# Pricing and Assortment Decision of Competitive Omnichannel Selling Strategy: Considering Online Return Cost 

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Received 21 June 2022; Revised 9 August 2022; Accepted 23 September 2022; Published 19 October 2022
Academic Editor: Xiang Peng
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#### Abstract

In the age of new retailing, consumers are discovering brands in new ways and seeking new conveniences to guide their purchasing decisions. Increasingly, retailers are offering consumers a cohesive and comprehensive experience across digital and physical touchpoints by integrating online and offline channels. This study considers a competitive market structure with two retailers selling four products via both online and offline channels. The retailers act as the decision maker of both the product assortment strategy and pricing strategy. Consumers are heterogenous in their horizontal fitness regarding each product, while they are common in the return probability when facing a deceptive product. Consumers' purchase decision of whether to purchase through physical stores or online stores and making a purchase from which retailers depend on not only the product assortment strategies across competitive retailers but also the return cost that the consumers are faced with if product return happens. Results show that the online product return cost plays an important role in the retailers' optimal pricing strategy design and product assortment strategy design. To be specific, the optimal prices of products that are sold through the online channel first increase in the return cost of the online product and then decrease it; while the optimal prices of products sold offline are always increasing in the online product return cost. Moreover, no matter what placement strategy sellers will choose, the optimal profit of both sellers is first decreasing the return cost of the online product and then increasing it. Our analyses also depict a two-dimensional market structure by considering sellers' return costs and consumers' misfit costs to investigate the optimal selling strategies under the cross-channel shopping platform.


## 1. Introduction

Consumers in their retail practice are now attaching importance to omnichannel behavior from their perspective. They are ready to take use of both online and offline retail channels in their searching behavior for product information [1, 2]. To be better adapted to this new environment, retailers of all industries are reexamining their strategies for delivering both information and products to their target consumers through channels. A business report points out that among the consumers conducting omnichannel behavior, $53 \%$ of them start researching digitally while $47 \%$ start gathering information in stores, and the two proportions are almost the same (Oracle Bronto 2018). These kinds of consumer searching behaviors are of great help for
consumers to gather product information from multidimensions in practice. Meanwhile, they give challenges to retailers to design their selling strategies including the pricing strategy and product placement strategy via the channel integration shopping platform, to devise their omnichannel selling strategies under the cross-channel shopping environment.

What attracts our attentions are the competitive pricing and product placement strategies among retailers selling similar kinds of products, with the only difference in their assortment methods via both online and offline channels. We assume consumers are heterogenous in their horizontal fitness regarding different products. While, they are common in the return probability facing a deceptive product due to quality dissatisfaction. To be more specific, we study the
issue by focusing on the difference remaining in consumers' online and offline shopping behavior, namely, the expected cost occurring during the selling process. Consumers' purchase the decision of whether to purchase through a physical store or online store and making a purchase from which retailer depends on not only the product assortment strategies across competitive retailers but also the return cost that the consumers are faced with if product return happens. We assume the return costs include not only financial expenditures but also psychological burdens and time costs undertaken by consumers. Results show that the online product return cost plays an important role in the retailers' optimal pricing strategy design and product assortment strategy design. Our analyses depict a two-dimensional market structure by considering sellers' return costs and consumers' misfit costs to investigate the optimal selling strategies under the cross-channel shopping platform.

From our analyses of the model depicting a competing market with two sellers selling products through both online and offline channels, we can derive the following three insights. First, no matter what the placement strategy both sellers choose, the optimal prices of products that are sold through the online channel are first increased in the return cost of the online product and then decreased in the online product return cost; while the optimal prices of products sold via offline are always increasing in the online product return cost. Second, no matter what placement strategy sellers will choose, the optimal profit of both sellers is first decreasing the return cost of the online product and then increasing it. With the increase in return cost, online product sales are guaranteed by avoiding consumers' arbitrary returning or exchanging behavior. Thus, the profits are increasing in the return cost. Third, if we consider the three placement strategies, given the optimal equilibrium results, the sellers choose three cases by considering the unit misfit cost of products, horizontal feature, and the return cost of online products simultaneously.

## 2. Literature Review

The first stream of literature that highly related to our research are those studying sellers' product assortment strategy and pricing strategy across channels. Brynjolfsson et al. [3] empirically studied the influence of assortment reduction via traditional sellers and assortment expansion via an online channel. Their results show that this strategy can increase consumer surplus. Dukes et al. [4] argue from the perspective of competitive incentives regarding assortment decisions. Results show that the strategic assortment reduction of traditional sellers, however, can cut down the consumer surplus. Bhatnagar and Syam [5] set up a model to study the product allocation for a hybrid retailer with both online and offline stores. In their model setting, the products can be withdrawn from offline stores and placed exclusively at online stores to save inventory costs. Taleizadeh et al. [6] investigated the best values for prices, quality levels, and effort decisions of the manufacturer, retailer, and third party under two types of closed-loop supply chains: (1) singlechannel forward supply chain with a dual-recycling channel
(SD model) and (2) dual-channel forward supply chain with a dual-recycling channel (DD model). Taleizadeh et al. [7] addressed coordinating and pricing decisions in two competitive reverse supply chains with different channel structures and provided two coordinated contracts considering that consumers can return e-waste through direct or traditional channels, while competitors can only collect obsolete products through traditional channels. Taleizadeh et al. [7] investigated the impact of market power structures and leadership between firms including two manufacturers and one retailer by addressing a pricing problem of two substitutable products in a two-echelon supply chain. Taleizadeh and Mokhtarzadeh [8] used a value-at-risk approach to address pricing and warranty policy optimization problems when manufacturers use online and offline channels to sell products and offer warranty policies for products sold through online channels. Kalantari et al. [9] used dynamic games to investigate the optimal pricing strategy problem in the second-hand market by considering a supply chain environment composed of a manufacturer and a retailer. The results show that a rich brand image is always beneficial to manufacturers and retailers and that improving physical utility can increase the demand for leased products, prompting retailers to actively participate in the market. Notably, online shopping preferences are critical to market segmentation and retailer decisions. Nevertheless, all preceding studies focus on the supply side factors impelling sellers' cross-channel product assortment strategies and none of them consider the sellers' product design feature strategies. Our research adds another motivation to sellers' product assortment strategy by considering the demand side factors of consumer omnichannel information searching behavior. Meanwhile, we make the sellers as decision makers of product features, which can dynamically depict the transformation of omnichannel sellers' product placement strategy.

Another stream of studies that are highly related to our study is that regarding product returns. Some of them examine this topic from the perspective of supply chain management. For instance, Majumder and Groenevelt [10] developed a two-period model of a competitive market to study the impact of remanufacturing costs on competing returned products; Ferguson and Toktay [11] set up models to support a manufacturer's recovery strategy in the competitive remanufactured product market; Savaskan, Bhattacharya et al. [12] show that simple coordination mechanisms can be designed to obtain the same level of retailer effort and supply chain profits as the centrally coordinated system; Savaskan and Van Wassenhove [13] focus on the interaction between a manufacturer's reverse channel choice to collect postconsumer products and the forward channel strategy to determine prices in a competitive market. Besides, Cachon [14] reviews the supply chain coordination with contracts in respect of inventory decisions and return contracts. Other researchers mainly focus on investigating the buyback pricing strategy in the durable product market [15-17]. Meanwhile, product returns are also brought about by the lack of information regarding product quality. It mainly results in the warranty returns of
damaged or low-quality products [3, 18, 19]. However, we focus our study on the product returns brought about by consumers' lack of information regarding both their preferences of product design features and product quality performance.

In the field of marketing, researchers also examine consumers' return behaviors in a multichannel shopping environment. Sarvary et al. [20] study a competitive market with dual channels and investigate how the pricing strategies and assistance levels in a physical store can change with the foundation of an online channel. Gao and Su [21] considered that only customers who purchased products through online channels have returned products, assumed that the prices of the two channels are the same, studied the inventory problem of omnichannel retailers, and compared retailers under different information strategies. Changes in profit and inventory, but the article does not consider the possibility of returns for customers who purchase products through offline channels. Javadi et al. [22] studied optimal pricing decisions for a dual-channel supply chain consisting of manufacturers and retailers and developed a flexible return policy including full refunds, returns through direct (i.e., manufacturer) channels, indirect (i.e., retailers) channel returns, and the results show that a full refund with an indirect channel policy under a price incentive mechanism can not only maximize the profits of a dual-channel supply chain but also satisfy the government's target function. Samorani et al. [23] argue that the return of a product is usually one of a series of transactions that consumers make to find an item, and analyze the return as part of the product search process: when returning a product, a consumer may buy another product. It may then be returned, followed by another replacement item, and the customer makes a series of purchases and returns until they eventually keep the product or make a final return. The study found that while higher average prices increased the probability of returns, they also increase the probability that a customer will keep the product, suggesting an opportunity for profit growth for retailers by allowing returns. Nageswaran et al. [24] studied both online and offline channels and analyzed the impact of two return strategies on retailers' profits when retailers adopt full-return returns, which can stimulate customers to bring additional benefits, and partial returns cannot bring additional benefits.

Ofek et al. [20] studied the pricing strategies of two competing retailers and the impact of adding online channels considering the impact of returns. Li et al. [25] studied the impact of online distributors' return policies, product quality, and pricing strategies on customers' purchase and return decisions, indicating that retailers should provide low-quality, low-price loose return policies or highquality, high-price returns' strict return policies. Hu et al.[26] studied the dynamic pricing problem of sellers when the customer return rate is high under limited inventory, and the authors also assumed that some of the returned items in good condition could be resold as new items, while the rest were processed at the end of the sales season. Ma et al. [27] pointed out that in addition to the traditional return channel, P2P secondary platforms have
gradually become another way to deal with improper products and established retailers to sell new products in the first stage and P2P platforms to operate second-hand products in the second stage A two-stage model of commodity markets and an analysis of the impact of returns on profits. Alaei et al. [28] explored the impact of return strategies on manufacturers' profits across three different retail channels. Dabaghian et al. [29] studied the impact of the return policy on the amount of profit and wholesaleretail prices in a three-echelon supply chain (manufacturers, distributors, and retailers) environment by considering social responsibility. However, we mainly focus our research on the competing sellers' pricing strategy and equilibrium profits with the interactive relations between the product demands and consumers' return behaviors.

Besides, we assume consumers are heterogenous in their preferences regarding product design features. This assumption leads to different implications compared to previous studies in respect of return policies. For example, Xie and Gerstner [30] studied the benefits of consumers' escape from prepurchase service contracts. Their results show that the refund policy for cancellation can reduce demand and improve capacity utilization. Guo [31] develops a model based on the preceding work to investigate how competition influences the equilibrium profits and refund policies through advance and spot selling. The result shows that competing sellers only adopt the partial refund policy for advanced selling if there is sufficiently constrained capacity. However, our model makes the return cost exogenous value, and we mainly focus on the competition between retailers under the assumption of consumers' heterogeneity in their initial valuations. Meanwhile, we take into consideration the product assortment strategy via its influence on competing sellers' pricing strategies.

## 3. Model Setting

3.1. Problem Description. Consumers in their retail practice are now attaching importance to omnichannel behavior from their perspective. They are ready to take use of both online and offline retail channels in their searching behavior for product information [1, 2]. To be better adapted to this new environment, retailers of all industries are reexamining their strategies for delivering both information and products to their target consumers through channels. Meanwhile, consumers' omnichannel searching behaviors give challenges to retailers to design their selling strategies including the pricing strategy and product placement strategy via the channel integration shopping platform, as they need to devise their omnichannel selling strategies under the crosschannel shopping environment. What attracts our attentions are the competitive pricing and product placement strategies among retailers selling similar kinds of products, with the only difference in their assortment methods via both online and offline channels.

We consider a competitive omnichannel selling market with four horizontally differentiated products sold by two competing retailers. We refer to the locations of the product $j$ as $x_{j}$ which is assumed evenly spaced out along a
unit circle [32]. More specifically, let the product $j$ be located under the competitive market structure. This assumption helps us get analytical results in our main model setting. However, in more general cases where the product spaces nonuniformly along the circle, the consumers with nonuniform preferences are introduced, which makes the model intractable. To eliminate the technical problems, we first consider the model by focusing on the interactions between sellers in the market. Each of the two competitive sellers owns two out of the four products and she can choose to place one product online and the other offline in the omnichannel environment. To be concrete, we assume one of the sellers sells two products with dual channels around the Salop circle, which is a variant of the traditional Hotelling [26] model of spatial competition derived from Lerner and Singer [32]. In this variant, the economy that is envisioned consists of two firms. Each one upon which owns two out of the four products. They are competitive in the overall market share. For instance, the competitive market allows each firm to sell products alternating in the location around the circle (i.e., one firm sells products located at $x_{0}=0$ and $x_{2}=2 / 4$, as the other firm sells $x_{1}=$ $1 / 4$ and $x_{3}=3 / 4$ ). Another placement strategy allows one firm sells $x_{0}$ and $x_{1}$ as the other firm sells $x_{2}$ and $x_{3}$. Figure 1 helps us to get a better understanding of the firms' locations around the Salop circle.

By taking into account the overall four products in the competitive market structure, we introduce consumers' the returning behavior of each good and the exchanging behavior between goods. That is to say, consumers in the competitive market can not only return their unsatisfied original goods but can also swap them for a more satisfactory ones. Each product has a common marginal cost of production $c$ in the vertically integrated systems. Moreover, we assume the products selling in the market are a kind of experience goods. Experience goods represent those products that consumers can only know whether their preferences match or not after they purchase the product or have a try in person [24]. We assume that this kind of consumers who observe the product through an online website without any personal inspection of product fitness before purchasing as online consumers, which account for $\omega$ of the overall consumers. Namely, this fraction of consumers are not sure if the product is a good fit with their preferences before purchase. We assume consumers are heterogenous in their taste of the product with an intrinsic preference parameter $\vartheta_{i}$, which is comprised of two main parts: an observable component and an unobservable component prior to purchase. To be more specific, $\vartheta_{i}=\theta_{i}+\varepsilon_{i}$, where $\theta_{i} \sim U[0,1]$ is perceived by consumers prior to their original purchase decision and $\varepsilon_{i}$ is uniformly distributed over $[-\delta, \delta]$, which is a common knowledge prior to purchase. However, the specific value of the unobservable component $\varepsilon_{i}$ is resolved only after the consumers obtain the experience products. Note that the value of $\delta$ is assumed to be less than $1 / 8$ in the market with four products to ensure that the uncertain component of consumers' preference will not affect the final judgment of


Figure 1: Salop circle of the market structure.
consumers' purchase decision. For instance, when the consumer is located at the exact position of a certain product with no misfit value in respect of $\theta_{i}$; then, no matter what the value of $\varepsilon_{i}$ is, he will always unambiguously choose this product rather than its adjacent counterpart. As for those consumers with store inspection before purchase that account for $1-\omega$ of overall consumers, we call them offline consumers. This kind of consumer understands the value of $\vartheta_{i}$ without any uncertainty after they try to experience products or observe the features of the goods such as colors or size. This finally resolves the uncertain component in consumers' taste before they make a purchase decision.

In addition to the uncertainty about products' horizontal design features to consumers' ideal preferences, there also exists an uncertain factor in terms of consumers' reservation value $v_{i}$, which is the utility gain a consumer obtains after consumption of a product. We can also demonstrate this factor as the vertical quality performance of the product that can augment consumers' utility gain with a rank-ordered preference. Both consumers with and without store inspection are uncertain about this reservation value because it will only be resolved after consumers' consumption of the product. This is the reason many retailers set up a return policy for free returns after seven days of usage or other return warrants like a quality guarantee for one year of usage. Concretely, the consumer obtains zero utility gain from possessing any one of the products offered by the retailer through either channel with probability $\alpha$, i.e., $v_{i}=0$. It corresponds to the scenario where the product is defective in quality after proper usage. However, with probability $1-\alpha$, the consumer obtains positive utility gain with $v_{i}=v$ and the consumer's consumption value equals to $v-t \mid x_{j}-$ $\vartheta_{i} \mid$ when the product is located at $x_{j}$. The parameter $t$ measures the unit misfit cost regarding the difference between the product design feature and consumers' ideal preferences. Both $v$ and $t$ are common knowledge to the consumers. Nevertheless, online purchases and offline purchases are differentiated in cost if the behaviors of returning defective goods or exchanging misfit products occur. Specifically, the return cost of online products includes the return freight insurance, the waiting cost of time, or the transportation cost, and we assume it to be $r$. While as for the offline products, this cost is the hassle cost involved with arguing with the salesclerks or shoe-leather

Table 1: Parameters and decision variables.

| Symbol | Definition |
| :--- | :---: |
| $c$ | The marginal cost of the product <br> $t$ |
| Consumer disutility per unit of deviation from the match <br> with preferences |  |
| $v$ | Consumer reservation utility for a perfect match with |
| preferences |  |
| $\alpha$ | Consumer probability of deriving zero utility from |
| $\mathcal{\vartheta}_{i}$ | product consumption |
| $\theta_{i}$ | Consumer $i$ 's ideal taste parameter |
| $x_{j}$ | Consumer $i$ 's prior belief about $\vartheta_{i}$ |
| $p_{j}$ | Location of product $j$ |
| $r$ | Price charged for the product $j$ |
| $h$ | The return cost of online product |

cost which is assumed to be $h$. Table 1 provides definitions of all parameters and decision variables for readers' better understanding of our model structure.
3.2. Sequence of the Game. In the competitive setting, each seller first chooses the placement strategy of the product location $x_{j}$ through online and offline channels simultaneously. Note that product locations are assumed evenly spaced out along a unit circle. We assume there are two placement strategies for two competing sellers: one seller's product in adjacent locations along the circle or one seller's product in the opposite locations along the circle. Specifically, we suppose one firm sells $x_{1}=1 / 4$ (online) and $x_{0}=0$ (offline), the other firm sells $x_{2}=2 / 4$ (online) and $x_{3}=3 / 4$ (offline) which can be noted as Case (i) or one firm sells $x_{1}=1 / 4$ (online) and $x_{0}=3 / 4$ (offline), the other firm sells $x_{2}=2 / 4$ (online) and $x_{3}=0$ (offline) which is noted as Case (ii). However, if we further consider the online and offline product assortment strategy as well, there is one more product placement strategy: one firm sells $x_{1}=1 / 4$ (online) and $x_{0}=0$ (offline), and the other firm sells $x_{2}=$ $3 / 4$ (online) and $x_{3}=2 / 4$ (offline) which is noted as Case (iii). Next, each firm sets its pricing strategy $p_{j}$ of the products it possesses in each case. We assume that the firm makes the product assortment strategy before the pricing strategy because it is generally supposed that the pricing strategy is more flexible and easier to change than the product assortment strategy. Therefore, the pricing strategy possesses a shorter time horizon than the product assortment strategy. In order to help readers get a better understanding of the two firms' placement strategies intuitively, we organize them around the Salop circle as Figure 2 from the Case (i) to Case (iii).
3.3. Demand Generation Process for Online Consumers. Each consumer makes his original purchase decision that maximizes his expected utility on account of his observation of the known part regarding preference parameter: $\theta_{i}$. We focus on studying the cases in which the two sellers are direct competitors in the market. Thus, all consumers make their original purchase decisions and can possess at most one good out of the four choices. This assumption will
hold naturally when the value of $v$ is high enough. However, a consumer will obtain zero utility gain from possessing any one of the products with probability $\alpha$. He can return this product with deceptive quality performance. On the other hand, consumers will obtain the utility gain of value $v$ with probability $1-\alpha$ from the quality dimension. They will choose to keep their original purchased product or exchange it for a more preferred one after the purchase has been made and they have observed the value of $\varepsilon_{i}$ [33, 34].

We first examine the demand and return behavior of online consumers when the sellers' pricing and product placement strategies are given. We assume consumers are forward-looking. They will take into consideration the chance of returning their original purchase or exchanging it for another at the beginning of their purchase decision. That is, consumers set their original purchase strategies on the strength of the expected utility by taking each probable post-purchase behavior into account. We can use the backward induction method to figure out which product will optimize each consumer's expected utility gain. After the consumer makes their original purchase decision, they obtain the product and has a try of it afterward. The consumer then makes the post-purchase return or exchange strategy based on the actual utility gain he obtains from consuming the product. In order to better understand the sequence of events, we consider a cell phone example with a trade-off between screen size and portability. A consumer may buy a phone with a large easy-toview screen and realize that the size makes it uncomfortable to carry around in a pocket. This experience would also allow the consumer to discern that a smaller phone (with a smaller screen) is better suited to match with his preferences. They can choose to exchange the initial purchase with the one smaller screen after the experience of the large screen.

We consider the demand generation process of the Case (i) as an example, and that of Case (ii) and Case (iii) can be derived by a similar method. In the scenario of Case (i), we assume seller one sells product 1 and product 0 , with product 1 through the online channel and product 0 through the offline channel. Meanwhile, seller two sells products 2 and product 3, with product 2 through the online channels and product 3 through the offline channels. The consumers' information searching behavior is demonstrated in Figure 3.

Following the demand generation process with the detailed analyses in Appendix A, we can obtain the online consumers' initial demand, return quantities, and exchange quantity for each product from the Case (i) to Case (iii). We only list those of Case (i) as follows, and other cases can be found in Appendix.

The sellers' product placement strategies in case (i) are as follows: one firm sells $x_{1}=1 / 4$ (online) and $x_{0}=0$ (offline) and the other firm sells $x_{2}=2 / 4$ (online) and $x_{3}=3 / 4$ (offline).

The initial demand of each product can be derived as below, with the subscript denoting the product number (see equation (1)):


Figure 2: Three placement strategies of two firms. (a)Case (i). (b) Case (ii). (c) Case (iii)


Figure 3: The sequence of events and payoffs for each party.
$D_{1}=\frac{2 p_{0}+(h-r)(-(h+r)(-1+\alpha)-4 t(1+\alpha) \delta)+2(h+r)(-1+\alpha)\left(-2 p_{1}+p_{2}-t x_{0}+t x_{2}\right) /(h+r)(-1+\alpha)}{4 t}$,
$D_{0}=\frac{-4 p_{0}+(h-r)((h+r)(-1+\alpha)+4 t(1+\alpha) \delta)+2(h+r)(-1+\alpha)\left(p_{1}+p_{3}+t x_{1}-t x_{3}\right) /(h+r)(-1+\alpha)}{4 t}$,
$D_{2}=\frac{2 p_{1}+(h-r)(-(h+r)(-1+\alpha)-4 t(1+\alpha) \delta)+2(h+r)(-1+\alpha)\left(-2 p_{2}+p_{3}-t x_{1}+t x_{3}\right) /(h+r)(-1+\alpha)}{4 t}$,
$D_{3}=\frac{2 p_{0}+(h-r)=((h+r)(-1+\alpha)+4 t(1+\alpha) \delta)+2(h+r)(-1+\alpha)\left(p_{2}-2 p_{3}+t x_{0}-t x_{2}\right) /(h+r)(-1+\alpha)}{4 t}$.

Equation (1) gives the initial sales quantities for each product $j$. Consumers make their initial purchase decisions based on expected utility. On purchasing, the consumer experiences the product and makes the post-purchase return decision based on the actual utility derived from owning the product. The exchange quantities of each product are derived as below, with the subscript denoting the exchange behavior happening between the two product numbers. To be more specific, the exchanges are from the left product number to the right one (see equation (2)). Besides, the return quantities are derived as follows (see equation (2)). Detailed analyses of consumers' post-purchase behaviors can be found in the appendix.

$$
\begin{align*}
& e_{01}=\frac{(h+r)(-1+\alpha) \delta}{2 t}-\frac{2(h-r)(1+\alpha) \delta^{2}}{h+r}, \\
& e_{10}=0 \\
& e_{32}=\frac{(h+r)(-1+\alpha) \delta}{2 t}-\frac{2(h-r)(1+\alpha) \delta^{2}}{h+r},  \tag{2}\\
& e_{23}=0 \\
& R_{j}=\alpha D_{j}
\end{align*}
$$

3.4. Demand Generation Process for Offline Consumers. We next examine the demand and return behavior of offline consumers when the sellers' pricing and product placement strategies are given. The consumers who take in-store inspection will have no uncertainty regarding the preference parameter $\vartheta_{i}$ as they can try the product prior to purchase, while all other behaviors are not affected compared to online consumers. We can also use the backward induction method to examine which product will optimize the consumers' expected utility gain. That is, consumers set their original purchase strategies on the strength of the expected utility by
taking each probable post-purchase behavior into account. After their original purchase decision, the consumers obtain the product and have a try it afterward. They then determine whether to return it on account of the actual utility they obtain from consuming the product. The derivation process of Case (i) can be seen as follows.

The consumers who take in-store inspections are uncertain only about $v_{i}$. The consumer will purchase the online product 1 if he can obtain his optimal utility from it under rational expectations: $(1-\alpha)\left(v-p_{1}-t\left|x_{1}-\vartheta_{i}\right|\right)-\alpha r$. It means the expected utility of purchasing product 1 should be greater than the expected utility of product 2 and product 0 simultaneously: both $(1-\alpha)\left(v-p_{1}-t\left|x_{1}-\vartheta_{i}\right|\right)-\alpha r>$ $(1-\alpha)\left(v-p_{2}-t\left|x_{2}-\vartheta_{i}\right|\right)-\alpha r$ and $(1-\alpha)\left(v-p_{1}-t \mid x_{1}-\right.$ $\left.\vartheta_{i} \mid\right)-\alpha r>(1-\alpha)\left(v-p_{0}-t\left|x_{0}-\vartheta_{i}\right|\right)-\alpha h$ should be satisfied. Therefore, the total demand of product 1 for offline consumers is $D_{1}^{\prime}=\left(-p_{1}+p_{2}+t\left(x_{1}+x_{2}\right) / 2 t\right)-((h-r) \alpha-$ $\left.(-1+\alpha) p_{0}+(-1+\alpha) p_{1}+t(-1+\alpha)\left(x_{0}+x_{1}\right) / 2 t(-1+\alpha)\right)$. Moreover, the corresponding returns of product 1 is $\alpha D_{1}^{\prime}$.

With the same method, we can derive the total demand of product 0 as $D_{0}^{\prime}=\left((h-r) \alpha-(-1+\alpha) p_{0}+(-1+\alpha) p_{1}+\right.$ $\left.t(-1+\alpha)\left(x_{0}+x_{1}\right) / 2 t(-1+\alpha)\right)-\left(-p_{3}+p_{0}+t\left(x_{0}+x_{3}\right) / 2 t\right)$. Moreover, the corresponding returns of product 0 is $\alpha D_{0}^{\prime}$. Moreover, note that the offline consumer has no uncertainty about his preference; thus, no exchange behavior will happen when he takes an in-store inspection before purchase.

The offline consumers' initial demand and return quantities for each product of case (i) can be obtained as below. The demand generation process of Case (ii) and Case (iii) can also be derived by a similar method, and their results of them can be found in Appendix A of this chapter.

The sellers' product placement strategies in case (i) are: one firm sells $x_{1}=1 / 4$ (online) and $x_{0}=0$ (offline), and the other firm sells $x_{2}=2 / 4$ (online) and $x_{3}=3 / 4$ (offline).

The demand and return quantity of each product can be derived as equation (3), with the subscript denoting the product number:

$$
\begin{align*}
& D_{1}^{\prime}=\frac{-h \alpha+r \alpha+(-1+\alpha) p_{0}-2(-1+\alpha) p_{1}-p_{2}+\alpha p_{2}+t x_{0}-t \alpha x_{0}+t(-1+\alpha) x_{2}}{2 t(-1+\alpha)}, \\
& D_{0}^{\prime}=-\frac{-h \alpha+r \alpha+2(-1+\alpha) p_{0}+p_{1}+p_{3}+t x_{1}-\alpha\left(p_{1}+p_{3}+t x_{1}\right)+t(-1+\alpha) x_{3}}{2 t(-1+\alpha)}, \\
& D_{2}^{\prime}=\frac{-h \alpha+r \alpha+(-1+\alpha) p_{1}-2(-1+\alpha) p_{2}-p_{3}+\alpha p_{3}+t x_{1}-t \alpha x_{1}+t(-1+\alpha) x_{3}}{2 t(-1+\alpha)},  \tag{3}\\
& D_{3}^{\prime}=\frac{h \alpha-r \alpha+(-1+\alpha) p_{0}+(-1+\alpha) p_{2}+2 p_{3}-2 \alpha p_{3}-t x_{0}+t \alpha x_{0}-t(-1+\alpha) x_{2}}{2 t(-1+\alpha)}, \\
& R_{j}^{\prime}=\alpha D_{j}^{\prime} .
\end{align*}
$$

After we have obtained the demand of both online consumers and offline consumers in all three cases of the product placement strategy, we then further analyze the
equilibrium results of each case. That is, we calculate the optimal pricing strategy on the condition that we are first given all possible placement strategies.

## 4. Model Analyses

We examine a market where there are two competing sellers, each selling two products that are horizontally differentiated from each other through either online or offline channels. The objective function of each seller is as shown below, where the product placement strategy has been divided into the three cases we have demonstrated. Meanwhile, we have listed the demand quantity, exchange quantity, and return
quantity of each case in the aforementioned section. Thus, we can derive the equilibrium results in each case from the following profit maximization problem, which is exactly the objective profit function of Case (i) (see equation (4)). The objective profit functions of Cases (ii) and (iii) are similar to Case (i) with only a change in the sources concerning product exchange quantities.

$$
\begin{align*}
& \frac{\max }{p_{1}, p_{0}, x_{1}, x_{0}} \omega\left[\left(p_{1}-c\right)\left(D_{1}+e_{01}+e_{21}\right)-(c-s)\left(e_{10}+e_{12}+R_{1}\right)+\left(p_{0}-c\right)\left(D_{0}+e_{10}+e_{30}\right)-(c-s)\left(e_{01}+e_{03}+R_{0}\right)\right] \\
& \quad+(1-\omega)\left[\left(p_{1}-c\right) D_{1}^{\prime}-(c-s) R_{1}^{\prime}+\left(p_{0}-c\right) D_{0}^{\prime}-(c-s) R_{0}^{\prime}\right]  \tag{4}\\
& \frac{\max }{p_{2}, p_{3}, x_{2}, x_{3}} \omega\left[\left(p_{2}-c\right)\left(D_{2}+e_{12}+e_{32}\right)-(c-s)\left(e_{21}+e_{23}+R_{2}\right)+\left(p_{3}-c\right)\left(D_{3}+e_{03}+e_{23}\right)\right. \\
& \left.-(c-s)\left(e_{30}+e_{32}+R_{3}\right)\right]+(1-\omega)\left[\left(p_{2}-c\right) D_{2}^{\prime}-(c-s) R_{2}^{\prime}+\left(p_{3}-c\right) D_{3}^{\prime}-(c-s) R_{3}^{\prime}\right] .
\end{align*}
$$

4.1. Equilibrium Pricing Strategy for Both Sellers. We obtain the analytical results for the profit maximization problem. The equilibrium results are tedious in their expressions; thus, we only put the results of Case (i) here, and others can be
found in Appendix B. The equilibrium results in Case (i) are listed in equation (5):

Firm 1: $x_{1}=1 / 4$ (online) and $x_{0}=0$ (offline)
Firm 2: $x_{2}=2 / 4$ (online) and $x_{3}=3 / 4$ (offline)

$$
\begin{align*}
p_{0}= & \frac{1}{10(h+r)(-1+\alpha)}\left((h+r)\left(t(-1+\alpha)+2 \alpha(-h+r+5 s-5 s \alpha)+10 c\left(-1+\alpha^{2}\right)\right)\right. \\
& +\left(h^{2}\left(1+\alpha+6(-1+\alpha)^{2} \delta\right)-4 h \delta\left(-3 r(-1+\alpha)^{2}+t\left(1+\alpha+6\left(-1+\alpha^{2}\right) \delta\right)\right)\right. \\
& \left.\left.+r\left(-r(1+\alpha)+6 r(-1+\alpha)^{2} \delta+4 t \delta\left(1+\alpha+6\left(-1+\alpha^{2}\right) \delta\right)\right)\right) \omega\right), \\
p_{1}= & \frac{1}{10(h+r)(-1+\alpha)}\left((h+r)\left(t-t \alpha+2 \alpha(h-r+5 s-5 s \alpha)+10 c\left(-1+\alpha^{2}\right)\right)\right. \\
& +\left(h^{2}\left(-1-\alpha+4(-1+\alpha)^{2} \delta\right)+r\left(4 t(1+\alpha) \delta(-1+4(-1+\alpha) \delta)+r\left(1+\alpha+4(-1+\alpha)^{2} \delta\right)\right)\right. \\
& \left.\left.+4 h \delta\left(2 r(-1+\alpha)^{2}+t\left(1+\alpha+4 \delta-4 \alpha^{2} \delta\right)\right)\right) \omega\right), \\
p_{2}= & \frac{1}{10(h+r)(-1+\alpha)}\left((h+r)\left(t(-1+\alpha)+2 \alpha(-h+r+5 s-5 s \alpha)+10 c\left(-1+\alpha^{2}\right)\right)\right.  \tag{5}\\
& +\left(h^{2}\left(1+\alpha+6(-1+\alpha)^{2} \delta\right)-4 h \delta\left(-3 r(-1+\alpha)^{2}+t\left(1+\alpha+6\left(-1+\alpha^{2}\right) \delta\right)\right)\right. \\
& \left.\left.+r\left(-r(1+\alpha)+6 r(-1+\alpha)^{2} \delta+4 t \delta\left(1+\alpha+6\left(-1+\alpha^{2}\right) \delta\right)\right)\right) \omega\right), \\
p_{3}= & \frac{1}{10(h+r)(-1+\alpha)}\left((h+r)\left(t(1-\alpha)+2 \alpha(h-r+5 s-5 s \alpha)+10 c\left(-1+\alpha^{2}\right)\right)\right. \\
& +\left(h^{2}\left(-1-\alpha+4(-1+\alpha)^{2} \delta\right)+4 h \delta\left(2 r(-1+\alpha)^{2}+t\left(1+\alpha+4\left(1-\alpha^{2}\right) \delta\right)\right)\right. \\
& \left.\left.+r\left(4 t(1+\alpha) \delta(-1+4(-1+\alpha) \delta)+r\left(1+\alpha+4(-1+\alpha)^{2} \delta\right)\right)\right) \omega\right) .
\end{align*}
$$

With the derivation of optimal pricing strategy in respect of both sellers, we can further do some sensitivity analyses regarding the relative parameters, such as the online product return cost. Conclusions are organized with the following several propositions.
4.2. Properties of the Equilibrium Price and Optimal Profits. With the results we have obtained from all three cases, we can further derive the following several propositions. Our conclusions are mainly focusing on the optimal pricing strategies and optimal profit values. Besides, we also take use of a twodimensional figure to illustrate the optimal product assortment strategy. We next clarify each result in detailed analyses.

Proposition 1. No matter what placement strategy sellers will choose, the prices of products that are sold through the online channel (Product 1 and Product 2) are first increasing in the return cost of the line product (i.e., $r$ ) and then decreasing in the return cost of the online product $r$; while the prices of products that sold via the offline channel (Product 0 and Product 3) are always increasing in the return cost of online product $r$. Figure 4 describes this trend, where the horizontal axis denotes the online product return cost and the vertical axis denotes the prices of both online and offline products.

This proposition demonstrates that in all three placement cases, the optimal pricing strategy of online products and that of offline products are changing in the same tendency with respect to the online product return cost, respectively. No matter which seller sells the specific product, as long as it is sold through a certain channel, the optimal price of this product is in the trajectory of change as depicted in the above figures. The authors can see clearly that the prices of online products are decreasing in the online product return cost, although there is a little interval where $r$ is low, the prices are increasing in it. However, it will not change the overall tendency of online products' prices being decreasing in $r$. On the contrary, the prices of offline products are increasing in the online product return cost no matter which seller sells them. This is quite intuitive since the online product return cost acts as a resistance for the consumers to make their purchase decisions of the online product when considering which product to buy. Thus, when the return cost $r$ is quite small, the seller can raise her online product price. Since the return cost indeed exists and cannot be avoided, it can be small enough for the consumers to ignore the disadvantages of purchasing online (i.e., uncertainty about the preference parameter, which will cause product exchange). However, when the online product return cost $r$ is relatively great, the consumers will be more prudent to realize their purchases via web stores. Thus, the seller should strive to cut down her online product price to appeal to consumers to accomplish their purchases via web stores. Otherwise, the operation cost of the online channel cannot be covered by its revenue when no consumers are purchasing online, and it will result in a waste of vacant channels.

Meanwhile, with the augmentation of online product return costs, the prices of their offline counterparts are increasing. The offline inspection helps consumers eliminate concerns about
product exchange due to the uncertainty of their preferences when making an online purchase. The seller that makes their products sold in the brick-and-mortar store will always have an incentive to raise their offline product prices, as consumers will accept the high price to avoid the possible exchange or return behavior that may happen through online purchases. We next analyze the optimal profits of each case.

Proposition 2. No matter what placement strategy the sellers will choose, the optimal profits of both sellers are first decreasing in the return cost of the online product (i.e., $r$ ) and then increasing in $r$. Figure 5 describes this trend, where the horizontal axis denotes the online product return cost and the vertical axis denotes the profits of both sellers.

This proposition shows us the property of the optimal profits for both sellers in all three placement strategies. They are first decreasing the online product return cost $r$ over a small interval and then increasing afterward. As Figure 5 depicts, when the online product return cost is quite small, the profits of both sellers decrease rapidly in value as this return cost cannot hinder the consumers' intention of returning products. It will impair the seller's profit as the possible negative effects brought about by the occurrence of exchanging and returning products. Thus, the return cost should not be too low for the seller to choose the online product selling strategy as long as the online return cost is not approaching zero. What is also intriguing is that when the return cost is approaching zero, which means the return cost is almost nonexistent, the profits of both sellers are approaching positive infinity. As under this circumstance, online selling goes smoothly like offline selling without any cost of product exchange; thus, the market degenerates into a transparent market with seamless product transactions. That is to say, any product without maximized fitness or good quality will be eliminated in the market, which will result in a market with no deceptive products. However, this is not true in practice. The authors focus our attention on reality by considering that the optimal profits increase in the online product return cost after the rapid decrease when $r$ is low. This is intuitive since the increase in return cost $r$ will guarantee online product sales by avoiding consumers' arbitrary returning or exchanging behavior. Namely, consumers that make an online purchase will have to balance their expected utility from exchanging a misfit product or returning a deceptive product with the utility of keeping the original product, which probably does not match well with their preference or even is a product with poor quality. This gives us the reason in practice, the online product's return or exchange should satisfy several conditions. These conditions will be demonstrated by sellers before consumers an online purchase. The restrictions of return or exchange make consumers consider their purchase more seriously, which avoids vicious or intentional online product return or exchange behavior. Meanwhile, they guarantee the sellers' profit to some extent.
4.3. Optimal Product Placement Strategy. We then consider the product placement strategies by further analyzing the sellers' optimal profit in each case (Case (i) to Case (iii)), and the optimal placement strategy can be derived in the following proposition.


Figure 4: Equilibrium prices from the Case (i) to Case (iii).(a) Optimal pricing strategy of Case (i). (b) Optimal pricing strategy of Case (ii). (c) Optimal pricing strategy of Case (iii).


Figure 5: Optimal profits from the Case (i) to Case (iii). Optimal profit value of each case (Profit 1-2 corresponds to Case (i); Profit 34 corresponds to Case (ii); Profit 5-6 corresponds to Case (iii)).

Proposition 3. If we consider the three placement strategies, given the optimal equilibrium results, the sellers choose three cases by considering the unit misfit cost of products, horizontal feature (i.e., $t$ ), and the return cost of the online product (i.e., $r$ ) simultaneously. Figure 6 describes the influence of these two elements on the optimal placement strategies, where the horizontal axis denotes the online product return cost and the vertical axis denotes the unit misfit cost of each product.

This proposition illustrates the relationship between the product placement strategy with the two costs contained in our model, i.e., the unit product misfit cost and the online product return cost. To be more specific, Case (i) depicts the


Figure 6: Optimal product placement strategies in the $r-t$ plane.
scenario when both online products (equivalent to both offline products) are adjacent in their horizontal locations and meanwhile as for a certain seller, the online product and the offline product she sells are also adjacent in their horizontal feature. With respect to Case (ii), it describes a scenario when both online products (equivalent to both offline products) are adjacent in their horizontal locations while a seller sells the online product and an offline product that is differentiated to the maximum extent, i.e., the two
goods are placed on the opposite locations along the unit circle. As for Case (iii), both online products and offline products are differentiated to the maximum extent, while for a certain seller, the online product and the offline product she sells are adjacent in their horizontal locations. We first explain Case (iii), as in Figure 6, which shows that when the return cost of an online product is quite low or quite high, no matter what the value of unit product misfit cost is, this product placement strategy dominates the other two. While when $r$ is in an intermediate range, the seller will choose this product placement strategy only if $t$ is quite low. The reason behind this phenomenon is that when $r$ is high or low, what we have derived from Proposition 2 has shown that the optimal profits for both sellers are higher than that when $r$ is in an intermediate range. Meanwhile, the optimal profit of both sellers in Case (iii) dominates that in Case (i) and Case (ii). As in this case, the seller sells similar products in their horizontal feature, while the online products of both sellers (equivalently the offline products of both sellers) are differentiated in their horizontal feature. This will make each seller focus on selling the products with a similar feature, thus resulting in more exchanges between the seller's products through the online and offline channels. However, the exchanges between different sellers' products placed online or offline will not happen then. Thus, the promotion effect of a certain seller's optimal profit brought about by the change in online product return cost is amplified in this scenario. Since the return cost highly affects the consumers' product exchange behavior within a certain seller rather than between the two sellers' products.

As for the other cases (Case (i) and Case (ii)), the main difference between them is that the two products sold by one seller are similar in Case (i), while they are differentiated a lot in Case (ii). Note that in both cases, the online products or offline products sold by both sellers are similar in their horizontal features. Thus, the change in return cost will affect the exchange quantities between both online products and offline products. It also affects the consumer's exchange behavior within a certain seller in Case (i). Nevertheless, its influence on the exchanges within a specific seller in Case (ii) is tiny as both sellers sell differentiated products in Case (ii) and exchange will not happen between two differentiated products in our setting. Thus, when the unit misfit cost $t$ is not too small, which means the mismatch between the product horizontal feature and the consumers' preference is
influential, Case (i) dominates the others when the return cost of the online product $r$ is exerting a positive effect on seller's optimal profit (i.e., $r$ is greater than the threshold when the optimal profit is lowest in the change of $r$ ). Otherwise, Case (ii) dominates the other two cases as $r$ has little influence on the optimal profits of both sellers in this range.
4.4. The Effect of Competing. In this section, we mainly focus on the influence of competition in our main model. As we have modeled an oligopolistic setting with two sellers, we then take the benchmark setting of one seller into consideration. All assumptions remain the same as the main model. However, we analyze a monopolistic scenario with one seller managing two products, one of them via an online store and the other via brick-and-mortar stores. Both products are evenly spaced out along a unit circle. Therefore, we can follow the same demand derivation process as our main model.

The equilibrium results of the monopolist's profit maximization problem should satisfy the following equation (6):

$$
\begin{aligned}
p_{1}^{*}= & \frac{C-D}{4(1-\alpha) \delta\left((h+r)^{2}(-1+\alpha)-4(h-r) t(1+\alpha) \delta\right)} \\
& +p_{0}^{*}
\end{aligned}
$$

Where,

$$
\begin{align*}
C= & (h-r)(h+r) t(1+\alpha) \\
& +2\left(t\left((h+r)^{2}+2(-h+r) t\right)\right) \\
D= & \left(\left((h-r)(h+r)^{2}+2(h+r)^{2} t+2(h-r) t^{2}\right) \alpha\right. \\
& \left.+(h+r)^{2}(h-r+t) \alpha^{2}\right) \delta \\
& +8(h-r) t(1+\alpha)(t-(h-r+t) \alpha) \delta^{2} . \tag{6}
\end{align*}
$$

With a similar method taken by our competitive main model structure, the existence and rationality of the solutions can be derived with the optimization of the monopolist's profit function. FOCs and SOCs all satisfy the global optimal conditions.

Meanwhile, the optimal profit should be obtained under the optimal conditions (see equation (7)):

$$
\pi_{m}^{*}=\frac{E+F}{\left.16 t(1-\alpha) \delta^{2}\left((h+r)^{2}(-1+\alpha)-4(h-r) t(1+\alpha) \delta\right)\right)\left(-(h+r)^{2}(-1+\alpha)^{2}+4(h-r) t\left(-1+\alpha^{2}\right) \delta\right)},
$$

Where, $E=16(c-s) t(1-\alpha)^{3} \delta^{2}\left((h+r)^{2}(-1+\alpha)-4(h-r) t(1+\alpha) \delta\right)^{2}$ and,

$$
\begin{align*}
F= & (h-r)(h+r) t(1+\alpha)+2 t\left((h+r)^{2}+2(-h+r) t\right) \\
& -\left((h-r)(h+r)^{2}+2(h+r)^{2} t+2(h-r) t^{2}\right) \alpha+(h+r)^{2}(h-r+t) \alpha^{2} \delta+8(h-r) t(1+\alpha)\left(t-((h-r+t) \alpha) \delta^{2}\right)^{2} \tag{7}
\end{align*}
$$

The monopolist's product placement strategy is as follows: $x_{0}=0$ (offline) and $x_{1}=1 / 2$ (online).

After we compare the equilibrium results of the two sellers structure and the one-seller structure, we can obtain several conclusions with further analyses of these equations. These differences and similarities reflect the influence of competition.

Proposition 4. In the market without competition, the optimal pricing strategies of online products and offline products are different with a constant and change in the same direction.

That is to say, although the two products placed by the seller via both the online channel and offline channels are competitive in their market share, we allow the exchange and return behavior after online purchase or in-store inspection. Thus, both products' characteristics are more transparent in the omnichannel selling market. This finally results in the same changing direction between $p_{1}^{*}$ and $p_{0}^{*}$. Namely, they are more likely to be complement goods rather than substitutes. This is intuitive since the seller with dual channels should guarantee her products in each channel to remain consistent, to avoid internal competition, which will not benefit the seller from expanding her market share.

While, in the competitive setting, the two online products and two offline products are changing in opposite directions in respect of return costs. The reason behind this phenomenon is that we allow exchange behavior in our oligopolistic setting, thus, the sellers will balance the demands in each channel in case of the existence of vacancy channels. The influence of return cost on online and offline products' prices reflects the sellers' objective to attract consumers' demand in each channel. Otherwise, the operation cost of online stores or physical stores cannot be covered when there is no consumer purchase in that channel. However, the two sellers' products in the same channel are changing in the same direction in respect of return costs. This reflects the products sold through a certain channel have a synergistic effect. They are not in malicious differentiated price competition, which will not benefit both sellers in the long run.

Although the pricing strategy in the market without competition is different from that with competition, the optimal profits are not affected by the inducement of competition.

Proposition 5. In a market without competition, the optimal profit of the seller is increasing in $r$ when $r$ is higher than a certain threshold (i.e., $r>\bar{r}$ ).

This result in respect of the optimal profit is the same as that in competition. That is to say, the optimal profits are always increasing in the online product return cost, as long as the return cost is higher than the threshold.

This is intuitive since the increase in return cost $r$ will guarantee online product sales by avoiding consumers' arbitrary returning or exchanging behavior. Namely, consumers that make an online purchase will have to balance their expected utility from exchanging a misfit product or returning a deceptive product with the utility of keeping the original product. This gives us the reason in practice, the online product's return or exchange should satisfy several conditions. These conditions will be demonstrated by sellers before consumers an online purchase. The restrictions of return or exchange behavior make consumers consider their purchase more seriously, which avoids vicious or intentional online product return or exchange behavior. Meanwhile, they guarantee the sellers' profit to some extent.
4.5. Single Channel vs Dual Channel Selling Strategy. In this section, we analyze when the single-channel retailer should stick to their original selling strategy, and when they should consider the dual-channel selling strategy as our main model depicts. We separate the single channel retailer from the web-only retailer and the store-only retailer. Nevertheless, we still take into account the competitive market structure with four goods sold by two firms, respectively. Therefore, the product placement strategy for the single-channel retailer can be divided into two cases. In case (1), the two products sold by one retailer are adjacent in their horizontal locations; in case (2), the two products sold by one retailer are differentiated in their horizontal locations.

We first take into account the web-only sellers with all their products sold via online shops, and the objective functions of each web-only seller can be obtained as equation (8):

$$
\begin{align*}
& \frac{\max }{p_{1}, p_{0}, x_{1}, x_{0}}\left(p_{1}-c\right)\left(D_{1}+e_{01}+e_{21}\right)-(c-s)\left(e_{10}+e_{12}+\alpha D_{1}\right)+\left(p_{0}-c\right)\left(D_{0}+e_{10}+e_{30}\right)-(c-s)\left(e_{01}+e_{03}+\alpha D_{0}\right)  \tag{8}\\
& \frac{\max }{p_{2}, p_{3}, x_{2}, x_{3}}\left(p_{2}-c\right)\left(D_{2}+e_{12}+e_{32}\right)-(c-s)\left(e_{21}+e_{23}+\alpha D_{2}\right)+\left(p_{3}-c\right)\left(D_{3}+e_{03}+e_{23}\right)-(c-s)\left(e_{30}+e_{32}+\alpha D_{3}\right)
\end{align*}
$$

We next analyze the store-only sellers' optimal selling strategy with all their products selling via brick-and-mortar
shops, and the objective functions of each store-only seller are as equation (9):

$$
\begin{align*}
& \frac{\max }{p_{1}, p_{0}, x_{1}, x_{0}}\left(p_{1}-c\right) D_{1}^{\prime}-(c-s) \alpha D_{1}^{\prime} \\
& \quad+\left(p_{0}-c\right) D_{0}^{\prime}-(c-s) \alpha D_{0}^{\prime} \\
& \frac{\max }{p_{2}, p_{3}, x_{2}, x_{3}}\left(p_{2}-c\right) D_{2}^{\prime}-(c-s) \alpha D_{2}^{\prime}  \tag{9}\\
& \quad+\left(p_{3}-c\right) D_{3}^{\prime}-(c-s) \alpha D_{3}^{\prime}
\end{align*}
$$

After the demand generation process for both types of retailers, we can derive the equilibrium results for web-only retailers in case (1) as equation (10):

Firm $\cdot 1: x_{1}=\frac{1}{4}($ online $) \cdot$ and $\cdot x_{0}=0($ online $) ;$

$$
\begin{align*}
p_{1}^{*} & =c+\frac{t}{10}+c \alpha-s \alpha \cdot \text { and } \cdot p_{0}^{*} \\
& =c+\frac{t}{10}+c \alpha-s \alpha ; \pi^{*}=\frac{3 t}{100}, \tag{10}
\end{align*}
$$

Firm $\cdot 2: x_{2}=\frac{2}{4}($ online $) \cdot$ and $\cdot x_{3}=\frac{3}{4} ;$

$$
\begin{aligned}
p_{2}^{*} & =c+\frac{t}{10}+c \alpha-s \alpha \cdot \text { and } \cdot p_{3}^{*} \\
& =c+\frac{t}{10}+c \alpha-s \alpha ; \pi^{*}=\frac{3 t}{100}
\end{aligned}
$$

The equilibrium results for web-only retailers in case (2) are as equation (11):

$$
\text { Firm } \begin{align*}
1: x_{1} & =\frac{1}{4}(\text { online }) \cdot \text { and } \cdot x_{0}=\frac{3}{4} \\
p_{1}^{*} & =c+\frac{t}{12}+c \alpha-s \alpha \cdot \text { and } \cdot p_{0}^{*} \\
& =c+\frac{t}{12}+c \alpha-s \alpha ; \pi^{*}=\frac{t}{12} \tag{11}
\end{align*}
$$

Firm $\cdot 2: x_{2}=\frac{2}{4}($ online $) \cdot$ and $\cdot x_{3}=0 ;$

$$
\begin{aligned}
p_{2}^{*} & =c+\frac{t}{6}+c \alpha-s \alpha \cdot \text { and } \cdot p_{3}^{*} \\
& =c+\frac{t}{3}+c \alpha-s \alpha ; \pi^{*}=\frac{5 t}{36}
\end{aligned}
$$

The equilibrium results for store-only retailers are similar to those of the web-only retailers; thus, we omit them here and only put them in Appendix B of this chapter.

We next make comparisons between the optimal equilibrium results when the retailers choose a single channel and dual channel selling strategy. We can obtain the conclusion by clarifying the condition when the retailers prefer clicks and mortar and when they stick to the single channel selling strategy.


Figure 7: Optimal selling channel strategy in $r-t$ plane.

Proposition 6. The sellers stick to the single channel selling strategy if and only if $\underline{\underline{x}} \leq r \leq \bar{r}$ and $t \geq \bar{t}$; otherwise, they prefer the dual channel selling strategy. Figure 7 illustrates the optimal selling channel strategy, where the horizontal axis denotes the online product return cost, and the vertical axis denotes the unit misfit cost of each product.

The web-only or store-only retailer will still choose their original single channel selling strategy only if the unit misfit cost of the horizontal feature is high, and the return cost of the online product is in an intermediate range. Otherwise, the click-and-mortar selling strategy is more attractive to the retailer. The reason is that when the misfit cost of product fitness is high, the relative disutility of the misfit in the horizontal feature is large. The consumers' prior purchasing utility has been greatly cut down in respect of the horizontal dimension. As consumers' online purchase is also faced with uncertainty regarding fitness prior to purchase, which will result in exchanges between offline products and online products, the single-channel retailer is difficult to benefit from the click-and-mortar selling strategy especially when the online product return cost is not too high. The online product return cost is relatively low compared with the horizontal misfit cost; thus, the returns or exchange of online products cannot be avoided with the limited return cost restriction. When both conditions are satisfied, namely, the dual channel selling strategy is not beneficial for the retailer to expand her market share (i.e., the return cost is not too high) and consumers experience a large disutility due to misfit (i.e., the unit misfit cost is high), the single-channel retailer still stick to their original selling strategy without taking use of clicks and mortar strategy.

However, in all other cases, the click-and-mortar selling strategy dominates the single-channel selling strategy. The disutility due to misfit is low in other parameter regions; thus, consumers can undertake the cost of uncertainty regarding the exchanges that may occur between online purchases and offline purchases. Besides, the online product
return cost is relatively high compared with the horizontal misfit cost, which helps the retailer expand her market share by avoiding arbitrary exchanges between offline and online purchases occurring in the click-and-mortar selling strategy.

## 5. Managerial Insights

In this oligopolistic setting, the two online products and two offline products are changing in opposite directions in respect of return costs. The reason behind this phenomenon is that we allow exchange behavior in our oligopolistic setting; thus, the sellers will balance the demands in each channel in case of the existence of vacancy channels. The influence of return cost on online and offline products' prices reflects the sellers' objective to attract consumers' demand in each channel. Otherwise, the operation cost of the online store or physical store cannot be covered when there is no consumer purchase in that channel. However, the two sellers' products in the same channel are changing in the same direction in respect of return costs. This reflects that the products sold through a certain channel have a synergistic effect on the seller's selling performance. They are not in malicious differentiated price competition, which will not benefit both sellers in long run. In practice, online product selling is always focused on the pursuit of a high-performance-cost ratio. With the augmentation of online product return costs, online product prices are decreasing significantly. Online selling should depend on providing products with good performance of low return risk; meanwhile, the reduced prices compared with offline products also attract consumers to purchase through online stores. On the contrary, offline selling should focus on improving the products' tastes and performance rather than competing with online products' prices. Many consumers stick to purchasing through the physical store by taking advantage of offline inspection and fitting experience with high-end products such as clothes. What they mainly focus on is whether the clothes fit them or not, while the price is put the second place. Thus, these kinds of experience products with high value are more suitable for consumers to purchase through a physical store, and the products' prices can be set to increase with the return difficulty of their online counterparts. Besides the high-end clothes market, many luxuries such as LV/Channel also stick to their offline selling by raising the prices of their bags or watches, while they still have great demands in the physical store. That is to say, offline selling should not compete with online selling in the pricing strategy, but it should consider providing consumers with a better purchasing experience and introducing high-end products with high prices to attract more consumers especially when the products are hard to return via online channels.

As for the optimal profits in our setting, the overall changing tendency of the sellers' optimal profits is increasing in online product return costs. However, in the oligopolistic setting, the optimal profits of both sellers first decrease in online product return cost over small intervals and then increase in it. As in the oligopolistic setting, we allow for the exchange behavior. When the return cost is approach zero, the profits of both sellers are approaching positive infinity,
which is not true in practice. In practice, we try to understand the variation of sellers' optimal profits in the online product return cost with some actions taken by sellers. By the year 2022, sales via online e-commerce are expected to account for nearly $35 \%$ of fashion retail (Forrester, 2021), and clothing is the most popular type among them. As for the sellers in this industry, it is significant for them to seek an efficient return process and avoid the resource consumption brought about by product returns. In the past five years, the return rate in e-commerce is increasing rapidly by $95 \%$ (Payments journal), and many retailers are reexamining their strategies to reduce return quantities. The key point for retailers to reduce returns is to learn about the reasons for return behaviors and to satisfy clients' real demands. According to a survey made by WBE, $59 \%$ of consumers return products for damaged goods, while $42 \%$ of them return for the reason of regret and $29 \%$ of them for the reason of information misleading. Some retailers consider increasing the barrier in the return process, such as charging for the returns or shortening the return periods. These strategies will reduce the arbitrary returns of many illogical consumer behaviors. Other retailers such as Zara, H\&M, and Aday resort provide consumers with better service and detailed descriptions of their products in order to help consumers make sensible purchasing decisions (Vogue Business). All these actions can reduce arbitrary product return behaviors and improve retailers' revenue performance in the long run.

## 6. Concluding Remarks and Discussion

From our analyses of the model depicting a competing market with two sellers selling products through both online and offline channels, we can derive the following three main conclusions as shown in our model analyses.

First, no matter what the placement strategy both sellers choose, the optimal prices of products that are sold through the online channel are increasing in the return cost of the online product and then decreasing in the online product return cost; while the optimal prices of products sold via offline are always increasing in the online product return cost. The online product return cost acts as a resistance for the consumers to make their final purchase decisions between the four products via both channels. When the return cost is quite small, the seller can raise their online product price. Since the return cost indeed exists and cannot be avoided, it can be small enough for the consumers to ignore the disadvantages of purchasing online (i.e., uncertainty about the preference parameter which will cause product exchange). However, when the online product return cost is at a great level, the consumers will be more prudent to realize their purchases via web stores. Thus, the seller should take into account the method of cutting down her online product prices to retain consumers via web stores.

Second, no matter what placement strategy sellers will choose, the optimal profit of both sellers is first decreasing the return cost of the online product and then increasing it.

When the online product return cost is quite small, the profits of both sellers decrease rapidly in value as this return cost cannot hinder the consumers' intention of returning products. With the increase in return cost, online product sales are guaranteed by avoiding consumers' arbitrary returning or exchanging behavior. Thus, the profits are increasing in the return cost.

Third, if we consider the three placement strategies, given the optimal equilibrium results, the sellers choose three cases by considering the unit misfit cost of products, horizontal feature, and the return cost of online products simultaneously. The placement strategy is that both online products or both offline products are differentiated to the maximum extent, while for a certain seller, the online product and the offline product she sells are adjacent in their horizontal locations and will dominate others when the return cost of the online product is quite low or quite high. Then, no matter what the value of unit product misfit cost is, this product placement strategy dominates the other two. While when the return cost is in an intermediate range, the seller will choose this product placement strategy only if the misfit cost is quite low. The performances of the other two placement strategies are also related to both unit misfit cost and the return cost of the online products.

Moreover, we would like to make some discussions on our model setting and conclusions with other studies in this section. According to recent research on sellers' omnichannel selling strategies ([21], they mainly focus on the optimal product assortment strategy by multichannel consideration. In their setting, the probability of product fitness is exogenously given. This induces the resulting conclusion that the optimal pricing strategy and product assortment strategy depend on two elements: the online product return cost (which is the same as our conclusion) and the fitness probability (which is distinct from ours). However, their assumptions are quite restricted as they failed to demonstrate the product attributes in a two-dimensional market structure, and the information revelation behaviors of consumers are over-simplified in their model setting. We thus generalize a more abundant model structure by depicting the consumer market profile with both vertical and horizontal feature locations. Our conclusions in the duopolistic model settings are intriguing and significant in respect of the optimal product assortment strategies with the consideration of both the unit misfit cost of products' horizontal features and the return cost of online products. Besides, according to the research on how competitive sellers should manage consumer returns [35], their research emphasis is on the optimal pricing and restocking fee strategies of competitive sellers. They mainly consider the horizontally differentiated goods with the exogenously given probability of product return, which is similar to our duopolistic setting. What we mainly care about is the product assortment strategy, while what they focus on is the equilibrium product prices and restocking fees, among which the latter one we do not take into consideration in our model structure. We plan to include the restocking fee strategies with the full image of the return policy in our future study.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities under grant no. 310422121.

## Supplementary Materials

In Appendix A, we describe the demand derivation process of both online and offline consumers separately. Then, in detail, we derive the online consumers' initial demands, return quantities and exchange quantities for each product in each case. In Appendix B, we just follow the results we have obtained in Appendix $A$ to derive the equilibrium results, i.e., pricing decision in each case. Finally, we consider the single channel selling strategy and the corresponding optimal pricing strategy, including both web-only and store-only seller. After which we make comparation between dual channel and single channel selling strategy to obtain the optimal channel decision for sellers. (Supplementary Materials)

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