follows [28]:

$$\mathbf{X} = \mathbf{CAS} + \mathbf{N}, \tag{2}$$

where $\mathbf{C}$ is the mutual coupling matrix. The mutual coupling matrix can be modelled as a B-banded symmetric Toeplitz

Figure 1: Array model.

matrix according to the assumption in the following equation [28]:

$$[\mathbf{C}]_{p,q} = \begin{cases} 0, & |z_p - z_q| > B \\ c_{|z_p - z_q|} & |z_p - z_q| \le B, \end{cases} \tag{3}$$

where $1 > |c_1| > \cdots > c_B > 0$, $c_1 = c_0 e^{j\pi/3}$, $c_s = c_1 e^{-j(s-1)/8}/s$, $s \in (0, B]$. c_0 is the mutual coupling constant and B denotes the maximum spacing of sensor pairs with mutual coupling. In addition, the mutual coupling is evaluated by the coupling leakage, i.e.,

$$\gamma = \frac{\|\mathbf{C} - \mathrm{diag}(\mathbf{C})\|_F}{\|\mathbf{C}\|_F}. \tag{4}$$

3. Mutual-Coupling-Optimized Array with Limited Aperture

In this section, we first show that the mutual coupling leakage of the sparse and uniform linear array is the smallest under the condition of finite aperture and number of elements. Then, we discussed how to change the position of the array elements, so that the mutual coupling leakage is still small, and there is no spatial aliasing in DOA estimation.

3.1. A Minimum Mutual Coupling Leakage Array. Considering the M- element array with an array aperture of N, we propose an array which has no spatial aliasing in DOA estimation and has the smaller mutual coupling leakage than most sparse arrays.

Lemma 1. *For an M element array with an array aperture of N, the mutual coupling leakage is minimized if and only if the array elements are equally spaced.*

$$z_{i+1}d - z_i d = z_i d - z_{i-1} d = \frac{N}{(M-1)}, \quad i = 2, 3, \cdots, M-1. \tag{5}$$

Proof. According to equation (3), the mutual coupling matrix can be expressed as follows:

$$\mathbf{C} = \begin{bmatrix} c_0 & c_{z_2-z_1} & \cdots & c_{z_M-z_1} \\ c_{z_2-z_1} & c_0 & \cdots & c_{z_M-z_2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{z_M-z_1} & c_{z_M-z_2} & \cdots & c_0 \end{bmatrix}. \tag{6}$$

Then, the expression of the coupling leakage can be calculated as follows:

$$\gamma = \frac{\|\mathbf{C} - \mathrm{diag}(\mathbf{C})\|_F}{\|\mathbf{C}\|_F} = \frac{\sqrt{2\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}\left|c_{z_j-z_i}\right|^2}}{\sqrt{M_{|c_0|}^2 + 2\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}\left|c_{z_j-z_i}\right|^2}}, \tag{7}$$

where

$$\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}\left|c_{z_j-z_i}\right|^2 = |c_1|^2 \sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \frac{1}{\left(z_j d - z_i d\right)^2} = |c_1|^2 S, \tag{8}$$

where $S = \sum_{i=1}^{M-1}\sum_{j=i+1}^{M}(z_j d - z_i d)^{-2}$. The value of z_i, $i = 1, 2, \cdots, M$ corresponding to the minimum value of S is the position of each array sensor when the mutual coupling leakage is minimum.

Denote the $M-1$ array spacings as $x_1, x_2, \cdots, x_{M-1}$ respectively. Then,

$$\begin{aligned} S &= \sum_{i=1}^{M-1}\sum_{j=i+1}^{M}\left(z_j d - z_i d\right)^{-2} = \sum_{i=1}^{M-1}(z_{i+1}d - z_i d)^{-2} \\ &\quad + \sum_{i=1}^{M-2}(z_{i+2}d - z_i d)^{-2} + \cdots + \sum_{i=1}^{1}(z_M d - z_i d)^{-2} \\ &= \sum_{i=1}^{M-1} x_i^{-2} + \sum_{i=1}^{M-2}(x_i + x_{i+1})^{-2} + \cdots \\ &\quad + \sum_{i=1}^{1}\left(x_i + x_{i+1} + \cdots + x_{i+(M-2)}\right)^{-2}. \end{aligned} \tag{9}$$

Calculate the minimum value of S by Lagrange multiplier method.

$$g(x_1, \cdots, x_{M-1}, \mu) = S + \mu(x_1 + x_2 + \cdots + x_{M-1} - N). \tag{10}$$

Take the partial derivative of each variable in the function and set the result equal to zero.

$$\begin{cases} \dfrac{\partial g(x_1,\cdots,x_{M-1},\mu)}{\partial x_1} = 0 \\ \vdots \\ \dfrac{\partial g(x_1,\cdots,x_{M-1},\mu)}{\partial x_{M-1}} = 0 \\ \dfrac{\partial g(x_1,\cdots,x_{M-1},\mu)}{\partial \mu} = 0. \end{cases} \quad (11)$$

There is an extreme point in (10) when $x_1 = x_2 = \cdots = x_{M-1} = N/(M-1)$ and the cost function S has a minimum value at this time. Therefore, the value of z_i, $i = 1, 2, \cdots, M$ corresponding to the minimum value of S is the position of each array element in the array when the mutual coupling leakage is minimum. From the above-given discussion, it can be seen that the M elements array with an aperture of the array N reach the minimum mutual coupling leakage when (5) holds. The array designed in (5) is a sparse uniform line array (SULA) when $N > (M-1)\lambda$.

However, SULA will cause spatial aliasing during DOA estimation [29] because the spacing of the adjacent sensors are larger than half-wavelength. Next, we discuss how to fine-tune the position of SULA's array elements to solve the angular ambiguity problem and maintain the advantage of low mutual coupling leakage. □

3.2. B Mutual-Coupling-Optimized Array (MCOA). Suppose that the first sensor is located at the origin without loss of generality, then the position of the array can be expressed as follows:

$$\mathbb{Z} = \{0, z_2, \cdots, z_M\}d. \quad (12)$$

In order to facilitate the subsequent discussion, z_i needs to be adjusted to integer by choosing an appropriate d.

Theorem 1 (see [29]). *ie array manifold* $\mathbf{a}(\theta) = [1, e^{j2\pi z_2 \sin\theta/\lambda}, \cdots, e^{j2\pi z_M \sin\theta/\lambda}]^T$ *is invertible if and only if the sensor locations* z_i *(assumed integers) are coprime.*

$$\gcd(z_2, z_3, \cdots, z_M) = 1. \quad (13)$$

According to Theorem 1, we design the array structure as follows:

$$\mathbf{d} = [x_1, x_2, \ldots, x_{M-1}] = \begin{cases} \Big[a-1, \underbrace{a, \ldots, a}_{[(M-2)/2]}, a+1, a, \ldots, a\Big], & N = a(M-1), \\ \Big[a, \ldots, \underbrace{a+1, \ldots, a+1}_{b}, \ldots, a\Big], & N = a(M-1) + b,\ b < M-1, \end{cases} \quad (14)$$

where $\mathbf{d}_{1\times(M-1)}$ represent the spacing of adjacent sensors in array and a, b are integers.

In fact, the different arrangement order of elements in $\mathbf{d}$ will also cause the structural change of $\mathbb{Z}$, which leads to different mutual coupling leakage. Due to the large number of repetitions of elements in $\mathbf{d}$, there will be many repeated combinations of corresponding $\mathbb{Z}$. Obviously, when the larger distance between adjacent sensors in the center of the array, the mutual coupling between the middle sensors and the sensors on both sides can be effectively reduced so that the mutual coupling leakage of the whole array degrades significantly. This is the reason why we make $\mathbf{d}$ as (14). The relationship between $\mathbb{Z}$ and $\mathbf{d}$ is $z_{i+1} - z_i = d_i$, then the expression of the array position $\mathbb{Z}$ can be written as follows:

$$\mathbb{Z} = \begin{cases} \{0, a-1, 2a-1, \cdots, N\}d, & N = a(M-1), \\ \{0, a, \cdots, ka+1, \cdots, N\}d, & N = a(M-1) + b, b < N+1. \end{cases} \quad (15)$$

We will prove that the position in (15) satisfies Theorem 1 in the following part.

Proof. When $N = a(M-1)$, then

$$\begin{aligned} \gcd(2a-1, a-1) &= \gcd(a-1, 2a-1 \bmod a-1) \\ &= 1 \\ &= 1, \end{aligned} \quad (16)$$

where $a \bmod b$ represents the remainder of dividing a by b.

When $N = a(M-1) + b, b < N+1$, then.

$$\begin{aligned} \gcd(a, ka+1) &= \gcd(a, ka+1 \bmod a) \\ &= \gcd(a, 1) = 1. \end{aligned} \quad (17)$$

In summary $\gcd(z_2, z_3, \cdots, z_M) = 1$, that is, the designed array structure satisfies Theorem 1. Suppose that an antenna array with array aperture $N = 8\lambda$ and the number of sensors

$M = 8$ needs to be designed. According to (15), since N is not divisible by $M-1$, we can calculate that $\mathrm{N} = 2(M-1)+2$ with $a=2, b=2$ and write the expression of $\mathbf{d}$ and $\mathbb{Z}$.

$$\begin{aligned}\mathbf{d} &= [2,2,3,3,2,2,2],\\ \mathbb{Z} &= \{0,2,4,7,10,12,14,16\}d.\end{aligned} \tag{18}$$

Figure 2 shows the example of the above MCOA. The array structure is very similar to that of the SULA. There are five spacing between the sensors are 2 d and the remaining two are 3 d. We place the 3 d in the middle of the array to make the mutual coupling between the middle sensors and sensor on both sides decreasing, which effectively reduces the coupling leakage of the whole array. The array locations also satisfy Theorem 1.

$$\gcd(2,4,7,10,12,14,16) = 1. \tag{19}$$

□

4. Sparse Bayesian-Based Compressed Sensing Method

In the sparse signal representation framework [23, 24], the direction matrix $\mathbf{A}$ in (2) should be replaced by a transfer matrix $\mathbf{A}_g$, thus the signal model in (2) can be rewritten as follows:

$$\mathbf{X} = \mathbf{C}\mathbf{A}_g\mathbf{S}_g + \mathbf{N}, \tag{20}$$

where $\mathbf{S}_g = [\mathbf{s}_1,\cdots,\mathbf{s}_J] \in \mathbb{C}^{G\times J}$ represent the complex signal amplitudes containing G DOAs and J snapshots. The transfer matrix $\mathbf{A}_g = [\mathbf{a}_1,\cdots,\mathbf{a}_G] \in \mathbb{C}^{M\times G}$ consists of all hypothetical DOAs. The likelihood function of the received signal $\mathbf{X}$ can be represented as follows:

$$p\left(\mathbf{X}|\mathbf{S}_g;\sigma^2\right) = \frac{\exp\left(-1/\sigma^2\left\|\mathbf{X}-\mathbf{C}\mathbf{A}_g\mathbf{S}_g\right\|_F^2\right)}{\left(\pi\sigma^2\right)^{NL}}. \tag{21}$$

The SBL algorithm treats $\mathbf{s}$ as a zero mean complex Gaussian random vector with unknown diagonal covariance $\Gamma = \mathrm{diag}(\gamma_1,\cdots,\gamma_M) = \mathrm{diag}(\gamma)$. The prior model is given by the following equation:

$$p\left(\mathbf{S}_g\right) = \prod_{j=1}^{J} p\left(\mathbf{s}_j\right) = \prod_{j=1}^{J}\mathrm{AC}(0,\Gamma). \tag{22}$$

For Gaussian prior and likelihood, the evidence $p(\mathbf{X})$ is Gaussian and represented as follows:

$$p(\mathbf{X}) = \int p\left(\mathbf{S}_g\right)p\left(\mathbf{X}|\mathbf{S}_g\right)d\mathbf{S}_g = \prod_{j=1}^{J}\mathrm{AC}\left(\mathbf{x}_j;\mathbf{0},\Sigma_{\mathbf{x}}\right), \tag{23}$$

where $\Sigma_{\mathbf{x}} = \sigma^2\mathbf{I} + \mathbf{C}\mathbf{A}_g\Gamma\mathbf{A}_g^H\mathbf{C}^H$ and $\mathbf{I}$ stands for the identity matrix of order $M\times M$. The SBL algorithm is to estimate the diagonal entries of Γ by maximizing the evidence

$$(\widehat{\gamma}_1,\cdots,\widehat{\gamma}_M) = \arg\max_{\gamma}\left\{-\sum_{j=1}^{J}\mathbf{x}_j^H\Sigma_y^{-1}\mathbf{x}_j - L\log\left|\Sigma_{\mathbf{x}}\right|\right\}, \tag{24}$$

the derivative of (24) is

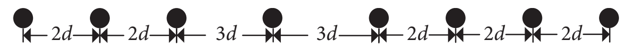

Figure 2: MCOA array with an array aperture of 8 wavelengths and a sensor count of 8.

$$\begin{aligned}&\frac{\partial\left(-\sum_{j=1}^{J}\mathbf{x}_j^H\sum_y^{-1}\mathbf{x}_j - L\log\left|\Sigma_{\mathbf{x}}\right|\right)}{\partial\gamma_m}\\ &= tr\left(\mathbf{X}^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m\mathbf{a}_m^H\sum_{\mathbf{x}}^{-1}\mathbf{X}\right) - L\mathbf{a}_m^H\Sigma_{\mathbf{x}}^{-1}\mathbf{a}_m\\ &= \left\|\mathbf{X}^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m\right\|_2^2 - L\mathbf{a}_m^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m\\ &= \left(\frac{\gamma_m^{\mathrm{old}}}{\gamma_m^{\mathrm{new}}}\right)^2\left\|\mathbf{X}^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m\right\|_2^2 - L\mathbf{a}_m^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m.\end{aligned} \tag{25}$$

The factor $(\gamma_m^{\mathrm{old}}/\gamma_m^{\mathrm{new}})^2$ is introduced to obtain an iterative equation in γ_m. Equate the derivatives to zero and we can get the fixed-point update rule [1, 2, 4].

$$\begin{aligned}\gamma_m^{\mathrm{new}} &= \gamma_m^{\mathrm{old}}\frac{1}{L}\frac{\left\|\mathbf{X}^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m\right\|_2^2}{\mathbf{a}_m^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m}\\ &= \gamma_m^{\mathrm{old}}\frac{\mathrm{Tr}\left[\mathbf{S}_{\mathbf{x}}\sum_{\mathbf{x}}^{-1}\mathbf{a}_m\mathbf{a}_m^H\Sigma_{\mathbf{x}}^{-1}\right]}{\mathbf{a}_m^H\sum_{\mathbf{x}}^{-1}\mathbf{a}_m}.\end{aligned} \tag{26}$$

where $\mathbf{S}_{\mathbf{x}} = 1/J\mathbf{X}\mathbf{X}^H$ is the sample covariance matrix.

The main steps of the SBL algorithm are summarized as follows:

Step 1: Set the parameters as $\varepsilon = 10^{-3}$ and get the input data $\mathbf{X}, \mathbf{A}_g, \sigma^2, k_{\max}$;

Step 2: Initialization the parameters $\gamma_m^{\mathrm{old}} = 1, \forall m$;

Step 3: Calculate $\Sigma_{\mathbf{x}} = \sigma^2\mathbf{I} + \mathbf{C}\mathbf{A}\Gamma^{\mathrm{old}}\mathbf{A}^H\mathbf{C}^H$;

Step 4: Update γ_m^{new} by using equation (27);

Step 5: Set $\gamma^{\mathrm{old}} = \gamma^{\mathrm{new}}, \Gamma^{\mathrm{old}} = \mathrm{diag}(\gamma^{\mathrm{old}}), k = k+1$;

Step 6: If $\|\gamma^{new} - \gamma^{\mathrm{old}}\|_1/\|\gamma^{\mathrm{old}}\|_1 > \varepsilon$ and $k < k_{\max}$, go back to step 3;

Step 7: The K largest peaks in γ are the required DOA values.

5. Performance Analysis

5.1. Comparison of Mutual Coupling Leakage between Different Arrays with the Same Aperture. We select some sparse arrays for comparison. In order to make the array aperture of all arrays consistent, we compress the element spacing of CA and ECA. The result is shown in Table 1, it can be seen that the mutual coupling leakage of the proposed array is the smallest except SULA, and its mutual coupling leakage is very close to SULA.

5.2. Crarmer–Rao Bound (CRB). According to the knowledge of the literature [32], the Crarmer–Rao Bound (CRB) matrix can be represented as follows:

TABLE 1: Comparison of mutual coupling leakage of different arrays.

	Array aperture	d	$\mathbb{Z}$	Mutual coupling leakage
Proposed	8 λ	0.5 λ	0,2,4,7,10,12,14,16	0.0826
CA	8 λ	0.5 λ	0 4 5 8 10 12 15 16	0.1076
ECA	8 λ	0.4444 λ	0 2 4 5 6 8 13 18	0.1140
NA	8 λ	0.4211 λ	0 1 2 3 7 11 15 19	0.1194
SULA	8 λ	1.1429 λ	0 1 2 3 4 5 6 7	0.0806
ULA	3.5 λ	0.5 λ	0 1 2 3 4 5 6 7	0.1819

$$CRB = \frac{\sigma_n^2}{2J}\left\{\mathrm{Re}\left[\mathbf{D}^H \prod_{\mathbf{A}}^{\perp} \mathbf{D}\widehat{P}^T\right]\right\}^{-1}, \tag{27}$$

where $\mathrm{Re}[\cdot]$ stands for the operation of taking the real part, $\mathbf{A}$ represents the manifold matrix of the array, $\Pi_{\mathbf{A}}^{\perp} = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$ is the orthogonal projection of $\mathbf{A}$, and $\mathbf{I}$ stands for the identity matrix of order $M \times M$, $\widehat{P} = 1/J\sum_{t=1}^{J} s(t)s^H(t)$, σ_n^2 denotes the average power of signal source, $\mathbf{D}$ can be written as follows:

$$\mathbf{D} = \left[\frac{\partial \mathbf{a}(\theta_1)}{\partial \theta_1}, \frac{\partial \mathbf{a}(\theta_2)}{\partial \theta_2}, \ldots, \frac{\partial \mathbf{a}(\theta_K)}{\partial \theta_K}\right], \tag{28}$$

where $\mathbf{a}(\theta_K)$ denotes steering vector.

5.3. Computational Complexity. In this section, we provide the complexity of the SBL method compared with OMP and MUSIC. Assuming that the number of array elements is M, there are G grid points and J snapshots, the maximum number of iterations is $k_{\max}$, then the computational complexity of main operations are as follows: (a) calculate the covariance matrix: $O(M^2J)$; (b) update the $\Sigma_{\mathbf{x}}$ in Step 3: $O(2M^2G + G^2M + M^3)$; (C) update γ_m^{new} in Step 4: $O[G(2M^3 + 3M^2 + M)]$. The computational complexity of SBL is $O[M^2J + k_{\max}(2GM^3 + 5GM^2 + G^2M + GM + M^3)]$.

6. Simulation Results

In this part, we provide numerical simulations of the performance of the proposed MCOA as well as a comparison with the other sparse arrays and the CRB. The array aperture is limited to 8 wavelengths and the number of array sensors is 8. Two coherent signals with equal power impinge on the array with directions $\theta_1 = 10°$, $\theta_2 = 40°$, and the correlation coefficient is set to $[\alpha_1, \alpha_2] = [1, e^{j\pi/4}]$. Define the Root Mean Square Error (RMSE) of the DOA estimates as follows:

$$\mathrm{RMSE} = \sqrt{\frac{1}{K}\frac{1}{Q}\sum_{k=1}^{K}\sum_{q=1}^{Q}\left(\widehat{\theta}_{q,k} - \theta_k\right)^2}, \tag{29}$$

where Q and K are the number of Monte Carlo trials and the total number of coherent signals, respectively. $\widehat{\theta}_{q,k}$ means the qth estimate of the real angle θ_k . Unless other stated, we assume that the mutual coupling constant is $c_0 = 0.12$ and the maximum spacing of sensor pairs is $B = 100$. For each Figure, 1000 Monte Carlo simulations were run to estimate the Root Mean Square Error (RMSE).

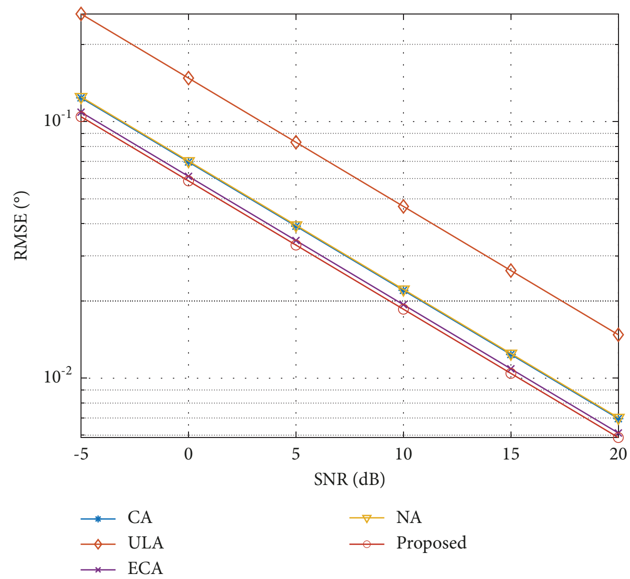

FIGURE 3: CRB comparison of different arrays.

First, we compare the CRB of different arrays in Figure 3. The result shows that the CRB of the coprime array is very close to the CRB of the proposed array, but the CRB of the proposed array is the smallest among all arrays, which indicates that its performance is optimal.

Figure 4 depicts the spatial spectrum of the SBL method with the proposed array. The signal-to-noise ratio (SNR) is 10 dB and the snapshot is $J = 200$. There are many spectral peaks in the estimation result, but the peak of the incident signal is the highest, which is 40 dB higher than other spectral peaks. It shows that the estimation result of this method is accurate enough to be used for the coherent signal estimation when there is mutual coupling between array sensors.

The proposed array is also compared with other arrays with different SNRs in Figure 5. The SNR varies from −5 dB to 20 dB and the number of snapshots is $J = 200$. All five kinds of arrays can accurately estimate the incident angle of the relevant signals. Due to the influence of mutual coupling, the angle estimation of the CA and ULA decreases greatly, and their RMSE value is larger than the proposed array. Because the influence of mutual coupling leakage is less than that of other arrays, the proposed array can better estimate the DOA of coherent signals.

Figure 6 shows the performance of different arrays with snapshots changing, the coherent signal is consistent with the previous simulation and the SNR is 5 dB. The snapshot varies from 10 to 600. When the snapshot is less than 100, all five arrays cannot work well and the performance is not good enough. However, the RMSE of all the arrays reduces with the snapshot increasing. When the snapshot is larger than 100, the curve of RMSE of the CA and ULA is almost a straight line, because the impact of the mutual coupling leakage at this time is greater than the performance

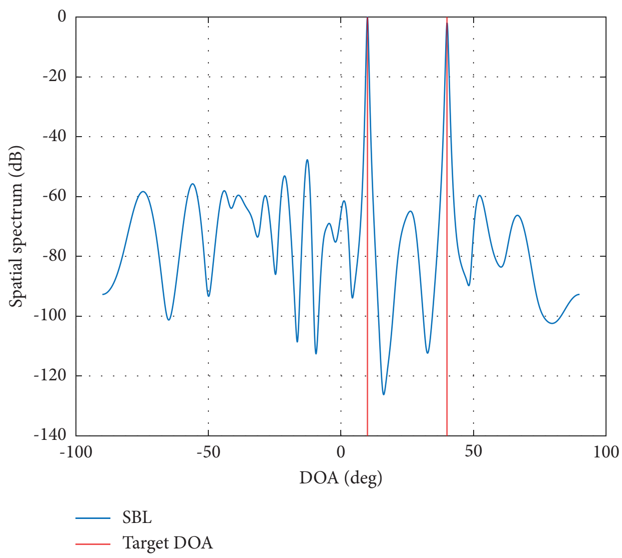

FIGURE 4: The spatial spectrum for DOA estimation.

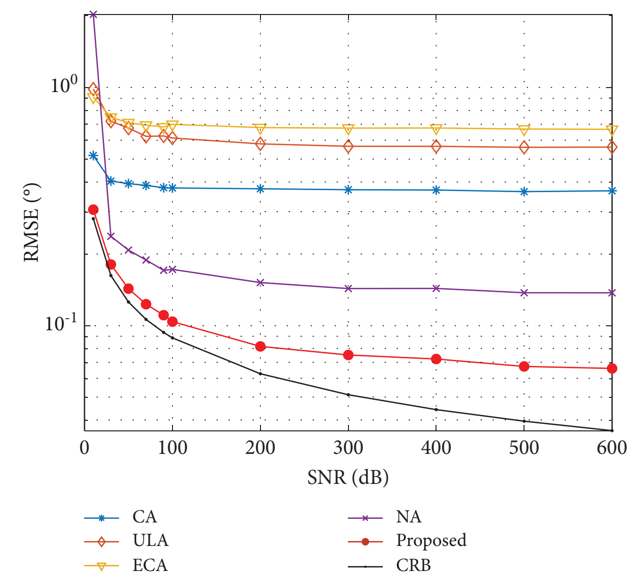

FIGURE 6: The DOA estimation performance with different snapshots.

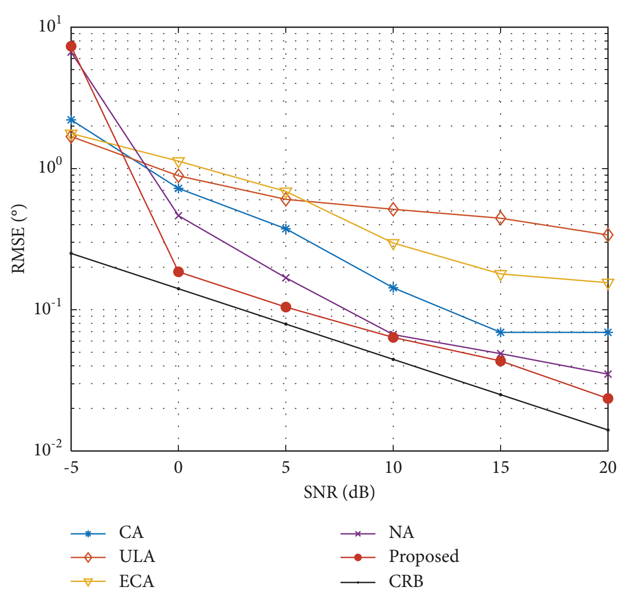

FIGURE 5: The DOA estimation performance with different SNRs.

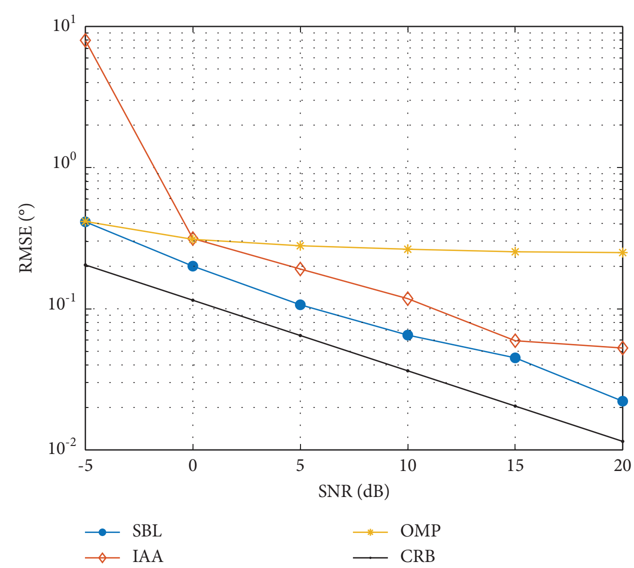

FIGURE 7: Comparison of RMSE of different algorithms with SNR.

improvement brought about by the increase of snapshots. On the other hand, the performance of the proposed array and NA is better than the other three arrays, and the proposed array has the best performance because the mutual coupling leakage of the proposed array is the smallest among these arrays.

Finally, we compared the SBL algorithm with two other compressed sensing algorithms including the IAA [20] and the OMP [21]. In Figure 7, the SNR changes from −5 dB to 20 dB and the snapshot is 200. The other simulation conditions were the same as before. At low SNR, the SBL has the same performance as IAA and OMP, but when the SNR increases, the performance of OMP hardly improves, and the performance of IAA is better than OMP. The curve of SBL is smoother than the other two algorithms, which means that in this case, its performance is more stable than other algorithms.

7. Conclusions

In this paper, the mutual coupling optimization array with a given number of sensors under the condition of the finite aperture is studied. Compared with the sparse arrays including CA, NA, and ECA, the mutual coupling leakage of the proposed array is smaller. When there is mutual coupling between array sensors, we apply the SBL to DOA estimation of coherent signals and compare its performance with other compressed sensing methods including IAA and OMP. Finally, various simulations are carried out to prove the superior performance of the proposed array for estimating coherent signals in the condition of mutual coupling. In fact, this paper mainly focuses on the coherent signal estimation problem of the sparse array in the 1D-DOA case. By using the previous related research, this result can be extended to an L-shaped array to realize DOA estimation of coherent signal in the 2D-DOA case. Reference [31].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] J. Li, Q. Zhang, W. Deng, Y. Tang, X. Zhang, and Q. Wu, "Source direction finding and direct localization exploiting UAV array with unknown gain-phase errors," *IEEE Internet of Things Journal*, vol. 2022, Article ID 3181450, p. 1, 2022.

[2] J. Li, P. Li, P. Li, L. Tang, X. Zhang, and Q. Wu, "Self-position awareness based on cascade direct localization over multiple source data," *IEEE Transactions on Intelligent Transportation Systems*, vol. 1, 9 pages, 2022.

[3] Y. Qian, Z. Yang, and H. Zeng, "Direct position determination for augmented coprime arrays via weighted subspace data fusion method," *Mathematical Problems in Engineering*, vol. 2021, Article ID 2825025, p. 1, 2021.

[4] S. E. Nai, W. Ser, Z. L. Yu, and H. Chen, "Iterative robust minimum variance beamforming," *IEEE Transactions on Signal Processing*, vol. 59, no. 4, pp. 1601–1611, April 2011.

[5] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, March 1986.

[6] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, & Signal Processing*, vol. 37, no. 7, pp. 984–995, 1989.

[7] C. Zhou, Y. Gu, S. He, and Z. Shi, "A robust and efficient algorithm for coprime array adaptive beamforming," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 2, pp. 1099–1112, 2018.

[8] S. Marcos, A. Marsal, and M. Benidir, "The propagator method for source bearing estimation," *Signal Processing*, vol. 42, no. 2, pp. 121–138, 1995.

[9] N. D. Sidiropoulos, G. B. Giannakis, and R. Bro, "Blind PARAFAC receivers for DS-CDMA systems," *IEEE Transactions on Signal Processing*, vol. 48, no. 3, pp. 810–823, March 2000.

[10] F. Wen, J. Shi, and Z. Zhang, "Generalized spatial smoothing in bistatic EMVS-MIMO radar," *Signal Processing*, vol. 193, Article ID 108406, 2022.

[11] T.-J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Transactions on Acoustics, Speech, & Signal Processing*, vol. 33, no. 4, pp. 806–811, August 1985.

[12] S. U. Pillai and B. H. Kwon, "Forward/backward spatial smoothing techniques for coherent signal identification," *IEEE Transactions on Acoustics, Speech, & Signal Processing*, vol. 37, no. 1, pp. 8–15, 1989.

[13] W. Du and R. L. Kirlin, "Improved spatial smoothing techniques for DOA estimation of coherent signals," *IEEE Transactions on Signal Processing*, vol. 39, no. 5, pp. 1208–1210, May 1991.

[14] W. X. Du and R. L. Kirlin, "Improved spatial smoothing techniques for DOA estimation of coherent signals," *IEEE Transactions on Signal Processing*, vol. 39, no. 5, pp. 1208–1210, 1991.

[15] J. Shi, Z. Yang, and Y. Liu, "On parameter identifiability of diversity-smoothing-based MIMO radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 58, no. 3, pp. 1660–1675, 2022.

[16] F.-M. Han and X.-Da Zhang, "An ESPRIT-like algorithm for coherent DOA estimation," *IEEE Antennas and Wireless Propagation Letters*, vol. 4, pp. 443–446, 2005.

[17] Z. Yang and L. Xie, "Exact joint sparse frequency recovery via optimization methods," *IEEE Transactions on Signal Processing*, vol. 64, no. 19, pp. 5145–5157, 2016.

[18] Z. Yang and L. Xie, "Enhancing sparsity and resolution via reweighted atomic norm minimization," *IEEE Transactions on Signal Processing*, vol. 64, no. 4, pp. 995–1006, 2016.

[19] Y. Yu, A. P. Petropulu, and H. V. Poor, "Measurement matrix design for compressive sensing–based MIMO radar," *IEEE Transactions on Signal Processing*, vol. 59, no. 11, pp. 5338–5352, 2011.

[20] L. Du, T. Yardibi, J. Li, and P. Stoica, "Review of user parameter-free robust adaptive beamforming algorithms," in *Proceedings of the 2008 42nd Asilomar Conference on Signals, Systems and Computers*, pp. 363–367, Beijing China, June 2008.

[21] S. K. Sahoo and A. Makur, "Signal recovery from random measurements via extended orthogonal matching Pursuit," *IEEE Transactions on Signal Processing*, vol. 63, no. 10, pp. 2572–2581, 2015.

[22] M. E. Tipping, "Sparse Bayesian learning and the relevance vector machine," *Journal of Machine Learning Research*, vol. 1, pp. 211–244, 2001.

[23] P. Pal and P. P. Vaidyanathan, "Nested arrays: a novel approach to array processing with enhanced degrees of freedom," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4167–4181, Aug, 2010.

[24] P. P. Vaidyanathan and P. Pal, "Sparse sensing with Co-prime samplers and arrays," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 573–586, 2011.

[25] A. Moffet, "Minimum-redundancy linear arrays," *IEEE Transactions on Antennas and Propagation*, vol. 16, no. 2, pp. 172–175, 1968.

[26] Z. Ye, J. Dai, X. Xu, and X. Wu, "DOA estimation for uniform linear array with mutual coupling," *IEEE Transactions on*

Aerospace and Electronic Systems, vol. 45, no. 1, pp. 280–288, 2009.

[27] J. Dai, D. Zhao, and X. Ji, "A sparse representation method for DOA estimation with unknown mutual coupling," *IEEE Antennas and Wireless Propagation Letters*, vol. 11, pp. 1210–1213, 2012.

[28] B. Liao, Z.-G. Zhang, and S.-C. Chan, "DOA estimation and tracking of ULAs with mutual coupling," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 1, pp. 891–905, 2012.

[29] P. P. Vaidyanathan and P. Pal, "Why does direct-MUSIC on sparse-arrays work?" in *Proceedings of the 2013 Asilomar Conference on Signals, Systems and Computers*, pp. 2007–2011, Pacific Grove, CA, USA, May 2013.

[30] P. P. Vaidyanathan and P. Pal, "Direct-MUSIC on sparse arrays," in *Proceedings of the 2012 International Conference on Signal Processing and Communications (SPCOM)*, pp. 1–5, Bangalore, India, August 2012.

[31] C. L. Liu and P. P. Vaidyanathan, "Super nested arrays: linear sparse arrays with reduced mutual coupling—Part I: fundamentals," *IEEE Transactions on Signal Processing*, vol. 64, no. 15, pp. 3997–4012, 2016.

[32] X. Lai, W. Chen, B. Li, and H. Zeng, "Improved DFT method for DOA estimation with extended coprime array:based on large difference coarray," *International Journal of Electronics*, vol. 2021, no. 5, Article ID 1941289, pp. 733–747, 2022.