Supply Chain Coordination with Capital-Constrained and Loss Aversion Retailer under Stochastic Demand and Supply

Lu Yueli,1 Shi Lu,1 and Zhao Xueyun2

1Guangxi Agricultural Vocational and Technical University, Nanning 530004, China
2Guangxi Police College, Nanning 530001, China

Correspondence should be addressed to Shi Lu; shilugxu@126.com

Received 31 May 2022; Revised 13 July 2022; Accepted 20 August 2022; Published 19 September 2022

Abstract

In the face of increasingly complex market environment, supply chain members pay attention to the possibility of obtaining expected profits and various risks while considering profit maximization. The behaviors of decision makers are often influenced by their psychological preferences, and loss aversion is one of the basic characteristics of human economic behavior. We research the supply chain coordination problem with capital constraint and loss-averse retailer in the case of stochastic production and demand. In order to address the loss aversion, a novel utility function is structured according to the theory of mental account. Under wholesale price contract, the optimal expected utility of supply chain under decentralized decision is smaller than that under centralized decision due to the retailer’s constrained capital. Based on that, a model is constructed by making revenue sharing contract act as the dual role of supply chain coordination and delayed payment strategy without seeking external financing or internal financing, and the contract conditions and optimal decisions of the coordinated supply chain are solved. Theoretical analysis indicates that when the contract parameters are properly selected, even though the retailer’s capital is constrained, the revenue sharing contract can not only coordinate the supply chain but also realize the arbitrary distribution of supply chain profits within a certain range. Finally, the change of decision variables and expected utility or profit with contract parameters is analyzed by several numerical examples.

1. Introduction

In recent decades, the management of supply chain has become a significant research field of management science, and has made great achievements both theoretically and practically. Due to the high-speed development of economic globalization and information technology, enterprises are facing a more complex and changeable market environment and increasingly fierce competition. Most of the traditional supply chain management research is based on the hypothesis of economic self-interest of neoclassical economics, and many research results cannot reflect the real supply chain management behavior. People do not always pursue the maximum “utility” in their behavior, but make “satisfactory choices” according to their cognition of the environment and limited thinking. People not only care about their own material interests but also the interests of others. People’s inner social preferences will change their behavior, such as loss aversion [1], altruistic preferences [3], and fairness concerns [4]. Loss aversion is one of the basic characteristics of human behavior. Kahneman and Tversky [5] found that most people are risk-averse in the face of gain, and people are more sensitive to loss than gain, and the pain of loss is much greater than the pleasure of gain. Decision maker has his own risk degree, which affects the final decision. Therefore, more and more scholars believe that only from the perspective of human behavior and psychological cognition to explore the complex environment of supply chain management, can effectively guide the practice.

Thaler [6] first proposed the concept of “psychological account” to explain why individuals are affected by “sunk costs” when making decisions. Sunk costs are the costs incurred by past decisions that cannot be changed by any present or future decisions. According to the perfectly
rational man assumption, sunk costs should be irrelevant to decision-making. But in real life, people’s decisions are often affected by sunk cost effect. Due to the irrational behavior of mental account in individual decision-making, Thaler [7] believes that the past investment and present investment will be placed in the same mental account, which will be taken as the total cost of the decision scheme, and then affect the decision maker’s choice. In newsupplier problem, retailers also have mental account behavior. The traditional newsupplier theory holds that the revenue from each product is fungible, that the $100 in revenue from product A is equivalent to the $100 in revenue from product B. However, as a matter of fact, due to the different demand and competitive environment of each product, the inventory decision maker will have different feelings about the $100 profit of different products. Retailer will spend part of the funds to buy some basic and competitive goods, which demand is generally stable and the profit will be low, belongs to the low expectations of retailer’s mental account. At the same time, retailers will allocate the other part of the money to higher-margin products, which belongs to the high expectations of retailer’s mental account.

Supply chain risks are multifaceted, including not only the uncertainty of demand but also the uncertainty of supply. Supply uncertainty is widespread in many industries. For example, in the supply chain of oil, coal, and other mining industries, the output of raw materials is uncertain due to the complexity of geological conditions. In the supply chain of agriculture and forestry, the output of agricultural products and forest products is uncertain due to the influence of climate, diseases, and insect pests. Some literature has focused on supply chain contract design under the condition of uncertain supply and demand [8–10].

In most previous literature of supply chain management, it has been assumed that the enterprises do not consider budget control in decision-making process. However, insufficient budget is common in the daily operation of small and medium enterprises (SMEs). The shortage of operating funds will restrict the daily operation of the enterprise and hinder the business development of the enterprise. In the context of supply chain, the capital status of member enterprises will also affect the capital operation of other enterprises on the chain. Supply chain is a unity of logistics and capital flow. The financial distress of node enterprises is transmitted to other enterprises on the chain along with the transaction activities, which may lead to the financial distress of the entire chain. Since SMEs do not have the appropriate qualifications and credit ratings required by the financial institutions, it is difficult to obtain external financing. Trade credit is based on commercial credit from previous transactions between member companies, allowing the cash-strapped buyer to defer repayment of loans under certain conditions, or the cash-strapped seller to demand partial payment in advance. As a typical internal financing method of supply chain, trade credit has been well developed in theory and practice [11–14], but the intensification of supply chain competition has increased the financing pressure of trade credit providers, and trade credit sometimes cannot be achieved.

Based on the above analysis, it can be seen that supply chain management model develops from considering risk neutrality to considering risk aversion [1, 6–9, 11, 15–22], from considering single market demand uncertainty to considering both supply and demand uncertainty [8–10, 23, 24], from not considering capital constraint to considering capital constraint [12–14, 25–33]. Risk not only causes economic losses but also has an impact on the psychology of decision makers, thus affecting their decision-making behavior. With the increasing complexity of supply chain problem, the coordination contract is also developing and evolving. Based on this, this paper focuses on the following questions: can supply chain coordination still be achieved in the face of uncertainty of demand and supply when a loss-averse retailer has constrained capital and cannot obtain external and internal financing? Are there contractual mechanisms to coordinate the supply chain?

To answer the above questions, this paper studies the supply chain coordination problem of retailer with capital constraints and loss aversion under the condition of uncertain supply and demand. A novel utility function based on the theory of mental account is built to describe the retailer’s loss-averse preferences. We discuss the optimal order quantity of retailer and optimal planned production of supplier in centralized decision model and distributed decision model, respectively. Then, in order to coordinate the supply chain, we formulate a revenue sharing contract. Finally, the obtained results are verified by numerical examples. Our innovations and contributions are as follows. First, we prove that the supply chain can be coordinated by revenue sharing contract when both production and demand are uncertain in an environment of capital constraint. Second, we apply the theory of mental account to figure the character and behavior of retailer with loss aversion and capital constraints.

The remaining structure is shown below. In the following section, we review the related literature. Section 3 gives the notations and assumptions. Section 4 shows the centralized decision model. Section 5 demonstrates the distributed decision model. In section 6, numerical examples and sensitivity analysis are carried out to verify the results obtained in the previous sections. Section 7 summarizes the research.

2. Literature Review

For the problem of supply chain coordination considering risk, many scholars have studied supply chain contract design with risk decision maker from the perspective of supply chain coordination. Schweitzer and Cachon [1] originally discussed the loss-averse newsupplier problem. They proved that the loss-averse newsvendor has lower optimal order quantity than the risk-neutral one. Moreover, with the increase of loss aversion, the optimal order quantity decreases. Base on their work, Wang and Webster [15] pointed out that a loss-averse retailer got bigger ordering quantity than a risk-neutral one when shortage cost was considered. Ma et al. [16] investigated a model that the retailer with risk aversion had two options to make order
decisions before the demand was accomplished. Xinsheng et al. [17] considered the loss of overorders and shortage cost by the CVaR measure. Yu et al. [18] presented a robust model and indicated that the optimal order policy of the loss-averse retailer was quite different from that of the risk-neutral one. According to CVaR criterion, a loss-avoiding newsvendor model was constructed by Xinsheng et al. [19], and they found that the increase of confidence coefficient would decrease the optimal order quantity and expected utility. Lam and Chang [20] proposed a value discounting model and showed that the ordering behaviors of buyback contract were different from that of revenue sharing contract. By incorporating risk-averse parameter and loss sharing into trade credit policy, Wu et al. [11] presented a supplier-Stackelberg model that a supplier provided loss sharing and trade credit to a risk-averse retailer. In the past few years, the theory of “mental account” has been applied to research the supply chain coordination in which the members have risk aversion. Sun et al. [21] proposed a novel utility function based on the theory of mental account to describe the newsvendor’s aversion level through a loss aversion parameter. They maximized CVaR of utility to acquire optimal order quantity and demonstrated that the expected utility was decreasing in the confidence level. Vipin and Amit [22] analyzed the retailer’s psychological behavior characteristics based on the theory of mental account, and pointed out that loss avoidance behavior of retailer can improve the performance of model.

The newsvendor models with loss aversion mentioned above only considered the uncertainty of demand, while the uncertainty of demand and supply recently has attracted the attention of some scholars. Liu et al. [8] established two newsvendor models under random yield and demand. The optimal order quantity of loss-averse newsvendor was lower than that of risk-neutral one when the shortage cost was ignored, but it was opposite when the shortage cost was considered. Ma et al. [9] analyzed the influence of loss aversion on the optimal decision under uncertainty of demand and supply with a situation that the retailer just paid for the actual items received. Luo et al. [10] studied the channel coordination by considering random yield, stochastic demand, and loss aversion under three traditional coordination contracts (buyback, quantity flexibility, and wholesale price), and presented the effects of loss aversion and random yield on decision-making behaviors and system performance. Luo and Chen [23] found that under the voluntary principle, a single revenue sharing contract could not coordinate the supply chain under uncertain supply and demand conditions, while the combined contract combining revenue sharing contract and residual compensation mechanism could coordinate the supply chain. Güler [24] investigated the coordination problem of supply chain composed of multiple suppliers and a single manufacturer under the condition of uncertain supply and demand, and the results showed that the combination of repurchase and punishment contract can realize the coordination of the supply chain. None of the above literature takes into account that the supply chain members are financially constrained.

Many academics have observed that the selection of supply chain contract affects the optimization of supply chain performance in a significant way under the capital constraint, and integrated the design of supply chain contract into the comprehensive decision-making process. Hwan and Rhee [12] found that financing costs affect supply chain coordination in various contracts. By assuming that the supplier granted trade credit and markdown allowance, Lee and Rhee [25] discussed a newsvendor model which showed that the markdown allowance cannot completely realize the coordination of supply chain unless the retailer is financing directly. Chen and Wan [26] researched the influence of financing on supply chain performance with a budget-constrained retailer through a three-stage Stackelberg model. Their study showed that financing was valuable for supplier and retailer. Chen and Wang [13] analyzed how trade credit impacted on supply chain performance with considering budget constraints. Since the accounts liability was limited, the retailer who had a lower original budget could order more products under trade credit contract. Based on the core enterprise commitment buyback and fair reference, Chen et al. [27] studied the pricing and ordering strategy for a fairness-concerned and capital-constrained retailer, and contradistinguish the supply chain performance under different financing conditions. Kouvelis and Zhao [28] investigated how to design contract and coordinate the supply chain under an assumption that supplier and retailer were both faced with restriction of capital and bankruptcy risk, and they pointed out that the revenue sharing contract can reach the goal of coordinating the capital-constrained supply chain. Xiao et al. [29] investigated the coordinating mechanisms for a supply chain, and they suppose that the retailer with financing constraint obtained trade credit from the supplier. The above researches considered risk-neutral supply chain member, while the following studies were under risk aversion. Based on the CVaR criterion, Chen and Zhou [14] studied the ordering strategy of the retailer with capital constraint under trade credit, and discovered that as risk aversion increases, the order quantity would decrease. Moreover, the growth of wholesale price would result in the increase of order quantity when the risk aversion was higher than a certain critical value. Yan et al. [30] constructed a Stackelberg model with a risk-averse manufacturer and a capital-constrained retailer, in which the manufacturer provided a credit guarantee for the retailer to get a loan from the bank. Li et al. [31] investigated the financing strategy for a supply chain which supplier was risk-averse and retailer was capital constrained, and analyzed the influence of supplier’s risk aversion and credit guarantee on the choice of financing strategy. Yan et al. [32] studied supplier finance and supplier investment, and retailer was supposed to be capital constrained. They found that both retailer and supplier would get a higher benefit from pure supplier finance or supplier investment when the retailer’s initial capital was lower. Yan et al. [33] compared loan and investment through a game theoretical approach when the supplier’s capital was insufficient. They found that the higher loss aversion of retailer would result in lower wholesale price.
under loan scheme or lower production quantity under investment scheme. Yan et al. [33] considered the value of factoring finance in a supply chain which supplier is loss-averse based on game theory. They found that low loss-averse supplier tended to the retailer-led factoring but the high loss-averse one tended to the factor-led factoring.

Our study is similar to the work by Luo et al. [10], but capital constraint was not considered in their work. The utility function used to describe loss aversion is also different, and we focus on the impact of out-of-stock losses on loss-averse decision makers. In addition, they analyzed the coordination of the supply chain under the buyback and quantity flexibility contracts, while we focused on the coordination of the supply chain under the revenue-sharing contract and the role of the revenue-sharing contract in a financially constrained environment.

3. Notations and Assumption

3.1. Notations. For convenient description, subscript $i$ denotes different members, $i = S$ denotes the supplier, $i = R$ denotes the retailer, and $i = C$ denotes the whole supply chain.

- $c$: the unit cost
- $w$: the unit wholesale price
- $p$: the unit retail price
- $v$: the unit salvage value
- $s$: the unit out of stock loss
- $\lambda$: the loss aversion factor of retailer
- $c_s$: the unit spot price
- $Q$: the order quantity, decision variable
- $M$: the planned production quantity, decision variable

3.2. Assumption. We discuss a two-level supply chain in which the supplier is risk-neutral and the retailer is loss-averse. The supplier makes products, and the retailer orders products and sells them to customers. The information is interchangeable among the supplier and retailer. A retailer-Stackelberg game is played between the supplier and retailer.

(i) The production is random and the effective production is $\epsilon M$, where $\epsilon (\epsilon \in [0,1])$ is the random factor and $M$ is the production. The probability density function and distribution function of $\epsilon$ are $g(\epsilon)$ and $G(\epsilon)$ separately. Moreover, $G(\epsilon)$ is continuous, differentiable, and increasing.

(ii) The market demand $x$ is a nonnegative and random variable, which probability density function and distribution function are $f(x)$ and $F(x)$ separately, and the mathematical expectation of $x$ is $\mu$, i.e., $\mu = E(x)$.

(iii) The retailer’s capital (denoted as $K$) is constrained, while the supplier’s capital is unconstrained.

(iv) Before the sales season, the supplier produces the products according to the order size of the retailer.

When the actual production is less than the order size, the supplier needs to purchase the deficient number from the spot market and supply to the retailer. If the retailer’s order size cannot satisfy the market demand, there will be a shortage loss. Conversely, if there is a surplus at the end of the sales season, the retailer will acquire the residual value though discounting them.

(v) A novel utility function based on the theory of mental account is adopted to describe the retailer’s loss-averse preferences: $U = Ra - \lambda La$, where $Ra$ is the revenue account of retailer, and $La$ is the loss account of retailer. Additionally, $\lambda$ reflects the retailer’s aversion to loss, and $\lambda \geq 1$. The larger the value of $\lambda$, the more the retailer’s aversion to the loss of stock out, conversely, the less the aversion to the loss of stock out.

(vi) According to the actual meaning, the parameters need to satisfy $p > w > c_s > c > 0$.

4. Centralized Decision Model

Since retailer and supplier are an economic entity in a centralized supply chain, the capital is unconstrained. The supply chain system is averse to the loss of shortage on account of the supply chain which is dominated by the loss-averse retailer. The order quantity and the planned production quantity are determined by maximizing the expected utility of the supply chain. The utility of the supply chain system is given by

$$U_C(Q, M) = p \min(x, Q) + v |Q - x|^\ast - \lambda s |Q - x|^\ast - cM - c_s |Q - \epsilon M|^\ast.$$  

In the above expression, the first part represents the sales revenue, the second part represents the residual value of the remaining products, the third part represents the loss of shortage, the fourth part represents the production cost, and the last part represents the cost of purchasing products from the spot market. So, the expected utility of the supply chain (the calculation see Appendix) is given by

$$E[U_C(Q, M)] = (p + \lambda s)Q - (p + \lambda s - v) \int_0^Q F(x)dx - cM - \lambda s \mu.$$  

Theorem 1. The expected utility of the supply chain $E[U_C(Q, M)]$ is a joint concave function of $(Q, M)$, and the optimal order quantity and optimal planned production are, respectively, given by

$$Q_c = F^{-1}\left(\frac{p + \lambda s - c_s G(z_0)}{p + \lambda s - v}\right),$$

$$M_c = \frac{Q_c}{z_0}.$$
where $z_0$ is the solution to the equation $\int_0^1 \varepsilon g(\varepsilon) d\varepsilon = (c/c_\ast)$. 

Proof. Take the first partial derivatives of $E[U_C(Q, M)]$ with respect to $Q$ and $M$, we have

$$\frac{\partial E[U_C(Q, M)]}{\partial Q} = (p + \lambda s) - (p + \lambda s - v)F(Q) - c_rG(Q/M),$$

$$\frac{\partial E[U_C(Q, M)]}{\partial M} = -c_\ast + c_r \int_0^{(Q/M)} \varepsilon g(\varepsilon) d\varepsilon.$$ (4)

Let $z = (Q/M)$, it is obvious that $z \in [0, 1]$, and we define the function $H(z) = -c + c_r \int_0^z \varepsilon g(\varepsilon) d\varepsilon$. Then, it follows from $H'(z) = c_rz g(z) \geq 0$ that $H(z)$ is monotonically increasing on $(0, 1)$. Moreover, since $H(0) = -c < 0$ and $H(1) = c_r \varepsilon(1) - c > 0$, implies that there is a unique $z_0 \in (0, 1)$ satisfies $H(z_0) = 0$. It means that $z_0$ is the solution to the equation $\int_0^1 \varepsilon g(\varepsilon) d\varepsilon = (c/c_\ast)$. Substituting $M = (Q/z_0)$ into $\partial E[U_C(Q, M)]/\partial Q = 0$, it follows that

$$(p + \lambda s) - (p + \lambda s - v)F(Q) - c_r G(z_0) = 0.$$ (5)

Let the left side of the above equation be denoted as $T(Q)$, then it follows from $T'(Q) = -(p + \lambda s - v)f(Q) < 0$ that $T(Q)$ is monotone decreasing. Furthermore, since $0 \leq G(z) \leq 1$, $p > c_r > v$, it is obvious that $T(0) = p + \lambda s - c_r G(z_0) > 0$, $T(\infty) = v - c_r G(z_0) < 0$. Hence, there exists a unique $Q_\ast \in (0, \infty)$ such that equality (6) is satisfied, and $Q_\ast = F^{-1}[(p + \lambda s - c_r G(z_0))/(p + \lambda s - v)]$. Therefore, the supplier’s optimal planned production $M_\ast = Q_\ast/z_0$, and $(Q_\ast, M_\ast)$ is the point that satisfies the first-order conditions.

Take the second partial derivatives of $E[U_C(Q, M)]$ with respect to $Q$ and $M$, we have

$$\frac{\partial^2 E[U_C(Q, M)]}{\partial Q^2} = -(p + \lambda s - v)f(Q) - c_r G(Q/M),$$

$$\frac{\partial^2 E[U_C(Q, M)]}{\partial Q \partial M} = \frac{\partial^2 E[U_C(Q, M)]}{\partial M \partial Q} = c_r Q/M g(Q/M),$$

$$\frac{\partial^2 E[U_C(Q, M)]}{\partial M^2} = -\frac{c_r Q^2}{M^3} g(Q/M).$$

(6)

Denote the Hessian matrix of $E[U_C(Q, M)]$ by $H^\ast$, then

$$H^\ast = \begin{bmatrix}
-(p + \lambda s - v)f(Q) - c_r G(Q/M) & c_r Q/M g(Q/M) \\
\frac{c_r Q}{M^2} g(Q/M) & -\frac{c_r Q^2}{M^3} g(Q/M)
\end{bmatrix}. $$ (7)

It is clear that $|H^\ast_{11}| < 0$ and $|H^\ast_{22}| = c_r Q^2 (p + \lambda s - v)f(Q)g(Q/M)M^3 \geq 0$, then $H^\ast$ is negative definite, that is to say, $E[U_C(Q, M)]$ is a concave function about $(Q, M)$. Since $(Q_\ast, M_\ast)$ is the point that satisfies the first-order conditions, $Q_\ast$ is the optimal order quantity, and $M_\ast$ is the optimal planned production. This completes the proof.

For convenience, noting $EU^\ast_C = E[U_C(Q_\ast, M_\ast)]$, then

$$EU^\ast_C = (p + \lambda s - c_r G(z_0))Q_\ast - (p + \lambda s - v) \int_0^{Q_\ast} F(x) dx - \lambda s\mu. $$ (8)

\quad

5. Distributed Decision Model

In the distributed supply chain, the retailer first determines the optimal order quantity to maximize his expected utility, and then the supplier determines the optimal planned production according to the retailer’s order quantity to maximize his expected profit.

5.1. Wholesale Price Contract Model. Under the wholesale price contract, the utility of the loss-averse retailer is

$$U_{R_1}(Q) = p \min[x, Q] + v[Q - x]^+ - \omega Q - \lambda s[x - Q]^+,$$

(9)

where $\lambda \geq 1$ is the loss-averse coefficient. Then, the expected utility of retailer is given by

$$E[U_{R_1}(Q)] = (p + \lambda s - w)Q - (p + \lambda s - v) \int_0^Q F(x) dx - \lambda s\mu. $$ (10)

Under the wholesale price contract, the decision problem of retailer with capital constraint is

$$\begin{align*}
\max & \quad E[U_{R_1}(Q)] \\
\text{s.t.} & \quad 0 \leq Q \leq \frac{K}{w}.
\end{align*} $$ (11)

The supplier’s expected profit is

$$E[\Pi_{S_1}(Q, M)] = wQ - cM - c_r M \int_0^{(Q/M)} G(\varepsilon) d\varepsilon.$$ (12)

\quad

Theorem 2. Under the wholesale price contract, the optimal order quantity of the capital-constrained retailer and the optimal planned production of supplier are, respectively, given by

$$Q^*_1 = \min\left\{Q_\ast, \frac{K}{w}\right\},$$

$$M^*_1 = \frac{Q^*_1}{z_0},$$

(13)

where $Q_\ast = F^{-1}[(p + \lambda s - w)/(p + \lambda s - v)]$.

The proof is shown in Appendix. Under the wholesale price contract, supplier’s optimal expected profit is

$$E[\Pi_{S_1}(Q_1^*, M_1^*)] = (w - c_r G(z_0))Q_1^*.$$ (14)

From $w - c_r G(z_0) > 0$, we know that
strained retailer’s decision problem is

\[ \frac{Q^*_1}{Q_0} = F^{-1}\left( \frac{p + \lambda s - w}{p + \lambda s - v} \right) < F^{-1}\left( \frac{p + \lambda s - c \cdot G(z_0)}{p + \lambda s - v} \right) = Q^*_e. \]

\[ M^*_1 = \frac{Q^*_1}{z_0} = M_e. \]

(15)

According to the above equations, compare to the optimal order quantity of the supply chain system, the capital-constrained and loss-averse retailer has a lower optimal order quantity under the wholesale price contract. Therefore, the capital constraint leads to the constrained order quantity, and the coordination of supply chain under the wholesale price contract is not achieved.

5.2. Revenue Sharing Contract Model. For coordinating the supply chain under the environment of uncertain supply and demand, on the one hand, before the sales season, since the retailer is loss averse, the supplier reduces the wholesale price to decrease the loss of out-of-stock for the retailer, so that the capital-constrained retailer is able to order with the optimal quantity. On the other hand, to encourage supplier to provide enough products to fill retailer’s order, at the end of sales cycle, the retailer pays the supplier \((1 - \varphi)\) proportion of his sales revenue to make up for the supplier’s economic losses caused by the reduction of wholesale prices and the uncertainty of production.

Under the revenue sharing contract, the retailer’s utility function is

\[ U_{R2}(Q) = \varphi p \min(x, Q) + v[Q - x]^+ - wQ - \lambda s[x - Q]^+. \]

(16)

Then, the retailer’s expected utility function is

\[ E[U_{R2}(Q)] = (\varphi p + \lambda s - w)Q - (\varphi p + \lambda s - v) \int_0^Q F(x)dx - \lambda s u. \]

(17)

Under the revenue sharing contract, the capital constrained retailer’s decision problem is

\[ \max E[U_{R2}(Q)], \]

\[ \text{s.t. } 0 \leq Q \leq \frac{K}{w}. \]

(18)

The supplier’ expected profit is

\[ E[\Pi_{S2}(Q, M)] = (1 - \varphi)p \left( Q - \int_0^Q F(x)dx \right) + wQ - cM - cM \int_{(Q,M)}^Q G(z)dz. \]

(19)

Theorem 3. Under the revenue sharing contract, the optimal order quantity of retailer and the optimal planned production of supplier are, respectively, given by

\[ Q^*_2 = \min\{Q_1, K/w\}, \]

\[ M^*_2 = (Q^*_2/z_0), \]

where \( Q_2 = F^{-1}\left( \varphi p + \lambda s - w/\varphi p + \lambda s - v \right). \]

The proof of Theorem 3 is similar to Theorem 2.

5.3. Supply Chain Coordination

Theorem 4. If contract parameters \((w, \varphi)\) satisfy \(w \leq K/Q_c\) and

\[ \varphi = \frac{(v - w)(p + \lambda s - v)}{p(v + c \cdot G(z_0)) + v - \lambda s} + \frac{\lambda s}{p} \]

(20)

The retailer and the supply chain can acquire their optimal expected utility, and the supplier can acquire the optimal expected profit.

Proof. When the contract parameters \((w, \varphi)\) satisfy equality (16), it is obvious that \(Q_1 = Q_e\). Since \(w \leq (K/Q_c)\), we have \(Q_2 = Q_c \leq K/w\). Hence, \(Q^*_2 = Q_c, Q^*_2 = Q_c\). According to Theorem 1 and Theorem 3, it follows that \(M^*_2 = Q^*_2/z_0 = Q_c/z_0 = M_c\).

To ensure that the contract is accepted by all the supply chain members, the contract parameters need to satisfy

\[ \begin{align*}
E[U_{R2}(Q^*_2)] & \geq N_R, \\
E[\Pi_{S2}(Q^*_2, M^*_2)] & \geq N_S,
\end{align*} \]

(21)

where \(E[\Pi_{S2}(Q^*_2, M^*_2)]\) is the expected retained utility of retailer and \(E[U_{R2}(Q^*_2)]\) is the expected retained profit of supplier under wholesale price contracts, i.e.,

\[ \begin{align*}
N_R & = E[U_{R1}(Q_1)], \\
N_S & = E[\Pi_{S1}(Q_1, M_1)].
\end{align*} \]

\( \Box \)

Theorem 5. If contract parameters \((w, \varphi)\) satisfy

\[ w = v - \frac{(v - c \cdot G(z_0))(\varphi p + \lambda s - v)}{p + \lambda s - v} \]

(22)

and \(\varphi_1 \leq \varphi \leq \min\{\varphi_2, \varphi_3\}\), coordination of the supply chain can be realized through the revenue sharing contract, where

\[ \varphi_0 = (v - K/Q_c)\left( p + \lambda s - v/p(v + c \cdot G(z_0)) + (v - \lambda s)/p \right), \]

\[ \varphi_2 = \left[ p(N_R + \lambda s\mu) + (\lambda s - v)(N_S - EU^*_C)/p(EU^*_C + \lambda s\mu) \right], \]

and \(\varphi_2 = 1 - N_S(p + \lambda s - v)/p(EU^*_C + \lambda s\mu)\).

Proof. If contract parameters \((w, \varphi)\) satisfy the equality (17), it is obvious that \(w \leq K/Q_c\) under the condition of \(\varphi \leq \varphi_0\). From Theorem 4, we have \(Q^*_2 = Q_c, M^*_2 = M_c\). Then, it follows that

\[ E[U_{R2}(Q^*_2)] = (\varphi p + \lambda s - w)Q_c - (\varphi p + \lambda s - v) \int_0^{Q^*_2} F(x)dx - \lambda s u \geq N_R. \]

(23)

That is to say,

\[ \frac{p + \lambda s - c \cdot G(z_0)}{p + \lambda s - v} Q_c - \int_0^{Q^*_2} F(x)dx \geq \frac{N_R + \lambda s\mu}{\varphi p + \lambda s - v} \]

(24)

which implies \(\varphi \geq \varphi_1\).

Similarly, we have
\[
E[\Pi_{S2}(Q^*_2, M^*_2)] = [(1 - \varphi)p + w - c_sG(z_0)]Q_c \\
- (1 - \varphi)p \int_0^{Q_c} F(x)dx \geq N_S.
\]

That is to say,
\[
p(1 - \varphi)\left[(p + \lambda s - c_sG(z_0))Q_c - (p + \lambda s - \nu)\int_0^{Q_c} F(x)dx\right] \geq N_S(p + \lambda s - \nu),
\]
which implies \(p(1 - \varphi)(EU^*_C + \lambda s\mu) \geq N_S(p + \lambda s - \nu),\) then \(\varphi \leq \varphi_2.\)

6. Numerical Examples

Suppose the market demand and the production rate subject to different uniform distribution, i.e., \(x \sim U(0, 1000),\) \(\varepsilon \sim U(0.75, 0.95).\) The other parameters are given by \(p = 18,\) \(c = 8,\) \(c_s = 10,\) \(\nu = 5,\) \(s = 6,\) and \(\lambda = 1.2.\) In the centralized decision model, the optimal order quantity \(Q_c = 778.68,\) the optimal planned production \(M_c = 828.90,\) and the optimal expected utility of the supply chain \(EU^*_C = 2524.\)

Under the wholesale price contract \((w = 13),\) according to Theorem 2, we get \(Q_1 = 603.96,\) \(M_1 = 642.91.\) Take the retailer’s expected utility without capital constraints and the supplier’s expected profit as their respective retained utility and profit, respectively, then \(N_R = EU_{R1}(Q_1) = 84.15,\) \(N_S = EU_{S1}(Q_1, M_1) = 2131.5.\) When the retailer’s capital is constrained, i.e., \(K \leq \omega Q_1,\) the impact of capital on the retailer’s optimal order quantity and optimal expected utility is shown in Figure 1. According to Figure 1, when the retailer is capital constrained, less capital will lead to lower the optimal order quantity and expected utility.

Under the wholesale price contract, the impact of loss aversion on the optimal order quantity is reflected in Figure 2. From Figure 2, we know that the optimal order quantity is increasing with the increasing of risk aversion level \(\lambda,\) which suggests that if the retailer is averse to the loss.
of shortages, the order quantity will be at a higher level. On
the contrary, if the retailer is close to risk neutral, the order
quantity will be lower.

Under the revenue sharing contract, let $K = 6500$. When
$\lambda \in [1,2]$ and $\varphi \in [0.4,0.9]$, the optimal order quantity is
calculated according to Theorem 3 as shown in Figure 3. As
can be seen from Figure 3, if $\lambda$ is constant, the optimal order
quantity increases with the increase of the $\varphi$; on the contrary,
if $\varphi$ is constant, the optimal order quantity increases with the
increase of the $\lambda$.

According to Theorem 5, we obtain $\varphi_0 = 0.7181$, $\varphi_1 = 0.5529$ , and $\varphi_2 = 0.6094$. Taking points with 0.008
intervals on $[0.55,0.61]$, the retailer’s expected utility and
the supplier’s expected profit are given in Table 1. The results
demonstrate that when $\varphi$ decreases, the expected utility of retailer increases while the supplier’s expected profit decreases. Furthermore, the impact of production uncertainty on the supply chain performance is shown in Table 2. The

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\omega$</th>
<th>$uQ_c$</th>
<th>$E[U_{R2}(Q^*_2)]$</th>
<th>$E[\Pi_{S2}(Q^<em>_2,M^</em>_2)]$</th>
<th>$\Delta_R$</th>
<th>$\Delta_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5500</td>
<td>7.6780</td>
<td>5978.69</td>
<td>68.34</td>
<td>2455.67</td>
<td>−15.82</td>
<td>324.12</td>
</tr>
<tr>
<td>0.5570</td>
<td>7.7059</td>
<td>6000.40</td>
<td>106.54</td>
<td>2417.47</td>
<td>22.37</td>
<td>285.92</td>
</tr>
<tr>
<td>0.5640</td>
<td>7.7338</td>
<td>6022.12</td>
<td>144.74</td>
<td>2379.27</td>
<td>60.57</td>
<td>247.73</td>
</tr>
<tr>
<td>0.5710</td>
<td>7.7617</td>
<td>6043.83</td>
<td>182.94</td>
<td>2341.07</td>
<td>98.77</td>
<td>209.53</td>
</tr>
<tr>
<td>0.5780</td>
<td>7.7896</td>
<td>6065.55</td>
<td>221.14</td>
<td>2302.87</td>
<td>136.97</td>
<td>171.33</td>
</tr>
<tr>
<td>0.5850</td>
<td>7.8174</td>
<td>6087.26</td>
<td>259.34</td>
<td>2264.67</td>
<td>175.17</td>
<td>133.13</td>
</tr>
<tr>
<td>0.5920</td>
<td>7.8453</td>
<td>6108.98</td>
<td>297.53</td>
<td>2226.47</td>
<td>213.37</td>
<td>94.93</td>
</tr>
<tr>
<td>0.5990</td>
<td>7.8732</td>
<td>6130.69</td>
<td>335.73</td>
<td>2188.27</td>
<td>251.57</td>
<td>56.73</td>
</tr>
<tr>
<td><strong>0.6060</strong></td>
<td><strong>7.9011</strong></td>
<td><strong>6152.41</strong></td>
<td><strong>373.93</strong></td>
<td><strong>2150.07</strong></td>
<td><strong>289.77</strong></td>
<td><strong>18.53</strong></td>
</tr>
<tr>
<td>0.6130</td>
<td>7.9290</td>
<td>6174.12</td>
<td>412.13</td>
<td>2111.87</td>
<td>327.97</td>
<td>−19.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$E[\epsilon]$</th>
<th>$\varphi$</th>
<th>$\omega$</th>
<th>$Q^*_2$</th>
<th>$M^*_2$</th>
<th>$E[U_{R2}]$</th>
<th>$E[\Pi_{S2}]$</th>
<th>$EU^*_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.6, 1]</td>
<td>0.80</td>
<td>0.62</td>
<td>8.32</td>
<td>752.48</td>
<td>752.48</td>
<td>195.54</td>
<td>1923.3</td>
<td>2118.8</td>
</tr>
<tr>
<td>[0.7, 1]</td>
<td>0.85</td>
<td>0.58</td>
<td>7.83</td>
<td>779.42</td>
<td>789.35</td>
<td>236.10</td>
<td>2288.1</td>
<td>2504.2</td>
</tr>
<tr>
<td>[0.8, 1]</td>
<td>0.90</td>
<td>0.55</td>
<td>7.38</td>
<td>802.49</td>
<td>819.03</td>
<td>283.19</td>
<td>2621.0</td>
<td>2904.2</td>
</tr>
</tbody>
</table>

results show that, with the increase of the production rate,
the optimal order quantity and optimal planned production,
the expected utility of retailer and supply chain, the sup-
plier’s expected profit are all increasing. Yield rate has a
significant impact on the performance of the supply chain,
and the supplier should use various technical means to
greatly improve the yield rate.

Then, the impact of the loss aversion level $\lambda$ on the
revenue sharing coefficient $\varphi$ is presented in Table 3. When $\lambda$
increases, $\varphi$ increases, and the value range of $\varphi$ becomes
smaller, which means that the retailer and the supplier do
not have much room for negotiation under a high level of \( \lambda \).

Next, the impacts of the loss aversion level \( \lambda \) on the optimal order quantity and optimal planned production, and retailer’s expected utility and supplier’s expected profit are shown in Figures 4 and 5, respectively. We vary \( \lambda \) from 1 to 1.5 in steps of 0.025, and take \( \phi = 0.58 \) from all of the intersection sets of \([\varphi_1, \varphi_2]\) while the other parameters are unchanged. It is obvious that the optimal order quantity and optimal planned production are all increasing in \( \lambda \). However, the retailer’s expected utility is decreasing in \( \lambda \) while the supplier’s expected profit is increasing in \( \lambda \). Finally, the impacts of the unit out of stock loss \( s \) on retailer’s expected utility are shown in Figure 6; the retailer’s expected utility is decreasing in \( s \). The results indicate that the retailer would like to increase the order quantity in order to avoid the out-of-stock losses. The stronger the willingness to avoid losses, the larger the order quantity may be. The retailer’s expected utility is closely related to its loss aversion attitude and the level of out-of-stock loss.

7. Conclusion and Management Inspiration

The behavior factors have been widely concerned in supply chain management. If the characteristics of these behaviors are ignored, the actual decision of supply chain members is quite different from the optimal decision obtained by mathematical model. Therefore, the research of supply chain management based on the perspective of behavior is significant. This paper deals with supply chain coordination with capital-constrained and loss-averse retailer under uncertain supply and demand. Based on the mental account theory, we designed a new utility function to account the loss aversion. Theoretical analysis indicates that when the contract parameters are properly selected, the revenue sharing contract can coordinate the supply chain. The sensitivity analysis is carried out by several numerical examples. The results show that retailer’s aversion to loss has a certain influence on the value range of revenue sharing ratio. The research enriches the theory of supply chain coordination and has certain guiding significance to supply chain management decisions when production and demand are uncertain.

### Table 3: The range of \([\varphi_1, \varphi_2]\) under different values of \( \lambda \).

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>([\varphi_1, \varphi_2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>[0.5495, 0.6118]</td>
</tr>
<tr>
<td>1.1</td>
<td>[0.5512, 0.6105]</td>
</tr>
<tr>
<td>1.2</td>
<td>[0.5529, 0.6094]</td>
</tr>
<tr>
<td>1.3</td>
<td>[0.5552, 0.6080]</td>
</tr>
</tbody>
</table>

![Figure 4: Optimal order quantity and optimal planned production under different values of \( \lambda \).](image1)

![Figure 5: Retailer’s expected utility and supplier’s expected profit under different values of \( \lambda \).](image2)

![Figure 6: Retailer’s expected utility with \( \lambda \) under different values of \( s \).](image3)
Management inspirations of this paper are as follows: (1) Revenue sharing contract can promote the realization of the overall revenue maximization goal of the supply chain and drive the coordinated operation of the supply chain by formulating the revenue sharing coefficient and reasonably distributing the revenue between the supplier and retailer. (2) Reasonable supply chain contract design can alleviate the conflict and competition between enterprises, mobilize the enthusiasm of the cooperation between supply chain enterprises, and effectively improve the profit level of the whole supply chain and all parties. (3) In reality, when making decisions, supply chain members do not always aim to maximize their own interests. Different decision makers have different risk preferences in the face of potential risks. Only from the perspective of human behavior and psychological cognition, can we effectively guide the practice of supply chain management in complex environment.

Some further research can be carried out based on this paper. For example, the risk aversion of supplier is not considered in this paper, but in reality, supplier may also be loss-averse due to the uncertainty of production. Another possible extension is that the retailer can address the shortfall of capital through external financing, such as bank loans. Other types of irrational behaviors, such as fairness concerns and overconfidence, can be considered under our supply chain structure.

Appendix

Calculation of mathematical expectations of supply chain system utility from supply chain system utility in Section 4 as follow

$$E[U_C(Q,M)] = p \left( \int_0^Q x f(x)dx \int_0^{\infty} Q f(x)dx \right) + \nu \left( \int_0^Q (Q-x)f(x)dx - \lambda s \right) \left( \int_0^{\infty} (x-Q)f(x)dx - c_1 \int_0^{Q/M} (Q-\varepsilon M)g(\varepsilon)d\varepsilon - cM, \right.$$

$$= p \left( \int_0^Q x df(x) + \nu \left( 1 - \int_0^Q x f(x)dx \right) \right) + \nu \left( \int_0^Q (Q-x)F(x)dx \right) - \lambda s \left( \int_0^{\infty} F(x)dx \right) - c_1 \int_0^{Q/M} (Q-\varepsilon M)g(\varepsilon)d\varepsilon - cM, \right.$$
Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the Research Basic Ability Improvement Project of Young and Middle-Aged Teachers in Guangxi Colleges and Universities with Grant 2021KY1557 and the Research Funds of Guangxi University in Guangxi Colleges and Universities with Grant Improvement Project of Young and Middle-Aged Teachers.

Zhis work was supported by the Research Basic Ability Improvement Project of Young and Middle-Aged Teachers.

Acknowledgments

regarding the publication of this paper.

Authors declare that there are no conflicts of interest Conflicts of Interest

References


