Modified Radial Malmquist–Luenberger Index Models for Infeasibility: An Application in Productivity Evaluation of Chinese Banks

Nannan Liang and Qian Zhang

1School of Business, Nanjing Audit University, Nanjing, Jiangsu 211815, China
2Leavitt School of Business, Southern Utah University, Cedar, UT 84720, USA
3Leavitt School of Business, Southern Utah University 351 W University Blvd, Cedar, UT 84720, USA

Correspondence should be addressed to Qian Zhang; qianvictorizhang@suu.edu

1. Introduction

There is a vast application of DEA in the context of bank efficiency and productivity. Based on the DEA application survey by Liu et al. [1], banking is one of the top five industries among the multifaceted DEA applications. Sharman and Gold [2] were the first to apply the DEA in the bank industry. They measured and evaluated the operating efficiency on the branch level. Since then, many researchers started to analyze the bank branch performance by DEA. For example, Al-Faraj et al. [3] applied to a set of branches of one of the largest commercial banks in Saudi Arabia to assess the efficiency via the DEA CCR model proposed by Charnes et al. [4]. To evaluate the efficiency from the branch level, a great portion of studies examined banking institutions as a whole to evaluate the efficiency. For example, Jemric and Vujcic [5] evaluated bank efficiency in Croatia from 1995 to 2000. Staub et al. [6] used DEA to compute the efficiency scores for Brazilian banks between 2000 and 2007.

Thus, Malmquist productivity index (MPI) was coined by Caves et al. [7]. The basic idea of MPI is to calculate the relative performance of a decision-making unit (DMU) at different periods using the technology of a base period. MPI is an effective approach for productivity measurement when the costs and prices are not observed [8]. It has been widely applied in various fields, such as productivity growth in industrialized countries [9, 10], the effect of deregulation on Spanish saving banks [11], production efficiency and productivity growth in Norwegian sawmilling [12], agriculture [13, 14], finance and economy [15, 16], and outsourcing decision of manufacturing processes [17].
Traditional MPI investigates that the measurement of the performance utilizes only the production factors while neglecting the production of by-products, especially undesirable outputs such as pollution. Such an approach may lead to biased measurements of productivity growth. Given its prevalence and importance, the production of by-products has received much attention by practitioners and researchers in economic and environmental management [18–21]. In other words, traditional MPI may not be able to give a credible identification of the efficiency and productivity growth of decision-making units (DMUs) when undesirable by-products exist. To overcome the undesirable output problem of MPI, Chung et al. [22] propose the Malmquist–Luenberger index (MLI) to offer firm reductions in undesirable outputs. Subsequently, MLI has been applied to assess productivity in various conditions by many researchers [20, 23–28]. In the banking industry, studies that applied DEA to evaluate the productivity also illustrated these issues [29–32].

Infeasibility and inconsistency are two main problems in MPI, which are also problems in MLI to do productivity evaluation. Aparicio et al. [33] identified the inconsistency problem of the Malmquist–Luenberger index (MLI) in interpreting the technical change component with its numerical value and illustrated this issue with a simple numerical example. In their research, they proposed a solution to solve the inconsistency in environmental productivity change under a certain assumption. Thus, it is also pointed out that inconsistency may still happen. In this paper, we mainly focus on the infeasibility of MLI, which occurs frequently when measuring cross-period productivity in MLI. Moreover, it cannot be eliminated substantially since DMUs are measured into two independent production sets in different periods. The infeasibility problem leads to the dilemma that a portion of DMUs under consideration cannot be projected onto the frontier with the directional vectors derived from the ordinary model. To tackle this problem, Färe et al. [20] propose reference technology by identifying the frontier in a successive period for all units. However, this way does not solve the infeasibility completely. In detail, some units may be located beyond the frontier in a period because they perform extremely well in their last period. Then, Färe et al. [34] develop a joint reference technology by incorporating the frontiers of two subsequent periods into an integrated period. In this way, DMUs under consideration are all located within the integrated frontier. Alternatively, Pastor and Lovell [35] introduce a Global Malmquist Productivity Index (GMPI) by using a base technology to estimate productivity change in different periods. Oh [26] extends the concept of GMPI to the MLI context by incorporating the negative effect of environmentally harmful by-products. Arabi et al. [24] propose a slack-based model to eliminate infeasibility and provide the scores for MLI for DMUs. Recently, Tzeremes [36] provides a robust (order-m based) Luenberger productivity indicator to evaluate hotels’ productivity. Similarly, Tzeremes [37] provide a robust (order-m based) MPI under different MPI decompositions. Such methodological treatments can minimize infeasibilities in productivity measurement derived from extreme data values or atypical data values.

However, the above methods do not identify the ultimate cause of the infeasibility. In addition, they fail to distinguish the super-efficiency and inefficiency in cross-period within a certain range correctly. Without a doubt, the incorrect cross-period efficiency evaluation will lead to an unreliable productivity analysis. It is worthwhile to re-examine the infeasibility of the MLI. Thus, our study contributes to providing more effective management messages and specific optimization suggestions for managers via answering the following three questions. First, how does infeasibility behave in a radial setting? Second, are DMUs beyond the frontier performing better than others that are on the frontier or within the frontier? Last, how does a well-defined cross-period inefficiency or cross-period super-efficiency perform in MLI?

Traditional MLI models suggest that the units can achieve an efficient status by increasing desirable outputs or/and decreasing undesirable outputs with an identical proportion. Existing studies on MPI/MLI models and super-DEA models define that the DMU beyond the frontier is super-efficient with a value larger than number one, which implies that this DMU is superior than the DMU on (within) the frontier in evaluation process. However, it may not fit the fact that the DMU beyond the frontier is derived from mutual impact of desirable variables and undesirable variables simultaneously when the undesirable variables are under consideration. In other words, just because a DMU is being considered and its value is higher than number one, does not mean it is better than other DMUs within or on the frontier. It is actually inefficient. To overcome the deviation of traditional efficiency score with efficiency score considering undesirable variables, we re-define the efficiency value, especially cross-period efficiency in MLI, according to the impacts of desirable and undesirable variables. Consequently, the DMU beyond the frontier may be superior, equivalent, or inferior to the DMU on (within) the frontier. Thus, we analyze the characteristics of desirable and undesirable outputs conditional on the infeasibility via classifying the region with desirable and undesirable outputs into four sub-regions. Specifically, our modified radial MLI models allow DMUs in different regions to reach the frontier with different projection directions, especially for the DMU with excessive desirable outputs and excessive undesirable outputs. Numerical analysis is provided to verify the availability of the proposed models.

The rest of the paper is organized as follows. Section 2 introduces MLI model and discusses the infeasible issue with four regions. Section 3 proposes modified radial MLI models to solve infeasibility problem and illustrates them with a numerical example, while Section 4 explores a productivity evaluation application of banks in China. Finally, Section 5 provides concluding remarks.

2. Methodology

2.1. Infeasibility in the Radial Malmquist–Luenberger Productivity Index Model. Suppose there are \( n \) DMUs over \( T \) time periods \( (t = 1, \ldots, T) \). The input, desirable output, and undesirable output vectors of DMU \( j \) in time period \( t \) are \( x^t_{i,j} \).
Undesirable outputs are modeled as a strongly or weakly disposable variable under different production situations [38–40]. In this study, we assume that they are weakly disposable because the reduction of undesirable output is “costly” in most cases [22] and infeasibility issues may have a higher possibility of occurrence.

The output-oriented directional distance function (DDF) model [22] is

\[
P' = \left\{ \begin{array}{l} \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq x_{im}^t, i = 1, \ldots, I, \\ I: \sum_{j=1}^{n} \lambda_j y_{ij}^t \geq y_{im}^t, r = 1, \ldots, R; \\ \sum_{j=1}^{n} \lambda_j z_{ij}^t = z_{im}^t, I = 1, \ldots, I; \lambda_j \geq 0, \quad j = 1, \ldots, n \end{array} \right. \quad (1)
\]

where \( g = (y_{ro}^t, -z_{ro}^t) \) is the directional vector, \( t \) denotes the period for the frontier, \( s \) denotes the period for DMU\( o \) under assessment, and \( \theta \) is the distance ratio of outputs from the frontier. Model (2) corresponds to the traditional DEA model if \( t \) is as same as \( s \) (i.e., in an identical period). Otherwise, it is equivalent to a super-efficiency DEA model because some of the DMUs may locate beyond the frontier. From this perspective, the DMU under evaluation may be inefficient (i.e., locates in the frontier, and the efficiency is lower than 1), efficient (i.e., locates on the frontier, and the efficiency is equal to 1), or super-efficient (i.e., locates beyond the frontier, and the efficiency is higher than 1). However, with the existence of undesirable output, a DMU beyond the frontier may not imply super-efficient as mentioned and discussed above, because it may be derived from the undesirable output. Considering this perspective, a further investigation is suggested to identify the reason why the DMU locates beyond the frontier.

Efficiency can be defined as \( \bar{E}_o^t(x^t, y^t, z^t) = 1/(1 + \bar{D}_o^t(x^t, y^t, z^t)) \) specifically, \( -1 < \bar{D}_o^t(x^t, y^t, z^t) < 1 \) (see the proof in Appendix). The value of DI for DMUs may be positive, negative, or zero. A positive value of DI indicates that the DMU under evaluation is suggested to reach the frontier by increasing desirable and decreasing undesirable output at the same time (denoted as Projection I). On the contrary, a negative value of DI implies that the DMU is suggested to reach the frontier by decreasing desirable and increasing undesirable output simultaneously (denoted as Projection II). Then, the Malmquist–Luenberger index of DMU\( o \) during time period \( t \) and \( t + 1 \) is

\[
MLI_o = \left[ \frac{1 + \bar{D}_o^t(x^t, y^t, z^t)}{1 + \bar{D}_o^{t+1}(x^{t+1}, y^{t+1}, z^{t+1})} \right]^{1/2}.
\]

In equation (3), \( \bar{D}_o^t(x^t, y^t, z^t) \) represents the distance of DMU\( o \) evaluated in a period \( t \) and the production frontier in period \( t \). \( MLI_o \) can be decomposed into two component measures: one is on accounting for efficiency change (MLEFFCH), and the other is measuring technical change (MLTECH). They are

\[
\text{MLEFFCH}_o = \frac{1 + \bar{D}_o^{t+1}(x^{t+1}, y^{t+1}, z^{t+1})}{1 + \bar{D}_o^t(x^t, y^t, z^t)},
\]

\[
\text{MLTECH}_o = \left[ \frac{1 + \bar{D}_o^{t+1}(x^{t+1}, y^{t+1}, z^{t+1})}{1 + \bar{D}_o^t(x^t, y^t, z^t)} \right]^{1/2}.
\]

Notice that infeasibility may occur in the MLI context. Then, we develop an approach to eliminate the infeasibility problem and provide a feasible estimation of the DMUs’ performances. Since the infeasibility issue only happens in cross-period efficiency evaluation, we focus on modeling the MLI score in two sequential periods. From this perspective, our model for infeasibility issues corresponds to the super-efficiency model via incorporating undesirable outputs. Similar to the super-efficiency setting, we define the DMU as inefficient (super-efficient) if the efficiency is lower (higher) than one, and efficient if the efficiency is equal to one.

2.2. Infeasibility in the Radial Malmquist–Luenberger Index Model. To illustrate the infeasibility in the radial MLI context, we use a simple example that each DMU consumes a single input to produce two outputs: one desirable and one undesirable. Table 1 lists the original data and feasible issue in model 2.

The infeasibility of MLI occurs when the desirable and undesirable outputs are beyond a certain range in cross-period efficiency evaluation. For example, the cross-period efficiency of A is unattainable with model 2. In an effort to describe the occurrence of infeasibility, we draw a virtual boundary (an approximate descriptive line) to distinguish the infeasible region (regions I, II, III, and IV) from the feasible one. As shown in Figure 1, the region is divided into four sub-regions by incorporating the frontier in period \( t \) and the boundary. Four regions and the characteristic of the DMU in each region are analyzed as follows, respectively.

The specific characteristic of DMU (such as \( B^{t+1} \)) in the region I can be described as follows: (1) it can reach the frontier by increasing desirable output and decreasing undesirable output with an identical proportion; (2) the optimal efficiency is obtained by calculating model (2), and it is
### 2.3. Modified Radial MLI Model for Infeasibility

As the above analysis, if the DMU locates in regions I, II, and III, the traditional radial model (3) proposes two paths for the inefficient or super-efficient DMUs to project onto the frontier. However, if the DMU locates in region IV, we cannot obtain a feasible solution for the DMU from the model (3). The reason is that the DMU produces relatively high desirable and undesirable outputs, and can only reach the frontier by decreasing desirable and undesirable outputs simultaneously.

Thus, we develop the modified radial MLI model to deal with the infeasibility:

\[
\theta - M \delta, \quad s.t. \sum_{j=1}^{n} \lambda_j x^*_{ij} \leq x^*_o, \quad i = 1, \ldots, I,
\]

\[
\sum_{j=1}^{n} \lambda_j y^*_{rj} \geq (1 + \theta) y^*_o, \quad r = 1, \ldots, R,
\]

\[
\sum_{j=1}^{n} \lambda_j z^*_{lj} = (1 - \theta - \delta) z^*_o, \quad l = 1, \ldots, L,
\]

\[
\lambda_j \geq 0, \quad j = 1, \ldots, n,
\]

\[
\delta \geq 0,
\]

where \( M \) is a user-defined larger positive number. In our application, \( M \) is set to be equal to \( 10^5 \theta^* \) and \( \theta^* + \delta^* \) refer to the distance ratio of desirable and undesirable outputs to the frontier, respectively. Following Cook et al. [41], we take the distance of DMU as \( \overline{D}_{o}(x^*, y^*, z^*): g \)

\[
= 1/2\theta^* + 1/2(\theta^* + \delta^*) - \theta^* - 1/2\delta^*.
\]

**Theorem 1.** Model (3) is infeasible if and only if \( \delta^* > 0 \), where \( \delta^* \) is the optimal solution in the model (5).

The Proof of Theorem 1 is in the Appendix. Theorem 1 indicates that the value of \( \delta^* \) is an indicator of whether model (3) is feasible. It has two implications. On the one hand, if the model (3) is feasible for the DMU, it implies that the DMU can achieve the frontier through projection I or II, and \( \delta^* = 0 \). Under this situation, the models (3) and (5) have an equivalent optimal value of \( \theta^* \). Otherwise, it has to make further adjustments to the outputs. That is, the DMU has to decrease desirable outputs and increase undesirable outputs with a proportion of \( |\theta^*|/(\theta^* < 0) \) following projection II, and then to reach the frontier with an additional decrease in...
undesirable outputs with a proportion of $\delta^*$. In this case, $D_o^*(x^*, y^*, z^*; g) > 0$ means that this DMU is beyond the frontier with excessive desirable and undesirable output. What’s more, the undesirable variable is more excessive and has a higher impact than the desirable output. If $\delta^* > 0$, $D_o^*(x^*, y^*, z^*; g) < 0$ means that this DMU is beyond the frontier with excessive desirable and undesirable output. However, on the contrary, the desirable variable is more excessive and has a higher impact than the undesirable variable.

Model (5) describes the additional adjustment of undesirable outputs for infeasible DMUs. If the decision-maker prefers an adjustment for desirable outputs, the corresponding model is as follows:

$$\theta - M \sigma,$$

subject to:

$$\frac{1}{n} \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij}^t, \quad i = 1, \ldots, I,$$

$$\frac{1}{n} \sum_{j=1}^{n} \lambda_j y_{ij} \geq (1 - \sigma) y_{ij}^t, \quad r = 1, \ldots, R,$$

$$\frac{1}{n} \sum_{j=1}^{n} \lambda_j z_{ij} = (1 - \theta) z_{ij}^t, \quad l = 1, \ldots, L,$$

$$\lambda_j \geq 0, \quad j = 1, \ldots, n,$$

$$\sigma \geq 0.$$

Thus, a proportion of $\sigma^*$ is suggested for the DMU to decrease desirable outputs additionally. The distance ratios of desirable and undesirable outputs are $\theta^* - \sigma^* \theta^* - \sigma^*$ and $\theta^*$, respectively. Similarly, the distance of DMU evaluated is defined as $D_o^*(x^*, y^*, z^*; g) = \theta^* - \sigma^* / 2 D_o^*(x^*, y^*, z^*; g) = \theta^* - \sigma^*/2$.

Theorem 2. Model (3) is infeasible if and only if $\sigma^* > 0$, where $\sigma^*$ is the optimal solution in the model (6).

Proof of Theorem 2 is similar to the Proof of Theorem 1.

Theorem 3. Model (5) and model (6) have the same optimal solution, i.e., $\theta^*_{model5} = \theta^*_{model5} - \sigma^*_{model5}$ and $\delta^*_{model4} = \theta^*_{model5} - \sigma^*_{model5}$.

The Proof of Theorem 3 is in Appendix. It is worth mentioning that the models (5) and (6) have the same implications for infeasibility. The difference and connections between them will illustrate in Table 2 and Figure 2 corresponding to the numerical example in Section 2.2, respectively.

Table 2: All banks.

<table>
<thead>
<tr>
<th>Bank type</th>
<th>Banks (for short)</th>
</tr>
</thead>
<tbody>
<tr>
<td>State-owned policy bank (SOPB)</td>
<td>China Development Bank (CDB), the Export-Import Bank of China (EXIMB), Agricultural Development Bank of China (ADBC)</td>
</tr>
<tr>
<td>State-owned bank (SOB)</td>
<td>Bank of China (BOC), China Construction Bank (CCB), Industrial And Commercial Bank of China (ICBC), Agricultural Bank of China (ABC)</td>
</tr>
<tr>
<td>Joint-stock bank (JSB)</td>
<td>Industrial Bank Co., Ltd (CIB), China Guangfa Bank (CGB), Bank of Communications (BCM), Shanghai Pudong Development Bank (SPDB), Ping An Bank Co., Ltd (PAB), China Minsheng Bank (CMB), China Merchants Bank (CMB), China CITIC Bank (CGB), Hua Xia Bank Co., Ltd (HXB)</td>
</tr>
<tr>
<td>Urban commercial bank (UCB)</td>
<td>Shanghai Rural Commercial Bank (SRCB), Bank of Chongqing (BCQ), Yinzhou Bank (YZB), Bank of Beijing Co., Ltd. (BBJ), Bank of Shanghai (BSH), Hengfeng Bank Co., Ltd. (FBH), China Zeshang Bank Co., Ltd (CZB), China Bohai Bank Co., Ltd (CBHB), Bank of Tianjin Co., Ltd (BTJ), Bank of Nanjing (BNJ), Huishang Bank (HSB), Bank of Ningbo (BNB)</td>
</tr>
</tbody>
</table>

3. Empirical Analysis

3.1. Empirical Implementation: Productivity of Banks in China. As important financial intermediation and payment channel, the banking industry plays an increasingly vital role in the economy of China [42–45]. To increase the competitiveness of banks, it is necessary to improve the
Operational performance of the banking system in China. In the bank industry, a nonperforming loan (or bad loan) can be taken as an undesirable output of the profitability stage as it harms the bank business and has negative impacts on returns directly. Specifically, the higher the proportion of nonperforming loans, the lower the operational performance of the bank.

In this section, we apply models proposed from the above section to investigate the productivity growth of banks in China with consideration of undesirable outputs from 2007 to 2016.

### 3.2. Data Sources and Descriptive Statistics

Based on the comprehensive literature reviews, we utilize two inputs (interest cost and operation cost), two desirable outputs (interest income and noninterest income), and one undesirable output (nonperforming loan). Most of the relevant data are collected from bank financial reports in China Stock Market Accounting Research (CSMAR). The rest of the data are obtained from the related bank’s annual report. Notice that operation costs are calculated from commission expenses, management fees, other business expenses, and nonbusiness expenses. Noninterest incomes are the sum of

---

![Figure 2: The variable growth rate from 2007 to 2016.](image)

### Table 3: Modified radial models.

<table>
<thead>
<tr>
<th>DMU</th>
<th>$D_{t+1}^*$</th>
<th>$y^*$</th>
<th>$z^*$</th>
<th>$E_{t+1}^*$</th>
<th>$D_{t+1}^*$</th>
<th>$y^*$</th>
<th>$z^*$</th>
<th>$E_{t+1}^*$</th>
<th>$y^*$</th>
<th>$z^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.069</td>
<td>-0.25</td>
<td>0.361</td>
<td>1.075</td>
<td>-0.069</td>
<td>0.111</td>
<td>0.361</td>
<td>1.075</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>0.167</td>
<td>0.167</td>
<td>0</td>
<td>0.857</td>
<td>0.167</td>
<td>0.167</td>
<td>0</td>
<td>0.857</td>
<td>2.33</td>
<td>0.83</td>
</tr>
<tr>
<td>C</td>
<td>-0.182</td>
<td>-0.182</td>
<td>0</td>
<td>1.222</td>
<td>-0.182</td>
<td>-0.182</td>
<td>0</td>
<td>1.222</td>
<td>3.27</td>
<td>1.77</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>-0.143</td>
<td>-0.143</td>
<td>0</td>
<td>1.167</td>
<td>-0.143</td>
<td>-0.143</td>
<td>0</td>
<td>1.167</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>0.643</td>
<td>0.643</td>
<td>0</td>
<td>0.609</td>
<td>0.643</td>
<td>0.643</td>
<td>0</td>
<td>0.609</td>
<td>3.29</td>
<td>1.79</td>
</tr>
<tr>
<td>G</td>
<td>0.029</td>
<td>-0.143</td>
<td>0.343</td>
<td>0.972</td>
<td>0.029</td>
<td>0.200</td>
<td>0.343</td>
<td>0.972</td>
<td>2.14</td>
<td>3.6</td>
</tr>
</tbody>
</table>

---

![Figure 3: Projection direction of DMUA with the modified model.](image)
commission income, investment income, other business income, and nonbusiness income. Our study focuses on evaluating the productivity change for 28 banks in China in ten years, from 2007 to 2016. In addition, according to the nature and ownership of banks, 28 banks are classified into 3 state-owned policy banks, 4 state-owned banks, 9 joint-stock banks, and 12 urban commercial banks. All banks are listed in Table 2.

Descriptive statistics (mean and standard deviation) of 28 banks from 2007 to 2016 are described in Table 4, while Figure 2 illustrates the variable change during this period.

We can find that variables fluctuated greatly around 2008. For example, interest income, the interest cost, and nonperforming loans decreased a lot, while noninterest income and operation costs increased in 2008-2009. Most variables showed a trend of slow annual growth after 2010.

4. Discussion of the Results

4.1. Productivity Analysis of All Banks. The evaluation result of productivity change for all 28 banks in China from 2007 to 2016 is calculated and obtained with the proposed model (5). The MLI of all banks is reported in Table 5.

According to Table 5, we find that most banks’ MLI is larger than 1, which means the productivity of the
corresponding banks is progressive over corresponding period. In detail, the productivities of 26 banks (except for EXIMB, CBHB, and BTJ) in the 2007-2008 period improved, while only 12, 15, and 7 banks have productivity progressive in 2008-2009, 2014-2015, and 2015-2016, respectively. The global economic crisis in 2008 is one of the reasons that most banks are productivity regressive in the period 2008-2009. Meanwhile, the economic downturn that began in 2015 causes another productivity decline from 2015.

An interesting finding from Figure 2 is that the growth rate of interest cost and interest income is 3.9% and 4.5% during 2014-2015, respectively. They are far below the growth rate before 2014 and even decreased (−5.6% and −5.5%) further during 2015-2016. This finding also provides proof of why productivity is regressive after 2014. In addition, we know that interest income is a key part of the income structure of Chinese banks and takes up a large proportion of gross income. The variable change during this period implies that the income structure of the banking system in China is slowly changing to some extent. The proportion of noninterest income is gradually increasing.

To analyze the influence of efficiency change and technical change on banks’ productivity, we consider the productivity of all banks in China during 2012 to 2013 and 2015 to 2016 periods as examples. According to the MLI, MLEFFCH, and MLTECH in Table 6, we find that (1) productivity change has been affected mutually by both efficiency change and technical change, and (2) the main influence factors are not the same under different time periods or among different banks. For example, the MLI of CDB during 2012-2013 is 1.061 (>1) with productivity progression. Correspondingly, MLEFFCH is 0.995 (<1) with regressive efficiency and MLTECH is 1.067 (>1) with technical improvement. Thus, the productivity progression of CDB during this period is mainly affected by technical improvement. Identically, the productivity of CDB during the 2015-2016 period is also mainly affected by technical change, but technical regression is not the reason of productivity regression anymore. For example, the MLI of YZB during the 2012-2013 period is 0.996 (<1) with regressive productivity, which is mainly influenced by efficiency regression since MLEFFCH is 0.938 (<1) with efficiency regression while MLTECH is 1.062 (>1) with technical progression, while the productivity of this bank during 2015-2016 period is mainly influenced by technical change. More specifically, productivity regression (MLI is 0.989) is the result of technical regression (MLTECH is 0.989) and no change in efficiency.

Average productivity index (MLI) and two decompositions of MLI, efficiency change index (MLE) and technical change index (MLTECH), are described in Figure 4. The average productivity of banks and corresponding MLTECH

### Table 6: MLI, MLEFFCH, and MLTECH during 2012-2013 and 2015-2016 periods.

<table>
<thead>
<tr>
<th>Bank</th>
<th>2012-2013</th>
<th>2015-2016</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MLI</td>
<td>MLEFFCH</td>
</tr>
<tr>
<td>CDB</td>
<td>1.061</td>
<td>0.995</td>
</tr>
<tr>
<td>EXIMB</td>
<td>1.052</td>
<td>1.000</td>
</tr>
<tr>
<td>ADGCC</td>
<td>1.125</td>
<td>1.000</td>
</tr>
<tr>
<td>BOC</td>
<td>1.038</td>
<td>1.038</td>
</tr>
<tr>
<td>CCB</td>
<td>1.008</td>
<td>1.000</td>
</tr>
<tr>
<td>ICBC</td>
<td>1.042</td>
<td>1.003</td>
</tr>
<tr>
<td>ABC</td>
<td>1.064</td>
<td>1.000</td>
</tr>
<tr>
<td>CIB</td>
<td>1.101</td>
<td>0.965</td>
</tr>
<tr>
<td>CGB</td>
<td>1.265</td>
<td>1.045</td>
</tr>
<tr>
<td>BCM</td>
<td>1.076</td>
<td>0.984</td>
</tr>
<tr>
<td>SPDB</td>
<td>1.124</td>
<td>0.940</td>
</tr>
<tr>
<td>PAB</td>
<td>1.200</td>
<td>1.038</td>
</tr>
<tr>
<td>CMBC</td>
<td>1.102</td>
<td>1.018</td>
</tr>
<tr>
<td>CMB</td>
<td>1.200</td>
<td>1.000</td>
</tr>
<tr>
<td>CCIB</td>
<td>1.095</td>
<td>0.948</td>
</tr>
<tr>
<td>HXB</td>
<td>1.013</td>
<td>1.045</td>
</tr>
<tr>
<td>SRCB</td>
<td>1.055</td>
<td>0.902</td>
</tr>
<tr>
<td>BCQ</td>
<td>1.101</td>
<td>1.000</td>
</tr>
<tr>
<td>YZB</td>
<td>0.996</td>
<td>0.938</td>
</tr>
<tr>
<td>BBJ</td>
<td>1.042</td>
<td>0.922</td>
</tr>
<tr>
<td>BSH</td>
<td>1.211</td>
<td>0.965</td>
</tr>
<tr>
<td>HFB</td>
<td>1.058</td>
<td>0.943</td>
</tr>
<tr>
<td>CZB</td>
<td>1.123</td>
<td>0.956</td>
</tr>
<tr>
<td>CBHB</td>
<td>0.914</td>
<td>1.000</td>
</tr>
<tr>
<td>BTJ</td>
<td>1.261</td>
<td>0.848</td>
</tr>
<tr>
<td>BNJ</td>
<td>1.159</td>
<td>0.936</td>
</tr>
<tr>
<td>HSB</td>
<td>0.853</td>
<td>1.000</td>
</tr>
<tr>
<td>BNB</td>
<td>1.234</td>
<td>0.958</td>
</tr>
<tr>
<td>Mean</td>
<td>1.092</td>
<td>0.977</td>
</tr>
</tbody>
</table>
during the 2007–2014 periods are all larger than 1, which means technically advanced. Meanwhile, some of MLEs are larger than 1, while others are less than 1. During the 2014–2016 periods, the average MLI, MLEFFCH, and MLTECH are all less than 1, which corresponds to both efficiency change and technical change being backward. Obviously, productivity change is an interactive result of technological change and efficiency change and is mainly affected by technical change.

4.2. Productivity Comparisons of Various Types of Banks. In this section, we analyze the productivity change in four different bank types. MLI is described in Figure 5. The dotted line "constant" in Figure 5 denotes that the productivity index is 1 and constant. The part above the dotted line, i.e., the productivity greater than 1, denotes that the corresponding bank is productivity progression. Conversely, a bank below the dotted line indicates that the bank is productivity declined. In addition, the further DMU is from the dotted line, the higher the rise or fall in productivity.

Obviously, there is no significant difference in productivity change of four types of banks over the past decade. Productivities of most banks are progressive before 2013 and started to decline after 2014. This is consistent with the change in the variables in the previous analysis. Combined with the statistical analysis, we further verify that the productivity difference among four types of banks is mainly influenced by period, but not by the types of banks. It is worth mentioning that the range of productivity changes in the SPOB is more stable, with little progress or decline. The reason may be because the SPOB is a policy bank (noncommercial bank), which is for the government to develop the economy, promote social progress, and conduct macroeconomic management. This type of bank will not have a large fluctuation range in order to stabilize the social economy.
Figure 6 depicts the changes and relationships of MLI, MLEFFCH, and MLTECH for the four types of banks in the past 10 years. We find that even though the MLEFFCH and MLTECH are mostly close to 1 after the 2014 year, the long-term change still shows that productivity changes of all kinds of banks are mainly influenced by MLTECH, i.e., technology change. The reason is the change range of MLEFFCH is small every year and close to the state of no change, while MLTECH has fluctuated even more. To some extent, this conforms to the law of "science and technology is the primary productive force"; that is, technological progress can bring more progress to the productive force. Simultaneously, we also verify that the change of MLEFFCH and MLTECH is not significantly affected by the type of banks.

5. Concluding Remarks

With the increasing prevalence of infeasibility in estimating productivity growth with the Malmquist–Luenberger index, a large amount of research in productivity and performance management has investigated various drives of infeasibility. An abundance of theoretical work has paid close attention to the integration of the frontiers. However, little attention has been paid to the characteristics of desirable and undesirable outputs. This paper fills the gap by building a framework model to overcome the infeasibility in the MLI context.

The main theoretical insight is that the production of desirable and undesirable outputs may exceed the technology in a certain time period, which causes infeasibility. To better illustrate the occurrence of infeasibility, in our study, we classify the region with desirable and undesirable outputs into four regions and investigate the characteristics of desirable and undesirable outputs. A special region for infeasibility is composed of the DMUs which produce a relatively high level of desirable and undesirable outputs. In addition, we propose modified radial MLI models to deal with the infeasibility by readjusting outputs. Theorems demonstrate the interactive relationship between the proposed models and traditional radial MLI models. At the same time, the availability of the proposed models is verified with a numerical example. Models for infeasibility can also be extended to deal with the infeasibility in the MPI setting and super-DEA model.

Further, the new model is used to evaluate the productivity of 28 banks in China during 2007–2016. We find that, first, the mean productivities of all banks are progressive from 2007 to 2014, and regressive after 2014. Second, more than half of banks are productivity regressive during 2008-2009 because of the global economic crisis 2008, and most of banks are productivity regressive after 2014 because of the economic decline. Third, the reasons for productivity progressive or regressive of 28 banks during
different periods are different. Fourth, time is the main influencing factor of productivity difference among the four types of banks. Last, productivity change is mainly influenced by technical change, which also conforms to the law of “science and technology is the primary productive force.”

Last, the model in this study addresses the impractical problem in MLI by differentiating between the effects of desirable and undesirable output. Practically, it assists bank managers in understanding the profitability of bank development and operation, that is, the ability to generate income and manage risk, specifically, and the management of nonperforming loans. Furthermore, it offers another way for other stakeholders to evaluate banks and guarantee the security of their own investments by examining and evaluating the productivity of the three different types of banks, as well as the operation of all Chinese commercial banks.

Note that the current study only investigates and solves infeasibility issue and does not consider the inconsistency. Aparicio et al. [46] also illustrated that most of the current literature ignores the inconsistency problem of MLI, while only a few researchers discussed this problem. Thus, in further research, we are investigating the inconsistency issue with nonradial MLI models since inconsistency often exists in nonradial settings. Simultaneously, more cases will be investigated and discussed, such as the undesirable output produced in this period is taken as input in the next period, each sub-system and system productivity evaluation in two-stage or more complex network structure, and so on.

Appendix

Proof of $-1 < \theta^* < 1$.

Denote $D_{\theta}^t(x^t, y^t, z^t)$ as $\theta^*$ which is the optimal solution of model (2). According to Chung et al. [22], $-1 < \theta^* < 1$.

Next, we prove $\theta^*$ cannot reach the bounds of $-1$ and $1$ by contradiction.

Assume $\theta^* = 1$, the third constraint in model (2) is simplified as $\sum_{j=1}^{n} \lambda_j^* y_{ij}^* = 0$, $i = 1, \ldots, L$. Since $z_{ij}^* > 0$ and $j \geq 1$ for any $l$ and $j$, we have $\lambda_j^* = 0$ for all $j$, and $\sum_{j=1}^{n} \lambda_j^* y_{ij}^* = 0$. However, it contradicts the second constraint $\sum_{j=1}^{n} \lambda_j^* y_{ij}^* \geq 2 z_{ij}^* > 0$, $r = 1, \ldots, R$.

Assume $\theta^* = -1$, and the second and third constraints are simplified as $\sum_{j=1}^{n} \lambda_j^* y_{ij}^* > 0$, $r = 1, \ldots, R$ and $\sum_{j=1}^{n} \lambda_j^* y_{ij}^* > 0$, $i = 1, \ldots, L$. Since $z_{ij}^* > 0$ and $j \geq 1$ for any $l$ and $j$, there exists at least one positive weight, i.e., $\lambda_j^* > 0$.

Denote a weight $\lambda_k^*$ satisfying $0 < \lambda_k^* < \lambda_1^*$, and we have $\sum_{j=1, j \neq k}^{n} \lambda_j^* x_{ij}^* + \lambda_k^* x_{ik}^* < \sum_{j=1}^{n} \lambda_j^* x_{ij}^* < x_{i0}^*$, $i = 1, \ldots, L$ and $\sum_{j=1, j \neq k}^{n} \lambda_j^* y_{ij}^* + \lambda_k^* y_{ik}^* = 2 x_{i0}^* - (\lambda_j^* - \lambda_k^*) x_{ik}^*$, $l = 1, \ldots, L$. Let $(\lambda_j^* - \lambda_k^*) x_{i0}^*$ and we have $\alpha = (\lambda_j^* - \lambda_k^*) x_{i0}^* / (\lambda_j^* - \lambda_k^*) x_{ik}^*$ and we have $\alpha = (\lambda_j^* - \lambda_k^*) x_{i0}^* / (\lambda_j^* - \lambda_k^*) x_{ik}^* - 1 > 1$ and $\sum_{j=1, j \neq k}^{n} \lambda_j^* y_{ij}^* \alpha > 1$ and $\sum_{j=1, j \neq k}^{n} \lambda_j^* y_{ij}^* + \lambda_k^* y_{ik}^* > 0$, $r = 1, \ldots, R$.

Then, for the evaluated DMU, there exists at least one feasible solution which is larger than $-1$, and for instance, $\theta = \alpha = (\lambda_j^* - \lambda_k^*) x_{i0}^* / (\lambda_j^* - \lambda_k^*) x_{ik}^* - 1$. It contradicts the assumption $\theta^* = -1$.

To sum up, we have $-1 < \theta^* < 1$.

Proof of Theorem 1. Note that $\delta > 0$ is constrained to be no less than $0$.

If model (3) is infeasible, then $\delta^* > 0$; if $\delta^* > 0$, then model (3) is infeasible.

Suppose model (3) is infeasible. Since a value of $\delta^*$ equal to $0$ implies that model (3) is feasible, it contradicts to the assumption. Therefore, $\delta^* > 0$. Now suppose that $\delta^* > 0$, and model (3) is feasible. This means $\delta^* = 0$ is a feasible solution to model (5), in contradiction to the fact that $\delta^* > 0$ is optimal. Therefore, model (3) is infeasible. Thus, above completes the proof.

Proof of Theorem 3. First, take $\theta + \delta = \alpha$ in model (5), and then model (5) can be converted into

$$\max \alpha - (1 + M)\delta,$$

s.t. $\sum_{j=1}^{n} \lambda_j^* x_{ij}^* \leq x_{ij}^*$, $i = 1, \ldots, I$,

$$\sum_{j=1}^{n} \lambda_j^* y_{ij}^* \leq (1 + \alpha - \delta) y_{ij}^*$, $r = 1, \ldots, R$,

$$\sum_{j=1}^{n} \lambda_j^* z_{ij}^* \leq (1 - \alpha) z_{ij}^*$, $l = 1, \ldots, L$,

$$\lambda_j^* \geq 0, j = 1, \ldots, n,$$

$$\delta \geq 0.$$

Since $M$ is a user-defined larger positive number, then $M + 1$ is also a larger positive number and can be regarded as $M$. Then, take $\alpha = \theta$ and $\delta = \sigma$ in model (A1). Now the model (A1) is converted into model (6).

Theorem 3 is proved.

Data Availability

The data generated or analyzed during this study are included in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Nannan Liang was responsible for conceptualization, methodology, data collection, result analysis, and original draft preparation. Qian Zhang carried out writing, reviewing, and editing.

Acknowledgments

The authors are grateful to Professor Joe Zhu for his suggestions and comments on earlier versions of the paper. This research was supported by Natural Science Fund for Colleges and Universities in Jiangsu Province (No. 18KJD120003), Philosophy and Social Science Research
Fund for Colleges and Universities in Jiangsu Province (No. 2019SJA0336), and National Natural Science Foundation of China (Nos. 71801132, 71828101, and 71701102).

References


