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Research Article

Sequential Test for Change-Point in Long Memory Process

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This paper proposes a modified kernel weighted variance ratio statistic to sequentially detect change-point that shifts from a stationary long memory process to a non-stationary long memory process. The limiting distribution of test statistic under the null hypothesis and its consistency under the alternative hypothesis are proved. Simulations indicate that the new method has better finite sample performance than the available method in the literature. Finally, we illustrate the new method by a set of U.S. inflation rate data.

1. Introduction

It is broadly accepted that time series with long memory, also known as long range dependencies, appear in many contexts, for example, in financial economics, networks traffic, hydrology, remaining useful life of some machines [1–4], and so on. A long memory time series $\{X_t\}$ can be written as $X_t \sim I(d)$ with long memory parameter $d \geq 0$. For a practitioner, it is of importance in terms of model building and forecasting to know whether a given time series has a certain kind of persistence, either stationary $I(d)$ with $0 \leq d < (1/2)$ or non-stationary $I(d)$ with $d > (1/2)$, or whether the persistent breaks from stationary to non-stationary persistence or vice versa. The classical $(I(0)/I(1))$ framework has been an issue of substantial empirical interest, especially concerning inflation rate series [5], foreign exchange rates [6], government budget deficits [7], and real output [8], and a number of testing procedures have been suggested that aim to distinguish such behavior. These include, inter alia, ratio tests [9], LBI tests [10], CUSUM of squares-based test [11], moving ratio tests [12], and Wilcoxon rank test [13].

Although the classical $(I(0)/I(1))$ persistent change problem has been well studied, long memory time series cannot be covered by this framework. Sibbertsen and Kruse [14] considered persistent change under the long memory innovation case via the CUSUM of squares-based test of Leybourne et al. [11] and found that this test undergoes serious size distortions. In order to overcome this problem, new critical values depending on the long memory parameter are necessary in the $I(d)$ framework. Hassler and Scheithauer [15] applied the ratio tests and the LBI tests to detect change-point that shifts from short memory to long memory process. Lavancier et al. [16] proposed a variance ratio statistic to test long memory parameter change-point. Iacone and Lazarová [5] studied the long memory parameter change-point detection problem via a semiparametric method.

While all above works are one shot tests, sequentially detecting change-point in long memory time series has also received some attention. Chen et al. [17] proposed a moving ratio statistic (MRS) to monitor change in persistence in long memory process. Chen et al. [6] proposed a two-stage moving ratio statistic to monitor long memory parameter change-point. Recently, Chen et al. [18] proposed the so-called DF difference statistic to sequentially detect unit root to long memory process change-point. It is well known that the kernel weighted variance ratio statistic (KWVRS) proposed by Steland [19] is a powerful method to sequentially detect $I(0)$ to $I(1)$ persistent change-point. An interesting question is whether the KWVRS still works for the more general persistent change-point problem. In this paper, we extend the KWVRS to sequentially test change-point that

shifts from a stationary long memory process to a non-stationary long memory process. However, the $I(0)$ process and the long memory process have different converging rates, and this will lead the KWVRS divergence to infinity under the long memory process null hypothesis. In order to guarantee the convergence of KWVRS, we proposed a modified KWVRS. We derive the limiting distribution of the modified KWVRS under the stationary long memory process null hypothesis and prove its consistency under the alternative hypothesis. We also will evaluate the finite sample performance of modified KWVRS and compare it with MRS via simulation.

The rest of the paper is organized as follows. Section 2 introduces the model, necessary assumptions, and the proposed sequential test statistic. The asymptotic distribution of test statistic and its consistency will be discussed in Section 3. In Section 4, we evaluate the finite sample performance of new proposed sequential test via simulation and an empirical application example. We conclude the paper in Section 5.

2. Model Assumption and Test Statistic

Let $y_1, y_2, \ldots$ be an observed time series that can be decomposed as

$$y_t = \mu + \varepsilon_t,\ (1-L)^d \varepsilon_t = e_t,\ t = 1, 2, \ldots, \tag{1}$$

where $\mu_t = E(y_t)$ is a deterministic component. For simplicity, we restrict the analysis to constant components, namely, $\mu_t = \mu$. An extension to polynomials in time would be possible. The random component ε_t is an integrated process, in which L is the lag operator, and e_t is the innovation process with mean zero and finite variance. The long memory parameter d is restricted to $0 \le d < (3/2)$. Note that the process y_t is a stationary long memory process if $0 < d < (1/2)$, the process y_t is a non-stationary long memory process if $(1/2) < d < (3/2)$, and y_t becomes a stationary short memory process if $d = 0$. To simplify the notation, we denote $y_t \sim I(d_1)$ with $0 \le d_1 < (1/2)$ if y_t is a stationary short/long memory process and $y_t \sim I(d_2)$ with $(1/2) < d_2 < (3/2)$ if y_t is a non-stationary long memory process.

Suppose we have observed samples $y_1, \ldots, y_{[T\tau]}$, $\tau \in (0,1)$, and call them as training samples. Here $[x]$ denotes the largest integer smaller than x. We start from the $([T\tau]+1)$th newly observed sample sequentially to detect $I(d_1)$ to $I(d_2)$ change-point until the time horizon T. That is, we want to test the following null and alternative hypothesis.

$$\begin{aligned} &H_0\colon\ y_t \sim I(d_1) \text{ with } 0 \le d_1 < \left(\frac{1}{2}\right), t \\ &H_1\colon\ y_t \sim I(d_1), t = [T\tau]+1, \ldots, [T\tau^*], \\ &\quad y_t \sim I(d_2) \text{ with } \left(\frac{1}{2d_2}\right) < \left(\frac{3}{2}\right), t = [T\tau^*]+1, \ldots, T. \end{aligned} \tag{2}$$

Here T denotes the prespecified largest monitoring sample size, $[T\tau]^*, \tau < \tau^* < 1$, is the unknown change-point. In order to study this hypothesis, we need the following assumption for the training samples.

Assumption 1. Assume $y_t \sim I(d_1)$, with $0 \le d_1 < (1/2)$, $t = 1, \ldots, [T\tau]$, $\tau \in (0,1)$.

Let $[Ts, s \in (0,1)]$, denote the current observed full sample size and $\widehat{\mu}_0$ and $\widehat{\mu}(s)$ denote the OLS estimators of μ in model (1) based on the training samples $y_1, \ldots, y_{[T\tau]}$ and all observed samples $y_1, \ldots, y_{[Ts]}$, respectively. We define the residuals $\widehat{\varepsilon}_j = y_j - \widehat{\mu}(s)$, $j = 1, \ldots, y_{[Ts]}$, and $\widetilde{\varepsilon}_j = y_j - \widehat{\mu}_0$, $j = 1, \ldots, y_{[T\tau]}$. We use the following modified kernel weighted variance ratio statistic to test the null hypothesis H_0 against the alternative hypothesis H_1.

$$U_T(s) = \frac{[Ts]^{-1-2d_1} \sum_{t=1}^{[Ts]} \left(\sum_{i=1}^{t} \widehat{\varepsilon}_i\right)^2 K(t - [Ts]/h)}{\sum_{j=1}^{[T\tau]} \widetilde{\varepsilon}_j^2}, \tag{3}$$

where $K(\cdot)$ is a Lipschitz continuous density function with mean 0 and finite variance and $h = h_T > 0$ is a sequence of bandwidth parameter that satisfies

$$\frac{T}{h_T} \longrightarrow \zeta \in [1, \infty), \quad \text{as } T \longrightarrow \infty. \tag{4}$$

A large value of $U_T(s)$ indicates that there occurs a stationary long memory process to a non-stationary long memory process change-point. At the α nominal level, the stopping time is defined as

$$S_T(n) = \min\left\{[T\tau] < n \le T\colon U_T\left(\frac{n}{T}\right) > c\right\}, \tag{5}$$

where c denotes the critical value that satisfies

$$\lim_{T \longrightarrow \infty} P_{H_0}\left(S_T(n) < T\right) = \alpha. \tag{6}$$

3. Asymptotic Properties

In this section, we derive the asymptotic properties of above modified KWVRS $U_T(s)$ when the time series y_t is generated by model (1).

Theorem 1. *Suppose Assumption 1 holds; then, under the null hypothesis H_0, if $T \longrightarrow \infty$, we have*

$$U_T(s) \Rightarrow \tau^{-1} s^{-1-2d_1} \int_0^s \left(W_{d_1}(u) - u s^{-1} W_{d_1}(s)\right)^2 K(\zeta(u-s)) \mathrm{d}u, \tag{7}$$

where $W_{d_1}(u)$ denotes the type I fractional Brownian motion with long memory parameter d_1.

Proof. Since $\varepsilon_t \sim I(d_1), 0 \le d_1 < 0.5$, according to [20], we have that if $T \longrightarrow \infty$,

$$T^{-(1/2)-d_1} \sum_{t=1}^{[Tr]} \varepsilon_t \Rightarrow \omega W_{d_1}(r),\ 0 < r \le 1, \tag{8}$$

where ω^2 denotes the long-run variance of long memory process ε_t. The notation $\Rightarrow$ denotes the weak convergence. Note that $W_0(r) = W(r)$ denotes the standard Wiener process.

Let $t = [Tu]$, and since $\widehat{\varepsilon}_i = \varepsilon_i - [Ts]^{-1}\sum_{j=1}^{[Ts]}\varepsilon_j$, then

$$T^{-(1/2)-d_1}\sum_{i=1}^{t}\varepsilon_i = T^{-(1/2)-d_1}\sum_{i=1}^{[Tu]}\varepsilon_i - \frac{[Tu]T^{-(1/2)-d_1}}{[Ts]}\sum_{j=1}^{[Ts]}\varepsilon_i$$
$$\Rightarrow \omega\left(W_{d_1}(u) - us^{-1}W_{d_1}(s)\right). \tag{9}$$

Recall that $K(\cdot)$ is Lipschitz continuous and $(T/h) \longrightarrow \zeta$ as $T \longrightarrow \infty$; therefore,

$$K\left(\frac{t-[Ts]}{h}\right) = K\left(\frac{T}{h}\left(\frac{t}{T} - \frac{[Ts]}{T}\right)\right) \longrightarrow K(\zeta(u-s)). \tag{10}$$

Hence,

$$[Ts]^{-2-2d_1}\sum_{t=1}^{[Ts]}\left(\sum_{i=1}^{t}\widehat{\varepsilon}_i\right)^2 K\left(\frac{t-[Ts]}{h}\right)$$
$$= \frac{T^{2+2d_1}}{[Ts]^{2+2d_1}}T^{-1}\sum_{t=1}^{[Ts]}\left(T^{-1/2-d_1}\sum_{i=1}^{t}\widehat{\varepsilon}_i\right)^2 K\left(\frac{t-[Ts]}{h}\right)$$
$$\Rightarrow s^{-2-2d_1}\omega^2\int_0^s\left(W_{d_1}(u) - us^{-1}W_{d_1}(s)\right)^2 K(\zeta(u-s))\mathrm{d}u. \tag{11}$$

On the other hand, since ε_t is a stationary process, ergodic theory of stationary process gives that

$$[Ts]^{-1}\sum_{j=1}^{[T\tau]}\widehat{\varepsilon}_j^2 = [Ts]^{-1}\sum_{j=1}^{[T\tau]}\left(\varepsilon_j^2 - \frac{2}{[T\tau]}\varepsilon_j\sum_{i=1}^{[T\tau]}\varepsilon_i + \frac{1}{[T\tau]^2}\left(\sum_{i=1}^{[T\tau]}\varepsilon_i\right)^2\right)$$
$$= \frac{[T\tau]}{[Ts]}[T\tau]^{-1}\left(\sum_{j=1}^{[T\tau]}\varepsilon_j^2 - \frac{1}{[T\tau]}\left(\sum_{i=1}^{[T\tau]}\varepsilon_i\right)^2\right)$$
$$\xrightarrow{p} \tau s^{-1}\omega^2. \tag{12}$$

Combining (4)–(6) and using the continuous mapping theorem, we obtain the null distribution of statistic $U_T(s)$. □

Theorem 2. *Suppose Assumption 1 holds; then, under the alternative hypothesis H_1, we have*

$$U_T(s) = O_p\left(T^{2(d_2-d_1)}\right), s \in (\tau^*, 1]. \tag{13}$$

Proof. We continue using the notations in the proof of Theorem 1. If $[Ts] > [T\tau^*]$, then as $T \longrightarrow \infty$, we have

$$\sum_{t=1}^{[Ts]}\left(\sum_{i=1}^{t}\widehat{\varepsilon}_i\right)^2 K\left(\frac{t-[Ts]}{h}\right)$$
$$= \sum_{t=1}^{[T\tau^*]}\left(\sum_{i=1}^{t}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=1}^{[T\tau^*]}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=[T\tau^*]+1}^{[Ts]}\varepsilon_i\right)^2 K\left(\frac{t-[Ts]}{h}\right)$$
$$+ \sum_{t=[T\tau^*]+1}^{[Ts]}\left(\sum_{i=1}^{[T\tau^*]}\varepsilon_i + \sum_{i=[T\tau^*]+1}^{[Tu]}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=1}^{[Ts]}\varepsilon_i\right)^2 K\left(\frac{t-[Ts]}{h}\right)$$
$$:= I_1 + I_2. \tag{14}$$

According to the proof of (10) and (11) and $d_2 > d_1$, we have

$$[Ts]^{-2-2d_2}I_1$$
$$= [Ts]^{-2-2d_2}\sum_{t=1}^{[T\tau^*]}\left(\sum_{i=1}^{t}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=1}^{[T\tau^*]}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=[T\tau^*]+1}^{[Ts]}\varepsilon_i\right)^2 K\left(\frac{t-[Ts]}{h}\right)$$
$$= [Ts]^{-2-2d_2}\frac{[Tu]^2[T\tau^*]}{[Ts]^2}\left(\sum_{i=[T\tau^*]+1}^{[Ts]}\varepsilon_i\right)^2 K\left(\frac{t-[Ts]}{h}\right) + o_p(1)$$
$$\Rightarrow u^2\tau^* s^{-4}\omega^2\left(\int_{\tau^*}^{s}W_{d_2-1}(r)\mathrm{d}r\right)^2 K(\zeta(u-s)),$$
$$[Ts]^{-2-2d_2}I_2$$
$$= [Ts]^{-2-2d_2}\sum_{t=[T\tau^*]+1}^{[Ts]}\left(\sum_{i=1}^{[T\tau^*]}\varepsilon_i + \sum_{i=[T\tau^*]+1}^{[Tu]}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=1}^{[Ts]}\varepsilon_i\right)^2$$
$$K\left(\frac{t-[Ts]}{h}\right)$$
$$= [Ts]^{-2-2d_2}\sum_{t=[T\tau^*]+1}^{[Ts]}\left(\sum_{i=[T\tau^*]+1}^{[Tu]}\varepsilon_i - \frac{[Tu]}{[Ts]}\sum_{i=[T\tau^*]+1}^{[Ts]}\varepsilon_i\right)^2$$
$$K\left(\frac{t-[Ts]}{h}\right) + o_p(1)$$
$$\Rightarrow s^{-2-2d_2}\omega^2\int_{\tau*}^{s}\left(\int_{\tau^*}^{u}W_{d_2-1}(r)\mathrm{d}r - us^{-1}\int_{\tau^*}^{s}W_{d_2-1}(r)\mathrm{d}r\right)^2$$
$$K(\zeta(u-s))\mathrm{d}u. \tag{15}$$

This indicates that the numerator of statistic $U_T(s)$ is $O_p(T^{1+2(d_2-d_1)})$. On the other hand, (6) gives that the denominator of statistic $U_T(s)$ is $O_p(T)$. Thus,

$$U_T(s) = O_p\left(T^{2(d_2-d_1)}\right). \tag{16}$$

So far, we assume that the value of long memory parameter d_1 is known. Because d_1 is unknown in practice, we estimate it via the local Whittle estimation based on the training samples $y_1, \ldots, y_{[T\tau]}$. Obviously, the above

Table 1: Empirical sizes of modified KWVRS $U_T(s)$ and MRS $\Gamma_T(s)$ at 5% nominal level.

	$U_T(s)$				$\Gamma_T(s)$			
$T\backslash d_1$	0	0.1	0.2	0.4	0	0.1	0.2	0.4
200	0.067	0.076	0.071	0.065	0.065	0.070	0.057	0.058
500	0.054	0.057	0.055	0.053	0.046	0.048	0.043	0.051

Table 2: Empirical powers of modified KWVRS $U_T(s)$ and MRS $\Gamma_T(s)$ at 5% nominal level.

			$U_T(s)$				$\Gamma_T(s)$			
T	k^*	$d_1\backslash d_2$	0.6	0.8	1	1.2	0.6	0.8	1	1.2
200	0.3	0	0.982	0.996	1	1	0.892	0.981	1	1
		0.1	0.952	0.990	1	1	0.766	0.945	0.995	1
		0.2	0.782	0.895	0.996	0.999	0.597	0.864	0.971	0.998
		0.4	0.559	0.754	0.895	0.997	0.251	0.554	0.850	0.967
	0.5	0	0.949	0.991	0.998	1	0.846	0.942	0.996	1
		0.1	0.849	0.947	0.993	0.999	0.713	0.917	0.983	1
		0.2	0.717	0.834	0.980	0.998	0.563	0.815	0.944	0.994
		0.4	0.470	0.684	0.904	0.987	0.228	0.483	0.788	0.943
500	0.3	0	1	1	1	1	0.992	1	1	1
		0.1	1	1	1	1	0.940	1	1	1
		0.2	0.985	1	1	1	0.805	0.986	1	1
		0.4	0.762	0.977	1	1	0.358	0.729	0.987	1
	0.5	0	0.997	1	1	1	0.981	0.998	1	1
		0.1	0.993	1	1	1	0.941	1	1	1
		0.2	0.968	0.998	1	1	0.742	0.857	0.997	1
		0.4	0.640	0.913	0.995	1	0.362	0.686	0.956	0.989

asymptotic properties still hold if we replace d_1 by its local whittle estimation for it is a consistent estimator. □

4. Simulation and Empirical Application

4.1. Simulation. In this section, we investigate the finite sample performance of our proposed modified KWVRS $U_T(s)$ and compare it with the MRS $\Gamma_T(s)$ of Chen et al. [17]. We use the following data-generating process to generate the simulation data.

$$y_t = \begin{cases} 1 + \varepsilon 1_t, & t = 1, \ldots, [T\tau^*], \\ 1 + \varepsilon 1_{[T\tau^*]} + \varepsilon 2_t, & t = [T\tau^*] + 1, \ldots, T, \end{cases} \tag{17}$$

where $\varepsilon 1_t$ follows a FARIMA $(0, d_1, 0)$ model with d_1 varying among $\{0, 0.1, 0.2, 0.4\}$ and $\varepsilon 2_t$ follows a FARIMA $(0, d_2, 0)$ model with d_2 varying among $\{0.6, 0.8, 1, 1.2\}$. We use the R package "fracdiff" to generate the FARIMA process. We assume that the change-point location $\tau^* = 1$ under the null hypothesis and τ^* varies among $\{0.3, 0.5\}$ under the alternative hypothesis. We set the largest monitoring sample size T varying among $\{200, 500\}$ and the training sample size $[T\tau] = [0.25T]$. In order to compute the critical values of monitoring statistics, we use the sieve bootstrap method proposed by Chen et al. [6]. The sieve bootstrap frequency was set to be $B = 300$, and all simulations are obtained by 1000 replications at $\alpha = 5\%$ nominal level. A lot of experimental tests indicate that $h = 0.7 * [T\tau]$ gives more satisfactory test results compared to other choice, so we set the bandwidth $h = 0.7 * [T\tau]$ throughout this section.

Table 1 reports the empirical sizes of two monitoring statistics. We can see that both statistics have some size distortions when the largest monitoring sample size $T = 200$, and all these size distortions become light when $T = 500$. The influence of long memory parameter d_1 on the modified KWVRS $U_T(s)$ is greater than the MRS $\Gamma_T(s)$. It is mainly because the statistic $U_T(s)$ contains this parameter and estimating d_1 based on the training sample is not very stable. A more robust way is estimating d_1 based on all available samples but not the training samples. However, for each newly observed sample, this method requires not only a reestimation of the long memory parameter d_1 but also a recalculation of the critical value. This, obviously, is computationally expensive. So, we still recommend using the previous approach. In addition, we also tried to calculate the critical value of statistic $U_T(s)$ via the bootstrap method proposed by Chen et al. [17]. We found that although the bootstrap method of Chen et al. [17] could control the empirical size better, it would lose more empirical power. This is the reason why we recommend using the sieve bootstrap method proposed by Chen et al. [6].

Tables 2 and 3 report the empirical powers and ARLs of two statistics, respectively. Three conclusions can be drawn from these two tables. First, the modified KWVRS $U_T(s)$ has higher empirical power and shorter ARL than the MRS $\Gamma_T(s)$ in most cases. This indicates that the newly proposed and

Table 3: ARLs of modified KWVRS $U_T(s)$ and MRS $\Gamma_T(s)$ at 5% nominal level.

			$U_T(s)$				$\Gamma_T(s)$			
T	k^*	$d_1 \backslash d_2$	0.6	0.8	1	1.2	0.6	0.8	1	1.2
200	0.3	0	19.3	15.2	12.2	9.52	31.1	25.2	18.6	13.5
		0.1	25.22	18.8	14.6	11.1	36.6	32.0	23.3	16.6
		0.2	28.1	24.2	18.6	13.5	41.8	39.0	28.9	20.4
		0.4	40.2	38.4	28.3	19.3	46.2	39.8	33.2	31.4
	0.5	0	19.5	17.8	14.0	10.1	26.2	23.0	17.9	12.9
		0.1	20.4	19.0	14.9	10.6	29.3	28.2	22.3	16.3
		0.2	21.1	20.5	16.9	11.8	32.5	30.3	26.7	19.4
		0.4	28.1	24.8	23.0	16.3	39.7	37.2	35.8	28.0
500	0.3	0	26.7	20.4	16.1	12.0	51.6	33.9	24.3	17.7
		0.1	37.2	27.2	20.3	14.7	75.5	50.9	34.9	22.8
		0.2	55.2	36.2	25.5	18.4	93.3	70.0	42.4	28.1
		0.4	96.1	72.8	44.7	28.9	116	96.8	85.4	51.7
	0.5	0	26.2	18.8	14.2	10.6	47.0	32.4	23.2	16.3
		0.1	38.7	26.7	18.5	13.4	67.7	48.3	30.6	19.8
		0.2	55.3	37.8	25.5	17.0	74.8	63.0	41.7	26.5
		0.4	74.3	72.7	49.8	30.2	90.1	86.6	68.4	47.8

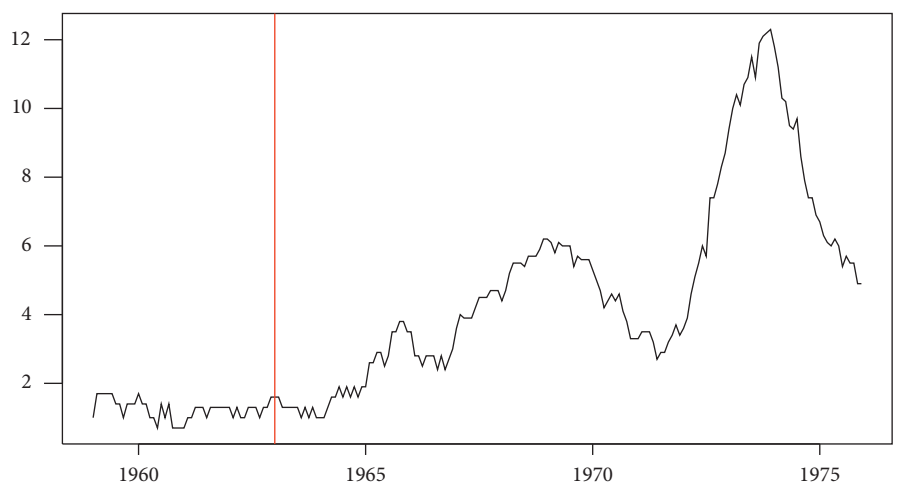

Figure 1: U.S. inflation rate monthly data from January 1959 to December 1975.

modified KWVRS has a significant advantage when it comes to sequential test stationary to non-stationary long memory process change-point. Second, the empirical power increases as T or change size increases. This verifies the consistency of modified KWVRS $U_T(s)$. Third, the later the change-point occurs, the lower the empirical power is. This is a general conclusion in most sequential change-point test problems.

4.2. Empirical Application. In this section, we illustrate the modified KWVRS $U_T(s)$ and MRS $\Gamma_T(s)$ using a set of U.S. inflation rate monthly data which were observed from January 1959 to December 1975. The data were downloaded from the U.S. Federal Reserve Bank official website. Figure 1 shows the raw dataset of a total of 204 observations. Under the same parametric assumptions as in the simulation section, we find that the modified KWVRS $U_T(s)$ stops at the 80th observation, and the MRS $\Gamma_T(s)$ stops at the 83th observation. This indicates that there occurs a persistent change-point that shifts from a stationary long memory process to a non-stationary long memory process before the 80th observation. Chen et al. [12] also studied this dataset and found that there exists an $I(0)$ to $I(1)$ persistent change-point. In fact, through the estimator proposed in [14], we can estimate this change-point at the 65th observation (see the vertical in line in Figure 1). The estimated long memory parameters before and after this change-point are 0.49 and 1.41, respectively. Obviously, compared to this change-point as an $I(0)$ to $I(1)$ persistent change-point, this is more like a stationary long memory process to a non-stationary long memory process persistent change-point.

5. Conclusion

In this paper, we have proposed a modified KWVRS to sequentially detect persistent change-point that shifts from a stationary long memory process to a non-stationary long memory process. We derived its limiting distribution under

the null hypothesis and proved its consistency under the alternative hypothesis. Simulations indicate that the new proposed sequential test procedure has satisfactory finite sample performances.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

All authors contributed equally to this paper and read and approved the final manuscript.

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