A Nonlinear Approach to Quantifying Investor Fear in Stock Markets of BRIC


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Abstract

We provide a rigorous investigation of information flow among stock markets of BRIC and the US VIX in a frequency-domain paradigm. Henceforward, the variation mode decomposition-based entropy approach is employed for the examination of diverse investment horizons and market conditions. First, we find that under stressed market conditions (lower quantiles), significant negative information flow exists between the BRIC constituents and the BRIC composite index. Also, under benign market conditions, we reveal similar dynamics as found at the lower quantiles, which enhances diversification. However, during market booms, we document more positive information flow between the assets and relevant to the redeployment of portfolios. Second, at low probability events representing market stress, we document potential negative information flow amid the stock markets and the US VIX for most investment horizons. Notwithstanding, the US VIX has the potential of transmitting positive information to the stock markets. However, at high market performance, we find more positive information flow amid the BRIC markets and VIX, generally implying long-term efficiency. Investors, portfolio managers, risk managers, and policy-makers should be wary of the heterogeneous and adaptive behaviour of BRIC stock markets with the VIX.

1. Introduction

The potency of stock markets is critical for boosting economic activity and driving growth and development among economies ([1–3], the extant literature emphasises the contribution of equities markets in enhancing the growth of economies, demonstrating that exuberant equities markets could promote liquidity in markets, lower the cost of sourcing capital, fortify governance mechanisms of corporations, and arouse cross-border risk-sharing, all of which help to support economic growth [4, 5]. Stock market capitalisation, according to Khan et al. [6]; and Tsaurai [7], characterises stock markets’ development.

In terms of commerce and investment, the BRIC nations, which constitute Brazil, Russia, India, and China, have risen significantly and have become increasingly intertwined with the industrialised world [8]. BRIC equities (stock) markets have grown in terms of magnitude and capacity of investments and have attracted a lot of attention from local and foreign investors [9–11]. BRIC stock markets have consistently produced strong average returns, attracting the interest of investors looking to build globally diversified portfolios. Thus, with intuitions from the financial market integration theory [12–14], & Darkwa, 2021), the potential of BRIC markets to be more integrated cannot be shelved.

As the uncertainties surrounding the growth of emerging financial markets such as the BRIC markets cannot be downplayed [11, 15], several significant question themes are raised concerning the growth prospects and attractiveness of BRIC markets to international portfolios. How do BRIC markets observe each other through mutual information...
flow? Do the fundamental market dynamics apply to the BRIC markets across diverse time scales? How does the aggregate BRIC market respond to market shocks from one of its constituents amid market uncertainties? Are there significant prospects for portfolio diversification across different investment horizons? These questions must be empirically addressed using the appropriate technique(s) (Appropriate techniques cover empirical methods which are data-driven and could produce robust results for different investment time scales). The impetus of this study is hinged on addressing these essential questions using the variational mode decomposition (VMD)-based (VMD means Variational Mode Decomposition. This is subsequently discussed in this paper) entropy analysis.

Generally, the materialisation of economic projections is hampered by tumult trading environments. The world has witnessed several financial market downturns over the last few decades, and these were mostly occasioned by pandemics (For extended highlights on the history of pandemics, visit https://www.visualcapitalist.com/history-of-pandemics-deadliest/) or the alike [15–18], which could introduce significant changes in traditional market dynamics between and within market blocs. The effect of such significant changes may be attributable to the severity of the pandemic and the deaths associated with these pandemics (see Figure 1), causing a reduction in the labour force and productivity levels across the global economy. The fore-running argument rekindles Bouri et al.’s [10, 19] conclusion that external variables significantly predict the performance of several economies, and the BRIC countries are no exception. BRIC markets are progressively gaining connections with industrialised markets, with indications of considerable financial flows from the advanced markets [10]. It stands to reason, therefore, that deteriorating macro-economic conditions impact the performance of BRIC stock markets by lowering the volume of exports and capital accumulation [20].

In reposeful and tumultuous epochs, the influence of the economic circumstances of the US, particularly volatilities in the US equities indices, on the equities markets of significant frontier economies, like BRIC, has been widely documented (see [21–23]; and the references therein). Consequently, international shocks, especially those emanating from the US equities market, could be transmitted to the BRIC equities markets but in diverse forms based on market conditions. The underlying discourse sheds light on the concerns about asymmetric volatility transmission vis-à-vis the US volatility index (VIX), which should not be overlooked in the context of stock markets as volatilities may erode investor confidence [24].

Rapach et al. [25] argue that stock market returns in the US significantly predict those of both advanced and emerging nations. Furthermore, Sarwar and Khan [26] concluded that increases in uncertainties in the US market reduce (increase) the stock market returns (return variance) in frontier markets which is suggestive that the stock markets of the BRIC economies could be affected by the uncertainty in the US. From the existing literature, evidence of how this may prevail across distinct scales and market conditions is almost extinct, particularly in periods extending into the COVID-19 health crisis. Junior et al.’s [9] study provides evidence of the comovement dynamics between VIX and BRIC markets in a time-frequency paradigm. Heliodoro et al. [27] investigated the connectedness between BRIC equities whereas Asafo-Adjei et al. [28] analysed the dynamics of spillovers between BRIC and the US. We extend this strand of literature by examining information flow between VIX and BRIC equities in an asymmetric and multi-scale effective transfer entropy (ETE) approach.

ETEs result from the related information shared between markets [29]. Quantification of the mutual information between the US VIX and BRIC equities is made possible using ETEs [15, 30–33]. That is, ETEs measure the amount of information that travels across markets. To demarcate the data series into their multi-scale components, we employ the VMD, which is a completely inherent, adaptive, and quasi-orthogonal decomposition approach [34]. After precisely decomposing the time series data into their variational mode functions (VMFs), the VMD-based entropy would give a robust technique to evaluate information flow across markets across different scales and market conditions. Previous studies on VIX and equities (see e.g., [9, 35–37]) have not employed this method.

We contribute to the body of knowledge in four major ways. First, using the VMD-based transfer entropy approach, we analyse the flow of information between the US VIX and the BRIC constituents’ stock markets at multi-scales and across different market conditions. Uniquely, this enables us to unveil asymmetries in the effect of the US VIX on developing markets’ equities. Note that we estimate ETEs for both the aggregate BRIC Index and its constituents. Hence, based on the market condition and investment scale, our findings should influence investors in choosing between investing in the overall BRIC index or specific constituents. Second, novel to the existing works, we quantify the flow of information between the BRIC constituents’ stock markets and the aggregate BRIC index at multi-scales. This would disclose the level of heterogeneity among BRICs by determining the quota of each BRIC constituent to the growth of the BRIC Composite Index. The growth of a market index is commensurate with a considerable information flow to its constituents [9, 38]. Hence, the reverse causation from the BRIC Composite Index to its constituents is analysed.

Third, we cover the COVID-19 pandemic era, which is noted to have caused significant changes in fundamental market dynamics [14, 20, 31, 39–42]. As the pandemic persists, covering this period is essential to produce findings that would influence effective hedging strategies. Fourth, by way of the econometric approach, because fat tails and information spillovers are particularly predominant in crisis periods, our VMD helps to curtail untrue signals by delineating the original series into its multi-scale parts. Similarly, specifying different weighting factors, to overcome tailed distributions and unveil asymmetries, is achieved by the transfer entropy paradigm.

At stressed market conditions, our results indicate significant negative information flow between the BRIC constituents and the BRIC Composite Index. Also, under benign
market conditions, we find similar dynamics as found at the lower quantiles which enhances diversification. However, specifically, at quantiles 0.9 and 1.0, representing market boom, we document more positive information flow between the variables, and relevant in the redeployment of portfolios. On the other hand, at low probability events representing market stress, we document potential negative information flow between the stock markets of BRIC and the US VIX at most investment horizons. However, at quantile 1.0, representing market boom, we found more positive information flow between the BRIC markets and the US VIX.

In addition to Section 1 are Sections 2–6. The literature review in Section 2, methodology in Section 3; empirical results and discussion in Section 4, theoretical and practical foundations in Section 5, and Section 6 concludes the study and guides for further research are provided.

2. Literature Review

Theoretically, the fast advancement of BRIC economies and their connectedness with developed markets is likely to result in a high extent of financial market integration between and among the BRIC markets and their counterpart developed markets [3, 16, 31, 43]. Inordinate integration of financial markets has implications for portfolio construction and thus, warrants that the diversification, safe haven, and hedging prospects are examined [31, 39], and [14, 44]. While BRIC markets have shown positive long-term prospects in recent decades, they have also been impacted by the global financial crisis, casting doubts on these expectations [10, 19, 45]. The turbulent market circumstances conditioned by the recent COVID-19 pandemic further cause doubts about these sweetening long-term prospects for the BRIC markets.

Corollary to the doubts, introduced by market stress, on these forecasts vis-à-vis the BRIC markets, empirical attention has been paid to the BRIC markets in recent period (see e.g., [3, 9–11, 23, 27, 31, 32, 45–49]) to examine plausible changes in fundamental dynamics that apply to the BRIC economies and their counterpart market blocs.

The extant literature is populated by studies that principally focus on the interconnection between the BRICs equities [50–52], the returns and spillovers of the US and BRIC equities markets [15], Owusu Junior, [53, 54], Rapach et al. [25, 26], or the lead-lag dynamics between the BRIC markets and the VIX [9]. Yet, a study that examines information flow between the fragments of BRIC and the aggregated BRIC Composite Index while evaluating the prospects and agitation of investors has gained a dearth of attention. The extant literature holds no record for a study that examines the BRIC Composite Index’s integration with its fragments while evaluating the flow of shocks between the markets as well as how the constituents respond to shocks from either of the markets.

The prior works of Kwon and Yang [55] and Osei and Adam [38] on the flow of information between aggregate...
stock market indices and their fragments, however, have underlined the relevance of settling this issue in various equities markets and economic blocs; yet, none of the existing studies has addressed information flow between the BRIC Composite Index and its constituents, and the US VIX, which quantifies investor sentiment and fear. This is significant in this circumstance because, in addition to the past financial crises, the recent financial turmoil occasioned by the COVID-19 pandemic has affected many financial and economic operations [39, 56–58], and therefore, a study that indicates a multi-scale discussion on stock markets cannot be unnoticed [59–61], etc. Additionally, empirical investigations of stock market volatilities generate reliable evidence that allows investors to modify their risk appetite, as advanced by Prasad, Bakry, and Varua [62]. Notwithstanding, market volatilities contribute to financial time series’ asymmetry, nonlinearity, and nonstationarity [39, 44, 63, 64].

From a methodological perspective, the extant literature holds that many naturally fluctuating signals tend to possess important attributes [65]. Because it could break down a multidimensional signal into numerous typical basic modes, signal decomposition is a valuable approach for overcoming nonstationary signals. Adam [63] and Owusu Junior et al. [44] argue that financial time series are bounded by noise and endure fast fluctuations as a result of the widespread behaviour of fluctuating signals. The cogency of the fundamental rule of scale-invariance belonging to a self-similar process corroborates the hypothesis of heterogeneous markets is catalysed by the complication of the price generation process [44, 66]. Market players react to information at distinct periods; as a result, market data is often noisy. Accordingly, stock price series are nonlinear and nonstationary [2, 3, 15, 39]. This shows that noise in the market may introduce difficulties for investors in determining the driving factors of a trend and whether volatilities in trends are a result of fundamental dynamics or they are merely temporal volatilities in the short term.

Decomposition is an indisputably useful line of action to pursue when working with financial data series due to the aforementioned considerations. Corollary to this, we decompose the data to show stock market participants’ various investing time scales, which is consistent with Müller et al.’s [66] heterogeneous markets hypothesis (HMH). Also, the adaptive market hypothesis (AMH) developed by Lo [67] proposes that markets evolve, and market efficiency differs in degree at separate periods, as a result of events and structural transformations. Both the HMH and the AMH corroborate the competitive market hypothesis engineered by Junior et al. [9]. The CMH suggests that there is an increased intensity of information flow and spillover between markets of the same and other asset classes during turbulent trading periods due to rational, albeit irrational, investors who are always looking for competing rewards and risks to meet their portfolio goals. As a result, we evaluate the level of competition in the BRIC markets incorporating investor fears, which are measured by the US VIX, to see if significant combinations may operate as a substitute or complement to one another. To enhance the study’s conclusions, the application of decomposition approaches would minimise noise (weak signals) while actual signals are retained [39, 44]. The use of decomposition techniques has been prominent in recent finance literature (see e.g., [15, 17, 29, 32, 39, 40, 43]).

Dragomiretskiy and Zosso [34] devised the Variational Mode Decomposition (VMD) approach, which we employ in this paper. As Li et al. [68] and Isham et al. [69] argued, the VMD is a viable approach for sampling and catering to signal noise that outperforms both the EMD and EEMD (EMD—empirical mode decomposition; EEMD—ensemble EMD) in respect of Huang et al. [70] and Wu and Huang [71]. According to Wu and Huang [71], mode mixing describes intra-mode functions (IMFs) that comprise fluctuations of extremely divergent amplitude, which is primarily generated by the driving mechanism’s intermittency. These IMFs are thought to be time scales (amplitude-modulated-frequency-modulated signals) [39, 40, 72]. The VMD decomposes input signals into main modes, termed variational mode functions (VMFs) that duplicate the input signal with changing sporadic quality. VMFs represent different investment time scales of short-term, intermediate-term, and long-term horizons in the context of this study.

In sum, given the several projections of the development of the BRIC market [11, 47, 73, 74], coupled with the fact that news items are more contagious in the few decades than ever [18], resulting from financial market turbulence, there is the need to assess the flow of information between this market bloc (BRIC) whiles integrating investor fear and sentiments, measured by the US VIX. This would give insights to facilitate assessments of the operability of fundamental market dynamics of the BRIC market, which would reveal the relationships between the markets across diverse investment horizons. Through the revealed relationships and market dynamics of the BRIC markets, reliable assessments of safe haven, hedges, and diversification prospects could be ascertained by investors (both individual and institutional), portfolio, and fund managers.

3. Methodology

First, we show the Variational Mode Decomposition (VMD) technique, and second, transfer entropy. The result obtained from the VMD will be used to conduct the effective transfer entropy at various values of quantiles (q).

3.1. VMD. According to Dragomiretskiy and Zosso [34], the $k_{th}$ mode $u_k(t)$ is shown as

$$u_k(t) = A_k(t)\cos(\phi_k(t)), \tag{1}$$

where $A_k(t)$ is the instant amplitude and $\phi_k(t)$ is the instantaneous phase, and its derivative $\omega_k(t) = \phi_k(t)$ is the instant scale.

The VMD generates, for every mode $u_k(t)$, the logical signal and approximates the independent frequency spectrum using the Hilbert transform. From the displacement property of the Fourier transform, there is a relocation to the baseband of the mode’s spectrum. Then, the bandwidth is
proposed by the $H^1$ Gaussian smoothness. There is an optimization whose existence is to minimise the addition of the entire spectral widths of the mode functions to an infinitesimal value as

$$\min_{\{u_k\}, \{w_k\}} \left\{ \sum_{k=1}^k \left\| \frac{\delta (t + j \pi t)}{\tau_k} \ast u_k (t) \right\|_2^2 \right\},$$

(2)

s.t. $\sum_{k=1}^k u_k = f$,

where $\{u_k\}$ is mode ensemble, $\{w_k\}$ is the comparable centre frequency ensemble $K$ is the mode observation. See [33, 44] and Adam et al. [14] for a detailed presentation of the technique. Hamilton and Ferry’s [75] package “VMD” contains the VMD code.

### 3.2. Rényi Transfer Entropy

The TE is a nonparametric measure of directed, asymmetric information flow between two processes. Prior to the highlights on the Rényi transfer entropy (RTE), it is important to understand Shannon entropy, which measures the uncertainty that transfer entropy (TE) is based on [63, 76]. We investigate a probability distribution with a variety of experiment $p_j$. From Hartley [77], the mean information of every symbol is provided as

$$H = \sum_{j=1}^n p_j \log_2 \left( \frac{1}{p_j} \right) \text{bits},$$

(3)

where the number of diverse symbols regarding the probabilities $p_j$ is represented by $n$.

The Shannon entropy (SE) (1948) provides for a discrete random variable ($J$) with probability distribution ($P(j)$), the mean number of bits desirable for encoding independent draws at the maximum [76] can be presented as

$$H_J = -\sum_{j=1}^n P(j) \log_2 P(j).$$

(4)

SE draws on the Kullback-Leibler distance (1951) concept to quantify information transmission between two-time series under the Markov framework, for two discrete random variables, $I$ and $J$, having respective marginal probabilities of $P(i)$ and $P(j)$. The joint probability is thus $P(i, j)$, with dynamic structures that resemble a stationary Markov process of order $k$ (Process $I$) and $l$ (Process $J$). The Markov property implies that the probability of observing $I$ at time $t + 1$ in state $i$ dependent on the $k$ prior observations is $P(i_{t+1} | i_t, \ldots, i_{t-k+1}) = P(i_{t+1} | i_t, \ldots, i_k)$. In encoding the observation in $t + 1$, the mean number of bits needed given that the-exante $k$ observations are known can be presented in the form

$$h_I (k) = -\sum_{i=1}^n P(i_{t+1} | i_t^{(k)}) \log P(i_{t+1} | i_t^{(k)}),$$

(5)

where $i_t^{(k)} = (i_t, \ldots, i_{t-k+1})$ (compatibly for process $J$). Under the Kullback-Leibler distance phenomenon in the context of two random variables, the flow of information from process $J$ to process $I$ is estimated through quantification of the deviation from the generalized Markov property $P(i_{t+1} | i_t^{(k)}) = P(i_{t+1} | i_t^{(k)}, j_t^{(l)})$. The SE can thus be presented as

$$T_{J\rightarrow I} (k, l) = \sum P(i_{t+1} | i_t^{(k)}, j_t^{(l)}) \log \frac{P(i_{t+1} | i_t^{(k)}, j_t^{(l)})}{P(i_{t+1} | i_t^{(k)})},$$

(6)

where $T_{J\rightarrow I}$ computes the flow of information from $J$ to $I$. Correspondingly, the information flow $T_{I\rightarrow J}$ can be deduced as from $I$ to $J$. Quantifying the differential can divulge the dominant direction of the information transmission between $T_{J\rightarrow I}$ and $T_{I\rightarrow J}$.

Following the SE, we now discuss the Rényi Transfer Entropy (RTE) (1970). The RTE is conditioned on a weighting factor $q$, which is estimated as

$$H^q_J = \frac{1}{1-q} \log \sum_j P^q (j).$$

(7)

With $q > 0$. For $q \rightarrow 1$, RTE converges to SE. For $0 < q < 1$, hence, extra weights are attributed to low probability events, while for $q > 1$ the weights benefit outcomes $j$ with a higher original probability. Consequently, based on factor $q$, RTE allows accentuating varied distribution areas [63, 76].

Applying the escort distribution [78]

$$\mathcal{Q}_q (j) = p^q (j) / \sum_j p^q (j) \text{ with } q > 0 \text{ to normalize the weighted distributions, the resultant RTE is expressed as}$$

$$RT_{J\rightarrow I} (k, l) = \frac{1}{1-q} \log \frac{\sum \mathcal{Q}_q (i_t^{(k)}) p^q (i_{t+1} | i_t^{(k)})}{\sum \mathcal{Q}_q (i_t^{(k)}) p^q (i_{t+1} | i_t^{(k)}, j_t^{(l)})}.$$

(8)

It is important to keep in mind that the computation of the RTE can produce negative findings. Based on this, knowing the record of $I$ presents noticeably more indecision than knowing the record of $I$ only would present.

The estimations from transfer entropies could be biased in tiny samples [79]. This bias could be possibly corrected, from which the effective transfer entropy can be computed as

$$ETE_{J\rightarrow I} (k, l) = T_{J\rightarrow I} (k, l) - T_{\text{shuffled} \rightarrow I} (k, l),$$

(9)

where $T_{\text{shuffled} \rightarrow I} (k, l)$ represents the transfer entropy using a shuffled version of the time series $J$; that is, through a random selection of observations from the actual time series $J$ and adjusting them to produce a fresh time series, destroying the dependencies in time series $J$, but not overlooking the statistical reliance between $J$ and $I$. This changes $T_{\text{shuffled} \rightarrow I} (k, l)$ to approach zero as the sample size increases, and any nonzero value of $T_{\text{shuffled} \rightarrow I} (k, l)$ is attributable to tiny sample effects. Recurring shuffles and the total replications of the mean of the transfer entropy shuffled estimates act as the small sample bias estimator, which is deducted from the estimated $ETE(s)$, to yield a bias-adjusted ETE estimate.
To assess the statistical significance of the TE estimates, the study relies on the Markov block bootstrap [80]. The Markov block bootstrap maintains the dependencies within each time series, in contrast to shuffling. It produces the distribution of transfer entropy estimates under the null hypothesis of no information transfer as a result, using blocks of a random process \( J \) are readjusted to form a simulated series, which maintains the univariate dependencies of \( J \) but eradicates the statistical dependencies between \( J \) and \( I \). The RTE is calculated under the simulated time series. The information flow has a null hypothesis of no information movement which is ascertained by repeated estimation of the RTE. Hence, the \( p\)-value linked to the null hypothesis of no information flow is shown as 1 – \( p_{TE} \) suggesting the quantile of the simulated distribution that links the unusual TE estimates.

The transfer entropy algorithms also rely on discrete data. The continuous data used in the study must be discretised to achieve this. To solve this problem, symbolic encoding is used, which divides the data into a limited number of bins [76]. For a given number of bins \( n \), with bounds \( q_1, q_2, \ldots, q_{n-1} \) and a continuous observed time series data \( y_t \), its partitioning is given by

\[
s_t = \begin{cases} 
1, & y_t \leq q_1 \\
2, & q_1 < y_t < q_2 \\
& \vdots \\
n - 1, & q_{n-2} < y_t < q_{n-1} \\
n, & y_t \geq q_{n-1}.
\end{cases}
\]

(10)

The number of bins is determined by the size and distribution of the observed time series. Binning is typically based on the left tail and right tail quantiles in empirical investigations that emphasise tail findings [9]. The process is made simpler by choosing the 5% and 95% empirical quantiles to represent the lower and upper boundaries of the bin, respectively. This results in three symbolic encodings: a lower tail with negative volatility shocks (5%), and upper tail with positive shocks (95%) and a normal shock in the second bin (middle 90%).

3.3. Data Sources and Description. We utilized daily prices made up of six indices. They include the US Volatility Index (VIX), Brazil’s Ibovespa Index, Russia’s Moscow Exchange Russia Index, India’s NIFTY 500 Index, China’s Shanghai Stock Exchange Composite Index, and the BRIC Composite Index. After removing missing data, the daily data ranges from 2012/12/11 to 2021/05/28, producing a total of 1733 observations. The recommended time frame includes the aftermath of the 2008 Global Financial Crisis (GFC), the Eurozone crisis, and the COVID-19 pandemic. The EquityRT database was used to compile daily data on the BRIC. The data was executed on daily returns as \( r_t = \ln P_t - \ln P_{t-1} \), where \( r_t \) is the outcome of the log returns, \( P_t \) and \( P_{t-1} \) are current and past prices respectively.

We provide both the time-varying prices and returns as shown in Figure 2. The price series show that following a downward spike in the early part of 2020, the price series for all markets trend upwards. That is, BRIC prices are rapidly increasing, which supports Zhang et al.’s [81] argument that markets will revive far along the COVID-19 period because most enterprises and economies have figured out how to survive. The VIX price series, on the other hand, is trending lower after a downward jump in the second half of 2020. The inverse association between stock prices and VIX is depicted graphically here. As a result, the financial markets examined in this study are critical to investigate, particularly, depicting heterogeneous dynamics. Figure 2 shows how the log-returns series is in line with the stylized facts of asset returns, demonstrating volatility clustering.

The summary of the statistical analysis of the variables is shown in Table 1. The skewness values indicate asymmetry, whilst the kurtosis values indicate leptokurtic market behaviour and investor concern. This indicates that the study’s data is not regularly distributed. The Augmented Dicky-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity tests are utilised. All of the data series expressly meet the stationarity requirements, according to both the ADF and the KPSS observations. This is consistent with numerous autoregressive studies’ assumptions, which assume global stationarity.

4. Results and Discussion

The study presents the flow of information between the BRIC Composite Index and its constituents, as well as information flow between the US VIX and the BRIC markets. Since information flow among BRIC constitutes a complex network, which needs comprehensive interpretations and analysis, we account for several quantiles to reveal different market conditions as provided in other techniques such as the quantile regression (see [57]). The selection of the quantiles for the information flow is made possible due to the RTE technique. The quantiles are presented from 0.3 to 1.0 with a step of 0.1. We do this to provide a potpourri of outcomes at various market conditions. At quantiles less than 0.5, we account for low probability events, which reveal a crash in the markets, quantiles above 0.5 indicate a boom, and at 0.5, benign market condition is experienced.

4.1. Information Flow between Stock Markets of BRIC Constituents and the BRIC Composite Index. Figure 3 depicts the information flow between stock markets of BRIC constituents and the BRIC Composite Index at diverse market conditions illustrated by the quantiles from 0.3 to 1.0. We notice that when there is stress in the markets (at \( q < 0.5 \)), significant negative information flows between the BRIC constituents (except Russia) and the BRIC Composite Index in the original series (signals) and short-term of the decomposed data (M1). Notwithstanding, there exist the potential for negative flows of information in the medium- and long-term (M2, M3, & MAgg). This implies that, in
times of chaotic conditions, the stock markets of BRIC respond negatively to the BRIC Composite Index and vice versa. Also, under benign market conditions, we notice similar dynamics as found at the lower quantiles. Indeed, information flow between the BRIC constituents (except Russia) and the BRIC Composite Index is negative, mostly from the original series (signal) and in the short-term (M1). At that point, investors who wish to diversify their portfolios may easily do so, especially in the short-term.

The consistency and persistence of similar dynamics of information flow occur until the extreme upper quantiles. Specifically, at quantiles 0.9 and 1.0, representing high market performance or market boom, we notice more positive information flow between the variables. This suggests that positive information flow increases monotonically with increases in quantiles with quantile 1.0 recording more positive information flow between the variables. This is not surprising because BRIC stock markets have been flaunted to be well developed [11, 47, 73, 74] and well-integrated, to exhibit self-similar behaviour. Accordingly, when the markets are highly performing, the contribution of each market towards the BRIC Composite Index is positive irrespective of the intrinsic time dimension: short-, medium-, and long-term, and vice versa. As a result, knowing the history of the BRIC constituents indicates considerably less uncertainty than knowing the history of the BRIC Composite Index alone, and vice versa, which could be relevant in the redeployment of portfolios. As found by Junior et al. [9]; the contribution of the stock markets of the BRIC constituents to the BRIC Composite Index is positive and significant, which is partly in line with the outcome of the current study in that their finding was revealed only at market boom from this study. This may be a result of differences in the estimation techniques, rendering our approach more robust to non-linear, asymmetric, and nonstationary relationships by

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<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Normtest W</th>
<th>ADF</th>
<th>KPSS</th>
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</thead>
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<tr>
<td>BRIC</td>
<td>0.0002</td>
<td>0.0126</td>
<td>-0.6276</td>
<td>6.0673</td>
<td>0.9296***</td>
<td>-10.429***</td>
<td>0.0860</td>
</tr>
<tr>
<td>Russia</td>
<td>0.0000</td>
<td>0.0176</td>
<td>-0.9869</td>
<td>9.6720</td>
<td>0.9132***</td>
<td>-11.1400***</td>
<td>0.1614</td>
</tr>
<tr>
<td>India</td>
<td>0.0004</td>
<td>0.0146</td>
<td>-1.1346</td>
<td>11.3683</td>
<td>0.9023***</td>
<td>-10.8560***</td>
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<tr>
<td>China</td>
<td>0.0003</td>
<td>0.0161</td>
<td>-1.3376</td>
<td>13.8307</td>
<td>0.8744***</td>
<td>-11.1930***</td>
<td>0.0740</td>
</tr>
<tr>
<td>VIX</td>
<td>0.0000</td>
<td>0.0888</td>
<td>1.0615</td>
<td>7.9000</td>
<td>0.9063***</td>
<td>-12.7400***</td>
<td>0.0118</td>
</tr>
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* , **, and *** Significance at 10%, 5%, and 1% levels respectively.
Figure 3: Continued.
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<tr>
<th>Flow towards BRIC Constituents</th>
<th>Flow towards BRIC Index</th>
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![Figure 3: Continued.](image)
Figure 3: Information flow between stock markets of BRIC constituents and index.
Renyi’s Effective Transfer Entropy between BRIC and VIX (Signal)

Effective Transfer Entropy

0.0
-0.1
-0.2

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

Flow towards VIX
Flow towards BRIC

Renyi’s Effective Transfer Entropy between BRIC and VIX (Signal)

Effective Transfer Entropy

0.0
-0.1
-0.2

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

Flow towards VIX
Flow towards BRIC

Renyi’s Effective Transfer Entropy between BRIC and VIX (M1)

Effective Transfer Entropy

0.1
0.0
-0.2

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

Flow towards VIX
Flow towards BRIC

Renyi’s Effective Transfer Entropy between BRIC and VIX (M2)

Effective Transfer Entropy

0.05
0.0
-0.10

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

Flow towards VIX
Flow towards BRIC

Renyi’s Effective Transfer Entropy between BRIC and VIX (MAgg)

Effective Transfer Entropy

0.1
0.0
-0.1

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

BRIC

India

China

Brazil

Russia

Flow towards VIX
Flow towards BRIC

(a)

Figure 4: Continued.
Figure 4: Continued.
Figure 4: Continued.
Figure 4: Information flow between BRIC stock markets and VIX.
revealing diverse market conditions. The heterogeneity in the study’s outcome confirms the findings of prior studies such as Tilfani et al. [61]; Asafo-Adjei et al. [54].

4.2. Information Flow between Stock Markets of BRIC and Investor Fear. This section shows the information flow between the stock markets of BRIC, in addition to the BRIC Composite Index and the CBOE Volatility Index (VIX). We do this to account for the nonlinear, asymmetric, and nonstationary relationship that exists between the stock markets of BRIC and the US VIX as an investor fear and expectation indicator. A recent study by Junior et al. [9] indicated that the US VIX transmits negative shocks to the BRIC stock markets at all time-frequency domains as the principal outcome. However, in this study, we investigate the extent, to which both BRIC and VIX transmit information among themselves due to irrational behaviours of investors rendering the markets competitive, adaptive, and heterogeneous at various investment horizons.

From Figure 4, at low probability events representing market stress (at \( q < 0.5 \)), we notice potential negative information flow between the BRIC stock markets and the US VIX at most investment horizons: short-, medium-, and long-terms. Notwithstanding, the US VIX has the potential to transmit positive information to the stock markets of BRIC, mostly for Russia and China. This implies that, in times of chaotic conditions, the stock markets of BRIC respond positively to shocks from the VIX. It is not daunting to see information flow between the BRIC stock markets and VIX depict self-similar behaviour at quantiles 0.3 and 0.4. This is because, despite capital flight due to tapering by the US Federal Reserve, BRIC nations foresee greater supremacy in the international arena, as well as significant shifts in the capital flow into their markets [8, 82, 83]. According to Piper [84], BRIC financial markets also share a development philosophy that is more partnership-oriented than donor-recipient-focused.

At normal market conditions (at \( q = 0.5 \)), we document similar dynamics as found at the lower quantiles. Thus, information transmission between the stock markets of BRIC and the US VIX is mostly negative, but the US VIX transmits potential positive information to BRIC, especially for China and Russia (at M1 and MAgg). This suggests that existing investors of the US VIX may less likely diversify from including stock markets of China and Russia in the short-, and long-term, but existing investors of BRIC markets (except India in the short-term) can at most times (except in the long-term at MAgg) rebalance their portfolios by including the US VIX under benign market phenomenon.

Similarities in the behaviour of information flow at the quantiles occur except at quantile 1.0 where the RTE approaches the Shannon entropy. Thus, at quantile 1.0, representing high market performance or market boom, we find more positive information flow between the BRIC markets and VIX. This is to say, knowing the history of one market indicates considerably less uncertainty than knowing the history of the other. This is strongly pertinent for the rebalancing of portfolios as diversification potentials during this point may be hindered.

5. Theoretical and Practical Underpinnings
We find that information flow between the markets is heterogeneous and adaptive regarding diverse investment horizons and market conditions to reveal the HMH [66] and AMH [67]. Also, as postulated by Junior et al. [20], “in part, the intensity of information flow and spillover between markets of the same and differing asset classes are exacerbated by rational, albeit irrational investors’ relentless search for competing rewards and risks to satisfy the portfolio goals,” the concentration of information flow between markets may lead to high uncertainties. This was mainly found in diverse market conditions except at the upper quantiles, and with varying sparsity of investment horizons. This is a result of the behavioural intentions of investors, varying risk preference, relative optimism, and information perception which impact the willingness to invest.

It is worthy of notice that while our findings support Müller et al.’s [66] HMH, Lo’s [67] AMH, and the CMH of Junior et al. [9] at diverse scales (except at upper quantiles), they are highly opposing the efficient market hypothesis (EMH) of Fama [85, 86]. Intuitively, varying direction and significance of transfer entropies across the short- and midterm horizons (M1–M3) as well as across quantiles suggests that BRIC markets are characterized by heterogeneous and adaptive dynamics, which oppose the principles of market efficiency propagated by Fama [85]. Notwithstanding, given that almost all markets respond similarly (positive, suggesting low risk) to information flow in the long-term (MAgg), the BRIC markets could be regarded as saturated with shocks emanating from information flow and, hence, the fundamental market dynamics are at play in the long-term. This reiterates the long-term efficiency of Fama [86].

From the Modern Portfolio theory of Markowitz [87]; investors who seek to minimise their portfolio risks can do so aside from a market boom (upper quantiles). Global and domestic investors of BRIC can find a safe haven with the BRIC composite index, especially in the short-term during chaotic market outcomes or stress as provided by Junior et al. [20, 33, 88] and Adam [14]. Existing investors of the US VIX may less likely diversify from including stock markets of China and Russia in the short-, and long-term, but existing investors of BRIC markets (except India in the short-term) can at most times (except in the long-term at MAgg) rebalance their portfolios by including the US VIX at normal market outcomes. Generally, we advocate that redeployment of portfolios at high markets performance irrespective of the investment horizons should be an appropriate course of action for investors of BRIC financial markets. We support the finding that stock markets of BRIC economies are highly integrated, but when impacted by a significant market force, arbitrage and diversification potentials may be hindered [89]. Notwithstanding, as averred by Junior et al. [9], BRIC markets have a distinguishing economic and financial system, capacities, and capabilities to withstand such shocks. We add that investors of BRIC markets can still find safe haven, hedge, or diversification benefits depending on the market outcomes, from these shocks, as long as they emanate from other financial markets, due to contagion.
6. Conclusions

This study offers new and abundant evidence on the directional nonlinear, nonstationary, and asymmetric causality between the Chicago Board Options Exchange (CBOE) Volatility Index (VIX) of the US and the stock markets of BRIC. The VMD-based RTE is employed to examine the flow of information between the variables from a frequency-domain paradigm, through the intrinsic time. We set \( q \) from the RTE from 0.3 to 1.0 with an increment of 0.1 to account for several quantiles to reveal different market conditions. With this, we document low-, normal, and high-probability events at various intrinsic times. Specifically, the objectives of the study are to, examine the flow of information between the BRIC Composite Index and its constituents and the flow of information between the stock markets of BRIC and the VIX.

We found some interesting results for all financial instruments when we considered various quantiles to reveal the nature of market conditions at different intrinsic times, which is lacking in the most empirical literature on the RTE. We revealed that under stressed market conditions (at \( q < 0.5 \)), significant negative information flow between the BRIC constituents (except Russia) and the BRIC composite index in the original series and short-term. Notwithstanding, there is potential for negative flows of information in the medium- and long-term. Also, under benign market conditions, we find similar dynamics as found at the lower quantiles. This implies that, in times of chaotic conditions, the stock markets from BRIC respond negatively to the BRIC Composite Index, which provides diversification advantages. However, specifically, at quantiles 0.9 and 1.0, representing a high market boom, we document more positive information flow between the variables, which are relevant to the redeployment of portfolios.

Moreover, at low probability events representing market stress (at \( q < 0.5 \)), we notice potential negative information flow between the stock markets of BRIC and the US VIX at most investment horizons. Notwithstanding, the US VIX has the potential to transmit positive information to the stock markets of BRIC, mostly for Russia and China. At normal market conditions (at \( q = 0.5 \)), we document similar dynamics as found at the lower quantiles. However, at quantile 1.0, representing high market performance, we find more positive information flow between the BRIC markets and VIX.

We contribute to the extant literature on the integration of BRIC markets by inferring from the similar dynamics most BRIC constituents exhibited regarding the bidirectional causality with the BRIC composite index. Moreover, we found that BRIC economies are not only well integrated but also demonstrate self-similar behaviour from one quantile to another at various investment horizons. This implies that the stock markets of BRIC do not instantaneously respond to market changes, but exhibit some delayed responses to market conditions. We advocate that investors, portfolio managers, and risk managers, among others, should be wary of the heterogeneous and adaptive behaviour of BRIC stock markets under diverse market conditions to appropriately rebalance portfolios as diversification potentials may vary between the constituents of BRIC, and the US VIX and BRIC Composite Index.

The study was limited to the use of one volatility index; however, future studies can employ other uncertainty indicators to assess their information flow with the BRIC stock markets with interesting dynamics. Also, the patterns of information flow could be assessed before and during the COVID-19 pandemic to reveal its impact on the BRIC markets and volatility dynamics. The current study considered the outcome of each VMF but did not account for the aggregated impact of the VMFs based on multi-frequencies via the cluster analysis approach as employed by extant literature [14, 90]. Studies can also be conducted on other emerging markets whose financial markets are integrated for comparison within the emerging markets region. To cater for the time dimension in such analysis, the sliding window approach (see [61]) may be considered [68, 91–96].

Data Availability

Data employed in this study are available upon reasonable request to the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

No funding was used in this study.

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Mathematical Problems in Engineering


