Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 9360726, 13 pages
https://doi.org/10.1155/2022/9360726

Research Article

A Scheme to Improve Network Performance Based on Traffic Restriction and Pricing in the Presence of Carpooling

Pengyun Chong,[1] **Min Lv**,[2] **Hao Zhu**,[3] **and Dong Ding**[3]

[1]*Yunnan Science Research Institute of Communication & Transportation Co., Ltd., Kunming 650011, China*
[2]*School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 611756, China*
[3]*School of Economics and Management, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*

Correspondence should be addressed to Dong Ding; dingdong@cqupt.edu.cn

Received 14 July 2022; Revised 4 November 2022; Accepted 9 November 2022; Published 7 December 2022

Academic Editor: Stefan Cristian Gherghina

For the purpose of optimizing the traffic network in the presence of carpooling, a traffic demand management scheme was proposed with the consideration of traffic restriction, carpooling, and road pricing. We establish a variational inequality describing the travelers' mode and route choice following the logit mode split and multimodal user equilibrium principle in the condition of elastic demand. A bilevel programming was designed with the variational inequality as the lower level and optimization objective function taking into account traffic congestion and social welfare as the upper level. The results show that this scheme is possible to be effective in improving network performance. The higher average occupancy of carpooling vehicles and the carpooling demand may not bring better network performance due to the topology, parameters of the network, and travel demand. The fare of carpooling travelers is possible to impact the network very slightly with this scheme. Besides, the group of travelers who is able to afford more than one vehicle is hardly affected by this scheme.

1. Introduction

In terms of the increasingly serious situation of traffic congestion, carpooling, which is defined as two or more persons sharing one vehicle on the same trip, has gradually been focused on due to the superiorities of reduction of the total number of vehicles, releasing the travel demand, and alleviating traffic congestion. Some previous studies show that it has positive impacts on mitigating traffic congestion [1, 2], while another issue is that some travelers are reluctant to carpool since they care about the privacy and security [3]. Vanoutrive et al. issues that compulsory measures are able to enhance carpooling more than mild measures [4]. Therefore, compulsory measures are better choices for us to encourage carpooling and thereby alleviate congestion and elevate social welfare.

As a widely-used traffic management policy, road pricing is implemented in several places all over the world and followed by a large body of studies in theory and practice for the purpose of analyzing the impacts on the traffic system and exploring optimization [5–8]. For instance, Yang and Huang proceed with systematic research to reveal the mathematical and economical elements in the road pricing field [9]. Zhang and Yang propose a model and algorithm to determine the pricing district and the toll level in the case of cordon-based congestion pricing [10]. The studies show that road pricing is an effective measure to alleviate traffic congestion and improve system efficiency by adjusting travel demands and travel flow patterns. Simultaneously, traffic restriction can be seen in worldwide cities such as Milan, Beijing, San Paulo, Santiago, etc., and brings positive feedback in mitigating traffic congestion. From the perspective of theoretical research, some studies detect the impacts of given traffic restriction schemes on the networks [11, 12], while others design the optimal restriction district and the proportion [13, 14]. However, there exist debates with respect to road pricing and traffic restriction. For example, both road pricing and traffic restriction lead to objections from the public since, by some populations, the roads are considered public resources, so the two managements are violations of people's rights. Therefore, in many cities, the policies are difficult to implement. Besides, traffic restriction is valid in managing congestion with the price of restraining the travel demand. Considering the value of the time, Nie establishes a model to describe the user equilibrium and system optimal in general road networks when implementing first-best pricing and traffic restriction, respectively, and analyzes the impacts on a network. The findings show that traffic restriction is almost impossible to achieve the results of road pricing, even not beneficial to the traffic system when using traffic restrictions since travelers are able to buy extra cars to avoid the traffic restriction, which is usually license plate rationing. Therefore, traffic restriction is supposed to be implemented with other policies matching up to optimize the system [15]. Nie explores the effects of the schemes taking tradable permits, vehicle quota, and tradable credits as the remedies for traffic restriction, respectively, and issues that tradable credit remedy brings the best effects [16]. Therefore, taking into account the superiorities and the drawbacks of road pricing and traffic restriction, the remedy policies are feasible choices.

Song et al. propose a Pareto-improving optimal scheme combined with road pricing and traffic restriction, which stipulates that the travelers who are restricted could pay to enter the restriction districts and provide the model and algorithm to capture the optimal restriction area, proportion, and payment. The result shows that it is capable of obtaining better effects with this scheme compared to the traffic restriction scheme and the existing Pareto-improving scheme [17]. Nevertheless, in this scheme, Song et al. have not considered the impacts of the case that some travelers are able to afford extra cars to avoid the traffic restriction. Neither has they set the restriction as a connected district, which brings inconvenience for implementing the scheme. In addition, carpooling is not supposed to be neglected, accounting for the gradual popularity of this travel mode.

Triggered by this scheme, this paper proposes a cordon-based traffic restriction scheme in the unrestricted solo-driving vehicles and carpooling vehicles are permitted to enter the connected district, and the restricted solo-driving travelers could either pay to enter the restriction area or bypass the area. Therefore, restricted solo-driving travelers could choose to carpool to save the travel cost without the willingness to pay. Up to now, the studies concerning carpooling can be divided into two streams: simulation and mathematical modeling. In the field of simulation, Balac et al. use MATSim to estimate the carsharing demand in Zurich and investigate the impacts of parking pricing on carsharing [18, 19]. Ciari et al. use simulation technology to detect the impacts of several road pricing schemes on carsharing in Zurich [20]. Through a complementarity model, Xu et al. made the first attempt to describe the ridesharing network with ungiven occupancy of the ridesharing vehicles and analyze the solution of the model [21]. Li et al. investigate the impacts of OD-based surging pricing on the ridesharing market [22]. Li et al. propose a convex programming approach to describe the ridesharing equilibrium network and provide the algorithm [23].

Few works of literature discuss the comprehensive impacts of carpooling, road pricing, and traffic restriction. This paper takes the case that some travelers are able to afford multiple vehicles and elastic demand into consideration and use a multimode model to describe the traffic system. In addition, bilevel programming is adopted to determine the restriction district, the proportion, and the toll level. The rest of this paper is organized as follows: Section 2 introduces the preliminaries of this paper. Section 3 introduces the basic multimode model of traffic restriction. Section 4 establishes the carpooling system equilibrium taking into account road pricing and traffic restriction. The optimal scheme and the algorithm are designed in Sections 5 and 6, respectively, and applied to numerical examples in Section 7. Section 8 concludes this paper.

2. Preliminaries

Considering a general network $G(Q, A)$, Q is the set of nodes, and A is the set of links. W is the set of OD pairs, M is the set of travel modes, and $a \in A, w \in W, m \in M$, W^r is the set of the OD pairs which have been encompassed in the restriction district. The set of path is L and $l \in L$. $f_{w,l}^m$ represents the path flow with mode m on path l between OD pair w. d_w^m denotes the travel demand with mode m between OD pair w and d_w denotes the travel demand between OD pair w. v_a is the link flow on link a.

3. Basic Multimode Model for Traffic Restriction

3.1. Travel Mode Split and Route Choice. In this paper, we use the logit model to split the modes of travelers. Each traveler determines the travel mode based on the utility, namely, the travel cost. The population of the two modes are calculated as follows:

$$d_w^m = d_w \cdot \frac{\exp\left(-\theta \mu_w^m\right)}{\sum_{m \in M} \exp\left(-\theta \mu_w^m\right)}, w \in W, m \in M, \quad (1)$$

where θ is the sensitive parameter and μ_w^m is the minimal cost of mode m between OD pair w. We use user equilibrium (UE) to describe the route choices of the travelers, which can be expressed as follows:

$$c_w^m - \mu_w^m \begin{cases} = 0, & \text{when } f_{w,l}^m \geq 0, \\ \geq 0, & \text{when } f_{w,l}^m = 0, \end{cases} \quad w \in W, m \in M, l \in L, \quad (2)$$

c_w^m is the travel cost of mode m between OD pair w. It is demonstrated that all travelers choose paths involving minimal travel costs, and no one is able to decrease his/her cost by choosing another path. Then, the network falls into the user equilibrium. In this paper, we consider elastic demand in more realistic circumstances. The actual demand is computed as follows:

$$d_w = \bar{d}_w \cdot \exp\left(-\eta_w \cdot \mu_w\right), w \in W, \quad (3)$$

where η_w is the sensitive coefficient, $\bar{d}_w$ is the potential demand, and μ_w is the minimal travel cost between the OD pair w. Note that μ_w is the minimal cost among all travel modes. Thus, it is calculated by the following:

$$\mu_w = -\frac{1}{\theta}\ln \sum_{m\in M} \exp\left(-\theta\mu_w^m\right), w \in W. \quad (4)$$

3.2. Multimodal Combined Variational Inequality with Traffic Restriction. The multimode travel network with traffic restriction can be described by the variational inequality (VI) as follows:

$$\sum_{w\in W}\sum_{m\in M} c_{w,l}^{m,u}(f)\left(f_{w,l}^{m,u} - f_{w,l}^{m,u*}\right) + \sum_{w\in W^r}\sum_{m\in M} c_{w,l}^{m,r}(f)$$

$$\left(f_{w,l}^{m,r} - f_{w,l}^{m,r*}\right) + \frac{1}{\theta}\sum_{w\in W}\sum_{m\in M} \ln\frac{d_w^{m^*}}{d_w}\left(d_w^m - d_w^{m^*}\right) \geq 0, \quad (5)$$

$$\text{s.t}, \quad \sum_{l\in L} f_{w,l}^{m,u} = (1-\gamma)d_w^m, w \in W, m \in M,$$

$$\sum_{l\in L_N} f_{w,l}^{m,r} = \gamma d_w^m, w \in W^r, m \in M, \quad (6)$$

$$\sum_{l\in L_R} \tilde{f}_{w,l}^{m,r} = \gamma d_w^m, \frac{w \in W^r}{W}, m \in M, \quad (7)$$

$$\sum_{m\in M} d_w^m = d_w, w \in W, \quad (8)$$

where γ is the traffic restriction proportion. For example, if $\gamma = 0.3$, it means 30% of the travelers are forbidden to enter the restriction district. $c_{w,l}^{m,u}$ and $c_{w,l}^{m,r}$ denote the travel cost of the unrestricted and restricted travelers with travel mode m on path l involving minimal cost between OD pair w on the equilibrium, respectively. L_N and L_R are the set of the paths when the restricted travelers have or have no path to bypass the district, respectively. W^r/W represents the OD pairs in which travelers have no path to bypass. Equations (5) and (6) demonstrate the flow conservation of the unrestricted and restricted travelers with the around paths. Equation (7) demonstrates that restricted travelers choose alternate modes without any around the path. $\tilde{f}_{w,l}^{m,r}$ represents the flow of alternatives.

The VI above follows the logit mode split and user equilibrium, and the Gauss-Seidel decomposition process can be applied to solve it to capture the travel demand of each mode and the flow patterns.

Proposition 1. *The traffic restriction on total demand can be decentralized into the demands of various travel modes in the multimode traffic network.*

Proof. From equation (8), we obtain the relationship$\gamma d_w = \gamma \sum_{m\in M} d_w^m = \sum_{m\in M} \gamma d_w^m$ by multiple the proportion γ simultaneously, which means the description of the traffic restriction on total demand in mathematical modeling can be replaced by that on the various demand of mode m.

By Proposition 1, we know that the impacts of traffic restriction on total demand d_w is the same as that on the various demand of mode m. This implies that when traffic restriction is implemented to a multimode network, it can be decentralized into various demand travel modes. Then, we are able to capture demand the flow patterns by solving the subproblem of VI with respect to each mode. Furthermore, the subproblems of VI are routine VI problems, which can readily be solved through an existing approach such as the F-W algorithm, and projection algorithm. This makes the procedure ore convenient. □

4. Mode Choice and User Equilibrium in the Presence of Traffic Restrictions and Road Pricing

A handful of studies have been devoted to traffic restrictions, road pricing schemes, alternate travel modes, and multiple car owners [15, 17, 24]. However, few studies consider those simultaneously. In a traffic system, if travelers fall into traffic restriction, they can choose an alternate mode, pay the price, or own more vehicles to avoid the traffic restriction, which influences the mode and route choices as well as the network performances. In this paper, we assume there exist solo-driving and carpooling since carpooling is advocated due to its superiorities such as time and energy saving. When implementing traffic restrictions, carpooling travelers and unrestricted solo-driving travelers are allowed to enter the restriction area. The restricted solo-driving travelers with alternate paths can bypass the restriction district, while those without alternate paths can avoid the traffic restriction by paying the price or owning multiple cars. The multicar owners are able to use the second car to avoid traffic restrictions. Therefore, we assume that these travelers would not choose carpooling since owning multiple cars plays the same role as carpooling. The relationship between the travel demands is illustrated in Figure 1.

For simplicity, this paper has the assumptions below:

(1) There exist two travel modes in the network: solo-driving and carpooling. And solo-driving travelers have the choice to own A car or multiple cars. Namely, the travel modes include solo-driving with A car denoted as $s1$, solo-driving with multiple cars denoted as $s2$, and the carpooling h.

(2) There is only one platform providing the carpooling service, which implies that the carpooling travelers in the same vehicle pay the same fare.

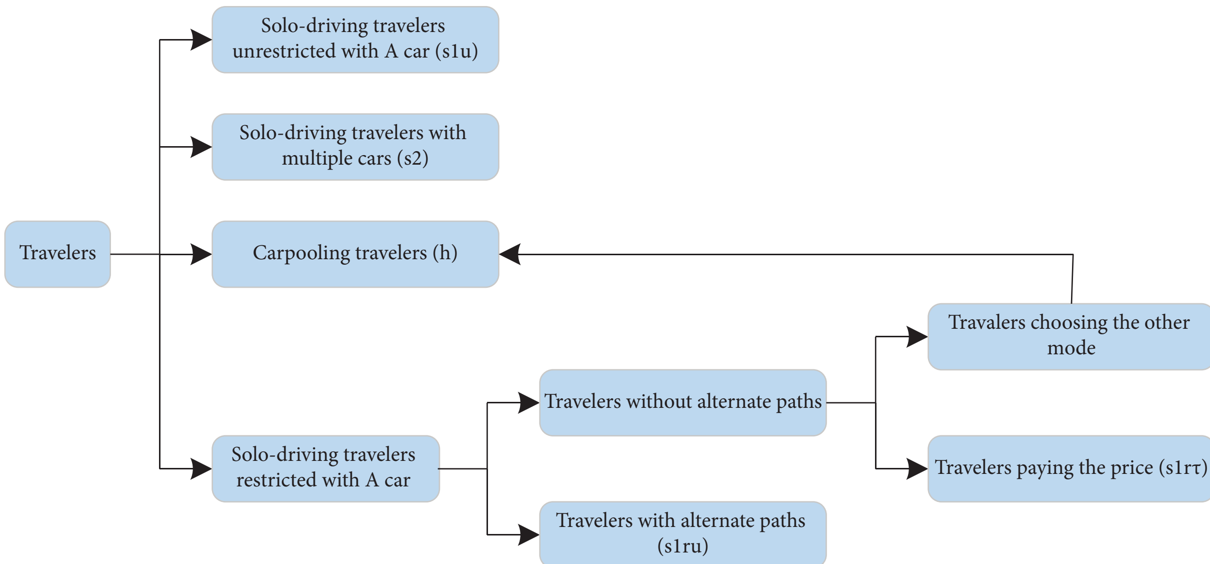

FIGURE 1: The relationship of the travel demands.

Then, the travel cost functions of the various type of travelers are expressed as follows:

$$c_{w,l}^{s1r} = \begin{cases} \lambda \sum\limits_{a\in A} t_a(v_a)\cdot\delta_a^l, & w\in W^r, l\in L^r, \\ \\ \sum\limits_{a\in A}\tau_a\cdot\delta_a^l + \lambda\sum\limits_{a\in A} t_a(v_a)\cdot\delta_a^l, & w\in \dfrac{W^r}{W}, l\in \widetilde{L}^{\tau}, \end{cases}$$

$$c_{w,l}^{s1u} = \lambda\sum_{a\in A} t_a(v_a)\cdot\delta_a^l, w\in W, l\in L,$$

$$c_{w,l}^{s2} = \phi + \lambda\sum_{a\in A} t_a(v_a)\cdot\delta_a^l, w\in W, l\in L,$$

$$c_{w,l}^{h} = \frac{\sigma}{o} + \Delta(o) + \lambda\sum_{a\in A} t_a(v_a)\cdot\delta_a^l, w\in W, l\in L. \tag{9}$$

W^r represents the OD pairs included in or separated by the restriction district, and in which there is at least one path around the restriction district, while W^r/W are the OD pairs included in or separated by the restriction district, and in which there is no path around the restriction district. It implies that the solo-driving travelers who do not prefer turning the travel mode have to pay to travel between the OD pairs W^r/W, and the solo-driving travelers between the OD pairs W^r are able to bypass the district. $c_{w,l}^{s1r\tau}$, $c_{w,l}^{s1u}$, $c_{w,l}^{s2}$, and $c_{w,l}^{h}$ denote the cost function of the solo-driving travelers restricted with A car and paying the price, the solo-driving travelers unrestricted with A car, solo-driving travelers with multiple cars, and the carpooling travelers. τ_a is the price that the restricted solo-driving travelers pay to use link a in the restriction district. ϕdenotes the amortized cost of owning multiple cars. o is the average occupancy of one car, and σ is the toll that the carpooling travelers pay to the carpooling platform. $\Delta(o)$ represents the nonfinancial additional cost, including matching and waiting time of carpooling riders. Yang and Huang assumed the additional cost consisting of the extra cost for the collection and distribution of riders and any other undesirable features as a constant [25]. We consider the cost $\Delta(o)$ as the function of carpooling occupancy. It is demonstrated that the average car occupancy is 1.6 in the German district [26]. Therefore, we set the average occupancy of carpooling vehicles as a constant varying from 2 to 5 normally. δ_a^l is a binary variable, which equals 1 if link a is on path l, and 0 otherwise. $\widetilde{L}^{\tau}$ represents the set of the paths on which the travelers restricted pay to use the restriction district.

The solo-driving travelers restricted without alternate paths follow the logit model for the mode split, namely, pay the price and carpooling. The two modes are expressed as $s1r\tau$ and $s1rh$, respectively. The demands can be computed as follows:

$$d_w^i = d_w^{\bar{s}1r}\cdot\frac{\exp\left(-\beta\mu_w^i\right)}{\sum\limits_i \exp\left(-\beta\mu_w^i\right)}, i = s1r\tau, s1rh, \frac{w\in W^r}{W}, \tag{10}$$

where $d_w^{\bar{s}1r}$ is the population of solo-driving travelers restricted with A car between OD pair w.

The solo-driving travelers restricted with A car and without alternate paths choose to pay or carpooling are denoted by $s1r\tau$ and $s1rh$, respectively. Note that the minimal cost of the alternate mode for solo-driving travelers restricted is the minimal cost of carpooling. In other words, $\mu_w^{s1rh} = \mu_w^h$ and $c_w^{s1rh} = c_w^h$. Therefore, between the OD pair w in which the solo-driving travelers are restricted have alternate paths, the minimal cost of the solo-driving travelers with A car can be computed by the following:

$$\mu_w^{s1} = (1-\gamma)\mu_w^{s1u} + \gamma\mu_w^{s1r}, w\in W^r, \tag{11}$$

where μ_w^{s1u} is the minimal cost of solo-driving travelers unrestricted and μ_w^{s1r} is the minimal cost of solo-driving travelers unrestricted having alternate paths. Between the OD pair w in which the solo-driving travelers are restricted and lack of alternate paths, the minimal travel cost of the solo-driving travelers with A car is calculated by the following:

$$\mu_w^{s1} = (1-\gamma)\mu_w^{s1u} + \gamma\bar{\mu}_w^{s1r}, w \in \frac{W^r}{W}. \quad (12)$$

$\bar{\mu}_w^{s1r}$ is the minimal cost of solo-driving travelers unrestricted with A car and lacking alternate paths, which can be obtained by the following:

$$\bar{\mu}_w^{s1r} = -\frac{1}{\beta}\ln\left[\exp\left(-\beta\mu_w^{s1r\tau}\right) + \exp\left(-\beta\mu_w^{h}\right)\right], w \in \frac{W^r}{W}. \quad (13)$$

Based on the logit mode split and UE principle, we formulate the VI to describe the traffic system. Let $\mathbf{f} = (f_{w,l}^{\bar{m}}, f_{w,l}^{s1u}, f_{w,l}^{s1r}, w \in W, l \in L, \bar{m} = s2, h)$, and $\mathbf{d} = (d_w^m, d_w^i, m \in M, i = s1r\tau, s1rh)$. We will show that we are able to capture the demand and flow patterns of the network via a VI.

Proposition 2. *The demand pattern* $\mathbf{d}$ *at the logit mode split and the aggregate flow pattern* $\mathbf{f}$ *at user equilibrium can be obtained by solving the VI in P1 as follows:*

$$\begin{aligned}
\text{P1:}\quad & \sum_{w\in W}\sum_{m} c_{w,l}^{\bar{m}}(f)\left(f_{w,l}^{\bar{m}} - f_{w,l}^{\bar{m}*}\right) + \sum_{w\in W} c_{w,l}^{s1u}(f)\left(f_{w,l}^{s1u} - f_{w,l}^{s1u*}\right) + \sum_{w\in W} c_{w,l}^{s1r}(f)\left(f_{w,l}^{s1r} - f_{w,l}^{s1r*}\right) \\
& + \frac{1}{\theta}\sum_{w\in W}\sum_{m\in M}\sum_{m}\ln\frac{d_w^{m*}}{d_w}\left(d_w^m - d_w^{m*}\right) + \frac{1}{\beta}\sum_{w\in W^r/W}\sum_{i}\ln\frac{d_w^{i*}}{d_w^{s1r}}\left(d_w^i - d_w^{i*}\right) \\
& + \frac{1}{\eta}\sum_{w\in W}\ln\frac{d_w^*}{d_w}\left(d_w - d_w^*\right) \geq 0,\ i = [\,s1r\tau \ \ s1rh\,], \bar{m} = [\,s2,\ h\,], \\
\text{s.t.}\quad & \sum_{l\in L} f_{w,l}^{s1u} = (1-\gamma)d_w^{s1}, w \in W, \\
& \sum_{l\in L} f_{w,l}^{s1r} = \gamma d_w^{s1}, w \in W^r, \\
& \sum_{l\in L} f_{w,l}^{s1r} = d_w^{s1r\tau}, w \in \frac{W^r}{W}, \\
& \sum_{l\in L} f_{w,l}^{h} = d_w^{h} + d_w^{s1rh}, w \in W, \\
& \sum_{l\in L} f_{w,l}^{s2} = d_w^{s2}, w \in W, \\
& \sum_{i} d_w^{i} = \gamma d_w^{s1}, i = [s1r\tau\, s1rh], w \in \frac{W^r}{W}, \\
& \sum_{m\in M} d_w^{m} = d_w, w \in W, \\
& (\mathbf{d},\mathbf{f}) \geq 0,
\end{aligned} \quad (14)$$

where $d_w^{s1r\tau}$ *and* d_w^{s1rh} *represent the demand of restricted solo-driving travelers owning A car between OD pair* w*, pay the price and turn to carpooling, respectively.* $f_{w,l}^{s1r}$ *is the flow of the travelers who owns A car and restricted on path* l *between OD pair* w*. Between the OD pairs* W^r/W*, the travelers of which flow can be denoted by* $f_{w,l}^{s1r}$ *prefer paying a toll, and the travelers of* *which flow can be denoted by $f_{w,l}^{s1r}$ between the OD pairs W can choose the alternate paths to avoid traffic restriction.*

Proof. Please see Appendix A.

Based on Proposition 2, it is shown that the VI above demonstrates the logit mode split and user equilibrium. In other words, the solution of the VI is the demand and the flow patterns. Then, we will explore the existence and the uniqueness of the solution. □

Proposition 3. *If the travel link time function $t_a(v_a)$ is continuous, the VI in P1 has one solution at least.*

Proof. Please see Appendix B.

The solutions might not be unique since we cannot prove the uniqueness of the solution. However, the nonunique solutions are path-based flow patterns. Various solutions probably generate the unique link-based flow pattern and one network performance since they are in the feasible set. Therefore, in terms of the various solutions, we can still calculate the network performances. □

5. Design of the Optimal Scheme

Obviously, elastic demand is more realistic. In the case of elastic demand, on the one hand, traffic restriction curtains the total number of vehicles on the network. On the other hand, the alleviation of congestion deriving from the decrease in travelers might induce more trips. Therefore, there is supposed to be an optimal scheme with the optimal traffic restriction district, proportion, and pricing. Considering a model to generate the rationing scheme, we are able to solve the VI to obtain the demand and flow patterns under the given scheme. If we can receive a comparative optimal scheme through the demands and flows, the optimal scheme is the final target. Therefore, bilevel programming is adopted for the design of the optimal scheme, in which the VI is the lower level, and we set an objective as the upper level. In this framework, the demand and flow patterns are captured from the lower level to receive the network performance as well as the optimal district, proportion, and price from the upper level built on the network performance. Since the network performances can be measured by the congestion and social welfare, we use the function of maximizing network congestion and social welfare as the objective of the upper level [13].

$$\begin{aligned} &\max \ Z, = -M \sum_{a\in A} \max\{0, v_a - C_a\} + \sum_{w\in W} \frac{d_w}{\eta_w}, \\ &\text{s.t.} 0 \le \gamma \le 1, \\ &\tau_{a,\min} \le \tau_a \le \tau_{a,\max}. \end{aligned} \tag{15}$$

The objective above implies taking the network congestion and social welfare into consideration simultaneously when we try to optimize the system, and the proportion and the price level are supposed to fall into the reasonable range. The first part of objective Z is the total overload flow, while the second part is the social welfare. M is a sufficiently large and positive coefficient connecting the two parts.

6. Algorithm

Bilevel programming is a mixed integer programming problem. Taking the convenience of implementation into consideration, we assume that the restriction district is connected. Some studies have explored a similar problem, such as how to optimize the connected area and the relevant variables. For instance, Zhang and Yang proposed an approach to design the cordon-based congestion pricing scheme to capture the optimal connected area and the congestion pricing level [10]. Shi et al. adopted the genetic algorithm to solve the optimal traffic restriction problem to obtain the optimal restriction district and the proportion [13]. Chen et al. used the surrogate assistant (SA) model to solve the bilevel congestion pricing problem and compared the effects of various SA models via a real case. It is known that the surrogate assistant model can help us reduce the expenses of the problems. Thus, in this paper, we propose an algorithm combined with a genetic algorithm and surrogate assistant model to capture the optimal restriction district, the proportion, and the link-based toll pattern. We train the SA model with the evaluation function and the variables, then apply the SA model to the genetic algorithm. The specific procedures are as follows:

Step 0. Generate the various initial solutions involving the traffic restriction districts, the proportions, and the toll levels.

Step 1. Construct the surrogate assistant function with the districts, the proportions, and the toll levels as the variable and the objectives as the results. In this paper, we use the quadratic polynomial function as the surrogate assistant model, which is written as follows:

$$y = \zeta_0 + \sum_{1\le i_y\le n} \zeta_{i_y} x_{i_y} + \sum_{1\le i_y\le j_y\le n} \zeta_{n-1+i_y+i_y} x_{i_y} x_{j_y}. \tag{16}$$

Where ξ_{i_y} is the parameters needed to be estimated, and n is the number of the variables.

Step 2. Use a genetic algorithm to solve the bilevel programming. Note that (i): The traffic restriction district is connected. In the code of the solution process, we use the binary variable ρ_q in the gene to denotes the determination of the restriction district. $\rho_q = 1$ means the node q is included in the restriction district and $\rho_q = 1$ otherwise. The proportion and the toll are also coded and connected to the gene consisting of ρ_q. (ii): The evaluation function in the process of the genetic algorithm is replaced by the surrogate assistant function built on the specific results of the district, the proportion, and the toll

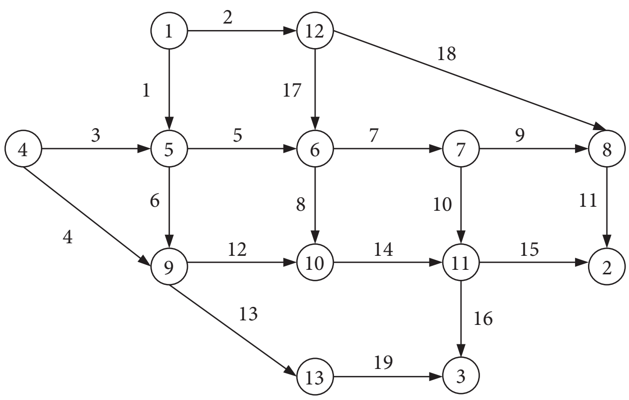

Figure 2: Topology of the simple network.

Table 1: Free flow travel time and the capacity.

Link	t_a^0 (min)	C_a (veh/h)
1	10	1000
4	20	1000
7	10	1000
10	10	1000
13	20	1000
16	10	1000
2	10	1000
5	10	1000
8	10	1000
11	10	1000
14	10	1000
17	10	1000
3	10	1000
6	10	1000
9	10	1000
12	10	1000
15	10	1000
18	30	1000

pattern. The parameters of the SA model can be calibrated through the results.

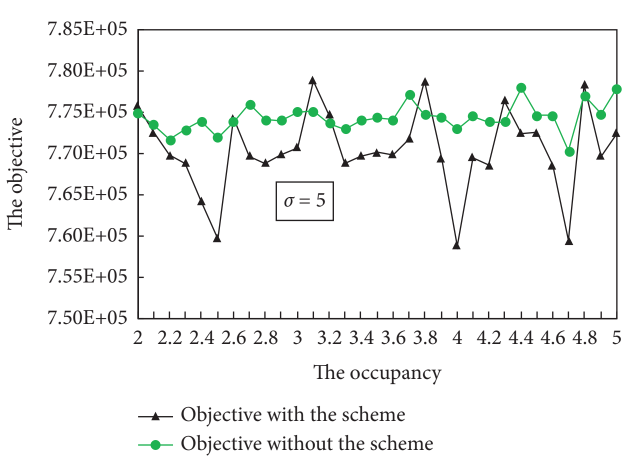

Figure 3: The objective versus the carpooling vehicle average occupancy.

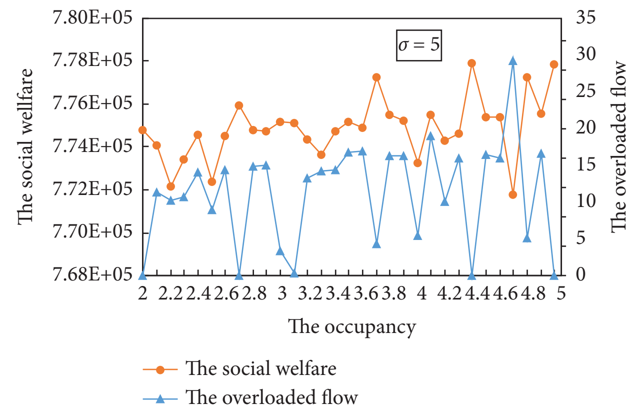

Figure 4: The social welfare and the overloaded flow against the carpooling vehicle average occupancy.

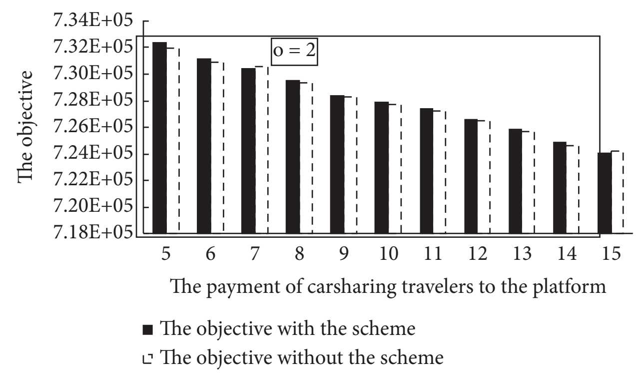

Figure 5: The objective against the various payments.

7. Numerical Examples

Example 1. We take the network in [12] as the example shown in Figure 2. The free-flow travel time t_a^0 and the capacity C_a are listed in Table 1. There are four OD pairs (1, 2), (1, 3), (4, 2), and (4, 3), and the potential travel demands are 1200, 900, 900, and 1200, respectively. The other parameters are as follows: $\phi = 15, \lambda = 2, \beta = 0.6, \eta_w = 0.04, \theta = 0.5, M = 50, \tau_{a,\min} = 0, \tau_{a,\max} = 10$. We adopt the BPR function to measure the link travel time, which is expressed as the following:

$$t_a(v_a) = t_a^0\left[1 + \alpha_1\left(\frac{v_a}{C_a}\right)^{\alpha_2}\right], a \in A, \tag{17}$$

where t_a^0 is the free-flow travel time on link a. α_1, α_2 are the parameters. Normally, we let $\alpha_1 = 0.15, \alpha_2 = 4$. The link flow v_a and the path flow $f_{w,l}^m$ satisfies the following:

$$v_a = \sum_w \sum_m \sum_l f_{w,l}^m \cdot \delta_a^l, w \in W, m \in M, l \in L. \tag{18}$$

Figure 3 illustrates the change of the objective against various average carpooling vehicle occupancy. It can be seen that the tendency of the objective change fluctuates

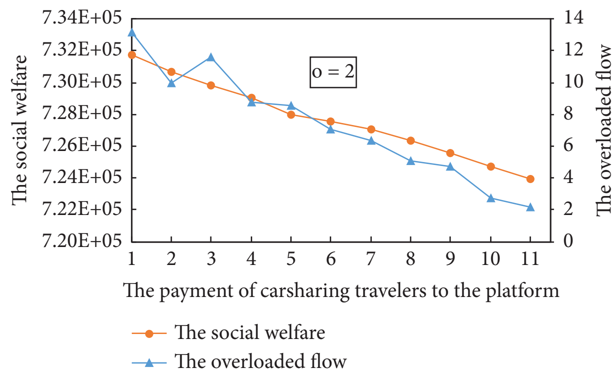

FIGURE 6: The social welfare and the overloaded flow with the various payments.

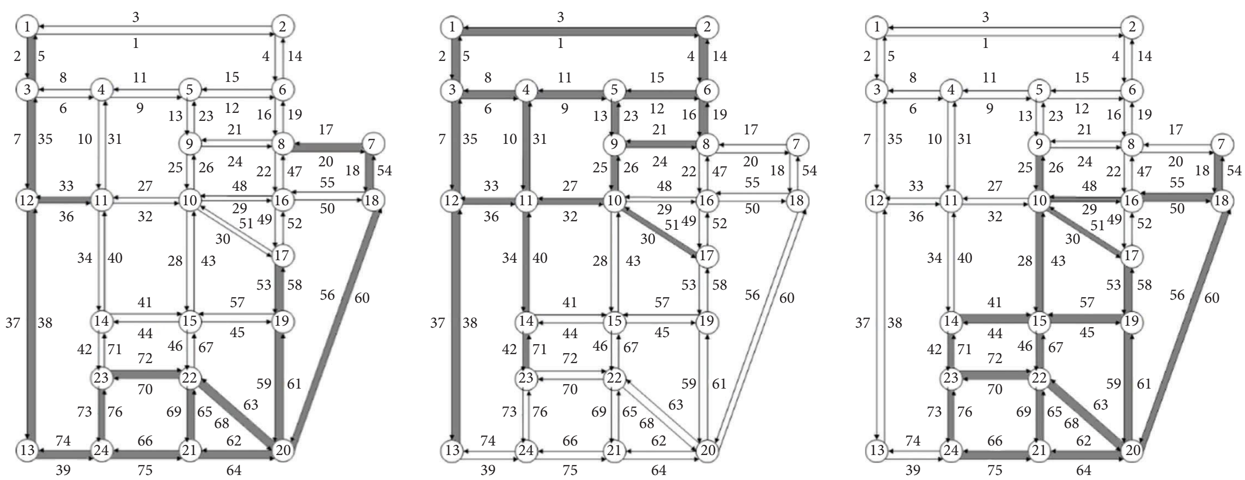

FIGURE 7: The various scenarios.

TABLE 2: The performances of the network with various scenarios.

Scenarios	Total demand	Demand for solo drivers with one car	Demand for solo drivers with multiple cars	Demand for carpooling travelers	Overloaded flow	Social welfare	Objective
No	17251	13246	6	3278	501	472017	446967
1	19136	10163	11	7816	139	465637	458687
2	21308	11275	9	9620	475	462314	438564
3	18863	13361	8	5753	482	471917	447817

instead of being linear with the occupancy. Furthermore, in terms of some occupancies, the objective value with the scheme is lower than that without the scheme. Figure 4 illustrates the changes in the social welfare and the overloaded flow with the occupancy are nonlinear neither. This is because the increasing average occupancy value means less vehicles, which releases congestion and thereby brings more trips. After the mode split and the traffic assignment, we have the results in Figures 3 and 4. Therefore, this scheme is beneficial to the network but not always accounting for the topology, the parameter of the network, and the demand patterns. Note that in Figure 4, when the overloaded flow is zero, the social welfare is on the top when the occupancy is 4.4, which means the network is not congested and the population travel most. It proves the rationality of the objective function. Namely, in some situations, this scheme is able to capture the maximal social welfare and uncongested roads.

Figure 5 depicts the change of the objective with the various payments of the carpooling travelers to the carpooling platform, and Figure 6 portrays the change the social welfare and the overloaded flow against the payment. Figure 5 illustrates that with the increase of the payment, the network performances with the scheme and without the scheme change alike so that the gap of the objectives between the cases with and without the scheme changes slightly. The reason is that the change of payment alters the travel demand and the congestion state of the network. The network performance maintains approximately due to the mode split and traffic assignment. Thus,

Table 3: The link toll level for the optimal scenario.

Link	Toll
1	0
8	0
15	0
22	0
29	0
36	4.2
43	0
50	0
57	0
64	2.05
71	0
2	8.27
9	0
16	0
23	0
30	0
37	8.28
44	0
51	0
58	8.28
65	2.05
72	2.05
3	0
10	0
17	2.05
24	0
31	0
38	8.28
45	0
52	0
59	1.36
66	8.28
73	4.2
4	0
11	0
18	4.2
25	0
32	0
39	2.01
46	0
53	1.36
60	9.36
67	0
74	1.36
5	1.36
12	0
19	0
26	0
33	9.36
40	0
47	0
54	6.11
61	0.72
68	4.2
75	4.2
6	0
13	0
20	2.05
27	0
34	0

Table 3: Continued.

Link	Toll
41	0
48	0
55	0
62	4.2
69	6.11
76	9.36
7	1.36
14	0
21	0
28	0
35	6.11
42	0
49	0
56	0.72
63	9.36
70	0.72

payment is not the key to improving the network performance under this scheme. The decision-makers could consider other factors for system optimization, e.g., the profit of the platform. Figure 6 shows that with the increase of the payment, the social welfare and the overloaded flow decrease with the scheme, implying that higher payment is not beneficial for carpooling encouragement.

Example 2. We adopt the Sioux Falls network in [27] as an example. The travel demand is set as 400 for each OD pair. The topology and the link attributes are shown in [27]. The other parameters are set as follows: $\lambda = 2, \phi = 15, \sigma = 5, o = 2, \beta = 0.6, \eta_w = 0.04, \theta = 0.5, M = 50, \tau_{a,\min} = 0, \tau_{a,\max} = 10$. In reality, plate-number-based odd-even rationing and connected district are considered for the convenience of the policy implementation. In addition, some places must be included in the restriction district due to some reasons, e.g., special events happening in the places. Therefore, we set three scenarios: (i) the theoretical optimal scheme, (ii) the optimal scheme including node 9, and (iii) the proportion is 50%, namely, plate-number-based odd-even rationing. We still use BPR function for computing the link travel time.

In Figure 7, the three scenarios illustrate the network with the optimal schemes, respectively, in which the restricted districts consist of the black links. From Table 2, we know that the carpooling demand is proportional to the total demand, which means the travel demand can be released by carpooling. However, the carpooling demand is negatively relevant to social welfare. It might derive from the sensitive parameter η_w. Thus, carpooling is not always beneficial for social welfare as the topology of the network and other policies have impacts on that. From Table 2, it can be seen that the relationship between the carpooling demand and the overloaded flow is not linear. When the carpooling demand is on the top, the overloaded flow is not minimal. This is because the various schemes bring different actual demands, which lead to the various travel mode demand and the assignment. In some OD pairs, the demands increase, and the paths become more congested. It implies that the carpooling demand cannot determine the network congestion. Furthermore, the demand for travelers owning more vehicles varies slightly, implying that those travelers are hardly influenced by the schemes. Table 3 shows the toll level with the theoretical optimal scheme, and the restriction proportion is 79%.

8. Conclusions and Future Directions

Road pricing and traffic restriction have superiorities and drawbacks for encouraging carpooling. In this paper, we propose a scheme that allows carpooling travelers and solo-driving travelers willing to pay the price to enter the traffic restriction district. In terms of this scheme, we describe the behaviors of travelers, design an optimal scheme, and explore the impacts of this scheme on the network. To be specific, firstly, we use a VI to describe the travelers' mode and route choice following the logit mode split and UE principle. Then taking the VI as the lower level, we use bilevel programming with the objective combined with social welfare and overloaded flows as the upper level. At last, we propose an algorithm consisting of a genetic algorithm and surrogate assistant model to determine the optimal scheme involving optimal connected restriction district, proportion, and the pricing, and apply the model and the algorithm to a simple network example as well as a sioux falls example with three scenarios: theoretical scenario, the scenario including a special node, and the scenario with plate-number-based odd-even rationing. From the results, we know that this scheme can be applied to reality. In our examples, this scheme is beneficial to the network performance, but not always. It implies this scheme could be an alternative for decision-makers with the purpose of improving the traffic system. On account of the topology of the network, the parameters, and the travel demand pattern, the higher average occupancy and carpooling demand are not always beneficial to the network with elastic demand case. Therefore, it is necessary to consider the individual case when encouraging carpooling to improve the network performance. In addition, the impacts of traffic policy on travelers owning multiple cars might be slight since the number of travelers are not too much. The decision-makers may decrease the weight of the travelers to some extent when making traffic policies.

This study offers a model and an algorithm to determine a traffic policy. Nevertheless, this model assumes the fixed carpooling average occupancy. For realistic circumstances, unfixed and nonaverage occupancy is supposed to be considered in this scheme. Besides, more travel modes, e.g., transit, need to be involved in the study in future directions.

Appendix

A. Proof of Proposition 2

Proof. The KKT condition of the VI in P1 is as follows:

$$c_{w,l}^{s1u} - \mu_w^{s1u} - \chi_{w,l}^{s1u} = 0, w \in W, l \in L, \tag{A.1}$$

$$c_{w,l}^{s1r} - \mu_w^{s1r} - \chi_{w,l}^{s1r} = 0, w \in W^r, l \in L, \tag{A.2}$$

$$c_{w,l}^{s1r} - \mu_w^{s1r\tau} - \chi_{w,l}^{s1r} = 0, w \in \frac{W^r}{W}, l \in L, \tag{A.3}$$

$$c_{w,l}^{h} - \mu_w^{h} - \chi_{w,l}^{h} = 0, w \in W, l \in L, \tag{A.4}$$

$$c_{w,l}^{s2} - \mu_w^{s2} - \chi_{w,l}^{s2} = 0, w \in W, l \in L, \tag{A.5}$$

$$\frac{1}{\theta}\ln\frac{d_w^{s1}}{d_w} + (1-\gamma)\mu_w^{s1u} + \gamma\mu_w^{s1r} - \pi_w = 0, w \in W^r, \tag{A.6}$$

$$\frac{1}{\theta}\ln\frac{d_w^{s1}}{d_w} + (1-\gamma)\mu_w^{s1u} + \bar{\gamma}\mu_w^{s1r} - \pi_w = 0, w \in \frac{W^r}{W}, \tag{A.7}$$

$$\frac{1}{\theta}\ln\frac{d_w^{h}}{d_w} + \varphi_w^{h} - \pi_w = 0, w \in W, \tag{A.8}$$

$$\frac{1}{\theta}\ln\frac{d_w^{s2}}{d_w} + \varphi_w^{s2} - \pi_w = 0, w \in W, \tag{A.9}$$

$$\frac{1}{\beta}\ln\frac{d_w^{s1r\tau}}{d_w^{s1r}}+\mu_w^{s1r\tau}-\overline{\mu}_w^{s1r}=0, w\in\frac{W^r}{W}, \tag{A.10}$$

$$\frac{1}{\beta}\ln\frac{d_w^{s1rh}}{d_w^{s1r}}+\mu_w^{h}-\overline{\mu}_w^{s1r}=0, w\in\frac{W^r}{W}, \tag{A.11}$$

$$\frac{1}{\eta_w}\ln\frac{d_w}{\overline{d}_w}+\pi_w=0, w\in W, \tag{A.12}$$

$$f_w^{\bar{n}}\cdot\chi_w^{\bar{n}}=0, w\in W, \bar{n}=(s2,h,s1r,s1u), \tag{A.13}$$

$$f_w^{\bar{n}}\geq 0, \chi_w^{\bar{n}}\geq 0, w\in W, \bar{n}=(s2,h,s1r,s1u). \tag{A.14}$$

From equations (A.1)–(A.5), equations (A.13) and (A.14), we know that if $f_w^{\bar{n}}>0$, then $\chi_w^{\bar{n}}=0$ and $c_w^{\bar{n}}=\mu_w^{\bar{n}}$. If $f_w^{\bar{n}}=0$, then $\chi_w^{\bar{n}}\geq 0$ and $c_w^{\bar{n}}\geq\mu_w^{\bar{n}}$. This can be expressed as follows:

$$\begin{cases} c_w^{s1u}=\mu_w^{s1u}, & \text{if } f_w^{s1u}>0, w\in W,\\ c_w^{s1u}\geq\mu_w^{s1u}, & \text{if } f_w^{s1u}=0, w\in W, \end{cases} \tag{A.15}$$

$$\begin{cases} c_w^{s1r}=\mu_w^{s1r}, & \text{if } f_w^{s1r}>0, w\in W^r,\\ c_w^{s1r}\geq\mu_w^{s1r}, & \text{if } f_w^{s1r}=0, w\in W^r, \end{cases} \tag{A.16}$$

$$\begin{cases} c_w^{s1r}=\overline{\mu}_w^{s1r}, & \text{if } f_w^{s1r}>0, w\in\dfrac{W^r}{W},\\ c_w^{s1r}\geq\overline{\mu}_w^{s1r}, & \text{if } f_w^{s1r}=0, w\in\dfrac{W^r}{W}, \end{cases} \tag{A.17}$$

$$\begin{cases} c_w^{h}=\mu_w^{h}, & \text{if } f_w^{h}>0, w\in W,\\ c_w^{h}\geq\mu_w^{h}, & \text{if } f_w^{h}=0, w\in W, \end{cases} \tag{A.18}$$

$$\begin{cases} c_w^{s2}=\mu_w^{s2}, & \text{if } f_w^{s2}>0, w\in W,\\ c_w^{s2}\geq\mu_w^{s2}, & \text{if } f_w^{s2}=0, w\in W. \end{cases} \tag{A.19}$$

Equations (A.15)–(A.19) demonstrate the UE equilibrium principle, which means each type of traveler follows its own equilibrium state. From equations (A.8) and (A.9), we have the following:

$$\exp\left[\beta\left(-\mu_w^{s1r\tau}+\overline{\mu}_w^{s1r}\right)\right]=\frac{d_w^{s1r\tau}}{d_w^{s1r}}, w\in\frac{W^r}{W}, \tag{A.20}$$

$$\exp\left[\beta\left(-\mu_w^{h}+\overline{\mu}_w^{s1r}\right)\right]=\frac{d_w^{s1rh}}{d_w^{s1r}}, w\in\frac{W^r}{W}. \tag{A.21}$$

Taking the sum of the two equations above on both sides leads to the following:

$$\exp\left(\beta\overline{\mu}_w^{s1r}\right)=\frac{1}{\exp\left(-\beta\mu_w^{s1r\tau}\right)+\exp\left(-\beta\mu_w^{h}\right)}, w\in\frac{W^r}{W}. \tag{A.22}$$

Substituting equation (A.22) into equations (A.20) and (A.21) gives rise to the following:

$$d_w^{s1r\tau} = d_w^{s1r} \cdot \frac{\exp\left(-\beta\mu_w^{s1r\tau}\right)}{\exp\left(-\beta\mu_w^{s1r\tau}\right) + \exp\left(-\beta\mu_w^{h}\right)}, w \in \frac{W^r}{W}, \tag{A.23}$$

$$d_w^{s1rh} = d_w^{s1r} \cdot \frac{\exp\left(-\beta\mu_w^{h}\right)}{\exp\left(-\beta\mu_w^{s1r\tau}\right) + \exp\left(-\beta\mu_w^{h}\right)}, w \in \frac{W^r}{W}, \tag{A.24}$$

which is consistent with the logit model for travel mode split. This implies that travelers in restriction district and have no path for their solo-driving trips have to travel either by carpooling or by paying pricing. Note that from equation (A.22), the expression of $\bar{\mu}_w^{s1r}$ falls into equation (13).

Based on equations (11) and (12), equations (A.6) and (A.7) can be rewritten as follows:

$$\frac{1}{\theta}\ln\frac{d_w^{s1}}{d_w} + \mu_w^{s1} - \pi_w = 0, w \in W^r, \tag{A.25}$$

$$\frac{1}{\theta}\ln\frac{d_w^{s1}}{d_w} + \mu_w^{s1} - \pi_w = 0, w \in \frac{W^r}{W}. \tag{A.26}$$

Combined with equations (A.8) and (A.9), we can deal with equations (A.8) and (A.9) and equations (A.25) and (A.26) by the procedure like that dealing with equations (A.20)–(A.24). From that, we have

$$d_w^m = d_w \cdot \frac{\exp\left(-\theta\mu_w^m\right)}{\sum_{m\in M}\exp\left(-\theta\mu_w^m\right)}, w \in W, m = (s1, s2, h). \tag{A.27}$$

This conforms to the logit-based model. Note that during the procedure, we can obtain the expression of μ_w as well.

From equation (A.12), we obtain

$$d_w = \bar{d}_w \cdot \exp\left(-\eta_w \cdot \mu_w\right), w \in W. \tag{A.28}$$

This is the calculation of actual demand from the potential demand. □

B. Proof of Proposition 3

Proof. The VI can be written as follows:

$$s.t. \quad con_j(\mathbf{g}) \geq 0, j = 1, 2, 3, \ldots \tag{B.1}$$

Then, the solutions of VI in $P1$ can be obtained by solving the mathematical programming $G(\mathbf{g})$, in which $\mathbf{g}$ is the variable vector, then the mathematical problem $P2$ is as follows:

$$\begin{aligned} &\min_{\mathbf{g}\in\mathbf{R}} G(\mathbf{g}), \\ &\mathbf{R} = \left\{\mathbf{g} | con_j(\mathbf{g}) \geq 0, j = 1, 2, 3, \ldots\right\}. \end{aligned} \tag{B.2}$$

The VI implies the existence of $\nabla G(\mathbf{g})$. Since the travel time function is continuous and $\nabla G(\mathbf{g})$ exists, the objective function $G(\mathbf{g})$ is continuous. The constraints $con(\mathbf{g})$ is linear, and the compactness of $con(\mathbf{g})$ is apparent. Then, we know that, the solution of the VI exists [24]. However, we are still concerned about the uniqueness of the solution. Let $\psi = \lambda \cdot t_a{}' \cdot \delta_a^l$, the Hesse matrix of the function $G(\mathbf{g})$ is calculated by

$$H(\mathbf{g}) = \begin{bmatrix} \psi & \psi & \psi & \frac{1}{o}\psi & \psi & 0 & 0 & 0 & 0 & 0 \\ \psi & \psi & \psi & \frac{1}{o}\psi & \psi & 0 & 0 & 0 & 0 & 0 \\ \psi & \psi & \psi & \frac{1}{o}\psi & \psi & 0 & 0 & 0 & 0 & 0 \\ \psi & \psi & \psi & \frac{1}{o}\psi & \psi & 0 & 0 & 0 & 0 & 0 \\ \psi & \psi & \psi & \frac{1}{o}\psi & \psi & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta d_w^{s1r\tau}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta d_w^{s1rh}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\theta d_w^{h}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\theta d_w^{s2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\eta_w d_w} \end{bmatrix}. \tag{B.3}$$

The second-order principal minor of $H(\mathbf{g})$ satisfies

$$H_2 = \begin{bmatrix} \psi & \psi \\ \psi & \psi \end{bmatrix} = 0, \tag{B.4}$$

which implies $H(\mathbf{g})$ is not strictly convex. Therefore, we cannot guarantee that the solution of the VI is unique. In conclusion, the VI has one optimal solution at least. □

Data Availability

The authors confirm that the data supporting the findings of this study are available in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by the Special Postdoctoral Science Fund Project for basic research and frontier exploration of the Chongqing Science and Technology Bureau (Grant number: cstc2021jcyj-bsh0210), the Science and Technology Research Program of the Chongqing Municipal Education Commission (Grant number: KJQN201900649, KJQN202000643), and Xingdian Talent Support Program, Major Social Research Program of the Chongqing University of Posts and Telecommunications (Grant number: 2018KZD11).

References

[1] W. Abrahamse and M. Keall, "Effectiveness of a web-based intervention to encourage carpooling to work: a case study of Wellington, New Zealand," *Transport Policy*, vol. 21, pp. 45–51, 2012.

[2] S. Shaheen and A. Cohen, "Growth in worldwide carsharing: an international comparison Transportation Research Record," *Journal of the Transportation Research Board*, no. 1992, pp. 81–89, 2007.

[3] P. Delhomme and A. Gheorghiu, "Comparing French carpoolers and non-carpoolers: which factors contribute the most to carpooling?" *Transportation Research Part D: Transport and Environment*, vol. 42, pp. 1–15, 2016.

[4] T. Vanoutrive, E. Van De Vijver, L. Van Malderen et al., "What determines carpooling to workplaces in Belgium: location, organisation, or promotion?" *Journal of Transport Geography*, vol. 22, pp. 77–86, 2012.

[5] L. Krabbenborg, E. Molin, J. A. Annema, and B. v. Wee, "Public frames in the road pricing debate: a q-methodology study," *Transport Policy*, vol. 93, 2020.

[6] H. Yang and H. J. Huang, "The multi-class, multi-criteria traffic network equilibrium and systems optimum problem," *Transportation Research Part B: Methodological*, vol. 38, no. 1, pp. 1–15, 2004.

[7] L. Han, D. Z. W. Wang, H. K. Lo, C. Zhu, and X. Cai, "Discrete-time day-to-day dynamic congestion pricing scheme considering multiple equilibria," *Transportation Research Part B: Methodological*, vol. 104, pp. 1–16, 2017.

[8] K. Qu, X. Xu, and A. Chen, "Static congestion pricing considering the cumulative impact of day-to-day dynamics with Weibit adjustment process," *Transportation Business: Transport Dynamics*, vol. 10, no. 1, pp. 237–265, 2022.

[9] H. Yang and H. J. Huang, *Mathematical and Economic Theory of Road pricing*, Elsevier, Oxford, UK, 2005.

[10] X. Zhang and H. Yang, "The optimal cordon-based network congestion pricing problem," *Transportation Research Part B: Methodological*, vol. 38, no. 6, pp. 517–537, 2004.

[11] D. Han, H. Yang, and X. L. Wang, "Efficiency of the plate-number-based traffic rationing in general networks," *Transportation Research Part E: Logistics and Transportation Review*, vol. 46, no. 6, pp. 1095–1110, 2010.

[12] X. L. Wang, H. Yang, and D. Han, "Traffic rationing and short-term and long-term equilibrium," *Transportation Research Record*, vol. 2196, no. 1, pp. 131–141, 2010.

[13] F. Shi, G. m Xu, B. Liu, and H. Huang, "Optimization method of alternate traffic restriction scheme based on elastic demand and mode choice behavior," *Transportation Research Part C: Emerging Technologies*, vol. 39, pp. 36–52, 2014.

[14] D. Chen, Y. Sun, and Z. Yang, "Optimization of the travel ban scheme of cars based on the spatial distribution of the last digit of license plates," *Transport Policy*, vol. 94, pp. 43–53, 2020.

[15] Y. M. Nie, "Why is license plate rationing not a good transport policy?" *Transportmetrica: Transportation Science*, vol. 13, no. 1, pp. 1–23, 2017.

[16] Y. M. Nie, "On the potential remedies for license plate rationing," *Economics of Transportation*, vol. 9, pp. 37–50, 2017.

[17] Z. Song, Y. Yin, and S. Lawphongpanich, "Optimal deployment of managed lanes in general networks," *International Journal of Sustainable Transportation*, vol. 9, no. 6, pp. 431–441, 2015.

[18] M. Balac, F. Ciari, and K. W. Axhausen, "Carsharing demand estimation: Zurich, Switzerland, area case study," *Journal of the Transportation Research Board*, vol. 2536, pp. 10–18, 2015.

[19] M. Balac, F. Ciari, and K. W. Axhausen, "Modeling the impact of parking price policy on free-floating carsharing: case study for Zurich, Switzerland," *Transportation Research Part C: Emerging Technologies*, vol. 77, pp. 207–225, 2017.

[20] F. Ciari, M. Balac, and M. Balmer, "Modelling the effect of different pricing schemes on free-floating carsharing travel demand: a test case for Zurich, Switzerland," *Transportation*, vol. 42, no. 3, pp. 413–433, 2015.

[21] H. Xu, J. S. Pang, F. Ordóñez, and M. Dessouky, "Complementarity models for traffic equilibrium with ridesharing," *Transportation Research Part B: Methodological*, vol. 81, pp. 161–182, 2015.

[22] M. Li, X. Di, H. X. Liu, and H. J. Huang, "A restricted path-based ridesharing user equilibrium," *Journal of Intelligent Transportation Systems*, vol. 24, pp. 1–21, 2019.

[23] M. Li, G. Hua, and H. Huang, "A multi-modal route choice model with ridesharing and public transit," *Sustainability*, vol. 10, no. 11, p. 4275, 2018.

[24] S. Boyd, L. Xiao, and A. Mutapcic, *Subgradient Methods. Lecture Notes of EE392o*, pp. 2004-2005, Stanford University, Stanford, CA, USA, 2003.

[25] H. Yang and H. J. Huang, "Carpooling and congestion pricing in a multilane highway with high-occupancy-vehicle lanes," *Transportation Research Part A: Policy and Practice*, vol. 33, no. 2, pp. 139–155, 1999.

[26] B. für Statistik and B. für Raumentwicklung, "Mobilität in der schweiz, ergebnisse des mikrozensus mobilität und verkehr 2010," *Mobililität und Verkehr. Schweiz, S. d. Neuchâtel und Bern, Bundesamt für Statistik*, pp. 1–120, 2012.

[27] W. H. K. Lam, H. Shao, and A. Sumalee, "Modeling impacts of adverse weather conditions on a road network with uncertainties in demand and supply," *Transportation Research Part B: Methodological*, vol. 42, no. 10, pp. 890–910, 2008.