

Research Article

Fractional Study for Transient Free Convection Flow in a Channel with Mittag-Leffler Memory

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Fractional-order convective transient flow of viscous and incompressible fluids transiting through two infinite hot parallel upright plates is investigated analytically in the presence of chemical reaction, radiative heat flux, and mass diffusion at the boundary. A physical model for transient incompressible unsteady flow is developed with a comparatively new fractional derivative, namely, Atangana–Baleanu with nonsingular, nonlocal kernel. The developed fractional model is studied with means of an integral transform, i.e., Laplace transform method. Results obtained for the concentration, velocity, and temperature are expressed in the form of generalized $M_q^p(y)$ function. The impact of various physical parameters like fractional and flow parametric quantities is demonstrated diagrammatically. At last, we envisioned that for the fractional model, temperature and concentration could be enhanced for smaller fractional parameter α values while velocity for larger values of α , respectively. The proposed model gives the better results in the presence of memory effect besides the Caputo and Caputo–Fabrizio model when compared with the existing literature.

1. Introduction

Processes affecting coupled mass and heat conveyance occur frequently in nature. This happens not only because of temperature gradient but also due to the concentration difference or a combination of these two differences mentioned earlier. Buoyancy forces stimulating the flow with the aggregated effect of mass and thermal diffusion have been of major concern for last three and a half decades. Nowadays, these specified problems have gained the attention of many researchers for numerous technological and engineering applications, specified as mass and heat transfer assorted with storage of nuclear fuel rubble, coal gasification below the earth's surface, hydrology of ground water, chemical engineering, cooling of processors, and so on.

Free convective flow through vertical channels had been studied extensively due to its important application in engineering and applied sciences [1]. Harris et al. [2, 3] had investigated the results of free convective transient flow transiting by an upright erect plate engrafted in poriferous media submitted to an abrupt transfer in heat flux and superficial temperature. The heat yielded by viscid dissipation gives an increment in temperature approaching the wall resulted in viscousness reduction and a substantial stratification in its profile, which bears upon H.T.R, i.e., heat transfer rate [4, 5]. Gupta underwent the study of steady, transient, free convection of an electrically transmitting fluids passing by a perpendicular plate, bearing the magnetic flux [6]. Toki applied the Laplace transform to solve transient convective rate of flow inside an oscillating poriferous. The

problem is solved by [7]. Sharma et al. had enquired the oscillatory reactive hydromagnetics innate or free convective boundary levels in poriferous media with hybrid rate of flow impressions [8]. An analysis of the transient, buoyancy effectuated stream, and heat conveyance in a Darcian fluid drenched in permeable medium adjoining to an abruptly heated infinite plate was presented by Haq and Mulligan [9]. Ramanaiah and Kumaran had discussed free convection of fluid passing through a passable cylinder and cone, with radiation boundary condition [10].

Results of the transient free convective flow across a propelling vertical or inclined plate and cylinder were incurred by Soundalgekar and Ganesan [11]. Takhar et al. had talked about the upshot of thermo-corporeal measures on free convective vaporous stream across an isothermic perpendicular cone in steady-state current, in which caloric conduction, absolute viscousness, and specific heat at unvarying pressure were developed as a power law magnetic fluctuation with inviolable temperature. In their research composition, they reasoned out the information of transfer of heat step-ups with suction or drop-offs with injectant [12].

Transient convection contains fundamental concern in several progressive and environmental situations as in air conditioner systems, in human consolation in buildings, in atmospherical flows, in thermal regulating processes, in cooling down of electronic machines, in protection of energy systems, and so on. It is pertinent to mention here about transient flow. What it is about? Flow is said to be unsteady or transient if velocity of fluid is just not function of space variable or position in xy -plane but also depending on time. A lot of work accounted in the literature dealt with fixed velocity fields and temperature domains but merely a small figure administer with time varying boundary checks, either in natural, forced, or mixed convection [13–17].

Ganesan and Muthucumaraswamy had numerically investigated incompressible, viscid, transient fluid's flow regime passing through a semi-infinite isothermal plate satisfying natural convection [18]. Siddiqa et al. [19] had investigated transient effects on free convection of heat transfer by natural convection along a vertical wavy surface. Singh [20] and Nanda and Sharma [21] had investigated the suction effects of free convective, transient flow passing through vertical porous plate. A numerical study to analyze transient effects of natural heat and mass conveyance or free convection in power law fluids passing through upright plate embedded in poriferous medium was carried out by Nasser [22].

In this modern era, the study of fractional calculus becomes hot topic because of its huge applications in all disciplines of science and engineering. Researchers have keen interest to formulate the physical problems with noninteger derivatives because fractional-order models provide the much efficient description of model [23–28]. Sarwar et al. studied the non-Newtonian fractional brinkman type fluid with AB derivative [29]. Aleem et al. had investigated channel flow of MHD Jeffrey fluid between two heated vertical parallel plates [30]. In another study, a

fractional heat and mass transferal model was developed and observed that MWCT-based nanofluids are efficiently good for heat transfer as compared with SWCTs-CMC-based nanofluids [31]. For significance of time fractional operator in heat transfer analysis, refer to [32–38]. Further literature could be seen in [39–44].

In this paper, our major objective is to demonstrate free convective, unsteady, flow of viscous, and incompressible fluids passing through two infinite hot parallel vertical plates. We have extended the classical derivative model to noninteger order differential Atangana–Baleanu model of order α . The method of Laplace transform is accustomed to acquire the results. Equations of dimensionless temperature, velocity, and concentration fields have been solved analytically, and results are compared. The graphical analysis is made to envision the effect of fractional and tangible flow parameters on velocity, concentration, and temperature by MathCad and Mathematica softwares. Moreover, the impact of fractional order and other parameters is presented graphically. The obtained results are compared with the existing literature for validation.

2. Problem Formulation

Let us undertake an incompressible, unsteady, free convective liquid passing through two parallel upright plates possessing the traits of temperature gradient and mass diffusion. We have fixed the plates in xy - plane of the Cartesian coordinate system. One of the plate is fixed along x - axis as shown in figure while y -axis is normal to the plate. The following assumptions have been made:

- (1) At time $t \leq 0$, the plates and the fluent possess ambient concentration C_d and temperature T_d
- (2) At time $t > 0$, concentration and temperature of the fluid at $y = 0$ are changed to C_w and T_w , respectively
- (3) The change in temperature and concentration has created free convection flows, presented in Figure 1.

$$u_t = \nu u_{yy} + g\beta(T - T_d) + g\beta^*(C - C_d), \quad (1)$$

$$\rho C_p T_t = kT_{yy} - q_{ry}, \quad y, t > 0, \quad (2)$$

Equations governing unsteady flow are obtained by Boussinesq's approximations [1]. where q_r is radiative heat flux and by Rosseland approximation [45], the above equation will get the form [46] as follows:

$$T_t = \frac{k}{\rho C_p} \left(1 + \frac{16\sigma^* T_d^3}{3kk^*} \right) T_{yy}, \quad y, t > 0, \quad (3)$$

where σ^* is the Stefan–Boltzmann constant, ρ is the density, k is the thermic conduction, k^* represents the mean value of heat absorption parameter, and C_p stands for the specific heat with invariant pressure.

$$C_t = DC_{yy} - K[C - C_d]. \quad (4)$$

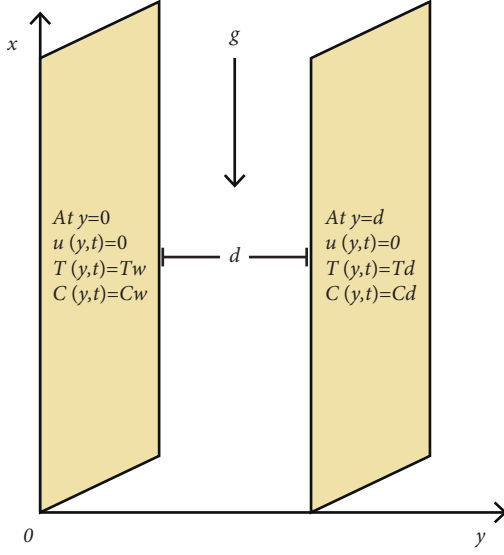


FIGURE 1: Problem orientation.

K is chemical reaction parameter. Appropriate IBCs are

$$\begin{aligned} t = 0: u(t, y) &= 0, \\ T(t, y) &= T_d, \\ C(t, y) &= C_d, \\ 0 \leq y &\leq d, \end{aligned} \quad (5)$$

$$\begin{aligned} 0 < t: u(t, y) &= 0, \\ T(t, y) &= T_w, \\ C(t, y) &= C_w, \\ \text{at } y &= 0, \end{aligned} \quad (6)$$

$$\begin{aligned} 0 < t: u(t, y) &= 0, \\ T(t, y) &= T_d, \\ C(t, y) &= C_d, \\ \text{at } y &= d. \end{aligned} \quad (7)$$

In order to make problem dimensionless, the following variables or parameters are used:

$$\begin{aligned} t^* &= \frac{\nu t}{d^2}, \\ y^* &= \frac{y}{d}, \\ u^* &= \frac{\nu u}{d^2 g \beta (T_w - T_d)}, \\ Gr &= \frac{g \beta (T_w - T_d) d^3}{\nu^2}, \\ N &= \frac{Gm}{Gr}, \end{aligned}$$

$$\begin{aligned} \theta &= \frac{T - T_d}{T_w - T_d}, \\ \Phi &= \frac{C - C_d}{C_w - C_d}, \\ Gm &= \frac{g \beta^* (C_w - C_d) d^3}{\nu^2}, \\ Sc &= \frac{\nu}{D}, \\ Pr &= \frac{\mu C_p}{\kappa}, \\ Nr &= \frac{16 \sigma^* T_d^3}{3 k k^*}, \\ Pr_{eff} &= \frac{Pr}{(1 + Nr)}, \\ K_r &= \frac{K d^2}{\nu}. \end{aligned} \quad (8)$$

Gm , Gr , Pr_{eff} , Nr , and K_r are dimensionless mass and thermal Grashof numbers, effective Prandtl number, and conduction-radiation and chemical reaction parameters, respectively.

Equations (1), (3), and (4) become

$$u_t - (\theta + NC + u_{yy}) = 0, \quad (9)$$

$$Pr_{eff} \theta_t - \theta_{yy} = 0, \quad (10)$$

$$Sc C_t - \Phi_{yy} + Kr C = 0, \quad (11)$$

with dimensionless IBCs (initial and boundary conditions) given in equations (5)–(7).

$$\begin{aligned} t = 0: u(t, y) &= 0, \\ \theta(t, y) &= 0, \\ C(t, y) &= 0, \end{aligned} \quad (12)$$

for $0 \leq y \leq 1$,

$$\begin{aligned} 0 < t: u(t, y) &= 0, \\ \theta(t, y) &= 1, \\ C(t, y) &= 1, \\ \text{at } y &= 0, \end{aligned} \quad (13)$$

$$\begin{aligned} 0 < t: u(t, y) &= 0, \\ \theta(t, y) &= 0, \\ C(t, y) &= 0, \\ \text{at } y &= 1. \end{aligned} \quad (14)$$

Equations (9)–(11) are nonhomogenous PDE's of second order. The fractional model is obtained by changing time derivative with fractional derivative given as follows:

$${}^{ABC}D_t^\alpha u(t, y) - (\theta(y, t) + NC(t, y) + u_{yy}(t, y)) = 0, \quad (15)$$

$${}^{ABC}D_t^\alpha \theta(t, y) - \frac{1}{Pr_{eff}} \theta_{yy}(t, y) = 0, \quad (16)$$

$$C_{yy} - Sc {}^{ABC}D_t^\alpha C(t, y) + K_r C(t, y) = 0. \quad (17)$$

In 2016, Atangana and Baleanu introduced a new fractional-order operator of differentiation in Riemann–Liouville and Caputo sense, which is called Atangana–Baleanu (AB) derivatives. This new AB derivative is nonsingular and has nonlocal kernel and possesses the long memory due to the existence of Mittag-Leffler kernel which is the generalization of the exponential kernel. This new AB fractional derivative in Caputo sense of order $\alpha > 0$ is defined as follows [47, 48]:

$${}^{ABC}D_{a,t}^\alpha f(t) = \frac{M(\alpha)}{1-\alpha} \int_a^t E_\alpha \left[\frac{-\alpha(t-\tau)^\alpha}{1-\alpha} \right] f'(\tau) d\tau, \quad t > a, \quad (18)$$

where $E_\alpha(x) = \sum_{k=1}^{\infty} x^k / (\Gamma(\alpha k + 1))$ is Mittag-Leffler function and $M(\alpha)$ is normalization function satisfying the conditions $M(0) = M(1) = 0$.

The Laplace transformation of ABC fractional derivative is

$$L\{{}^{ABC}D_t^\alpha f(t, y)\} = \frac{s^\alpha L\{f(t, y)\} - s^{\alpha-1} f(y, 0)}{s^\alpha(1-\alpha) + \alpha}. \quad (19)$$

3. Fractional-Order IBV Problem of Fluid

In this section, we develop the fractional model of equations (15)–(17) by using relation given in equation (19), and we get partial differential equations with associated initial and boundary conditions:

$$\frac{s^\alpha \xi}{s^\alpha + \xi \alpha} \bar{u}(y, s) = \frac{\partial^2 \bar{u}(y, s)}{\partial y^2} + \bar{\theta}(y, s) + N \bar{C}(y, s), \quad (20)$$

$$Pr_{eff} \frac{s^\alpha \xi}{s^\alpha + \xi \alpha} \bar{\theta}(y, s) = \frac{\partial^2 \bar{\theta}(y, s)}{\partial y^2}, \quad (21)$$

$$Sc \frac{s^\alpha \xi}{s^\alpha + \xi \alpha} \bar{C}(y, s) = \frac{\partial^2 \bar{C}(y, s)}{\partial y^2} - K_r \bar{C}(y, s), \quad (22)$$

associated with initial and boundary conditions:

$$\begin{aligned} \bar{u}(y, 0) &= 0, \\ \bar{T}(y, 0) &= 0, \end{aligned} \quad (23)$$

$$\begin{aligned} \bar{C}(y, 0) &= 0, \\ \bar{u}(0, s) &= 0, \\ \bar{T}(0, s) &= \frac{1}{s}, \\ \bar{C}(0, s) &= \frac{1}{s}, \end{aligned} \quad (24)$$

$$\begin{aligned} \bar{u}(1, s) &= 0, \\ \bar{T}(1, s) &= 0, \\ \bar{C}(1, s) &= 0, \end{aligned} \quad (25)$$

where $\xi = 1/1 - \alpha$.

4. Analytical Solution of Fractional-Order IBV Problem of Fluid

Equations (21) and (22) are solved separately, and then their results are substituted in equation (20), i.e., momentum equation.

4.1. Results of Fractional Temperature Field. Utilizing the Laplace transform to equation (21), we obtain

$$\begin{aligned} \left[\frac{\partial^2}{\partial y^2} - Pr_{eff} \frac{s^\alpha \xi}{s^\alpha + \xi \alpha} \right] \bar{\theta}(y, s) &= 0, \\ \bar{\theta}(y, s) &= c_3 \exp \left(-y \sqrt{Pr_{eff} \frac{s^\alpha \xi}{s^\alpha + \xi \alpha}} \right) \\ &+ c_4 \exp \left(y \sqrt{Pr_{eff} \frac{s^\alpha \xi}{s^\alpha + \xi \alpha}} \right). \end{aligned} \quad (26)$$

Using conditions from equations (24) and (25) in equation (26), we have

$$\bar{\theta}(y, s) = \frac{1}{s} \frac{\sinh \left((1-y) \sqrt{Pr_{eff} (s^\alpha \xi / s^\alpha + \xi \alpha)} \right)}{\sinh \left(\sqrt{Pr_{eff} (s^\alpha \xi / s^\alpha + \xi \alpha)} \right)}, \quad (27)$$

$$\bar{\theta}(y, s) = \frac{1}{s} \sum_{m=0}^{\infty} \left[e^{-(2m+y) \sqrt{Pr_{eff} (s^\alpha \xi / s^\alpha + \xi \alpha)}} - e^{-(2m+2-y) \sqrt{Pr_{eff} s^\alpha \xi / s^\alpha + \xi \alpha}} \right].$$

Equation (27) can be written in suitable form as

$$\bar{\theta}(y, s) = \frac{1}{s} + \sum_{m=0}^{\infty} \sum_{m_1=1}^{\infty} \sum_{m_2=0}^{\infty} \frac{(-2m+y)^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2} \Gamma(m_1+m_2)}{m_1!m_2!s^{1+\alpha m_2}} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)} - \sum_{m=0}^{\infty} \sum_{m_3=1}^{\infty} \sum_{m_4=0}^{\infty} \frac{(y-2m-2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4} \Gamma(m_3+m_4)}{m_3!m_4!s^{1+\alpha m_4}} \frac{\Gamma(m_3+m_4)}{\Gamma(m_3)}.$$

Now, by applying inverse Laplace transform to equation (28),

$$\theta(t, y) = 1 + \sum_{m=0}^{\infty} \sum_{m_1=1}^{\infty} \sum_{m_2=0}^{\infty} \frac{(-2m+y)^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2} t^{\alpha m_2} \Gamma(m_1+m_2)}{m_1!m_2!} \frac{t^{\alpha m_2} \Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(1+\alpha m_2)} - \sum_{m=0}^{\infty} \sum_{m_3=1}^{\infty} \sum_{m_4=0}^{\infty} \frac{(y-2m-2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4} t^{\alpha m_4} \Gamma(m_3+m_4)}{m_3!m_4!} \frac{t^{\alpha m_4} \Gamma(m_3+m_4)}{\Gamma(m_3)\Gamma(1+\alpha m_4)}.$$

Further, the solution can be expressed in more general M-function [49] form as

$$\theta(t, y) = 1 + \sum_{m=0}^{\infty} \sum_{m_1=1}^{\infty} \frac{(-2m+y)^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2}}{m_1!} M_1^1 \left[(-\xi\alpha)t^\alpha \middle| \begin{matrix} (m_1,1) \\ (m_1,0), (1,\alpha) \end{matrix} \right] - \sum_{m=0}^{\infty} \sum_{m_3=1}^{\infty} \frac{(y-2m-2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2}}{m_3!} M_2^1 \left[(-\xi\alpha)t^\alpha \middle| \begin{matrix} (m_3,1) \\ (m_3,0), (1,\alpha) \end{matrix} \right].$$

In case of ordinary temperature, when $\alpha \rightarrow 1$ and $Nr = 0$, our solution is reduced to the known results obtained in [1], and for the validation, Figure 2 is presented.

4.2. Results of Fractional Concentration Field. Solving equation (22) by using conditions from (24) and (25) in a similar pattern as presented for temperature field, solution for concentration field is

$$C(t, y) = 1 + \sum_{l=0}^{\infty} \sum_{l_1=1}^{\infty} \sum_{l_2=0}^{\infty} \frac{(-2l+y)^{l_1} (K_r)^{(l_1/2)-l_2} (Sc\xi)^{l_2}}{l_1!l_2!} M_2^1 \left[(-\xi\alpha)t^\alpha \middle| \begin{matrix} (l_2,1) \\ (l_2,0), (1,\alpha) \end{matrix} \right] - \sum_{l=0}^{\infty} \sum_{l_4=1}^{\infty} \sum_{l_5=0}^{\infty} \frac{(y-2l-2)^{l_4} (K_r)^{(l_4/2)-l_5} (Sc\xi)^{l_5}}{l_4!l_5!} M_2^1 \left[(-\xi\alpha)t^\alpha \middle| \begin{matrix} (l_5,1) \\ (l_5,0), (1,\alpha) \end{matrix} \right].$$

In case of ordinary concentration when $\alpha \rightarrow 1$ and $K_r = 0$, our solution is reduced to the known results obtained in [1], and for validation, Figure 3 is presented.

4.3. Results of Fractional Velocity Field. Solving equation (20) after substituting equations (27) and (31), we get

$$\left[\frac{\partial^2}{\partial y^2} - \frac{s^\alpha \xi}{s^\alpha + \xi\alpha} \right] \bar{u}(y, s) = -\frac{1}{s} \sum_{m=0}^{\infty} \left[e^{-(2m+y)\sqrt{\text{Pr}_{\text{eff}}(s^\alpha \xi/s^\alpha + \xi\alpha)}} - e^{-(2m+2-y)\sqrt{\text{Pr}_{\text{eff}}s^\alpha \xi/s^\alpha + \xi\alpha}} \right] - \frac{N}{s} \sum_{l=0}^{\infty} \left[e^{-(2l+y)\sqrt{Sc(s^\alpha \xi/s^\alpha + \xi\alpha) + K_r}} - e^{-(2l+2-y)\sqrt{Sc(s^\alpha \xi/s^\alpha + \xi\alpha) + K_r}} \right].$$

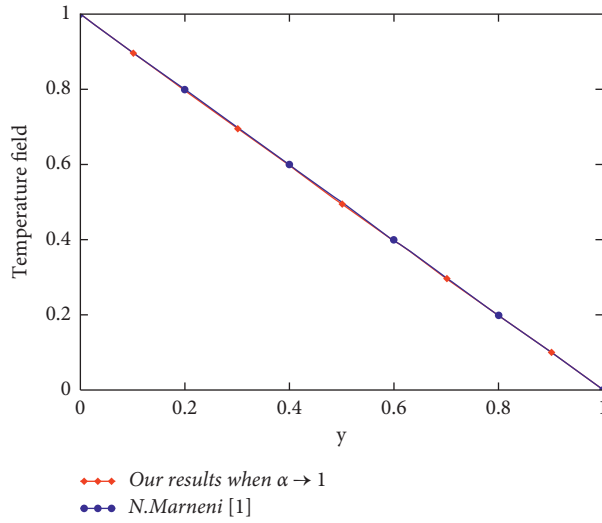


FIGURE 2: Comparison for $\theta(t, y)$.

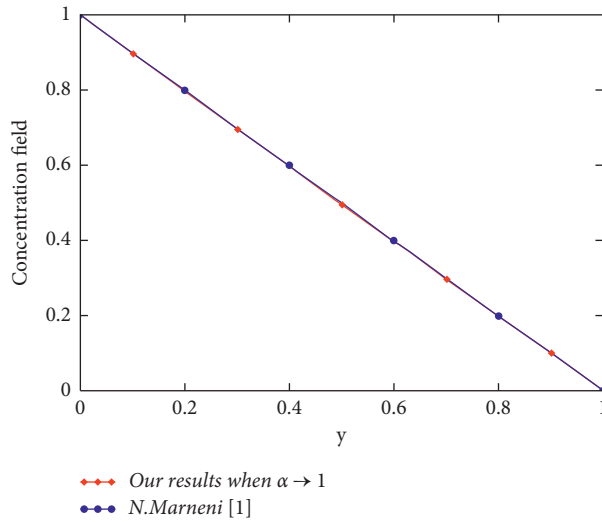


FIGURE 3: Comparison for $C(t, y)$.

Solving equation (32) subject to conditions (24) and (25),

$$\begin{aligned} \bar{u}(y, s) = & \frac{N}{s} \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{e^{-(2l+y)\sqrt{Sc(s^\alpha \xi/s^\alpha + \xi\alpha) + K_r}} - e^{-(2l+2-y)\sqrt{Sc(s^\alpha \xi/s^\alpha + \xi\alpha) + K_r}}}{K_r + (Sc - 1)(s^\alpha \xi/s^\alpha + \xi\alpha)} \right] \\ & \cdot \left[e^{-(2n+y)\sqrt{(s^\alpha \xi/s^\alpha + \xi\alpha)}} - e^{(y-2n-2)\sqrt{(s^\alpha \xi/s^\alpha + \xi\alpha)}} \right] \\ & + \frac{1}{s} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{e^{-(2m+y)\sqrt{Pr_{eff}(s^\alpha \xi/s^\alpha + \xi\alpha)}} - e^{-(2m+2-y)\sqrt{Pr_{eff}(s^\alpha \xi/s^\alpha + \xi\alpha)}}}{(Pr_{eff} - 1)(s^\alpha \xi/s^\alpha + \xi\alpha)} \right] \end{aligned}$$

$$\begin{aligned}
 & \cdot \left[e^{-(2n+y)\sqrt{(s^\alpha \xi/s^\alpha + \xi \alpha)}} - e^{(y-2n-2)\sqrt{(s^\alpha \xi/s^\alpha + \xi \alpha)}} \right] \\
 & - \frac{N}{s} \sum_{l=0}^{\infty} \left[\frac{e^{-(2l+y)\sqrt{Sc(s^\alpha \xi/s^\alpha + \xi \alpha) + K_r}} - e^{-(2l+2-y)\sqrt{Sc(s^\alpha \xi/s^\alpha + \xi \alpha) + K_r}}}{K_r + (Sc-1)(s^\alpha \xi/s^\alpha + \xi \alpha)} \right] \\
 & - \frac{1}{s} \sum_{m=0}^{\infty} \left[\frac{e^{-(2m+y)\sqrt{Pr_{eff}(s^\alpha \xi/s^\alpha + \xi \alpha)}} - e^{-(2m+2-y)\sqrt{Pr_{eff}(s^\alpha \xi/s^\alpha + \xi \alpha)}}}{(Pr_{eff}-1)(s^\alpha \xi/s^\alpha + \xi \alpha)} \right].
 \end{aligned} \tag{33}$$

Equation (33) can be written as

$$\begin{aligned}
 \bar{u}(y, s) = & \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{\infty} \sum_{l_3=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[N \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \bar{\Phi}_1 - N \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \bar{\Phi}_2 \right] \\
 & - \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l_4=0}^{\infty} \sum_{l_5=0}^{\infty} \sum_{l_6=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[N \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \bar{\Phi}_3 - N \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \bar{\Phi}_4 \right] \\
 & + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \sum_{m_3=0}^{\infty} \sum_{m_4=0}^{\infty} \left[\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \bar{\Phi}_5 - \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \bar{\Phi}_6 \right] \\
 & - \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m_3=0}^{\infty} \sum_{m_4=0}^{\infty} \sum_{m_5=0}^{\infty} \left[\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \bar{\Phi}_7 + \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \bar{\Phi}_8 \right] \\
 & - N \sum_{l=0}^{\infty} \sum_{l_1=1}^{\infty} \sum_{l_2=0}^{\infty} \sum_{l_3=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[\frac{(-(2l+y))^{l_1} (Sc\xi)^{l_2} (\xi(1-Sc))^{l_7} (-\xi\alpha)^{l_3+l_8}}{l_1!l_2!l_3!l_8!(K_r)^{1-(l_1/2)+l_2+l_7} s^{1+al_3+al_8}} \frac{\Gamma(l_2+l_3)\Gamma(l_7+l_8)}{\Gamma(l_2)\Gamma(l_7)} \right] \\
 & + N \sum_{l=0}^{\infty} \sum_{l_4=1}^{\infty} \sum_{l_5=0}^{\infty} \sum_{l_6=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[\frac{(y-2l-2)^{l_4} (Sc\xi)^{l_5} (\xi(1-Sc))^{l_7} (-\xi\alpha)^{l_6+l_8}}{l_4!l_5!l_6!l_8!(K_r)^{1-(l_4/2)+l_5+l_7} s^{1+al_6+al_8}} \frac{\Gamma(l_5+l_6)\Gamma(l_7+l_8)}{\Gamma(l_5)\Gamma(l_7)} \right] \\
 & - \sum_{m=0}^{\infty} \sum_{m_1=1}^{\infty} \sum_{m_2=0}^{\infty} \sum_{m_3=0}^{\infty} \left[\frac{(-(2m+y))^{m_1} (Pr_{eff}\xi)^{m_1/2} (-\xi\alpha)^{m_2+m_3}}{m_1!m_2!(Pr_{eff}-1)s^{1+am_2+am_3}} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)} \right] \\
 & + \sum_{m=0}^{\infty} \sum_{m_3=1}^{\infty} \sum_{m_4=0}^{\infty} \sum_{m_5=0}^{\infty} \left[\frac{(y-2m-2)^{m_3} (Pr_{eff}\xi)^{m_3/2} (-\xi\alpha)^{m_4+m_5}}{m_3!m_4!s^{1+am_4+am_5}} \frac{\Gamma(m_3+m_4)}{\Gamma(m_3)} \right],
 \end{aligned} \tag{34}$$

where

$$\begin{aligned}
 \bar{\Phi}_1 = & \left[\frac{(-(2l+y))^{l_1} (Sc\xi)^{l_2} (-\xi\alpha)^{l_3+l_8+n_2}}{l_1!l_2!l_3!l_8!n_1!n_2!(K_r)^{1-(l_1/2)+l_2+l_7}} \frac{(\xi(1-Sc))^{l_7} (-2n+y)^{n_1} (\xi)^{n_1/2} \Gamma(l_2+l_3)\Gamma(l_7+l_8)\Gamma((n_1/2)+n_2)}{s^{1+al_3+al_8+an_2} \Gamma(l_2)\Gamma(l_7)\Gamma(n_1/2)} \right], \\
 \bar{\Phi}_2 = & \left[\frac{(-(2l+y))^{l_1} (Sc\xi)^{l_2} (-\xi\alpha)^{l_3+l_8+n_4}}{l_1!l_2!l_3!l_8!n_3!n_4!(K_r)^{1-(l_1/2)+l_2+l_7}} \frac{(\xi(1-Sc))^{l_7} (y-2n-2)^{n_3} (\xi)^{n_3/2} \Gamma(l_2+l_3)\Gamma(l_7+l_8)\Gamma((n_3/2)+n_4)}{s^{1+al_3+al_8+an_4} \Gamma(l_2)\Gamma(l_7)\Gamma(n_3/2)} \right], \\
 \bar{\Phi}_3 = & \left[\frac{(y-2l-2)^{l_4} (Sc\xi)^{l_5} (-\xi\alpha)^{l_6+l_8+n_2}}{l_4!l_5!l_6!l_8!n_1!n_2!(K_r)^{1-(l_4/2)+l_5+l_7}} \frac{(\xi(1-Sc))^{l_7} (-2n+y)^{n_1} (\xi)^{n_1/2} \Gamma(l_5+l_6)\Gamma(l_7+l_8)\Gamma((n_1/2)+n_2)}{s^{1+al_6+al_8+an_2} \Gamma(l_5)\Gamma(l_7)\Gamma(n_1/2)} \right],
 \end{aligned}$$

$$\begin{aligned}
\bar{\Phi}_4 &= \left[\frac{(y-2l-2)^{l_4} (Sc\xi)^{l_5} (-\xi\alpha)^{l_6+l_8+n_4}}{l_4!l_5!l_6!l_8!n_3!n_4! (K_r)^{1-(l_4/2)+l_5+l_7}} \frac{(\xi(1-Sc))^{l_7} (y-2n-2)^{n_3} (\xi)^{n_3/2} \Gamma(l_5+l_6) \Gamma(l_7+l_8) \Gamma((n_1/2)+n_2)}{s^{1+\alpha l_6+\alpha l_8+\alpha n_4} \Gamma(l_5) \Gamma(l_7) \Gamma(n_1/2)} \right], \\
\bar{\Phi}_5 &= \left[\frac{(-(2m+y))^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2+m_5+n_2}}{m_1!m_2!n_1!n_2! (\text{Pr}_{\text{eff}}-1)} \frac{(-2n+y)^{n_1} (\xi)^{n_1/2} \Gamma(m_1+m_2) \Gamma((n_1/2)+n_2)}{s^{1+\alpha m_2+\alpha m_5+\alpha n_2} \Gamma(m_1) \Gamma(n_1/2)} \right], \\
\bar{\Phi}_6 &= \left[\frac{(-(2m+y))^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2+m_5+n_4}}{m_1!m_2!n_3!n_4! (\text{Pr}_{\text{eff}}-1)} \frac{(y-2n-2)^{n_3} (\xi)^{n_3/2} \Gamma(m_1+m_2) \Gamma((n_3/2)+n_4)}{s^{1+\alpha m_2+\alpha m_5+\alpha n_4} \Gamma(m_1) \Gamma(n_3/2)} \right], \\
\bar{\Phi}_7 &= \left[\frac{(y-2m-2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4+m_5+n_2}}{m_3!m_4!n_1!n_2! (\text{Pr}_{\text{eff}}-1)} \frac{(-2n+y)^{n_1} (\xi)^{n_1/2} \Gamma(m_3+m_4) \Gamma((n_1/2)+n_2)}{s^{1+\alpha m_4+\alpha m_5+\alpha n_2} \Gamma(m_3) \Gamma(n_1/2)} \right], \\
\bar{\Phi}_8 &= \left[\frac{(y-2m-2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4+m_5+n_4}}{m_3!m_4!n_3!n_4! (\text{Pr}_{\text{eff}}-1)} \frac{(y-2n-2)^{n_3} (\xi)^{n_3/2} \Gamma(m_3+m_4) \Gamma((n_3/2)+n_4)}{s^{1+\alpha m_4+\alpha m_5+\alpha n_4} \Gamma(m_3) \Gamma(n_3/2)} \right].
\end{aligned} \tag{35}$$

Taking Laplace inverse, we have

$$\begin{aligned}
u(y, t) &= \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{\infty} \sum_{l_3=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[N \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \Phi_1 - N \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \Phi_2 \right] \\
&\quad - \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l_4=0}^{\infty} \sum_{l_5=0}^{\infty} \sum_{l_6=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[N \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \Phi_3 - N \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \Phi_4 \right] \\
&\quad + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \sum_{m_5=0}^{\infty} \left[\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \Phi_5 - \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \Phi_6 \right] \\
&\quad - \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m_3=0}^{\infty} \sum_{m_4=0}^{\infty} \sum_{m_5=0}^{\infty} \left[\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \Phi_7 + \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \Phi_8 \right] \\
&\quad - N \sum_{l=0}^{\infty} \sum_{l_1=1}^{\infty} \sum_{l_2=0}^{\infty} \sum_{l_3=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[\frac{(-(2l+y))^{l_1} (Sc\xi)^{l_2} (\xi(1-Sc))^{l_7} (-\xi\alpha)^{l_3+l_8} t^{\alpha l_3+\alpha l_8}}{l_1!l_2!l_3!l_8! (K_r)^{1-(l_1/2)+l_2+l_7}} \right. \\
&\quad \times \left. \frac{\Gamma(l_2+l_3) \Gamma(l_7+l_8)}{\Gamma(1+\alpha l_3+\alpha l_8) \Gamma(l_2) \Gamma(l_7)} \right] \\
&\quad + N \sum_{l=0}^{\infty} \sum_{l_4=1}^{\infty} \sum_{l_5=0}^{\infty} \sum_{l_6=0}^{\infty} \sum_{l_7=0}^{\infty} \sum_{l_8=0}^{\infty} \left[\frac{(y-2l-2)^{l_4} (Sc\xi)^{l_5} (\xi(1-Sc))^{l_7} (-\xi\alpha)^{l_6+l_8} t^{\alpha l_6+\alpha l_8}}{l_4!l_5!l_6!l_8! (K_r)^{1-(l_4/2)+l_5+l_7}} \right. \\
&\quad \times \left. \frac{\Gamma(l_5+l_6) \Gamma(l_7+l_8)}{\Gamma(1+\alpha l_6+\alpha l_8) \Gamma(l_5) \Gamma(l_7)} \right] \\
&\quad - \sum_{m=0}^{\infty} \sum_{m_1=1}^{\infty} \sum_{m_2=0}^{\infty} \sum_{m_5=0}^{\infty} \left[\frac{(-(2m+y))^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2+m_5}}{m_1!m_2! (\text{Pr}_{\text{eff}}-1)} \frac{t^{\alpha m_2+\alpha m_5} \Gamma(m_1+m_2)}{\Gamma(1+\alpha m_2+\alpha m_5) \Gamma(m_1)} \right] \\
&\quad + \sum_{m=0}^{\infty} \sum_{m_3=1}^{\infty} \sum_{m_4=0}^{\infty} \sum_{m_5=0}^{\infty} \left[\frac{(y-2m-2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4+m_5}}{m_3!m_4!} \frac{t^{\alpha m_4+\alpha m_5} \Gamma(m_3+m_4)}{\Gamma(1+\alpha m_4+\alpha m_5) \Gamma(m_3)} \right],
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
 \Phi_1 &= \left[\frac{(-2l + y)^{l_1} (Sc\xi)^{l_2} (-\xi\alpha)^{l_3+l_8+n_2}}{l_1!l_2!l_3!l_8!n_1!n_2! (K_r)^{1-(l_1/2)+l_2+l_7}} \right. \\
 &\quad \left. \times \frac{(\xi(1 - Sc))^{l_7} (-2n + y)^{n_1} (\xi)^{n_1/2} t^{\alpha l_3+\alpha l_8+\alpha n_2} \Gamma(l_2 + l_3) \Gamma(l_7 + l_8) \Gamma((n_1/2) + n_2)}{\Gamma(1 + \alpha l_3 + \alpha l_8 + \alpha n_2) \Gamma(l_2) \Gamma(l_7) \Gamma(n_1/2)} \right], \\
 \Phi_2 &= \left[\frac{(-2l + y)^{l_1} (Sc\xi)^{l_2} (-\xi\alpha)^{l_3+l_8+n_4}}{l_1!l_2!l_3!l_8!n_3!n_4! (K_r)^{1-(l_1/2)+l_2+l_7}} \right] \\
 &\quad \times \left[\frac{(\xi(1 - Sc))^{l_7} (y - 2n - 2)^{n_3} (\xi)^{n_3/2} t^{\alpha l_3+\alpha l_8+\alpha n_4} \Gamma(l_2 + l_3) \Gamma(l_7 + l_8) \Gamma(n_3/2 + n_4)}{\Gamma(1 + \alpha l_3 + \alpha l_8 + \alpha n_4) \Gamma(l_2) \Gamma(l_7) \Gamma(n_3/2)} \right], \\
 \Phi_3 &= \left[\frac{(y - 2l - 2)^{l_4} (Sc\xi)^{l_5} (-\xi\alpha)^{l_6+l_8+n_2}}{l_4!l_5!l_6!l_8!n_1!n_2! (K_r)^{1-(l_4/2)+l_5+l_7}} \right] \\
 &\quad \times \left[\frac{(\xi(1 - Sc))^{l_7} (-2n + y)^{n_1} (\xi)^{n_1/2} t^{\alpha l_6+\alpha l_8+\alpha n_2} \Gamma(l_5 + l_6) \Gamma(l_7 + l_8) \Gamma((n_1/2) + n_2)}{\Gamma(1 + \alpha l_6 + \alpha l_8 + \alpha n_2) \Gamma(l_5) \Gamma(l_7) \Gamma(n_1/2)} \right], \\
 \Phi_4 &= \left[\frac{(y - 2l - 2)^{l_4} (Sc\xi)^{l_5} (-\xi\alpha)^{l_6+l_8+n_4}}{l_4!l_5!l_6!l_8!n_3!n_4! (K_r)^{1-(l_4/2)+l_5+l_7}} \right] \\
 &\quad \times \left[\frac{(\xi(1 - Sc))^{l_7} (y - 2n - 2)^{n_3} (\xi)^{n_3/2} t^{\alpha l_6+\alpha l_8+\alpha n_4} \Gamma(l_5 + l_6) \Gamma(l_7 + l_8) \Gamma((n_1/2) + n_2)}{\Gamma(1 + \alpha l_6 + \alpha l_8 + \alpha n_4) \Gamma(l_5) \Gamma(l_7) \Gamma(n_1/2)} \right], \\
 \Phi_5 &= \left[\frac{(-2m + y)^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2+m_5+n_2}}{m_1!m_2!n_1!n_2! (\text{Pr}_{\text{eff}} - 1)} \right] \\
 &\quad \times \left[\frac{(-2n + y)^{n_1} (\xi)^{n_1/2} t^{\alpha m_2+\alpha m_5+\alpha n_2} \Gamma(m_1 + m_2) \Gamma((n_1/2) + n_2)}{\Gamma(1 + \alpha m_2 + \alpha m_5 + \alpha n_2) \Gamma(m_1) \Gamma(n_1/2)} \right], \\
 \Phi_6 &= \left[\frac{(-2m + y)^{m_1} (\text{Pr}_{\text{eff}}\xi)^{m_1/2} (-\xi\alpha)^{m_2+m_5+n_4}}{m_1!m_2!n_3!n_4! (\text{Pr}_{\text{eff}} - 1)} \right] \\
 &\quad \times \left[\frac{(y - 2n - 2)^{n_3} (\xi)^{n_3/2} t^{\alpha m_2+\alpha m_5+\alpha n_4} \Gamma(m_1 + m_2) \Gamma((n_3/2) + n_4)}{\Gamma(1 + \alpha m_2 + \alpha m_5 + \alpha n_4) \Gamma(m_1) \Gamma(n_3/2)} \right], \\
 \Phi_7 &= \left[\frac{(y - 2m - 2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4+m_5+n_2}}{m_3!m_4!n_1!n_2! (\text{Pr}_{\text{eff}} - 1)} \right] \\
 &\quad \times \left[\frac{(-2n + y)^{n_1} (\xi)^{n_1/2} t^{\alpha m_4+\alpha m_5+\alpha n_2} \Gamma(m_3 + m_4) \Gamma((n_1/2) + n_2)}{\Gamma(1 + \alpha m_4 + \alpha m_5 + \alpha n_2) \Gamma(m_3) \Gamma(n_1/2)} \right], \\
 \Phi_8 &= \left[\frac{(y - 2m - 2)^{m_3} (\text{Pr}_{\text{eff}}\xi)^{m_3/2} (-\xi\alpha)^{m_4+m_5+n_4}}{m_3!m_4!n_3!n_4! (\text{Pr} - 1)} \right] \\
 &\quad \times \left[\frac{(y - 2n - 2)^{n_3} (\xi)^{n_3/2} t^{\alpha m_4+\alpha m_5+\alpha n_4} \Gamma(m_3 + m_4) \Gamma((n_3/2) + n_4)}{\Gamma(1 + \alpha m_4 + \alpha m_5 + \alpha n_4) \Gamma(m_3) \Gamma(n_3/2)} \right].
 \end{aligned}
 \tag{37}$$

In case of ordinary velocity when $\alpha \rightarrow 1$, $Nr = 0$, and $Kr = 0$, our solution is reduced to the known results obtained in [1], and for validation, Figure 4 is presented.

5. Graphical Results and Discussion

Transient, viscid, incompressible, free convective flow between upright parallel plates of infinite length bearing the traits of mass dissemination and invariant temperature has been studied in this manuscript. The obtained model is solved analytically by Laplace transform technique executing all boundary and initial conditions. The results for temperature, concentration, and velocity expressions are envisioned graphically.

Figure 5 visualises the effect of fractional parameter α on fluid's concentration regime. By fixing other parameters as constant and increasing the value of α , concentration field the associated boundary layer thickness is increased. Figure 6 shows the effect of Sc on concentration domain. It is observed, with the increase in Sc , the denseness of fluent decreases which affects the rate of molecular diffusion and it tends to minimise thickness of boundary layer. Figure 7 is presented to envision the impact of chemical reaction parameter Kr . It can be seen clearly that as Kr increases, fluent concentration decreases. Figure 8 shows the consequences of time fractional-order concentration and observes that fluid concentration increases with time.

In Figure 9, we vary α and fix other parameters on temperature domain. As α increases, liquid temperature decreases and the boundary level heaviness increases. Figure 10 shows varied effective Prandtl number Pr_{eff} against y by limiting $t = 0.01$ and $\alpha = 0.65$. Graphically, it is to the point that as Prandtl number Pr_{eff} heightens, liquid temperature minimises. As anticipated, raising Pr_{eff} , abbreviates the thermic conduction because the high viscosity reduces the thickness of the caloric boundary stratum. Figure 11 is drawn by fixing $Pr_{eff} = 1.5$ and $\alpha = 0.65$, and effect of fractional time is observed on $\theta(t, y)$.

Figure 12 is drawn for different values of alpha by keeping other parameters fixed. It is a well-known fact that the integer order differential operator is a local operator whereas the fractional-order differential operator (ABC derivative) is nonlocal in the sense that the next stage of the system depends not only upon its current stage but also upon all of its proceeding stages. It describes the behaviour of the function in better way because it holds Mittag-Leffler kernel which stores memory factor and therefore better in describing fluid's flow fields. It is rather visible from the graphical record that velocity increases as α value upshots, and when fractional parameter approaches to one, its velocity will be closer to the classical values. The effects on velocity by Pr_{eff} are shown in Figure 13. One can see that by increasing Pr_{eff} , the fluid velocity decreases, and likely the stratum boundary thickness shrinks because Pr_{eff} increases the viscousness and decreases thermic stratum boundary in slower movement. Figure 14 is given for respective measures of the buoyancy ratio parametric quantity N which interprets the ratio between thermic and aggregative buoyancy drives. For $N = 0$, there is no mass transference, and

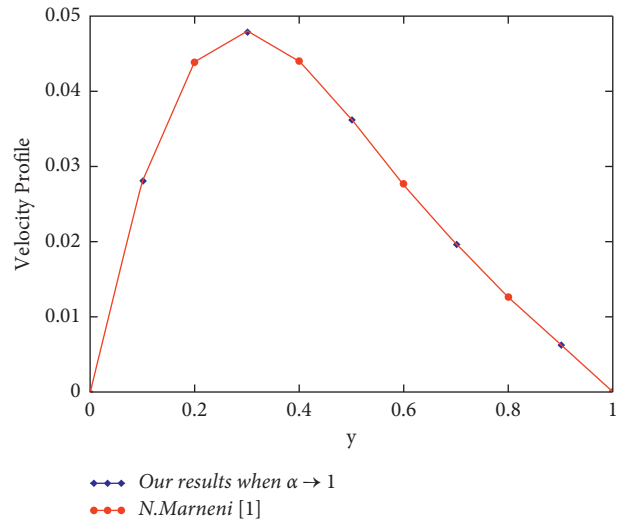


FIGURE 4: Comparison for $u(t, y)$.

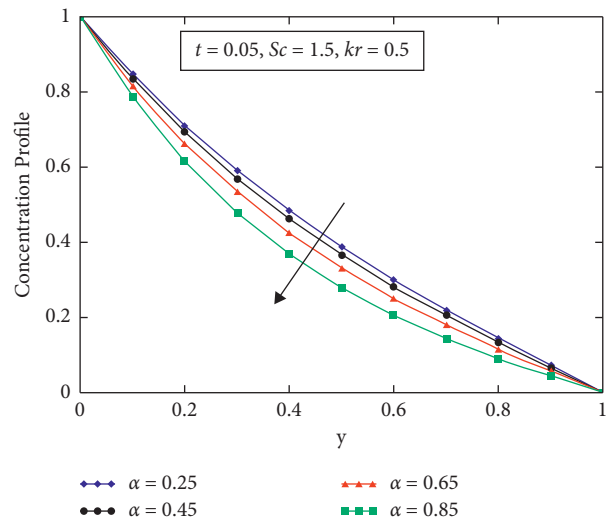


FIGURE 5: $C(t, y)$ profiles for α .

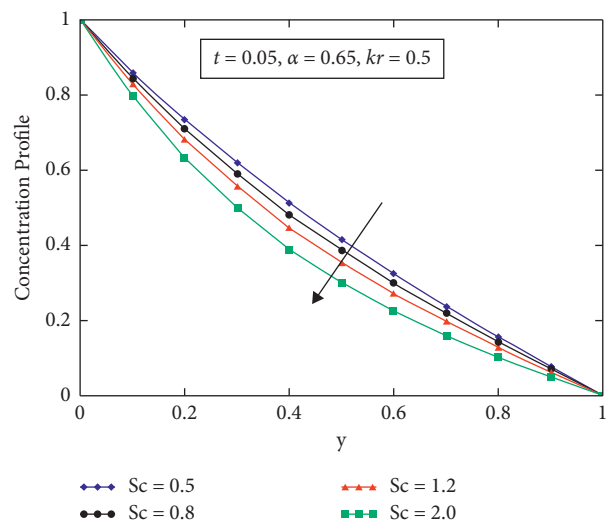


FIGURE 6: $C(t, y)$ profiles for Sc .

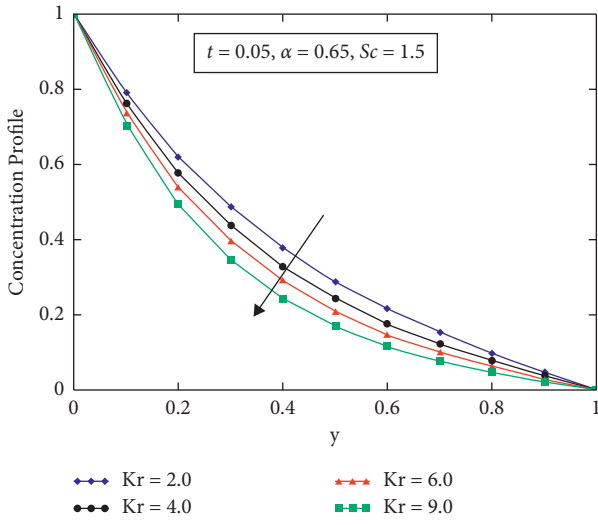


FIGURE 7: $C(t, y)$ profiles for Kr .

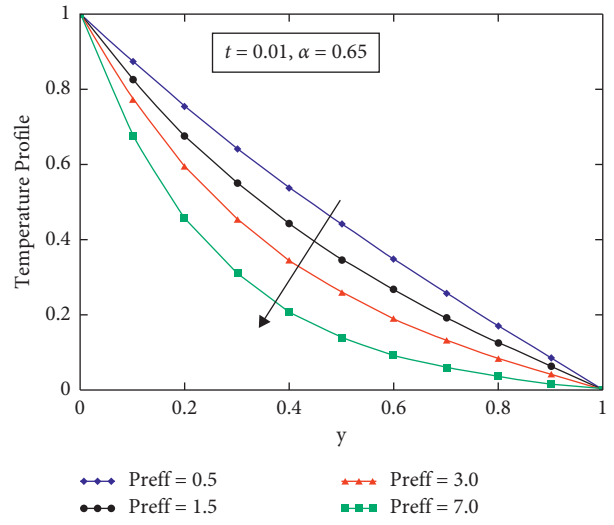


FIGURE 10: $\theta(t, y)$ profiles for Pr .

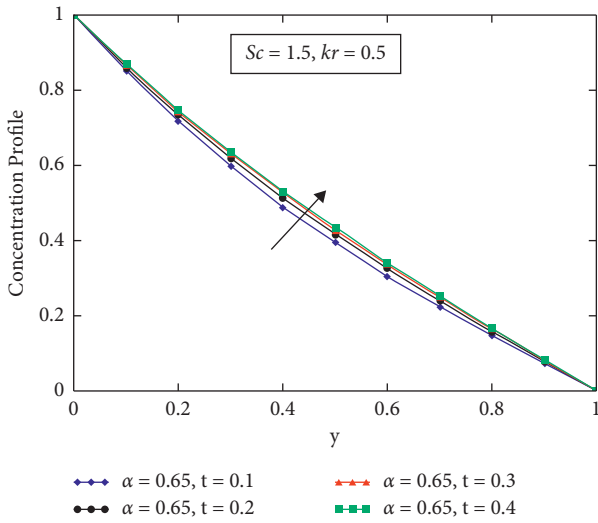


FIGURE 8: $C(t, y)$ profiles for t .

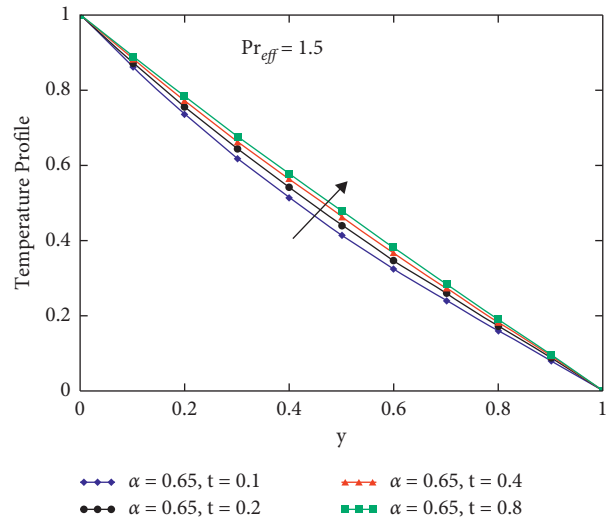


FIGURE 11: $\theta(t, y)$ profiles for t .

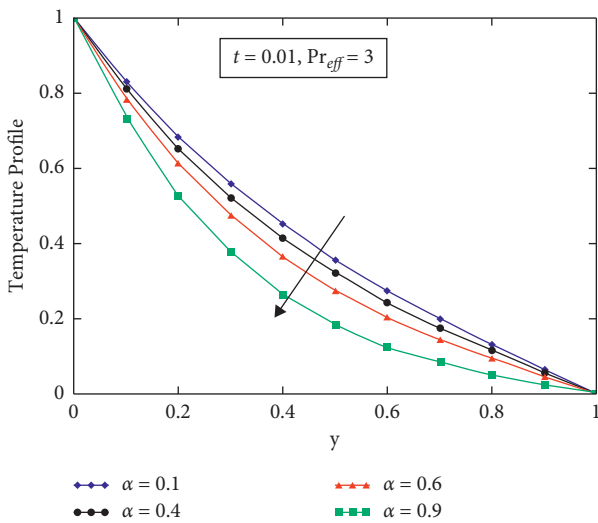


FIGURE 9: $\theta(t, y)$ profiles for α .

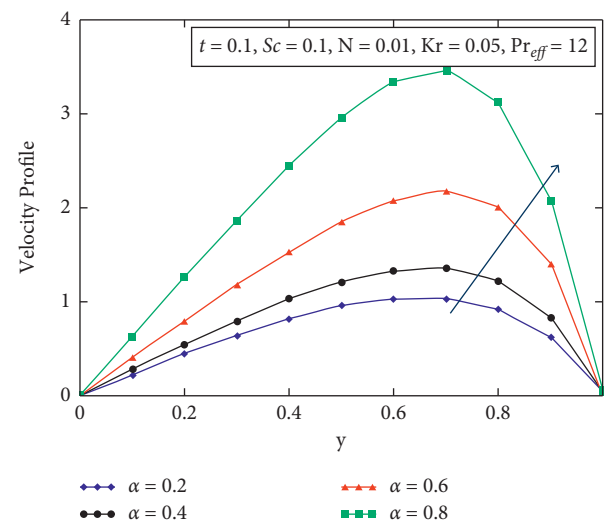


FIGURE 12: $\theta(t, y)$ profiles for α .

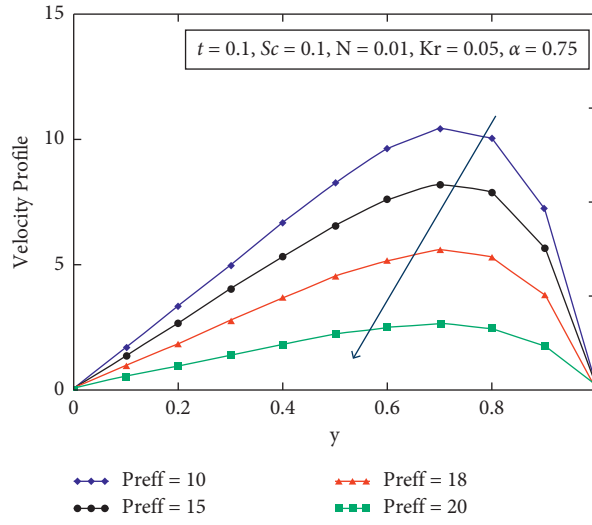


FIGURE 13: $u(t, y)$ profiles for Pr.

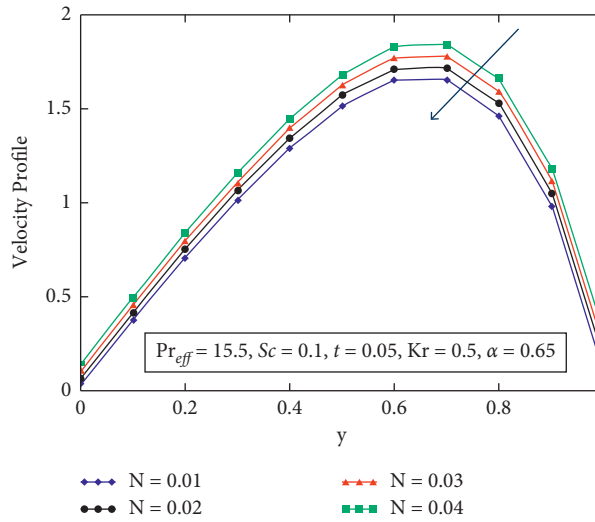


FIGURE 14: $u(t, y)$ profiles for N .

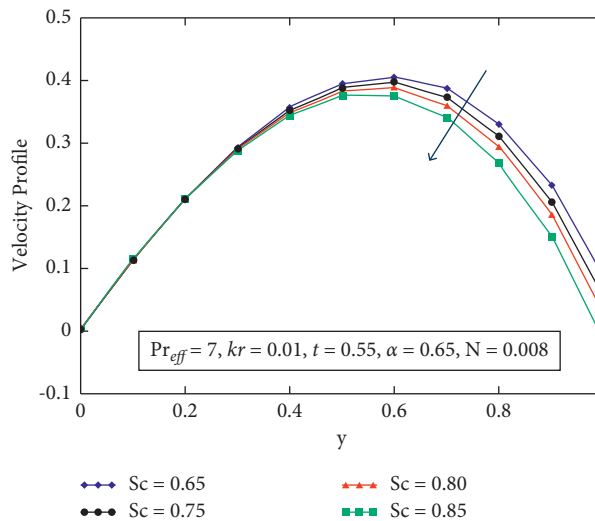


FIGURE 15: $u(t, y)$ profiles for Sc .

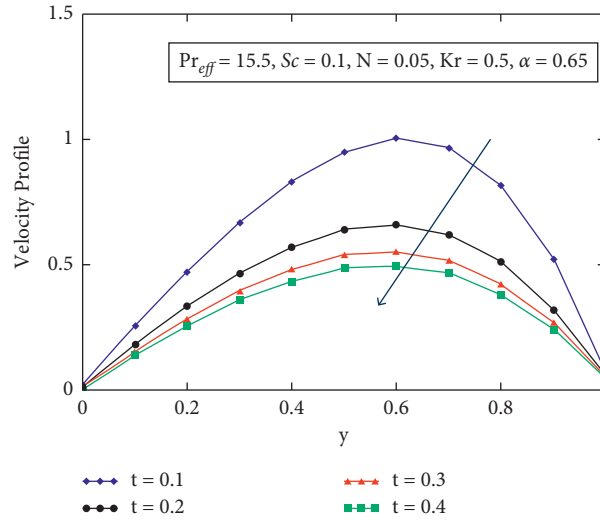


FIGURE 16: $u(t, y)$ profiles for t .

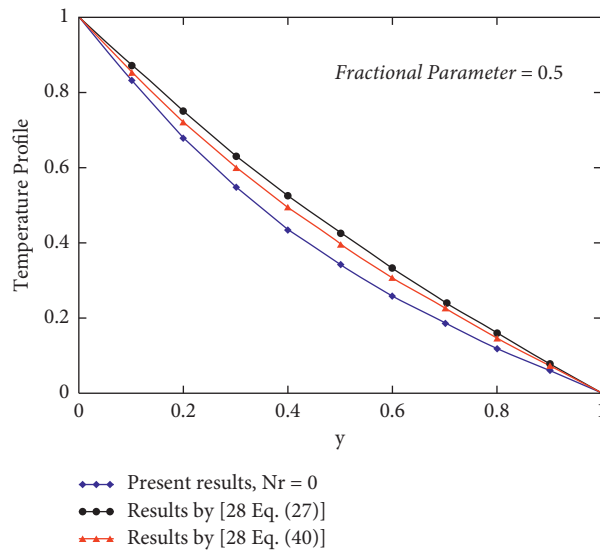
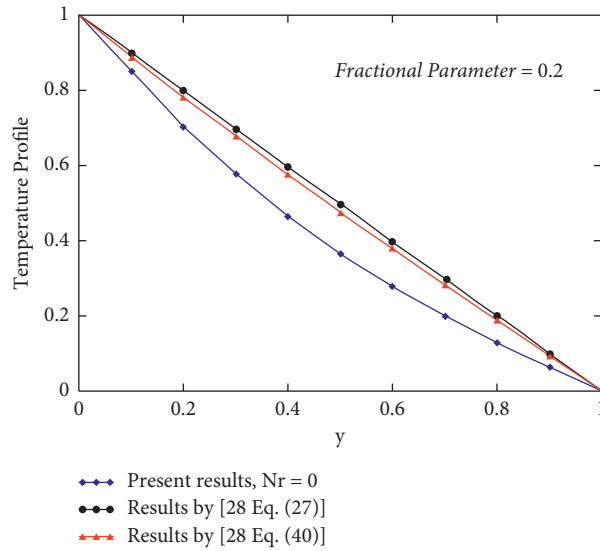


FIGURE 17: Comparison for $\theta(t, y)$.

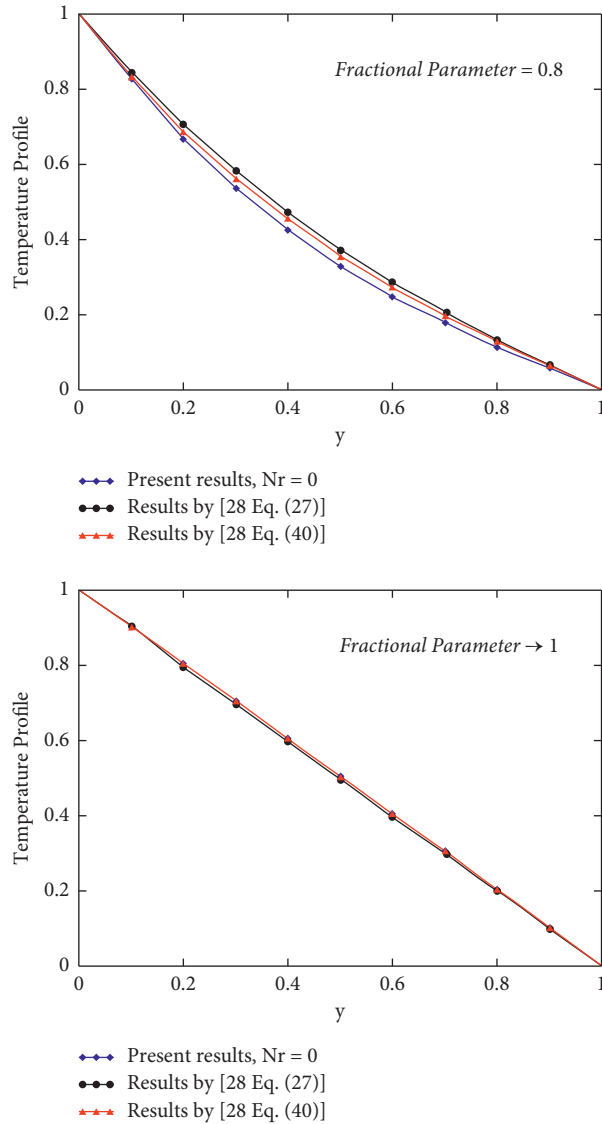


FIGURE 18: Comparison for $\theta(t, y)$.

buoyancy effect is primarily attributable to caloric diffusion rate. Therefore, for dissimilar N values, the thickness of boundary stratum and velocity get enhanced.

Figure 15 shows the effect of Sc number on fluid speed by fixing the parameters. It is observed that increase in Sc decreases the velocity. Figure 16 is given to experience the temperature of time on velocity when $Sc = 0.22$, $Pr_{eff} = 7$, $N = 0.015$, and $\alpha = 0.65$. As time t increments the fluent velocity, likewise the boundary stratum thickness diminishes as seen from graph. Figures 14–16 are drawn to ascertain the validness of obtained for concentration, velocity, and temperature disciplines compared with [1] in the absence of radiative heat flux and chemical reaction parameter. The overlapping arcs distinctly signal the integration of found results. Another comparison is drawn to validate our obtained results. Results are compared with

[26], in the absence of MHD, radiative heat flux, and chemical reaction. Heat and mass transfer model with three fractional approaches such as (i) Caputo, (ii) Caputo–Fabrizio, and (iii) Atangana–Baleanu is compared graphically and presented in Figures 17–22. How fractional parameter controls fluid flow can be seen in this comparison? From these displays, we concluded that the Caputo–Fabrizio model exhibits similar behaviour as the classical model and Atangana–Baleanu model is well suited in stimulating the history function due to Mittag-Leffler kernel which can describe full memory effect for a given system. By increasing the value of fractional parameter, all three profiles display decreasing behaviour. The overlapping curves depicts the integrity of three models when fractional parameter $\rightarrow 1$.

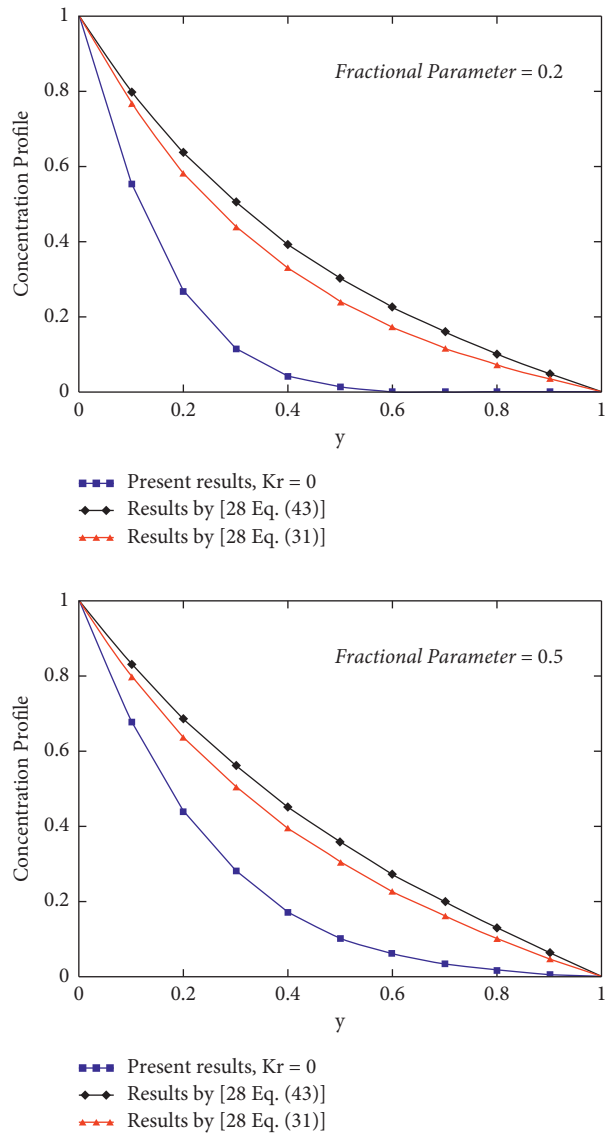


FIGURE 19: Comparison for $C(t, y)$.

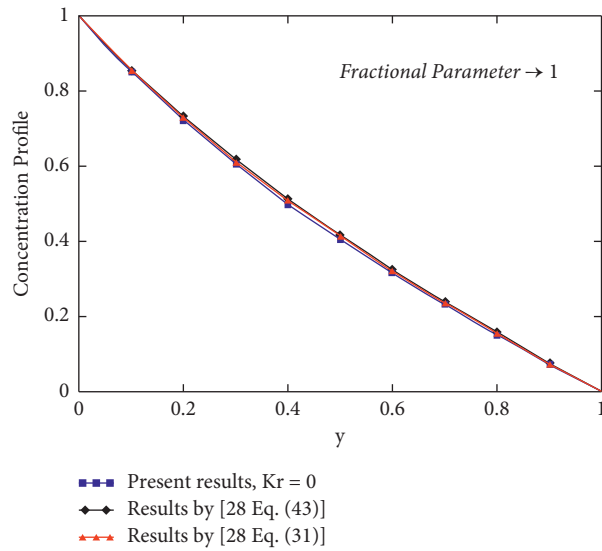
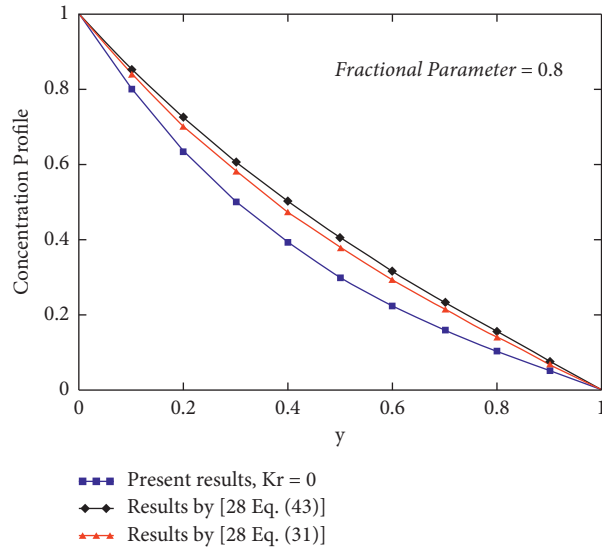
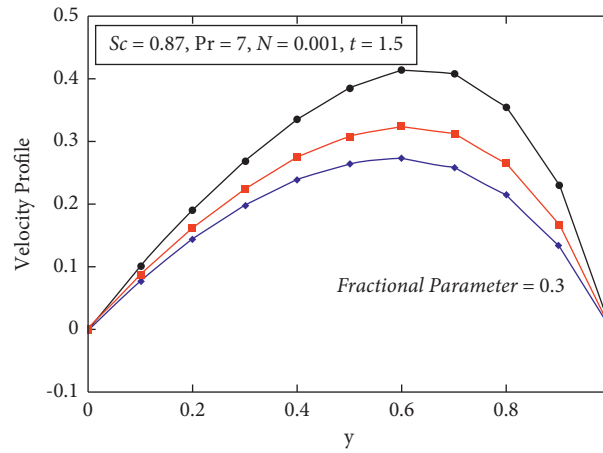
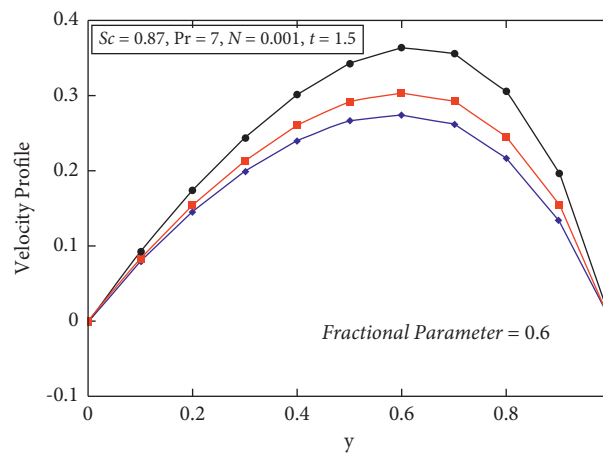


FIGURE 20: Comparison for $C(t, y)$.

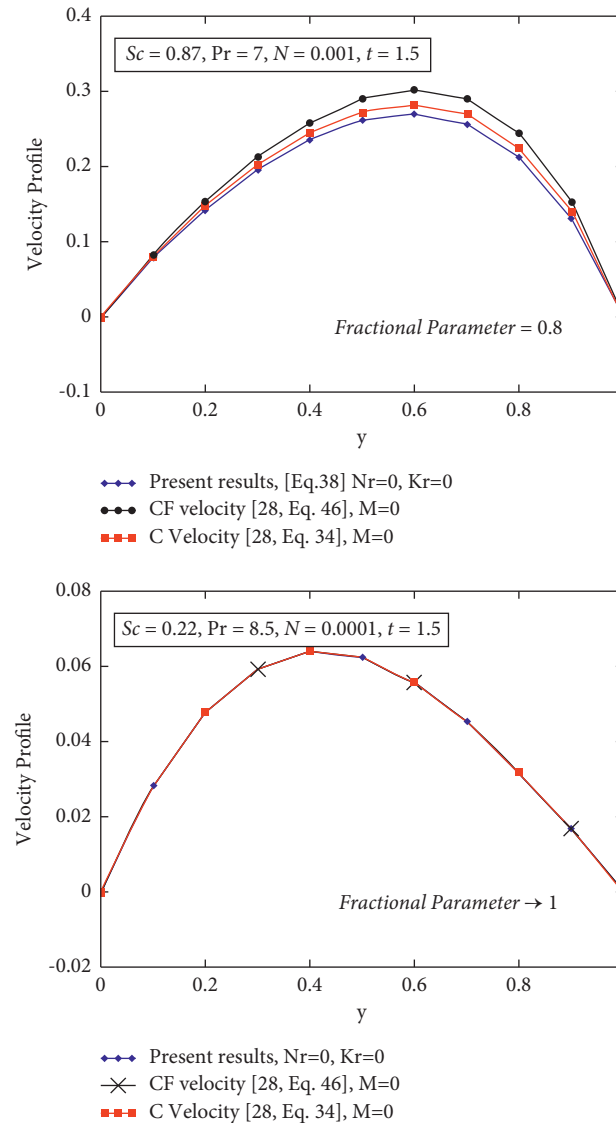


- ◆ Present results, [Eq.38] $Nr=0, Kr=0$
- CF velocity [28, Eq. 46], $M=0$
- C Velocity [28, Eq. 34], $M=0$



- ◆ Present results, [Eq.38] $Nr=0, Kr=0$
- CF velocity [28, Eq. 46], $M=0$
- C Velocity [28, Eq. 34], $M=0$

FIGURE 21: Comparison for $u(t, y)$.

FIGURE 22: Comparison for $u(t, y)$.

6. Conclusion

Free convective transient flow of viscous and incompressible fluids transiting through two infinite hot parallel upright plates is investigated analytically in the presence of chemical reaction, radiative heat flux, and mass diffusion at the boundary. A fractional model is developed with comparatively new (ABC) derivative. The following observations are made:

- (i) In this model, the fractional-order parameter α observes the concentration thickness of thermal and momentum boundary layers.
- (ii) The fluid temperature is enhanced for smaller values of α and larger values of time t while decreases for greater Pr_{eff} .
- (iii) Concentration grows by decreasing the values of fractional parameter α and for larger values of time

t . Additionally, concentration abbreviates by raising Sc and Kr likewise the boundary layer thickness.

- (iv) Velocity increases by raising the parametric values of N, α, Nr , and time t whereas it decreases by raising Pr, Kr , and Sc values, respectively. The proposed model with ABC derivative gives results as compared with Caputo and Caputo–Fabrizio model because of memory effect.

Data Availability

The data used to support this study are included within the article

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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